2015年11月9, 16, 30日

集中講義@名古屋大学

「宇宙大規模構造と精密宇宙論」

第５講

種々の非線形性

内容

赤方偏移空間ゆがみの非線形性

修正重力理論における摂動論

ニュートリノが質量を持つ場合の非線形クラスタリング

重力レンズ効果における非線形性の低減

赤方偏移空間ゆがみの非線形性

For a review, Hamilton, astro-ph/9708102

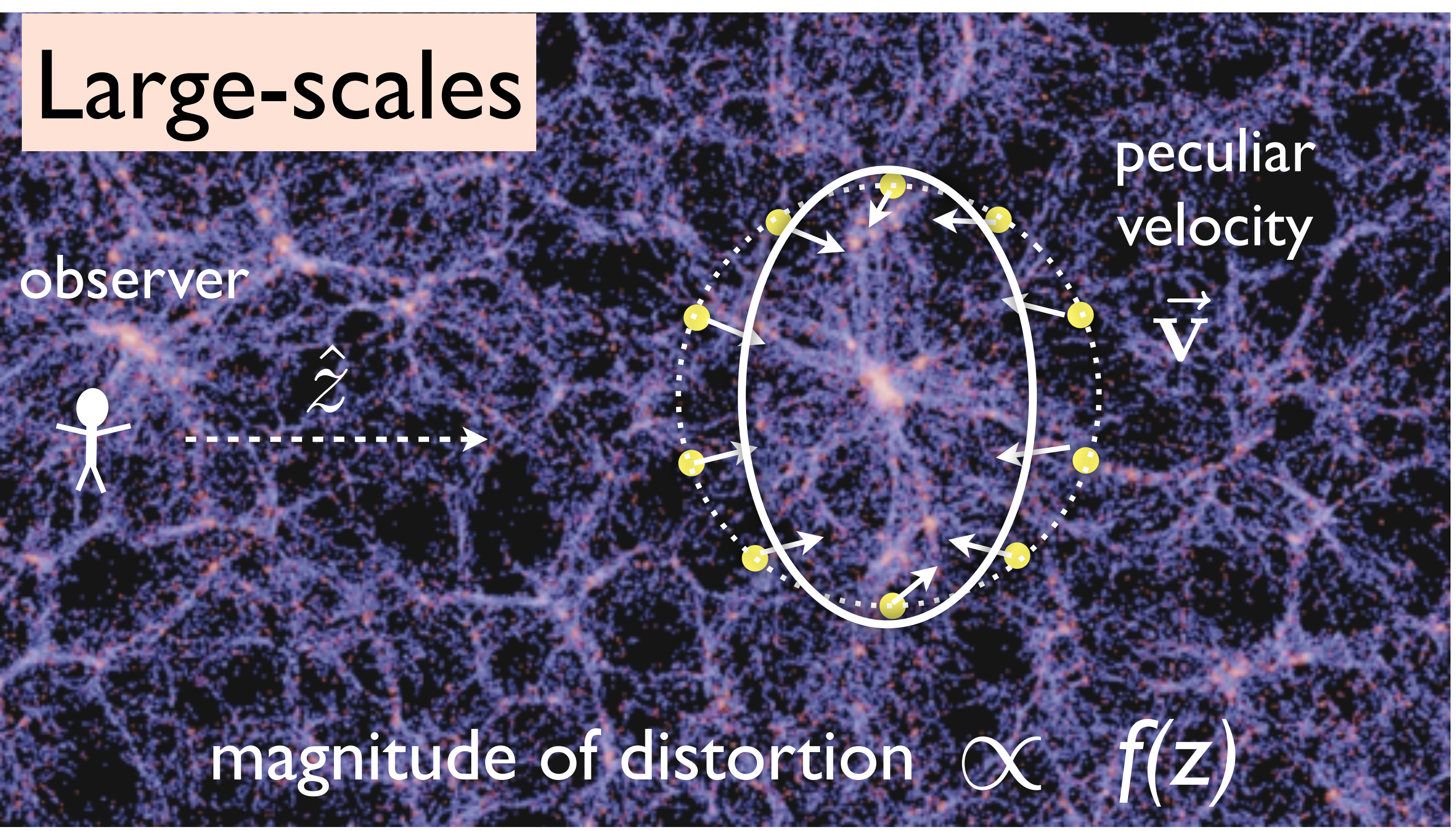
Refs. AT, Nishimichi & Saito, PRD 82, 063522 ('10)
Nishimichi & AT, PRD 84, 043526 ('11)

Redshift-space distortions

- Clustering anisotropies induced by line-of-sight velocity fields
- Unavoidable systematics in 3D galaxy clustering via spectroscopic measurement

redshift space / real space / observer's line-of-sight

$$\vec{s} = \vec{r} + \frac{1+z}{H(z)}(\vec{v}\cdot\hat{z})\,\hat{z} \equiv \vec{r} - f\,u_z(\vec{r})\,\hat{z}$$



growth-rate parameter

$$f(z) \equiv \frac{d\ln D_+}{d\ln a} \simeq \{\Omega_{\rm m}(z)\}^{\gamma}$$

$$\gamma \approx 0.55\ (\mathrm{GR}),\quad 0.68\ (\mathrm{DGP})$$

Linder ('05)

Measurement of f(z) offers a test of gravity on cosmological scales

Effects on clustering amplitude

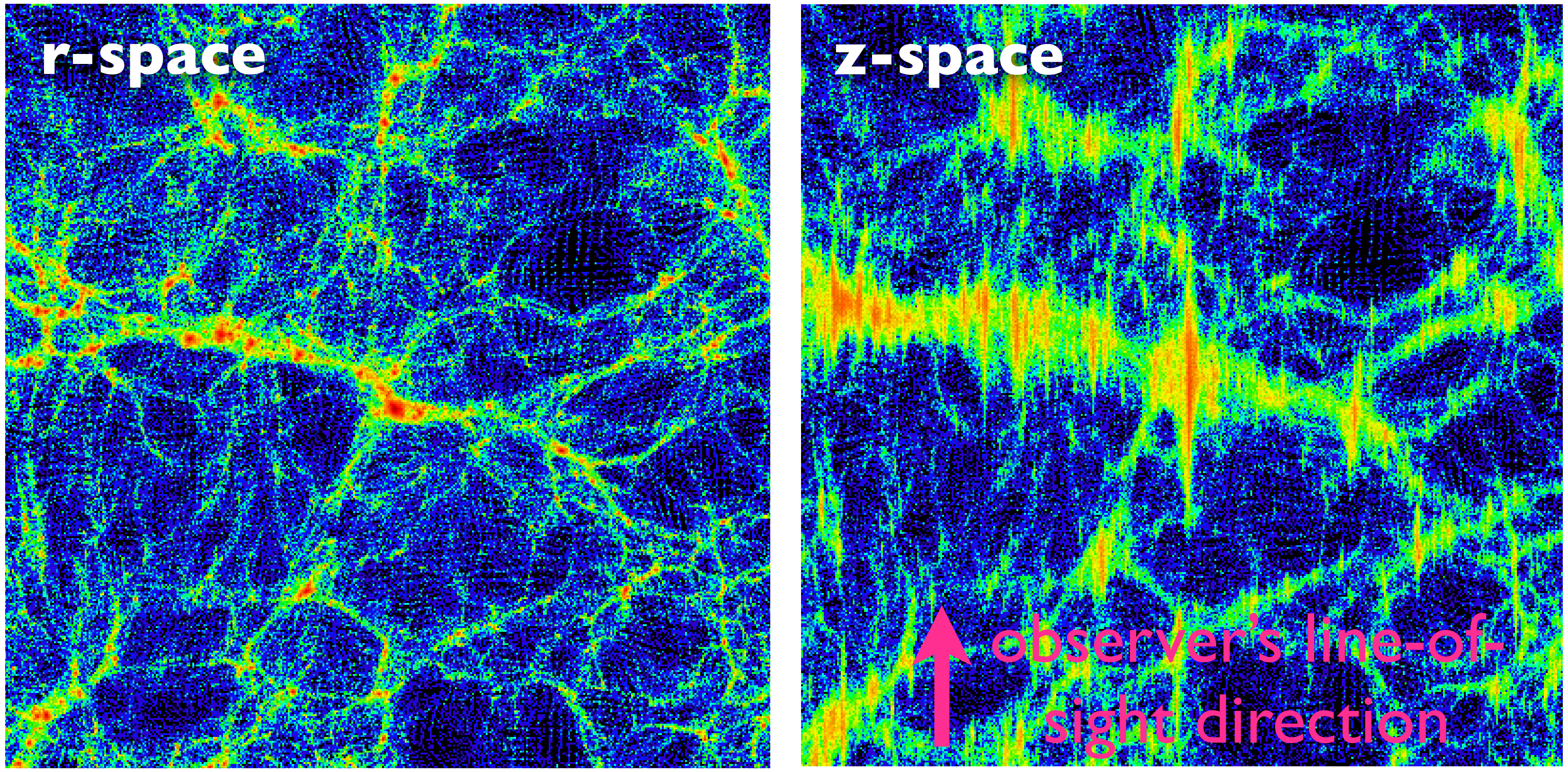
Large scales: *Kaiser effect* ⟶ probe of gravity

mild enhancement of clustering amplitude

Small scales: *Finger-of-God effect*

clustering pattern is blurred, and amplitude is suppressed

These two effects exhibits nonlinear corrections, and cannot be separately treated



simulation: T. Nishimichi

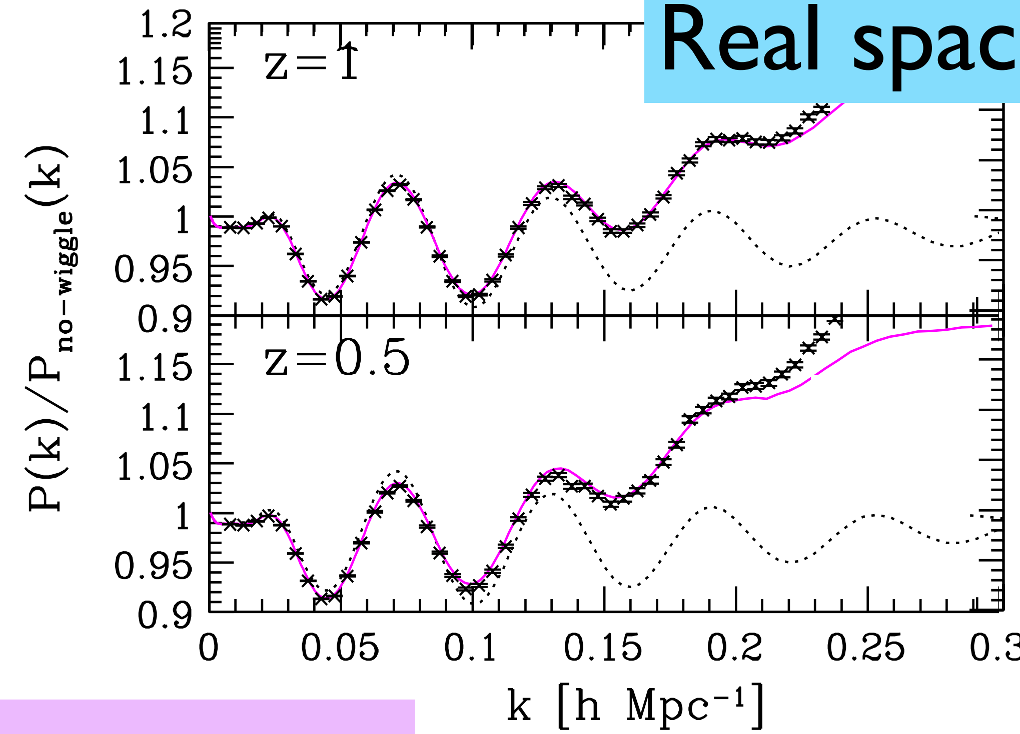
Redshift-space power spectrum

AT, Nishimichi, Saito & Hiramatsu ('09)

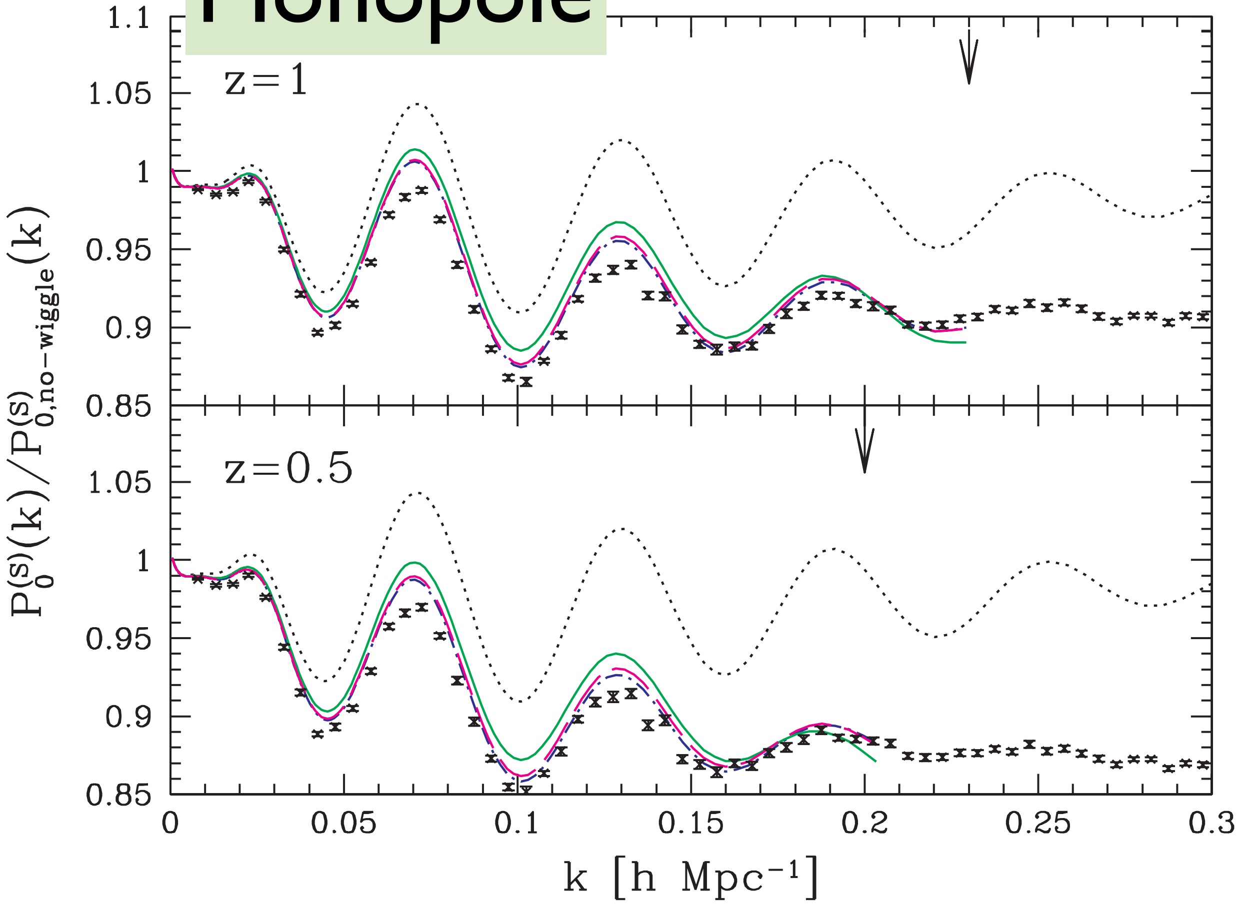
AT, Nishimichi & Saito ('10)

$$P^{(\mathrm{S})}(k,\mu) = \sum_{\ell=\mathrm{even}} P_\ell^{(\mathrm{S})}(k)\,\mathcal{P}_\ell(\mu)$$

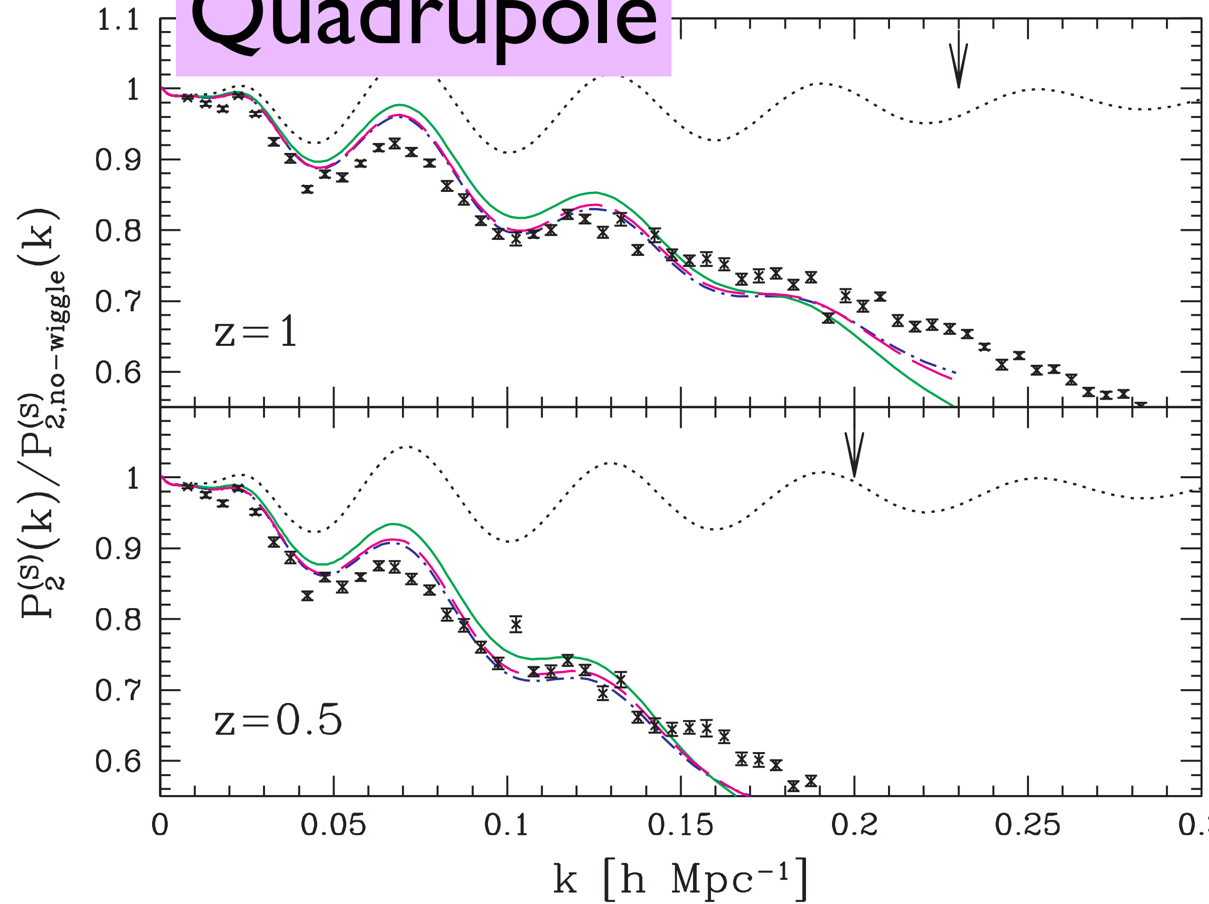
Real space



Monopole

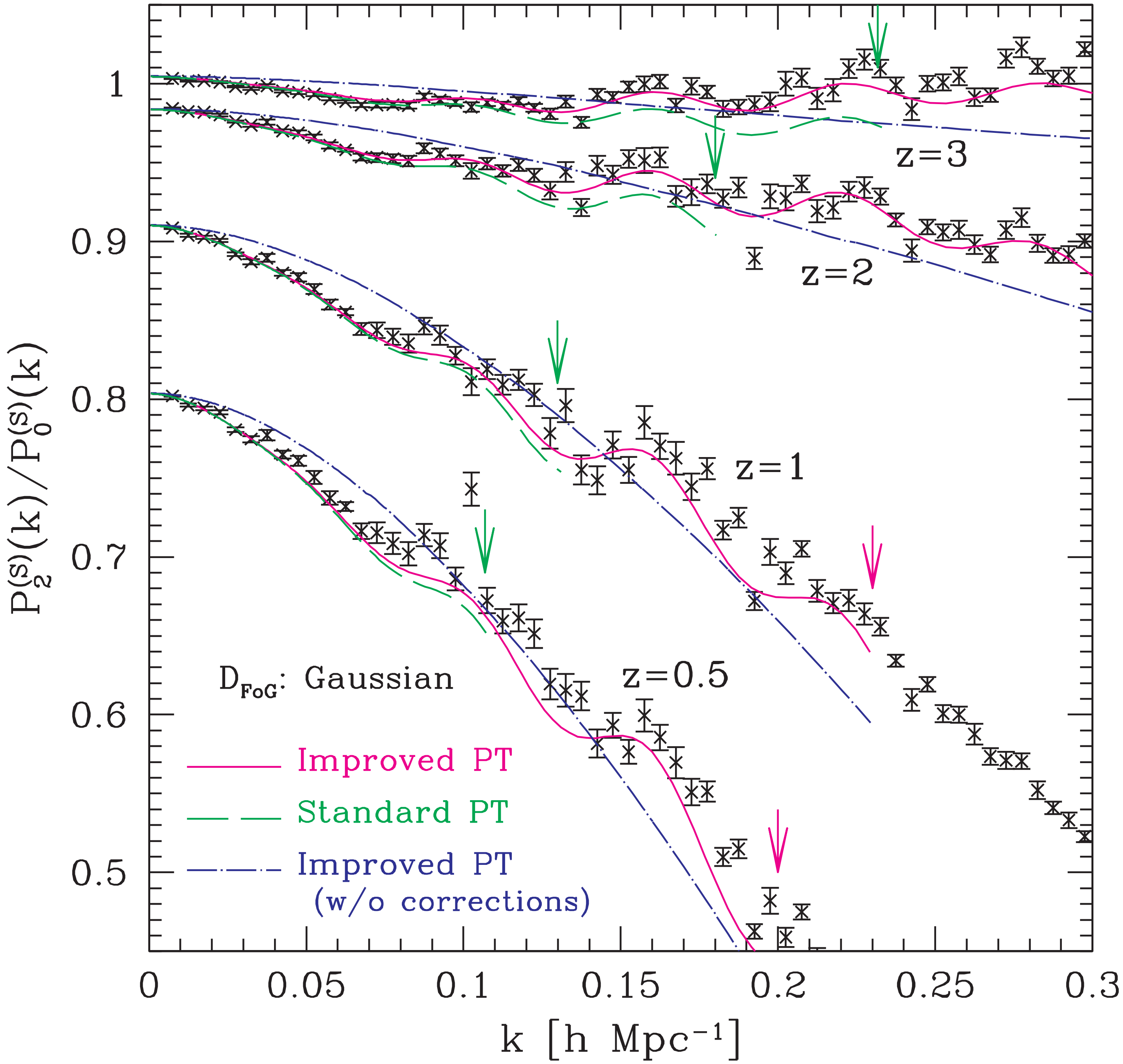


Quadrupole



Redshift-space power spectrum

Quadrupole-to-monopole ratio



In linear theory,

$$\frac{P_2^{(S)}(k)}{P_0^{(S)}(k)} = \frac{1+(2/3)f+f^2/5}{(4/3)f+(4/7)f^2}$$

······· independent of scale

AT, Nishimichi & Saito ('10)

Redshift-space power spectrum

Exact formula

$$P^{(\mathrm{S})}(\mathbf{k}) = \int d^3\mathbf{x}\, e^{i\mathbf{k}\cdot\mathbf{x}} \left\langle e^{-ik\mu\,\Delta u_z} \{\delta(\mathbf{r}) + f\nabla_z u_z(\mathbf{r})\}\{\delta(\mathbf{r}') + f\nabla_z u_z(\mathbf{r}')\} \right\rangle$$

$$P^{(\mathrm{S})}(k,\mu) = \int d^3\boldsymbol{x}\, e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \langle e^{j_1 A_1} A_2 A_3 \rangle$$

$$\begin{aligned} j_1 &= -ik\mu f, \\ A_1 &= u_z(\boldsymbol{r}) - u_z(\boldsymbol{r}') \\ A_2 &= \delta(\boldsymbol{r}) + f\nabla_z u_z(\boldsymbol{r}), \\ A_3 &= \delta(\boldsymbol{r}') + f\nabla_z u_z(\boldsymbol{r}'). \end{aligned}$$

In terms of cumulants,

$$\langle e^{\boldsymbol{j}\cdot\boldsymbol{A}} \rangle = \exp\{\langle e^{\boldsymbol{j}\cdot\boldsymbol{A}} \rangle_c\}$$

$$P^{(\mathrm{S})}(k,\mu) = \int d^3\boldsymbol{x}\, e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \exp\{\langle e^{j_1 A_1} \rangle_c\} [\langle e^{j_1 A_1} A_2 A_3 \rangle_c + \langle e^{j_1 A_1} A_2 \rangle_c \langle e^{j_1 A_1} A_3 \rangle_c].$$

Factorized model

e.g., Scoccimarro ('04)

$$P^{(\mathrm{S})}(k,\,\mu)=\int d^3\boldsymbol{x}\,e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\exp\{\langle e^{j_1A_1}\rangle_c\}[\langle e^{j_1A_1}A_2A_3\rangle_c+\langle e^{j_1A_1}A_2\rangle_c\langle e^{j_1A_1}A_3\rangle_c].$$

ignoring spatial correlation

$$j_1=-ik\mu f,$$
$$A_1=u_z(\boldsymbol{r})-u_z(\boldsymbol{r}')$$
$$A_2=\delta(\boldsymbol{r})+f\nabla_z u_z(\boldsymbol{r}),$$
$$A_3=\delta(\boldsymbol{r}')+f\nabla_z u_z(\boldsymbol{r}').$$

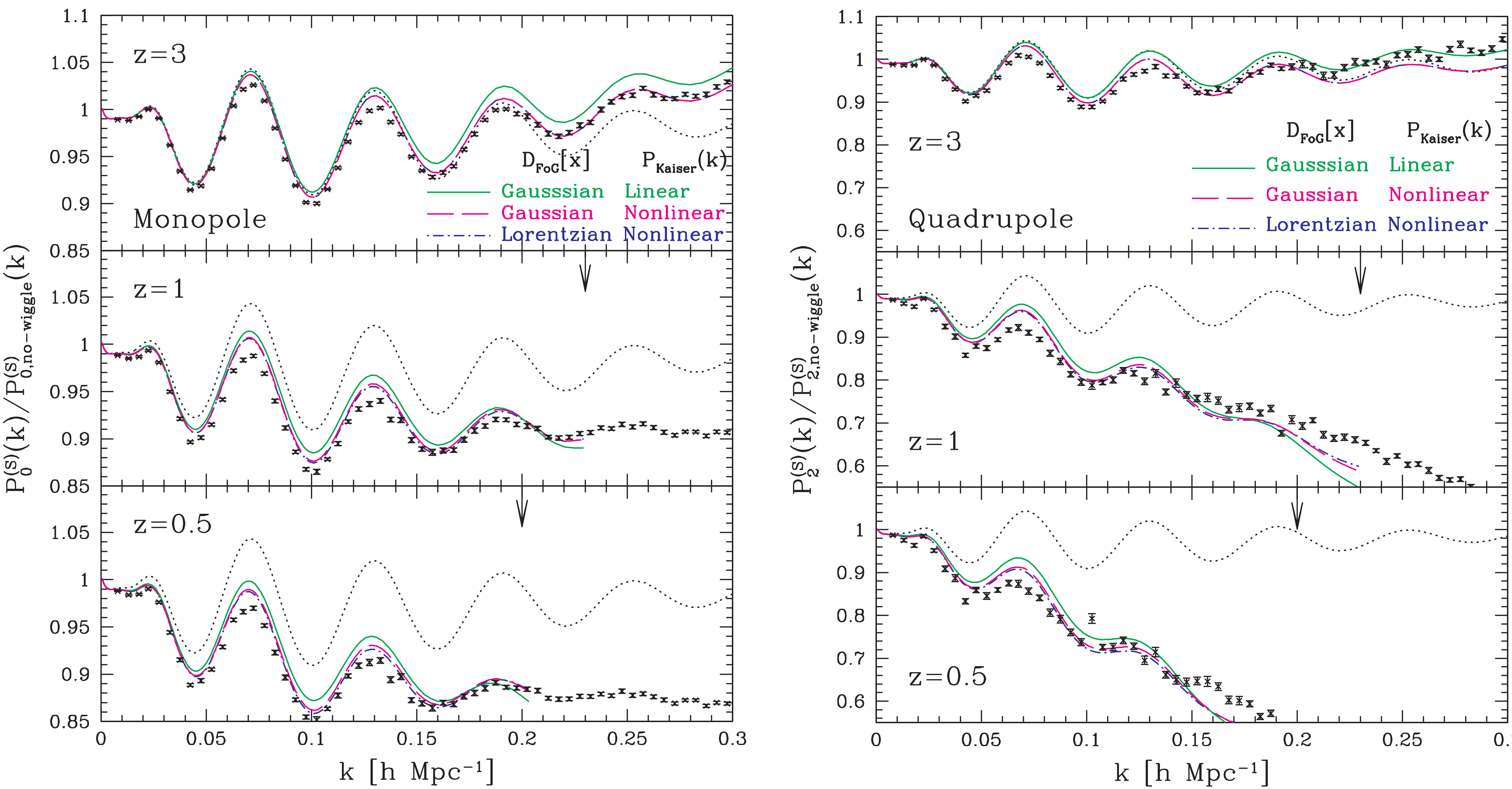
$$P^{(\mathrm{S})}(k,\mu)=D[k\mu\,f\,\sigma_{\mathrm{v}}]\left[P_{\delta\delta}(k)+2\,\mu^2\,P_{\delta\theta}(k)+\mu^4\,P_{\theta\theta}(k)\right]$$

σ_{v} : 1D velocity dispersion (fitting parameter)

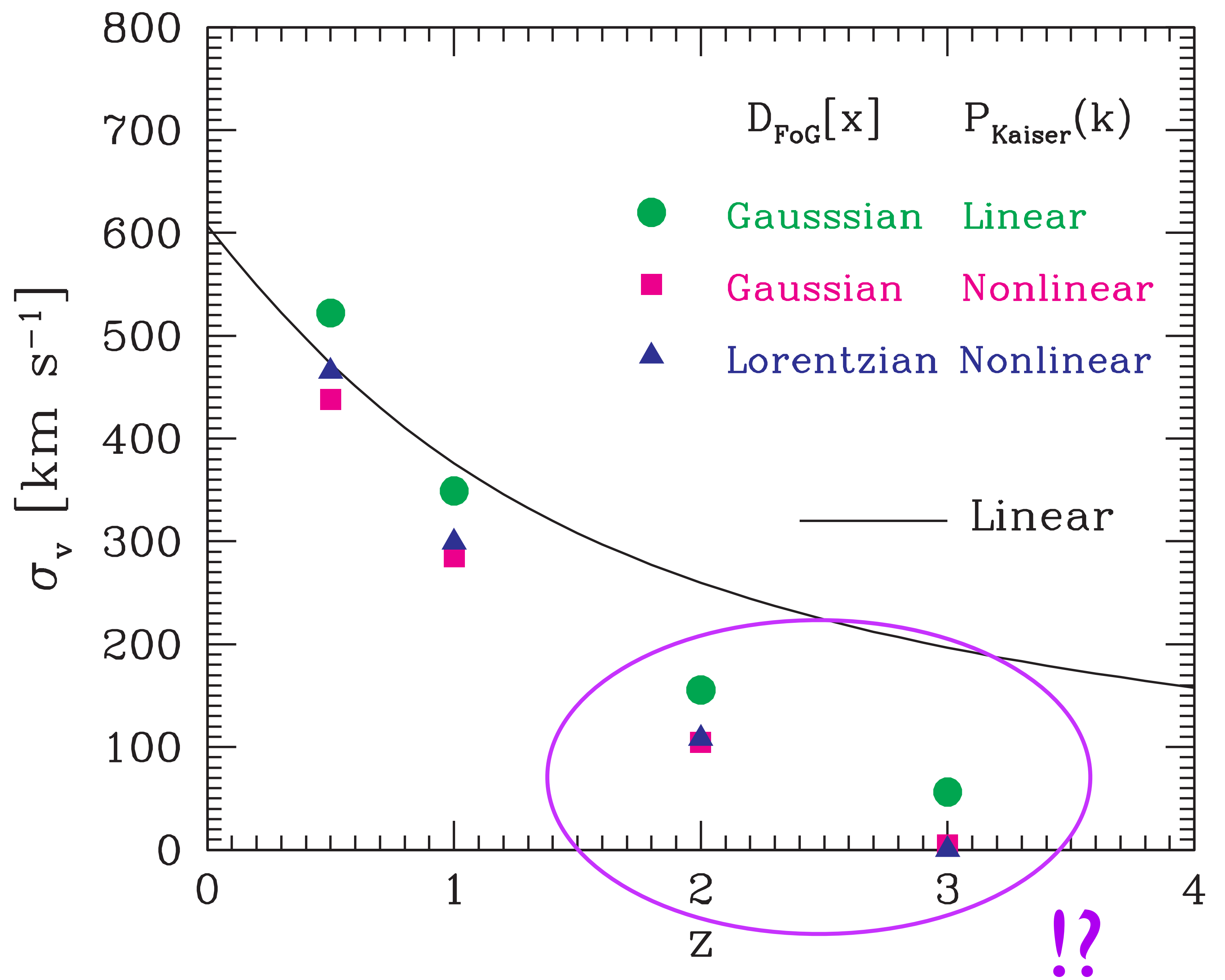
Similar forms have been frequently used in the last two decades

(Fisher et al. '93, Peacock & Dodds '94, Cole et al. '95, ...)

