Evolution of a Dissipative Self-Gravitating Particle Disk
or
Terrestrial Planet Formation

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Outline

Sakagami-san and Me

Planetesimal Dynamics

- Viscous stirring
- Dynamical friction
- Orbital repulsion

Planetesimal Accretion

- Runaway growth of planetesimals
- Oligarchic growth of protoplanets
- Giant impacts

Introduction

Terrestrial Planets

Planets

- Mercury, Venus, Earth, Mars

Alias

- rocky planets

Orbital Radius

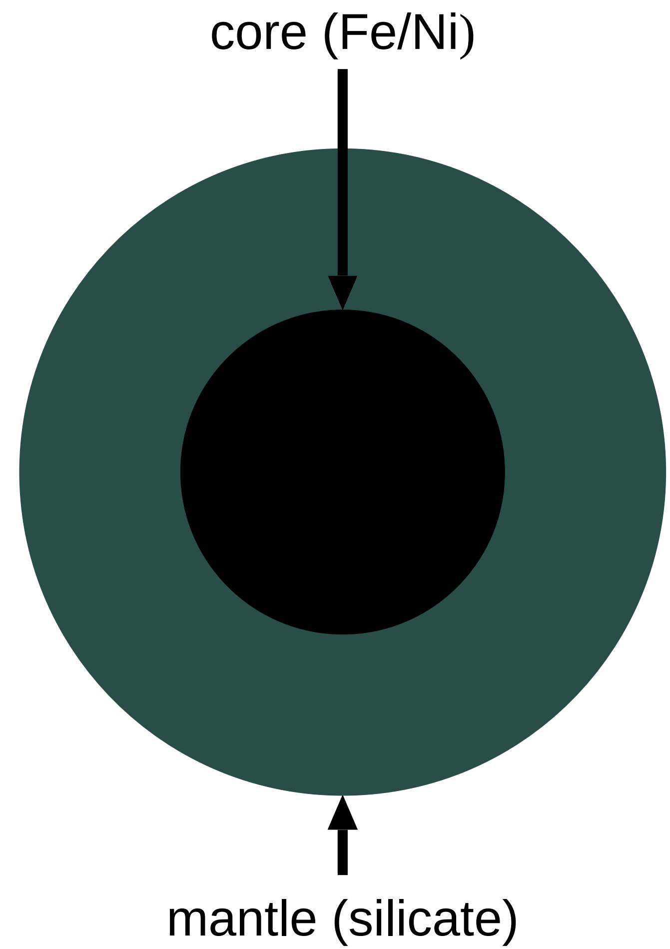
- $\simeq 0.4$-1.5 AU (inner solar system)

Mass

- ~ 0.1-$1\ M_\oplus$

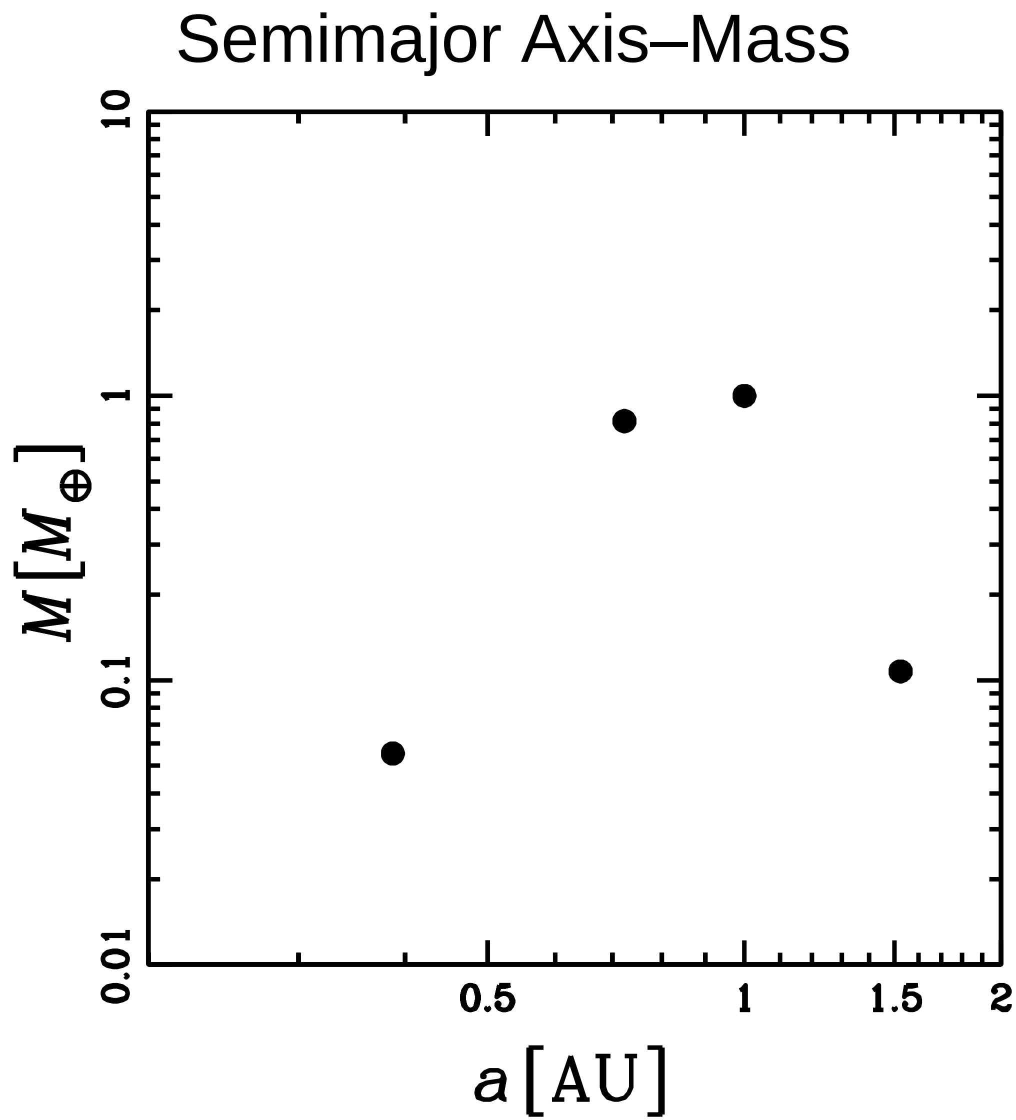
Composition

- rock (mantle), iron (core)



Close-in super-Earths are most common!

