Computing CMB bispectra

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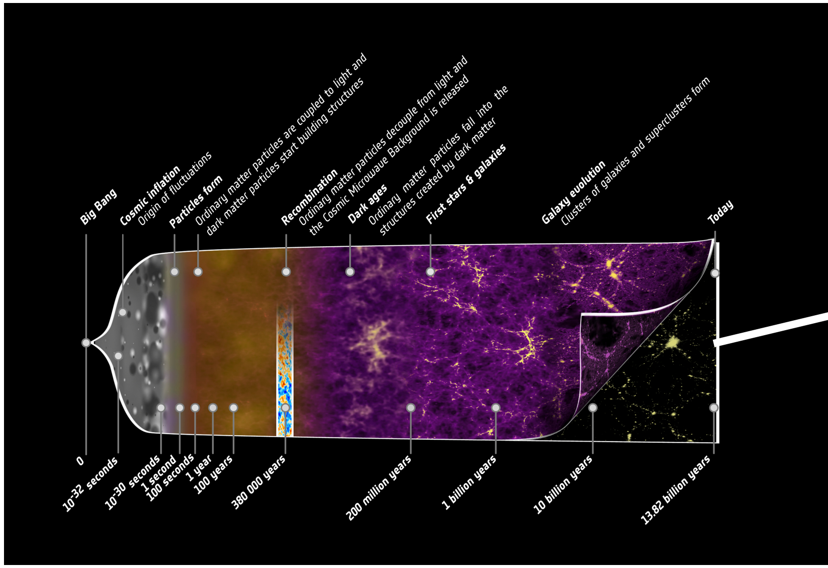
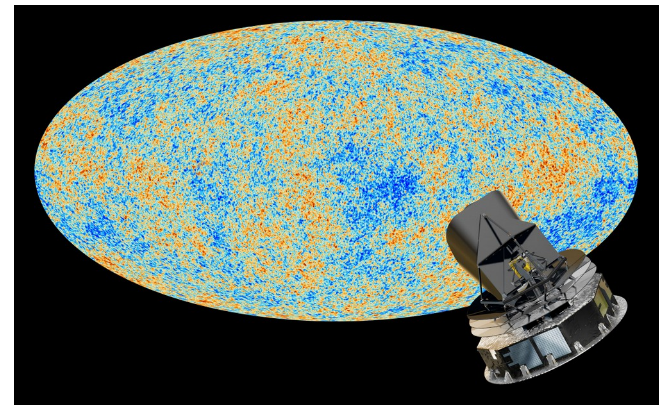


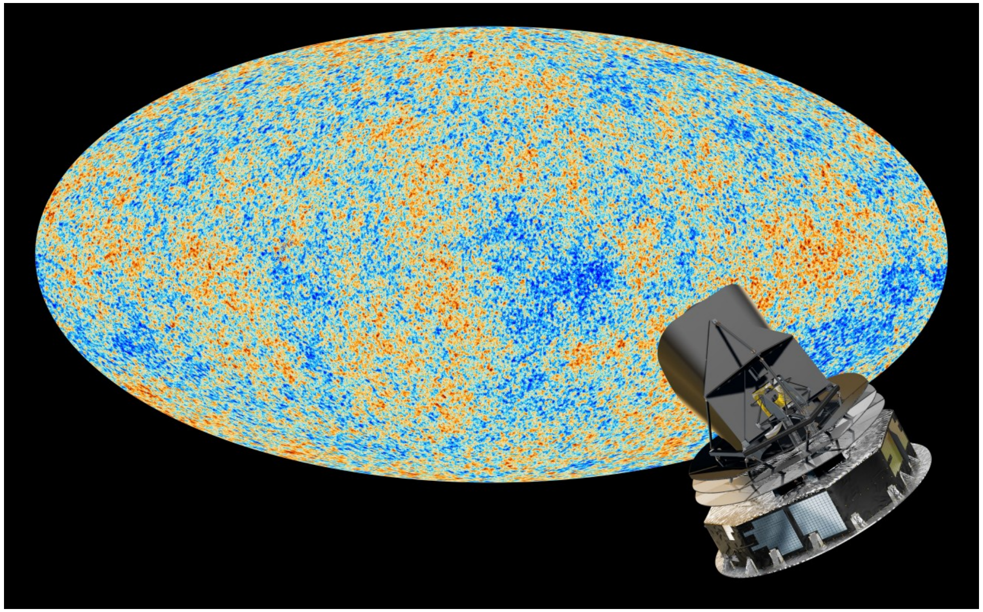
Microwave radiation from last-scattering surface

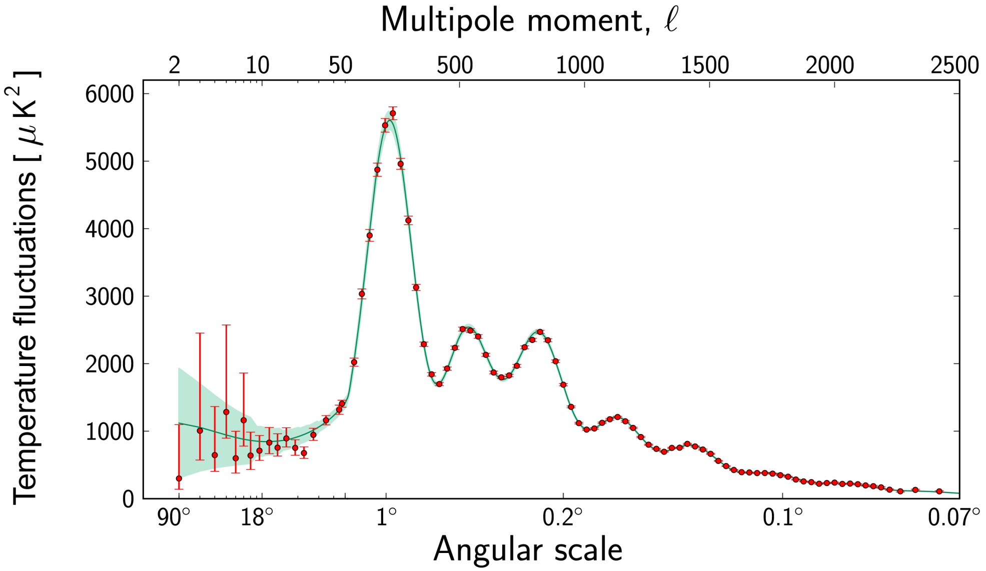


- Almost isotropic
- Almost complete blackbody radiation with $2.726\mathrm{K}$ (cf. COBE)

- Tiny anisotropic fluctuations induced by quantum fluctuations in inflation

$$\Theta \equiv \frac{\delta T(t, \boldsymbol{k})}{T} \sim \mathcal{O}(10)\mu\mathrm{K}$$





http://www.sciops.esa.int

Theoretical prediction ⟺ Observational data

for example...