Comparison with simulations



Fitted results of sigma_v



An improved modeling

AT, Nishimichi & Saito ('10)

$$P^{(\mathrm{S})}(k,\mu)=\int d^3\boldsymbol{x}\,e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\exp\{\langle e^{j_1A_1}\rangle_c\}\left[\langle e^{j_1A_1}A_2A_3\rangle_c+\langle e^{j_1A_1}A_2\rangle_c\langle e^{j_1A_1}A_3\rangle_c\right].$$

Slight mismatch with simulation may be caused by a naive treatment in the bracket

low-k expansion

$$\langle e^{j_1A_1}A_2A_3\rangle_c+\langle e^{j_1A_1}A_2\rangle_c\langle e^{j_1A_1}A_3\rangle_c\simeq\langle A_2A_3\rangle+j_1\langle A_1A_2A_3\rangle_c$$
$$+\,j_1^2\{\tfrac{1}{2}\langle A_1^2A_2A_3\rangle_c+\langle A_1A_2\rangle_c\langle A_1A_3\rangle_c\}+\mathcal{O}(j_1^3).$$

regarding "j_1" as expansion parameter

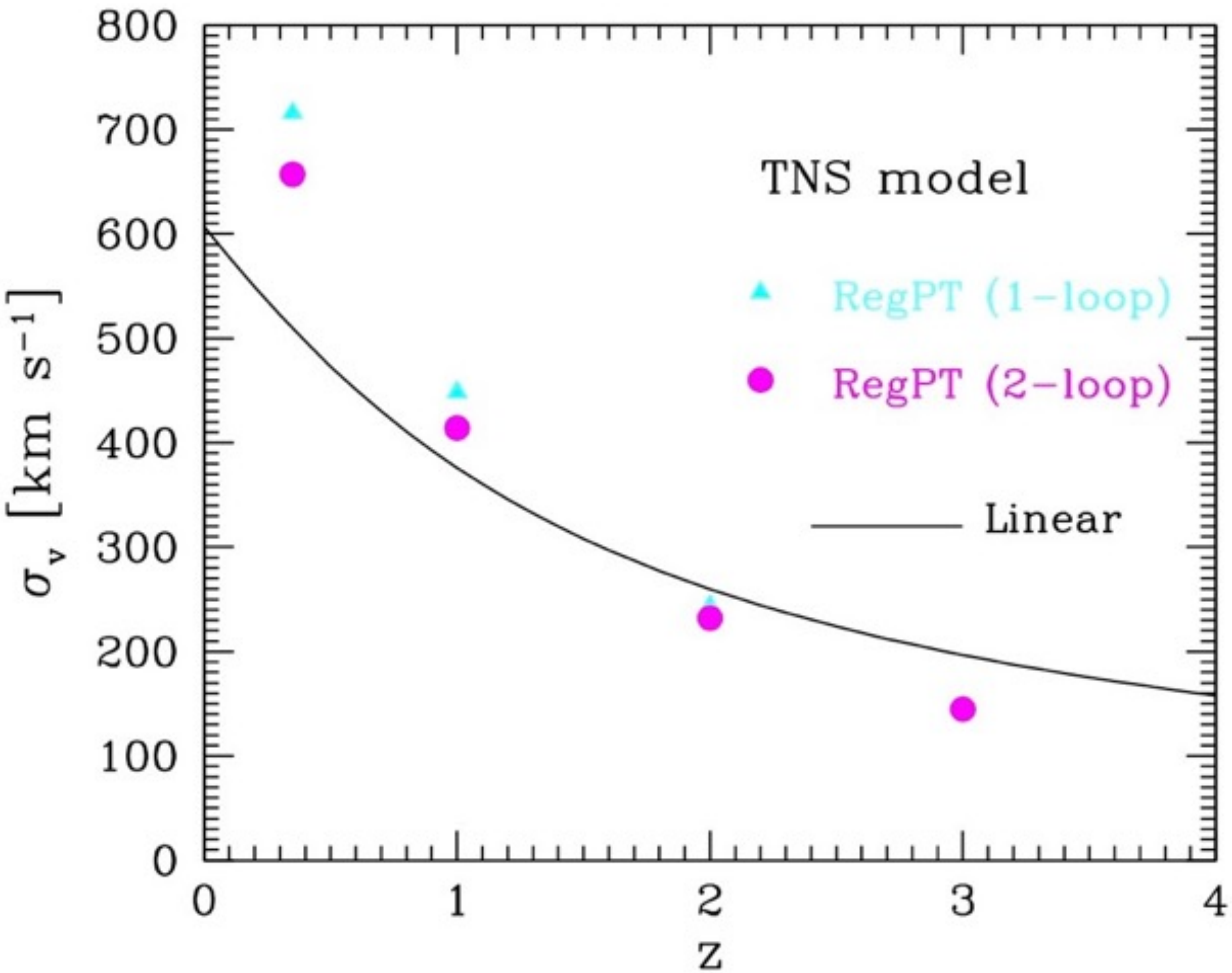
$$P^{(\mathrm{S})}(k,\mu)=D_{\mathrm{FoG}}[k\mu f\sigma_{\mathrm{v}}]\left[P_{\delta\delta}(k)-2f\mu^2P_{\delta\theta}(k)+f^2\mu^4P_{\theta\theta}(k)+A(k,\mu)+B(k,\mu)\right]$$

$$A(k,\mu)=-2\,k\,\mu\int\frac{d^3\mathbf{p}}{(2\pi)^3}\,\frac{p_z}{p^2}B_\sigma(\mathbf{p},\mathbf{k}-\mathbf{p},-\mathbf{k})$$

$$\left\langle\theta(\mathbf{k}_1)\left\{\delta(\mathbf{k}_2)-\mu_2^2\,\theta(\mathbf{k}_2)\right\}\left\{\delta(\mathbf{k}_3)-\mu_3^2\,\theta(\mathbf{k}_3)\right\}\right\rangle=(2\pi)^3\delta_D(\mathbf{k}_{123})\,B_\sigma(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$$

$$B(k,\mu)=(k\mu)^2\int\frac{d^3\boldsymbol{p}}{(2\pi)^3}F(\boldsymbol{p})F(\boldsymbol{k}-\boldsymbol{p})\quad;\quad F(\boldsymbol{p})\equiv\frac{p_z}{p^2}\left\{P_{\delta\theta}(p)-\frac{p_z^2}{p^2}P_{\theta\theta}(p)\right\}$$

Fitted results of sigma_v



Performance of TNS model

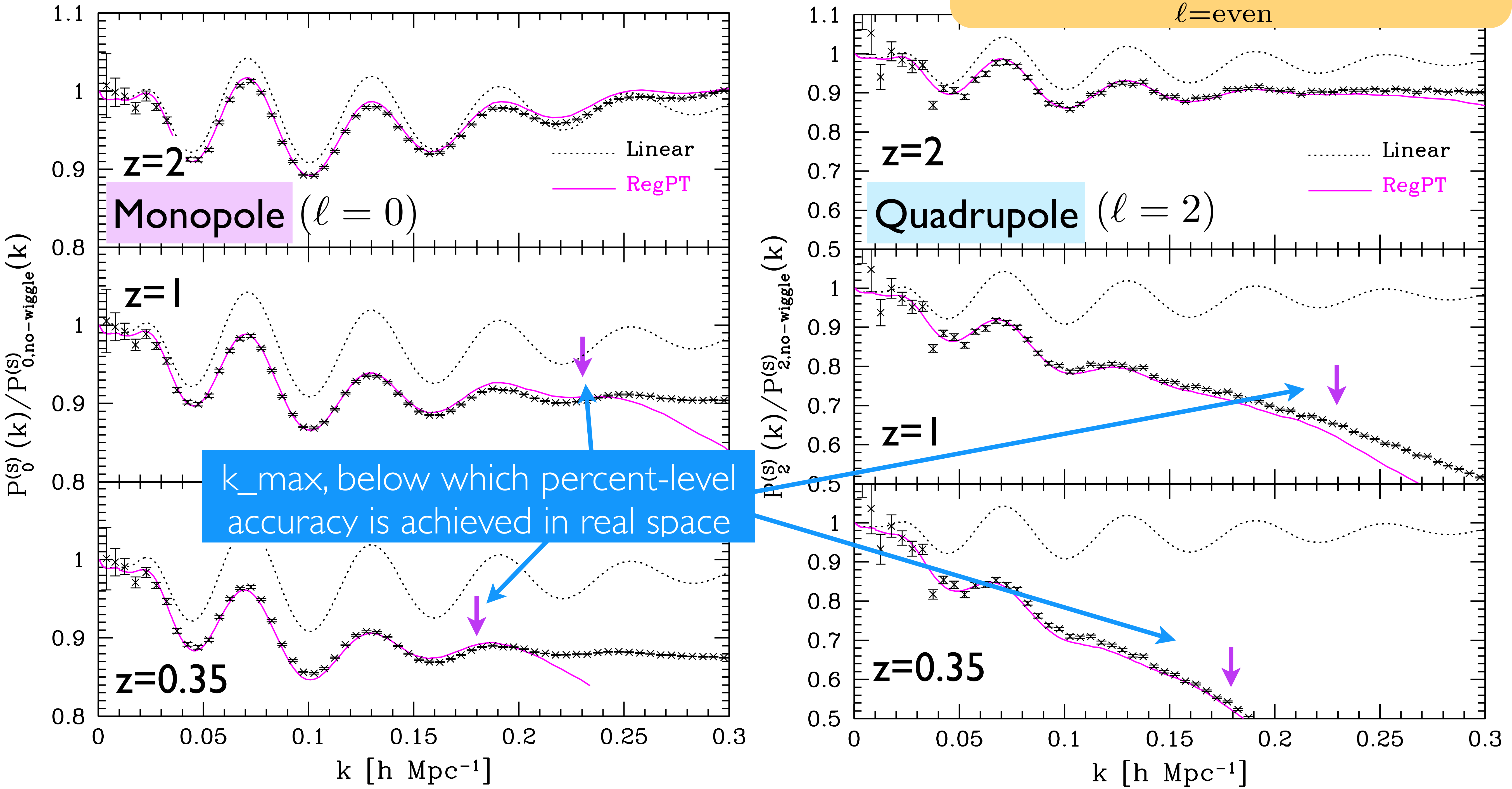
AT, Bernardeau & Nishimichi ('13)

TNS

Power spectrum

computed with RegPT 2-loop

$$P^{(\mathrm{S})}(k,\mu) = \sum_{\ell=\mathrm{even}} P_\ell^{(\mathrm{S})}(k)\,\mathcal{P}_\ell(\mu)$$



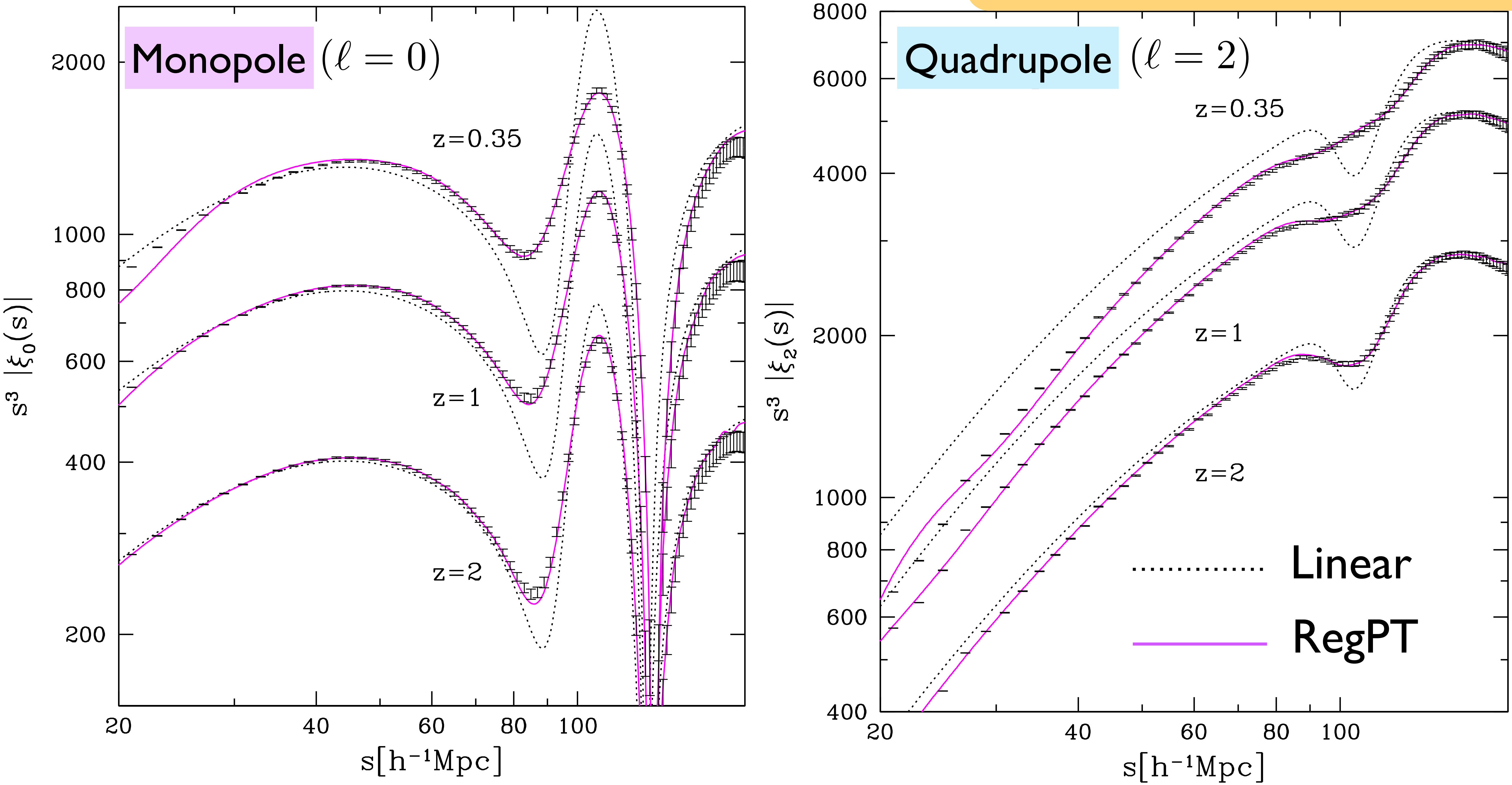
Performance of TNS model

correlation function

AT, Bernardeau & Nishimichi ('13)

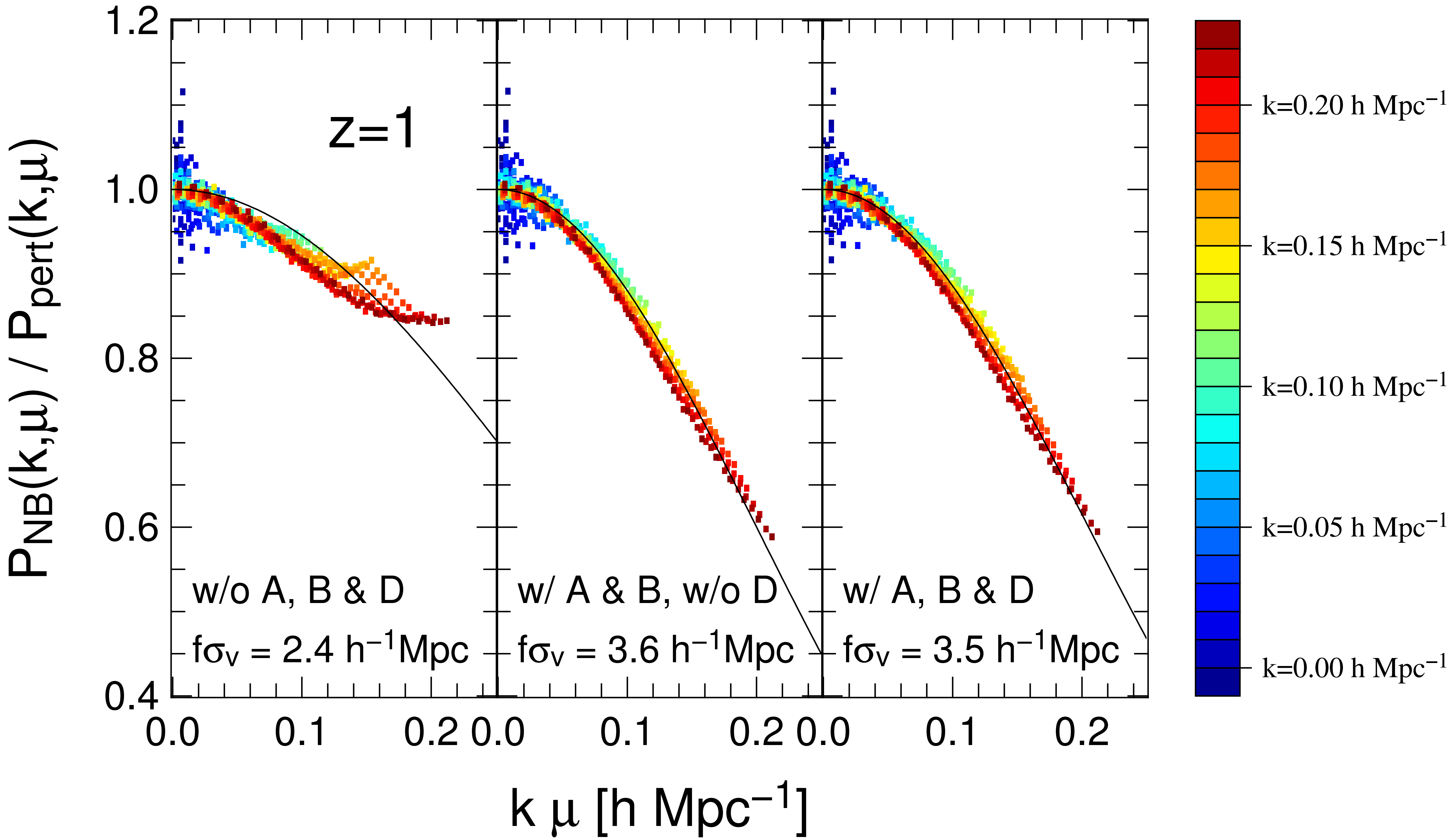
computed with RegPT 2-loop

$$\xi^{(S)}(s,\mu)=\sum_{\ell=\text{even}}\xi_\ell^{(S)}(s)\,\mathcal{P}_\ell(\mu)$$



Validity of PT modeling

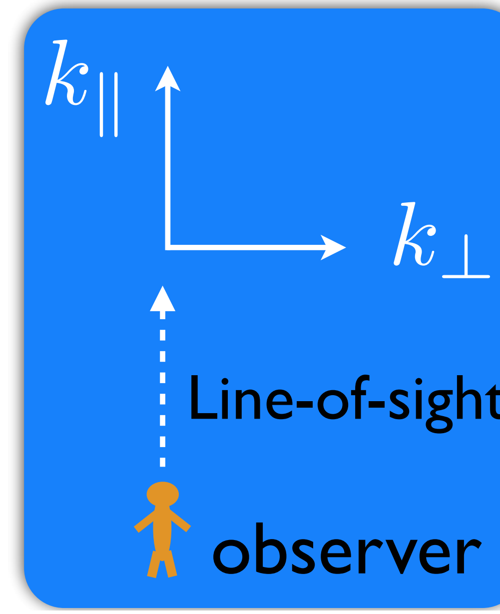
$$\frac{P_{\text{N-body}}(k,\mu)}{P_{\text{Kaiser}}(k,\mu)+A(k,\mu)+B(k,\mu)+D(k,\mu)}$$



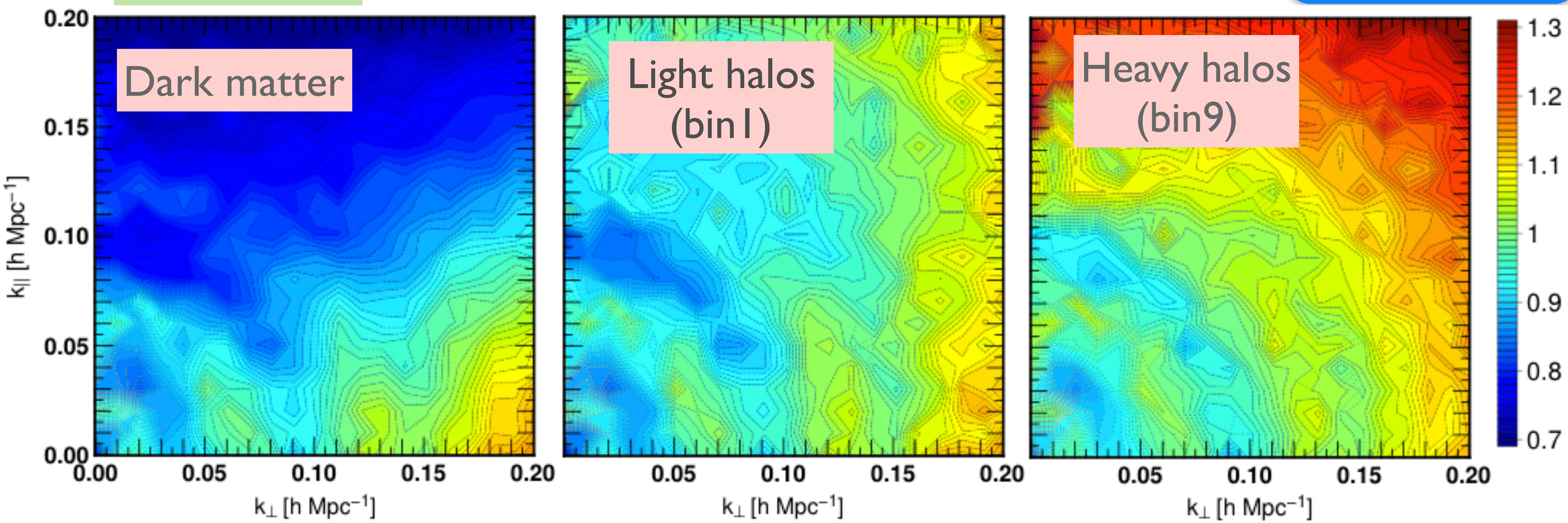
Halo power spectrum in 2D

Nishimichi & AT ('11)

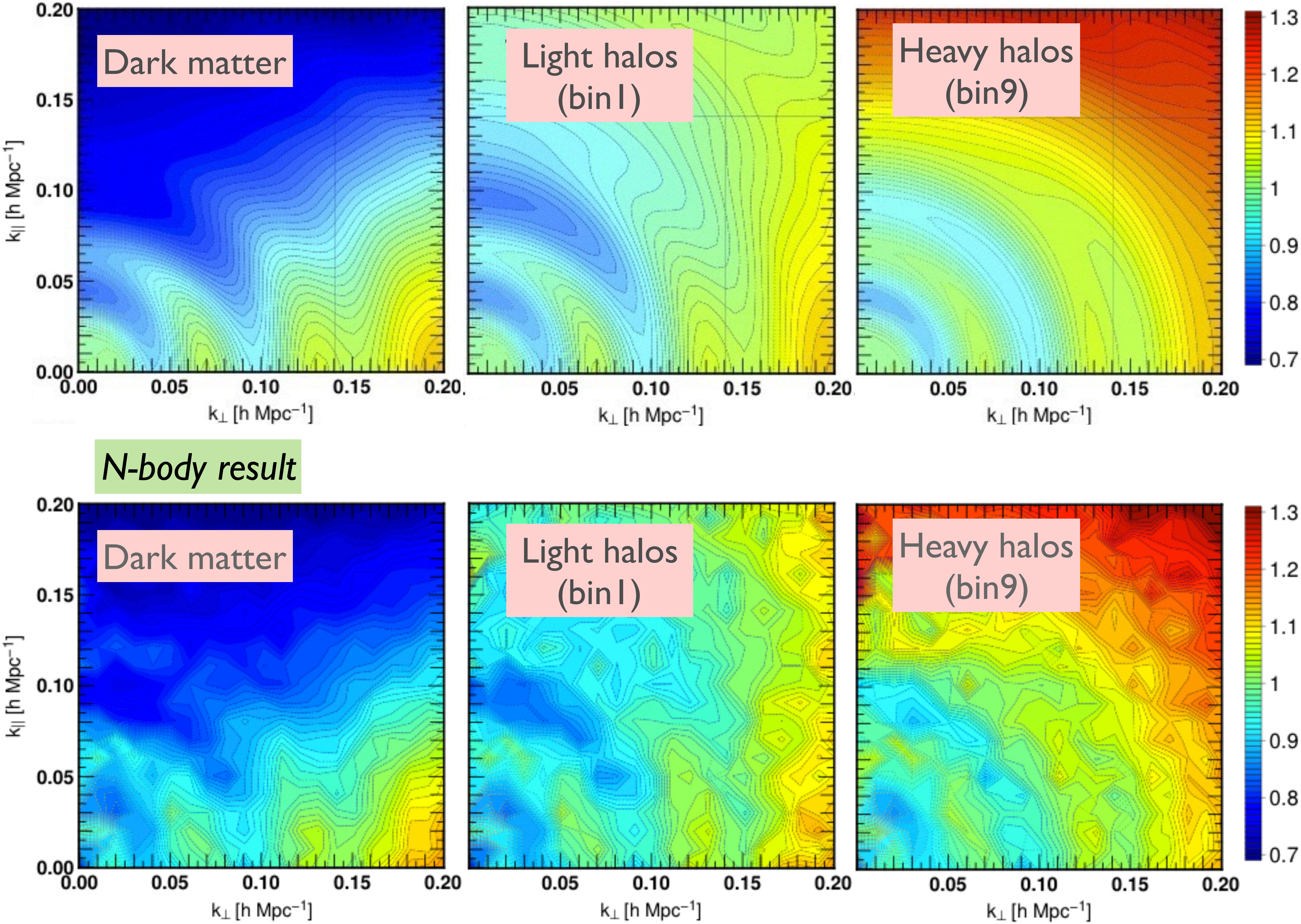
$$\frac{P_{\rm halo}(k_{||}, k_\perp)}{(b^2 + f\,\mu^2)^2 P_{\rm lin,no\text{-}wiggle}(k)}$$



