Nonlinear structure formation

Contents

- Spherical collapse model
- Zel'dovich approximation
- Eulerian Perturbation theory

Spherical collapse model (SCM)

Halo formation

Beyond spherical collapse model (SCM)

SCM only describes the onset of halo formation

In particular, SCM fails to describe phase-space structure of halo (continuous matter accretion)

Tractable analytic Self-Similar collanse e.g., treatment

Self-Similar collapse

Fillmore & Goldreich ('84) Bertschinger ('85)

Self-similar collapse *M*(*r,t*) = *MtM*(*r/rt*)*,* (5.41) where *M* is a dimensionless mass profile. Since *M*(*r,t*) contains all the mass in the mass shells with current radii smaller than *r*(*r*i*,t*), we have for *r < rt* that $$ *H* & where *H* (*x*) is the Heaviside step function: *H* (*x*) = 1 for *x* ≥ 0 and *H* (*x*) = 0 otherwise. In terms of the dimensionless variables, \sim

∎
Fillmc *r*(*r*i*,t*)−*r*(*r* ⁱ*,t*) Fillmore & Goldreich ('84)

- Einstein-de Sitter background, a(t) [«] t^{2/3} $\sum_{i=1}^{n}$ δ itter background, $a(t) \propto t^{2/3}$ \sim the dimensionless variables, \sim the dimensionless variables, \sim $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ (f) $\frac{1}{2}$ (7)
	- Scale-free initial density perturbation, $\delta_{\rm i} \propto M_{\rm i}^{-\epsilon}$ • Scale-free initial density perturbation, $\delta_i \propto M^{-\epsilon}$ *^M*(*r/rt*) = ¹ ale-f \mathbf{A} initial density perturbation λ , α , $M^{+\epsilon}$ • Scale-free initial density perturbation, $\delta_{\rm i} \propto M_{\rm i}^{-\epsilon}$ where *r*max and *t*max are the radius and time at the *first* apocenter, respectively, the equation of
- •Motion of continuously infall shells at は自己相似的であり- その球殻の転回点半径 ֏ \mathbf{L} • Motion of continuously infall shells at $r < r_\ast$ \bullet Motion of continuously infall shalls at $r < r$ terms of the dimensionless variables, ontinuously infall shells at $r < r_*$ motion, Eq. (5.36), can be written as dτ² ا² at $\frac{1}{2}$ $\left\langle \right\rangle$

turn-around radius *,* (5.44)

Self-similar
\n
$$
r(t, t_*) = r_*(t_*) \lambda(t/t_*)
$$
\n
$$
\Delta(\tau) = \tau^{2/3+2/9\varepsilon}
$$
\n
$$
\frac{d^2\lambda}{d\tau^2} = -\frac{\pi^2}{8} \frac{\tau^{2/3\varepsilon}}{\lambda^2} \mathcal{M}\left[\frac{\lambda}{\Lambda(\tau)}\right]; \quad \mathcal{M}(x) = \frac{2}{3\varepsilon} \int_1^\infty \frac{dy}{y^{1+2/3\varepsilon}} \mathcal{H}\left[x - \frac{\lambda(y)}{\Lambda(y)}\right]
$$

ɱ - 2010 ዓ.ም. 2010 ዓ.ም. 2010 ዓ.ም. 2010 ዓ.ም. 2010 ዓ.ም. 2010 ዓ.ም. 2010 ዓ.ም. 2010 ዓ.ም. 2010 ዓ.ም. 2010 ዓ.ም. 2010 ዓ.ም
- 2010 ዓ.ም. 2010 $\tau \equiv t/t_*~$: time normalized by turn-around time $~$ Heaviside step func. *x* → *v*/ *v** → curie Hormanzer $\frac{1}{2}$ alized by turn-around time $\frac{1}{2}$ $\frac{1}{2}$ and $\frac{1}{2}$ and $\frac{1}{2}$

H

^M(*x*) = ²

Heaviside step func. and Λ(τ) = ^τ2*/*3+2*/*9^ε *.* (5.45) is independent of *r*i, and so it applies to all mass shells that have turned around before time *t*.

Mass:
$$
M(r,t) = M_t \mathcal{M}\left(\frac{\lambda}{\Lambda(t)}\right)
$$
; $M_t \propto a(t)^{1/\epsilon}$

Solutions \blacksquare \sim

mass shell

 \mathcal{L}

Tracing multi-stream flow with particle trajectories in *N*-body simulation

修論 by 杉浦宏夢

Keeping track of apocenter passage(s) for particle trajectories, number of apocenter passages, *p*, is stored for each particle

> (Diemer'17; Diemer et al.'17) $=$ SPARTA algorithm $+$ α

> > Tiling phase-space streams with *p*

N-body simulation Y. Rasera@ (Observatoire de Paris)

- \cdot L=316Mpc/h, N=512^3
- 60 snapshots at 0<z<1.43
- Einstein-de Sitter universe

Comparison with self-similar solution

(Master thesis by H. Sugiura)

Zel'dovich approximation

Lagrangian PT **1 BASIC EQUATIONS** We begin by writing down equations of motion for mass ∇*^x · x* + 2*H*^{*x*} ∇*^x* × ! *x*¨ + 2*Hx*˙ \Box ij¹ a σκαπσία η \Box ⎪⎪⎨ ⎪⎪⎩ *x* + 2*H*^{*x*} ∇*^x* × *x*¨ + 2*Hx*˙

quations to be solved with $\frac{1}{2}$ and $\frac{1}{2}$ and $\frac{1}{2}$ and $\frac{1}{2}$ $\ddot{x} + 2H\dot{x} = -\frac{1}{a^2}$ $\frac{1}{a^2} \nabla_x \phi(\boldsymbol{x}),$ $\nabla_x^2\,\phi(\boldsymbol{x})=4\pi\,Ga^2$ $\overline{\rho}_{\rm m}\,\delta(\boldsymbol{x}).$ **Basic equations** $L=\frac{1}{2}m\,a^2\dot{x}^2-m\,\phi(\boldsymbol{x})$ \mathbf{w} describe the motion of mass element in \mathbf{w} space, *x*. International W, the displacement field Ψ, the displacement field Ψ, the relationship $\mathcal{L} = \mathcal{L} \times \mathcal{L}$ \mathbf{x} . $Basic equations$ $\ddot{\alpha} + 2H\dot{\alpha} = \frac{1}{2} \nabla A$ $\sum^2 \phi(m) = 4\pi G a^2 \overline{q}$ $\mathbf{v}_x \cdot \mathbf{w}_y = \mathbf{w}_x \cdot \mathbf{w}_y$ \mathbf{r} and \mathbf{r} and \mathbf{r} and \mathbf{r} represent the \mathbf{r} $\mathcal{L} = \frac{\mathcal{L}}{2} \mathcal{H} u \mathcal{L} = \mathcal{H} u \varphi(\mathcal{X})$ symbols are summarized as \sim $\frac{1}{\sqrt{2}}$ $L = -m a^2 \dot{x}^2 - m \phi(x)$ space, *x*. International W, the displacement field \mathcal{A} *x* + *q* $L =$ 1 2 $m\,a^2\dot{\boldsymbol{x}}^2-m\,\phi(\boldsymbol{x})$