2-point function (Power spectrum) of primordial curvature perturbation

$$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\rangle = (2\pi)^3 P_\zeta(k_1)\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2)$$

$$P_\zeta(k) = A\left(\frac{k}{k_*}\right)^{n_s-1}$$

$$A = 2.196^{+0.080}_{-0.078}\times 10^{-9}$$
$$n_s = 0.968 \pm 0.006$$

Planck Collaboration, arXiv://1502.01589

Non-Gaussianity

3-point function (Bispectrum)

$$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = (2\pi)^3 B_\zeta(k_1,k_2,k_3)\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)$$

If the PDF of ζ is Gaussian,

$$P(\zeta) = \frac{1}{\sqrt{2\pi\sigma^2}}e^{-\zeta^2/2\sigma^2} \quad \Rightarrow \quad \langle\zeta\zeta\zeta\rangle = 0$$

Bispectrum measures the non-Gaussianity, deviation from Gaussian PDF

Gaussian : $\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = 0$

Non-Gaussian : $\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle \neq 0$

Parametrised by $f_{\rm NL}$,

$$f_{\rm NL}^{(i)} = \frac{(B_\zeta \cdot B^{{\rm temp}(i)})}{(B^{{\rm temp}(i)} \cdot B^{{\rm temp}(i)})}$$

Bispectrum

How is non-zero bispectrum signal produced ?
→ Non-linear process

- primordial

  Higher-order operators of inflaton Lagrangian

  Maldacena, JHEP0305 (2003) 013 [arXiv:astro-ph/0210603]
  e.g. Domenech et al. JCAP05 (2017) 034 [arXiv:1701.05554]

- post-inflation

  * Non-linearity of Einstein's equation and fluid equations.
  * Non-linear collision term

    Bartolo, Matarrese, Riotto, JCAP 0606 (2006) 024 [astro-ph/0604416]
    Pitrou, CQG 26 (2009) 065006 [arXiv:0809.3036]
    Beneke, Fidler, PRD82 (2010) 063509 [arXiv:1003.1834]

  * Non-linear propagation

    Fidler, Koyama, Pettinari, JCAP 04 (2015) 037 [arXiv:1409.2461]
    R.Saito, Naruko, Hiramatsu, Sasaki, JCAP10(2014)051 [arXiv:1409.2464]

  etc...

Primordial origin

Maldacena, JHEP0305 (2003) 013 [arXiv:astro-ph/0210603]

$$S = \int \left[M_{\rm pl}^2 R - \partial_\mu \phi \partial^\mu \phi - V(\phi) \right] \sqrt{-g}\, d^4x$$

$$R \ni \zeta, N_i, h_{ij} \qquad \phi(t, \boldsymbol{x}) = \phi_0(t) + \varphi(t, \boldsymbol{x})$$

$$S = \int \mathcal{L}(\zeta, N_i, h_{ij}, \varphi)\, d^4x$$

in-in formalism

Bispectrum from slow-roll inflaiton

$$B_\zeta(k_1, k_2, k_3) = \frac{(2\pi)^4}{8} \mathcal{P}_\zeta \frac{1}{\prod k_i^3} \left[(3\epsilon - 2\eta) \sum_i k_i^3 + \epsilon \sum_{i \neq j} k_i k_j^2 + 8\epsilon \frac{\sum_{i>j} k_i^2 k_j^2}{k_1 + k_2 + k_3} \right]$$

Primordial origin

Temperature anisotropy bispectrum

$$a_{\ell m} = 4\pi(-i)^\ell \int \frac{d^3k}{(2\pi)^3}\zeta(\boldsymbol{k})\mathcal{T}_\ell(k)Y^*_{\ell m}(\hat{k})$$

$$B^{m_1m_2m_3}_{\ell_1\ell_2\ell_3} := \langle a_{\ell_1m_1}a_{\ell_2m_2}a_{\ell_3m_3}\rangle$$
$$\propto \int \frac{d^3k_1d^3k_2d^3k_3}{(2\pi)^9}\cdots\langle\zeta(\boldsymbol{k}_1)\zeta(\boldsymbol{k}_2)\zeta(\boldsymbol{k}_3)\rangle$$

↑

$$\langle\zeta(\boldsymbol{k}_1)\zeta(\boldsymbol{k}_2)\zeta(\boldsymbol{k}_3)\rangle = (2\pi)^3\delta(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)B_\zeta(k_1,k_2,k_3)$$

Reduced bispectrum

$$B_{\ell_1\ell_2\ell_3} := \sum_{m_1m_2m_3}\begin{pmatrix}\ell_1 & \ell_2 & \ell_3\\ m_1 & m_2 & m_3\end{pmatrix}B^{m_1m_2m_3}_{\ell_1\ell_2\ell_3}$$
$$= \left(\frac{2}{\pi}\right)^3\sqrt{\frac{(2\ell_1+1)(2\ell_2+1)(2\ell_3+1)}{16\pi}}\begin{pmatrix}\ell_1 & \ell_2 & \ell_3\\ 0 & 0 & 0\end{pmatrix}$$
$$\times\int dk_1dk_2dk_3\,k_1^2k_2^2k_3^2\mathcal{T}^{(S)}_{\ell_1}(k_1)\mathcal{T}^{(S)}_{\ell_2}(k_2)\mathcal{T}^{(S)}_{\ell_3}(k_3)J_{\ell_1\ell_2\ell_3}(k_1,k_2,k_3)$$

$$\left(J_{\ell_1\ell_2\ell_3}(k_1,k_2,k_3) = \int dr\,r^2 j_{\ell_1}(k_1)j_{\ell_2}(k_2)j_{\ell_3}(k_3)\right)$$

Tensor-Scalar-Scalar case

Domenech et al. JCAP05 (2017) 034 [arXiv:1701.05554]

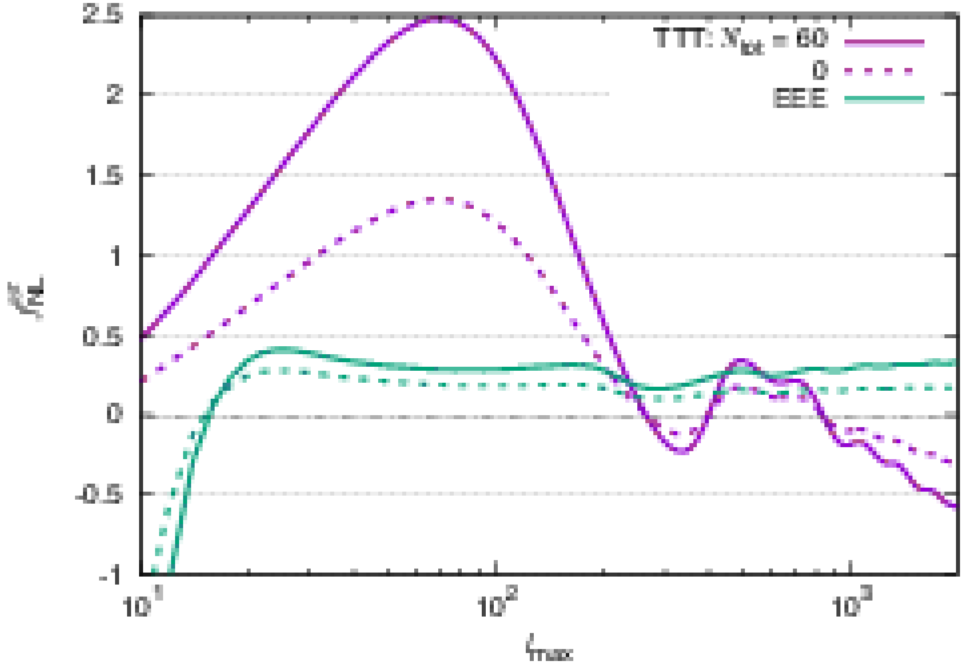
$$\langle \gamma^s(\boldsymbol{k}_1)\zeta(\boldsymbol{k}_2)\zeta(\boldsymbol{k}_3)\rangle = (2\pi)^3\delta(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)F_\zeta(k_1,k_2,k_3)e^{-s}_{ij}(\hat{k}_1)\hat{k}_{2i}\hat{k}_{3j}$$