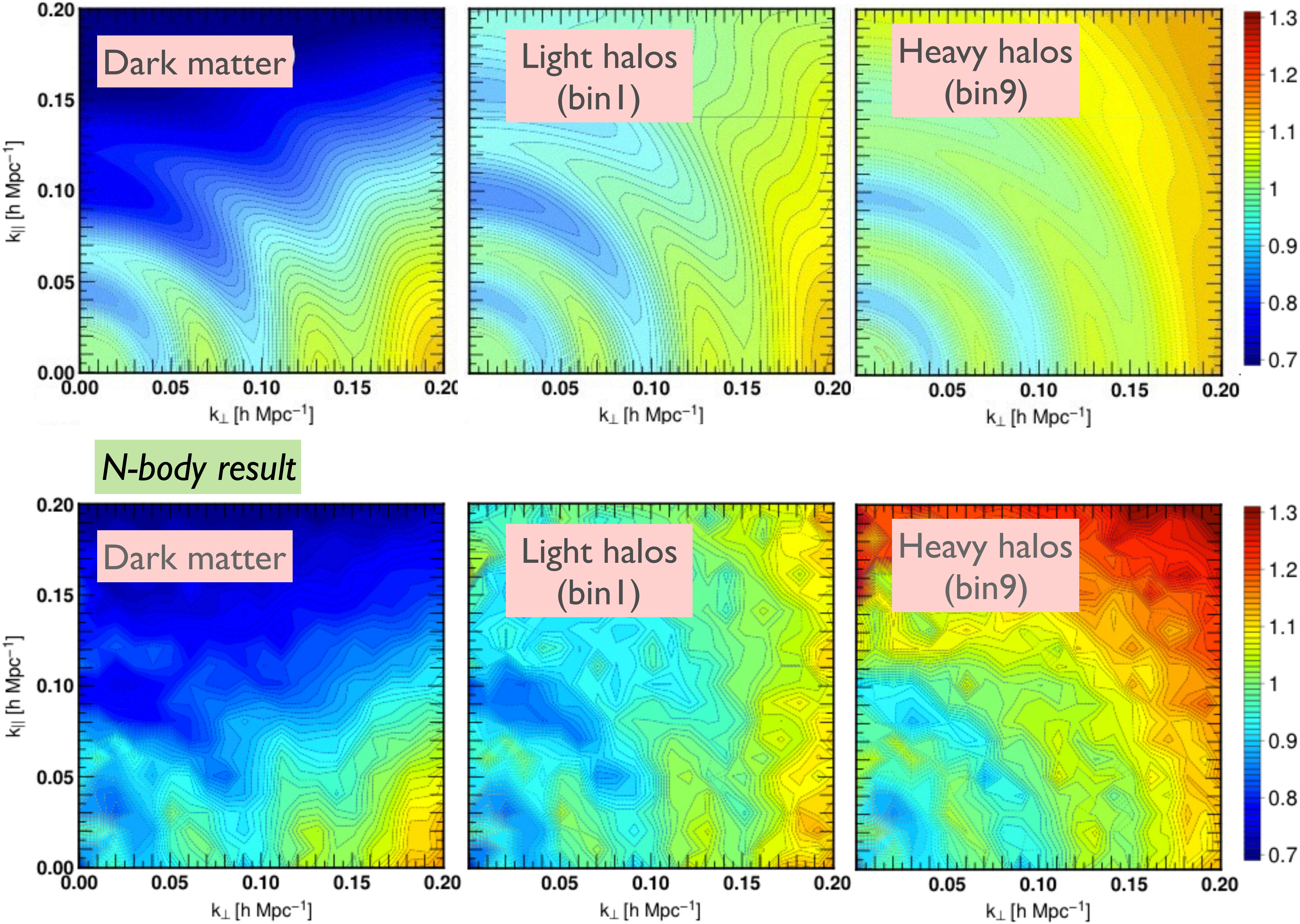
N-body result ‘shot-noise’ corrected



TNS ※ Use scale-dept linear bias measured from simulation



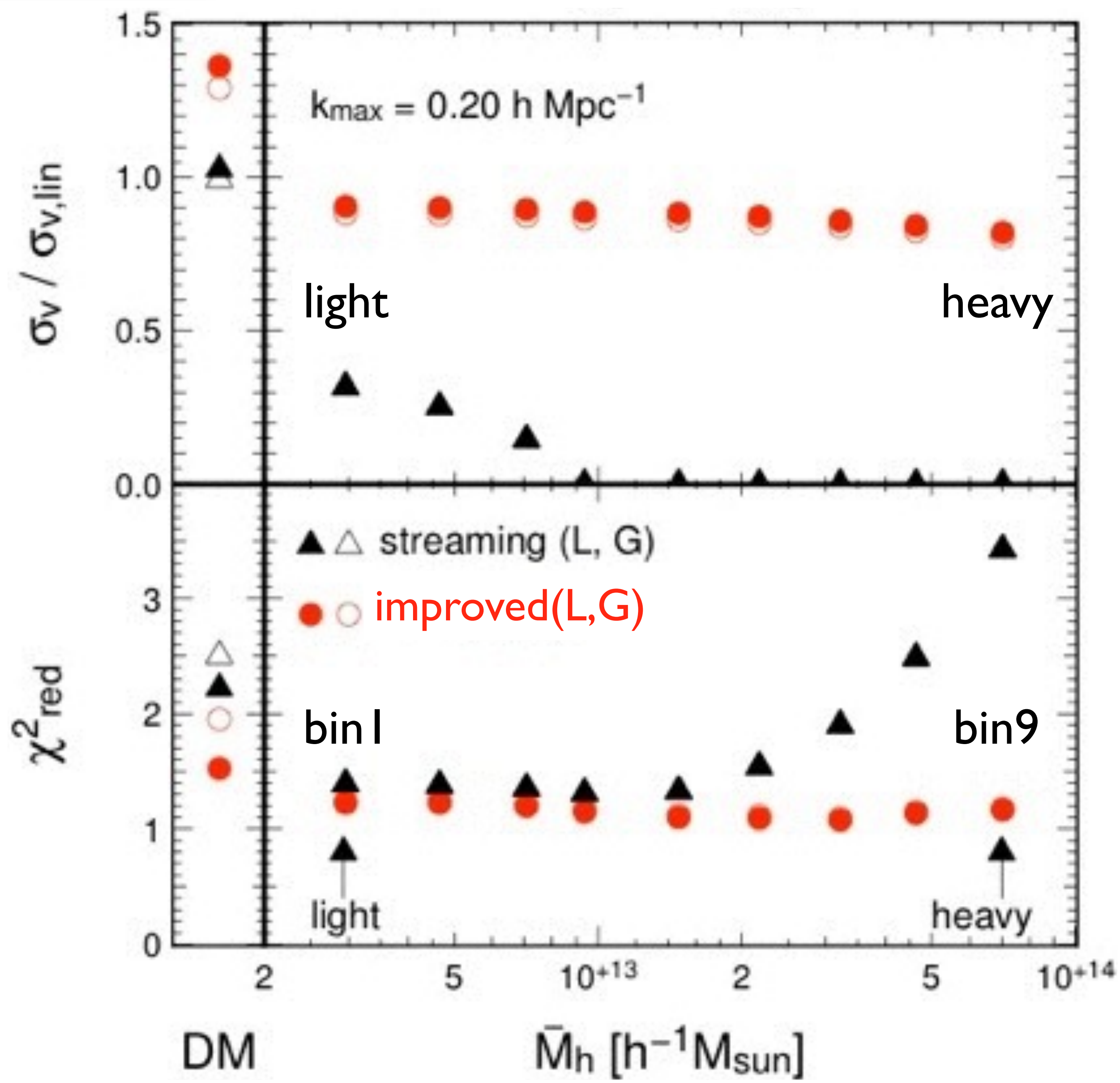
Factorized model   ※ Use scale-dept linear bias measured from simulation



Halo mass dependence

Choice of damping func.

L: Lorentzian, G: Gaussian

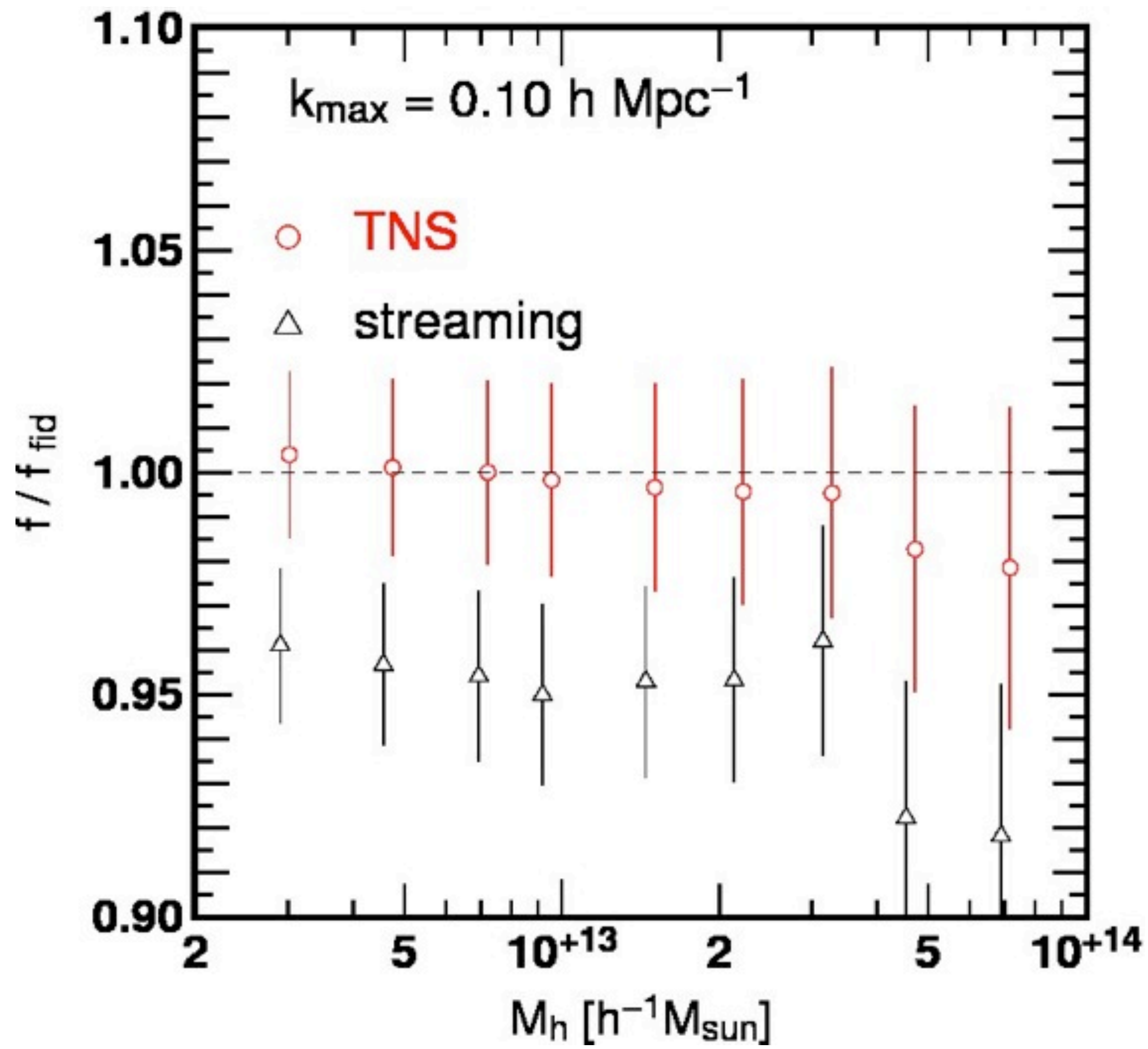


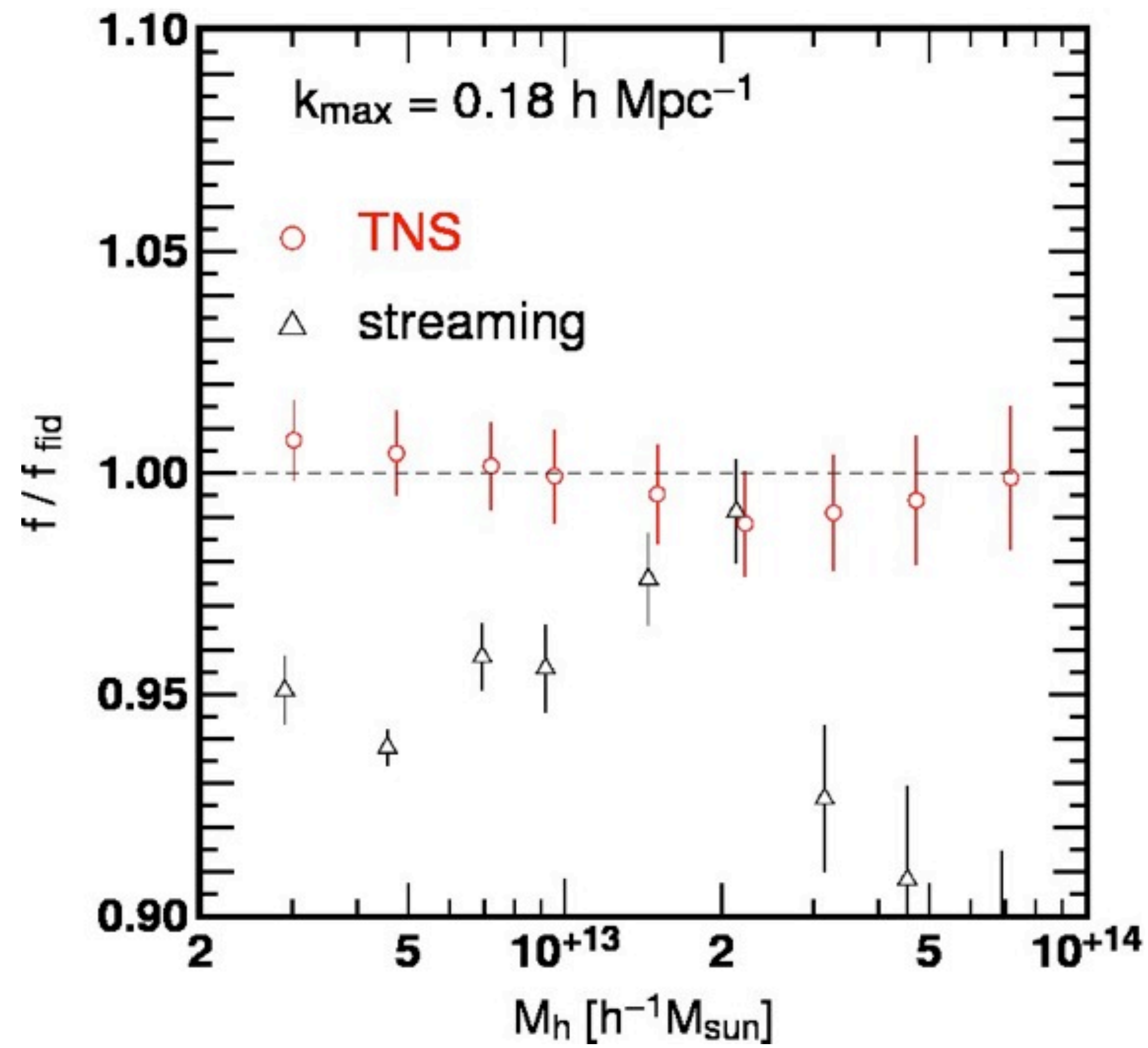
Fitted results of velocity dispersion

Goodness of fit

Recovery of f(z)

fitting parameters: sigma_v, f(z)





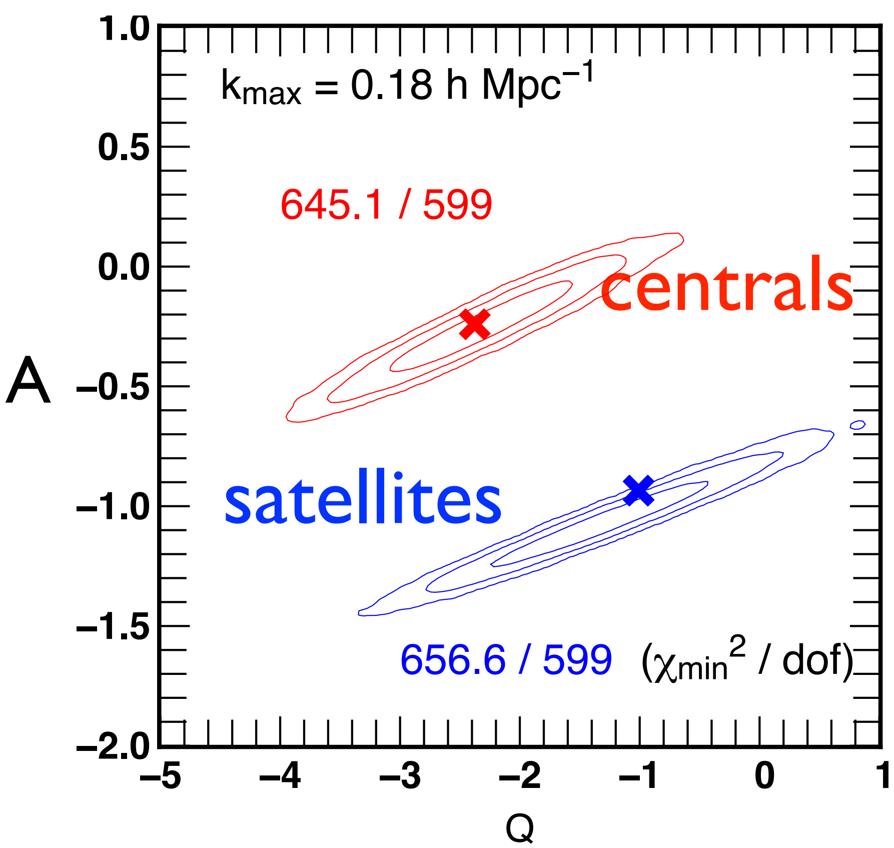
Viewgraph by T. Nishimichi

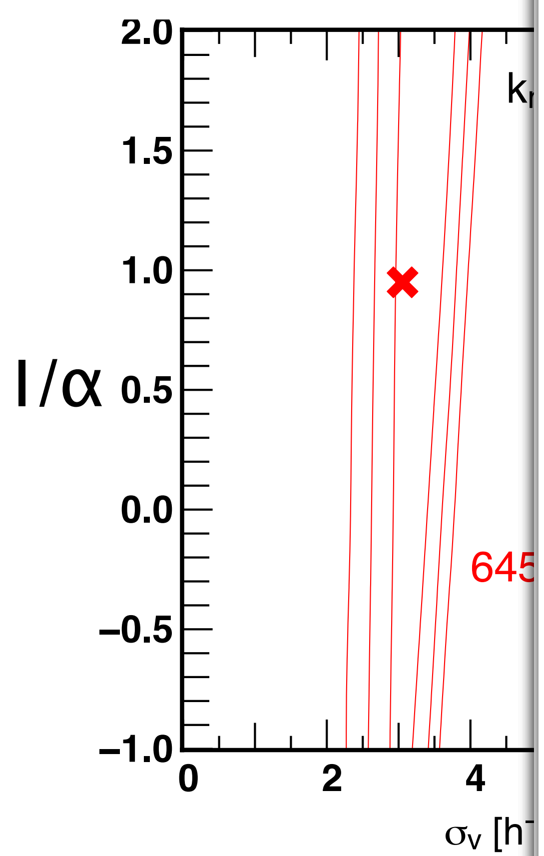
Impact of satellites

fitting parameters: b, A, Q, alpha, sigma_v, f(z)

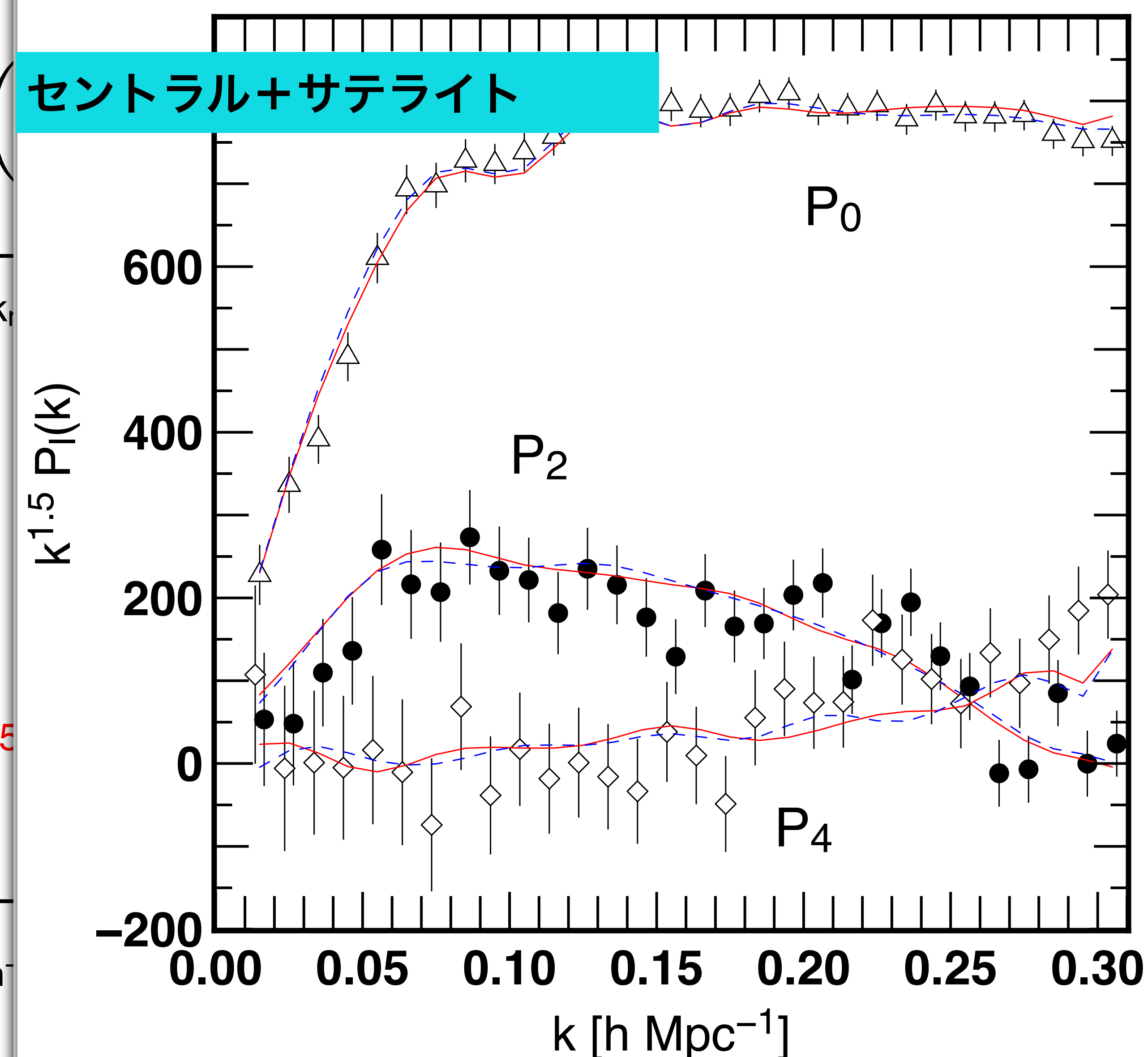
$$b(k) = b\,\frac{1+Qk^2}{1+Ak}$$

$$D_{\rm FoG}(x) = ($$





Nishimichi & Oka ('13)

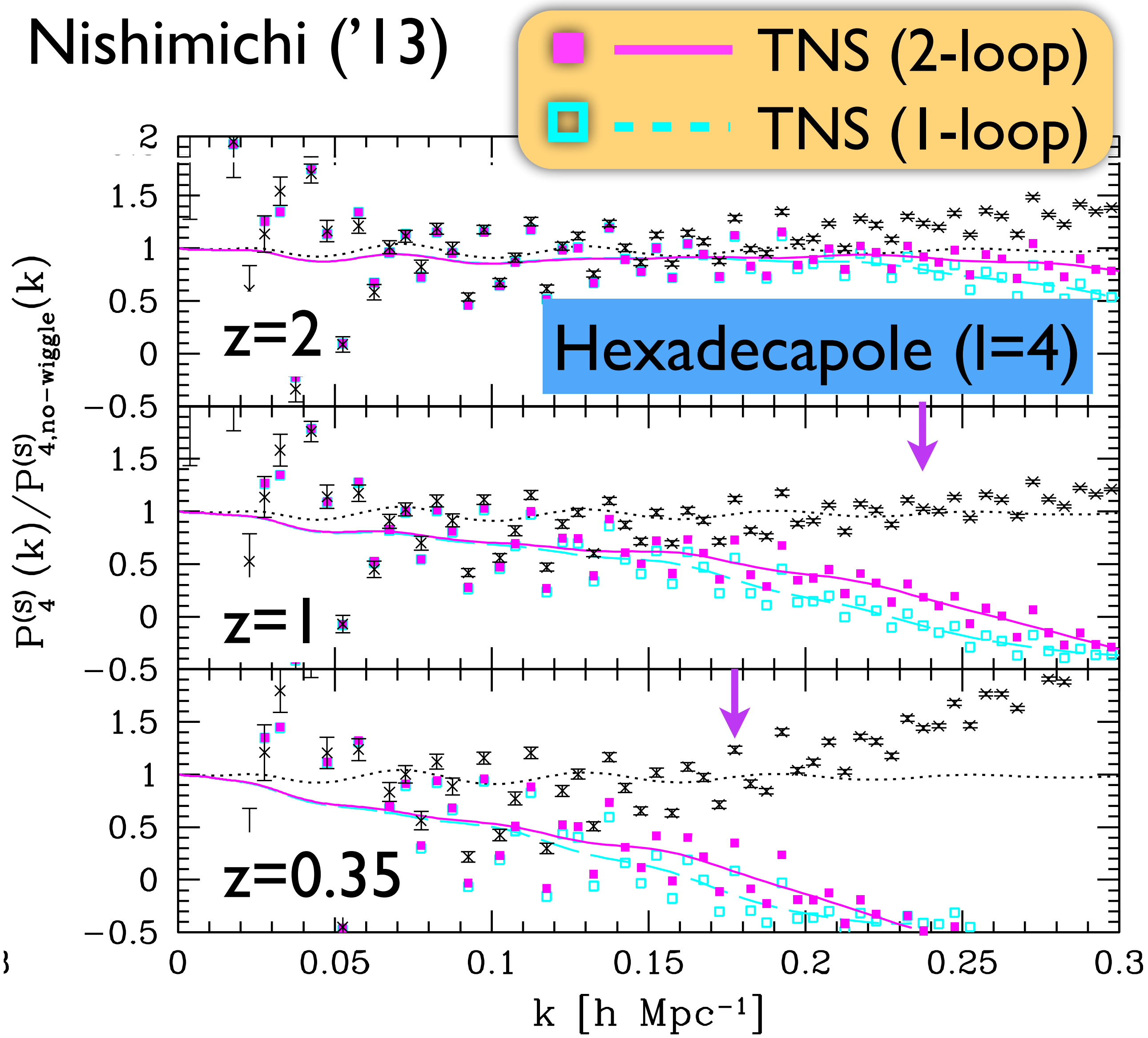


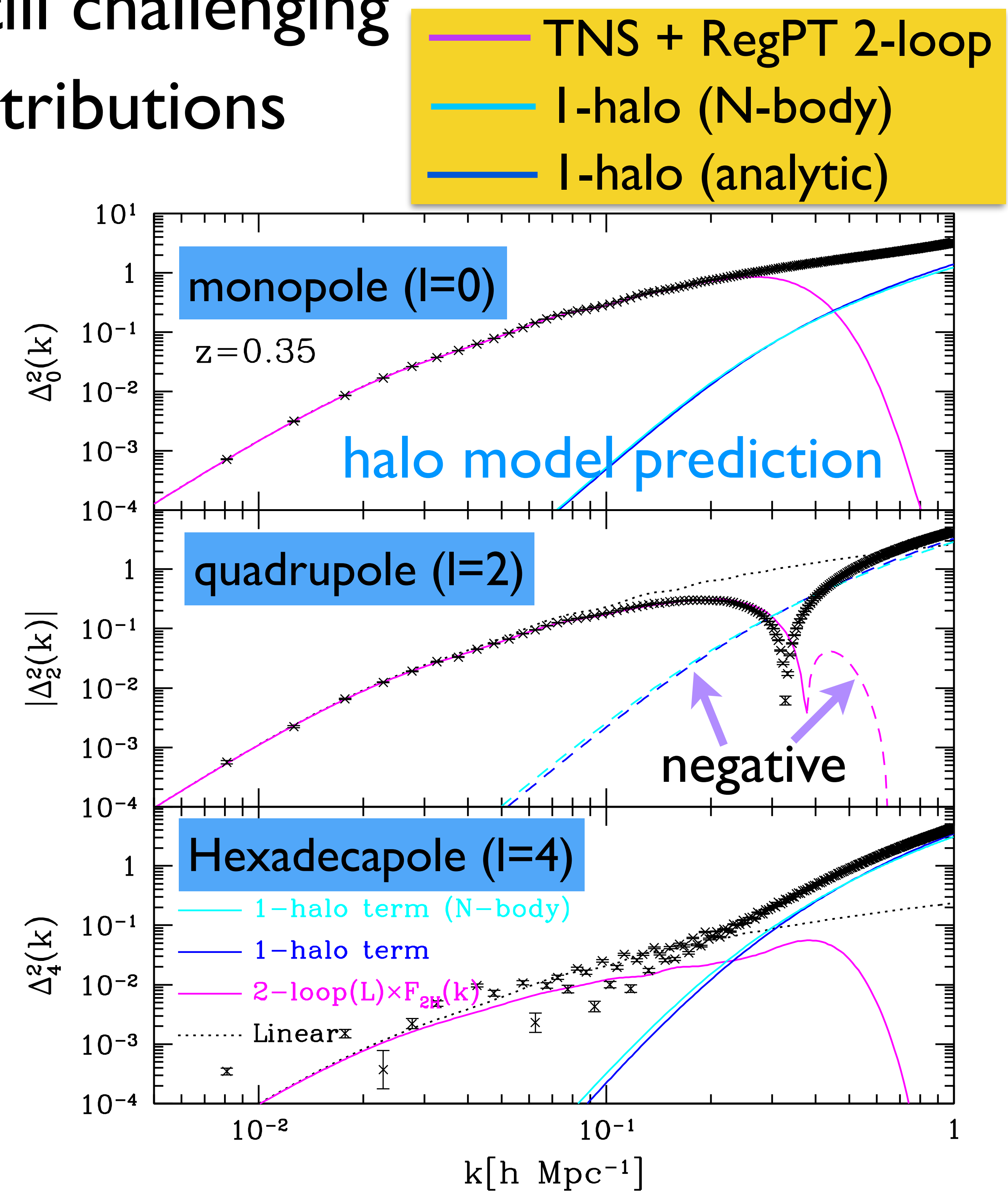
Viewgraph by T. Nishimichi

Impact of 1-halo contributions

Prediction of higher-multipoles is still challenging in the presence of one-halo contributions

AT, Bernardeau & Nishimichi ('13)





Summary

- RSD measurement is renewed with great interest in test of gravity on cosmological scales
- Growth rate parameter 'f(z)' can be measured through linear Kaiser effect in both spec/photo-z obs.
- Complication: non-linearity of RSD/gravity

improved RSD model based on perturbation theory (PT)

Still, PT-based model has limitation:

- galaxy bias (impact of satellites)
- impact of 1-halo contributions

→ Hybrid modeling

Okumura et al. ('15)
arXiv:1506.05814

修正重力理論における摂動論

For a concise review, Koyama, arXiv:1504.04623

Refs. Koyama, AT, & Hiramatsu, PRD 79, 123512 ('09)
AT, Koyama, Hiramatsu & Oka, PRD 89, 043509 ('14)
AT et al. PRD 90, 123515 ('14)

Motivation

Origin of late-time cosmic acceleration

If GR correctly describes cosmic expansion,

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} \quad \Rightarrow \quad \frac{\ddot{a}}{a} = \frac{\Lambda}{3} - \frac{4\pi G}{3c^2}(\rho + 3P)$$

$P < -\rho/3$

Dark energy (or cosmological constant)

If this is not the case,

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + D_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} + T^{\rm DE}_{\mu\nu}$$

Modified gravity / Deviation from GR

Gravity test

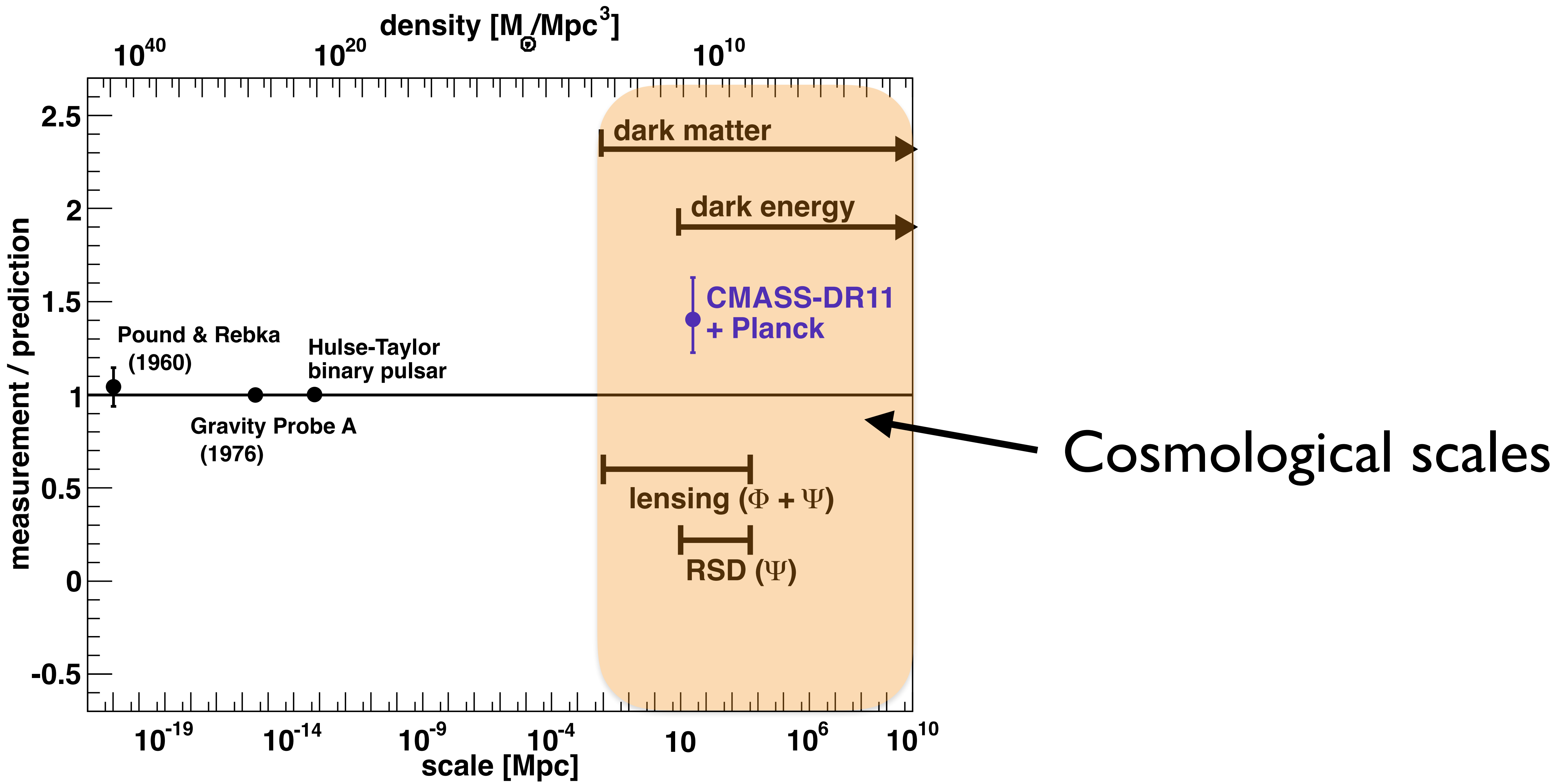


Figure 1. Summary of different tests of General Relativity (GR) as a function of distance scale (bottom axis) and densities (top axis). The standard model of cosmology seems to run into problems (dark matter, dark energy) at large scales. Because these problems could indicate a breakdown of GR we need to test GR on large scales. Two probes which can do this are redshift space distortions (RSD) and lensing. While RSD measures the Newtonian potential Ψ, lensing measures the sum of the metric potentials $\Phi + \Psi$. However, any modification of gravity needs to pass the very precise tests on smaller scales (Pound & Rebka experiment Pound & Rebka 1960, Gravity Probe A, Vessot et al. 1980, Hulse-Taylor binary pulsar Hulse & Taylor 1975, see Will 2006 for a complete list). Note that the error bars for Gravity Probe A and the Hulse-Taylor binary pulsar are smaller than the data points in this plot. In this analysis we perform a ΛCDM consistency test (blue data point), where we use the CMASS-DR11 power spectrum multipoles together with Planck (Ade et al. 2013a) to tests GR on scales of $\sim 30\,\mathrm{Mpc}$ (see section 9.1).

