MODIFIED GRAVITY AND MATTER BISPECTRUM

Shin'ichi Hirano Rikkyo U. D1

based on

SH, T. Kobayashi, S. Yokoyama (Rikkyo U.), T. Hiroyuki (Nagoya U.) **1801. 07885**

SH, T. Kobayashi, D. Yamauchi (Kanagawa U.), S. Yokoyama, in preparation

バイスペクトル研究会 2018年 3/29 ~ 30

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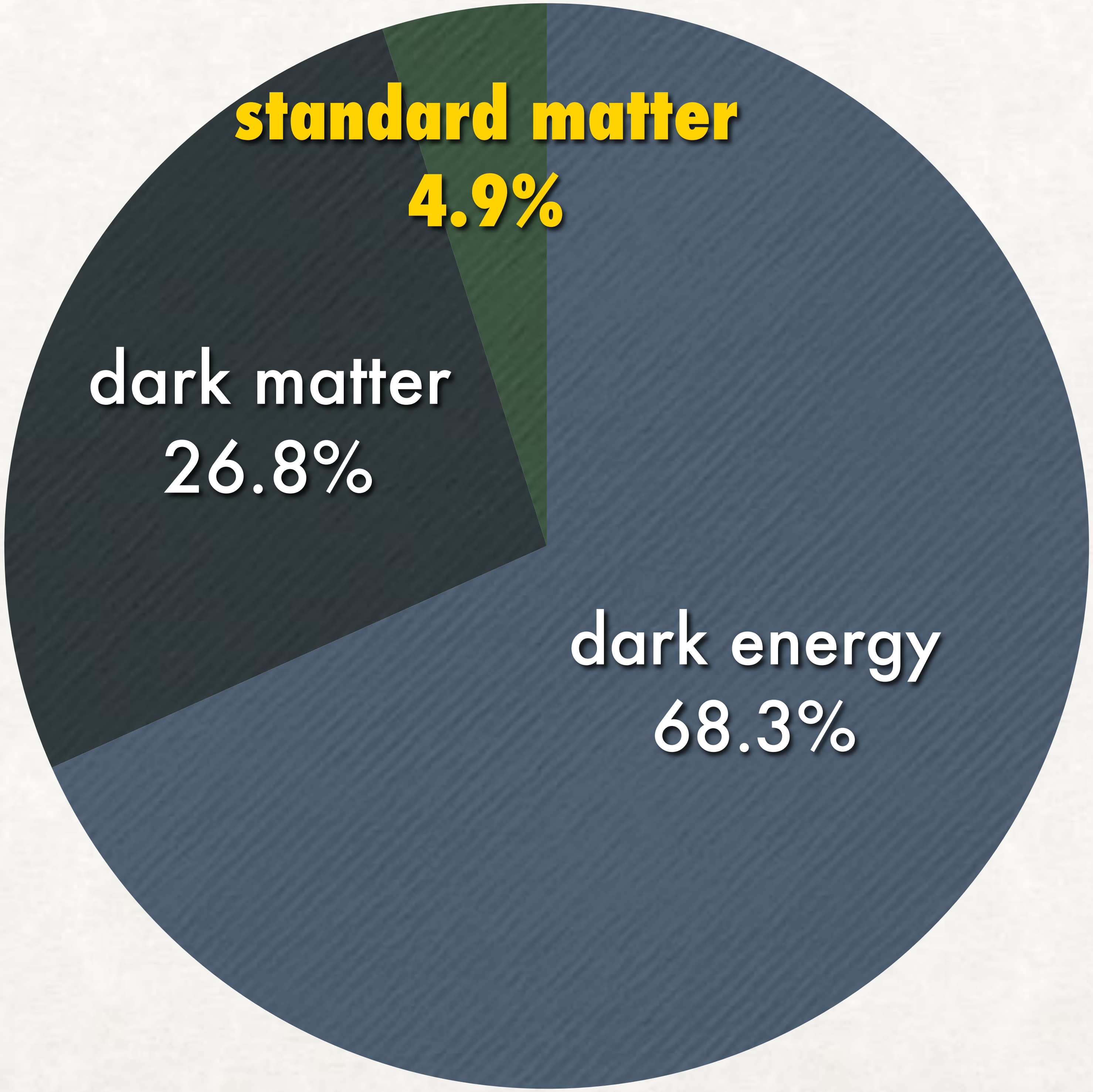
Plan of talk

- Modified Gravity: review and progress, our work
- Our setup
- Cosmological perturbations
- Matter bispectrum in MG with Vainshtein screening
- Summary

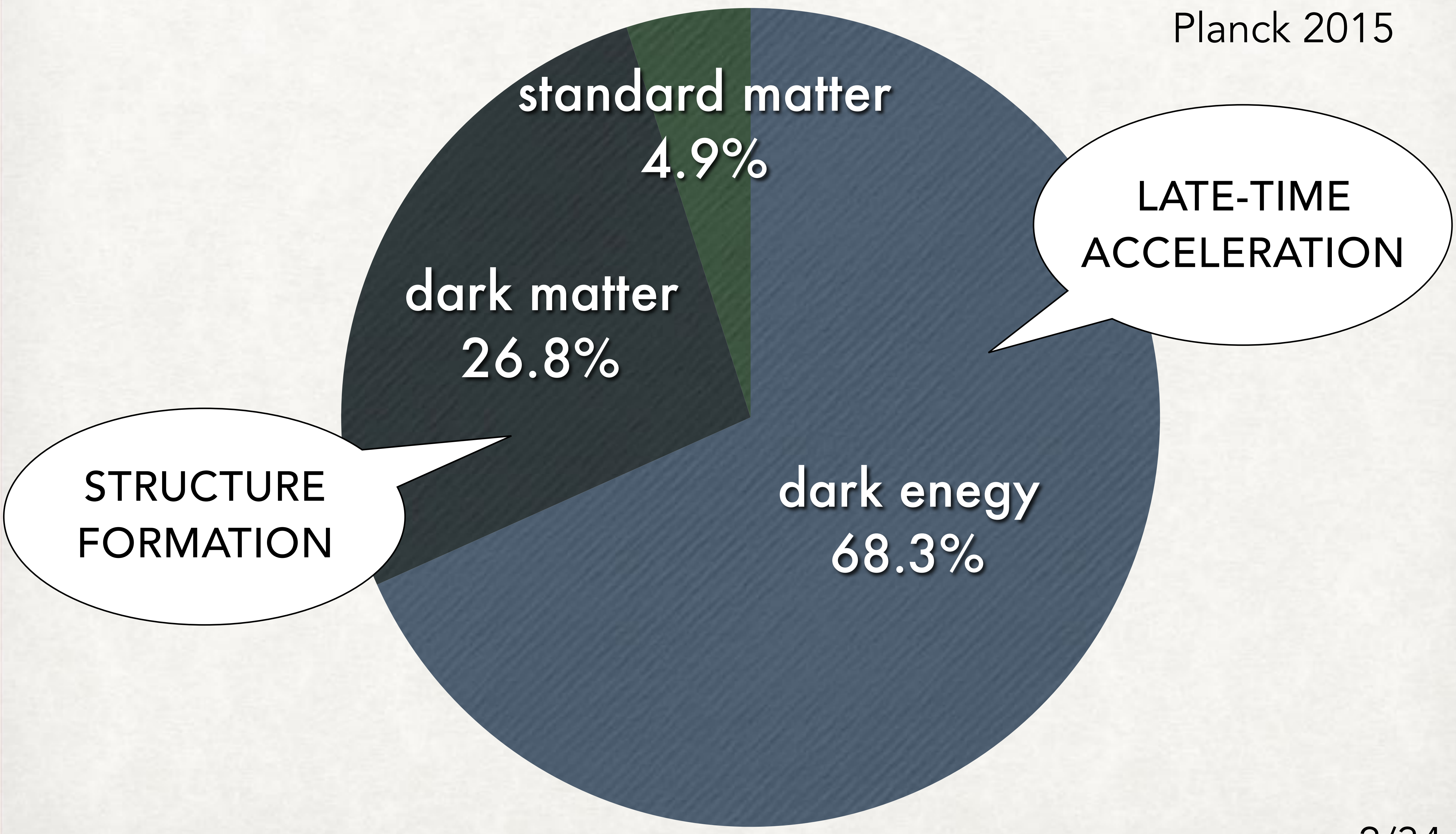
MODIFIED GRAVITY: REVIEW & PROGRESS, AND OUR WORK

Components of universe

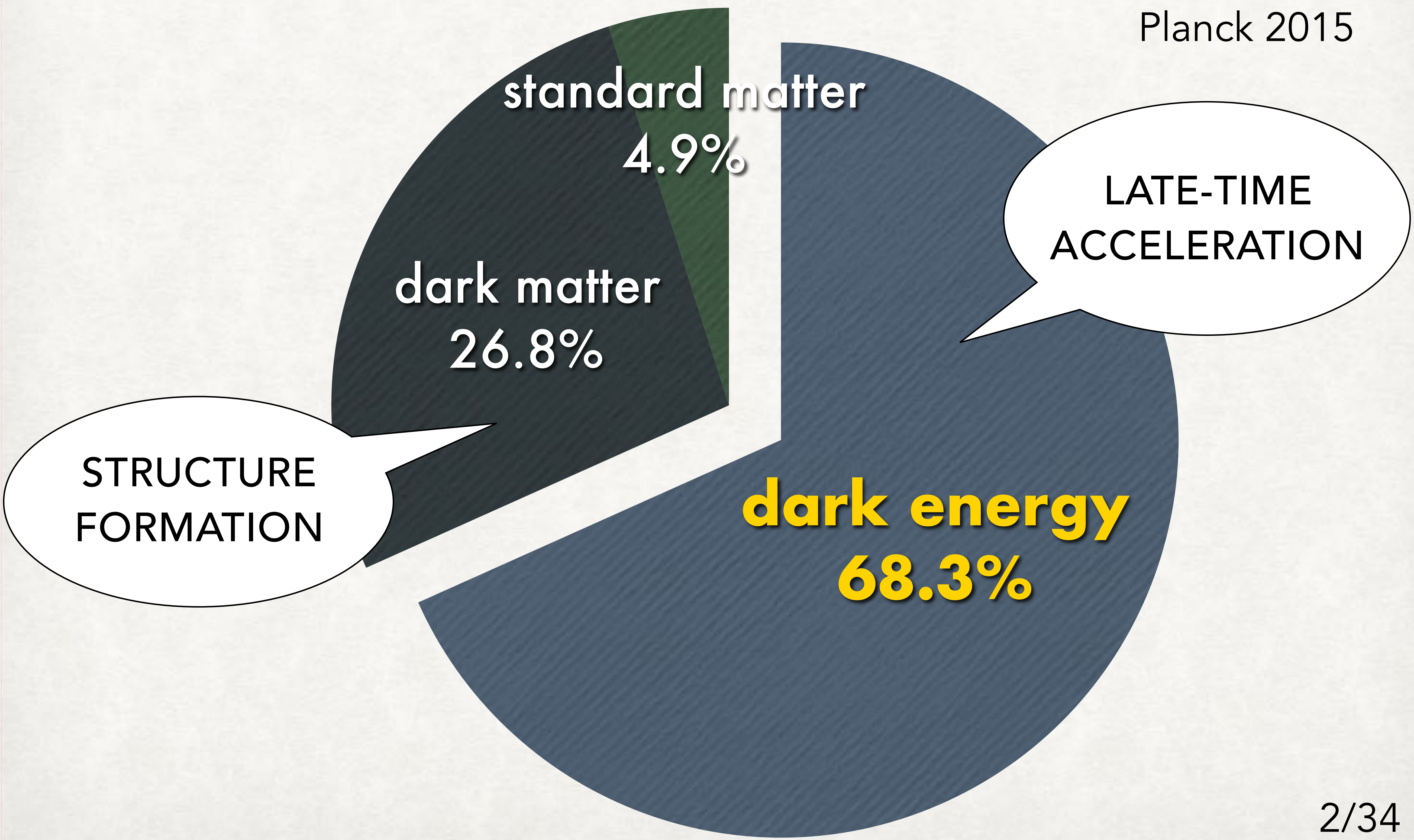
Planck 2015



Components of universe



Components of universe



Simplest candidate of DE

- Simplest candidate: **Cosmological Constant** Λ

$$G_{\mu\nu} = M_{\rm pl}^{-2}(T_{\mu\nu} - \Lambda g_{\mu\nu})$$

- Cosmological background:

$$3M_{\rm pl}^2 H^2 = \Lambda$$

$$-M_{\rm pl}^2(3H^2 + 2\dot{H}) = -\Lambda$$

$\Rightarrow$

acceleration sol. $a \propto e^{Ht}$ $\omega = -1$

"de Sitter solution"