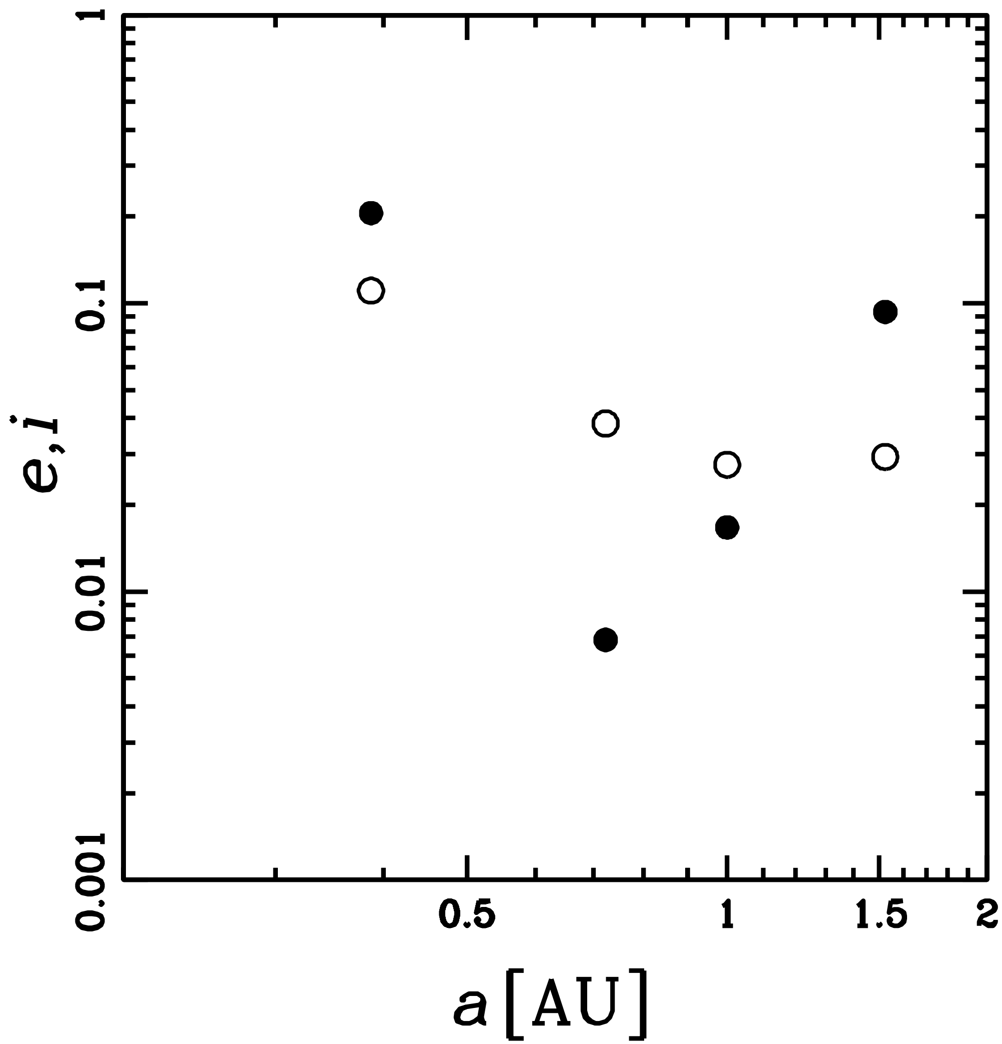
Semimajor Axis-Mass



"two mass populations"

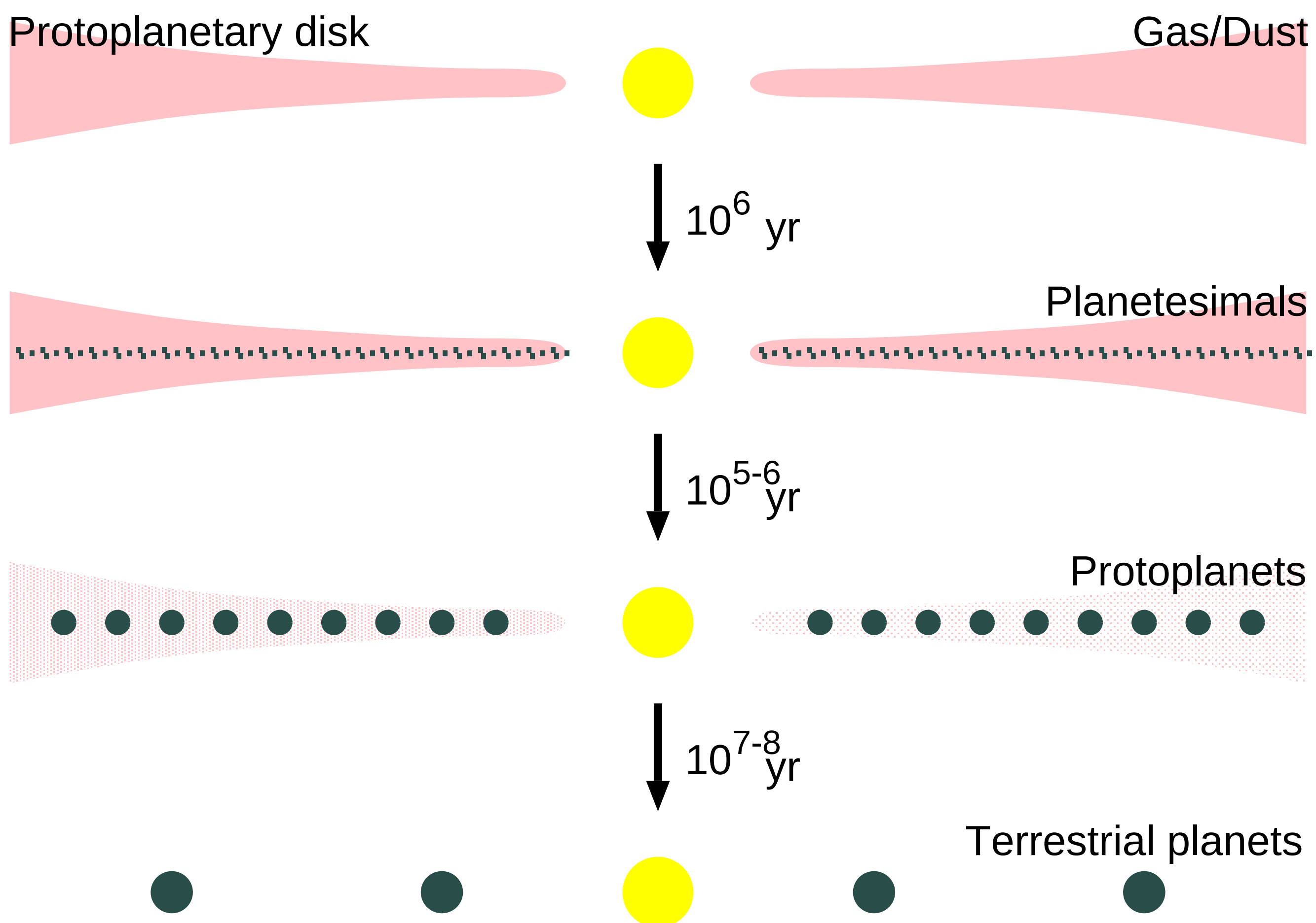
Orbital Elements

Semimajor Axis–Eccentricity (●), Inclination (○)



“nearly circular coplanar”

Terrestrial Planet Formation



Act 1 Dust to planetesimals (gravitational instability/binary coagulation)

Act 2 Planetesimals to protoplanets (runaway-oligarchic growth)

Act 3 Protoplanets to terrestrial planets (giant impacts)