Beutler, Saito et al. ('14)

Cosmological probe of gravity

Suppose metric theory of gravity is still valid:

spacetime metric

$$ds^2 = -(1 + 2\Psi)dt^2 + a^2(t)(1 + 2\Phi)d\vec{x}^2$$

$a(t)$: cosmic expansion ⟷ geometric distances $H(z),\ D_A(z),\ D_L(z)$

$\Phi(\vec{x}),\ \Psi(\vec{x})$: potentials ⟷ large-scale structure

In particular,

RSD Ψ

Weak lensing $\Phi - \Psi$

Also, structure and abundance of halos are powerful probe

(Schmidt et al.'09; Terukina et al. '14)

Modified gravity effect on LSS

Linear theory (on sub-horizin scales)

第１講の板書より

Eqs. Ⓓ Ⓕ $\dot{\delta}_{\rm m}-\frac{k}{a}v=0$

Eqs. Ⓔ Ⓖ $\dot{v}+Hv+\frac{k}{a}\Psi=0$

} Conservation law of energy-momentum tensor

Eq. Ⓐ $\frac{k^2}{a^2}\Phi=4\pi\,G\,\rho_{\rm m}\,\delta_{\rm m}$

Eq. Ⓑ $\Phi+\Psi=0$

} Einstein equations

$$\ddot{\delta}_{\rm m}+2H\,\dot{\delta}_{\rm m}-4\pi\,G\,\rho_{\rm m}\,\delta_{\rm m}=0$$ in GR (ΛCDM)

Modification of Eqs. Ⓐ&Ⓑ generally implies

$$\ddot{\delta}_{\rm m}+2H\,\dot{\delta}_{\rm m}-4\pi\,G_{\rm eff}(k,\,t)\,\rho_{\rm m}\,\delta_{\rm m}=0$$ in modified gravity

time- & scale-(in)dependence of growth will be changed

Nonlinearity in modified gravity

Beyond linear theory, $G_{\rm eff}$ should has non-trivial dependence on *k* and *t* (should be non-linear func. of $\delta_{\rm m}$)

'Fifth force' mediated by new degree of freedom (scalar field)

Indeed,

For consistent modified gravity models that can pass solar-system tests, nonlinearity plays a crucial role to recover GR on small scales

.... *Screening effect* (e.g.,Vainshtein/Chameleon mechanism)

Taking account of nonlinearity is essential for cosmological test of gravity beyond linear scales

f(R) gravity

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa} \{R + f(R)\} + \mathcal{L}_{\rm m} \right]$$

$$f_R \equiv \frac{df(R)}{dR}$$

(Modified) Einstein equation

$$G_{\mu\nu} + f_R R_{\mu\nu} - \left(\frac{1}{2} f - \Box f_R \right) g_{\mu\nu} - \nabla_\mu \nabla_\nu f_R = 8\pi G T^{\rm m}_{\mu\nu},$$

→ **trace part:** $3\Box f_R - R + f_R R - 2f = -8\pi G \rho_{\rm m}$

Pressure ignored

This can be viewed as scalar field coupled with matter density field:

$$\Box\phi - \frac{\partial V_{\rm eff}}{\partial \phi} = 0$$

$$\phi \longleftrightarrow f_R \equiv \frac{df(R)}{dR}$$

$$\frac{\partial V_{\rm eff}}{\partial \phi} \longleftrightarrow \frac{1}{3}\left(R - f_R R + 2f + 8\pi G \rho_{\rm m}\right)$$

fifth force

f(R) gravity

Perturbation $f_R \longrightarrow \overline{f}_R + \delta f_R$ $\qquad ds^2 = -(1+2\Psi)dt^2 + a^2(t)\,(1+2\Phi)d\vec{x}^2$

Assuming $|\overline{f}_R| \ll 1$ & $|\overline{f}/\overline{R}| \ll 1$ (cosmic expansion close to ΛCDM)

Scalar-field eq. (quasi-static limit) ⟶

$$\frac{3}{a^2}\nabla^2\delta f_R = -8\pi\,G\,\rho_{\rm m}\delta_{\rm m} + \delta R \;;\; \delta R \equiv R(f_R) - R(\bar{f}_R)$$

nonlinear function of δf_R

Further, Modified Einstein eq. ⟶

$$\left\{\begin{array}{l} \dfrac{1}{a^2}\nabla^2\Psi = \dfrac{16\pi\,G}{3}\,\rho_{\rm m}\,\delta_{\rm m} - \dfrac{1}{6}\,\delta\,R \\[2ex] \nabla^2(\Psi+\Phi) = -\nabla^2\,\delta f_R \end{array}\right.$$

modified Poisson eq.

- Standard Poisson eq. is recovered if $\delta R = 8\pi\,G\rho_{\rm m}\,\delta_{\rm m}$ (screening effect)
- $\dfrac{1}{a^2}\nabla^2(\Psi-\Phi) = 8\pi\,G\,\rho_{\rm m}\delta_{\rm m}$ → weak-lensing effect unchanged

Viable models

Hu & Sawicki ('07) $$f(R) = -\lambda\, R_0 \frac{(R/R_0)^{2n}}{(R/R_0)^{2n}+1}$$

Starobinsky ('07) $$f(R) = -\lambda\, R_0 \left\{ 1 - \left(1 + \frac{R^2}{R_0^2}\right)^{-n} \right\}$$

Both models realize cosmic acceleration at late time, and recover GR at small scales

In particular, at $R \gg R_0$

$$f(R) \simeq -16\pi\, G\, \rho_\Lambda - f_{\mathrm{R0}} \frac{R_0^{(2n+1)}}{R^{2n}} \quad ; \; f_{R0} < 0$$

Formulation

Koyama, AT & Hiramatsu ('09)

Assumptions

- (Standard) fluid treatment of matter fluctuations (CDM+Baryon)

  ➡ Modification of gravity sector only appears in Poisson eq.

- Theory effectively looks like Brans-Dicke gravity on sub-horizon scales

Poisson eq. :

Newton potential

$$\frac{1}{a^2}\nabla^2\Psi = 4\pi\,G\,\overline{\rho}_m\,\delta_m - \frac{1}{2a^2}\nabla^2\varphi$$

new d.o.f for gravity

Brans-Dicke scalar

E.O.M for scalar field (under quasi-static approx.):

$$-(3+2\omega_{\rm BD})\frac{1}{a^2}\nabla^2\varphi = 8\pi\,G\,\overline{\rho}_m\,\delta_m - \mathcal{I}(\varphi)$$

Vainshtein/Chamelon mechanism

perturbative expansion (see next slide)

Formulation

Expansion of $\mathcal{I}(\varphi)$

$$\mathcal{I}(\varphi) = M_1(k)\varphi + \frac{1}{2}\int \frac{d^3\boldsymbol{k}_1 d^3\boldsymbol{k}_2}{(2\pi)^3}\delta_D(\boldsymbol{k}-\boldsymbol{k}_{12})M_2(\boldsymbol{k}_1,\boldsymbol{k}_2)\varphi(\boldsymbol{k}_1)\varphi(\boldsymbol{k}_2)$$
$$+\frac{1}{6}\int \frac{d^3\boldsymbol{k}_1 d^3\boldsymbol{k}_2 d^3\boldsymbol{k}_3}{(2\pi)^6}\delta_D(\boldsymbol{k}-\boldsymbol{k}_{123})M_3(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)\varphi(\boldsymbol{k}_1)\varphi(\boldsymbol{k}_2)\varphi(\boldsymbol{k}_3) + \dots$$

Examples

f(R) : $(\omega_{\rm BD} = 0)$ $\quad M_1 = \frac{dR(f_R)}{df_R}, \quad M_2 = \frac{d^2R(f_R)}{df_R^2}, \quad M_3 = \frac{d^3R(f_R)}{df_R^3}$

DGP : $\quad M_1 = 0, \quad M_2 = 2\frac{r_c^2}{a^4}\{(k_1k_2)^2 - (\mathbf{k}_1\cdot\mathbf{k}_2)^2\}, \quad M_3 = 0$

Most of Horndeski theory is also described by this formalism

Solving the scalar-field equation perturbatively,

new scalar field is expressed in terms of density fluctuation

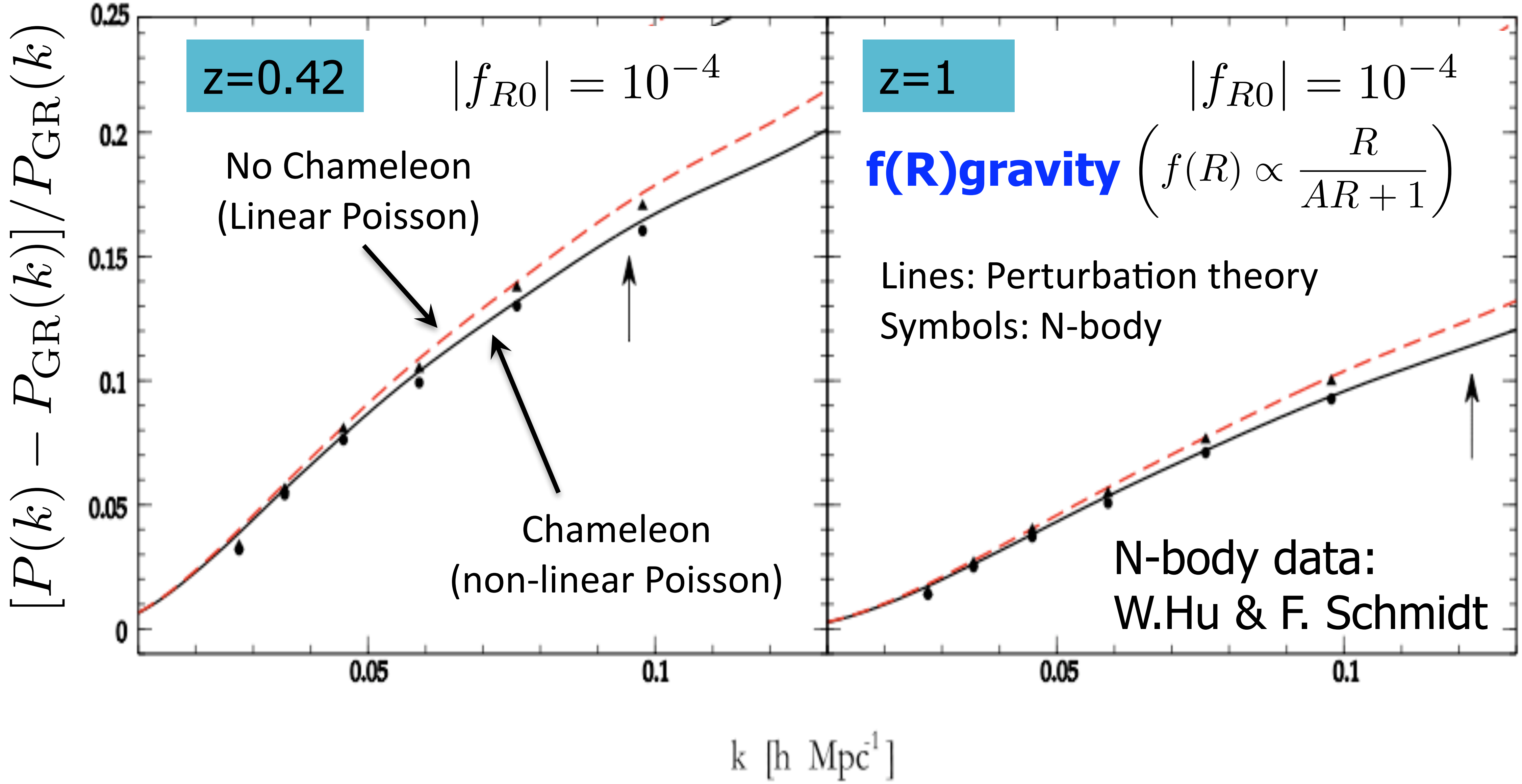
Total system: Non-linear Poisson eq. + Fluid eqs. (Euler eq., Continuity eq.)

Standard PT 1-loop

Standard perturbation theory (PT)

Koyama, AT & Hiramatsu ('09)

$$\delta = \delta_1 + \delta_2 + \delta_3 + \cdots \quad \rightarrow \quad \langle \delta(\vec{k})\delta(\vec{k}')\rangle = (2\pi)^3\,\delta_{\rm D}(\vec{k}+\vec{k}')\,P(k)$$



Impact of screening effect is small (~1%), but is not entirely negligible

Effective Newton constant

<u>Linearized equation:</u>

Continuity eq.
Euler eq. → $\ddot{\delta}_{\rm m} + 2H\dot{\delta}_{\rm m} + \frac{k^2}{a^2}\Psi = 0$

Poisson eq.
Scalar-field eq. → $-\frac{k^2}{a^2}\Psi = 4\pi G_{\rm eff}(k,t)\,\delta_{\rm m}$

$$G_{\rm eff} \equiv G\left\{1 + \frac{(k/a)^2}{(3+2\omega_{\rm BD})(k/a)^2 + M_1(k)}\right\}$$

For f(R), $G_{\rm eff} = G\left\{1 + \frac{(k/a)^2}{3(k/a)^2 + dR/df_R}\right\} \xrightarrow{k\to\infty} \frac{4}{3}G$

Linear growth factor becomes scale-dependent in f(R) gravity
→ Scale- & time-dependence is no longer separable in PT calculation

A novel PT scheme

Standard PT kernels

$\delta = \delta^{(1)} + \delta^{(2)} + \cdots$

$\theta = \theta^{(1)} + \theta^{(2)} + \cdots$

$\theta \equiv \dfrac{\nabla \cdot \boldsymbol{v}}{a\,H}$

$$\delta^{(n)}(\boldsymbol{k};t) = \int \frac{d^3\boldsymbol{k}_1 \cdots d^3\boldsymbol{k}_n}{(2\pi)^{3(n-1)}}\, \delta_{\rm D}(\boldsymbol{k}-\boldsymbol{k}_{12\cdots n})\, F_n(\boldsymbol{k}_1,\cdots,\boldsymbol{k}_n;t)\, \delta_0(\boldsymbol{k}_1)\cdots\delta_0(\boldsymbol{k}_n),$$

$$\theta^{(n)}(\boldsymbol{k};t) = \int \frac{d^3\boldsymbol{k}_1 \cdots d^3\boldsymbol{k}_n}{(2\pi)^{3(n-1)}}\, \delta_{\rm D}(\boldsymbol{k}-\boldsymbol{k}_{12\cdots n})\, G_n(\boldsymbol{k}_1,\cdots,\boldsymbol{k}_n;t)\, \delta_0(\boldsymbol{k}_1)\cdots\delta_0(\boldsymbol{k}_n),$$

In GR, kernels (F_n, G_n) are analytically constructed from recursion relation (e.g., Goroff et al. '86)

But this is not possible in non-separable case like f(R) gravity

⟶ Numerical reconstruction of standard PT kernels

A novel PT scheme

Solving evolution eqs. for PT kernels numerically:

Linear operator

$$\widehat{\boldsymbol{\mathcal{L}}}(k_{1\cdots n})\begin{pmatrix} F_n(\boldsymbol{k}_1,\cdots,\boldsymbol{k}_n;a) \\ G_n(\boldsymbol{k}_1,\cdots,\boldsymbol{k}_n;a)\end{pmatrix} = \begin{pmatrix} S_n(\boldsymbol{k}_1,\cdots,\boldsymbol{k}_n;a) \\ T_n(\boldsymbol{k}_1,\cdots,\boldsymbol{k}_n;a)\end{pmatrix}$$

nonlinear source term

$$\sum_{j=1}^{n-1}\begin{pmatrix} -\alpha(\boldsymbol{k}_{1\cdots j},\boldsymbol{k}_{j+1\cdots n})\,G_j(\boldsymbol{k}_1,\cdots,\boldsymbol{k}_j)\,F_{n-j}(\boldsymbol{k}_{j+1},\cdots,\boldsymbol{k}_n) \\ -\frac{1}{2}\beta(\boldsymbol{k}_{1\cdots j},\boldsymbol{k}_{j+1\cdots n})\,G_j(\boldsymbol{k}_1,\cdots,\boldsymbol{k}_j)\,G_{n-j}(\boldsymbol{k}_{j+1},\cdots,\boldsymbol{k}_n)\end{pmatrix} + \square$$

$$\widehat{\boldsymbol{\mathcal{L}}}(k) \equiv \begin{pmatrix} a\frac{d}{da} & 1 \\ \frac{3}{2}\left(\frac{H_0}{H(a)}\right)^2\frac{\Omega_{\mathrm{m},0}}{a^3} + \square & a\frac{d}{da} + \left(2+\frac{\dot{H}}{H^2}\right) + \square \end{pmatrix}$$

modification is easy

scale factor as time variable

A novel PT scheme

Recipes

1. Solve these equations with initial conditions at a_i <<1:

$$F_1 = a_i, \quad G_1 = -a_i \,, \quad \textit{otherwise zero}$$

2. Symmetrized : $F_n^{(\mathrm{sym})}(\boldsymbol{k}_1, \cdots, \boldsymbol{k}_n) = \frac{1}{n!} \sum_{\{n\}} \{F_n(\boldsymbol{k}_1, \cdots, \boldsymbol{k}_n) + \mathrm{perm}\}$

3. Store the output in multi-dim arrays

special technique is unnecessary
it can be parallelized

For power spectrum at 1-loop order,
what we need is just the 3D arrays of kernels up to 3rd order
(typical size ~100x100x10)

kernels up to 3rd order → application to → resmmed PT and/or RSD calculations

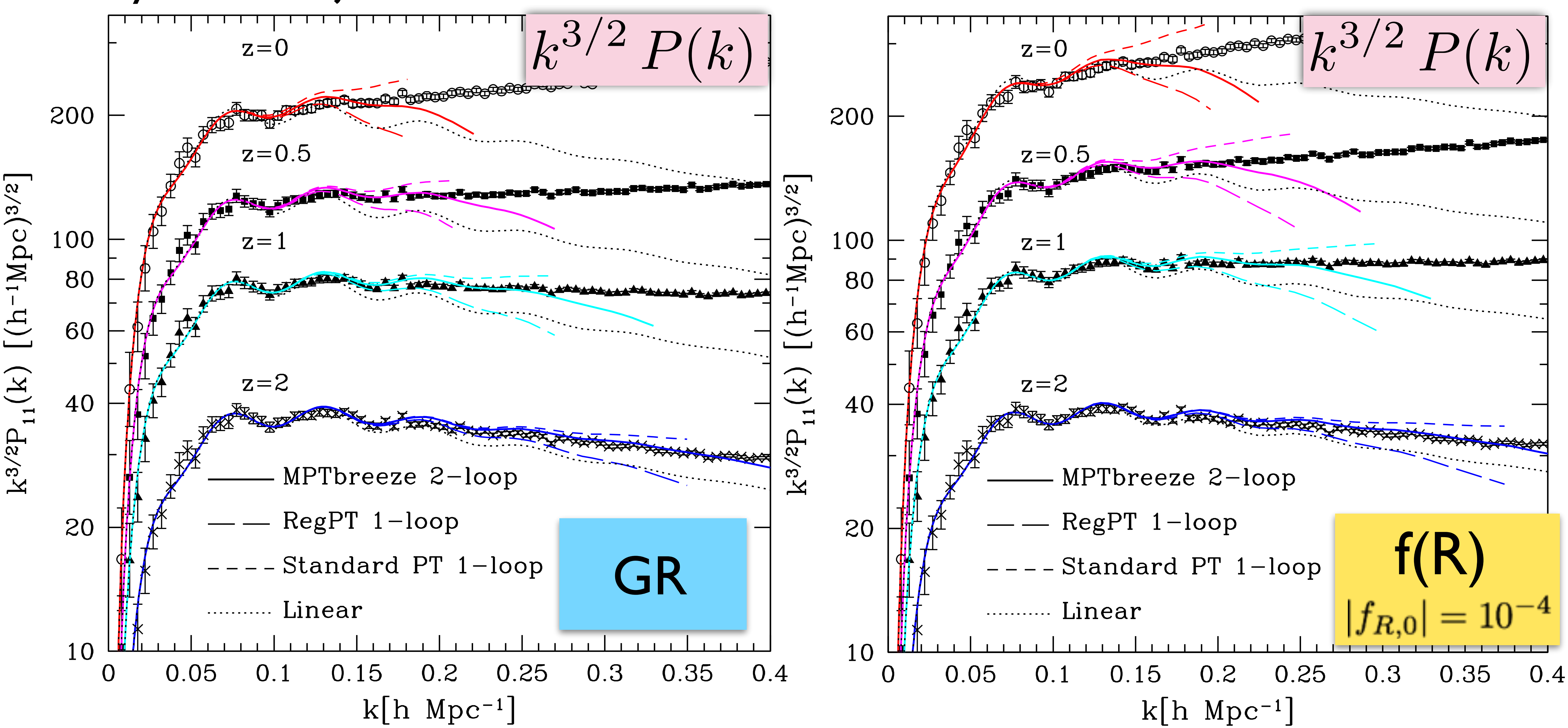
Application: f(R) gravity

All predictions are made from
kernels up to 3rd order (i.e., F_2, F_3)

$$f(R) \simeq -16\pi G \rho_\Lambda + |f_{R,0}| \frac{R_0^2}{R}$$

$|f_{R,0}| = 10^{-4}$

N-body data: Baojiu Li



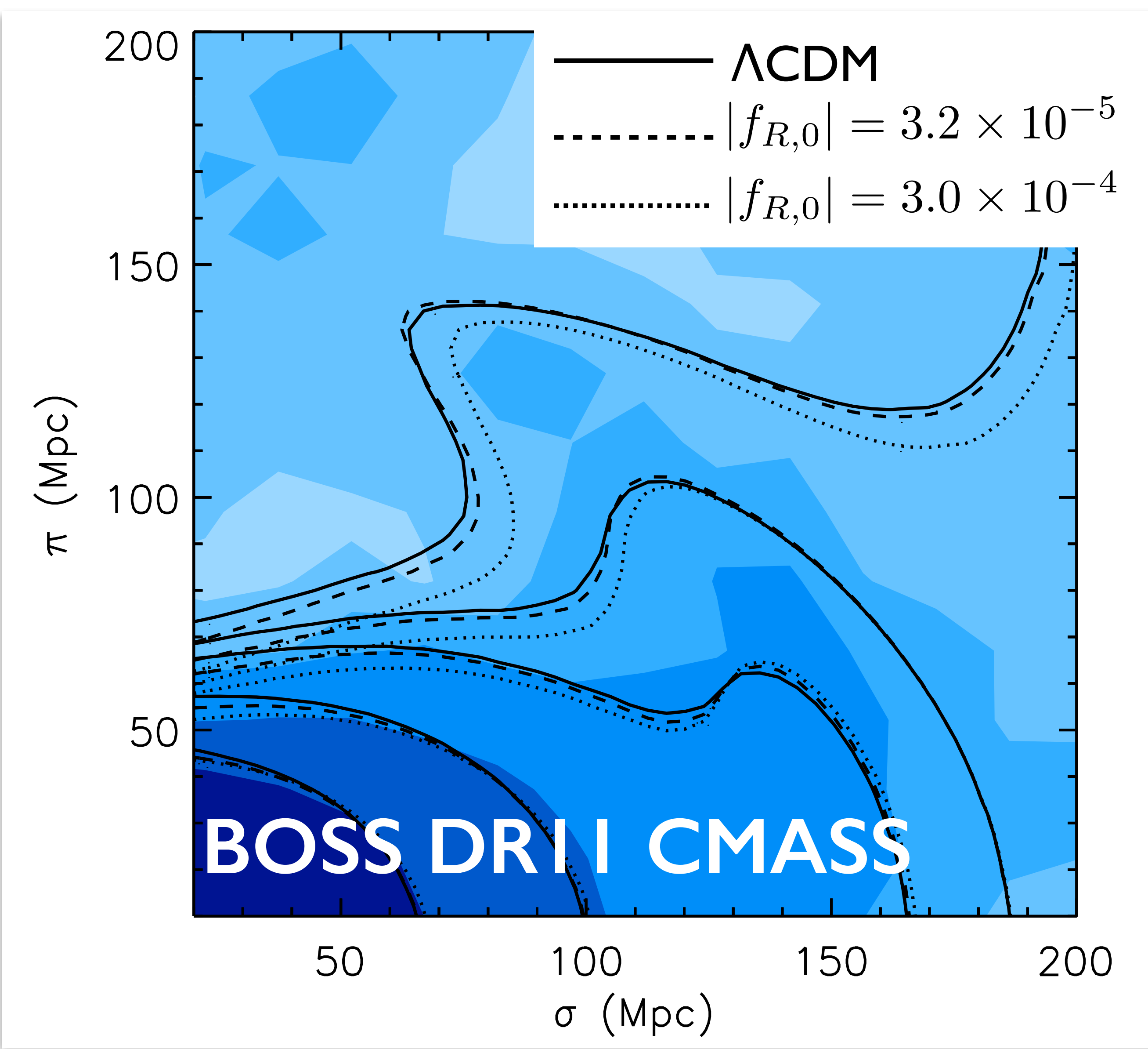
Consistent modified gravity analysis

Y-S.Song, AT, Linder, Koyama et al.

arXiv:1507.01592

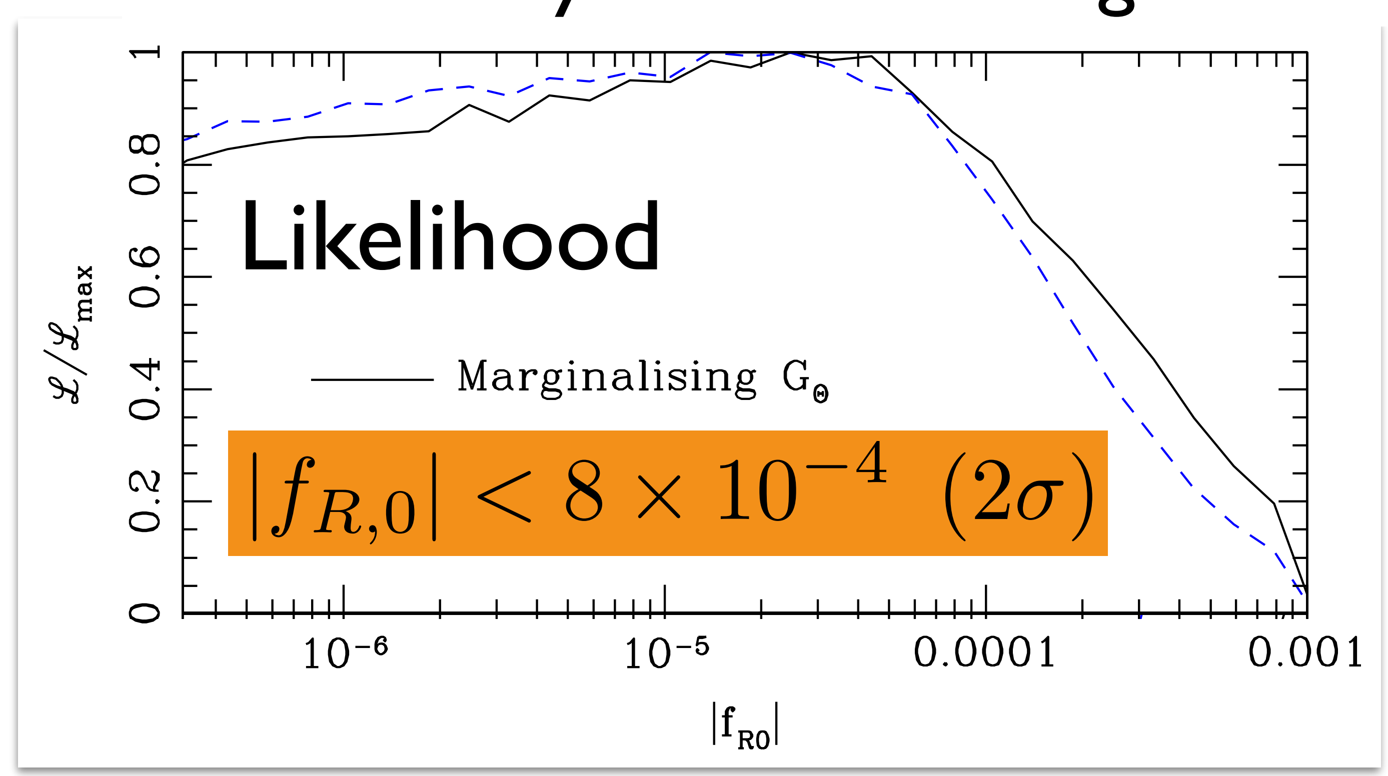
Combining TNS model of RSD,

anisotropic correlation function is consistently computed in f(R) gravity → BOSS DR11 CMASS



$$f(R) \simeq -16\pi\, G\, \rho_\Lambda + |f_{R,0}|\, \frac{R_0^2}{R}$$

Alcock-Paczynski effect marginalized



ニュートリノが質量を持つ場合の非線形クラスタリング

For review & pedagogical paper,

Lesgourgues & Pastor, Phys.Rep. 429, 307 ('06)
Shoji & Komatsu, PRD 81, 123516 ('10)

Refs.