$$e^{-s}_{ij}\hat{k}_{2i}\hat{k}_{3j} = \frac{(8\pi)^{3/2}}{6}\sum_{Mmm'} Y^{s*}_{2M}(\hat{k}_1)Y^*_{1m}(\hat{k}_2)Y^*_{1m'}(\hat{k}_3)\begin{pmatrix} 2 & 1 & 1 \\ M & m & m' \end{pmatrix}$$

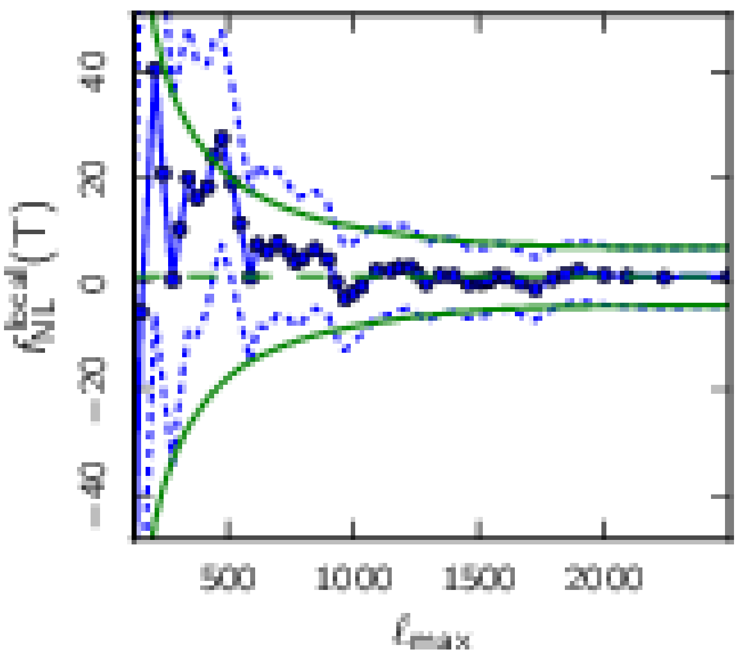
$$\begin{aligned} B^{(TSS)}_{\ell_1\ell_2\ell_3} &= \frac{4}{3}\left(\frac{8}{\pi}\right)^{3/2}(-i)^{\ell_1+\ell_2+\ell_3} \\ &\times \sum_{L_1L_2L_3} i^{L_1+L_2+L_3}(I^{2-20}_{\ell_1 2 L_1} + I^{-220}_{\ell_1 2 L_1})I^{000}_{\ell_2 1 L_2}I^{000}_{\ell_3 1 L_3}I^{000}_{L_1L_2L_3}\begin{Bmatrix} \ell_1 & \ell_2 & \ell_3 \\ L_1 & L_2 & L_3 \\ 2 & 1 & 1 \end{Bmatrix} \\ &\times \int dk_1dk_2dk_3\, k_1^2k_2^2k_3^2\mathcal{T}^{(T)}_{\ell_1}(k_1)\mathcal{T}^{(S)}_{\ell_2}(k_2)\mathcal{T}^{(S)}_{\ell_3}(k_3)F(k_1,k_2,k_3)J_{\ell_1\ell_2\ell_3}(k_1,k_2,k_3) \end{aligned}$$

Shiraishi et al., PTP125 (2011) 795 [arXiv:1012.1079]
Shiraishi, thesis [arXiv:1210.2518]

Tensor-Scalar-Scalar case

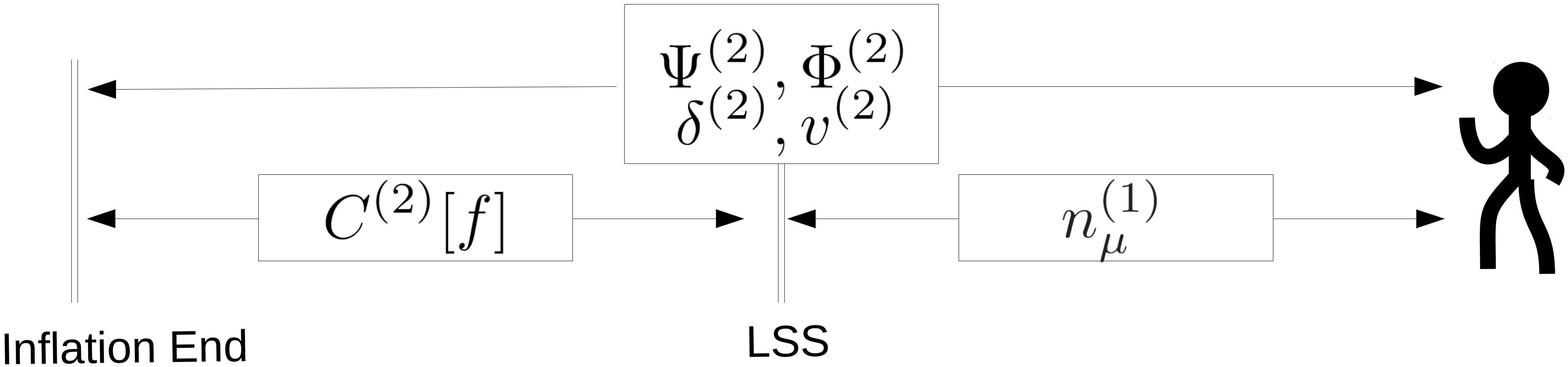


Domenech et al. JCAP05 (2017) 034 [arXiv:1701.05554]



Planck collaboration, A&A594 (2016) A17 [arXiv:1502.01592]]

Post-inflationary origin



$$\Theta := \frac{\delta f}{f} \quad \Rightarrow \quad \frac{d\Theta}{d\eta} = \frac{\partial \Theta}{\partial \eta} + p^\mu \frac{\partial \Theta}{\partial x_\mu} + \frac{dp^\mu}{d\eta}\frac{\partial \Theta}{\partial p_\mu} = C[f]$$

$\Psi^{(2)}, \Phi^{(2)}$ $\qquad$ $C^{(2)}[f]$

$$\Theta_\ell(k, \eta_0) = \int_0^{\eta_0} S(k, \eta) j_\ell[k(\eta_0 - \eta)]\, d\eta$$