Planetesimal Disks

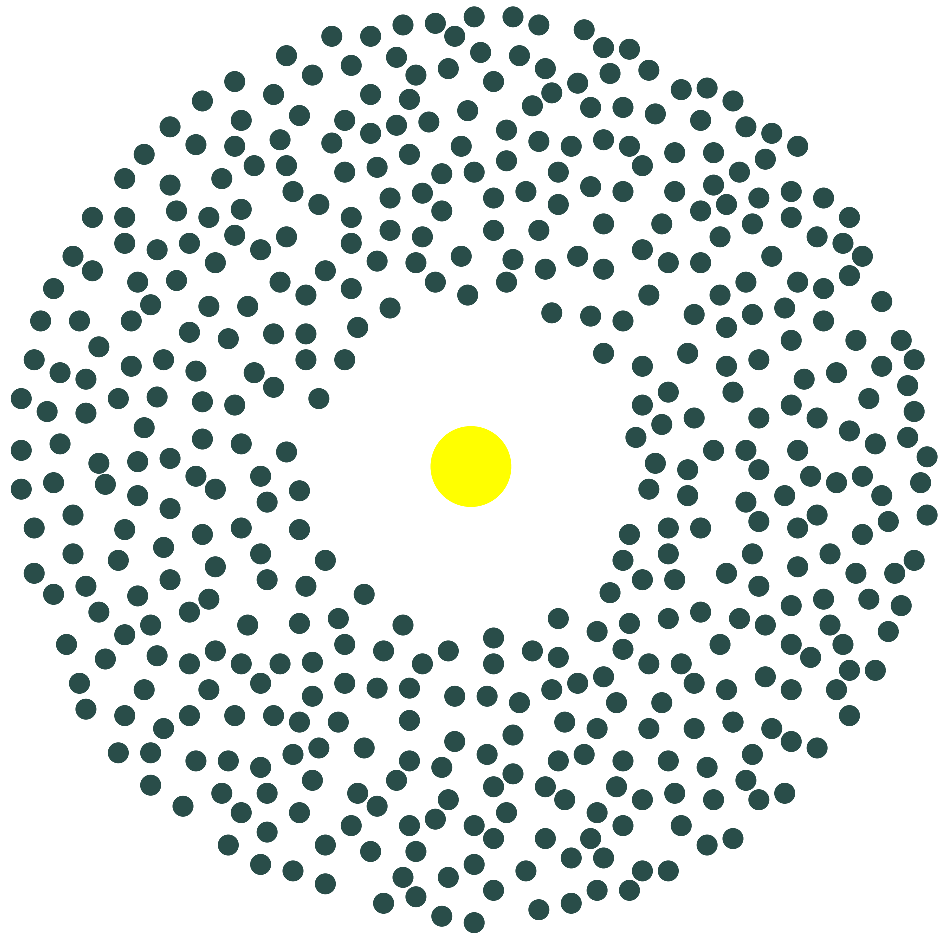
Disk Properties

- many-body (particulate) system
- rotation
- self-gravity
- dissipation (collisions and accretion)

Planet Formation as Disk Evolution

- evolution of a dissipative self-gravitating particulate disk
- velocity and spatial evolution $\leftrightarrow$ mass evolution

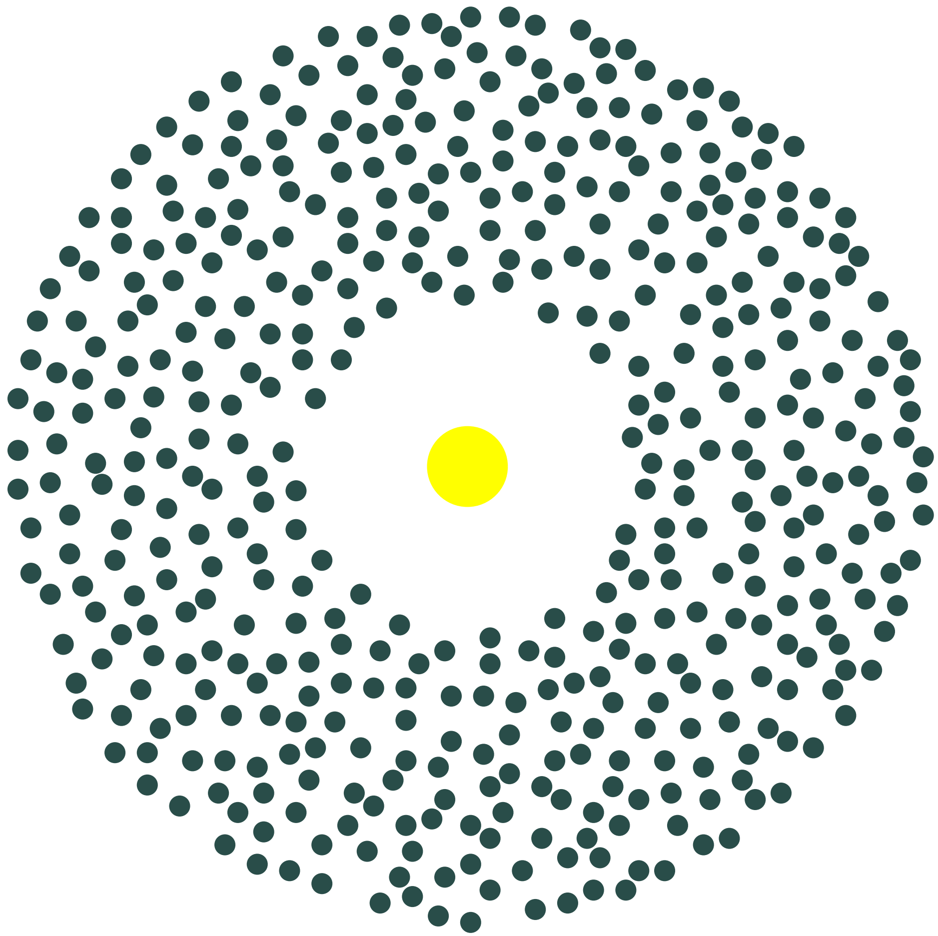
Question



How does a dissipative self-gravitating particulate disk evolve?

Planetesimal Dyanamics

Question



How do particle orbits (a, e, i) evolve?

Terminology

Random Velocity

- deviation velocity from a non-inclined circular orbit

$$v_{\rm ran} \simeq \left(e^2 + i^2\right)^{1/2} v_{\rm K}$$

$$\sigma_R \propto \sigma_e, \ \ \sigma_z \propto \sigma_i$$

e : eccentricity, i : incination, $v_{\rm K}$: Kepler circular velocity

Hill (Roche/Tidal) Radius

- radius of the potential well of an orbiting body

$$r_{\rm H} = \left(\frac{m}{3M_{\rm c}}\right)^{1/3} a$$

$M_{\rm c}$: central body mass, m : orbiting body mass, a : semimajor axis

Disk Properties

Dynamics

- central gravity dominant (nearly Keplerian orbit)
- differential rotation (shear velocity)
- “collisional” system (evolution by two-body encounters)

Structure

- disk thickness $\propto$ velocity dispersion ($\sigma_z \propto \sigma_i$)

Equation of Motion

$$\frac{\mathrm{d}\boldsymbol{v}_i}{\mathrm{d}t} = \underbrace{-GM_\mathrm{c}\frac{\boldsymbol{x}_i}{|\boldsymbol{x}_i|^3}}_{\text{central gravity}} + \underbrace{\sum_{j\neq i}^{N} Gm_j \frac{\boldsymbol{x}_j - \boldsymbol{x}_i}{|\boldsymbol{x}_j - \boldsymbol{x}_i|^3}}_{\text{mutual interaction}} + \underbrace{\boldsymbol{f}_\mathrm{gas}}_{\text{gas drag}} + \underbrace{\boldsymbol{f}_\mathrm{col}}_{\text{collision effect}}$$

- central gravity (dominant) $\Rightarrow$ nearly Keplerian orbits
- mutual interaction $\Rightarrow$ random velocity$\Uparrow$
- gas drag $\Rightarrow$ random velocity$\Downarrow$
- collision $\Rightarrow$ random velocity$\Downarrow$

mutual interaction + gas drag/collision $\Rightarrow$ equilibrium random velocity

Equation of Motion

$$\frac{\mathrm{d}\boldsymbol{v}_i}{\mathrm{d}t} = \underbrace{-GM_\mathrm{c}\frac{\boldsymbol{x}_i}{|\boldsymbol{x}_i|^3}}_{\text{central gravity}} + \underbrace{\sum_{j\neq i}^{N} Gm_j \frac{\boldsymbol{x}_j - \boldsymbol{x}_i}{|\boldsymbol{x}_j - \boldsymbol{x}_i|^3}}_{\text{mutual interaction}}$$

- central gravity (dominant) $\Rightarrow$ nearly Keplerian orbits
- mutual interaction $\Rightarrow$ random velocity $\Uparrow$