$w_{\rm de} = -1.00^{+0.04}_{-0.05}$ (wCDM model)

DES collaboration (1708)

C. C. problem

Weinberg (1989)

cf) Martin (2012), Padilla (2015)

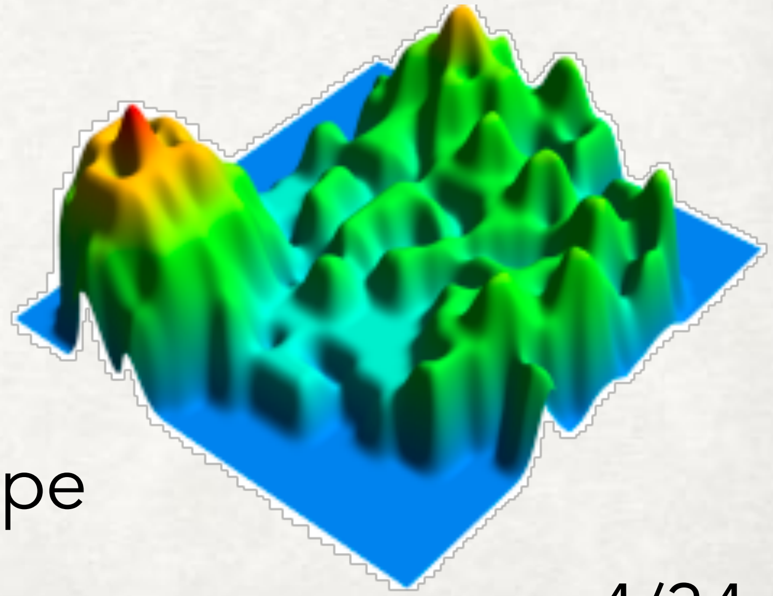
- General Relativity (classical field theory of metric)

cosmological constant ⇔ vacuum expectation value

$$\rho_{\rm vac} = \frac{1}{2}\int \frac{d^3k}{(2\pi)^3}\sqrt{|\boldsymbol{k}|^2 + m^2} \sim M_{\rm pl}^4$$

$$\Rightarrow \quad \Omega_{\rm vac0} = \frac{\rho_{\rm vac}}{3M_{\rm pl}^2 H_0^2} \sim 10^{123} h^{-2}$$

Observation: $\Omega_{\Lambda 0} \sim 0.7$ **not consistent**



- We need anthropic principle? cf) string landscape

C. C. problem

Weinberg (1989)

cf) Martin (2012), Padilla (2015)

Dark energy scenarios

- **Exotic matter (so-called dark energy)**

Quitessence, k-essence int. between SM? relation to UV physics?

$$V(\phi) \propto 1/\phi^n$$

- **Modified Gravity** cf) Lovelock theorem (1971)

~ extra degree of freedom ex) **Scalar-Tensor theory**

✓ Cosmological scale: late-time acceleration, structure formation

✓ Small scale: satisfying test of gravity Perihelion motion of Mercury, Shapiro time delay

"**Screening mechanism**"

Frame transformation

- ex) f(R) gravity

(Jordan) $S^{\rm J} = \int d^4x \sqrt{-\tilde{g}} f(\tilde{R}) + S_{\rm m}[\tilde{g}_{\mu\nu}, \psi]$

equivalent ! $\tilde{g}_{\mu\nu} = A^2(\phi) g_{\mu\nu}$ (conformal trans.)

(Einstein) $S^{\rm E} = \int d^4x \sqrt{-g} \left[\frac{M_{\rm pl}^2}{2} R - \frac{1}{2}(\partial\phi)^2 - V(\phi) \right] + S_{\rm m}[A(\phi)^2 g_{\mu\nu}, \psi]$

Extra d.o.f. couples to matter in most MG models.

Screening mechanism

- Static spherically symm. + weak field

$$\mathcal{L} = -\frac{1}{2} Z^{\mu\nu}(\phi, \partial\phi, \partial\partial\phi)\nabla_\mu\phi\nabla_\nu\phi - V(\phi) + \beta(\phi)T$$

Large kinetic term $Z^{\mu\nu}$

・Kamouflage (1st-order deri.)

・Vainshtein (2nd order deri.)

Lagre mass $V(\phi)$

・Chameleon

$$\phi \propto \frac{e^{-mr}}{r}$$

Small coupling $\beta(\phi)$

・Symmetron

・Dilaton

Screening mechanism

- Static spherically symm. + weak field

$$\mathcal{L} = -\frac{1}{2} Z^{\mu\nu}(\phi, \partial\phi, \partial\partial\phi)\nabla_\mu\phi\nabla_\nu\phi - V(\phi) + \beta(\phi)T$$

Large kinetic term $Z^{\mu\nu}$

- Kamouflage (1st-order deri.)
- **Vainshtein (2nd order deri.)**

✓ Suppression during early universe

Chow, Khoury (2009)

✓ Appearance at EFT

ex) Massive gravity

Vainshtein mechanism

Vainshtein (1972)

- **ex) Cubic Galileon, DGP**

(Einstein) $$\mathcal{L}_\varphi = -\frac{1}{16\pi G}\left[(\partial\varphi)^2 + \frac{r_\mathrm{c}^2}{2}(\partial\varphi)^2\Box\varphi\right] - \varphi T$$

2nd order, **non-linear**

SSS, weak field $\Rightarrow$ $$\left(\frac{\partial_r\varphi}{r}\right) + r_\mathrm{c}^2\left(\frac{\partial_r\varphi}{r}\right)^2 = \frac{r_g}{r^3}\ ,\ r_g = 2GM$$

Vainshtein mechanism

Vainshtein (1972)

- **ex) Cubic Galileon, DGP**

(Einstein) $$\mathcal{L}_\varphi = -\frac{1}{16\pi G}\left[1 + \frac{r_c^2}{2}\Box\varphi\right](\partial\varphi)^2 - \varphi T$$

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Vainshtein mechanism

Vainshtein (1972)

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(Einstein) $$\mathcal{L}_\varphi \sim -\frac{1}{2}(\partial\phi)^2 - \cancel{\frac{\phi}{M_{\rm Pl}\sqrt{1+r_{\rm c}^2\Box\phi/(2M_{\rm Pl})}}}T$$ ← **huge !**

$$r_{\rm c} \sim H_0^{-1}$$

SSS, weak field $\Rightarrow$ $$\left(\frac{\partial_r\varphi}{r}\right) + r_{\rm c}^2\left(\frac{\partial_r\varphi}{r}\right)^2 = \frac{r_g}{r^3}\ ,\ r_g = 2GM$$

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(Einstein) $$\mathcal{L}_\varphi \sim -\frac{1}{2}(\partial\phi)^2 - \cancel{\frac{\phi}{M_{\rm Pl}\sqrt{1+r_{\rm c}^2\Box\phi/(2M_{\rm Pl})}}}T$$ ⟵ **huge !**

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SSS, weak field $\Rightarrow$ $$\left(\frac{\partial_r\varphi}{r}\right) + r_{\rm c}^2\left(\frac{\partial_r\varphi}{r}\right)^2 = \frac{r_g}{r^3}\ ,\ r_g = 2GM$$

$$\partial_r\varphi \sim \frac{r_g}{r^2} \sim \partial_r\Phi \quad (r \gg r_V)$$

$$\partial_r\varphi \sim \cancel{\frac{r_g}{r^2}\left(\frac{r}{r_V}\right)^{3/2}} \ll \partial_r\Phi \quad (r \ll r_V)$$

suppress!

Vainshtein radius

$$r_V = (r_g r_{\rm c}^2)^{1/3}$$

$$r_V = \mathcal{O}(1)\mathrm{Mpc}$$ (cluster)

Horndeski theory

Horndeski (1972), Kobayashi+ (2011), Deffaiyet+ (2011)

$$\frac{\mathcal{L}}{\sqrt{-g}} = G_2(\phi, X) - G_3(\phi, X)\Box\phi$$
$$+ G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - \phi_{\mu\nu}^2\right]$$
$$+ G_5(\phi, X)G^{\mu\nu}\phi_{\mu\nu} - \frac{G_{5X}}{6}\left[(\Box\phi)^3 - 3(\Box\phi)\phi_{\mu\nu}^2 + 2\phi_{\mu\nu}^3\right]$$

($\nabla_\mu\phi = \phi_\mu, \nabla_\nu\nabla_\mu\phi = \phi_{\mu\nu}, \phi^\mu{}_\mu = \Box\phi$)

✓ Most general ST theory with 2nd order EoM (no Ostrogradski ghost)

✓ **Vainshtein screening**

✓ Propagation speed of graviton changes from that of photon

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Horndeski theory

Horndeski (1972), Kobayashi+ (2011), Deffaiyet+ (2011)

$$\frac{\mathcal{L}}{\sqrt{-g}} = G_2(\phi, X) - G_3(\phi, X)\Box\phi$$

much constrained?

$$+ G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - \phi_{\mu\nu}^2\right]$$

$$+ G_5(\phi, X)G^{\mu\nu}\phi_{\mu\nu} - \frac{G_{5X}}{6}\left[(\Box\phi)^3 - 3(\Box\phi)\phi_{\mu\nu}^2 + 2\phi_{\mu\nu}^3\right]$$

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✓ Propagation speed of graviton changes from that of photon

$\Rightarrow$ $|c_T - 1| < 10^{-15}$ (GW170817)

Recent progress & our work

- **Beyond Horndeski** (higher-order EoM, no Ostrogradski ghost)

GLPV theory Gleyzes+ (2014), Gleyzes+ (2015)

DHOST theory Langlois & Noui (2015), Achour+ (2016), Achour+ (2016)

✓ Models with $(\partial\partial\phi)^2$ and $c_T = 1$

✓ Partially breaking of Vainshtein screening inside matter ($\delta > 1$)

non-linear int.

Kobayashi+ (2015), Langlois+ (2017), …

Recent progress & our work

- **Beyond Horndeski** (higher-order EoM, no Ostrogradski ghost)

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non-linear int. Kobayashi+ (2015), Langlois+ (2017), …

Our aim

How much is the effect of non-linear int. at "*cosmological scale*" ?