$n_\mu^{(1)}$

Other sources : $\Delta = 4\Theta + 6\Theta\Theta$ $\qquad$ $n_e^{(1)}(\eta)$

Bispectrum templates

Quantify the magnitude of NG

$$B_\Theta(k_1,k_2,k_3) = \sum_i f_{\rm NL}^{(i)} B^{(i)}(k_1,k_2,k_3)$$

signals templates

Bispectrum templates

$$B_\Phi^{\rm local}(k_1,k_2,k_3) = 2 f_{\rm NL}^{\rm local}\left[P_\Phi(k_1)P_\Phi(k_2) + 2\ {\rm perms}\right]$$

Gangui et al., APJ 430 (1994) 447
Verde et al., MNRAS 313 (2000) L141
Komatsu, Spergel, PRD63 (2001) 063002

$$B_\Phi^{\rm equil} = 6 f_{\rm NL}^{\rm equil}\Big[-\{P_\Phi(k_1)P_\Phi(k_2) + 2\ {\rm perms}\} - 2P_\Phi(k_1)^{2/3}P_\Phi(k_2)^{2/3}P_\Phi(k_3)^{2/3} + P_\Phi(k_1)^{1/3}P_\Phi(k_2)^{2/3}P_\Phi(k_3) + 5\ {\rm perms}\Big]$$

Babich et al., JCAP 0408 (2004) 009

$$B_\Phi^{\rm ortho} = 6 f_{\rm NL}^{\rm ortho}\Big[-3\{P_\Phi(k_1)P_\Phi(k_2) + 2\ {\rm perms}\} - 8P_\Phi(k_1)^{2/3}P_\Phi(k_2)^{2/3}P_\Phi(k_3)^{2/3} + 3\left\{P_\Phi(k_1)^{1/3}P_\Phi(k_2)^{2/3}P_\Phi(k_3) + 5\ {\rm perms}\right\}\Big]$$

Senatore et al., JCAP 1001 (2010) 028

$$B_\Phi^{\rm folded} = 6 f_{\rm NL}^{\rm folded}\Big[\{P_\Phi(k_1)P_\Phi(k_2) + 2\ {\rm perms}\} + 3P_\Phi(k_1)^{2/3}P_\Phi(k_2)^{2/3}P_\Phi(k_3)^{2/3} - \left\{P_\Phi(k_1)^{1/3}P_\Phi(k_2)^{2/3}P_\Phi(k_3) + 5\ {\rm perms}\right\}\Big]$$

Chen et al., JCAP 0701 (2007) 002

Komatsu, Spergel, PRD63 (2001) 063002

Using the least-square method, we determine the fitting parameter $f_{\rm NL}^{(i)}$ so that

$$\chi^2 \equiv \sum_{2\le\ell_1\le\ell_2\le\ell_3}^{\ell_{\rm max}} \frac{\left(B_{\ell_1\ell_2\ell_3} - \sum_i f_{\rm NL}^{(i)} B^{(i)}_{\ell_1\ell_2\ell_3}\right)^2}{\sigma^2_{\ell_1\ell_2\ell_3}}$$

is minimised.

- local-type
- equilateral-type
- orthogonal-type

$$F^{ij} \equiv \sum_{2\le\ell_1\le\ell_2\le\ell_3} \frac{B^{(i)}_{\ell_1\ell_2\ell_3} B^{(j)}_{\ell_1\ell_2\ell_3}}{\sigma^2_{\ell_1\ell_2\ell_3}}$$

$$G^{j} \equiv \sum_{2\le\ell_1\le\ell_2\le\ell_3} \frac{B_{\ell_1\ell_2\ell_3} B^{(j)}_{\ell_1\ell_2\ell_3}}{\sigma^2_{\ell_1\ell_2\ell_3}}$$

$$f_{\rm NL}^{(i)} = (F^{-1})^{ij} G^j$$

e.g. Propagation effect yields $f_{\rm NL}^{\rm local} = 9.3$ $f_{\rm NL}^{\rm equil} = -2.4$ Hanson et al., PRD 80 (2009) 083004

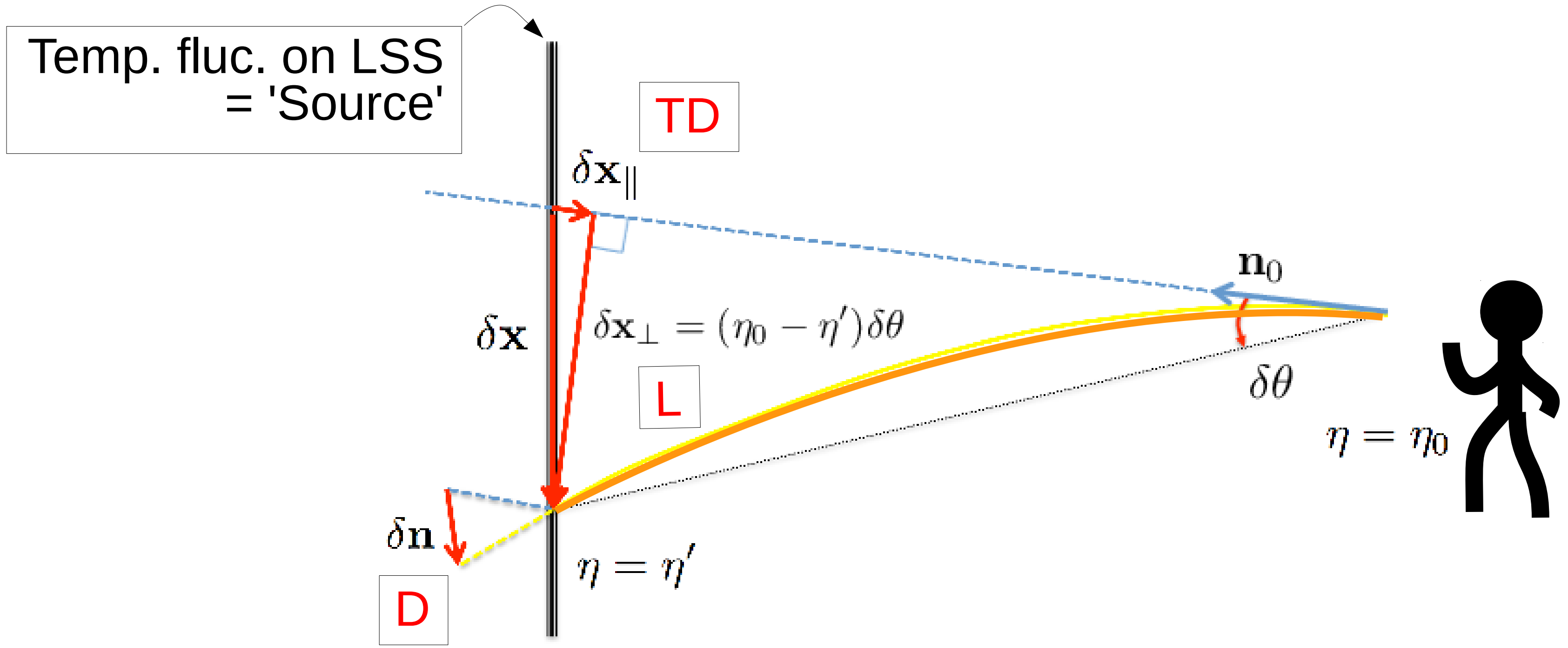
$$f_{\rm NL}^{\rm loc(T)} \approx 10.2 \pm 5.7 \ \rightarrow\ 2.5 \pm 5.7 \qquad f_{\rm NL}^{\rm loc(T+E)} \approx 6.5 \pm 5.0 \ \rightarrow\ 0.8 \pm 5.0$$

$$f_{\rm NL}^{\rm equ(T)} \approx -13 \pm 70 \ \rightarrow\ -16 \pm 70 \qquad f_{\rm NL}^{\rm equ(T+E)} \approx 3 \pm 43 \ \rightarrow\ -4 \pm 43$$