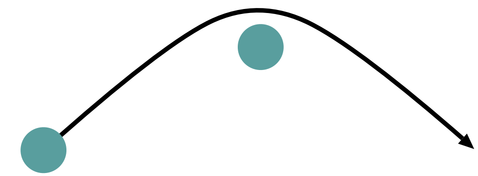
$\Uparrow$

Two-Body Relaxation

Two-Body Relaxation

Elementary Process

- Two-body gravitational scattering



Chandrasekhar's Two-Body Relaxation Time

- Timescale to forget the initial orbit

$$t_{\rm relax} \equiv \frac{v^2}{{\rm d}v^2/{\rm d}t} \simeq \frac{1}{n\pi {r_{\rm g}}^2 v \ln\Lambda} = \frac{v^3}{n\pi G^2 m^2 \ln\Lambda}$$

n: number density, $r_{\rm g}$: gravitational radius, $\ln\Lambda$: Coulomb logarithm

(Chandrasekhar 1949)

Relaxation of Particulate Disks

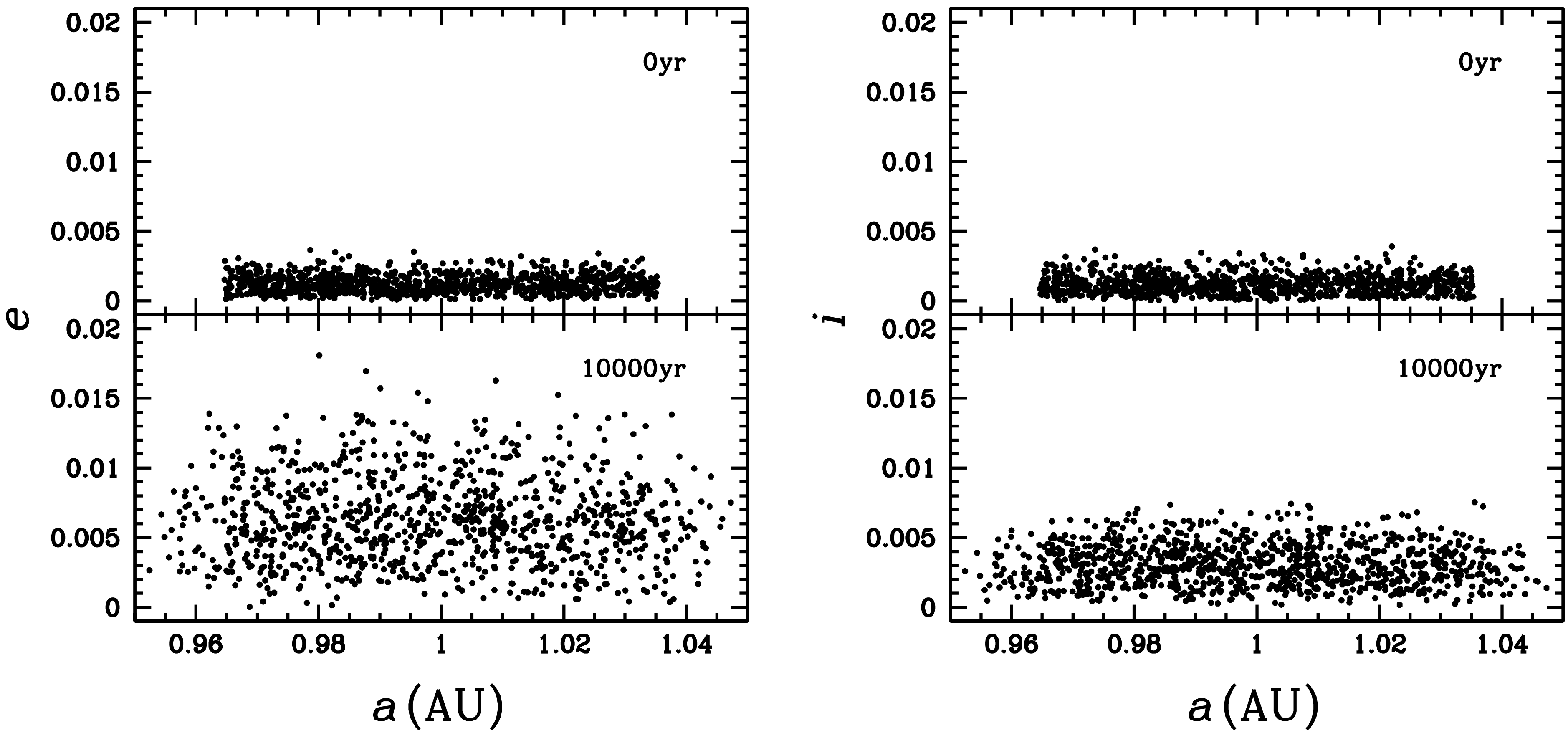
Viscous Stirring (Disk Heating)

- increase of random velocity $v_{\rm ran}$ (e and i)

Dynamical Friction

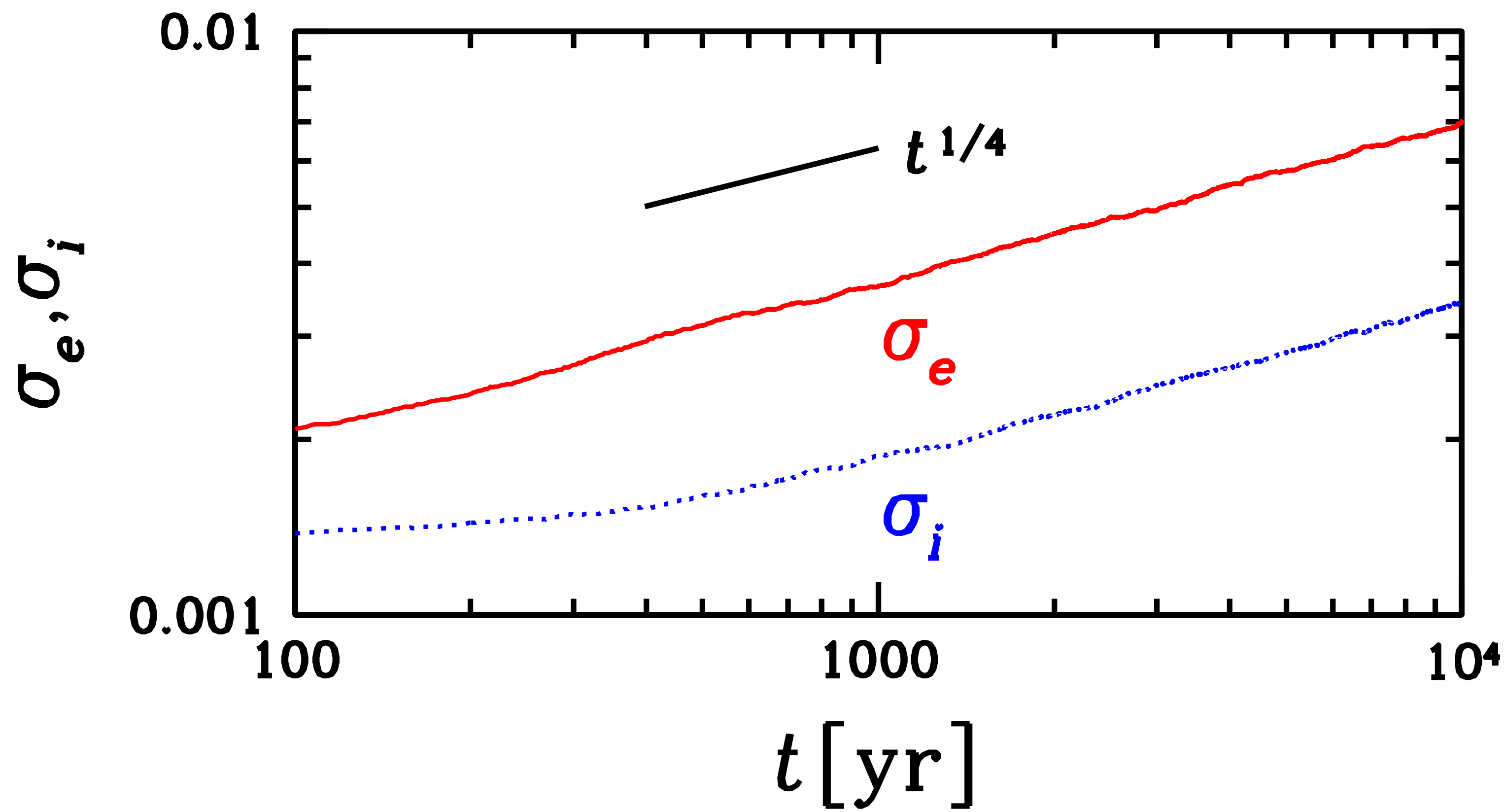
- equiparation of random energy $mv_{\rm ran}^2 \propto m(e^2 + i^2)$