To condinate $(a)^{n}$ $\alpha(a + b) = a \pm \Psi(a + b)$ $\overline{}$ Lagrangian coordinate (q) : $\bm{x}(\bm{q},t) = \bm{q} + \bm{\Psi}(\bm{q},t)$ space, *x*. International W, the displacement field λ Lagrangian coordinate (**q**). $\overline{\mathbf{J}}$ *i* $\left(\begin{array}{c} 1 \end{array} \right)$ $I(t) = \alpha + \Psi(\alpha t)$ $(\mathbf{y}, \mathbf{v}) = \mathbf{q} + \mathbf{r}(\mathbf{q}, \mathbf{v})$

> ∇*^x ·* **.** *x* oordinate, " nass density is assumed to be uniform: \mathcal{L} . In Lagrangian coordinate, mass element is supposed to be a supposed to be homogeneously distributed, i.e., ρ^m *dⁿq* = ρm(*x*) *dⁿx* with *n* ϵ*ij* ϵ*ik* = δ*jk,* ϵ*ij* ϵ*ij* = 2 In Lagrangian coordinate, mass density is assumed to be uniform: μ is assumed to be uniform;

$$
\overline{\rho}_{\rm m} d^n \mathbf{q} = \rho_{\rm m}(\mathbf{x}) d^n \mathbf{x} \longrightarrow \delta(\mathbf{x}) = \frac{\rho_{\rm m}(\mathbf{x})}{\overline{\rho}_{\rm m}} - 1 = \left| \frac{\partial \mathbf{x}}{\partial \mathbf{q}} \right|^{-1} - 1
$$

 ν witing quantities in Eulerian coose ν ith we will describe the motion of mass element in Eulerian ^ϵ*ikp*ϵ*jqrJkqJpr*(^ˆ [−] ⁴^π *^G* space, *x*. International Hydroducing the displacement of the relationship of the **n Eulerian spac**
ngian quantities **u**
D those in Lagrangian quantities Below, following Matsubara (2015), we will derive the Rewriting quantities in Eulerian space with

Lagrangian PT *x* $\frac{1}{2}$ $\frac{1$ ρ^m δ(*x*)*.* (2) Taking the divergence and rotation, Eq. (1) with a set of the divergence and rotation, $\frac{1}{2}$ Noting that *Jij* is expressed as *Jij* = δ*ij* + Ψ*i,j* , the evolutions derived and the contract of the contract are functions of the contract and the contract are functions of the contract of the contract of the contract are functions of the contract of the contract of the contract \log l displacement fields of α \blacksquare \blacksquare

i.e., $\frac{d}{d}$ **Platsubara ('**l. 2 *Authors* Matsubara ('15)

Using the Lagrangian quantities defined above, Eqs. (3)

$$
\nabla_x \cdot \left[\dot{\vec{x}} + 2H\dot{\vec{x}} \right] = -4\pi G \, \overline{\rho}_{\rm m} \, \delta,
$$

$$
\nabla_x \times \left[\dot{\vec{x}} + 2H\dot{\vec{x}} \right] = 0.
$$

$$
\hat{\mathcal{T}}f(t) \equiv \ddot{f}(t) + 2H\dot{f}(t)
$$

Longitudinal:
$$
\begin{aligned} \left(\hat{\mathcal{T}} - 4\pi G \overline{\rho}_{\rm m}\right) \Psi_{k,k} \\ &= -\epsilon_{ijk}\epsilon_{ipq} \Psi_{j,p} \left(\hat{\mathcal{T}} - 2\pi G \overline{\rho}_{\rm m}\right) \psi_{k,q} \\ &- \frac{1}{2} \epsilon_{ijk}\epsilon_{pqr} \Psi_{i,p} \Psi_{j,q} \left(\hat{\mathcal{T}} - \frac{4\pi G}{3} \overline{\rho}_{\rm m}\right) \Psi_{k,r}, \end{aligned}
$$

Transverse:
$$
\epsilon_{ijk} \hat{\mathcal{T}} \Psi_{j,k} = -\epsilon_{ijk} \Psi_{p,j} \hat{\mathcal{T}} \Psi_{p,k}.
$$

\nLevi-Civita symbol

 $\mathbf{Y}(\mathbf{q}, \mathbf{u}) = \mathbf{Y}(\mathbf{q}, \mathbf{u}) + \mathbf{Y}(\mathbf{q}, \mathbf{u}) + \mathbf{Y}(\mathbf{q}, \mathbf{u}) + \cdots$ $\mathbf{F}\cdot\mathbf{\Psi}(\bm{q},\,t)=\mathbf{\Psi}^{(1)}(\bm{q},\,t)+\mathbf{\Psi}^{(2)}(\bm{q},\,t)+\mathbf{\Psi}^{(3)}(\bm{q},\,t)+\cdots,$ $\iota_j + \cdots$ PT expansion: $\Psi(q, t) = \Psi^{(1)}(q, t) + \Psi^{(2)}(q, t) + \Psi^{(3)}(q, t) + \cdots$

Zel'dovich solution: 1st-order LPT expressed in terms of the scale factor *a*(*t*), are recast as the function of linear growth factor *D*¹ by simply replacing *a* $\frac{1}{2}$ $\frac{1}{2}$ 6 ϵ*ijk*ϵ*pqrJipJjqJkr* ≃ 1 + ∇*^q ·* ψ*,* (4.16) $\overline{}$ ²*^J* ^ϵ*jkp*ϵ*iqrJkqJpr* [≃] ^δ*ij* ⁺ *^O*(ψ)*.* (4.17)

$$
\mathbf{\Psi}^{(1)} = \mathbf{\Psi}^{(1L)} + \mathbf{\Psi}^{(1T)} \, ;
$$

$$
\eta \equiv \ln D_1(t)
$$

$$
\left(\frac{\partial^2}{\partial \eta^2} + \frac{1}{2} \frac{\partial}{\partial \eta} - \frac{3}{2}\right) \Psi_{k,k}^{(1L)} = 0
$$

$$
\left(\frac{\partial^2}{\partial \eta^2} + \frac{1}{2} \frac{\partial}{\partial \eta}\right) \epsilon_{ijk} \Psi_{j,k}^{(1T)} = 0
$$

 $\mathcal{A}(\mathcal{A}, \mathcal{B})$ define of $\mathcal{A}(\mathcal{B})$ gives of $\mathcal{A}(\mathcal{B})$ ${\bf Z}$ el'dovich approximation : $| \mathbf{\Psi}^{(1T)} = 0 \rangle$ \mathbf{E} is nothing but the evolution equation for linear density field. Since \mathbf{H} Zel'dovich approximation : $\mathbf{\Psi}^{(1T)} = 0$ and take growing-mode only

$$
\mathbf{\Psi}^{(1)} = \mathbf{\Psi}^{(1L)} = - D_1(a) \, \nabla_q \varphi(\mathbf{q}), \quad \nabla_q^2 \varphi(\mathbf{q}) = \delta_0(\mathbf{q})
$$

: initial density field

$$
\therefore \quad 1 + \delta_{\mathrm{m}}(\boldsymbol{x}) = \left| \frac{\partial \boldsymbol{x}}{\partial \boldsymbol{q}} \right|^{-1} \equiv \frac{1}{J} \, \simeq \, 1 - \nabla_{\boldsymbol{q}} \cdot \boldsymbol{\psi}
$$

Particle trajectories in ZA

Figure 3. A family of trajectories corresponding to the model presented in Fig. 1 is shown for the first-order (upper panel) and second-order (lower panel) approximations. The trajectories end in the Eulerian space-time section $(y=0.5, t)$ centred at a cluster. These plots illustrate that the three-stream system that develops after the first shell-crossing performs a self-oscillation due to the action of self-gravity.