Planck collaboration, A&A594 (2016) A17 [arXiv:1502.01592]]

Non-linear nature of geodesics

R.Saito, Naruko, Hiramatsu, Sasaki, JCAP10(2014)051 [arXiv:1409.2464]



$\Theta_{\ell}^{\mathrm{T}(2)}$ is sourced by

- 1st -order x Lensing (= ISW-Lensing)
- 1st -order x Time-delay
- 1st -order x Deflection

(Time-delay and Deflection: quite tiny...)

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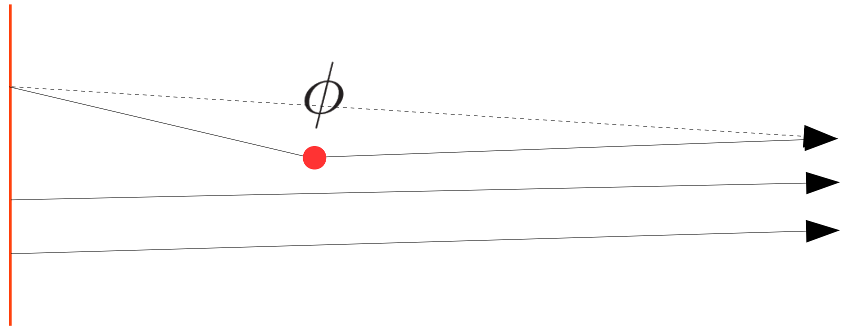
Remapping approach



Lensing potential

$$\phi(\mathbf{n}) = -2\int dD\, g_\phi(D)\Psi(\mathbf{x}, D)$$

Goldberg, Spergel, PRD 59 (1999) 103002
Hu, PRD 62 (2000) 043007
Zaldarriaga, PRD 62 (2000) 063510
Review : Lewis, Challinor, PR 429 (2006) 1

Lensed photon is expanded in terms of lensing potential

$$\widetilde{\Theta}(\mathbf{n}) \approx \Theta(\mathbf{n}) + \nabla_i\phi(\mathbf{n})\nabla^i\Theta(\mathbf{n}) + \cdots$$

Leading contribution to lensing bispectrum

Hanson et al., PRD 80 (2009) 083004

$$B_{\ell_1\ell_2\ell_3} \approx \langle \widetilde{\Theta}^T_{\ell_1}\Theta^T_{\ell_2}\Theta^T_{\ell_3}\rangle = C^{T\phi}_{\ell_1}C^{TT}_{\ell_2} + \text{ 5 perms.}$$

→

$$f_{\rm NL}^{\rm local} = 9.3$$
$$f_{\rm NL}^{\rm equil} = -2.4$$

CMB lensing

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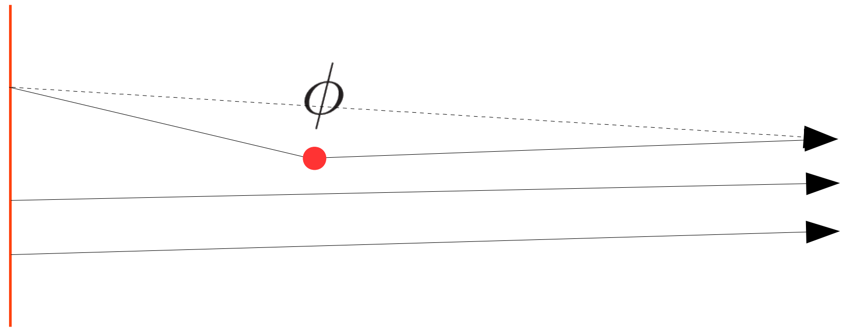
Deflection angle



$$\widetilde{\Theta}(\hat{n}) \equiv \Theta(\hat{n}_i + d_i)$$

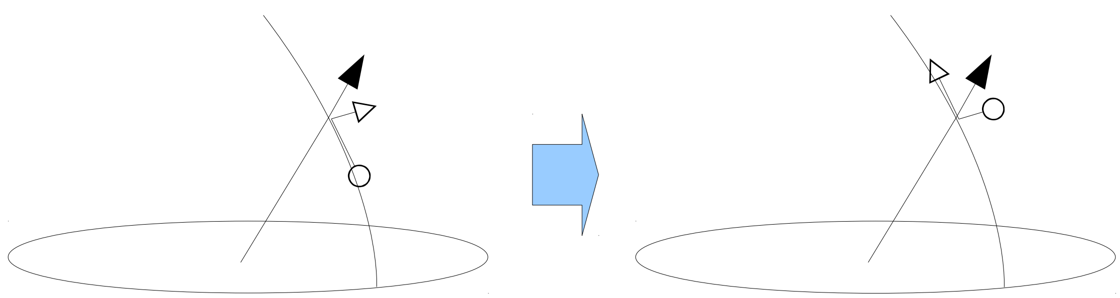
$$\Theta(\hat{n})$$

$$\Theta(\hat{n})$$

gradient mode   curl mode

$$d_i = \nabla_i \phi(\hat{n}) + \star \nabla_i \varpi(\hat{n}) \qquad \left(\nabla = \hat{e}_\theta \frac{\partial}{\partial \theta} + \hat{e}_\varphi \frac{1}{\sin\theta}\frac{\partial}{\partial \varphi}\right)$$

$$\star :\ \hat{e}_\theta \rightarrow \hat{e}_\varphi, \hat{e}_\phi \rightarrow -\hat{e}_\theta$$



Easy to generalise : $\widetilde{\Theta}(\hat{n}) \longrightarrow \widetilde{X}(\hat{n}) \equiv X(\hat{n}_i + d_i)$

$X = \Theta, E, B$ for scalar, vector, tensor modes

CMB lensing

Taylor expansion

$$\widetilde{X}(\hat{n}) = X(\hat{n}) + d_i(\hat{n})\nabla^i X(\hat{n}) + \frac{1}{2}d_i(\hat{n})d_j(\hat{n})\nabla^i\nabla^j X(\hat{n}) + \cdots$$