Viscous Stirring



- increase of e and i ($\sigma_e > \sigma_i$)
- diffusion in a

Viscous Stirring



- $\sigma_e, \sigma_i \propto t^{1/4}$ (two-body relaxation timescale)
- $\sigma_e/\sigma_i = \sigma_R/\sigma_z \simeq 2$ (anisotropic velocity dispersion)

Viscous Stirring

Elementary Process

- two-body scattering: shear velocity $\rightarrow$ random velocity

Timescale

$$t_{\mathrm{VS}} \equiv \frac{\sigma^2}{\mathrm{d}\sigma^2/\mathrm{d}t} \simeq \frac{\sigma^3}{n\pi G^2 m^2 \ln\Lambda} \Rightarrow t_{\mathrm{VS}} \propto \sigma^4 \Rightarrow \sigma \propto t^{1/4}$$

$$n \propto (\text{thickness})^{-1} \propto \sigma^{-1}$$

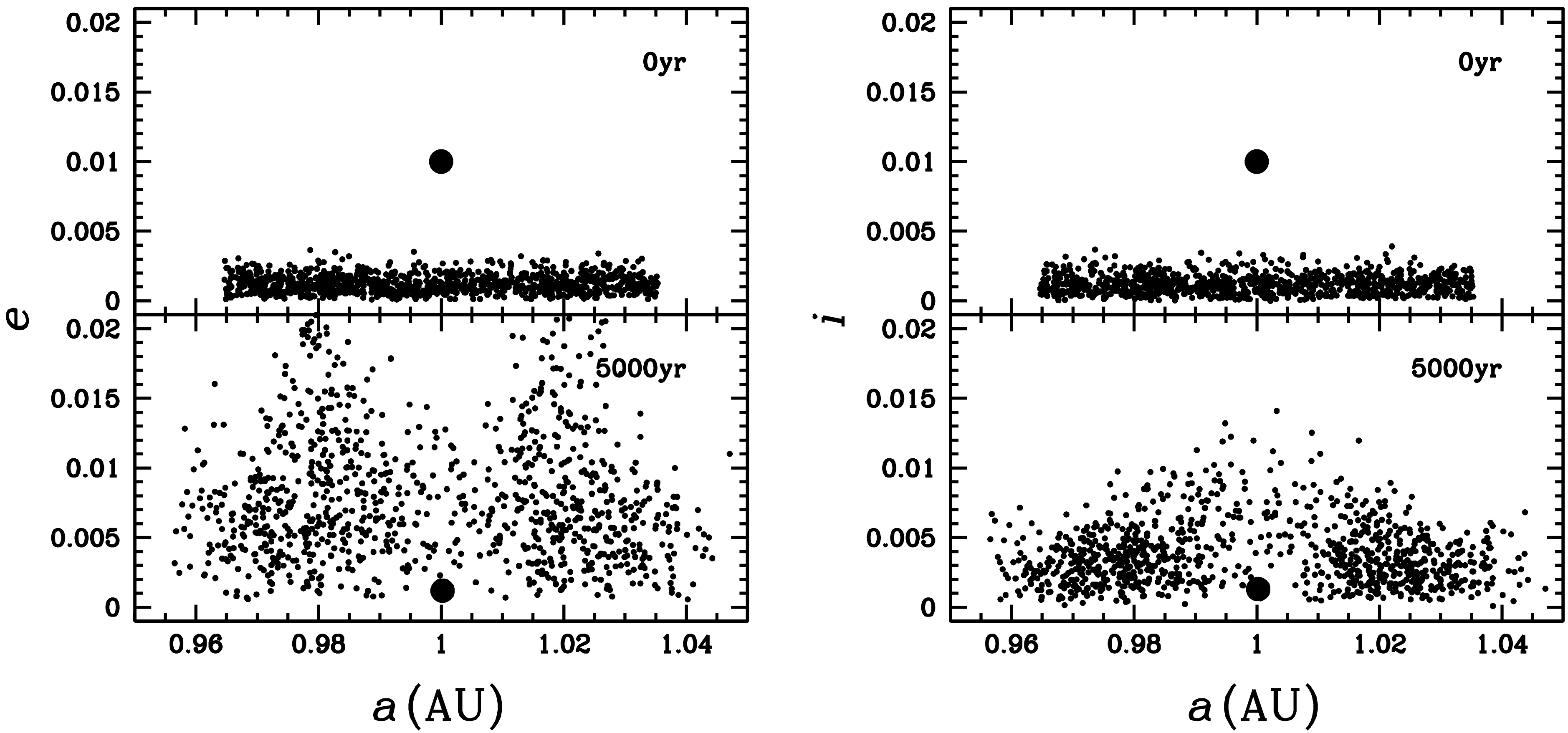
(Ida & Makino 1992; EK & Ida 1992)

Anisotropic Velocity Dispersion

- $\sigma_e/\sigma_i \propto$ shear strength
  (cf. $\sigma_R/\sigma_z \simeq 1.4$ for the Galactic disk)