Zel'dovich近似の応用

N体シミュレーションの初期条件生成に有用

粒子を格子状に並べてZel'dovich近似でずらす:

² *Authors* Higher-order Lagrangian PT **Higher-order Lagrangian PI** *^T*ˆ*f*(*t*) [≡] ¨*f*(*t*)+2*^H* ˙*f*(*t*). \sim \sim \sim \sim \sim \sim \sim \sim \mathbf{T} \blacksquare Higher, order Lagrangian PT Noting that *Jij* is expressed as *Jij* = δ*ij* + Ψ*i,j* , the **di Lazianzi** ment in 300 and 200 an TUIUCI LAXIAIIXIAII I for perturbative solutions in a specific initial conditions in a specific initial condition: \mathcal{O}

e.g,. Matsubara ('15) $e.g.,$ M_{other} is approximation. $(21F)$ $\frac{1}{\sqrt{2}}$. The Einstein definitions of $\frac{1}{\sqrt{2}}$ S

function of linear growth factor *D*¹ by simply replacing *a*

PT expansion:
$$
\Psi(q, t) = \Psi^{(1)}(q, t) + \Psi^{(2)}(q, t) + \Psi^{(3)}(q, t) + \cdots
$$

displacement field are obtained:

in 2D case. Here, the differential operator, *^T*ˆ, is defined as

Under $\mathsf{stein}\text{-}\mathsf{de}$ Sitter annroximation: $\mathbf{u}^{(n)}(a_1,a_2(t)) \longrightarrow \mathbf{u}^{(n)}(t)$ \mathbf{S} $\frac{q^{(n)}(q)}{\mathsf{EdS}}$ 2 $a(t)$) — Under Einstein-de Sitter approximation: $\mathbf{\Psi}^{(n)}_{{\sf EdS}}(q;\, a(t)) \longrightarrow \mathbf{\Psi}^{(n)}(q;\, D_1(t))$ of longitudinal and transverse parts of \mathbb{R}^n \mathbf{C} <mark>∢imat</mark> $\overline{}$ \mathbf{p} : $\mathbf{\Psi}^{(n)}$ $\text{ker approximation: } \Psi^{(n)}(q; a(t)) \longrightarrow \Psi^{(n)}(q; D_1(t))$ ΛCDM model, we employ the Einstein-de Sitter approxi-

 $\int \partial^2$ $rac{\delta}{\partial \eta^2}$ + 1 2 $\frac{\partial}{\partial\eta}-\frac{3}{2}% {\displaystyle\sum\limits_{\sigma\in S_{2n-1}}} \left(\frac{\partial\chi_{\sigma\sigma}^{\sigma}}{\partial\eta}-\frac{\partial\chi_{\sigma\sigma}^{\sigma}}{\partial\eta}\right) \frac{\partial\chi_{\sigma\sigma}^{\sigma}}{\partial\eta} \label{rho2}%$ $\left(\Psi^{(n)}_{k,k}\right)$ $=$ \sum *m*1+*m*2=*n* $\epsilon_{ijk}\epsilon_{ipq}\,\Psi^{(m_1)}_{j,p}\Big(\frac{\partial^2}{\partial p^2}\Big)$ $rac{\delta}{\partial \eta^2}$ + 1 2 $\frac{\partial}{\partial\eta}-\frac{3}{4}% {\displaystyle\sum\limits_{\sigma\in S_{2n-1}}} \left(\frac{\partial\sigma}{\partial\eta}\right) ^{\sigma\left(\sigma\right) }\left[\left[\left[T_{A_{\sigma\left(1\right) }},...,T_{A_{\sigma\left(n\right) }}\right] \right] \right] =0. \label{eq2.3}%$ $\bigg) \psi_{k,q}^{(m_2)}$ $-\frac{1}{2}$ 2 \sum *m*1+*m*2+*m*3=*n* $\epsilon_{ijk}\epsilon_{pqr}\,\Psi_{i,p}^{(m_1)}\Psi_{j,q}^{(m_2)}$ $\frac{2}{1}$ $\frac{1}{2}$ \cdots \cdots θ^2 θ θ \cdots $\frac{m_1 + m_2 = n}{1}$ \cdot ^{η} $\frac{2}{m_1+m_2}$ $+m_2 + m$ $=n$ $\eta \equiv \ln D_1(t)$ $\eta \equiv \ln D_1(t)$ $\frac{1}{2} \frac{\partial}{\partial n}$ η $\frac{1}{2}$ *m*1+*m*2+*m*3=*n vanished in ID* \mathcal{L} $\int \partial^2$ $rac{\delta}{\partial \eta^2}$ + 1 2 $\frac{\partial}{\partial\eta}-\frac{1}{2}% {\displaystyle\sum\limits_{\sigma\in S_{2n-1}}} \left(\frac{\partial\chi_{\sigma\sigma}^{\sigma}}{\partial\eta}-\frac{\partial\chi_{\sigma\sigma}^{\sigma}}{\partial\eta}\right) \frac{\partial\chi_{\sigma\sigma}^{\sigma}}{\partial\eta} \label{rho2}%$ $\Big)\Psi_{k,r}^{(m_{3})},$ inka : *angre*
alikuwa
 val Longitudinal: \approx 3² $\frac{\partial^2}{\partial \overline{\partial} z} +$ $\overline{1}$ $\frac{1}{2}$ $rac{\partial}{\partial}$ - $rac{3}{2}$ $\begin{array}{cc} 1 \ \frac{\partial}{\partial t} & \frac{3}{\sqrt{t}} \end{array}$ Γ *m*1+*m*2=*n* $\frac{1}{\sqrt{1}}(m_1) \left(\frac{\partial^2}{\partial n_1} \right)$ $\frac{\partial^2}{\partial x^2} +$ $\overline{1}$ $\frac{1}{\Omega}$ $\frac{\partial}{\partial}$ - $\frac{3}{4}$ $\binom{m_2}{ }$ $\frac{1}{-}$ $\frac{1}{2}$ # *m*1+*m*2+*m*3=*n* $\epsilon_{ijk} \epsilon_{lm} \ln^{(m_1)} \ln^{(m_2)}$ $\frac{1}{2}$ | $\frac{1$ $\overline{}$. .
. Ω^2 Ω Ω $\overline{}$ Ψ*k,r,* $\hat{\theta}^2$ *j,p* ! [∂]² $\eta \equiv \ln D_1(t)$ 1
1940
1940 ∂ " ψ(*m*2) $\binom{m_1}{m_1}$ $\frac{1}{2}$ $\sqrt{\partial \eta^2}$ $2 \partial \eta$ $\lambda = \frac{3}{2}$ $\sqrt{m_2}$ vanished in 1D ε $\frac{1}{2}$ $\Psi^{(m_1)}_{i,p}$ ۱ $\overline{1}$ \boldsymbol{j} ${n_2)\over q}\bigg(\frac{\partial^2}{\partial r^2}$ $\frac{1}{2} + \frac{1}{2} \frac{\partial}{\partial x} - \frac{1}{2} \psi_n^{(m_3)}$ ^ϵ*ijk* ! [∂]² 1 ∂ " Ψ(*n*) $π = 2D$ $t_2 \partial \eta$ obtained in the Einstein definition definition definition of \mathcal{L} expressed in $\mathbb{U}^{(m_1)}$ ($\frac{\partial^2}{\partial}$, 1∂ $3 \big\rangle_{ab}(m_2)$ vanished in **ID** $\sum_{m_1+m_2=n} a_m a_p a_p q - j, p \quad \big(\partial \eta^2 + 2 \partial \eta - 4 \big)^{m} \kappa, q$ Γ Γ ^{(m₁)</sub> (m_0) θ ² 1θ 1)}