Harmonic expansion

$$_sX_{\ell m} = \int d\hat{n}\, X(\hat{n})\, {}_sY^*_{\ell m}(\hat{n})$$

$$X(\hat{n}) = \sum_{\ell m} {}_sX_{\ell m}\, {}_sY_{\ell m}(\hat{n})$$

Same for ϕ, ϖ

Should perform the integrations like $\int d\hat{n}\, \nabla_i Y_{\ell' m'}(\hat{n}) \nabla^i {}_sY_{\ell'' m''}(\hat{n})\, {}_sY^*_{\ell m}(\hat{n})$

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Derivative → Harmonics

$$\nabla_i Y_{\ell m} = {}_0L_\ell^+ {}_1Y_{\ell m} m_i^+ + {}_0L_\ell^- {}_{-1}Y_{\ell m} m_i^-$$

$$i \star \nabla_i Y_{\ell m} = -{}_0L_\ell^+ {}_1Y_{\ell m} m_i^+ + {}_0L_\ell^- {}_{-1}Y_{\ell m} m_i^-$$

$$\boldsymbol{m}^\pm = \frac{1}{\sqrt{2}}(\hat{e}_\theta \mp i\hat{e}_\varphi) \qquad \star\boldsymbol{m}^\pm = \pm i\boldsymbol{m}^\pm$$

$${}_sL_\ell^\pm = \pm\sqrt{\frac{(\ell \mp s)(\ell \pm s + 1)}{2}}$$

In general, n[th]-order derivatives can be decomposed into a sum of products of ${}_sY_{\ell m}$

$$\nabla_{i_1}\nabla_{i_1}\cdots\nabla_{i_1} Y^{(s)} = \sum_{\mathrm{all}\{p_i\}} \left(\prod_{k=1}^{n} L^{p_k}_{(s+\sum_{i=1}^{k-1} p_i)}\right) Y^{(s+\sum_{i=1}^{k} p_i)} m_{i_1}^{p_1} m_{i_2}^{p_2} \cdots m_{i_n}^{p_n}$$

Integration of products of harmonics

$$\int d\hat{n}\, \nabla_i Y_{\ell' m'}(\hat{n}) \nabla^i {}_s Y_{\ell'' m''}(\hat{n}) {}_s Y^*_{\ell m}(\hat{n})$$

$$= \sum (\text{product of } {}_s L^{\pm}_{\ell}) \underbrace{\int d\hat{n}\, {}_s Y^*_{\ell m}(\hat{n}) {}_{s'} Y_{\ell' m'}(\hat{n}) {}_{s''} Y_{\ell'' m''}(\hat{n})}$$

$$(-1)^{s+m} \sqrt{\frac{(2\ell+1)(2\ell'+1)(2\ell''+1)}{4\pi}} \begin{pmatrix} \ell & \ell' & \ell'' \\ s & -s' & -s'' \end{pmatrix} \begin{pmatrix} \ell & \ell' & \ell'' \\ -m & m' & m'' \end{pmatrix}$$

Wigner's 3j symbol
(=Clebsch-Gordan coeff.)

Non-zero if $\begin{cases} s - s' - s'' = 0 \\ |\ell - \ell'| \le \ell'' \le \ell + \ell' \end{cases}$

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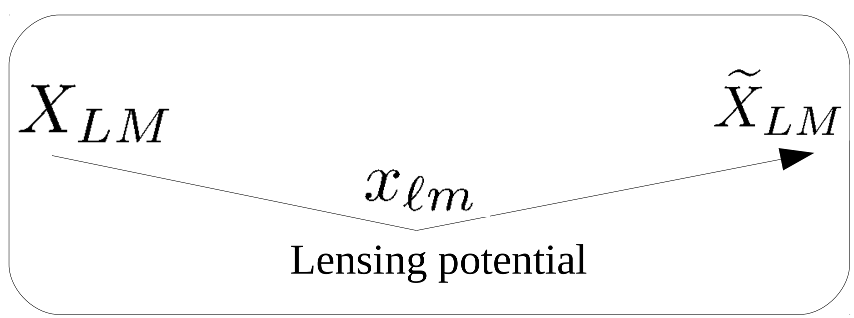
Lensed signal

$$\widetilde{X}_{LM} = X_{LM} + \sum_{\ell\ell' mm' \overline{X} x} \mathcal{M}^{L\ell\ell';x;X\overline{X}}_{Mmm'} x_{\ell m} \overline{X}_{\ell' m'}$$

$$+\frac{1}{2} \sum_{\ell\ell'\ell'' mm'm'' \overline{X} xy} \mathcal{M}^{L\ell\ell'\ell'';xy;X\overline{X}}_{Mmm'm''} x_{\ell m} y_{\ell' m'} \overline{X}_{\ell'' m''} + \cdots$$

$$X = \Theta, E, B$$

$$x = \phi, \varpi$$



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Coefficient at second-order

$$\mathcal{M}^{L\ell\ell';x;X\overline{X}}_{Mmm'} = \begin{pmatrix} J^{\{L\};x(0)}_{\{M\}} & 0 & 0 \\ 0 & J^{\{L\};x(+)}_{\{M\}} & -J^{\{L\};x(-)}_{\{M\}} \\ 0 & J^{\{L\};x(-)}_{\{M\}} & J^{\{L\};x(+)}_{\{M\}} \end{pmatrix} \begin{matrix} (X) \\ \Theta \\ E \\ B \end{matrix}$$

$$(\overline{X}) \quad \Theta \quad E \quad B$$

$$J^{L\ell\ell';x(s)}_{Mmm'} = (-1)^M \begin{pmatrix} L & \ell & \ell' \\ -M & m & m' \end{pmatrix} S^{(s)x}_{L\ell\ell'}$$

$$S^{(0)\varpi}_{L\ell\ell'} = c_{L\ell\ell'}\overline{e}\mathcal{S}^{(0)\varpi}_{L\ell\ell'}$$
$$S^{(0)\phi}_{L\ell\ell'} = c_{L\ell\ell'}e\mathcal{S}^{(0)\phi}_{L\ell\ell'}$$
$$S^{(+)\varpi}_{L\ell\ell'} = c_{L\ell\ell'}\overline{e}\mathcal{S}^{\varpi}_{L\ell\ell'}$$
$$S^{(+)\phi}_{L\ell\ell'} = c_{L\ell\ell'}e\mathcal{S}^{\phi}_{L\ell\ell'}$$
$$S^{(-)\varpi}_{L\ell\ell'} = c_{L\ell\ell'}e\mathcal{S}^{\varpi}_{L\ell\ell'}$$
$$S^{(-)\phi}_{L\ell\ell'} = c_{L\ell\ell'}\overline{e}\mathcal{S}^{\phi}_{L\ell\ell'}$$

$$e\mathcal{S}^{(0)\phi}_{L\ell\ell'} = [-L(L+1)+\ell(\ell+1)+\ell'(\ell'+1)]\begin{pmatrix} L & \ell & \ell' \\ 0 & 0 & 0 \end{pmatrix}$$

$$\overline{e}\mathcal{S}^{(0)\varpi}_{L\ell\ell'} = 2\sqrt{\ell(\ell+1)\ell'(\ell'+1)}\begin{pmatrix} L & \ell & \ell' \\ 0 & -1 & 1 \end{pmatrix}$$