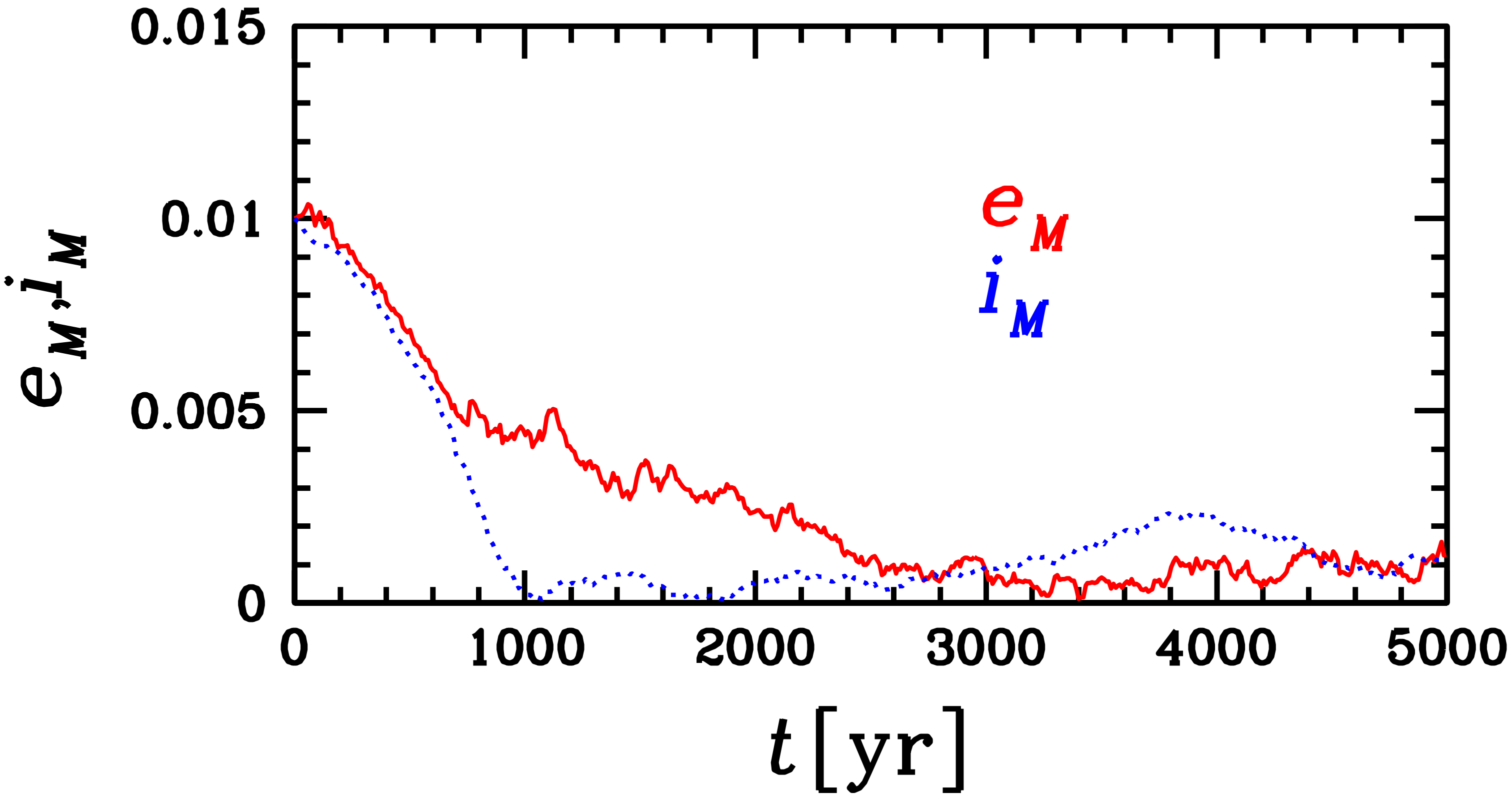
(Ida, EK, & Makino 1993)

Dynamical Friction



- decrease of e_M and i_M ($\leftrightarrow$ increase of local e and i)
- almost constant a_M

Dynamical Friction



- $e_M, i_M \to 0$ (non-inclined circular orbit)

Dynamical Friction

Chandrasekhar's Formula

A large particle with M and v_M in a swarm of small particles with m and v_m

$$\frac{1}{v_M}\frac{\mathrm{d}v_M}{\mathrm{d}t} \sim \frac{G^2 M m n_m}{v_M^3}$$

$$(v_M > v_m)$$

(Chandrasekhar 1949)

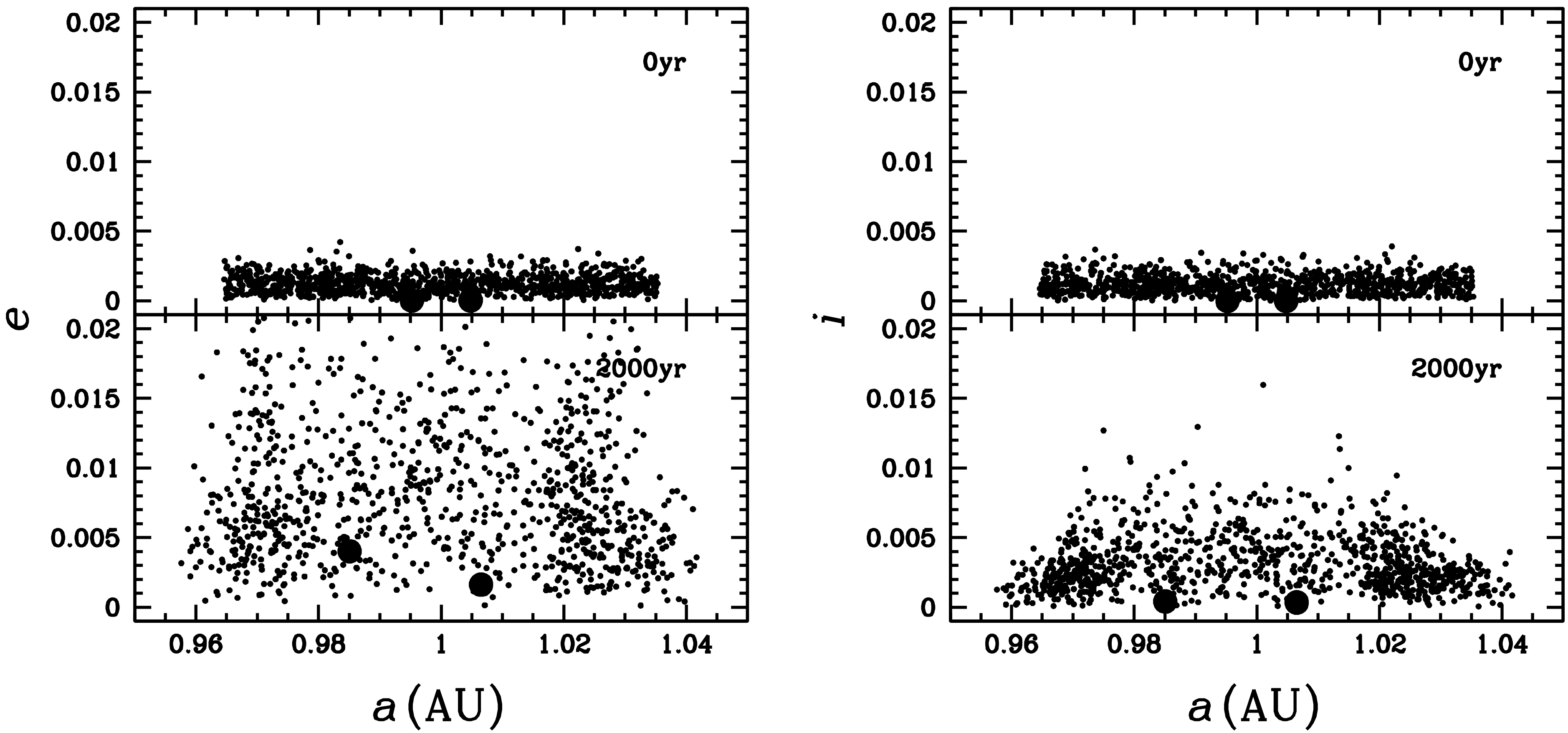
Application to a Particulate Disk

$$\frac{1}{e_M}\frac{\mathrm{d}e_M}{\mathrm{d}t} \sim \frac{G^2 M m n_{\mathrm{s},m}}{2 i_m a e_M^3 a^3 \Omega^3} \sim \frac{G^2 M \Sigma}{e_M^4 a^4 \Omega^3}$$

$$(\Sigma = m n_{\mathrm{s},m},\ v_M \simeq e_M a \Omega,\ n_m \simeq n_{\mathrm{s},m}/2 a i_M,\ i_m < i_M)$$