$\delta \ll 1$

Matter bispectrum, beyond Horndeski

cf) f(R), DGP Koyama+ (2009), Horndeski Takushima+ (2013)

OUR SETUP

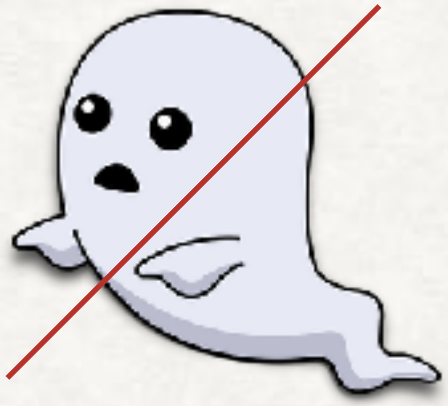
Degenerate theory

Langlois, Noui (2015,2016), Koyama+ (2016)

- **Higher derivatives (more than 2nd order)**

✓ Ostrogradski ghost … extra d.o.f due to higher derivatives

✓ degeneracy condition … evading Ostrogradski ghost



Trivially degenerate: Horndeski

Non-trivially degenerate: **beyond Horndeski** (GLPV, DHOST)

quadratic DHOST

Langlois, Noui (2015,2016), Koyama+ (2016), de Rham, Matas (2016)

$$\frac{\mathcal{L}_{\rm qD}}{\sqrt{-g}} = G_2(\phi, X) - G_3(\phi, X)\Box\phi + G_4(\phi, X)R + C_{(2)}^{\mu\nu\rho\sigma}\phi_{\mu\nu}\phi_{\rho\sigma}$$

$$C_{(2)}^{\mu\nu\rho\sigma}\phi_{\mu\nu}\phi_{\rho\sigma} = a_1(\phi, X)\phi_{\mu\nu}^2 + a_2(\phi, X)(\Box\phi)^2 + a_3(\phi, X)\Box\phi(\phi^\mu\phi_{\mu\nu}\phi^\nu)$$
$$+ a_4(\phi, X)\phi^\mu\phi_{\mu\nu}\phi^{\nu\rho}\phi_\rho + a_5(\phi, X)(\phi^\mu\phi_{\mu\nu}\phi^\nu)^2$$

($X = -\frac{1}{2}(\nabla\phi)^2, \phi_\mu = \nabla_\mu\phi, \Box\phi = \nabla^2\phi, \phi_{\mu\nu} = \nabla_\nu\nabla_\mu\phi$)

non-trivial
non-linear ints.

✓ includes Horndeski and GLPV at Lagrangian level

Horndeski: $a_1 = -a_2 = -G_{4X},\ a_3 = a_4 = a_5 = 0$

GLPV: $\dots$

✓ degeneracy condition ⇒ the relation between arbitrary funcs. $(G_4, C_{(2)})$

Our setup

- stable cosmological solution: **class I qD** de Rham & Matas (2016)

$$S_{\mathrm{classI}}[\tilde{g},\phi]+S_{\mathrm{m}}[\tilde{g},\psi] \longleftrightarrow S_{\mathrm{H}}[g,\phi]+S_{\mathrm{m}}[\tilde{g},\psi]$$

$$\tilde{g}_{\mu\nu}=\Omega(\phi,X)g_{\mu\nu}+\Gamma(\phi,X)\phi_\mu\phi_\nu$$

conformal/ disformal coupling to matter

- $c_T=1$(all eras)

- No $\phi^3_{\mu\nu}$: ① viable Vainshtein solutions could not exist?

  Horndeski × Kimura et al. (2011)

  ② the speed of GWs directly depends on BG dynamics

Parametrization

Bellini & Sawicki (2014), Gleyzes et al. (2015)
Langlois+ (2017), Dima & Vernizzi (2017)

$$S^{\rm eff} = \int d^4x \sqrt{\gamma}\, \frac{M^2}{2} \Bigg[-\mathcal{K}_2 + c_T^2 R^{(3)} + H^2 \alpha_K \delta N^2 + 4H\alpha_B \delta K \delta N$$

$$+ (1+\alpha_H) R^{(3)} \delta N + (1-\alpha_H) \delta N \delta \mathcal{K}_2 + 4\beta_1 \delta K \tilde{V} + \beta_2 \tilde{V}^2 + \beta_3 a_i^2 \Bigg].$$

$\mathcal{K}_2 := K_{ij}^2 - K^2,\ \tilde{V} := \frac{1}{N}(\dot{N} - N^i \partial_i N),\ a_i := \partial_i N/N$

depend on β_1
through degeneracy cond.

- **Parameters**

α_K : kineticity … non-standard kinetic terms

α_B : braiding … kinetic mixing between scalar and metric

α_M : time evolution of M

α_H : disformal coupling to matter → **GLPV**

β_1 : conformal & disformal coupling to matter → **DHOST**

Cosmological models

	α_K	α_B	α_M	α_H	β_1
ΛCDM					
Quintessence	✓				
KGB, cubic Galileon	✓	✓			
f(R)		✓	✓		
Generalized Brans-Dicke	✓	✓	✓		
Horndeski after GW	✓	✓	✓		
GLPV	✓	✓	✓	✓	✓
DHOST	✓	✓	✓	✓	✓

Cosmological models

with Vainshtein screening	α_K	α_B	α_M	α_H	β_1
ΛCDM					
Quintessence	✓				
KGB, cubic Galileon	✓	✓			
f(R)		✓	✓		
Generalized Brans-Dicke	✓	✓	✓		
Horndeski after GW	✓	✓	✓		
GLPV	✓	✓	✓	✓	✓
DHOST	✓	✓	✓	✓	✓

Cosmological models

with Vainshtein screening	α_K	α_B	α_M	α_H	β_1
ΛCDM					
Quintessence	✓				
KGB, cubic Galileon	✓	✓			
f(R)		✓	✓		
Generalized Brans-Dicke	✓	✓	✓		
Horndeski after GW	✓	✓	✓		
GLPV	✓	✓	✓	✓	✓
DHOST	✓	✓	✓	✓	✓

Viable cosmology

Kimura et al. (2011)
Kobayashi et al. (2015)

- **Early time**

Inflation

RD era

MD era

- **Late time**

late-time acceleration

primordial fluc. ~ Gaussian

CMB photon (ISW)

matter density fluc.,
structure formation

Viable cosmology

Kimura et al. (2011)
Kobayashi et al. (2015)

- **Early time**

Inflation → RD era → MD era → late-time acceleration

primordial fluc. ~ Gaussian

※ **We do not consider early DE scenario quintessential inflation.**

- **Late time**

CMB photon (ISW)

matter density fluc.,
structure formation

MG affects here

Viable conditions

Kimura et al. (2011)
Kobayashi et al. (2015)

- **Early time**

$M_{\rm pl}^2/2$ (GR)

$$M^2 \approx 2G_4 := \mathcal{O}(M_{\rm pl}^2),\ \ (\alpha_i, \beta_1) \ll 1$$

$$\Rightarrow\ \ 3M^2H^2 \approx \rho_{\rm m},\ \ \rho_{\rm m} \gg \rho_\phi$$, cosmological Vainshtein

- **Late time** (after MD)

$$\phi \sim M_{\rm pl},\ \ \dot{\phi} \sim M_{\rm pl}H_0,\ \ \ddot{\phi} \sim M_{\rm pl}H_0^2,$$
$$G_2 \sim M_{\rm pl}^2 H_0^0,\ \ G_3 \sim M_{\rm pl},\ \ G_4 \sim M_{\rm pl}^2,\ \ \cdots$$

$$\Rightarrow\ \ 3M^2H^2(1-\alpha_H-3\beta_1) \approx \rho_\phi \gg \rho_{\rm m},\ \ \alpha_i = \mathcal{O}(1),\ \ \beta_1 = \mathcal{O}(1)$$

Vainshtein screening around matter, **its breaking inside matter**

DENSITY FLUCTUATIONS

Hirano+ in prep.

Cosmological perturbations

Sub-horizon ($aH \ll k$) , late time (after MD)

$$Q = H\pi/\dot{\phi}$$

- **perturbations**

$$ds^2 = -(1+2\Phi)dt^2 + a^2(t)(1-2\Psi)d\boldsymbol{x}^2.$$

$$\phi(t,\boldsymbol{x}) = \phi(t) + \pi(t,\boldsymbol{x}),\ \ \rho(t,\boldsymbol{x}) = \rho(t)[1+\delta(t,\boldsymbol{x})].$$

- **Quasi-static approximation (QSA)** ← ansatz

$$k_{sh} := \frac{aH}{c_s} \ll k \quad \Rightarrow \quad |\dot{\epsilon}| \approx |H\epsilon|,\ \ \epsilon = \Psi, \Phi, Q$$

$$\cancel{|\dot{\Psi}|^2, |\dot{\Phi}|^2, |\dot{Q}|^2} \ll k^2\Psi^2, k^2\Phi^2, k^2Q^2$$

Note: $0 \neq \alpha_i \ll 1 \quad \Rightarrow H^2\epsilon^2 \sim \alpha_i k^2 \epsilon^2$ In this work

cf) f(R) $\quad G_{\rm eff} = G_{\rm eff}(k,t)$ $\qquad \alpha_i \sim \alpha_j = \mathcal{O}(1)$

Koyama, Hiramatsu, Taruya (2009)

Evolution of density fluctuations

- Perturbative expansion: $\epsilon = \epsilon_1 + \epsilon_2, \ \epsilon = \Psi, \Phi, Q, \delta$

- **EoMs**: $\delta\Psi, \delta\Phi, \delta Q \quad \Rightarrow \quad \Phi_1, \Phi_2$

  ↑ include the effect of modified gravity

- Fluid equations:
  continuity/ Eular
  (usual forms)

$$\frac{\partial \delta(t,\boldsymbol{x})}{\partial t} + \frac{1}{a}\partial_i[(1+\delta)u^i(t,\boldsymbol{x})] = 0,$$
$$\frac{\partial u^i}{\partial t} + Hu^i + \frac{1}{a}u^j\partial_j u^i = -\frac{1}{a}\partial^i \Phi(t,\boldsymbol{x})$$

$\Rightarrow$ **evolution equation of density contrast**

Effective Lagrangian

- **Effective Lagrangian:** variation ⇒ EoM

Sub-horizon ($aH \ll k$) , late time, **QSA**, ${}^{\forall}\alpha_i, \beta_1 := \mathcal{O}(1)$

$$\mathcal{L}^{\rm eff} = \frac{M^2 a}{2}\Bigg[(c_1\Phi + c_2\Psi + c_3 Q)\nabla^2 Q + c_4\Psi\nabla^2\Phi + c_5\Psi\nabla^2\Psi + c_6\Phi\nabla^2\Phi$$

$$+\left(c_7\frac{\dot\Psi}{H} + c_8\frac{\dot\Phi}{H} + c_9\frac{\ddot Q}{H^2}\right)\nabla^2 Q\Bigg] - a^3\rho_{\rm m}\Phi\delta$$

$$\frac{M^2 a}{2}\Bigg[\frac{b_1}{a^2H^2}\left(-\frac{1}{2}(\nabla Q)^2\right)\nabla^2 Q + \frac{1}{a^2H^2}b_2\Phi\left[(\nabla^2 Q)^2 - (\nabla_i\nabla_j Q)^2\right]$$

$$+\frac{1}{a^2H^2}\left(b_4\nabla_i\Psi + b_5\nabla_i\Phi + b_6\frac{\dot H}{H^2}\nabla_i Q + b_6\frac{\nabla_i\dot Q}{H}\right)\nabla_j Q\nabla^i\nabla^j Q\Bigg]$$