Ψ*k,r,* $\overline{\mathbf{u}}$ ϵ_{ijk} $\left(\frac{\partial}{\partial\eta}\right)$ $\partial \eta^2$ $\left(\frac{\overline{a}}{2}\frac{\partial}{\partial\eta}\right)\Psi_{j,k}^{(n)}$ 2 ∂η $\text{Transverse:} \quad \text{for} \quad \left(\frac{\partial^2}{\partial x^2} + \frac{1}{x}\frac{\partial}{\partial y}\right) \text{ if } (n)$ $\partial \eta^2 = 2 \partial \eta^f$ η^m η^m $\epsilon_{ijk}\,\Psi^{(m_1)}_{p,j}\Big(\frac{\partial}{\partial\eta^2}+\frac{1}{2}\frac{\partial}{\partial\eta}\Big)\Psi^{(m_1)}_{p,\chi}$ *m*1+*m*2=*n* $\frac{P}{p,k}$. Transverse: ϵ_{ijk} $\left(\frac{\partial^2}{\partial x^2}\right)$ $rac{\delta}{\partial \eta^2}$ + 1 2 ∂ $\partial \eta$ $\Big)$ $\Psi_{j,k}^{(n)}$ Transverse: $\epsilon_{ijk} \left(\frac{\partial^2}{\partial \eta^2} + \frac{1}{2} \frac{\partial}{\partial \eta} \right) \Psi_{j,k}^{(n)} = - \sum$ $m_1 + m_2 = n$ $\epsilon_{ijk}\,\Psi^{(m_1)}_{p,j}\Big(\frac{\partial^2}{\partial n^2}\Big)$ $rac{\delta}{\partial \eta^2}$ + 1 2 ∂ $\partial \eta$ $\left(\frac{\partial^2}{\partial x^2}+\frac{1}{2}\frac{\partial}{\partial x}\right)\Psi^{(n)}_{j,k}\;\;=\;-\;\;\;\sum\;\;\;\; \epsilon_{ijk}\,\Psi^{(m_1)}_{p,j}\Big(\frac{\partial^2}{\partial n^2}+\frac{1}{2}\frac{\partial}{\partial n}\Big)\Psi^{(m_2)}_{p,k}\,.$

$\frac{1}{2}$ ✒ ✑ vanished in 1D

Performance of Lagrangian PT

(1)

Saga, AT & Colombi, arXiv:1805.08787

初期条件 (Zel'dovich近似解)

$$
\boldsymbol{\Psi}(\boldsymbol{q}) = a_{\text{init}} \left(\begin{array}{c} \epsilon_{\text{x}} \sin q_x \\ \epsilon_{\text{y}} \sin q_y \\ \epsilon_{\text{z}} \sin q_z \end{array} \right)
$$

10次まで計算

Results

Saga, AT & Colombi, arXiv:1805.08787

Vlasov simulation vs LPT $\overline{}$

After shell-crossing,

quasi-1D collapse quasi-2D collapse 3D collapse

マルチストリーム構造が発達(ハローが形成)

Cosmological *N*-body simulations

Directly solve equation of motion for *N* particles

Run *N*-body simulation many times with a large \rightarrow number of particles in a huge box

To reduce $O(N^2)$ operation for force calculation,

- •Tree algorithm
- Particle-Mesh algorithm (using FFT) O(*N* log *N*)

For cosmological study

N~1,000^3, *L*~1,000 Mpc, >50 runs

Still extensive but very useful for practical purposes : mock data analysis, locating 'galaxies' in dark matter halo, …

Tree-PM method for force calculation of a synthesis of the PM method and the tree algorithm. GADGET-2's mathematical implementation of this so-called TreePM method The new version of GADGET-2 used in this study option of GADGET-2 used in this study optionally allows the stu the pure tree algorithm to be replaced by a hybrid method consisting of a synthesis of the PM method and the tree algorithm. GADGET-Troo DM moothing the first part of the SPH computation (i.e. the SPH computation (i.e. the SPH computation (i.e. the density lo α d for force calculat with r describing the spatial scale of the force spatial scale of the force split \mathcal{L} but not smaller than h . We solve the solve than h n nacional management occurring among all an occurring among all an among all an among all an among all an amo particles represented by the node. Note that we update these values of Bagla & Ray (2003). The potential of equation (3) is explicitly split in Fourier space into a long-range part and a short-range part **k** \mathbf{F} \mathbf{F} \mathbf{F} \mathbf{F}

In Fourier space, of Bagla & Ray (2003). The potential of equation (3) is explicitly space, in Fourier space, Fourier space, Ostriker & Xu 2002 of Bagla & Ray (2003). The potential of equation (3) is explicitly such that all interacting pairs in the SPH force computation are the first part of the SPH computation (i.e. the density loop). Using In Fourier space. such that all interacting pairs in the SPH force computation are

for determining the desired SPH smoothing lengths of each parti-

 $\phi_k = \phi_k^{\text{long}} + \phi_k^{\text{short}}$ $\forall k$ $\forall k$: $\frac{1}{\sqrt{1-\frac{1$ $\oint \phi_k = \phi_k^{\text{max}} + \phi_k^{\text{short}}$ cle in the first place. For simplicity, and to allow a straightforward \log $\gamma_k - \varphi_k$ for φ_k

$$
\phi_k^{\text{long}} = \phi_k \exp(-k^2 r_s^2)
$$

\n
$$
\phi_k^{\text{short}} = \phi_k [1 - \exp(-k^2 r_s^2)] \longrightarrow \phi^{\text{short}}(x) = -G \sum_i \frac{m_i}{r_i} \text{erfc}\left(\frac{r_i}{2r_s}\right)
$$

\n
$$
r_i = \min(|x - r_i - nL|)
$$

plementary error function rapidly suppresses the force for distances

plementary error function rapidly suppresses the force for distances

large compared to *r*^s (the force drops to about 1 per cent of its

• long-range: PM method with FFT solution of equation (3) is given by onlarge compared to a few per cent accuracy, and the force of its compared to a few perfect of its compared to a
Large compared to a few per cent of its compared to about 1 per cent of its compared to about 1 per cent of i ⃝^C 2005 The Author. Journal compilation ⃝^C 2005 RAS, MNRAS **364,** 1105–1134

• short-range: Tree algorithm *.* (21) • short-range: Tree \overline{a} 11goriunn Here, *ri* = min(|*x* − *rⁱ* − *nL*|) is defined as the smallest distance of any of the internal in
The component α is the point of the point α internal internal internal internal internal internal in (Barnes-Hut oct-tree) http://arborjs.org/docs/barnes-hut

Here, *ri* = min(|*x* − *rⁱ* − *nL*|) is defined as the smallest distance **Performance of each method is O(N Ic**. large compared to *r*^s (the force drops to about 1 per cent of its Performance of each method is O(N log N)

Cosmological initial condition

Note $\boldsymbol{v} = a\boldsymbol{\dot{x}} = a\dot{\boldsymbol{\Psi}}(\boldsymbol{q})$

For particle assigned on each grid:

 $\bm{x} = \bm{q} + \bm{\Psi}(\bm{q})^\top$

initial position Lagrangian displacement

'q' is called Lagrangian coordinate (homogeneous mass dist)

 $\Psi(\bm{k}) \simeq$

leading order $\Psi(\bm{k}) \simeq \frac{i\,\bm{k}}{k^2}\, D_+(z) \overline{\delta_0(\bm{k})}$.
I'dovich approx.) (Zel'dovich approx.) $\mathcal{L}(\mathcal{L}) = \frac{1}{k^2} D + (2) \frac{\partial O(\mathcal{L})}{\partial \mathcal{L}}$ initial density field (random)

with Lagrangian PT (2LPT code)

General procedure Improved initial condition generator

1. generate random field $\delta_0(\mathbf{k})$

2. calculate displacement field $\Psi(k)$ FFT, $\Psi(q)$

3. move particles according to displacement field $\; \Psi(q) \; \dot \Psi(q) = \frac{\dot D_+(z)}{D_+(z)}$ $\frac{D_+(z)}{D_+(z)}\,\bm{\Psi}(\bm{q})$

Perturbation theory of large-scale structure

Nonlinear gravitational evolution 104

Regime of our interest

Most of interesting cosmological information (BAO, RSD, signature of massive neutrinos, ...) lies at k<0.2-0.3 h/Mpc

Weakly nonlinear regime

Dimensionless $10⁵$ **X** Based on linear theory power spectrum **Nonlinear** $(2\pi^2)$ $z=0$ z=0.5 $10⁴$ $\equiv k^3 P(k)$ $P(k)$ [Mpc³] $\overline{}$ Weakly nonlinear 2 3 10^{-1} $\Delta^2({\bf k})$ $10³$ Linear theory N-body simulations Linear by T. Nishimichi 10^{2} 10^{-2} 0.05 0.01 0.1 10^{-1} 10^{-8} k $[Mpc^{-1}]$ $k \left[h \text{ Mpc}^{-1} \right]$

Range of applicability

Methods (Gravitational evolution) Other systematics

Fully nonlinear

weakly nonlinear $(\Delta^2 \lesssim 1)$

linear $(\Delta^2\ll 1)$

N-body simulation

 $(\Delta^2 > 1)$ | | \sim 8 time-consuming most powerful, but extensive

(c.f. fitting formula)

Perturbation theory **• Galaxy bias**

limited range of application, but analytical & very fast

Linear theory (CMB Boltzmann code) *very difficult*

Baryon physics (weak lensing)

• Redshift-space distortion (galaxy surveys)

relatively easy

Perturbation theory (PT)

Theory of large-scale structure based on gravitational instability

Juszkiewicz ('81), Vishniac ('83), Goroff et al. ('86), Suto & Sasaki ('91), Jain & Bertschinger ('94), ...

Cold dark matter + baryons = pressureless & irrotational fluid

$$
\frac{\partial \delta}{\partial t} + \frac{1}{a} \vec{\nabla} \cdot \left[(1 + \delta) \vec{v} \right] = 0
$$

Basic eqs.

$$
\frac{1}{a^2}\nabla^2\Phi = 4\pi G \,\overline{\rho}_{\rm m}\,\delta
$$

Single-stream approx. of collisionless Boltzmann eq.

standard PT

 $|\delta| \ll 1$

 $\delta = \delta^{(1)} + \delta^{(2)} + \delta^{(3)} + \cdots$ $\langle f, t \rangle$ = $(2\pi)^3 \delta_D(\mathbf{k} + \mathbf{k}') P(|\mathbf{k}|; t)$

Equations of motion This is what is what is what is what is verified to the subject of Formal SPT. In the subject see Ref. \sim we shall merely recall well well and interventies and interventies and interventies of \mathcal{L}_1 @*jvⁱ* ⁺ ⁴ ⁼ ⌧✓ *,* (2.7) we shall merely recall merely recall well known results and introduce our notation. It is most convenient of \mathcal{L} **FOURTIONS OF M** @⌧ ✓ ⁺ *^H* ✓ ⁺ *^v^j* @*j*✓ + @*iv^j* @*jvⁱ* ⁺ ⁴ ⁼ ⌧✓ *,* (2.7) this work we will neglect the vorticity since at the linear order it decays as ⇠ 1*/a* (see Refs. [8, 9] for a discussion of the vorticity in the EFTofLSS). It is therefore useful to rewrite the Euler equation for the velocity divergence ✓ ⌘ r *· v* F_{All} same notation as \blacksquare for the smoothing of a smoothing to the smoot $B_{\rm eff}$ and the equation, we can derive the equations of motion for the smoothed quantities of motion for ⇡(*x,* ⌧) ⌘ [⇡˜] ⇤ = z
Z $\frac{1}{2}$