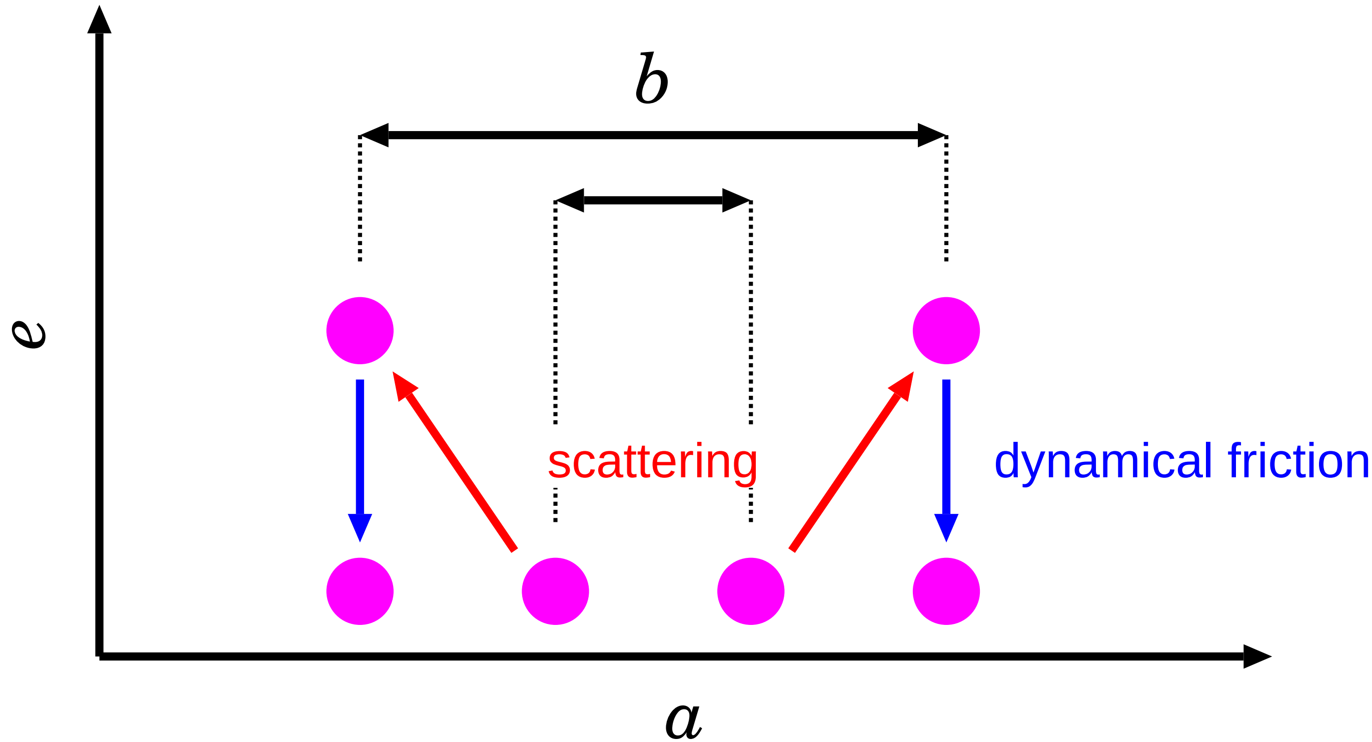
$$t_{\mathrm{DF}} \equiv \frac{e_M}{\mathrm{d}e_M/\mathrm{d}t} \sim \frac{e_M^4 a^4 \Omega^3}{G^2 M \Sigma}$$

Orbital Repulsion



- expansion of orbital separation b: $b = 3r_{\rm H} \to b \simeq 8r_{\rm H}$
- keeping e_M and i_M small

Orbital Repulsion Mechanism



1. protoplanet-protoplanet scattering (e_M, $b \Uparrow$)
2. dynamical friction from planetesimals ($e_M \Downarrow$, a_M constant)

$$b \gtrsim 5r_{\mathrm{H}}$$

(EK & Ida 1995)

Summary

Two-Body Relaxation of Particulate Disks

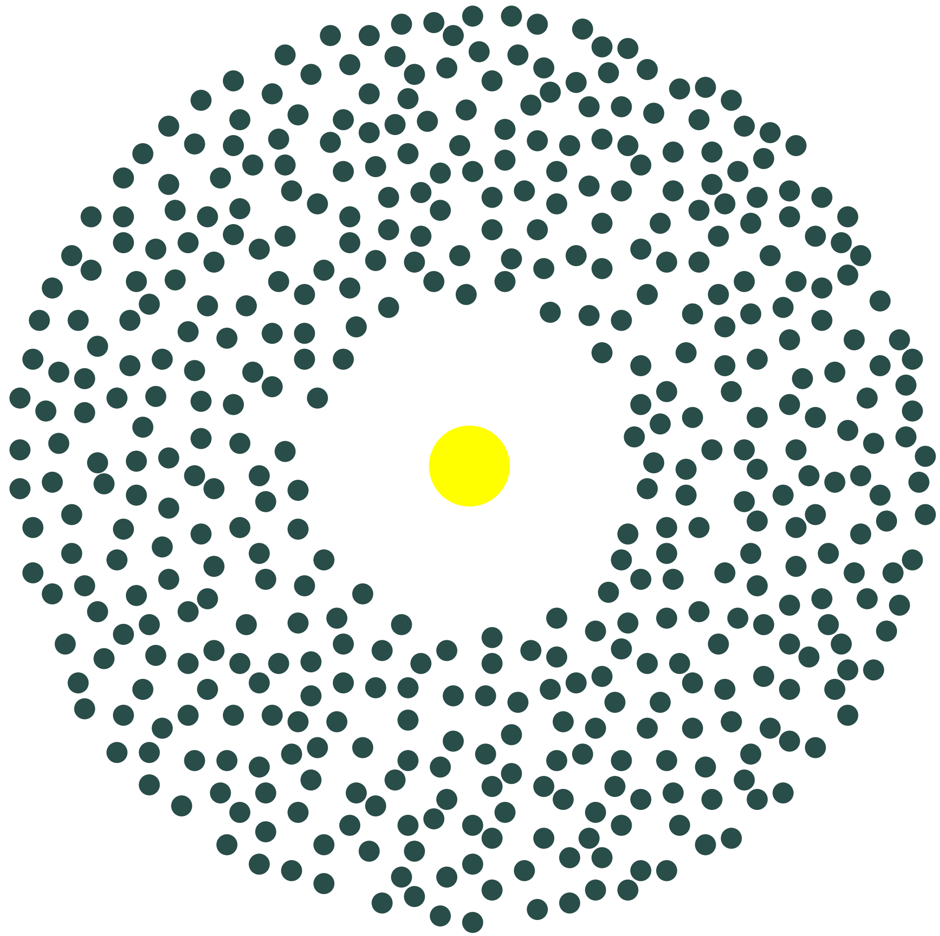
Disk Evolution:

- Viscous stirring
  - $\sigma_e, \sigma_i \propto t^{1/4}$ ($\Leftarrow$ disk)
  - $\sigma_e/\sigma_i \simeq 2$ ($\Leftarrow$ differential rotation)
- Dynamical friction
  - $e, i \propto m^{-1/2}$ ($\Leftarrow$ energy equipartition)
- Orbital repulsion
  - $b \gtrsim 5 r_\mathrm{H}$ ($\Leftarrow$ scattering, dynamical friction)

All these elementary processes control the basic dynamics and structure of particulate disks!

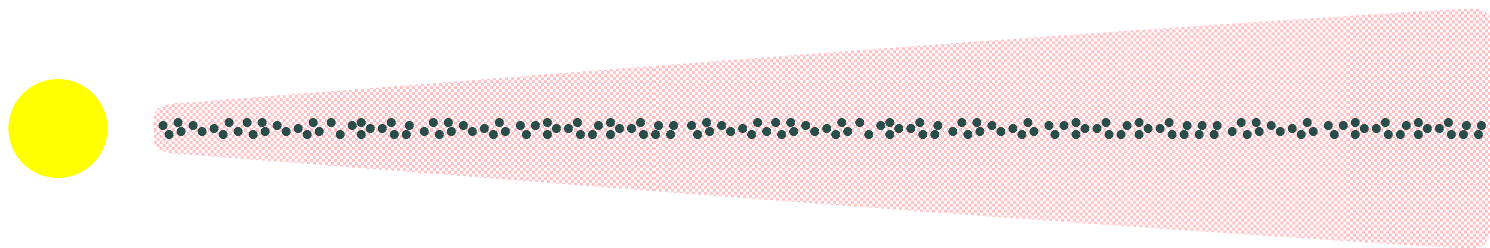
Planetesimal Accretion

Question



How does particle mass distribution evolve by accretion?