Green: GLPV, **Red**: DHOST $\quad b, c \supset \alpha_i, \beta_1$

EoMs of gravitational fields

$(\delta\Phi)$

$$\mathcal{G}_T\partial^2\Psi+\tilde{A}_2\partial^2 Q-A_6\partial^2\Phi+A_8\frac{\partial^2\dot{Q}}{H}-\frac{a^2}{2}\rho_{\rm m}\delta=-\frac{B_2}{2a^2H^2}\mathcal{Q}_2+\frac{B_5}{a^2H^2}\left[(\partial_i\partial_j Q)^2+\partial_i Q\partial^i\partial^2 Q\right]$$

$(\delta\Psi)$

$$\mathcal{F}_T\partial^2\Psi-\mathcal{G}_T\partial^2\Phi-\tilde{A}_1\partial^2 Q+A_4\frac{\partial^2\dot{Q}}{H}=\frac{B_1}{2a^2H^2}\mathcal{Q}_2+\frac{B_4}{a^2H^2}\left[(\partial_i\partial_j Q)^2+\partial_i Q\partial^i\partial^2 Q\right]$$

(δQ)

$$\begin{aligned}
&A_0\partial^2 Q-A_1\partial^2\Psi-A_2\partial^2\Phi-A_4\frac{\partial^2\dot{\Psi}}{H}+A_8\frac{\partial^2\dot{\Phi}}{H}-\tilde{A}_9\frac{\partial^2\dot{Q}}{H}-A_9\frac{\partial^2\ddot{Q}}{H^2}\\
&=-\frac{B_0}{a^2H^2}\mathcal{Q}_2+\frac{B_2}{a^2H^2}(\partial^2\Phi\partial^2 Q-\partial_i\partial_j\Phi\partial^i\partial^j Q)\\
&\quad-\frac{B_4}{a^2H^2}\left(\partial^2\Psi\partial^2 Q+\partial_i Q\partial^i\partial^2\Psi\right)+\frac{B_5}{a^2H^2}(\partial^2\Phi\partial^2 Q+\partial_i Q\partial^i\partial^2\Phi)\\
&\quad-\frac{\tilde{B}_6}{a^2H^2}\left[(\partial_i\partial_j Q)^2+\partial_i Q\partial^i\partial^2 Q\right]\\
&\quad-\frac{B_6}{a^2H^2}\frac{1}{H}\left(\partial^2 Q\partial^2\dot{Q}+2\partial_i Q\partial^i\partial^2\dot{Q}+2\partial_i\partial_j Q\partial^i\partial^j\dot{Q}+\partial_i\partial^2 Q\partial^i\dot{Q}\right)
\end{aligned}$$

EoMs of gravitational fields

- **linear level, GR** $\mathcal{G}_T = \mathcal{F}_T = M_{\rm pl}^2$

$(\delta\Phi)$ Poisson equation

$$\mathcal{G}_T \partial^2 \Psi - \frac{a^2}{2}\rho_{\rm m}\delta = 0$$

$(\delta\Psi)$ trace component of Einstein tensor

$$\mathcal{F}_T \partial^2 \Psi - \mathcal{G}_T \partial^2 \Phi = 0$$

(δQ) $\delta Q = 0$

EoMs of gravitational fields

- **linear level, Horndeski**

contribution of scalar field

$(\delta\Phi)$

$$\mathcal{G}_T\partial^2\Psi + \tilde{A}_2\partial^2 Q - \frac{a^2}{2}\rho_{\rm m}\delta = 0$$

$(\delta\Psi)$

anisotropic stress

$$\mathcal{F}_T\partial^2\Psi - \mathcal{G}_T\partial^2\Phi - \tilde{A}_1\partial^2 Q = 0$$

$$(\delta Q) \quad A_0\partial^2 Q - A_1\partial^2\Psi - A_2\partial^2\Phi$$

$$= 0$$

$\Rightarrow$ **increase of gravitational constant**

EoMs of gravitational fields

- linear level, beyond Horndeski

$\mathcal{O}(\dot{\delta})$ で寄与

$(\delta\Phi)$

$$\mathcal{G}_T\partial^2\Psi + \tilde{A}_2\partial^2 Q - A_6\partial^2\Phi + A_8\frac{\partial^2\dot{Q}}{H} - \frac{a^2}{2}\rho_{\rm m}\delta = 0$$

$(\delta\Psi)$

$\mathcal{O}(\ddot{\delta})$ で寄与

$$\mathcal{F}_T\partial^2\Psi - \mathcal{G}_T\partial^2\Phi - \tilde{A}_1\partial^2 Q + A_4\frac{\partial^2\dot{Q}}{H} = 0$$

$$(\delta Q)\quad A_0\partial^2 Q - A_1\partial^2\Psi - A_2\partial^2\Phi - A_4\frac{\partial^2\dot{\Psi}}{H} + A_8\frac{\partial^2\dot{\Phi}}{H} - \tilde{A}_9\frac{\partial^2\dot{Q}}{H} - A_9\frac{\partial^2\ddot{Q}}{H^2}$$

$$= 0$$

$\Rightarrow$ **increase of gravitational constant,**

additional friction term

1st-order solution

previous works: Kobayashi+ (2015), D'Amico+ (2017), Chrisostomi & Koyama (2017)

$$\ddot{\delta}_1 + (2+\varsigma)H\dot{\delta}_1 - 4\pi G_{\rm eff}\rho_{\rm m}\delta_1 = 0$$

✓ $G_{\rm eff}(t)$: G (GR), $G \rightarrow G_{\rm eff}$ (Horndeski, beyond Horndeski)

$\varsigma(t) \propto \alpha_H, \beta_1:$ 0 (GR, Horndeski), $0 \rightarrow \varsigma$ (beyond Horndeski)

✓ growing mode: $\delta_1(\mathbf{p},t) = D_+(t)\delta_L(\mathbf{p})$

$D_+(t)$: growth factor, $\delta_L(\mathbf{p})$: initial density fluc.

✓ change in the growth of density fluctuation due to ς

cf.) improvement of $f\sigma_8$ Tsujikawa (2015), D'Amico+ (2017)

✓ studying the linear evolution on typical BG sols. beyond H.

Hirano, Kobayashi, Yamauchi, Yokoyama, on going

EoMs of gravitational fields

- **non-linear level, GR** $\mathcal{G}_T = \mathcal{F}_T = M_{\rm pl}^2$

$(\delta\Phi)$ Poisson equation

$$\mathcal{G}_T\partial^2\Psi - \frac{a^2}{2}\rho_{\rm m}\delta = 0$$ remains usual form

$(\delta\Psi)$ trace component of Einstein tensor

$$\mathcal{F}_T\partial^2\Psi - \mathcal{G}_T\partial^2\Phi = 0$$ remains usual form

(δQ) $\delta Q = 0$

$\Rightarrow$ **Only non-linearity from fluid equations**

EoMs of gravitational fields

- non-linear, Horndeski

$$\mathcal{Q}_2 = (\partial^2 Q)^2 - (\partial_i \partial_j Q)^2$$

$(\delta\Phi)$

$$\mathcal{G}_T \partial^2 \Psi + \tilde{A}_2 \partial^2 Q - \frac{a^2}{2}\rho_{\rm m}\delta = -\frac{B_2}{2a^2H^2}\mathcal{Q}_2$$

$(\delta\Psi)$

$$\mathcal{F}_T \partial^2 \Psi - \mathcal{G}_T \partial^2 \Phi - \tilde{A}_1 \partial^2 Q = \frac{B_1}{2a^2H^2}\mathcal{Q}_2$$

(δQ) $\quad A_0 \partial^2 Q - A_1 \partial^2 \Psi - A_2 \partial^2 \Phi$

$$= -\frac{B_0}{a^2H^2}\mathcal{Q}_2 + \frac{B_2}{a^2H^2}(\partial^2\Phi\partial^2 Q - \partial_i\partial_j\Phi\partial^i\partial^j Q)$$

additional non-linearity from scalar field

⇒ novel probe of modified gravity !

(quasi non-linear regime)

2nd-order solution

Horndeski: Takushima+ (2013)
GLPV: Hirano+ (2018)

Hirano+ in prep.

$$\delta_1^2$$

$$\ddot{\delta}_2 + (2+\varsigma)H\dot{\delta}_2 - 4\pi G_{\rm eff}\rho_{\rm m}\delta_2 = S_\delta$$

Primordial fluc. : Gaussian $\Rightarrow$ inhomogeneous sol.

$$\Rightarrow \quad \delta_2(\mathbf{p},t) = D_+^2(t)[\kappa(t)\mathcal{W}_\alpha(\mathbf{p}) + \lambda(t)\mathcal{W}_\gamma(\mathbf{p})]$$

$$\mathcal{W}_i(\boldsymbol{p}) = \frac{1}{(2\pi)^3}\int d^3k_1 d^3k_2\,\delta^{(3)}(\boldsymbol{k}_1+\boldsymbol{k}_2-\boldsymbol{p})\mathcal{E}(\boldsymbol{k}_1\cdot\boldsymbol{k}_2)\delta_L(\boldsymbol{k}_1)\delta_L(\boldsymbol{k}_2)$$

$$i = \alpha,\gamma\ ,\ \alpha(\mathbf{k}_1,\mathbf{k}_2) = 1 + \frac{(\boldsymbol{k}_1\cdot\boldsymbol{k}_2)(k_1^2+k_2^2)}{2k_1^2k_2^2}\ ,\ \gamma(\mathbf{k}_1,\mathbf{k}_2) = 1 - \frac{(\boldsymbol{k}_1\cdot\boldsymbol{k}_2)^2}{k_1^2k_2^2}$$

$\lambda(t)$: 1 (GR) , $1 \to \lambda_0 \neq 1$ (Horndeski, beyond Horndeski)

New! $\kappa(t) \supset \alpha_H, \beta_1$: 1 (GR, Horndeski), $1 \to \kappa_0 \neq 1$ (beyond Horndeski)

MATTER BISPECTRUM IN MG WITH VAINSHTEIN SCREENING

Takushima+ (2013, 2015)
Hirano+ (2018)

Matter bispectrum

cf) Scoccimarro+ (1998)
Barnardeau+ (2000)

- **Correlation function**

$$\langle \delta(t,\boldsymbol{k}_1)\delta(t,\boldsymbol{k}_2)\delta(t,\boldsymbol{k}_3)\rangle := (2\pi)^3\delta(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)B(t,\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)$$

- **Leading order (tree-level)**

$$B(t,k_1,k_2,k_3) = 2D_+^4 F_2(t,\boldsymbol{k}_1,\boldsymbol{k}_2)P_{11}(k_1)P_{11}(k_2) + 2 \text{ cyclic terms}$$

Kernel $\quad F_2(t,\mathbf{k}_1,\mathbf{k}_2) = \kappa(t)\alpha(\mathbf{k}_1,\mathbf{k}_2) - \frac{2}{7}\lambda(t)\gamma(\mathbf{k}_1,\mathbf{k}_2)$

$P_{11}(k)$: initial power spectrum

$W_\alpha,\ W_\gamma$

Matter bispectrum

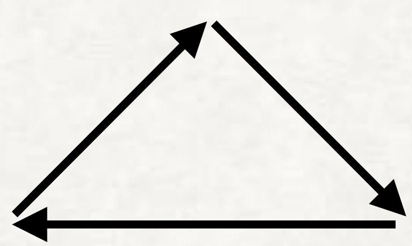
cf) Scoccimarro+ (1998)
Barnardeau+ (2000)

- **Reduced bispectrum**

$$Q_{123}(t,k_1,k_2,k_3) = \frac{B(t,k_1,k_2,k_3)}{D_+^4(t)[P_{11}(k_1)P_{11}(k_2) + 2 \text{ cyclic terms}]}$$

does not depend on matter growth D_+

✓ $\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 = 0 \quad \Rightarrow \quad \boldsymbol{k}_1 = (0, 0, k_1),\ \boldsymbol{k}_2 = (0, k_2 \sin\theta_{12}, k_2 \cos\theta_{12}),$
$\boldsymbol{k}_3 = (0, -k_2 \sin\theta_{12}, -k_1 - k_2 \cos\theta_{12})$



✓ We give κ, λ at z = 0 instead of solving the evolution of these.