 $\bm{q}_1 \cdot (\bm{q}_1 + \bm{q}_2) = \bm{q}_1 \cdot (\bm{q}_1 + \bm{q}_2)$ 1 @⌧ (*k,* ⌧) + ✓(*k,* ⌧) = *S*↵(*k,* ⌧) *,* 2 $= 0$ (*k,* ⌧) = *S*(*k,* ⌧) *,* where $\frac{u \cdot q}{\equiv}$ is a slight abuse of the same notation, we use the same notation for the fields in Fourier $\int d^2\Omega_m \delta$. We will most in $\int d\mathbf{q} = \int (2\pi)^3$ arguments of the fields for streamlining the notation. The two source terms *S*↵ and *S* ϵ $\mathcal{H}\theta(\mathbf{k},\tau)+\frac{3}{\Gamma}\mathcal{G}$ *q* $\partial_{\tau} \theta(\mathbf{k}, \tau) + \mathcal{H} \theta(\mathbf{k}, \tau) + \frac{3}{2} \Omega_{m} \mathcal{H}^{2} \delta(\mathbf{k}, \tau)$ $=$ $\sqrt{ }$ *q* $\beta(\boldsymbol{q}, \boldsymbol{k} - \boldsymbol{q}) \, \theta(\boldsymbol{q}, \tau) \theta(\boldsymbol{k} - \boldsymbol{q}, \tau)$ $\beta(\boldsymbol{q}_1 \ \boldsymbol{q}_2) = \frac{1}{2} (\boldsymbol{q}_1 + \boldsymbol{q}_2)^2 \frac{\boldsymbol{q}_1 \cdot \boldsymbol{q}_2}{2}$ $\partial_{\tau}\delta + \partial_i \left[(1+\delta)u \right]$ The quest of finding a perturbative solution for the equations of motion in Eq. (2.6) is $\partial_{\tau}v^{i} + \mathcal{H}v_{l}^{i} + \partial^{i}\phi + v_{l}^{j}\partial_{i}v^{i} = 0$ $\int_{0}^{t} d^{3}a$ $\frac{u}{\lambda}$, $\frac{3}{2}$, $\frac{20}{\lambda}$ $\alpha \varphi = \frac{1}{2} \pi \Omega_m$ in Fourier space. $\partial_{\tau}\delta(\boldsymbol{k},\tau) + \theta(\boldsymbol{k},\tau) = -\int \alpha(\boldsymbol{q},\tau)$ $\frac{c}{\lambda}$ $\Omega \quad 2^{2} \delta(k, \tau)$ \int $\theta \equiv \nabla \cdot \bm{v} \qquad \qquad \nonumber \ = \ - \int \; \beta (\bm{q}, \bm{k} - \bm{q}) \, \theta (\bm{q}, \tau) \theta (\bm{k} - \bm{q}, \tau)$ arguments of the fields for streamlining the notation. The two source terms *S*↵ and *S* contain the non-linear terms of $\mathbf{q}_1 \cdot (\mathbf{q}_1 + \mathbf{q}_2)$ as the e \mathbf{q}_2 @⌧ (*k,* ⌧) + ✓(*k,* ⌧) = *S*↵(*k,* ⌧) *,* @⌧ ✓(*k,* ⌧) + *^H* ✓(*k,* ⌧) + ³ ⌦*mH*² (*k,* ⌧) = *S*(*k,* ⌧) *,* τ: conformal time space as in real space as in real space. We will most in $\mathcal{C} = \mathcal{C} = \mathcal{C}$ the space and we shall often drop the space and we shall often drop the space and we shall often drop the space of $\mathcal{C} = \mathcal{C}$ $c_0^2 \mathcal{H}^2 \Omega_m \delta$. $\int \mathbf{q} \int (\mathbf{2}\pi)^3$ *q* $\alpha(\boldsymbol{q},\boldsymbol{k}-\boldsymbol{q})\,\theta(\boldsymbol{q},\tau)\delta(\boldsymbol{k}-\boldsymbol{q},\tau)~,$ $\mathcal{H} \theta(\boldsymbol{k}, \tau) + \frac{3}{2} \Omega_m \mathcal{H}^2 \delta(\boldsymbol{k}, \tau)$ We use the abbreviation R $\boldsymbol{d} = \int_{\boldsymbol{q}} \ \beta(\boldsymbol{q},\boldsymbol{k}-\boldsymbol{q})\, \theta(\boldsymbol{q},\tau) \theta(\boldsymbol{k}-\boldsymbol{q},\tau) \nonumber$ *k* = *|k|*. The kernel functions ↵ and are ⌧✓ ⌘ @*ⁱ a* ⇢ and 4⌘r *·* r = @*i*@*ⁱ* $\frac{1}{\sqrt{2}}$ in the approximation where the e $\frac{1}{\sqrt{2}}$ This is usually called SPT. For a thorough review on the subject see Ref. $\frac{a^{\circ}q}{\circ}$. $\Delta \phi = \frac{3}{2} \mathcal{H}^2 \Omega_m \delta$. $\int \mathbf{q}^2 \mathbf{q}^2 \mathbf{r}^3 \mathbf{r}^4 \mathbf{r}^3$ to rewrite \mathcal{L} , and (2.7) in Fourier space \mathcal{L} in Fourier space \mathcal{L} $\int_{\bm{q}}$ $\partial_{\tau} \theta(\boldsymbol{k}, \tau) + \mathcal{H} \, \theta(\boldsymbol{k}, \tau) + \frac{3}{2}$ 2 $\Omega_m \mathcal{H}^2 \delta(\boldsymbol{k},\tau)$ $\mathbf{v} = - \int \beta(\mathbf{q}, \mathbf{k} - \mathbf{q}) \theta(\mathbf{q}, \tau) \theta(\mathbf{k} - \mathbf{q}, \tau)$ $\mathcal{S} = \mathcal{S}$ in Fourier space. We shall of $\mathcal{S} = \mathcal{S}$ are $q_1 \cdot (q_1 + q_2)$ streamlining the notation. The two source terms **S** $q_1 \cdot q_2$ @⌧ ✓ ⁺ *^H* ✓ ⁺ *^v^j* @*j*✓ + @*iv^j* @*jvⁱ* ⁺ ⁴ ⁼ ⌧✓ *,* (2.7) $U_{\tau}U + U_i$ [(1) ⌧✓ ⌘ @*ⁱ* 1 *a* ⇢ The quest of finding a perturbative solution for the equation for the equations of \mathcal{L} \sim in the approximation where the e \sim Fourier $\partial_\tau \delta({\bm k}, \tau) + \theta({\bm k}, \tau) \ = \ - \ \int \ \alpha({\bm q}, {\bm k} - {\bm q}) \, \theta({\bm q}, \tau) \delta({\bm k} - {\bm q}, \tau) \ ,$ α expansion recall merely σq $\frac{3}{\sqrt{2}}$ and $= -\int_{\mathbb{R}} \beta(\mathbf{q})$ $a_1 \cdot (a_1 + a_2)$ (*k,* ⌧) = *S*(*k,* ⌧) *,* Fourier expansion **Equipm** $\partial_x \delta(\mathbf{k}, \tau) + \theta(\mathbf{k}, \tau) =$ complication of the Newtonian potential (see e.g. e.g. e.g. (34) of Ref. [5] and, [5] and, [5] and, [5] and, [5] $\sigma_{\tau}(\mathbf{a}, t) + \mu(\mathbf{a}, t) + \sigma_{\tau}(\mathbf{a}, t)$ Refs. [8, 9] for a discussion of the vorticity in the EFTofLSS). It is therefore useful to $\theta \equiv \nabla \cdot \bm v$ $\alpha(\bm{q}_1,\bm{q}_2) \; \equiv \; \frac{\bm{q}_1 \cdot (\bm{q}_1 + \bm{q}_2)}{2} \; ,$ $\partial_{\tau}\delta + \partial_i \left[(1+\delta)v^i \right] = 0$, T: conformal tim $\partial_{\tau}v^{i} + \mathcal{H}v^{i}_{l} + \partial^{i}\phi + v^{j}_{l}\partial_{j}v^{i} = 0$ *,* 3 2 $=\frac{3}{2} \mathcal{H}^2 \Omega_m \delta$. is the Newtonian potential which is generated by the ℓ Fourier $\partial_\tau \delta({\bm k},\tau) + \theta({\bm k},\tau) \ = \ - \ \int \ \alpha({\bm q},{\bm k} - {\bm q}) \, \theta({\bm q},\tau) \delta({\bm k} - {\bm q})$ same notation as Ref. [2] for the smoothing of a quantity). Applying the smoothing to the *a* ⇢ $\frac{4}{3}$ $\tilde{\zeta}$ $\mathbf{L} \rightarrow 0$ $\frac{q}{3}$ expansion density contrast $\frac{q}{3}$ $\partial_{\tau}\theta(\boldsymbol{k},\tau)+\mathcal{H}\,\theta(\boldsymbol{k},\tau)+\frac{3}{2}\Omega_{m}\mathcal{H}^{2}\delta(\boldsymbol{k},\tau)$ $(ad\tau = dt)$ in the contract of the contrac *q* \equiv $\int d^3q$ $(2\pi)^3$

$$
\alpha(\bm{q}_1,\bm{q}_2) \; \equiv \; \frac{\bm{q}_1\cdot(\bm{q}_1+\bm{q}_2)}{q_1^2} \; , \qquad \qquad \beta(\bm{q}_1,\bm{q}_2) \; \equiv \; \frac{1}{2} \, (\bm{q}_1+\bm{q}_2)^2 \, \frac{\bm{q}_1\cdot\bm{q}_2}{q_1^2 \, q_2^2} \; .
$$

. (2.11)

Standard perturbation theory *q*1 *Fⁿ k Fⁿ* the density contrast is small, i.e. (1) *<* 1. In particular, this is the case for the smoothed fields. This allows allows the equations of the extension perturbative the extension perturbative motion pertu e vealidate portege stress the recovery E_n $\sum_{n=1}^\infty$ which is obtained by replacing the right hand side of \mathbb{R}^2). The Green's functions for the Green's functions for α

qn

 $\frac{1}{\sqrt{2}}$ is safe to assume that the linear solution is dominant and the linear solution is dominant and that the linear solution is dominant and the linear solution is dominant and the linear solution is dominant and