Saito, Takada & AT, PRL 100, 191301 ('08)
Saito, Takada & AT, PRD 80, 038528 ('09)

Linearized Boltzmann equation

For massive neutrinos,

第１講の板書
Eq. (H) →

$$\dot{\mathcal{N}} + i\frac{k\mu}{a}\left(\frac{p}{\sqrt{p^2+m^2}}\mathcal{N} + \frac{\sqrt{p^2+m^2}}{p}\Psi\right) + \dot{\Phi} = 0$$

Legendre expansion: $\mathcal{N}(\boldsymbol{k},\mu,p) = \sum_\ell (-i)^\ell (2\ell+1)\mathcal{N}_\ell(k,p)\mathcal{P}_\ell(\mu)$

$$\dot{\mathcal{N}}_0 + \frac{k}{a}\frac{p}{\sqrt{p^2+m^2}}\mathcal{N}_1 + \dot{\Phi} = 0$$

$$\dot{\mathcal{N}}_1 + \frac{k}{3a}\left\{\frac{p}{\sqrt{p^2+m^2}}(2\mathcal{N}_2 - \mathcal{N}_0) - \frac{\sqrt{p^2+m^2}}{p}\Psi\right\} = 0$$

$$\dot{\mathcal{N}}_2 + \frac{k}{5a}\frac{p}{\sqrt{p^2+m^2}}(3\mathcal{N}_3 - 2\mathcal{N}_1) = 0$$

$$\dot{\mathcal{N}}_\ell + \frac{k}{(2\ell+1)a}\frac{p}{\sqrt{p^2+m^2}}\{(\ell+1)\mathcal{N}_{\ell+1} - \ell\mathcal{N}_{\ell-1}\} = 0 \;;\; (\ell \geq 2)$$

$$\underbrace{g_{ij}p^ip^j}_{p^2} + \underbrace{g_{00}(p^0)^2}_{-E^2} = m^2$$

Note— 'p' is physical momentum

Relation to fluid quantities

$$\delta_\nu(\boldsymbol{k}) = \frac{1}{\rho_\nu}\int \frac{d^3\boldsymbol{p}}{(2\pi)^3}\sqrt{p^2+m^2}\,\boxed{\delta f_\nu(\boldsymbol{k},\mu,p)}$$

$$\delta f_\nu(\boldsymbol{k},\mu,p) = -\frac{\partial f_\nu^{(0)}(p)}{\partial \ln p}\,\mathcal{N}(\boldsymbol{k},\mu,p)$$

$$v_\nu(\boldsymbol{k}) = \frac{1}{\rho_\nu + P_\nu}\int \frac{d^3\boldsymbol{p}}{(2\pi)^3}\,p\mu\;\delta f_\nu(\boldsymbol{k},\mu,p)$$

To describe the perturbed distribution function for neutrinos, we need 3 variables (i,e., k, p, and mu)

It is thus difficult to analytically treat Linearized Boltzmann eq.

(also bit difficult for numerical computation)

Characteristic scales

Masses of (each) neutrino are known to be very light, and their distribution is close to thermal distribution (i.e., Fermi-Dirac)

⟶ Neutrinos were relativistic until recently

⟶ They have large velocity dispersion

Two important characteristic scales: $k_{\rm nr}$, $k_{\rm FS}$

1. Non-relativistic scale

Fermi-Dirac

$$\langle E\rangle \equiv \frac{\int d^3q\, q\, f_\nu(q)}{\int d^3q\, f_\nu(q)} = \frac{7\pi^4}{180\zeta(3)} T_\nu \simeq 3.15\, T_\nu(z)$$

Transition redshift :
(relativistic → non-relativistic)
$E \simeq m$

$$1 + z_{\rm nr} \simeq 1890 \left(\frac{m}{1\ {\rm eV}}\right)$$

non-relativistic scale :
$k_{\rm nr} = a_{\rm nr} H(a_{\rm nr})$

$$k_{\rm nr} \simeq 0.018\, \Omega_{\rm m}^{1/2} \left(\frac{m}{1\ {\rm eV}}\right)^{1/2} h\, {\rm Mpc}^{-1}$$

Characteristic scales

2. free-streaming scale

$$\sigma_\nu^2(z) \equiv \frac{\int d^3q\,(q/m)^2\, f_\nu(q)}{\int d^3q\;\, f_\nu(q)} = \frac{15\,\zeta(5)}{\zeta(3)}\left(\frac{4}{11}\right)^{2/3}\frac{T_{\rm cmb}^2(1+z)^2}{m^2}$$

$$\longrightarrow\quad \sigma_\nu \simeq 181\,(1+z)\,\left(\frac{m}{1\ \mathrm{eV}}\right)^{-1}\ \mathrm{km\,s^{-1}}$$

In analogy with Jeans instability,

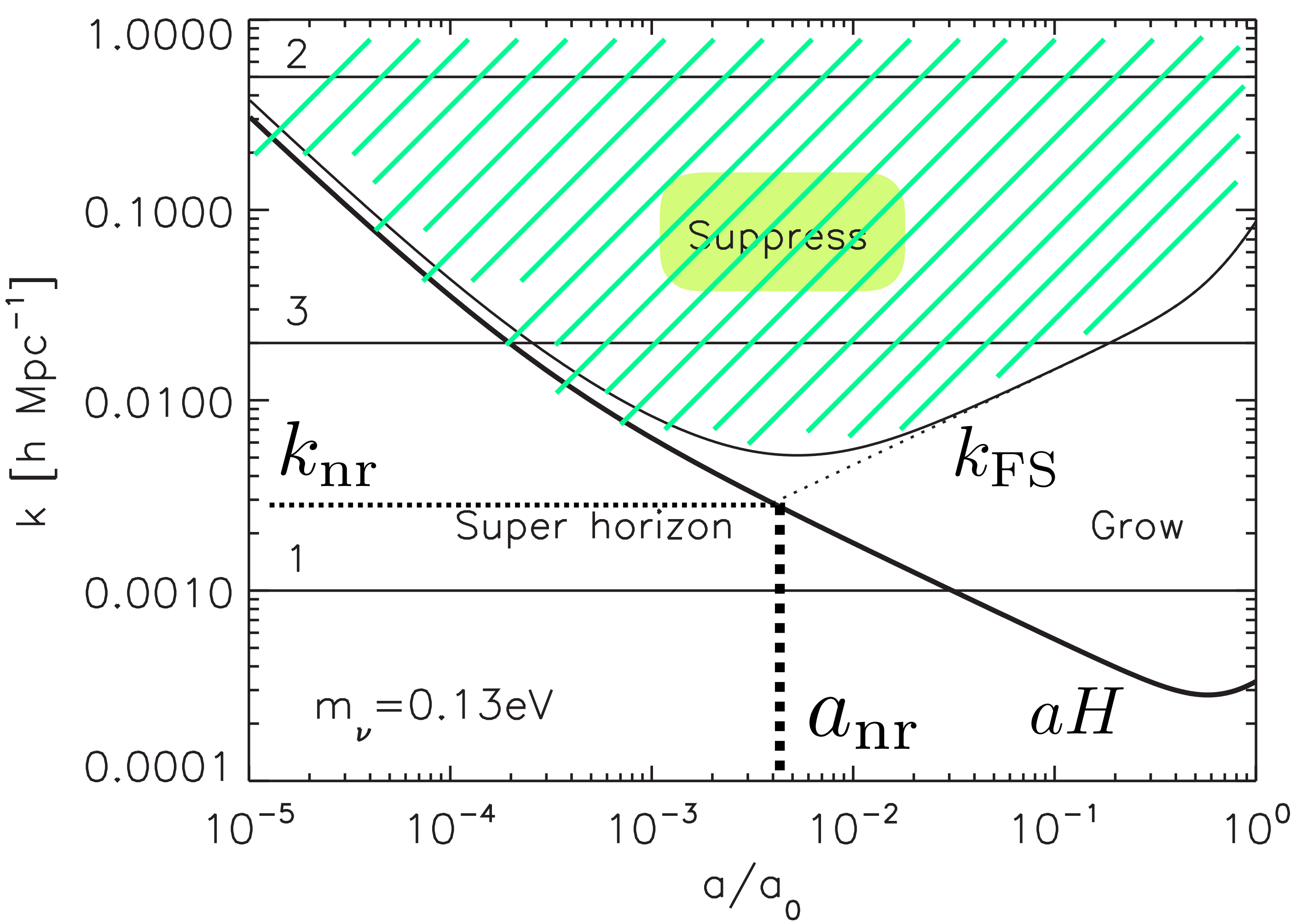
density perturbation δ_ν does not grow (rather suppressed)

Characteristic scale: $k_{\rm FS} \equiv \sqrt{\frac{3}{2}}\,\frac{a\,H}{\sigma_\nu}$ ⟷ Jeans scale $\left(\frac{\frac{3}{2}H^2}{c_s^2}\right)^{1/2} \equiv \frac{k_{\rm J}}{a}$

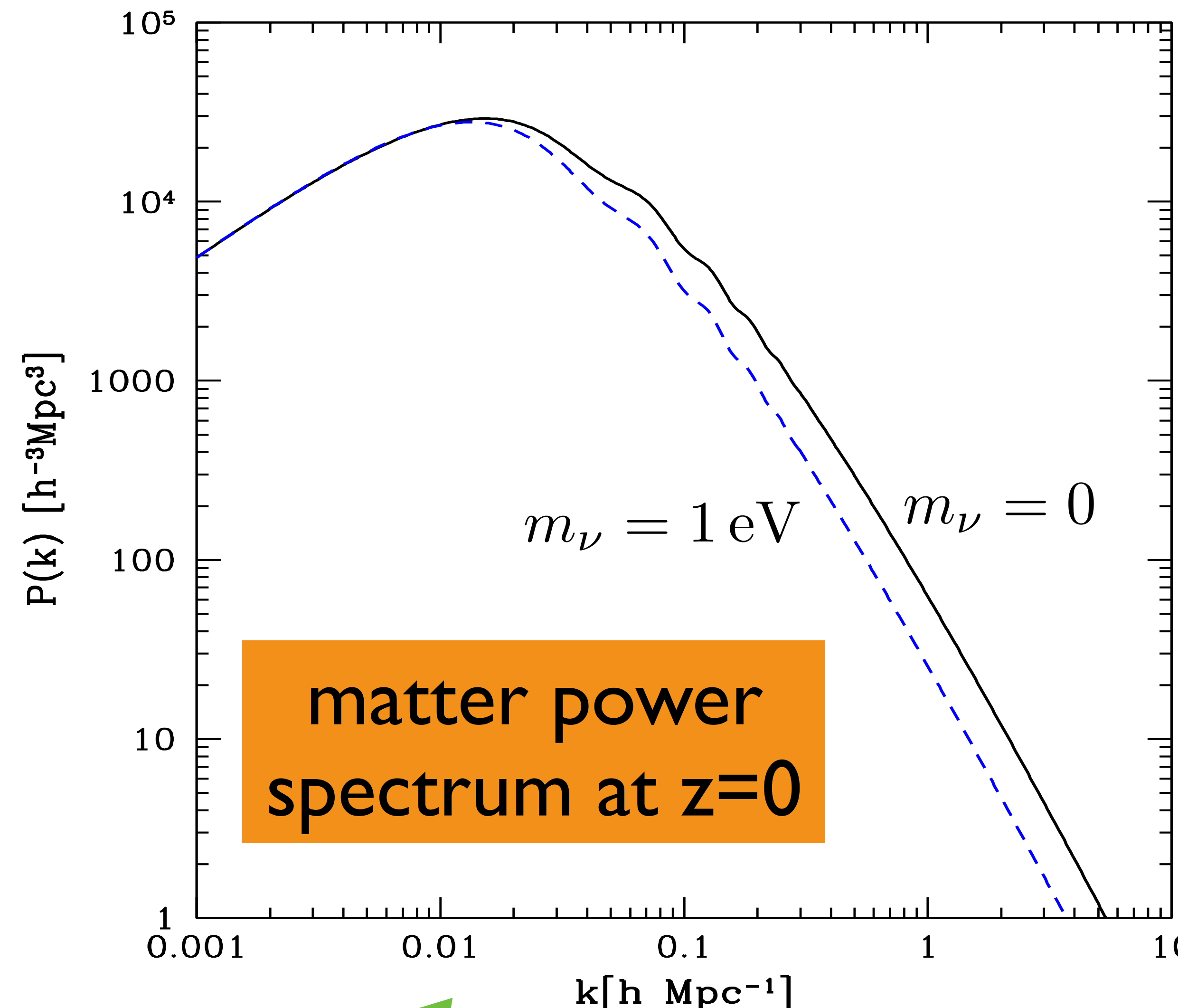
$$= \frac{0.677}{(1+z)^2}\,\frac{m}{1\ \mathrm{eV}}\sqrt{\Omega_{\rm m}(1+z)^3+\Omega_\Lambda}\ \left[h\,\mathrm{Mpc}^{-1}\right]$$

Free-streaming scale

Qualitative behaviors



Shoji & Komatsu ('10)



suppressed

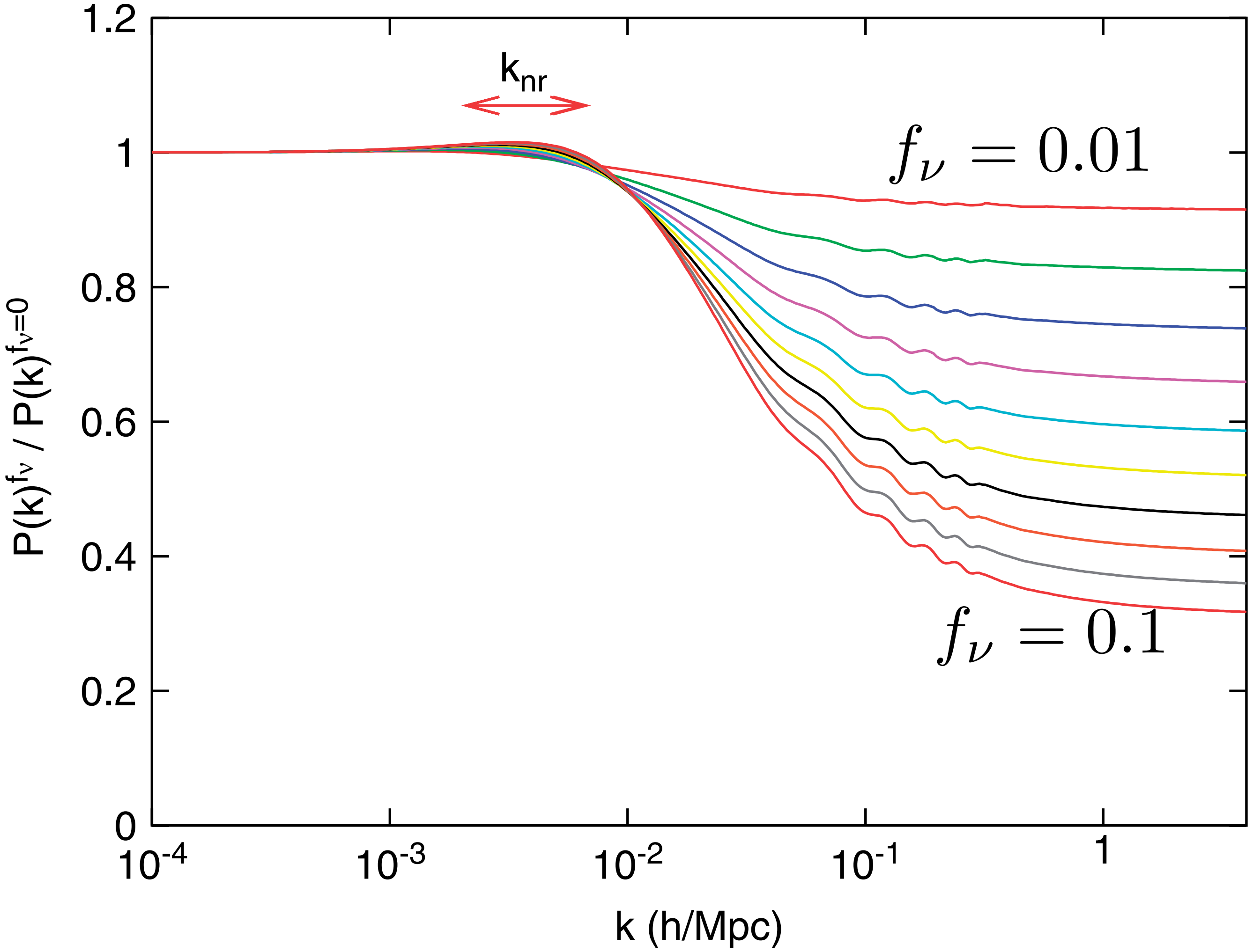
$$\delta_{\rm m} = \frac{\Omega_{\rm c}}{\Omega_{\rm m}}\delta_{\rm c} + \frac{\Omega_{\rm b}}{\Omega_{\rm m}}\delta_{\rm b} + \frac{\Omega_\nu}{\Omega_{\rm m}}\delta_\nu$$

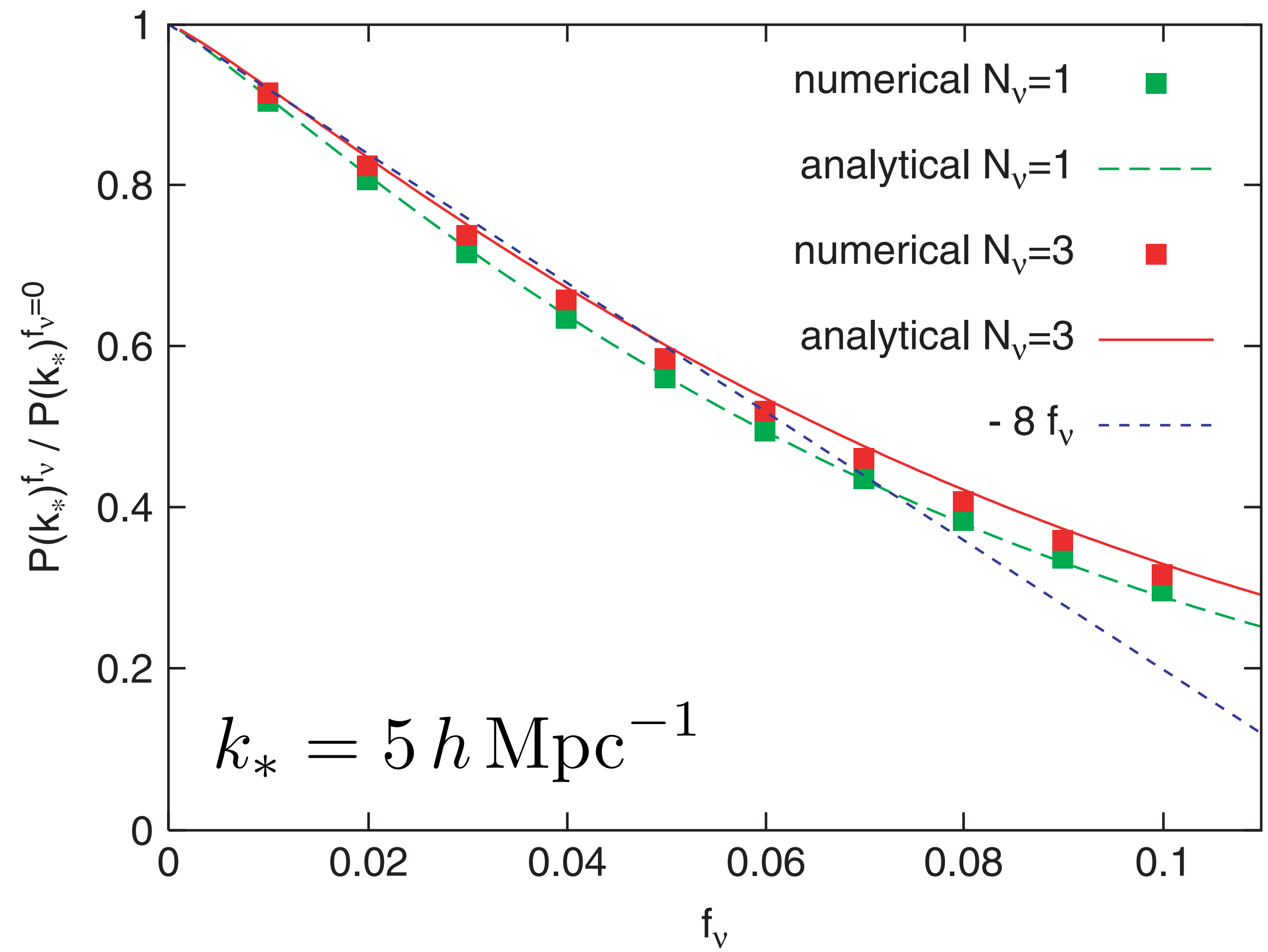
Below free-streaming scale $(k > k_{\rm FS})$

Free-streaming suppression

ratio of matter power spectrum

$$f_\nu = \frac{\Omega_\nu}{\Omega_m} \simeq 0.075 \left(\frac{0.1426}{\Omega_m h^2}\right)\left(\frac{m}{1\ \mathrm{eV}}\right)$$





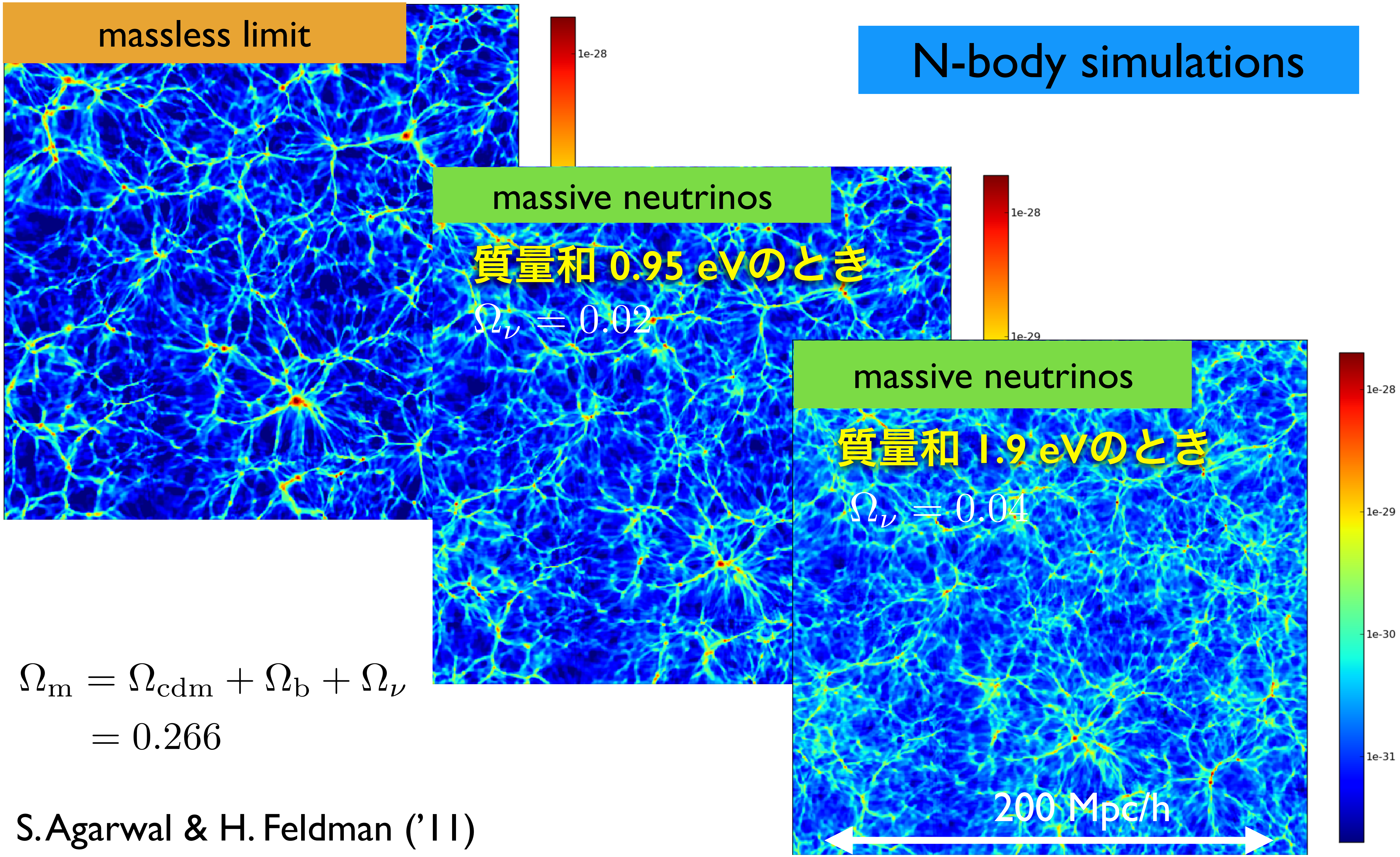
Empirical formula

$$\frac{P_{f_\nu \neq 0}(k_*)}{P_{f_\nu = 0}(k_*)} \simeq 1 - 8\, f_\nu$$

Lesgourgues & Pastor ('06)

(at z=0)

Impact on large-scale structure



$$\Omega_{\rm m} = \Omega_{\rm cdm} + \Omega_{\rm b} + \Omega_\nu$$
$$= 0.266$$

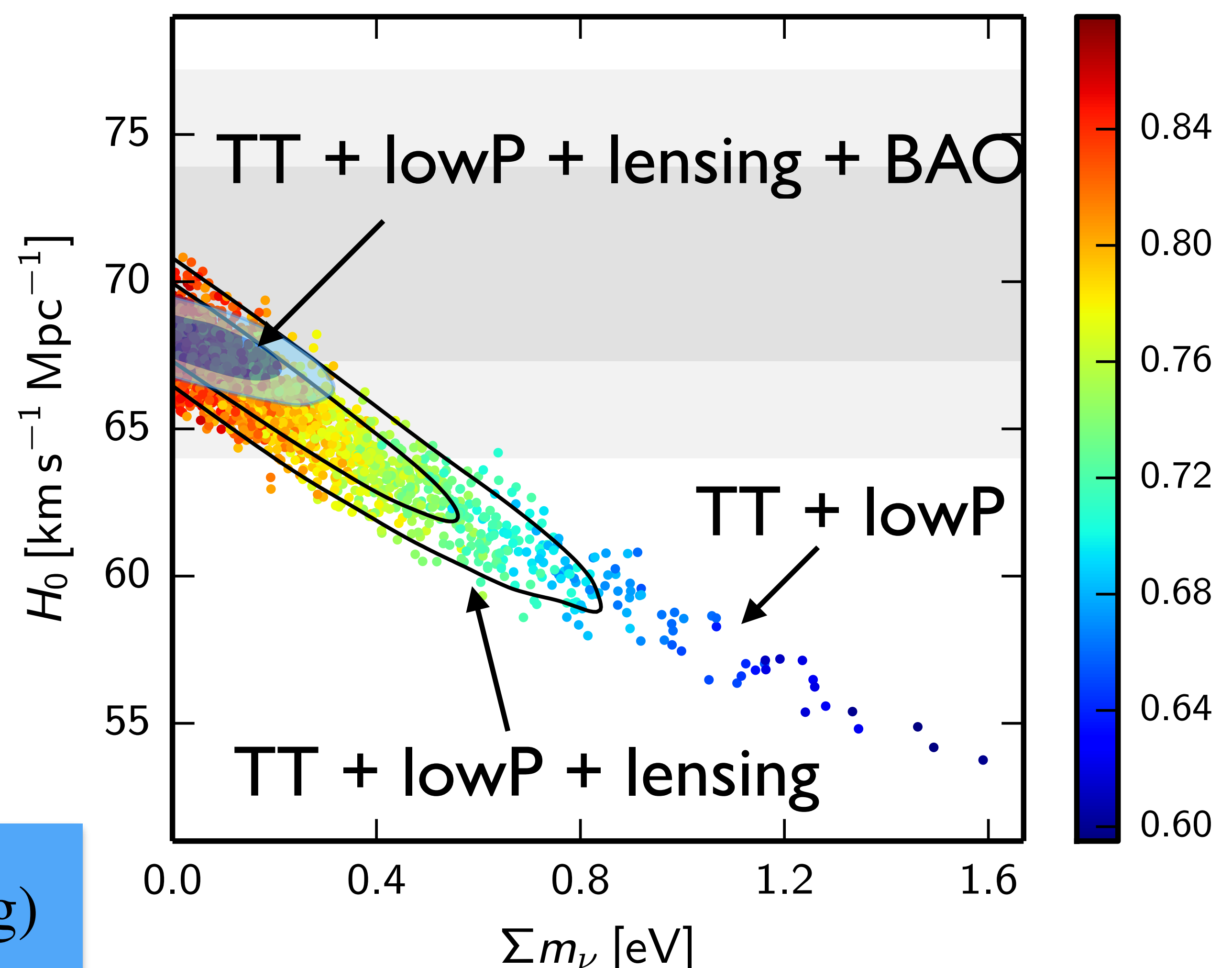
S. Agarwal & H. Feldman ('11)

Constraints on neutrino masses

Detection of *free-streaming suppression* is the key to weigh the neutrino masses

Combining CMB with

- Galaxy surveys
- Weak lensing & CMB lensing
- Others: Cluster count
  Lyman-α forest
  21cm (future)



Current bounds

$$\sum m_\nu < 0.68 \text{ eV} \quad (95\%, \textit{Planck} \text{ TT+lowP+lensing})$$

$$\left.\begin{array}{l} \sum m_\nu < 0.23 \text{ eV} \\ \Omega_\nu h^2 < 0.0025 \end{array}\right\} \quad 95\%, \textit{Planck} \text{ TT+lowP+lensing+ext}$$

(ext=BAO+H0+JLA)