We estimate matter bispectrum and plot the case of
$k_1 = k_2 = 0.01h/\mathrm{Mpc}$ and $k_1 = 5k_2 = 0.05h/\mathrm{Mpc}$.

(cosmological parameters: Planck 2015)

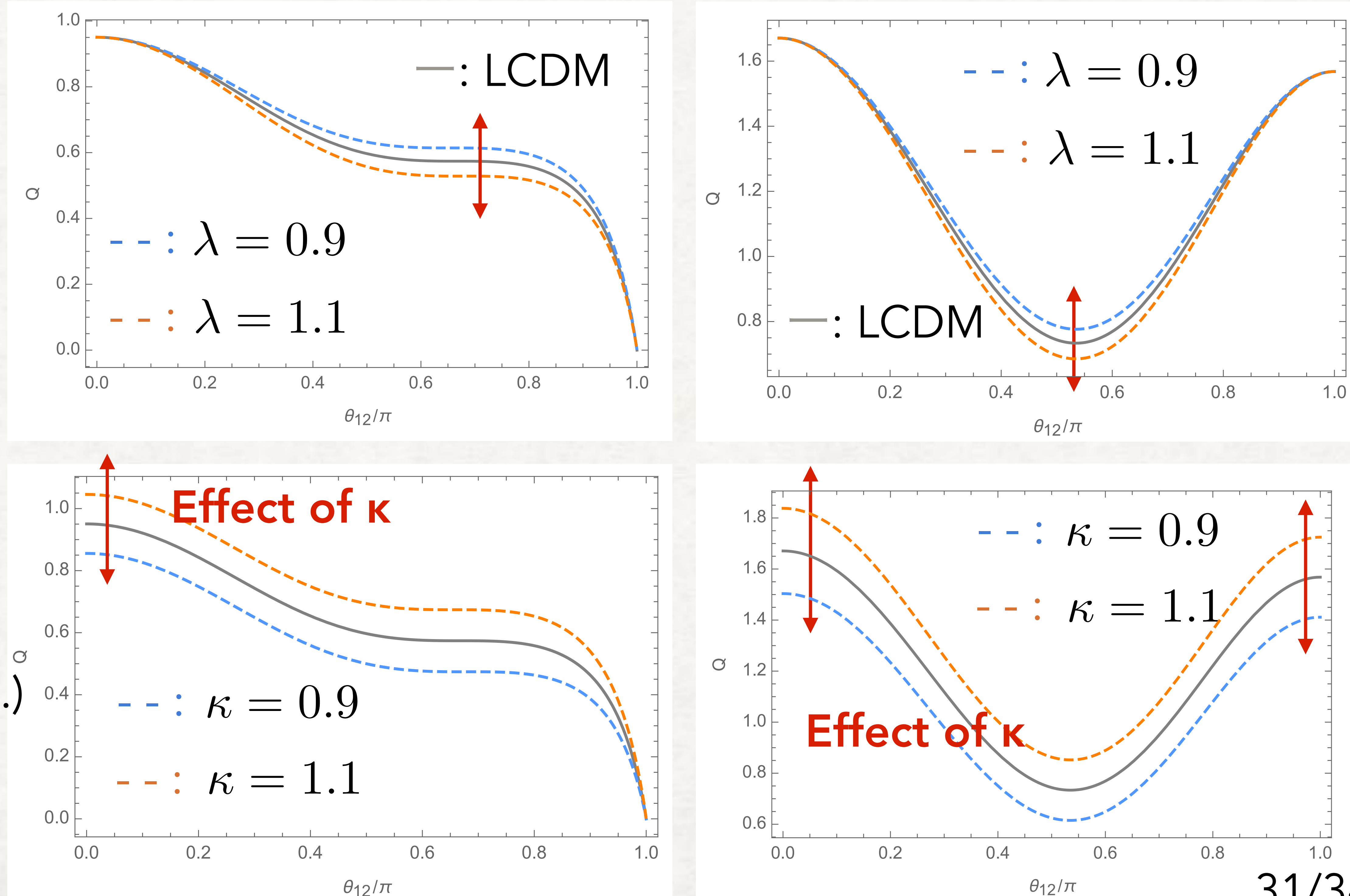
k-dependence

Horndeski: Takushima+ (2013)
beyond H.: Hirano+ (2018)

$k_1 = k_2 = 0.01 h/Mpc$ $\quad$ $k_1 = 5k_2 = 0.05 h/Mpc$

$\kappa = 1$ (Horndeski)

$\lambda = 1$ (beyond H.)

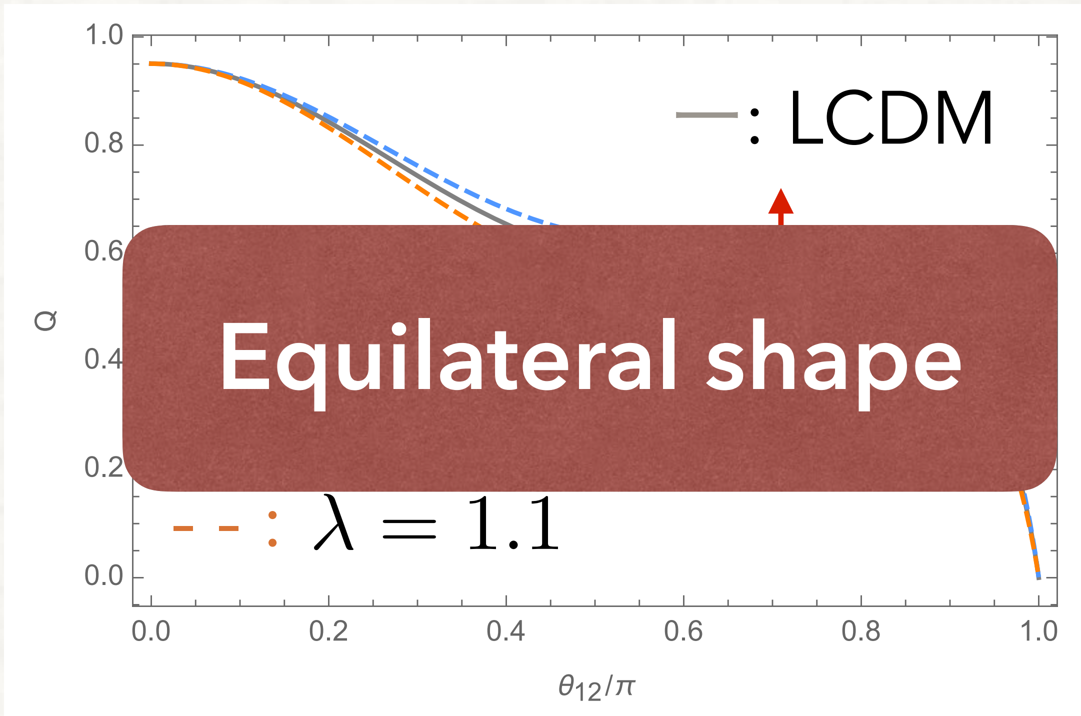


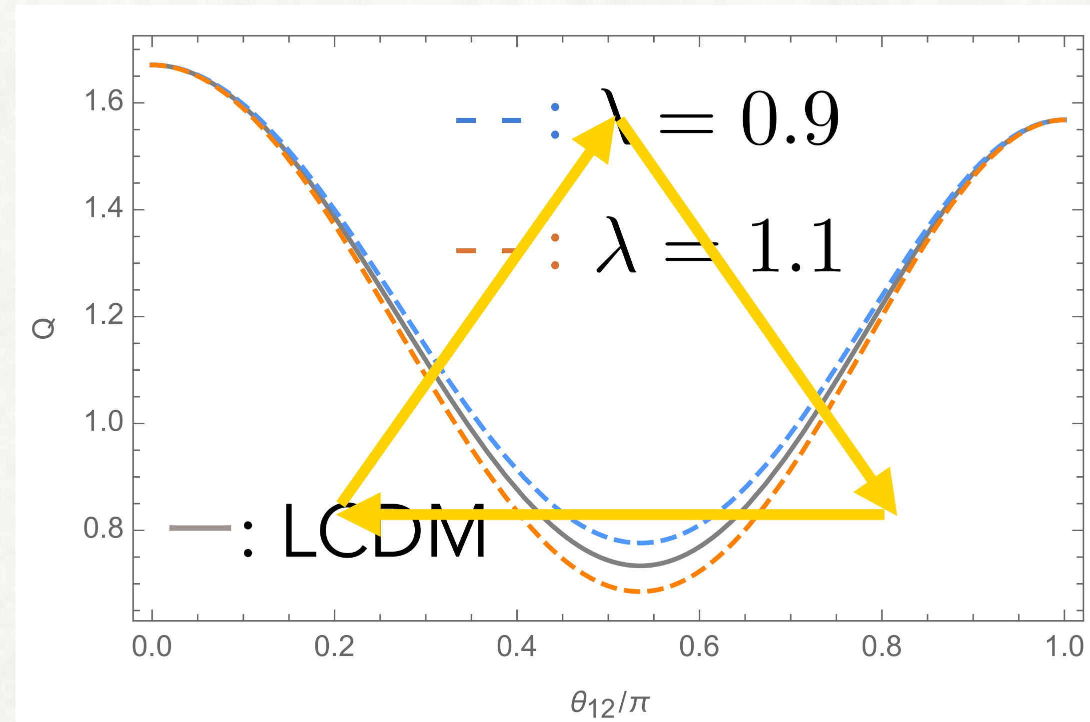
k-dependence

Horndeski: Takushima+ (2013)
beyond H.: Hirano+ (2018)