*^H*³ *,* (2.14)

Recursion relation for PT kernels the quantity DðtÞ is the linear growth factor. Then, in terms orursian relation fo ecursion relatio P^+ UT Izarnale is is neighborhood in the internal terms of the initial state \overline{a}

$$
\mathcal{F}_a^{(n)}(\boldsymbol{k}_1,\cdots,\boldsymbol{k}_n)\equiv \left[\begin{array}{c} F_n(\boldsymbol{k}_1,\cdots,\boldsymbol{k}_n) \\ G_n(\boldsymbol{k}_1,\cdots,\boldsymbol{k}_n) \end{array} \right]
$$

$$
\mathcal{F}_a^{(n)}(\bm{k}_1,\cdots,\bm{k}_n) = \sum_{m=1}^{n-1} \sigma_{ab}^{(n)} \, \gamma_{bcd}(\bm{q}_1,\bm{q}_2) \, \mathcal{F}_c^{(m)}(\bm{k}_1,\cdots,\bm{k}_m) \, \mathcal{F}_d^{(n-m)}(\bm{k}_{m+1},\cdots,\bm{k}_n)
$$

(4)

$$
q_{1} = k_{1} + \dots + k_{m}
$$
\n
$$
q_{2} = k_{m+1} + \dots + k_{n}
$$
\n
$$
\sigma_{ab}^{(n)} = \frac{1}{(2n+3)(n-1)} \begin{pmatrix} 2n+1 & 2 \\ 3 & 2n \end{pmatrix}
$$
\nin
\n
$$
m
$$

$$
\gamma_{abc}(\mathbf{k}_1, \mathbf{k}_2) = \begin{cases} \frac{1}{2} \left\{ 1 + \frac{k_2 \cdot k_1}{|k_2|^2} \right\}; & (a, b, c) = (1, 1, 2) \\ \frac{1}{2} \left\{ 1 + \frac{k_1 \cdot k_2}{|k_1|^2} \right\}; & (a, b, c) = (1, 2, 1) \\ \frac{(k_1 \cdot k_2)|k_1 + k_2|^2}{2|k_1|^2|k_2|^2}; & (a, b, c) = (2, 2, 2) \\ 0; & \text{otherwise.} \end{cases}
$$

Note—. repetition of the α subscripts (a h c) \mathcal{E} subscripts (a, b, c) are solutions are solutions are solutions are solutions are shown \mathcal{E} indicates the sum over all same subscripts (a,b,c) multiplet components

PT kernels constructed from recursion relation should be *symmetrized*

Power spectrum *P*13(*k*)=6*Plin*(*k*) **D** *Plin*(*q*) *F*3(*k, q, q*) *,* Fower spectrum possibility to connect the three external points without invoking the three-point vertex of a detailed discussion of the e \sim 10 and 4. The e \sim 3 and 4. The two- and 4. The two- and three-stress tensor to Sec \sim P_{\bigwedge} in this paper. In Fourier space, the power- and bispectrum are defined as

$$
\langle \delta(\mathbf{k}_1, a) \delta(\mathbf{k}_2, a) \rangle \equiv (2\pi)^3 \delta_D^{(3)}(\mathbf{k}_1 + \mathbf{k}_2) P(k_1, a)
$$

3

(3)

↵ ⌘ (2⇡)

 $P_{SPT}(k) = P_{lin}(k) + P_{22}(k) + P_{13}(k) +$ higher order loops *.* $P(A) = P(A) + P(A) + P(A) + P(A) + P(A)$ linear 1-loop

These integrals can be divergent when the loop momentum *q* becomes large and the renor-

$$
P_{22}(k) = 2 \int_{\mathbf{q}} P_{lin}(q) P_{lin}(|\mathbf{k} - \mathbf{q}|) F_2^2(\mathbf{q}, \mathbf{k} - \mathbf{q}),
$$

$$
P_{13}(k) = 6 P_{lin}(k) \int_{\mathbf{q}} P_{lin}(q) F_3(\mathbf{k}, \mathbf{q}, -\mathbf{q}),
$$

Next-to-next-to leading order

$P^{(mn)} \simeq \langle \delta^{(m)} \delta^{(n)} \rangle$ $\Bigg)$

up to 2-loop order

$$
P(k) = P^{(11)}(k) + \left(P^{(22)}(k) + P^{(13)}(k)\right) + \left(P^{(33)}(k) + P^{(24)}(k) + P^{(15)}(k)\right) + \cdots
$$

Linear (tree) 1-loop 2-loop

Crocce & Scoccimarro ('06)

Calculation involves multi-dimensional numerical integration (time-consuming)

Comparison with simulations

Standard PT qualitatively explains scale-dependent nonlinear growth, however,

1-loop : overestimates simulations

2-loop : overestimates at high-z, while it turn to underestimate at low-z

Standard PT produces illbehaved PT expansion !!

… need to be improved

Density field in standard PT

AT, Nishimichi & Jeong (to appear soon) $x[h^{-1}Mpc]$

 $x[h^{-1}Mpc]$

Density field in standard PT FIG. 1. 2D density field at *z* = 0 smoothed with Gaussian filter of *R* = 10 *h*−¹Mpc. The results generated with GridSPT code are shown (from top left to bottom middle), averaging over the 10 *h*−¹Mpc depth in each grid. Here, the color scale represents **Density field in standard Fig.** \blacksquare right panel shows the density field from *N*-body simulation, evolved with the same random seed as used in grid PT calculations.

AT, Nishimichi & Jeong (to appear soon) \mathbf{x} [h⁻¹Mpc]

Correlation between N-body and SPT

Improving PT predictions

Basic idea

Reorganizing standard PT expansion by introducing non-perturbative statistical quantities

Initial power spectrum **Observables** $\delta_0(\boldsymbol{k})$ $P_0(k)$

from linear theory

P(*k*; *z*) $\delta(\mathbf{k}; z)$ $B(k_1, k_2, k_3; z)$ $T(k_1, k_2, k_3, k_4; z)$ *···* initial density field (Gaussian) **Evolved density field (non-Gaussian)**

(CMB Boltzmann code) of dark matter/galaxies/halos

Concept of '*propagator*' in physics/mathematics may be useful

Nonlinear

mapping

Propagator in physics

✦ Green's function in linear differential equations

✦ Probability amplitude in quantum mechanics

 $\left(-i\hbar\frac{\partial}{\partial t}+H_x\right)\psi(x,t)=0\qquad G(x,t;x',t')$ Schrödinger Eq.