$$\mathcal{S}^{\phi}_{L\ell\ell'} = [-L(L+1)+\ell(\ell+1)+\ell'(\ell'+1)]\begin{pmatrix} L & \ell & \ell' \\ \pm 2 & 0 & \mp 2 \end{pmatrix}$$

$$\mathcal{S}^{\varpi}_{L\ell\ell'} = \mathcal{S}^{\phi}_{L\ell\ell'} + 2\sqrt{\ell(\ell+1)(\ell'-1)(\ell'+2)}\begin{pmatrix} L & \ell & \ell' \\ 2 & -1 & -1 \end{pmatrix}$$

$$c_{\ell\ell' L} = \sqrt{\frac{(2\ell+1)(2\ell'+1)(2L+1)}{16\pi}}$$

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Lensed spectra

$$\langle X^*_{LM} Y_{\hat{L}\hat{M}} \rangle = C^{XY}_L \delta_{L\hat{L}} \delta_{M\hat{M}}$$

$$\langle X_{L_1M_1} Y_{L_2M_2} Z_{L_3M_3} \rangle = B^{XYZ}_{L_1L_2L_3;M_1M_2M_3}$$

$$\langle W_{L_1M_1} X_{L_2M_2} Y_{L_3M_3} Z_{L_4M_4} \rangle = T^{WXYZ}_{L_1L_2L_3L_4;M_1M_2M_3M_4}$$

Reduced spectra

$$B^{XYZ}_{L_1L_2L_3} = \sum_{M_1M_2M_3} \begin{pmatrix} L_1 & L_2 & L_3 \\ M_1 & M_2 & M_3 \end{pmatrix} B^{XYZ}_{L_1L_2L_3;M_1M_2M_3}$$

$$T^{WXYZ}_{L_1L_2L_3L_4;L} = (2L+1) \sum_{\{M_i\}M} (-1)^M \begin{pmatrix} L_1 & L_2 & L \\ M_1 & M_2 & M \end{pmatrix} \begin{pmatrix} L_3 & L_4 & L \\ M_3 & M_4 & -M \end{pmatrix} \times T^{WXYZ}_{L_1L_2L_3L_4;M_1M_2M_3M_4}$$

Lensed spectra

$$C_L^{\widetilde{X}\widetilde{Y}} = \frac{1}{2L+1}\sum_{\ell\ell'}\sum_{xy\overline{X}\overline{Y}} M_{L\ell\ell'}^{X\overline{X},x}\left(M_{L\ell\ell'}^{Y\overline{Y},y}C_{\ell'}^{\overline{X}\overline{Y}}C_{\ell}^{xy} + (-1)^{L+\ell+\ell'}M_{L\ell'\ell}^{Y\overline{Y},y}C_{\ell'}^{\overline{X}y}C_{\ell}^{\overline{Y}x}\right)$$

$$B_{L_1L_2L_3}^{\widetilde{X}YZ,s_1s_2s_3} = \sum_{\overline{X}x}\Big(M_{L_1L_3L_2}^{X\overline{X},x}C_{L_2}^{\overline{X}Y(s_2)}C_{L_3}^{xZ(s_3)}\delta_{s_1s_2} + (-1)^{L_1+L_2+L_3}M_{L_1L_2L_3}^{X\overline{X},x}C_{L_3}^{\overline{X}Z(s_3)}C_{L_2}^{xY(s_2)}\delta_{s_1s_3}\Big)$$

$$\begin{aligned}\frac{1}{2L+1}T_{L_1L_2L_3L_4;L}^{\widetilde{W}\widetilde{X}YZ} &= \delta_{L_1L_2}\delta_{L_3L_4}\delta_{L0}(-1)^{L_1+L_3}\sqrt{\frac{2L_3+1}{2L_1+1}}\sum_{\ell_1\ell_2}\sum_{wx\overline{W}\overline{X}}\left[\mathcal{U}^{(1)WX;wx,\overline{W}\overline{X},YZ}_{L_1\ell_1\ell_2,L_2\ell_1\ell_2;\ell_1\ell_2L_3} + \mathcal{U}^{(0)WX;w\overline{X},x\overline{W},YZ}_{L_1\ell_1\ell_2,L_2\ell_2\ell_1;\ell_1\ell_2L_3}\right] \\ &+\sum_{\ell_1}\sum_{wx\overline{W}\overline{X}}\left[\mathcal{U}^{(3)WX;wx,\overline{W}Z,\overline{X}Y}_{L_1\ell_1L_4,L_2\ell_1L_3;\ell_1L_4L_3} + \mathcal{U}^{(2)WX;w\overline{X},xY,\overline{W}Z}_{L_1\ell_1L_4,L_2L_3\ell_1;\ell_1L_3L_4} + \mathcal{U}^{(0)WX;wZ,\overline{W}\overline{X},xY}_{L_1L_4\ell_1,L_2L_3\ell_1;L_4\ell_1L_3} + \mathcal{U}^{(1)WX;wZ,x\overline{W},\overline{X}Y}_{L_1L_4\ell_1,L_2\ell_1L_3;L_4\ell_1L_3}\right]\begin{Bmatrix}L_1 & L_2 & L \\ L_3 & L_4 & \ell_1\end{Bmatrix}, \\ &+\sum_{\ell_1}\sum_{wx\overline{W}\overline{X}}\left[\mathcal{U}^{(3)WX;wx,\overline{W}Y,\overline{X}Z}_{L_1\ell_1L_3,L_2\ell_1L_4;\ell_1L_3L_4} + \mathcal{U}^{(2)WX;w\overline{X},xZ,\overline{W}Y}_{L_1\ell_1L_3,L_2L_4\ell_1;\ell_1L_4L_3} + \mathcal{U}^{(0)WX;wY,\overline{W}\overline{X},xZ}_{L_1L_3\ell_1,L_2L_4\ell_1;L_3\ell_1L_4} + \mathcal{U}^{(1)WX;wY,x\overline{W},\overline{X}Z}_{L_1L_3\ell_1,L_2\ell_1L_4;L_3\ell_1L_4}\right]\begin{Bmatrix}L_1 & L_2 & L \\ L_4 & L_3 & \ell_1\end{Bmatrix}\end{aligned}$$

$$\mathcal{U}^{(0)AB;\lambda_1\lambda_2,\lambda_3\lambda_4,\lambda_5\lambda_6}_{p_1p_2p_3,q_1q_2q_3;r_1r_2r_3} := M^{A\overline{A},a}_{p_1p_2p_3}M^{B\overline{B},b}_{q_1q_2q_3}C^{\lambda_1\lambda_2}_{r_1}C^{\lambda_3\lambda_4}_{r_2}C^{\lambda_5\lambda_6}_{r_3}$$