Planck 2015

Modeling LSS including massive ν

To detect small masses of ν,

precision modeling of large-scale structure (LSS) is crucial

→ Nonlinear gravitational clustering of CDM + baryon + ν

Difficulties

- Co-existence of *very* hot & *very* cold components
- Tiny amount of neutrinos : $\delta = f_\nu \delta_\nu + (1 - f_\nu)\delta_{cb}$

$$f_\nu \equiv \frac{\Omega_\nu h^2}{\Omega_m h^2} = \frac{1}{\Omega_m h^2} \frac{\sum_i m_{\nu,i}}{94.1\,\mathrm{eV}} \lesssim 0.02 \quad \text{for} \quad \sum_i m_{\nu,i} < 0.3\ \mathrm{eV}$$

------→ different dynamic range in phase-space

Simulations with massive ν

In addition to CDM+baryon treated as N-body particles,

- massive ν : N-body particles

e.g., Brandbyge et al. ('08), Viel et al. ('10), …

- massive ν : Linear Boltzmann on grids

e.g., Brandbyge & Hannestad ('09), Ali-Haimoud & Bird ('12), …

- massive ν : Linear Boltzmann on grids + N-body particles

'Hybrid' approach Brandbyge & Hannestad ('10)

- massive ν : SPH particles (treated as fluid with pressure)

Hannestad, Haugbølle, Schultz ('12)

Alternative method

'N-one body approach'

ignore neutrino's self-gravity e.g., Ringwald & Wong ('04)

Perturbation theory with massive ν

Strictly speaking,

single-stream approximation is invalid for massive neutrinos
(neutrinos are 'hot' dark matter)

⟶ Need a further approximate treatment

<u>A simple recipe by Saito, Takada & AT ('08, '09)</u>

$$P_{\rm m}(k) = (1-f_\nu)^2\, P_{\rm cb}(k) + 2f_\nu(1-f_\nu)\, P_{{\rm cb},\nu}(k) + f_\nu^2\, P_\nu(k)$$

Standard (or resummed) PT
with a slight modification :

Linear Boltzmann

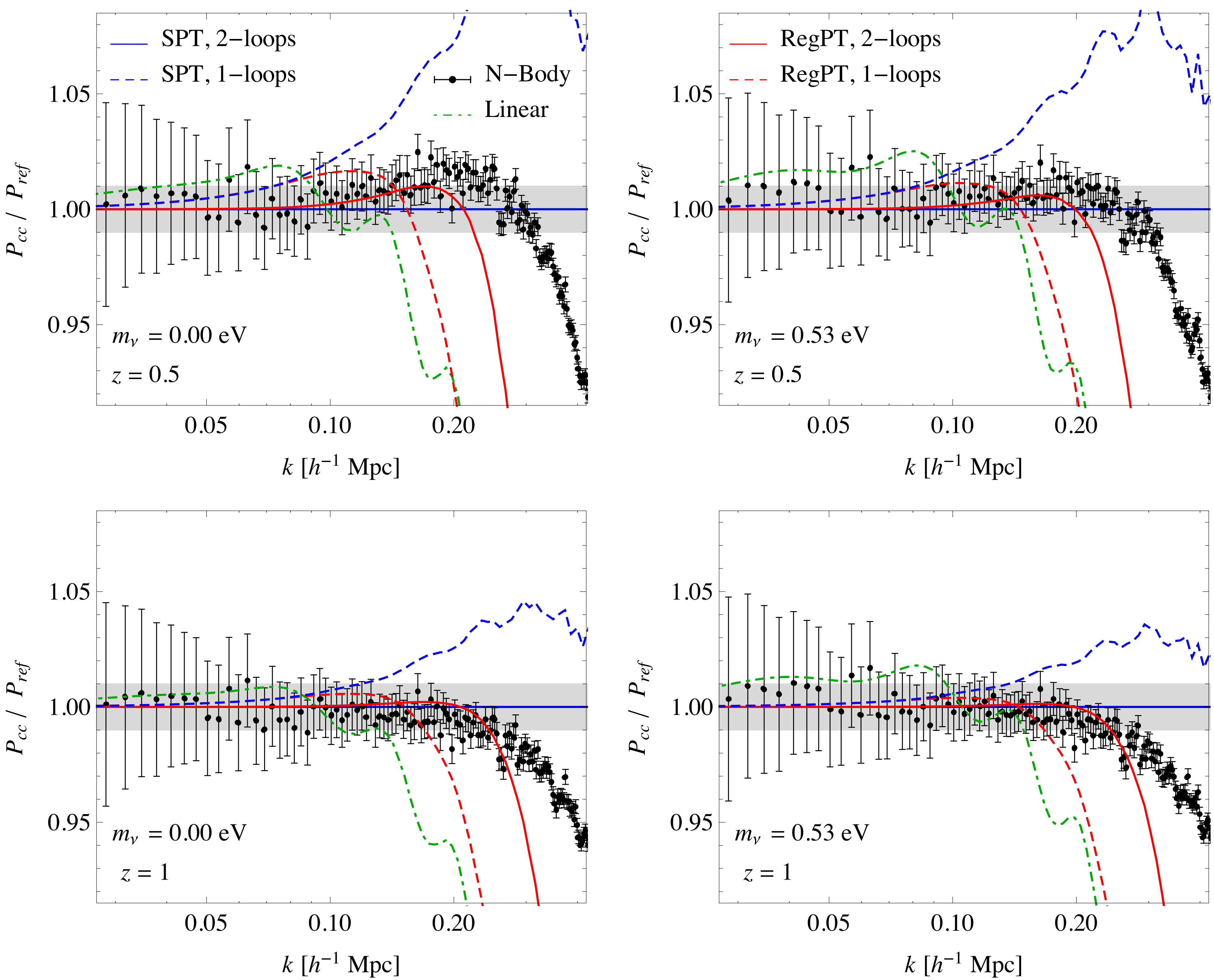
replacing all $D_+^2(z)\,P_0(k)$ in PT expressions with $P_{\rm cb}(k;z)$

linear matter
power spectrum

linear power spectrum
of CDM + baryon

For other approach, multi-component fluid system by Blas et al. ('14)

Simulation vs PT

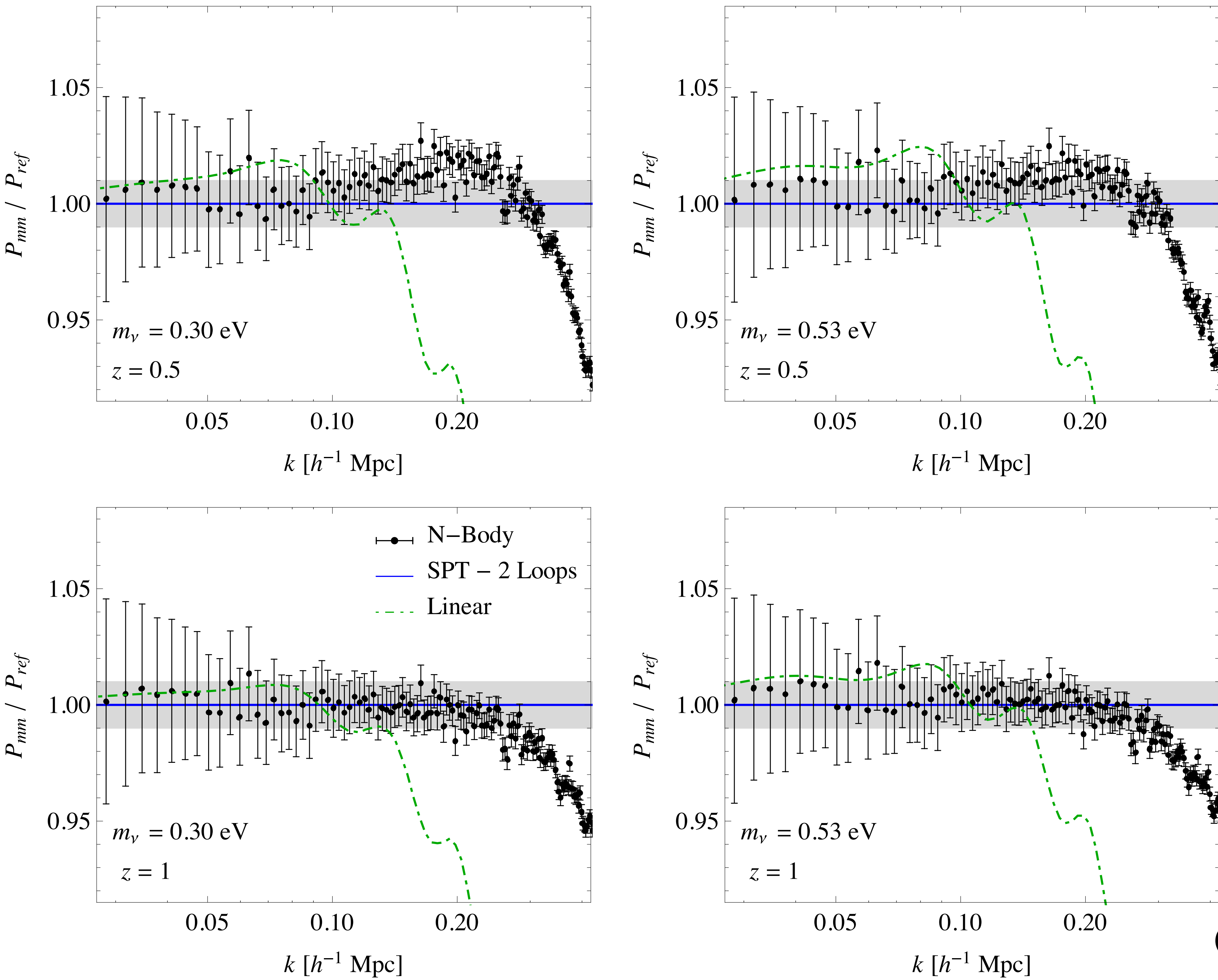


DEMNUni

CDM power spectrum

Castorina et al. ('15)

Simulation vs PT



DEMNUni

matter power spectrum

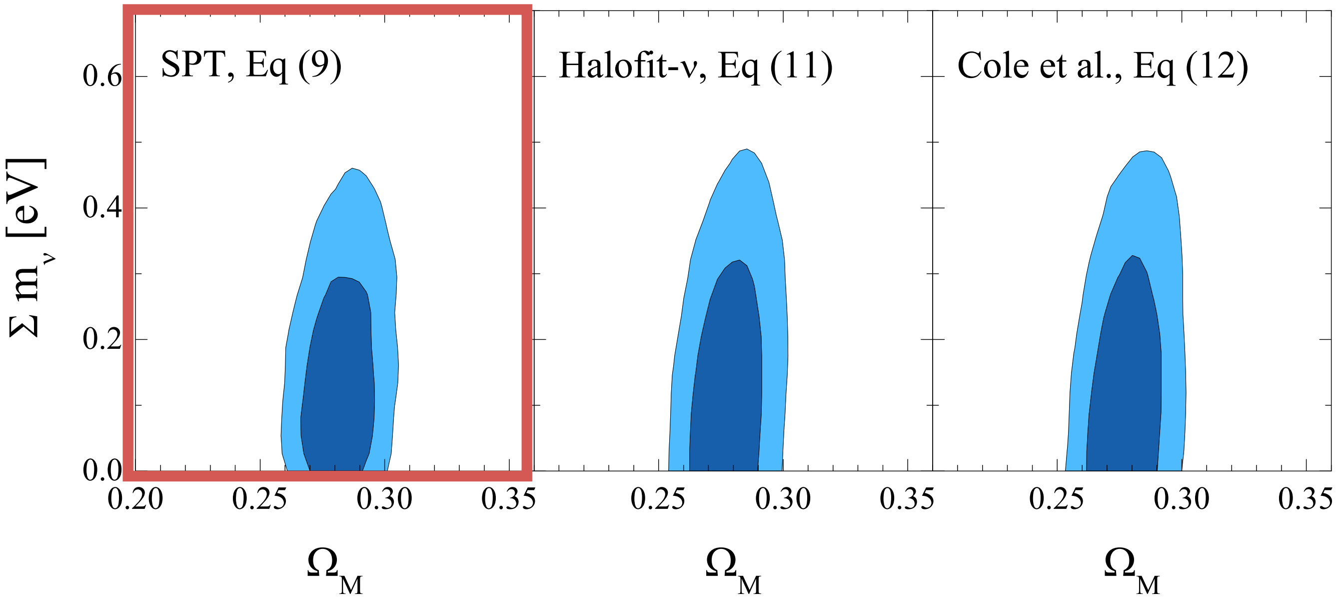
$$P_{mm}^{PT}(k) = (1-f_\nu)^2\, P_{cc}^{PT}(k) + 2\,(1-f_\nu)\, f_\nu\, P_{c\nu}^{L}(k) + f_\nu^2\, P_{\nu\nu}^{L}(k)$$

Castorina et al. ('15)

Application to observations

BOSS DR9

flat νΛCDM



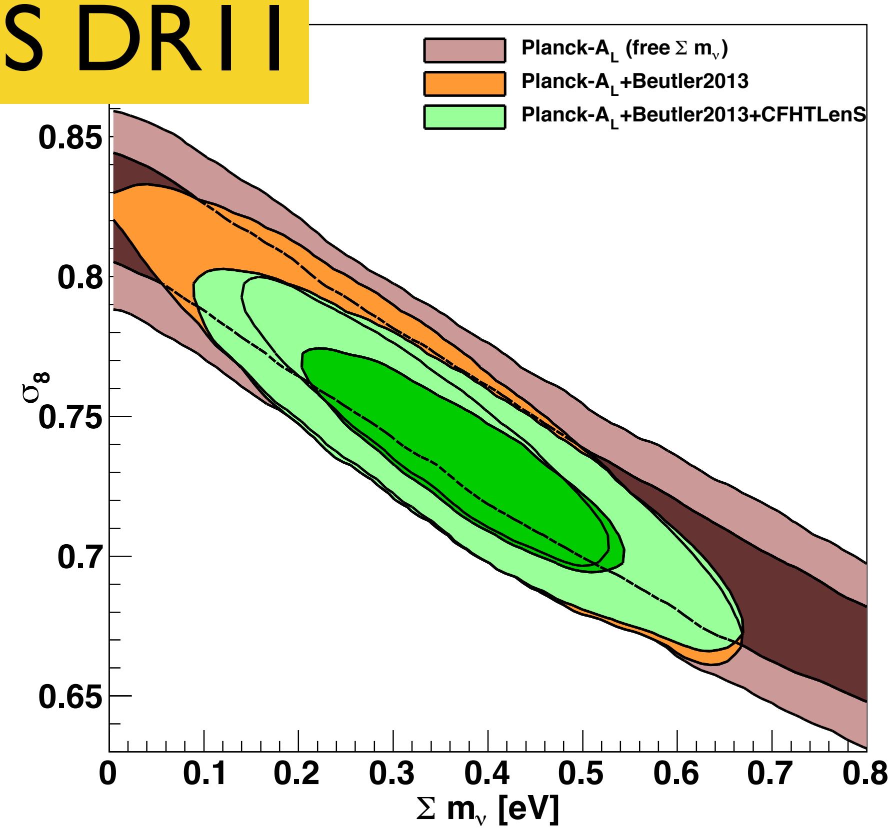
Standard PT 1-loop + McDonald bias

k_max=0.1h/Mpc

$$\sum m_\nu < 0.340\,\mathrm{eV} \;\; (95\%\mathrm{CL})$$

Zhao, Saito et al. ('13)

BOSS DR11



RegPT 2-loop + TNS + non-local bias

k_max=0.2h/Mpc

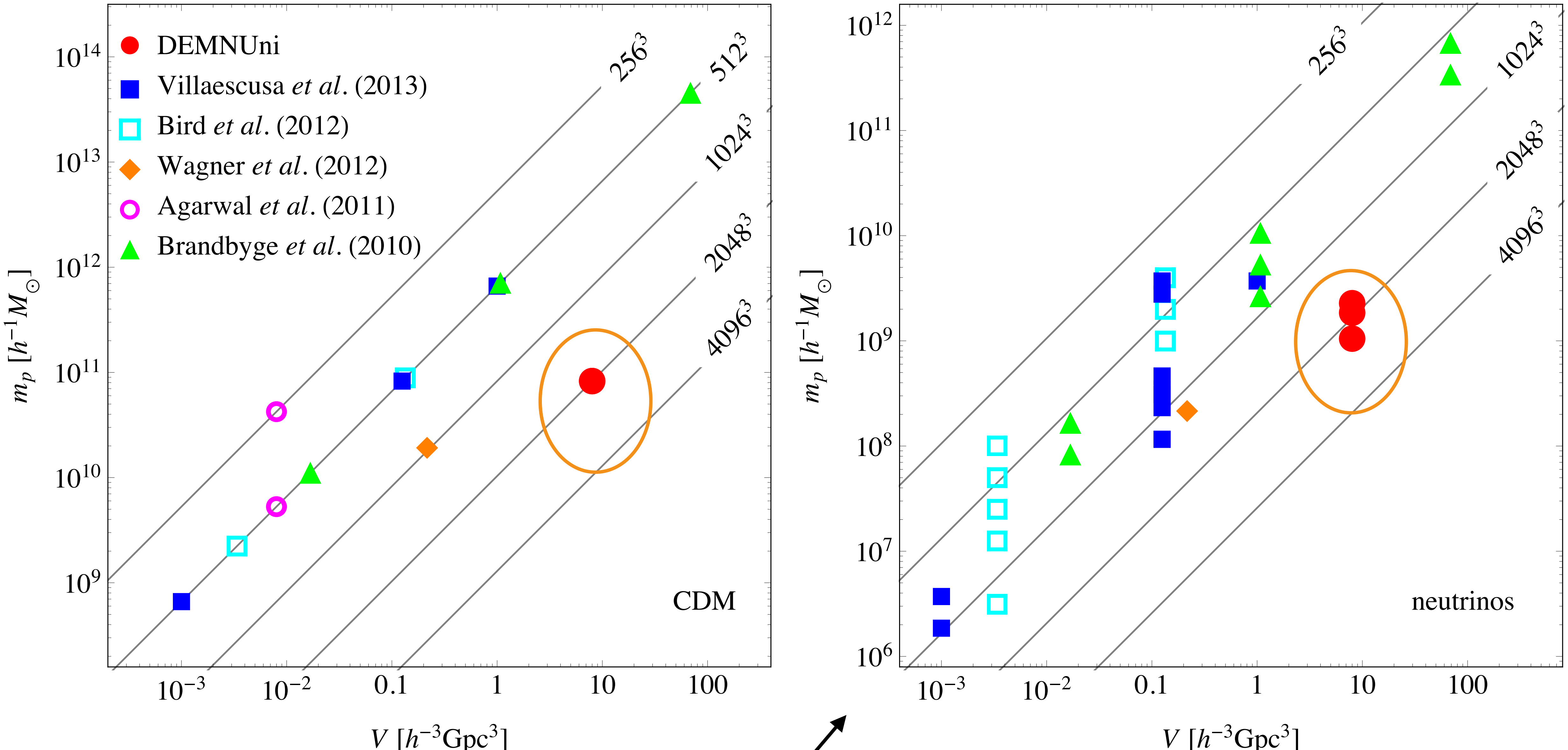
$$\sum m_\nu = 0.34 \pm 0.14\,\mathrm{eV} \;\; (68\%\mathrm{CL})$$

Sign of massive neutrino ?

Beutler, Saito et al. ('14)

Resolution of simulations

Castorina et al. ('15)

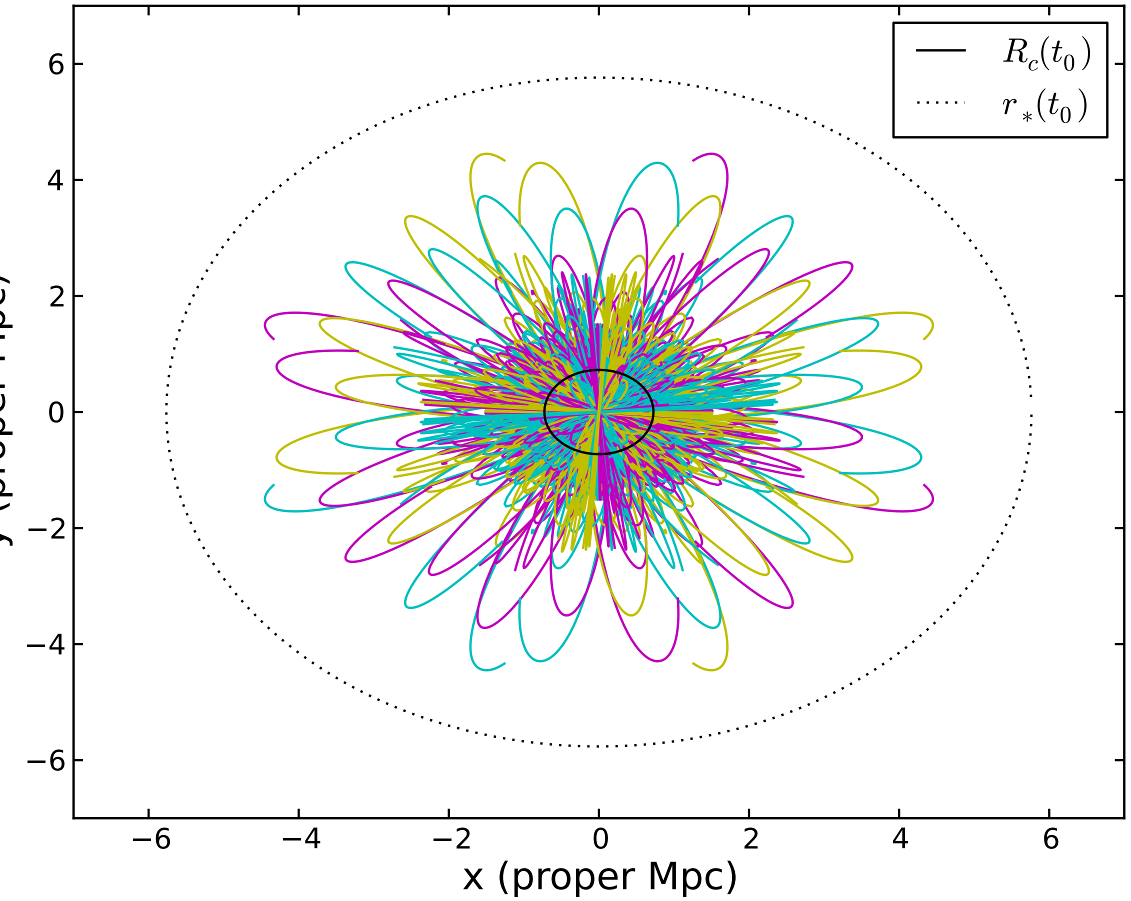


Neutrinos are still described with large-size macro particles

N-one-body simulation

Full-Boltzmann simulation, but ignoring self-gravity of neutrinos

(e.g., Ringwald & Wong '04)



For a spherical top-hat CDM halo,

$$\delta M_\nu(<r_*, t_0) \approx \sum_i (3.4 \times 10^9 M_\odot) \left(\frac{m_{\nu i}}{0.05\ \mathrm{eV}}\right)^{2.6} \left(\frac{M}{10^{14} M_\odot}\right)^{1.5}$$

$$\delta M_\nu(<r_*, t_0)|_{\mathrm{bound}} \approx \sum_i (1.2 \times 10^8 M_\odot) \left(\frac{m_{\nu i}}{0.05\ \mathrm{eV}}\right)^{3.8} \left(\frac{M}{10^{14} M_\odot}\right)^{1.9}$$

LoVerde & Zaldarriaga ('14)

… comparable to the mass of neutrino's macro particle

Current simulation with massive ν does not have sufficient resolution to properly describe the neutrino clustering

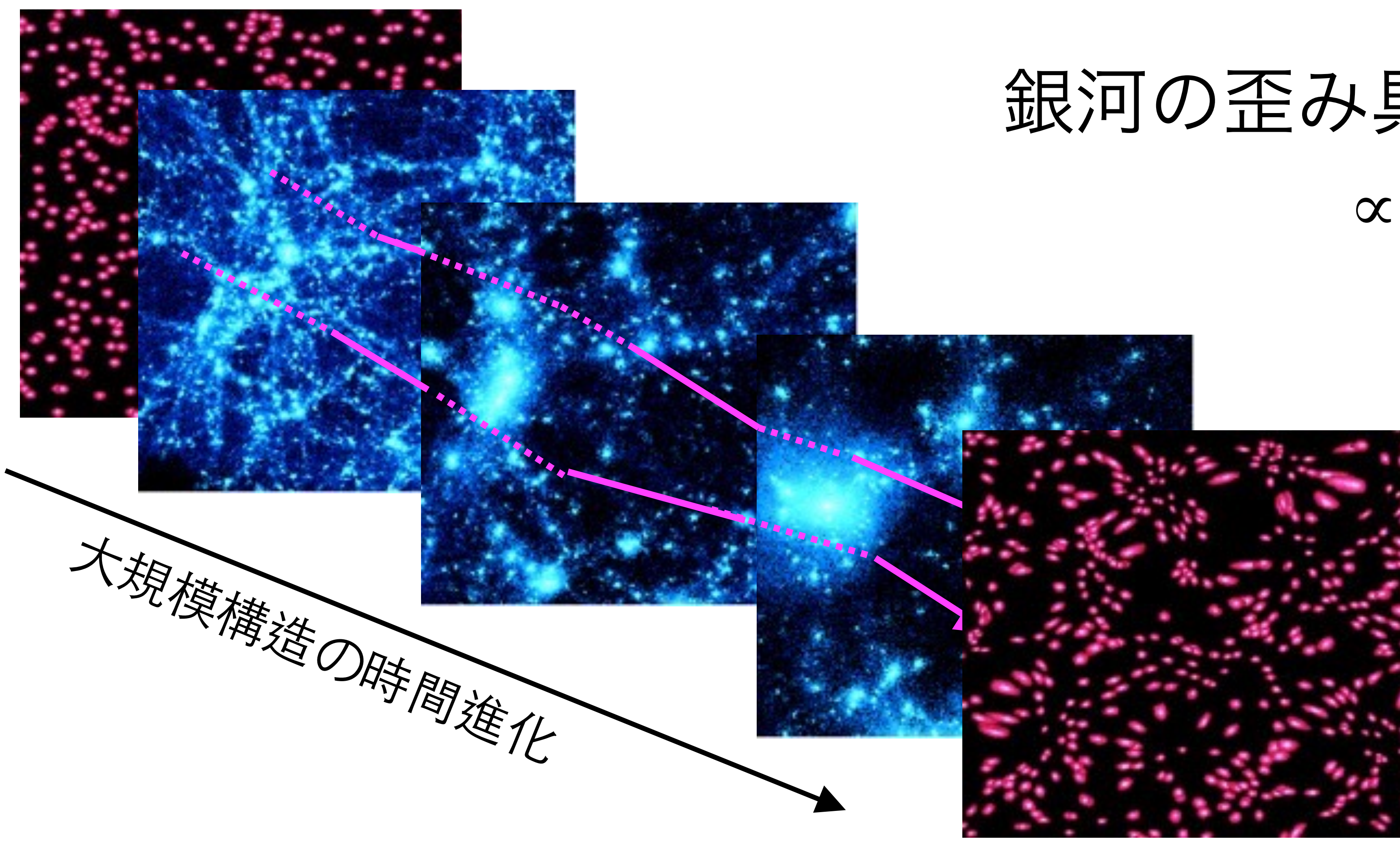
重力レンズ効果における非線形性の低減

Ref.