 $\left(-i\hbar\frac{\partial}{\partial t}+H_{x}\right) G(x,t;x',t')=-i\hbar\,\delta_{D}(x-x')\delta_{D}(t-t')\,,$

 $\psi(x,t)=\int_{-\infty}^{+\infty}dx' G(x,t;x',t')\,\psi(x',t')\,\,;\quad t>t'$

Cosmic propagators

non-linear evolution & statistical properties Propagator should carry information on

Evolved (non-linear) density field

Crocce & Scoccimarro ('06)

$$
\left\langle \frac{\delta \delta_{\rm m}(\boldsymbol{k};t)}{\delta \delta_{\rm 0}(\boldsymbol{k'})} \right\rangle \equiv \delta_{\rm D}(\boldsymbol{k}-\boldsymbol{k'}) \Gamma^{(1)}(k;t) \text{ Propagator}
$$

Initial density field

Ensemble w.r.t randomness of initial condition

Contain statistical information on *full-nonlinear* evolution

(Non-linear extension of Green's function)

Multi-point propagators

Bernardeau, Crocce & Scoccimarro ('08) Matsubara ('11) \longrightarrow integrated PT

As a natural generalization,

Multi-point propagator

$$
\left\langle \frac{\delta^n \delta_{\mathbf{m}}(\mathbf{k};t)}{\delta \delta_0(\mathbf{k}_1) \cdots \delta \delta_0(\mathbf{k}_n)} \right\rangle = (2\pi)^{3(1-n)} \delta_{\mathbf{D}}(\mathbf{k}-\mathbf{k'}) \Gamma^{(n)}(\mathbf{k}_1,\cdots,\mathbf{k}_n;t)
$$

With this multi-point prop.

Γ-expansion or Wiener-Hermite expansion • Building blocks of a new perturbative theory (PT) expansion • A good convergence of PT expansion is expected (c.f. standard PT)

Power spectrum \$ **Fower spectrum Power spectrum Power spectru** \mathbf{m}

Bi

Initial power spectrum

$$
P(k; t) = \left[\Gamma^{(1)}(k; t)\right]^2 P_0(k) + 2 \int \frac{d^3q}{(2\pi)^3} \left[\Gamma^{(2)}(q, k-q; t)\right]^2 P_0(q) P_0(|k-q|)
$$

+6
$$
\int \frac{d^6p d^3q}{(2\pi)^6} \left[\Gamma^{(3)}(p, q, k-p-q; t)\right]^2 P_0(p) P_0(q) P_0(|k-p-q|) + \cdots
$$

initial $P(k)$
 k
 k

+
$$
\left[8\int d^3q \frac{\Gamma^{(2)}(\mathbf{k}_1-\mathbf{q},\mathbf{q})\Gamma^{(2)}(\mathbf{k}_2+\mathbf{q},-\mathbf{q})\Gamma^{(2)}(\mathbf{q}-\mathbf{k}_1,-\mathbf{k}_2-\mathbf{q})P_0(|\mathbf{k}_1-\mathbf{q}|)P_0(|\mathbf{k}_2+\mathbf{q}|)P_0(q)\right]
$$

+ $6\int d^3q \frac{\Gamma^{(3)}(-\mathbf{k}_3,-\mathbf{k}_2+\mathbf{q},-\mathbf{q})\Gamma^{(2)}(\mathbf{k}_2-\mathbf{q},\mathbf{q})\Gamma^{(1)}(\mathbf{k}_3)P_0(|\mathbf{k}_2-\mathbf{q}|)P_0(q)P_0(k_3)+\text{cyc.}\right].$

Generic property of propagators

FIG. 7. This figure is the effect of the effect of the time-ordering exchanges (the time-ordering exchanges). The time-ordering exchanges (the time-ordering exchanges, the time-ordering exchanges (the time-ordering exchang

Constructing regularized propagators

• UV property (k >>1) :

$$
\Gamma^{(n)} \quad \stackrel{k \to +\infty}{\longrightarrow} \quad \Gamma^{(n)}_{\text{tree}} \, e^{-k^2 \sigma_v^2/2} \quad ; \quad \sigma_v^2 = \int \frac{dq}{6\pi^2} \, P_{\theta\theta}(q)
$$

Bernardeau, Crocce & Scoccimarro ('08), Bernardeau, Van de Rijt, Vernizzi ('11)

1

 $k \to +\infty$

 $\left(-\frac{k^2\sigma_v^2}{2}\right)$

2

v

 \setminus ^{*p*}

 $\Gamma^{(n)}_{\text{tree}}$

p!

• IR behavior (k<<1) can be described by standard PT calculations :

$$
\Gamma^{(n)} = \Gamma^{(n)}_{\text{tree}} + \Gamma^{(n)}_{\text{1-loop}} + \Gamma^{(n)}_{\text{2-loop}} + \cdots
$$

 $\Gamma_{n-\mathbf{k}}^{(n)}$ $p\text{-loop}$ \longrightarrow Importantly, each term behaves like

A regularization scheme that reproduces both UV & IR behaviors Bernardeau, Crocce & Scoccimarro ('12)

Regularized propagator

Bernardeau, Crocce & Scoccimarro ('12)

A global solution that satisfies both UV (k>>1) & IR (k<<1) properties:

$$
\Gamma_{\text{reg}}^{(n)} = \left[\Gamma_{\text{tree}}^{(n)} \left\{ 1 + \frac{k^2 \sigma_v^2}{2} \right\} + \Gamma_{\text{1-loop}}^{(n)} \right] \exp \left\{ -\frac{k^2 \sigma_v^2}{2} \right\}; \quad \sigma_v^2 = \int \frac{dq}{6\pi^2} P_{\theta\theta}(q)
$$

counter term
............ *IR behavior is valid at 1-loop level*

Precision of IR behavior can be systematically improved by including higher-loop corrections and adding counter terms

e.g., For IR behavior valid at 2-loop level,

$$
\Gamma_{\text{reg}}^{(n)} = \left[\Gamma_{\text{tree}}^{(n)} \left\{ 1 + \frac{k^2 \sigma_v^2}{2} + \frac{1}{2} \left(\frac{k^2 \sigma_v^2}{2} \right)^2 \right\} + \Gamma_{\text{1-loop}}^{(n)} \left\{ 1 + \frac{k^2 \sigma_v^2}{2} \right\} + \Gamma_{\text{2-loop}}^{(n)} \right] \exp \left\{ -\frac{k^2 \sigma_v^2}{2} \right\}
$$

Propagators in N-body simulations

compared with '*Regularized'* propagators constructed analytically

Bernardeau, AT & Nishimichi ('12)

Bernardeau et al. ('12)

RegPT: fast PT code for P(k) & ξ(r) few sec.

(regularized)

A public code based on multi-point propagators at *2-loop order*

http://www2.yukawa.kyoto-u.ac.jp/~atsushi.taruya/regpt_code.html

Why improved PT works well? \blacksquare are prediction of two-point correlation of two-point correlation of two-point correlation of two-point correlation of the set of two-point correlation of the set of the s **v** v B. Results in real space characteristic wave numbers, and they are shifted to the higher k modes as increasing the order of PT. These trends clearly indicate that the improved PT with closure approximation has a better convergence property. Qualitative be-

AT, Bernardeau, Nishimichi, Codis ('12) predictions of AT et al. ('09) Now, let us focus on the behavior of BAOs, and BAO

- Before addressing a quantitative comparison between • All corrections become α converge properties of the improved PT, and conside the comparable at low-z.
- Positivity is not guaranteed.

Corrections are positive & localized, the WMAP3 cosmological parameters, we plot the ratio of the shifted to higher-k for higher-loop

RegPT in modified gravity

Good convergence is ensured by a generic damping behavior in propagators $\Gamma^{(n)} \stackrel{k\to\infty}{\longrightarrow} \Gamma^{(n)}_{\text{tree}} e^{-k^2 \sigma_{\text{d}}^2/2}$

Even in modified gravity, well-controlled expansion with RegPT

¹ (*k*), measured in *N*-body simulations at *z* = 0, 0*.*5, 1, and 2. Left and right