Lensed spectra

Observed bispectra

$$\widehat{B}^{XYZ,s_1s_2s_3}_{L_1L_2L_3} = B^{\widetilde{X}YZ,s_1s_2s_3}_{L_1L_2L_3} + B^{X\widetilde{Y}Z,s_1s_2s_3}_{L_1L_2L_3} + B^{XY\widetilde{Z},s_1s_2s_3}_{L_1L_2L_3}$$

$$\begin{aligned}\widehat{B}^{XYZ,s_1s_2s_3(211)}_{L_1L_2L_3} = &\sum_{\overline{X}x}\left[M^{X\overline{X},x}_{L_1L_3L_2}C^{\overline{X}Y(s_2)}_{L_2}C^{xZ(s_3)}_{L_3}\delta_{s_1s_2} + (Y \leftrightarrow Z)\right] \\ &+ \sum_{\overline{X}x}\left[M^{Y\overline{X},x}_{L_2L_1L_3}C^{\overline{X}Z(s_3)}_{L_3}C^{xX(s_1)}_{L_1}\delta_{s_2s_3} + (X \leftrightarrow Z)\right] \\ &+ \sum_{\overline{X}x}\left[M^{Z\overline{X},x}_{L_3L_2L_1}C^{\overline{X}X(s_1)}_{L_1}C^{xY(s_2)}_{L_2}\delta_{s_1s_3} + (X \leftrightarrow Y)\right]\end{aligned}$$

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Relevant combinations

$$\widehat{B}^{XYZ,sss'}_{L_1L_2L_3} \qquad (XY) = (\Theta\Theta, \Theta E, \Theta B, EE, EB, BB)$$
$$(Z) = (\Theta, E, B)$$

$$\widehat{B}^{XYZ,sss}_{L_1L_2L_3} \qquad (XYZ) = (\Theta\Theta\Theta, \Theta\Theta E, \Theta\Theta B, \Theta EE, \Theta EB, \Theta BB, EEE, EEB, EBB, BBB)$$

Taking into account the fact that scalar mode yields no B,
we have totally 102 kinds of bispectra.

Current status of CMB2ND

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- Linear power spectra of Scalar/Vector/Tensor Θ/E/B-modes
- Linear power spectra of Gradient/Curl-modes induced by S/V/T perturbations
- Lensed power spectra and bispectra of all possible combinations up to 2×2 and $2 \times 1 \times 1$ (50 C_ℓ's and 102 $B_{\ell_1 \ell_2 \ell_3}$'s)
- $f_{\rm NL}$ estimator for Local/Equilateral/Orthogonal/Folded templates

Test results and discussion

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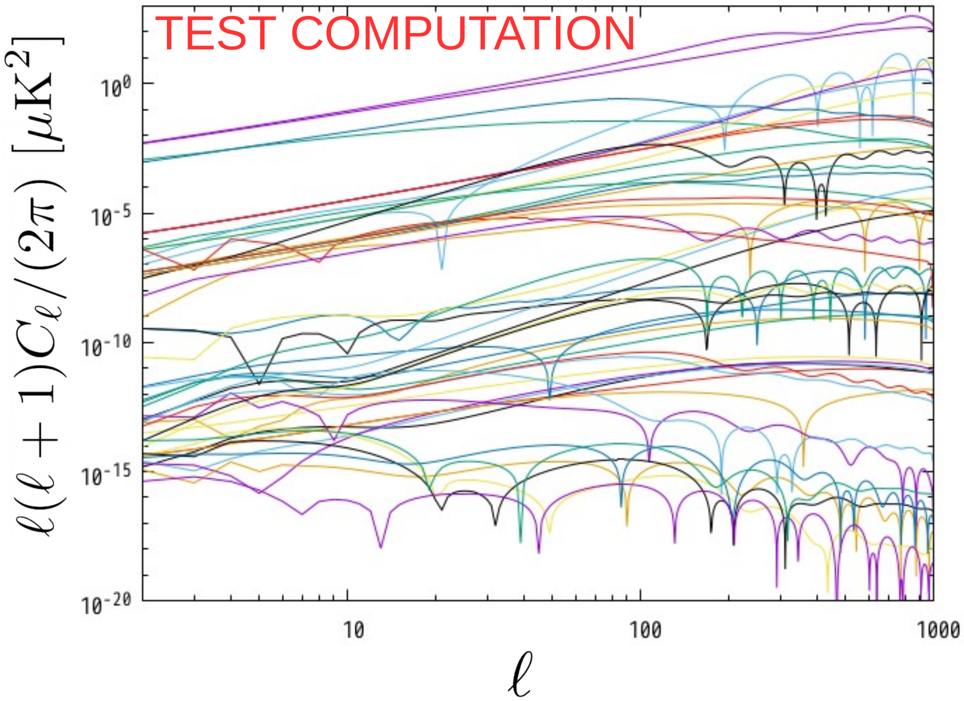
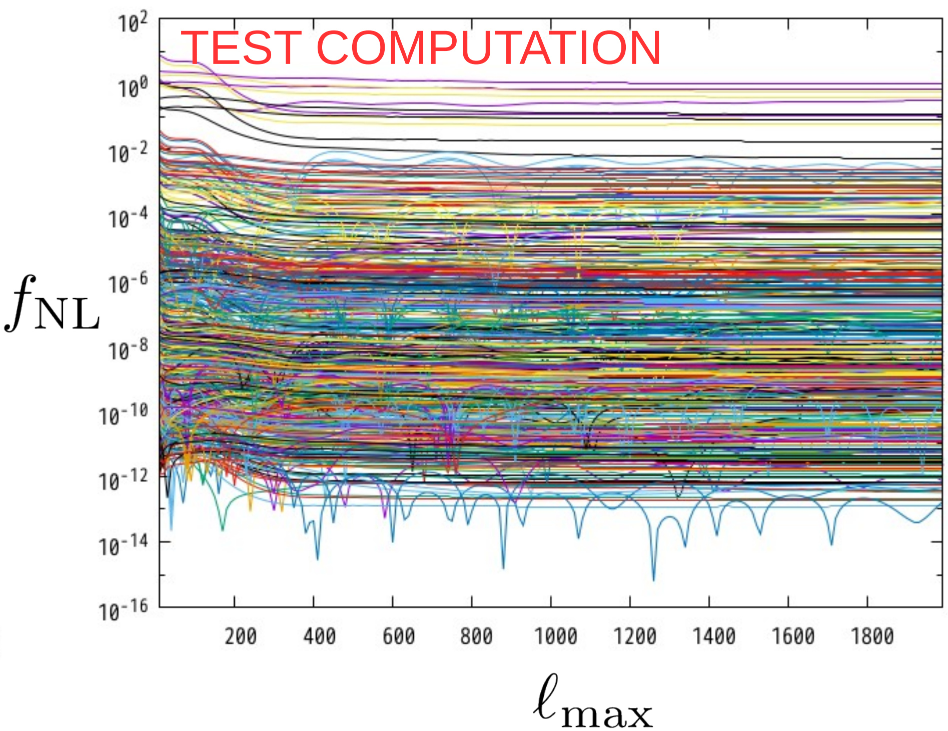
50 power spectra



102 bispectra × 4 templates



Using CMB2nd, we study...

- Cosmic strings, inducing the unequal-time correlation $\mathcal{P}(k, \eta_1, \eta_2)$

e.g. Daveiro et al., PRD93 (2016) 085014, arXiv:1510.05006

- Improve lensing potential estimator and delensing scheme ?
- Prove beyond-GR effects through CMB lensing ?