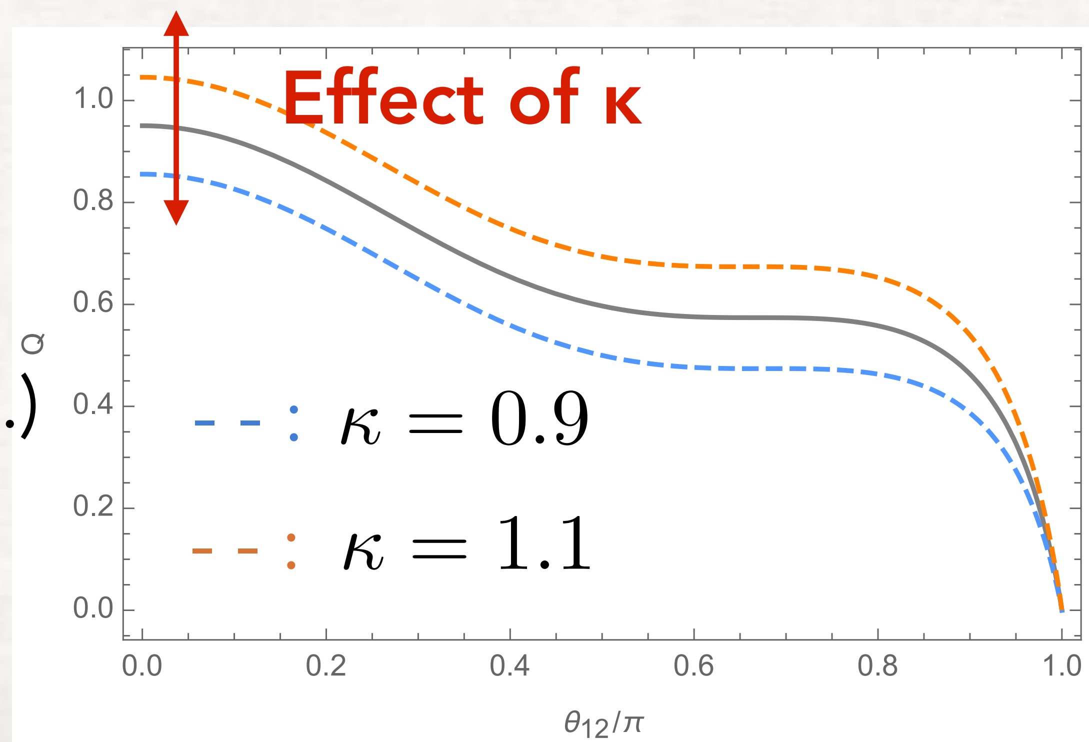
$k_1 = k_2 = 0.01h/Mpc$ | $k_1 = 5k_2 = 0.05h/Mpc$

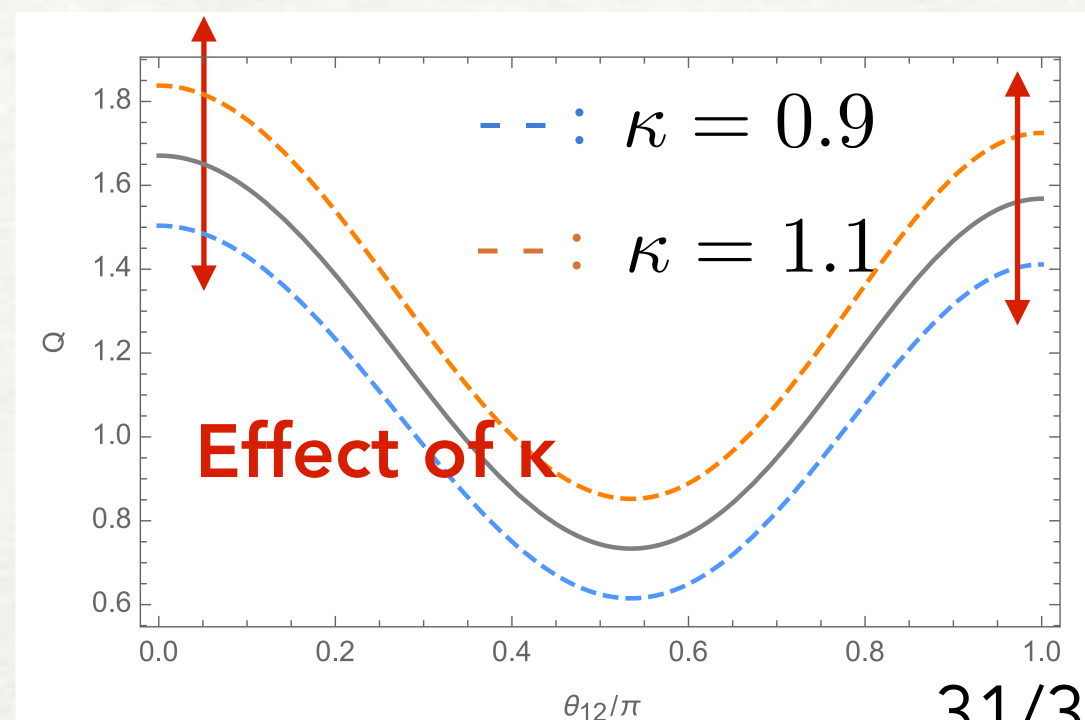
$\kappa = 1$ (Horndeski)





$\lambda = 1$ (beyond H.)





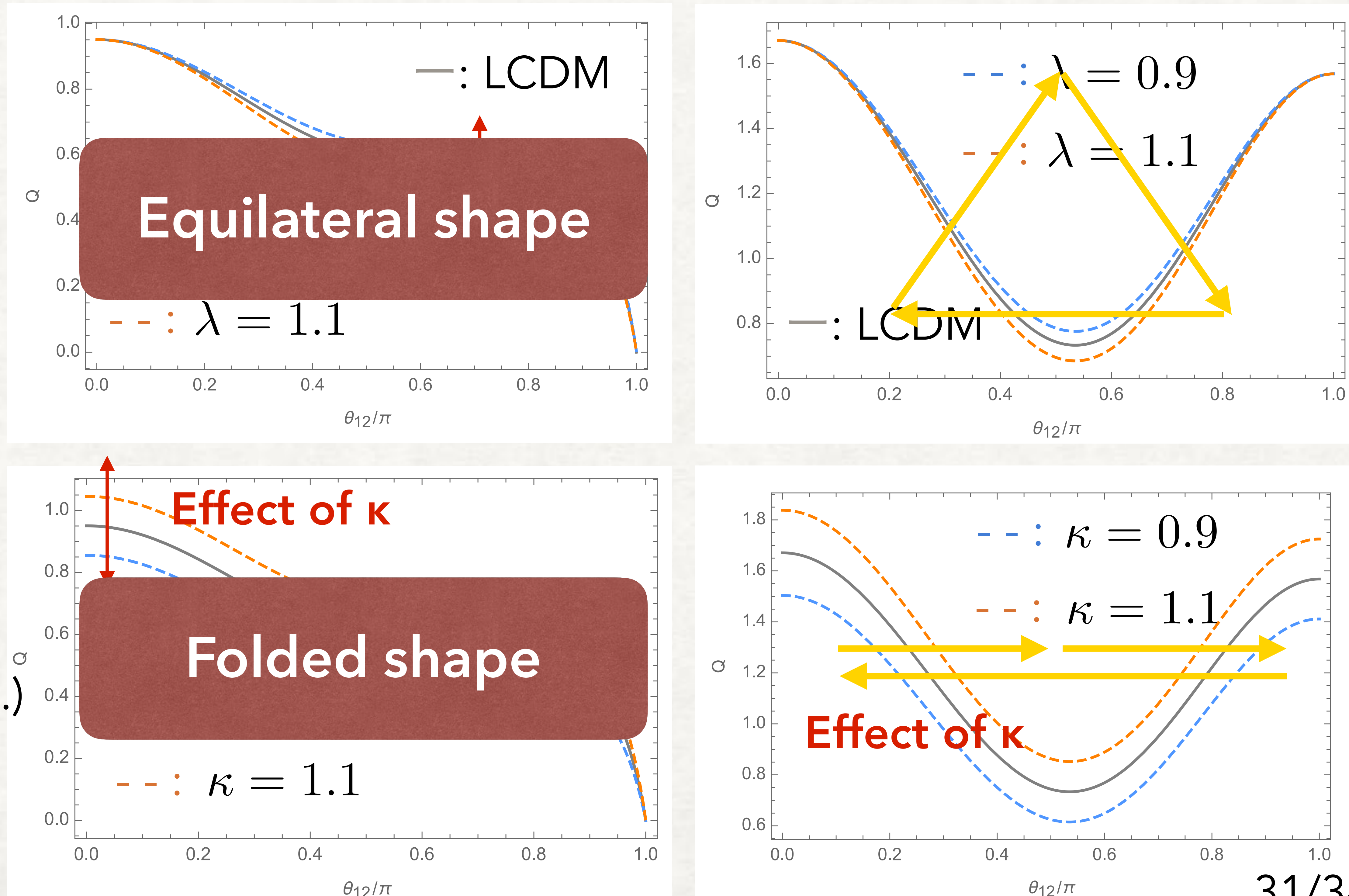
k-dependence

Horndeski: Takushima+ (2013)
beyond H.: Hirano+ (2018)

$k_1 = k_2 = 0.01h/Mpc$ | $k_1 = 5k_2 = 0.05h/Mpc$

$\kappa = 1$ (Horndeski)

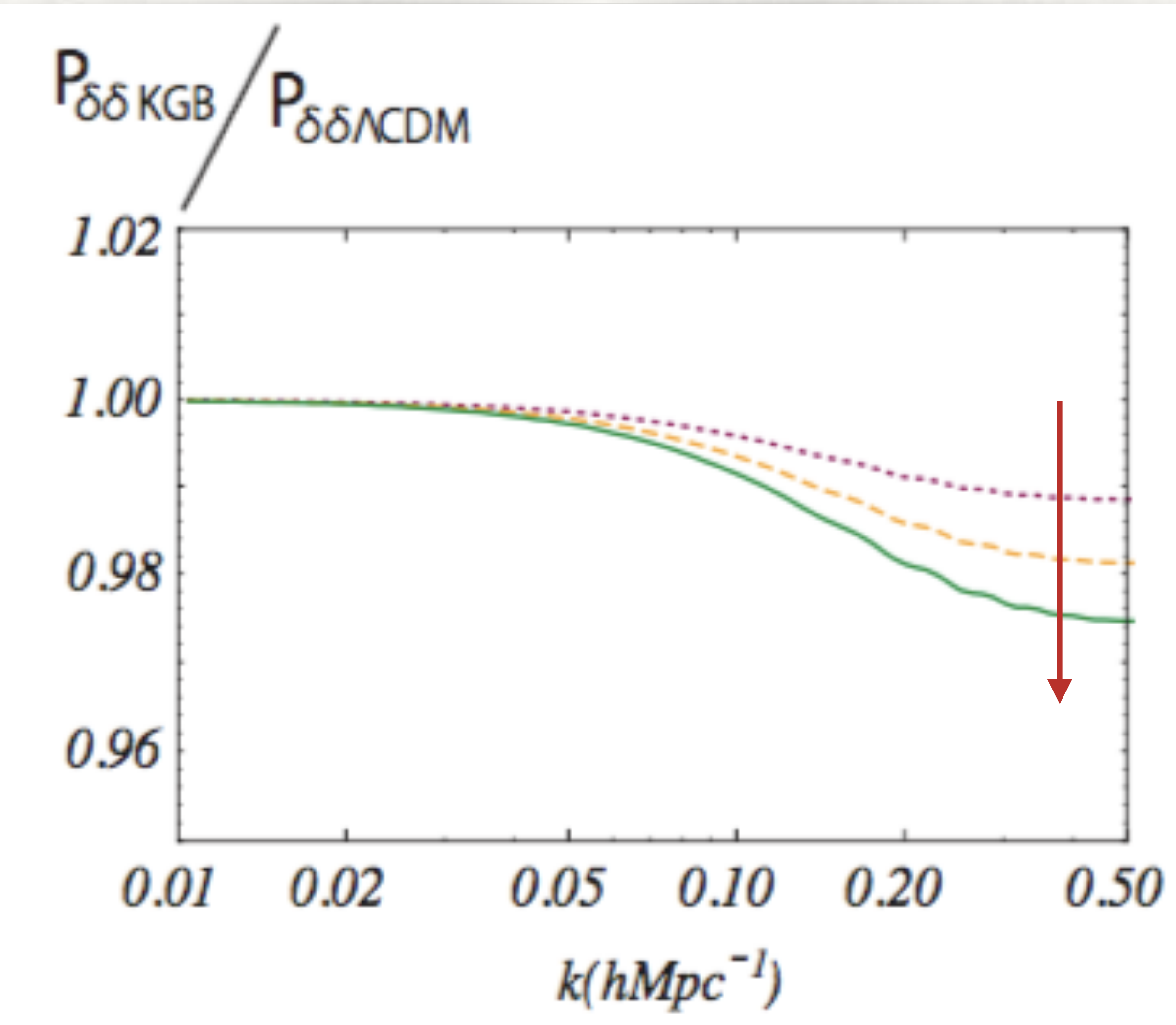
$\lambda = 1$ (beyond H.)