Bernardeau, Nishimichi & Taruya, MNRAS 445, 1526 ('14)

Cosmic shear

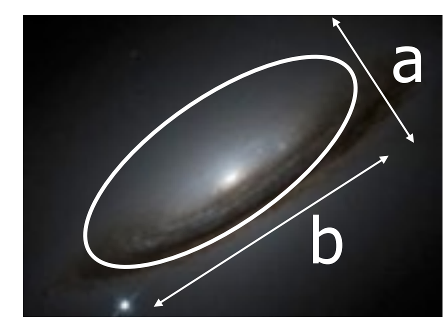
手前に存在する宇宙大規模構造が作る（弱い）重力レンズ効果により、遠方の背景銀河のイメージが歪む現象



銀河の歪み具合（楕円率）

∝ 幾何学的重み

×密度ゆらぎの振幅



$$\gamma = \frac{a-b}{a+b}$$

イメージの歪みの空間相関から、宇宙大規模構造のもつ宇宙論的情報を引き出せる → 精密宇宙論の基本観測量

Cosmic shear statistics : theory

E-mode cosmic shear

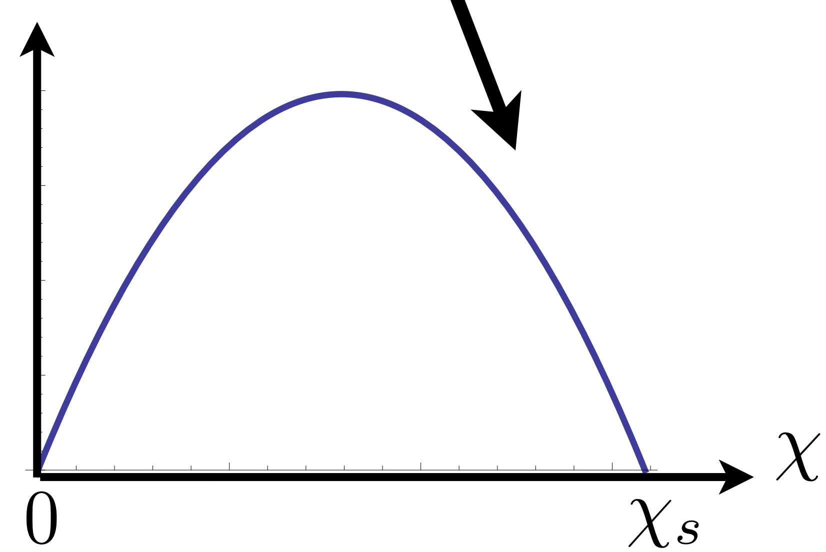
"Convergence field" を考える：　　　　(平坦宇宙の場合)

$$\kappa(\vec{\theta}) = \frac{3}{2}\Omega_{\rm m}\frac{H_0^2}{c^2}\int_0^{\chi_\infty} d\chi_s\, n(\chi_s)\int_0^{\chi_s} d\chi\, \frac{\chi(\chi_s-\chi)}{\chi_s}\,\frac{\delta(\vec{\theta},\chi)}{a(\chi)}$$

背景銀河の分布　　レンズカーネル　　質量分布（大規模構造）

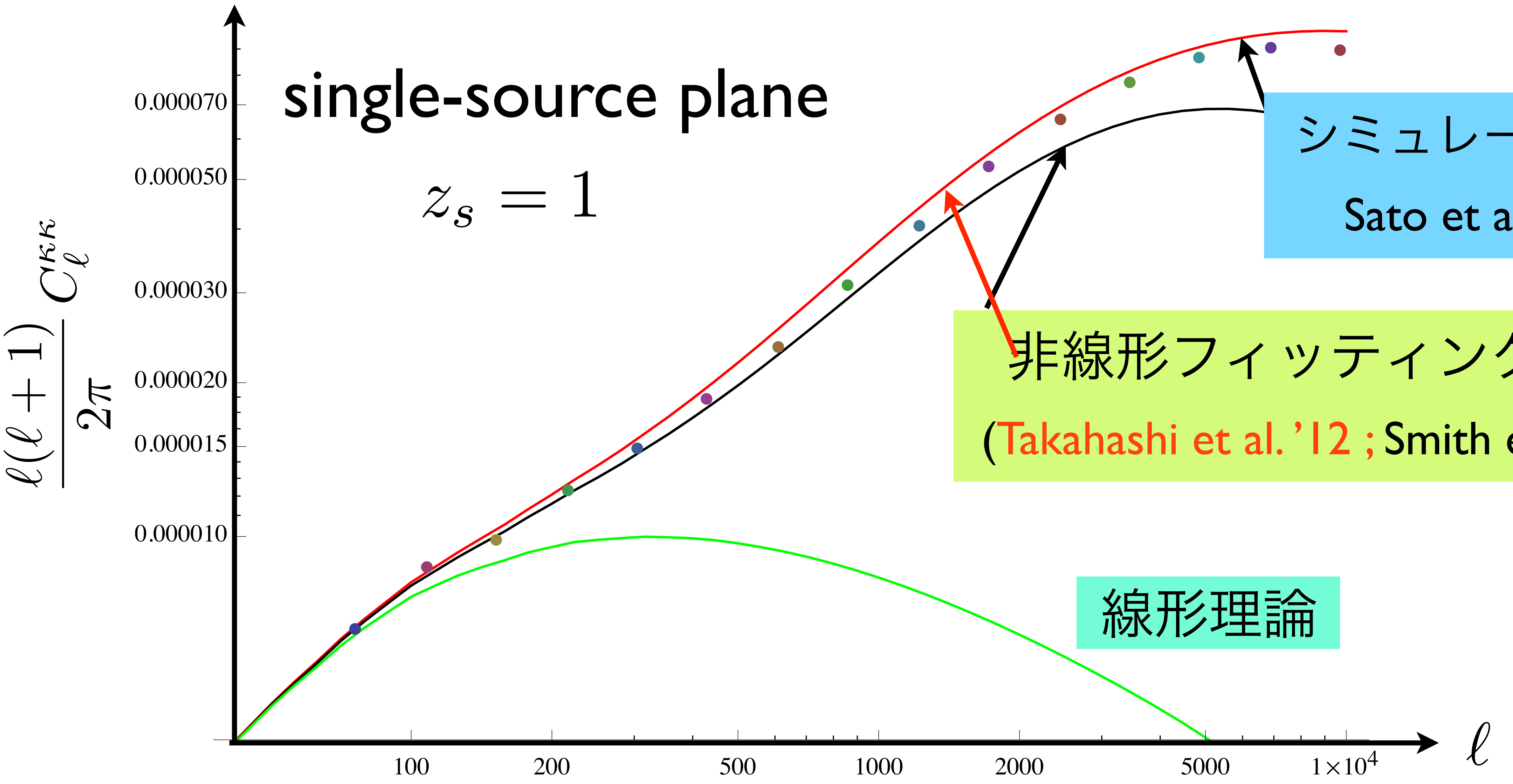
（共動）動径距離：

$$\chi(z) = \int_0^z \frac{c\,dz}{H(z)}$$



いろんな赤方偏移からの寄与が混じる (projection effect)

Cosmic shear power spectrum



線形理論からのずれが顕著

Impact of small-scale nonlinearity

角度パワースペクトル（Limber近似）：

$$C_\ell^{\kappa\kappa} = \int_0^{\chi_\infty} \frac{d\chi}{\chi^2} \{g(\chi)\}^2 P_{\delta\delta}\left(k = \frac{\ell+1/2}{\chi}; z(\chi)\right);$$

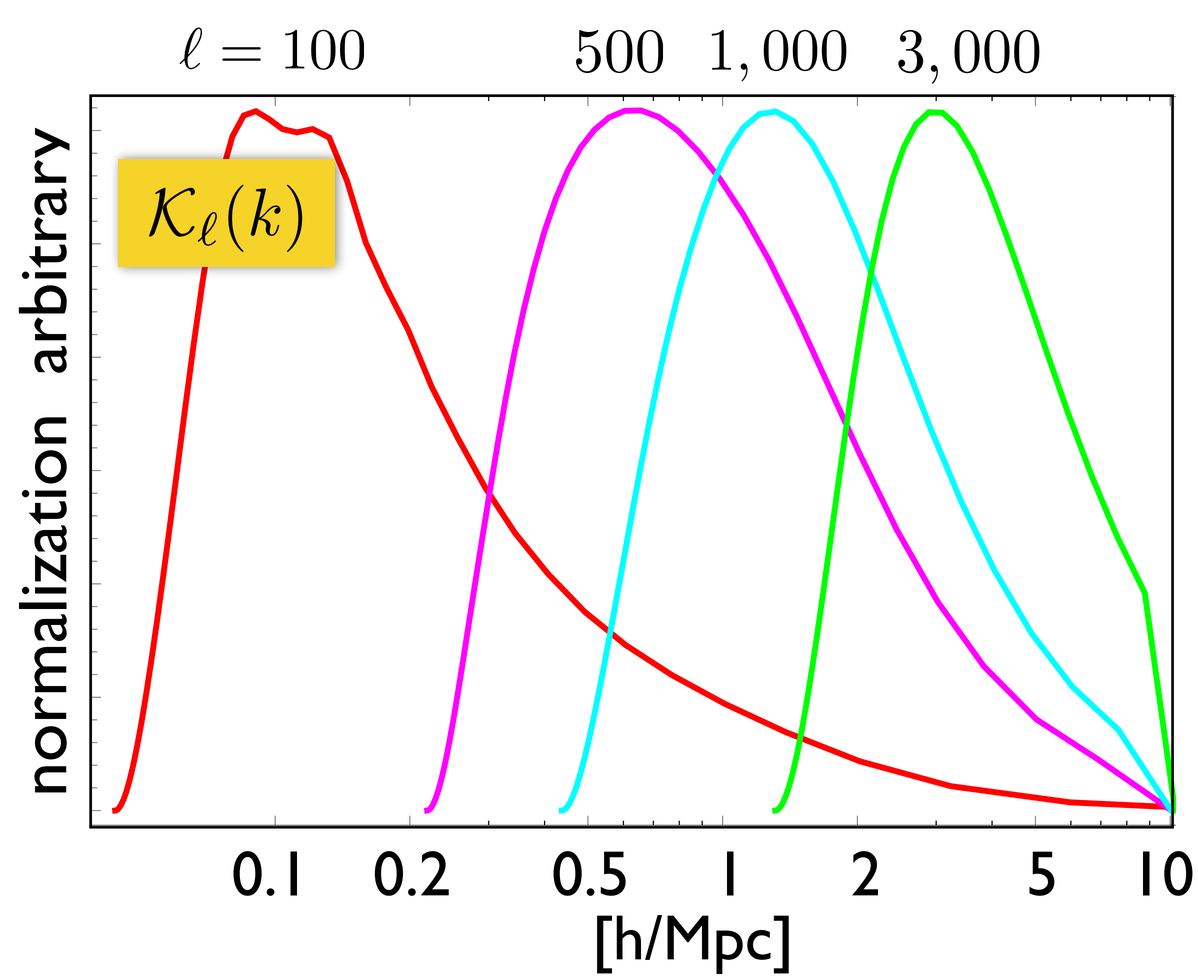
レンズカーネル　質量パワースペクトル

$$g(\chi) = \frac{3}{2}\frac{\Omega_{\rm m} H_0^2}{a(\chi)} \times \int_\chi^\infty d\chi_s \frac{(\chi_s-\chi)\chi}{\chi_s} n(\chi_s)$$

$$= \int_0^{\chi_\infty} d\chi\, \mathcal{K}_\ell(\ell/\chi)$$

single-source plane

$$z_s = 1$$



Motivation

小スケールからの寄与が大きすぎる!!

- 摂動論をこえる重力進化の理論モデルが不可欠

  高精度予言が困難（e.g., フィッティング公式）

- バリオン物理の影響（物理的不定性）

  宇宙論パラメーター推定をバイアス

ここでの話

小スケールからの非線形性を何とか低減して、
摂動論レベルの理論予言でパワースペクトルを記述できないか？

➡ その方法論の開発

Nulling low-z contribution

基本的なアイデア

適当な「重み関数」をかけてレンズカーネルの形を変える

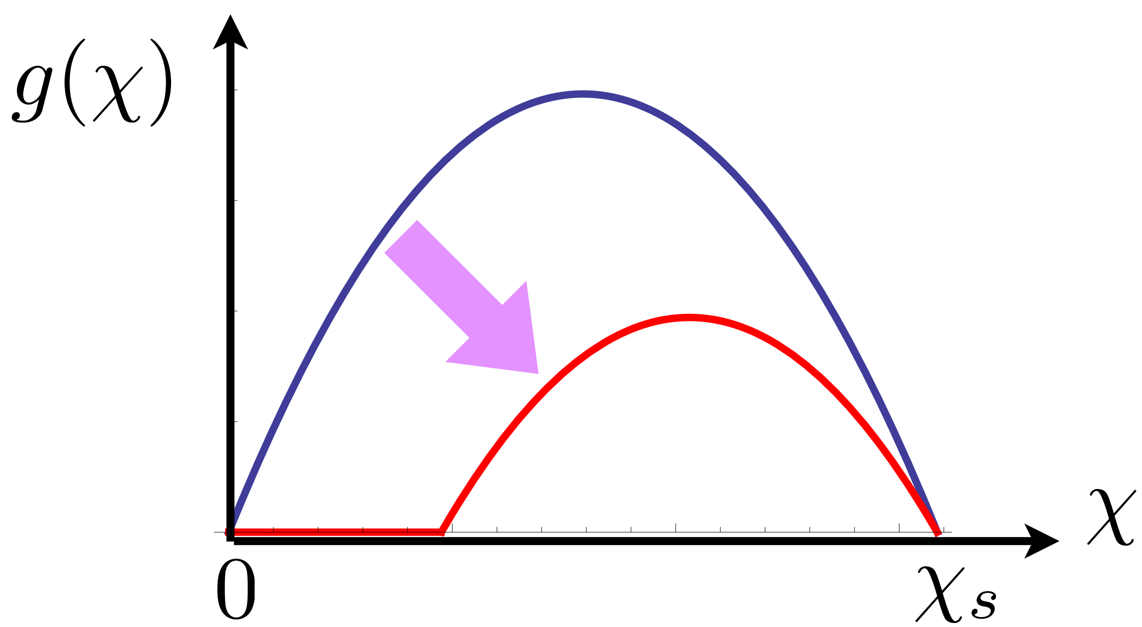
（背景銀河の赤方偏移はわかっているとして）

$$C_\ell^{\kappa\kappa} = \int_0^{\chi_\infty} \frac{d\chi}{\chi^2} \{g(\chi)\}^2 P_{\delta\delta}\left(k = \frac{\ell + 1/2}{\chi}; z(\chi)\right) ;$$

レンズカーネル

$$g(\chi) = \frac{3}{2}\frac{\Omega_{\rm m} H_0^2}{a(\chi)} \int_\chi^\infty d\chi_s \frac{(\chi_s - \chi)\chi}{\chi_s} w(\chi_s) n(\chi_s)$$

重み関数



low-z からの寄与を低減

Simplified setup (1)

3 source-plane solution

背景銀河が離散的に、$\chi_s = \chi_i \ (i = 1, 2, 3)$ の赤方偏移面にいる場合

レンズカーネル

$$g(\chi) \longrightarrow \frac{3}{2}\frac{\Omega_{\mathrm{m}} H_0^2}{a(\chi)} \sum_{i=1}^{3} \frac{(\chi_i - \chi)\chi}{\chi_i} w_i \Theta(\chi_i - \chi)$$

ステップ関数

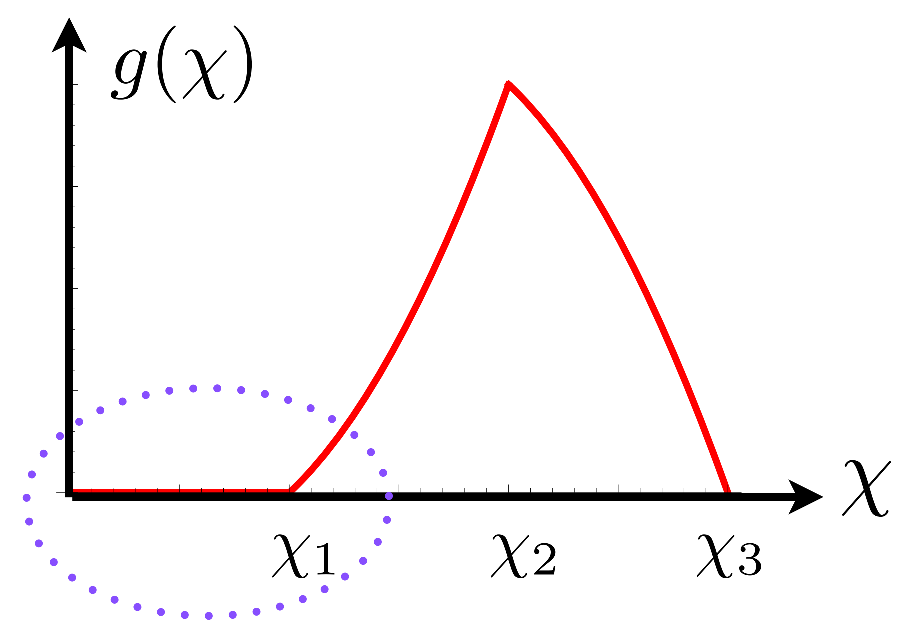
この時、

$$\begin{cases} w_1 = (\chi_3 - \chi_2)\chi_1 \\ w_2 = (\chi_1 - \chi_3)\chi_2 \\ w_3 = (\chi_2 - \chi_1)\chi_3 \end{cases}$$

と選ぶと

$$g(\chi) = 0 \quad \text{at} \quad 0 \le \chi \le \chi_1$$

nulling low-z signal



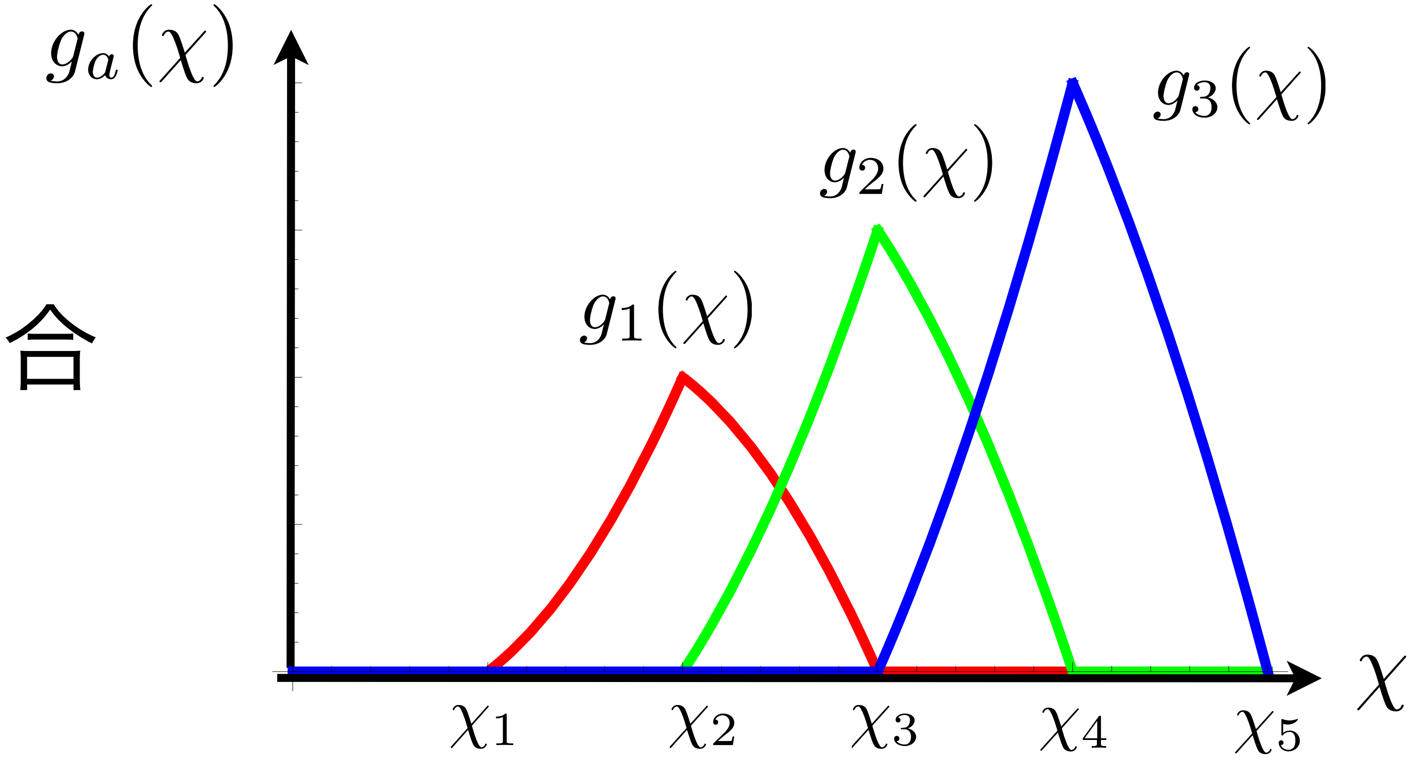
Simplified setup (2)

Multiple source-plane solution

背景銀河の赤方偏移面が3枚以上ある場合　　$\chi_s = \chi_i \ (i = 1, \cdots, n)$

いろいろな nulling low-z signals が作れる

5枚の場合



$$C_\ell^{(ab)} = \int_0^{\chi_\infty} \frac{d\chi}{\chi^2}\, g_a(\chi) g_b(\chi)\, P_{\delta\delta}\left(k = \frac{\ell + 1/2}{\chi}; z(\chi)\right)$$

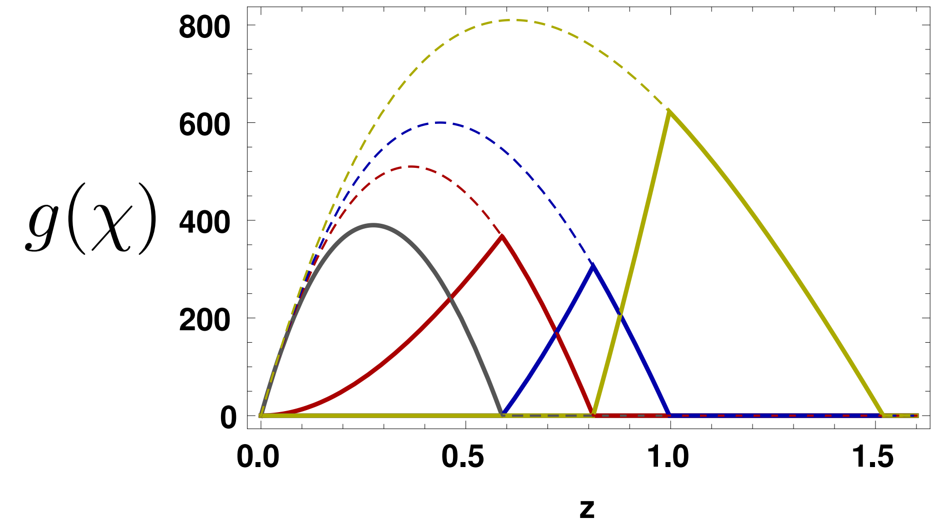
このうち、

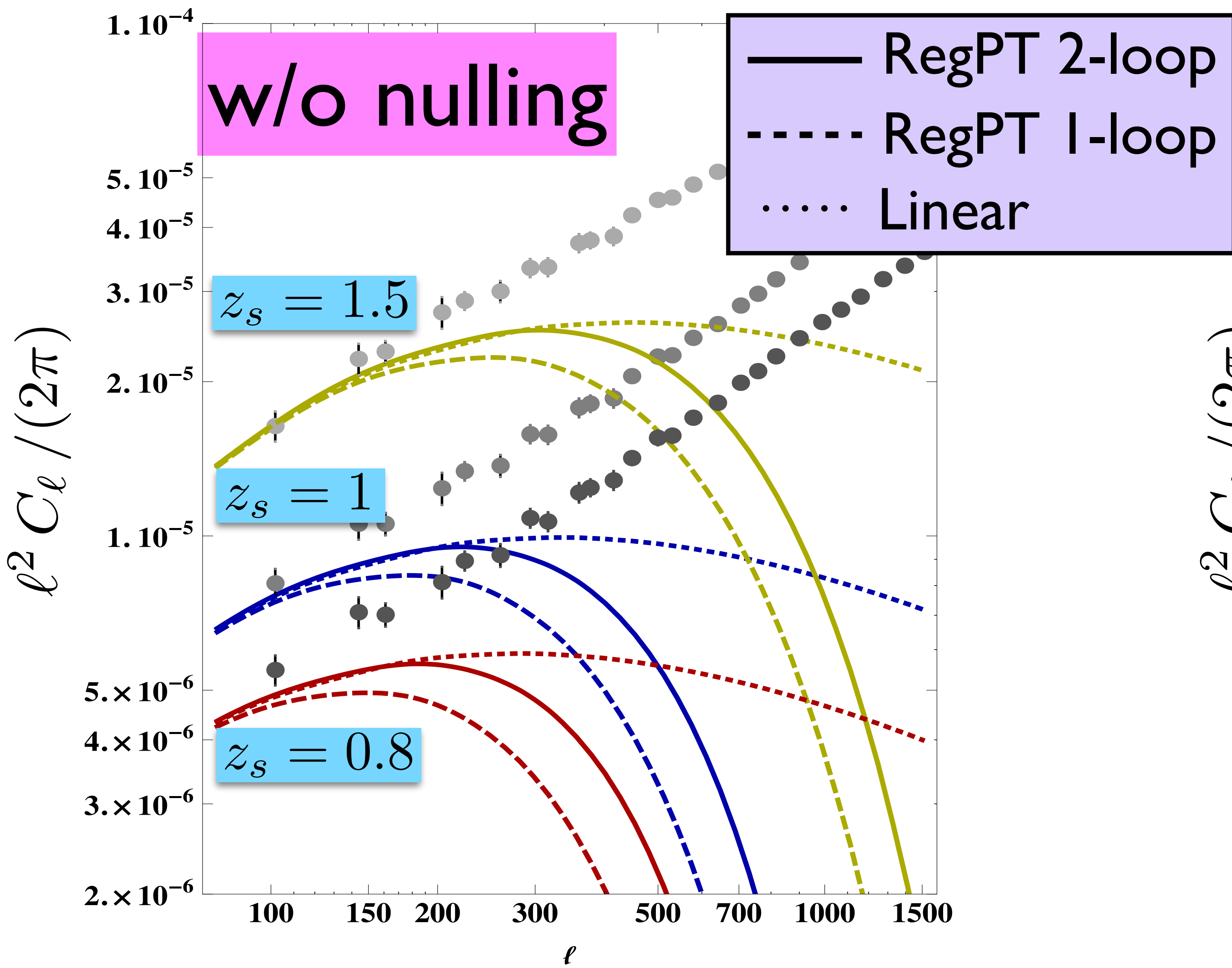
$$C_\ell^{(13)} = 0$$

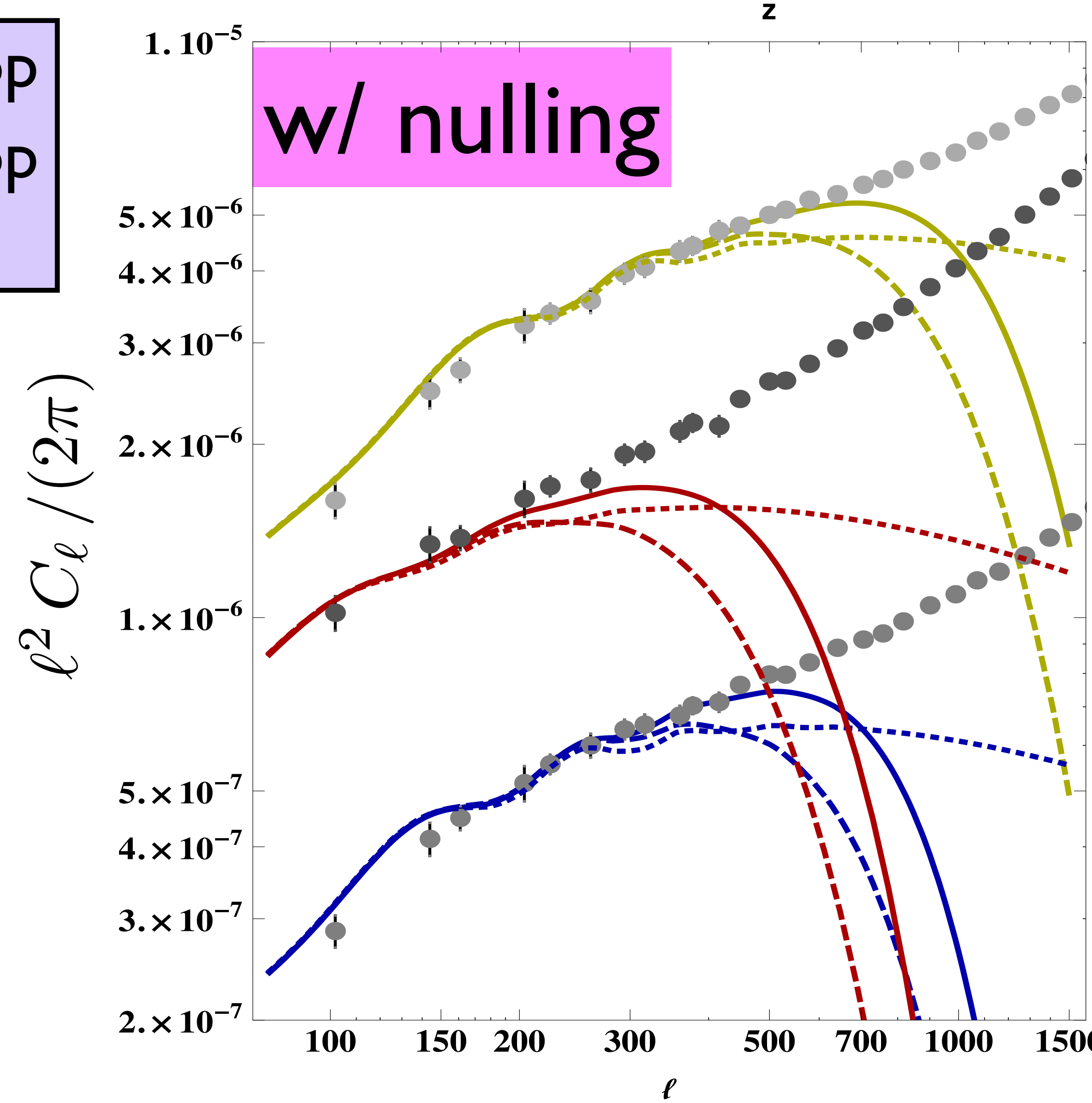
Simulations vs. Perturbation theory

小スケール（large-ℓ）まで
摂動論の適用範囲が広がった

シミュレーション: Sato et al. ('09)







Realistic setup (1)

1. 背景銀河の赤方偏移分布は ‘連続的’

2. 背景銀河の赤方偏移には誤差が含まれる（photo-z error）

1. 背景銀河分布 $n(\chi_s)$ に対して、

$g(\chi) = 0$ **at** $\chi \le \chi_1, \ \chi_2 \le \chi$

$$\begin{cases} \displaystyle\int_{\chi_1}^{\chi_2} d\chi_s \, w(\chi_s) \, n(\chi_s) = 0 \\ \displaystyle\int_{\chi_1}^{\chi_2} d\chi_s \, \frac{w(\chi_s)}{\chi_s} \, n(\chi_s) = 0 \\ w(\chi_s) = 0 \text{ at } \chi \le \chi_1, \ \chi_2 \le \chi_2 \end{cases}$$

ただしこの条件だけではユニークに決まらない：

付加的
拘束条件

$$\frac{\int_{\chi_1}^{\chi_2} d\chi_s \, w(\chi_s) n(\chi_s) \, \chi_s^2}{\left[\int_{\chi_1}^{\chi_2} d\chi_s \, w^2(\chi_s) n(\chi_s)\right]^{1/2}}$$