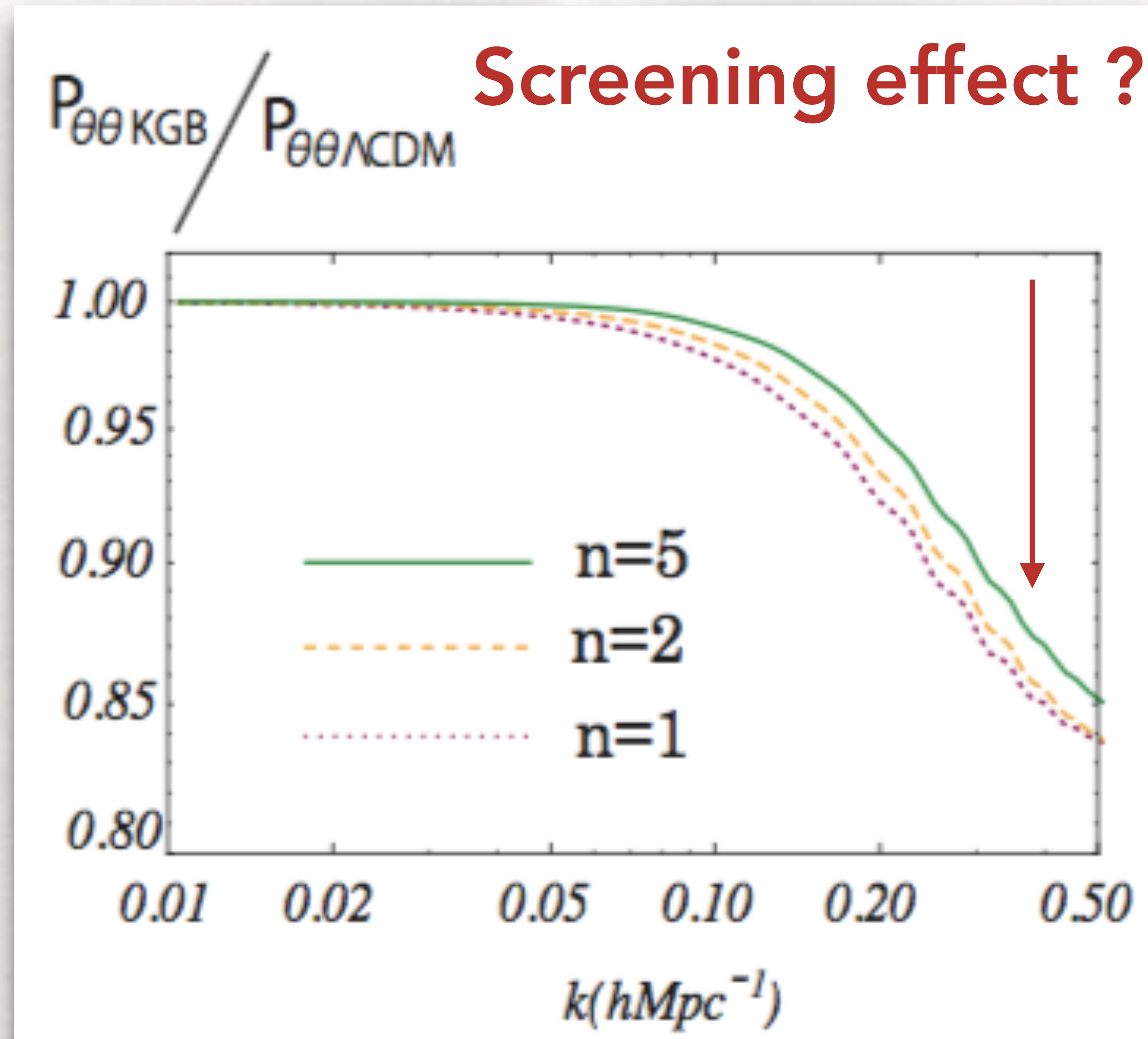


Screening in Power spectrum

- 1 loop power spectrum Takushima+ (2015)

  KGB (n→∞: Λ-CDM)





~ **depending on time-derivative of λ,** $\lambda_\theta = \lambda + \frac{\dot{\lambda}}{2fH}$

Matter bispectrum in RSD

DM ⇒ galaxy: $\delta_g(a,\boldsymbol{x}) = b_1\delta(a,\boldsymbol{x}) + \frac{1}{2}b_2\delta^2(a,\boldsymbol{x}) + \cdots$

$$\delta_g^s(a,\boldsymbol{k}) = D_+ Z_1(\boldsymbol{k};a)\delta_L(\boldsymbol{k}) + D_+^2\int\frac{d^3k_1 d^3k_2}{(2\pi)^3}\delta(\boldsymbol{k}_1+\boldsymbol{k}_2-\boldsymbol{k})Z_2(\boldsymbol{k}_1,\boldsymbol{k}_2;a)\delta_L(\boldsymbol{k}_1)\delta_L(\boldsymbol{k}_2)+\cdots$$

$$Z_1(\boldsymbol{k};a) = b_1 + f\mu^2,$$

kernel of θ_2

$$Z_2(\boldsymbol{k}_1,\boldsymbol{k}_2;a) = bF_2(\boldsymbol{k}_1,\boldsymbol{k}_2) + f\mu_{12}^2 G_2(\boldsymbol{k}_1,\boldsymbol{k}_2) + \frac{b_2}{2} + \frac{f\mu_1 2\mu_1 2}{2}\left[\frac{\mu_2}{k_2}(b+f\mu_1^2) + \frac{\mu_1}{k_1}(b+f\mu_2^2)\right]$$

$$\mu = \boldsymbol{k}\cdot\hat{z}/k,\ \ \mu_i = \boldsymbol{k}_i\cdot\hat{z}/k_i,\ \ k_i = |\boldsymbol{k}_i|\ [i=1,2,12(=k_1+k_2)]$$

$$G_2(t,\boldsymbol{k}_1,\boldsymbol{k}_2) = \kappa_\theta(t)\alpha(\boldsymbol{k}_1,\boldsymbol{k}_2) - \frac{4}{7}\lambda_\theta(t)\gamma(\boldsymbol{k}_1,\boldsymbol{k}_2)$$

$$\kappa_\theta(t) = 2\kappa - 1 + \frac{\dot{\kappa}}{fH},\ \ \lambda_\theta(t) = \lambda + \frac{\dot{\lambda}}{2fH}$$

Time-derivatives of κ and λ changes RSD bispectrum largely?

SUMMARY

Summary

- We discuss <u>beyond Horndeski</u> on matter density fluctuations at cosmological scale under some assumptions (QSA, $\alpha_i \sim \beta_1 = \mathcal{O}(1)$)

- Non-linear int. … (small scale, early universe) Vainshtein screening

  (cosmological scale) Matter bispectrum, …

- Cosmological perturbations

  linear level: friction term $\varsigma(t)$

  non-linear level: new time-evolution $\kappa(t)$ on matter bispectrum

  k-dependence … folded shape (beyond Horndeski)

<Future direction>

- studying **the linear evolution on typical BG sols.** beyond Horndeski
- typical behavior of κ and λ beyond Horndeski?

  $f = \Omega_\mathrm{m}(a)^\gamma,\ \lambda = \Omega_\mathrm{m}(a)^\xi,\ \kappa = \Omega_\mathrm{m}(a)^\varpi$ Hirano+ in preparation

Appendix

ISW effect in MG

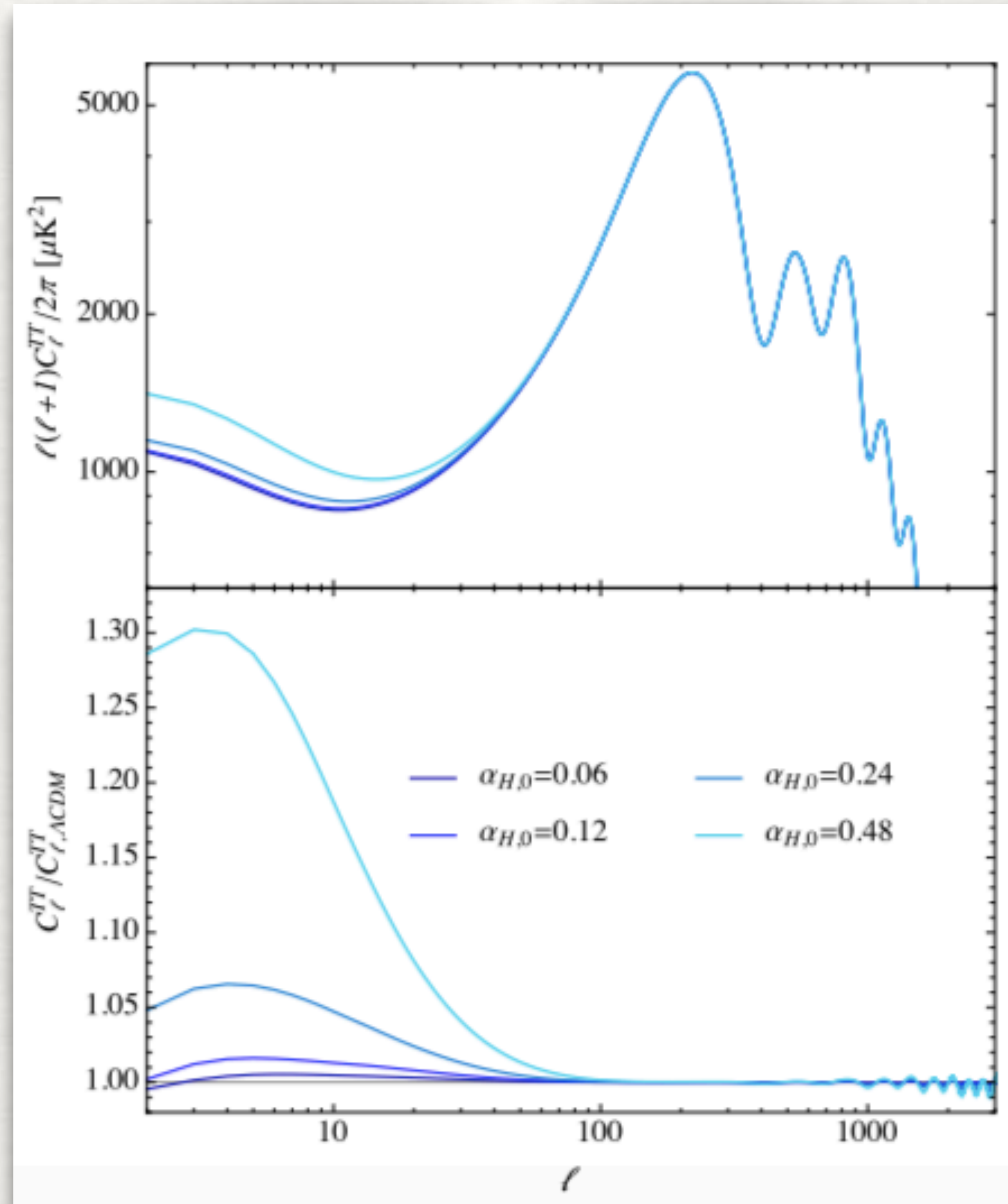
cf) Kimura+ (2011), D'amico (2017)

ex) Einstein-Hilbelt + α_H

D'amico (2017)

$$\frac{T^{\rm ISW}}{T}(\hat{n}) = -\int_0^{z_\star} dz\{\partial_z\Phi(\chi z, n) + \partial_z\Psi(\chi z, n)\}$$

$$\frac{d\ln(\Phi+\Psi)}{d\ln a} = f - 1 - 3\alpha_H + \mathcal{O}(\Omega_\phi^2)$$



ISW effect beyond Horndeki

cf) Kimura+ (2011)

$$\Phi_1 = \left(-\frac{a^2H^2}{k^2}\right)\left(\mu_\Phi \frac{\ddot{\delta}_1}{H^2} + \nu_\Phi \frac{\dot{\delta}_1}{H} + \kappa_\Phi \delta_1\right)$$

$$\Psi_1 = \left(-\frac{a^2H^2}{k^2}\right)\left(\mu_\Psi \frac{\ddot{\delta}_1}{H^2} + \nu_\Psi \frac{\dot{\delta}_1}{H} + \kappa_\Psi \delta_1\right)$$

$$\frac{\kappa_\Phi}{1-\mu_\Phi} = \frac{3}{2}\frac{G_{\rm eff}}{G_{\cos}}\Omega_{\rm m}$$

$$\frac{2\mu_\Phi - \nu_\Phi}{1-\mu_\Phi} = \varsigma$$

これの時間微分

Friedmann eq.上の重力定数

$$-k^2\frac{\Phi+\Psi}{2} = \frac{1}{2}\left\{\kappa_\Psi + \frac{3}{2}\frac{G_{\rm eff}}{G_{\cos}}(\mu_\Psi+1)\Omega_{\rm m} - \underline{[(2\mu_\Psi - \nu_\Psi) + (\mu_\Psi+1)\varsigma]f}\right\} a^2H^2\delta_1$$

$$\to \frac{3}{2}\frac{G_{\rm eff}}{G}\Omega_{\rm m} a^2 H^2 \delta_1(a,k)$$ (cubic Galileon, $\kappa_\Phi = \kappa_\Psi$ その他0)

摩擦項のおかげでlensing potentialの減衰が抑えられる. D'Amico+ (2017)

Ostrogradski ghost

cf.) Woodard (2015)

- **Ostrogradski ghost** Ostrogradski (1850)

$$L = L(q, \dot{q}, \ddot{q})$$

4th order

$\Rightarrow$ Eular-Lagrange eq. $$\frac{\partial L}{\partial q} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} + \frac{d^2}{dt^2}\underbrace{\frac{\partial L}{\partial \ddot{q}}}_{\neq 0} = 0$$

$Q_1 := q$, $Q_2 := \dot{q}$, $P_1 = \cdots$, $P_2 = \cdots$ physical dof **2**

$$H = P_1 Q_2 + \cdots$$ linear dependence of momentum

- Hamiltonian unbounded from below ($\Leftrightarrow$ negative kinetic energy)
- 4th-order equation $\Rightarrow$ extra dof = Ostrogradski ghost

Ostrogradski ghost

cf.) Woodard (2015)

- **Ostrogradski ghost** Ostrogradski (1850)

$$L = L(q, \dot{q}, \ddot{q})$$

$\Rightarrow$ Eular-Lagrange eq. $$\frac{\partial L}{\partial q} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} + \cancel{\frac{d^2}{dt^2}\frac{\partial L}{\partial \ddot{q}}} = 0$$

can avoid extra dof

$$Q_1 := q \,,\; Q_2 := \dot{q} \,,\; P_1 = \cdots \,,\; P_2 = \cdots$$ physical dof 2

$$H = P_1 Q_2 + \cdots$$ linear dependence of momentum

- Hamiltonian unbounded from below ($\Leftrightarrow$ negative kinetic energy)
- 4th-order equation $\Rightarrow$ extra dof = Ostrogradski ghost

Degenerate theory

cf) Langlois, Noui (2015,2016)

- **Degeneracy condition (in the system of multiple dof)**

$$L = \frac{1}{2}a\,\ddot{\phi}^2 + \frac{1}{2}k_0\,\dot{\phi}^2 + \frac{1}{2}k_{ij}\dot{q}^i\dot{q}^j + b_i\ddot{\phi}\dot{q}^i + c_i\dot{\phi}\dot{q}^i - V(\phi, q)$$

$$\phi(t), q^i(t)(i = 1, 2, 3)\ \ ,\ \ a, k_0, k_{ij}, b_i, c_i = \text{const.}$$

1 + 3 dof

Degenerate theory

cf) Langlois, Noui (2015,2016)

- **Degeneracy condition (in the system of multiple dof)**

$$L = \underline{\frac{1}{2} a\,\dot{Q}^2 + \frac{1}{2} k_{ij}\dot{q}^i\dot{q}^j + b_i\dot{Q}\dot{q}^i} \quad + c_i Q\dot{q}^i + \frac{1}{2}k_0\,Q^2 - V(\phi, q) - \lambda(Q - \dot{\phi})$$

λ : Lagrange multiplier, $\dot{\phi} \to Q$

EL eq. $\Rightarrow$

$$\begin{aligned} \boxed{a\ddot{Q} + b_i\ddot{q}^i} &= c_i\dot{q}^i + k_0 Q - \lambda \\ \boxed{b_j\ddot{Q} + k_{ij}\ddot{q}^i} &= -c_i\dot{Q} - \frac{\partial V}{\partial q^i} \end{aligned}$$

and $\dot{\phi} = Q\,,\ \dot{\lambda} = -\dfrac{\partial V}{\partial \phi}$

・invertible kinetic matrix ⇒ 4 + 1

Degenerate theory

cf) Langlois, Noui (2015,2016)

- **Degeneracy condition (in the system of multiple dof)**

$$L = \underline{\frac{1}{2}a\,\dot{Q}^2 + \frac{1}{2}k_{ij}\dot{q}^i\dot{q}^j + b_i\dot{Q}\dot{q}^i} + c_i Q\dot{q}^i + \frac{1}{2}k_0\,Q^2 - V(\phi, q) - \lambda(Q - \dot{\phi})$$

λ : Lagrange multiplier, $\dot{\phi} \to Q$

EL eq. $\Rightarrow$

$$\begin{aligned} a\ddot{Q} + b_i\ddot{q}^i &= c_i\dot{q}^i + k_0 Q - \lambda \\ b_j\ddot{Q} + k_{ij}\ddot{q}^i &= -c_i\dot{Q} - \frac{\partial V}{\partial q^i} \end{aligned}$$

and $\dot{\phi} = Q\,,\ \dot{\lambda} = -\dfrac{\partial V}{\partial \phi}$

Degeneracy condition: determinant of this matrix is zero

$$\det M = 0 \ \ , \ \ M = \begin{bmatrix} a & b_i \\ b_j & k_{ij} \end{bmatrix} \ \Leftrightarrow \ \det k \cdot [\underbrace{a - b_i b_j (k^{-1})^{ij}}_{=0}] = 0$$

Degenerate theory

Langlois, Noui (2015,2016),
Koyama+ (2016)

- **Degeneracy condition (in scalar-tensor theory)**

$$X = -\frac{1}{2}(\nabla\phi)^2, \phi_\mu = \nabla_\mu\phi, \Box\phi = \nabla^2\phi, \phi_{\mu\nu} = \nabla_\nu\nabla_\mu\phi$$

$$S[\phi, g] = \int d^4x\sqrt{-g}\left[G_2(\phi, X) - G_3(\phi, X)\Box\phi + G_4(\phi, X)R + C_{(2)}^{\mu\nu\rho\sigma}\phi_{\mu\nu}\phi_{\rho\sigma}\right]$$

Degenerate theory

Langlois, Noui (2015,2016),
Koyama+ (2016)

- **Degeneracy condition (in scalar-tensor theory)**

$$X=-\frac{1}{2}(\nabla\phi)^2, \phi_\mu=\nabla_\mu\phi, \Box\phi=\nabla^2\phi, \phi_{\mu\nu}=\nabla_\nu\nabla_\mu\phi$$

$$S[\phi,g]=\int d^4x\sqrt{-g}\left[G_2(\phi,X)-G_3(\phi,X)\Box\phi+G_4(\phi,X)R+C_{(2)}^{\mu\nu\rho\sigma}\phi_{\mu\nu}\phi_{\rho\sigma}\right]$$

$$\Rightarrow\quad S_{\rm kin}=\int d^4x\sqrt{-g}\left[G_4(\phi,X)R+C_{(2)}^{\mu\nu\rho\sigma}\phi_{\mu\nu}\phi_{\rho\sigma}\right]$$

$$L_{\rm kin}=\mathcal{A}(\phi,X,A)\dot{A}^2+2\mathcal{B}^{ij}(\phi,X,A)\dot{A}K_{ij}+\mathcal{K}^{ijkl}(\phi,X,A)K_{ij}K_{kl}$$

$$Q\to A,\ \dot{q}^i\to K_{ij},\ a\to\mathcal{A},\ b_i\to\mathcal{B}^{ij},\ k_{ij}\to\mathcal{K}^{ijkl}$$

extrinsic curvature $\supset \dot{h}_{ij}$

Degenerate theory

Langlois, Noui (2015,2016), Koyama+ (2016)

- **Degeneracy condition (in scalar-tensor theory)**

$$X = -\frac{1}{2}(\nabla\phi)^2, \phi_\mu = \nabla_\mu\phi, \Box\phi = \nabla^2\phi, \phi_{\mu\nu} = \nabla_\nu\nabla_\mu\phi$$

$$S[\phi, g] = \int d^4x\sqrt{-g}\left[G_2(\phi, X) + G_3(\phi, X)\Box\phi + f_2(\phi, X)R + C_{(2)}^{\mu\nu\rho\sigma}\phi_{\mu\nu}\phi_{\rho\sigma}\right]$$

$$\Rightarrow \quad S_{\text{kin}} = \int d^4x\sqrt{-g}\left[f_2(\phi, X)R + C_{(2)}^{\mu\nu\rho\sigma}\nabla_\mu A_\nu\nabla_\rho A_\sigma + \lambda^\mu(\phi_\mu - A_\mu)\right]$$

$$L_{\text{kin}} = \mathcal{A}(\phi, X, A)\dot{A}^2 + 2\mathcal{B}^{ij}(\phi, X, A)\dot{A}K_{ij} + \mathcal{K}^{ijkl}(\phi, X, A)K_{ij}K_{kl}$$

$$Q \to A,\ \ \dot{q}^i \to K_{ij},\ \ a \to \mathcal{A},\ \ b_i \to \mathcal{B}^{ij},\ \ k_{ij} \to \mathcal{K}^{ijkl}$$

Degeneracy condition: $\mathcal{A} - \mathcal{B}_{ij}\mathcal{B}_{kl}(\mathcal{K}^{-1})^{ijkl} = 0$

- $\mathcal{A} = 0\,,\ \mathcal{B}_{ij} = 0$: Horndeski
- $\mathcal{A} = 0\,,\ \mathcal{B}_{ij} \neq 0$: GLPV
- $\mathcal{A} \neq 0\,,\ \mathcal{B}_{ij} \neq 0$: DHOST

Non-trivial models !