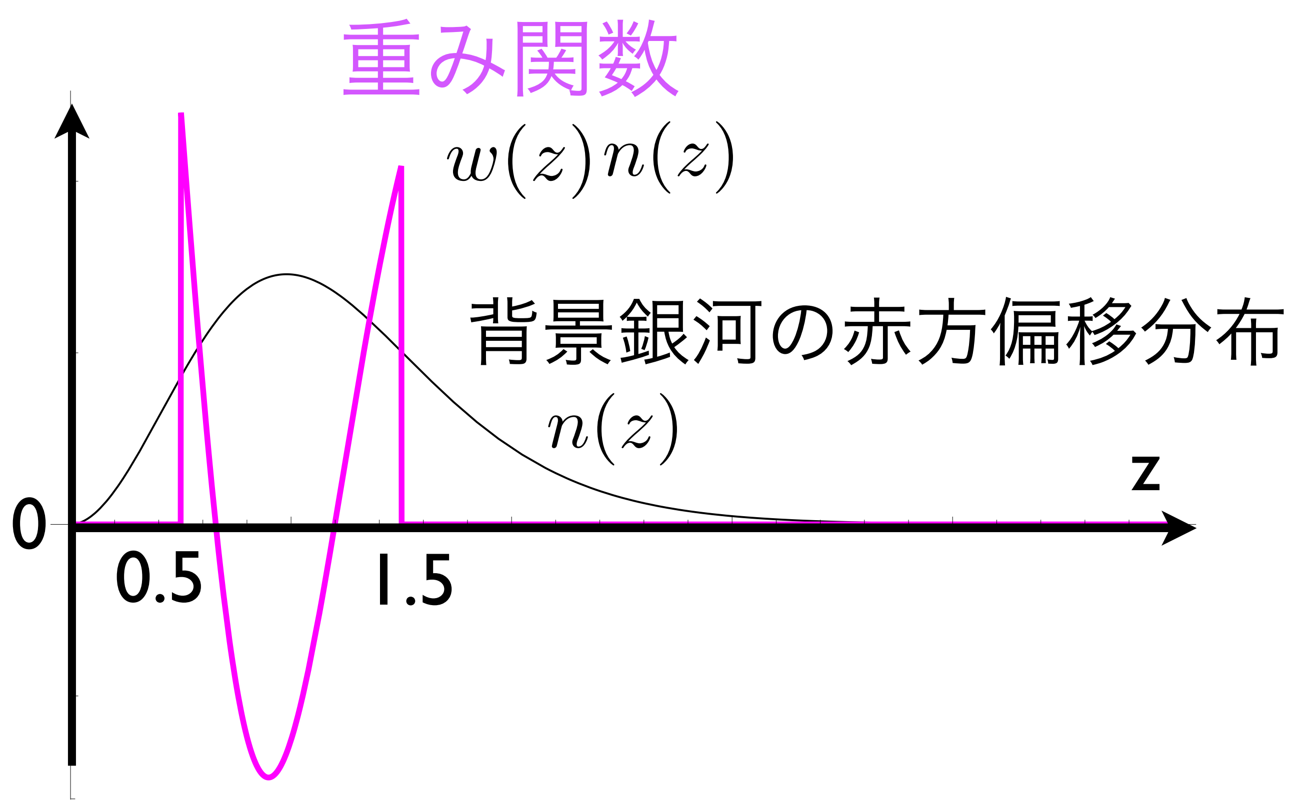
を最大化

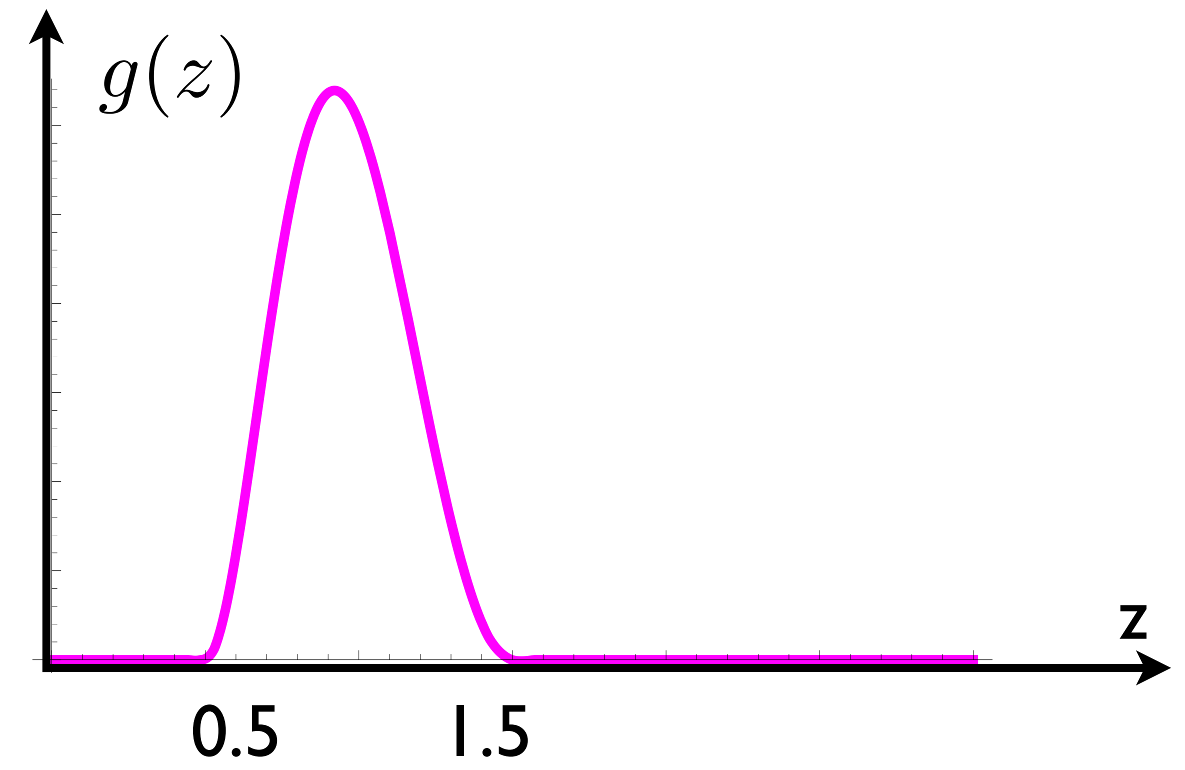
Realistic setup (2)

重み関数の解析的表式

$$w(\chi) = p_2(\chi) - \frac{\int_{\chi_1}^{\chi_2} d\chi\, p_2(\chi) n(\chi)}{\int_{\chi_1}^{\chi_2} d\chi\, p_{-1}^2(\chi) n(\chi)}\, p_{-1}(\chi) \,; \quad p_\alpha(\chi) = \chi^\alpha - \frac{\int_{\chi_1}^{\chi_2} d\chi\, \chi^\alpha n(\chi)}{\int_{\chi_1}^{\chi_2} d\chi\, n(\chi)}$$

例



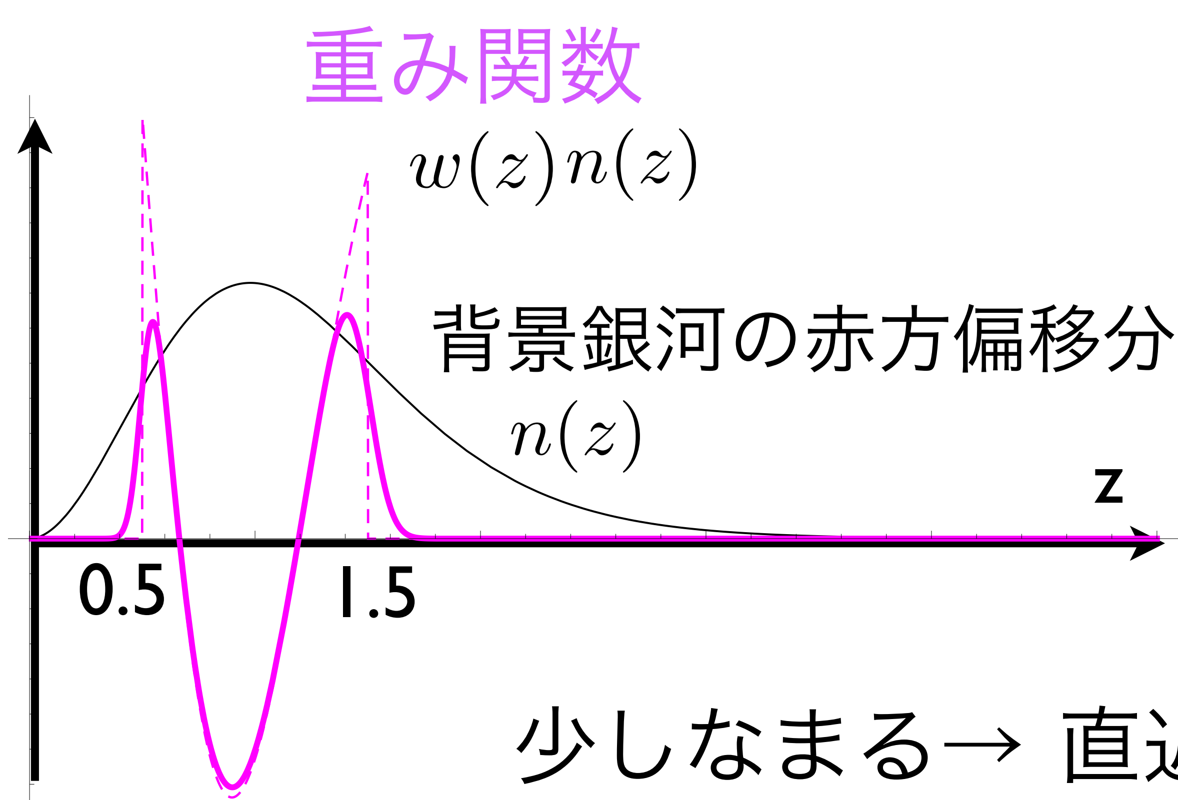


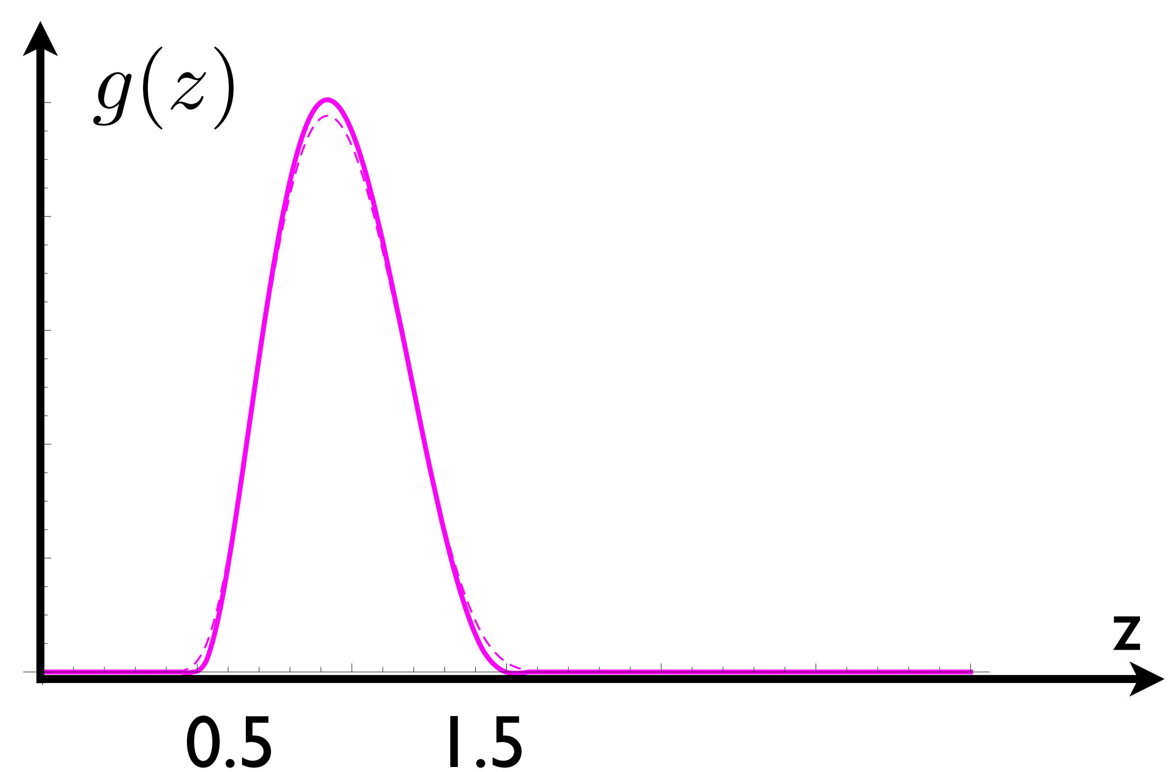
Realistic setup (3)

Photo-z error の影響

先ほど求めた重み関数に photo-z errorの影響を考慮
（ガウス分布を仮定して畳み込み）

photo-z errorの分散: $\sigma_z(z) = \sigma_0\,(1+z)\ ;\quad \sigma_0 : \mathrm{const.}$





少しなまる→ 直近の赤方偏移ビンへシグナルが混入

Impact on lensing tomography

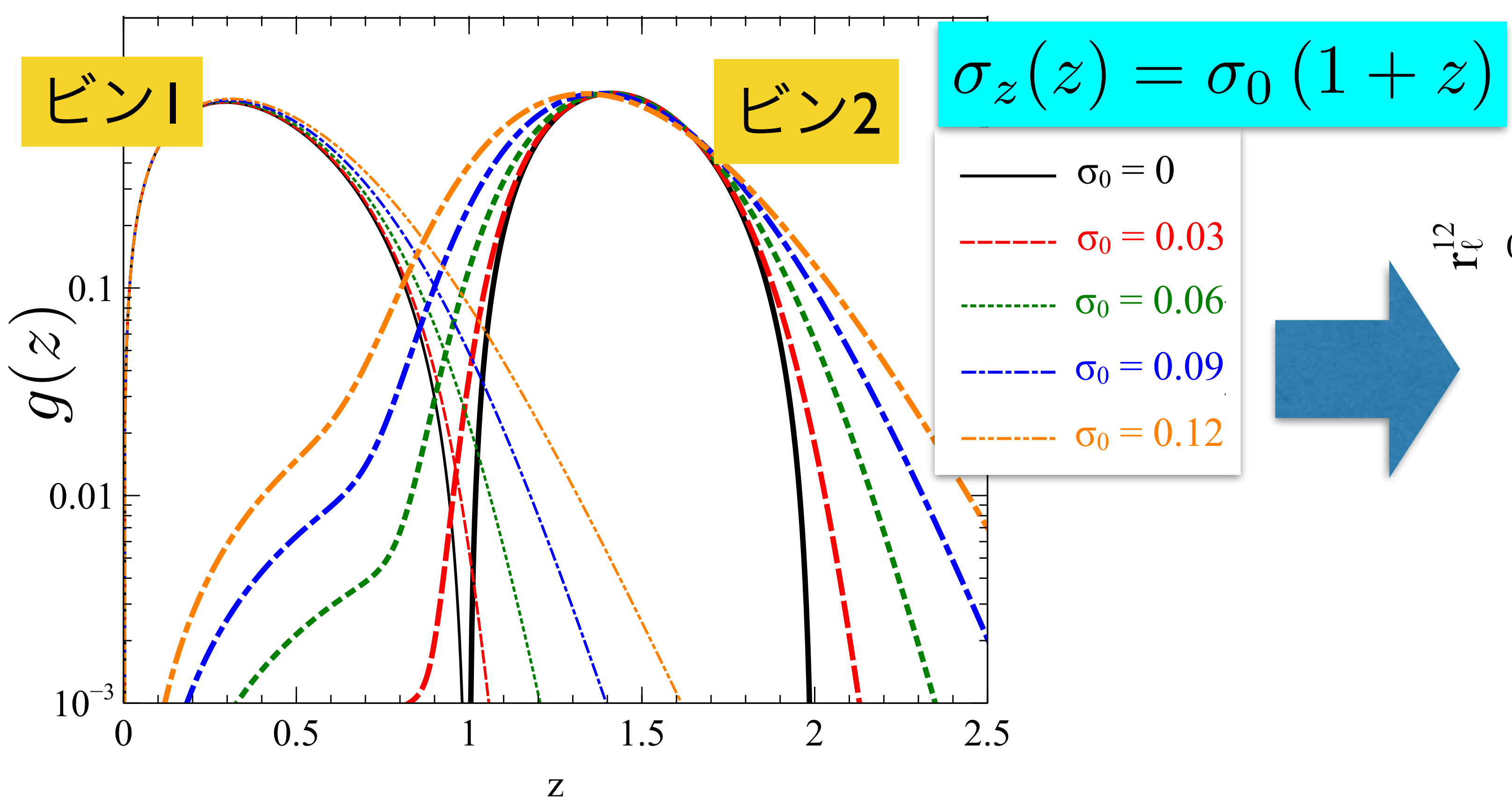
Photo-z error による 隣り合うビン同士の spurious な相関

→ 重力レンズトモグラフィーへの影響

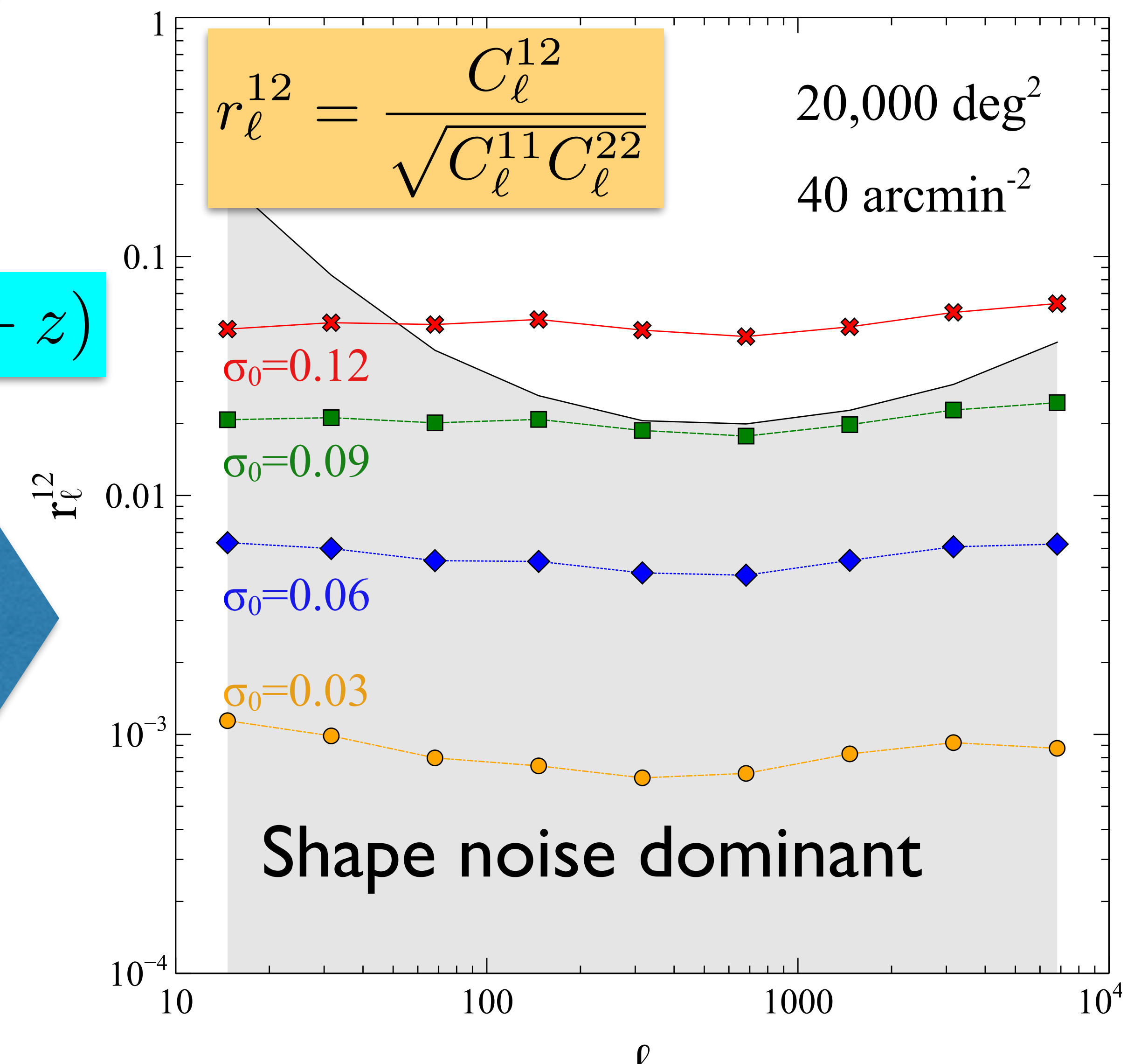
背景銀河の赤方偏移分布 $n(z) \propto \left(\frac{z}{z_0}\right)^2 e^{-(z/z_0)^{3/2}}$; $z_0 = 0.8$

ビン1（0<z<1）：一様な重み

ビン2（1<z<2）： nulling profile を適用



ビン1・2間の相関係数



Impact of wrong cosmological prior

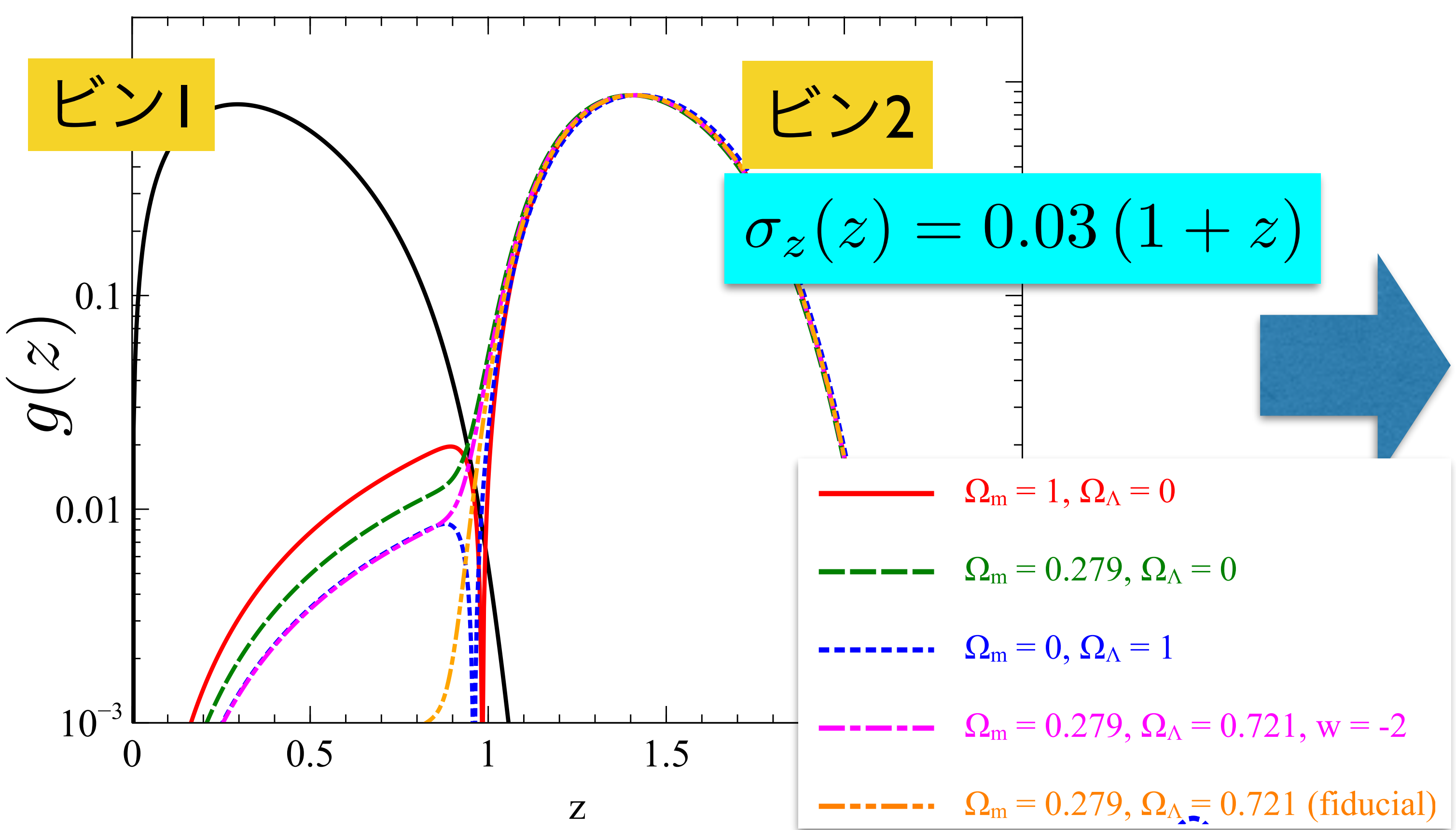
Nulling profile 構築には、宇宙論モデルを事前に知る必要あり

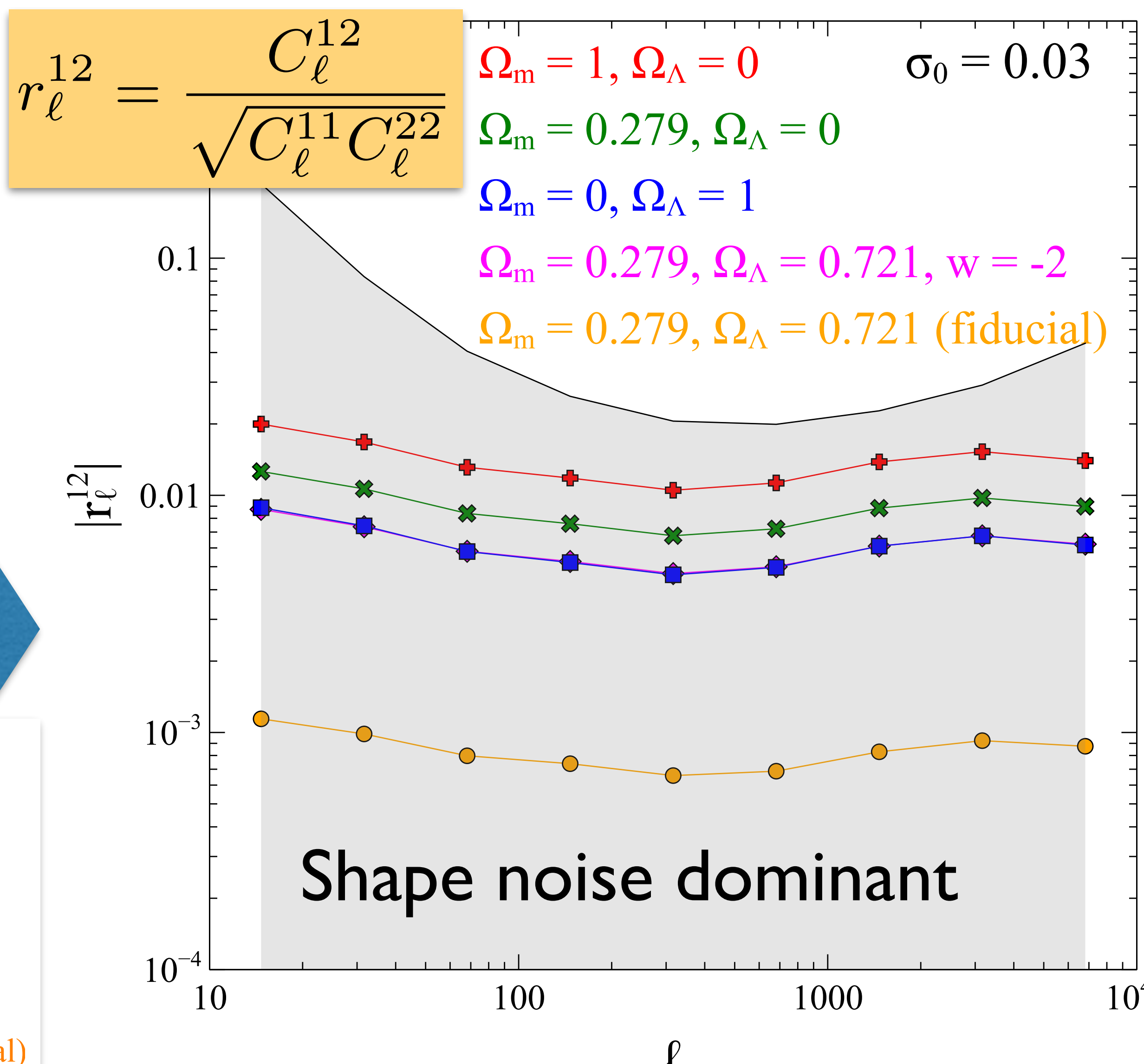
→ 宇宙論を間違うと パーフェクトな "nulling" ができない

→ 隣り合うビン同士の spurious な相関

ビン1（0<z<1）: 一様な重み

ビン2（1<z<2）: nulling profile を適用





Summary

重力レンズ観測における、Nulling テクニックの開発

- 小スケールの非線形性の影響を低減する
- 背景銀河のphoto-z に対して、"重み"をかけるだけ

重み関数

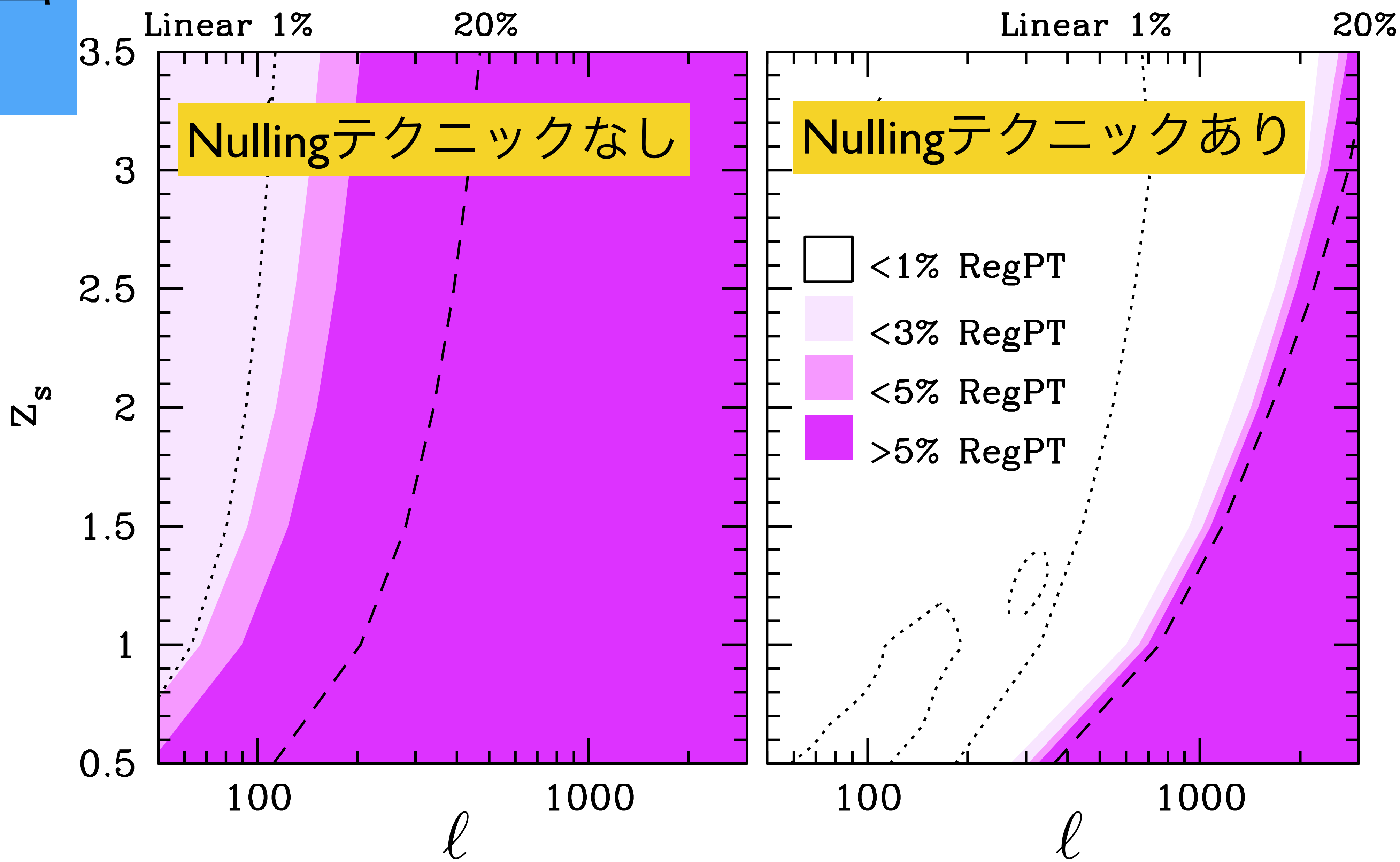
$$w(\chi) = p_2(\chi) - \frac{\int_{\chi_1}^{\chi_2} d\chi\, p_2(\chi) n(\chi)}{\int_{\chi_1}^{\chi_2} d\chi\, p_{-1}^2(\chi) n(\chi)}\, p_{-1}(\chi)\ ;\quad p_\alpha(\chi) = \chi^\alpha - \frac{\int_{\chi_1}^{\chi_2} d\chi\, \chi^\alpha n(\chi)}{\int_{\chi_1}^{\chi_2} d\chi\, n(\chi)}$$

Impact of photo-z error / wrong cosmological prior
on nulling lensing technique ………… 影響小さい

⇒ 摂動論を用いた理論テンプレートの適用範囲を広げる

Applicable range of perturbation theory

摂動論の適用範囲
チャート



RegPT: 摂動論パブリックコード (AT, Bernardeau, Nishimichi & Codis '13)

http://www2.yukawa.kyoto-u.ac.jp/~atsushi.taruya/regpt_code.html