Planetesimals



planetesimals

Mass (size)

- $m \sim 10^{18}\,\mathrm{g}\ (r \sim 1\,\mathrm{km})$

Surface density distribution

$$\Sigma_{\mathrm{solid}} = \Sigma_1 \left(\frac{a}{1\,\mathrm{au}}\right)^{-\alpha}\ \mathrm{g cm^{-2}}$$

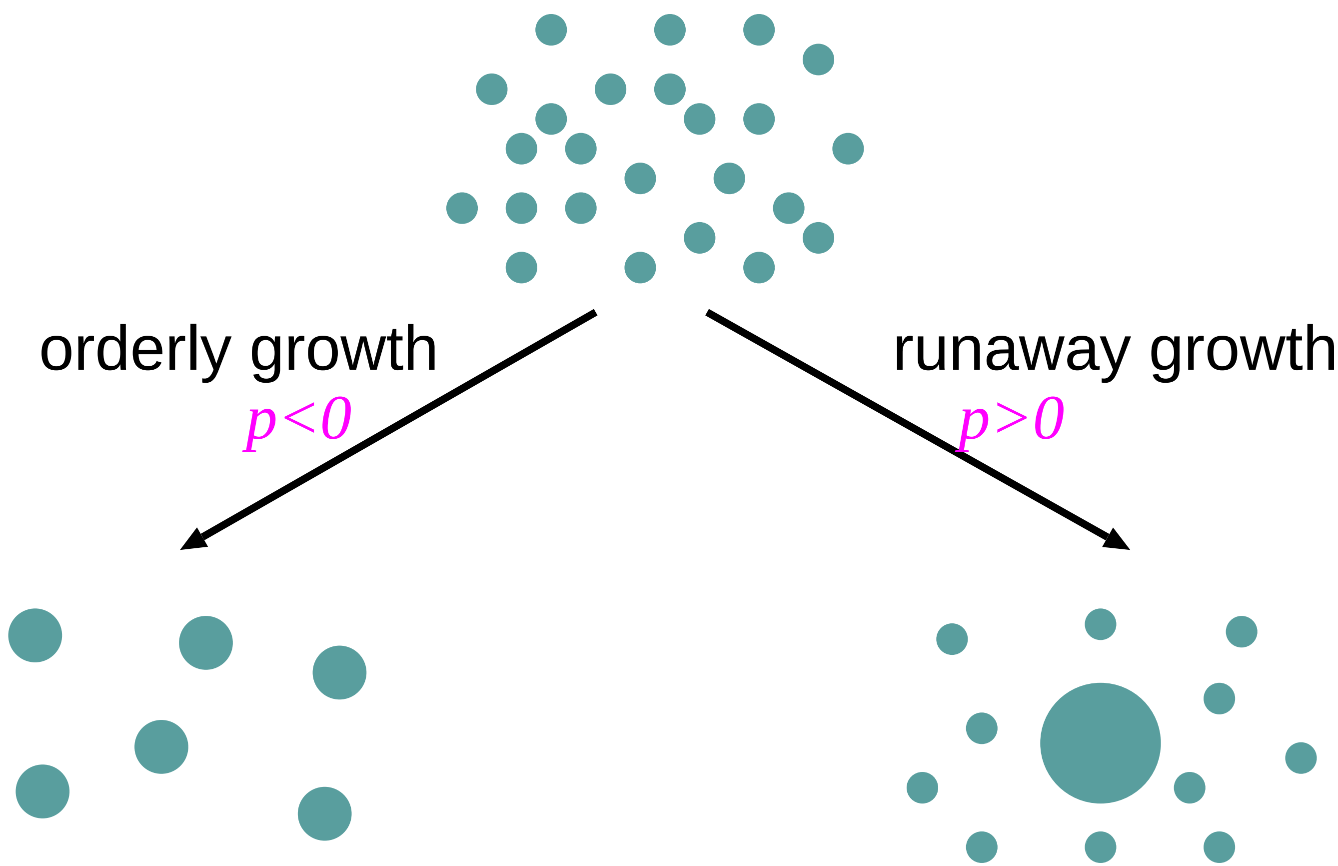
$1 \le \Sigma_1 \le 100$, $1/2 \le \alpha \le 5/2$

standard protosolar disk: $\Sigma_1 \simeq 10$, $\alpha = 3/2$ (Hayashi 1981)

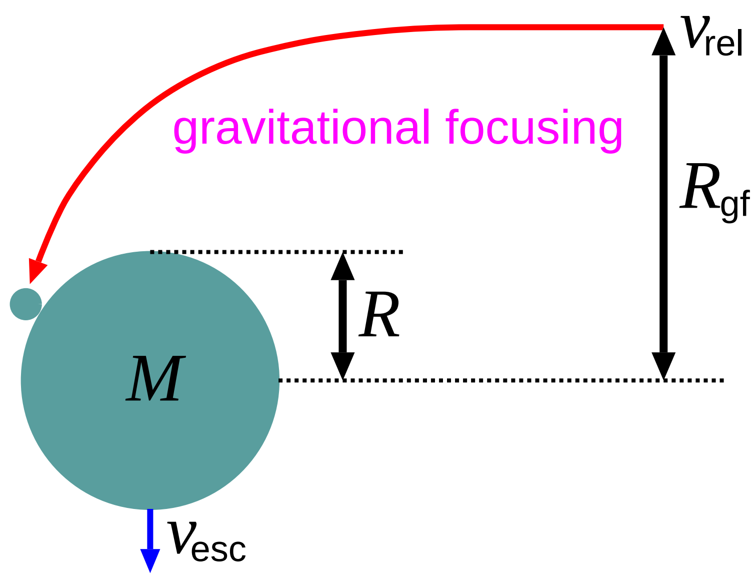
Growth Mode

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{M_1}{M_2}\right) = \frac{M_1}{M_2}\left(\frac{1}{M_1}\frac{\mathrm{d}M_1}{\mathrm{d}t} - \frac{1}{M_2}\frac{\mathrm{d}M_2}{\mathrm{d}t}\right)$$

relative growth rate: $\frac{1}{M}\frac{\mathrm{d}M}{\mathrm{d}t} \propto M^p$



Collisional Cross-Section



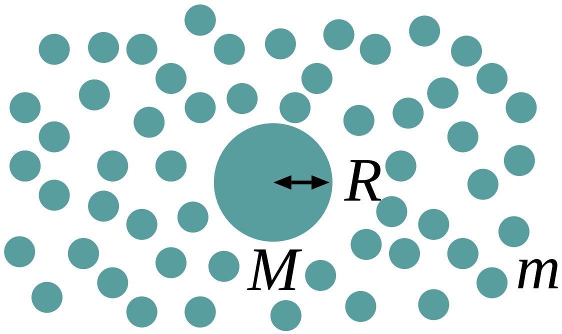
Gravitational focusing

$$R_{\rm gf} = R\left(1 + \frac{2GM}{Rv_{\rm rel}^2}\right)^{1/2} = R\left(1 + \frac{v_{\rm esc}^2}{v_{\rm rel}^2}\right)^{1/2}$$

Collisional cross-section

$$S_{\rm gf} = \pi R_{\rm gf}^2 = \pi R^2\left(1 + \frac{v_{\rm esc}^2}{v_{\rm rel}^2}\right)$$

Growth Rate



Test body: $M, \; R, \; v_{\rm esc}$

Field bodies: $n\,($number density$), \; m$

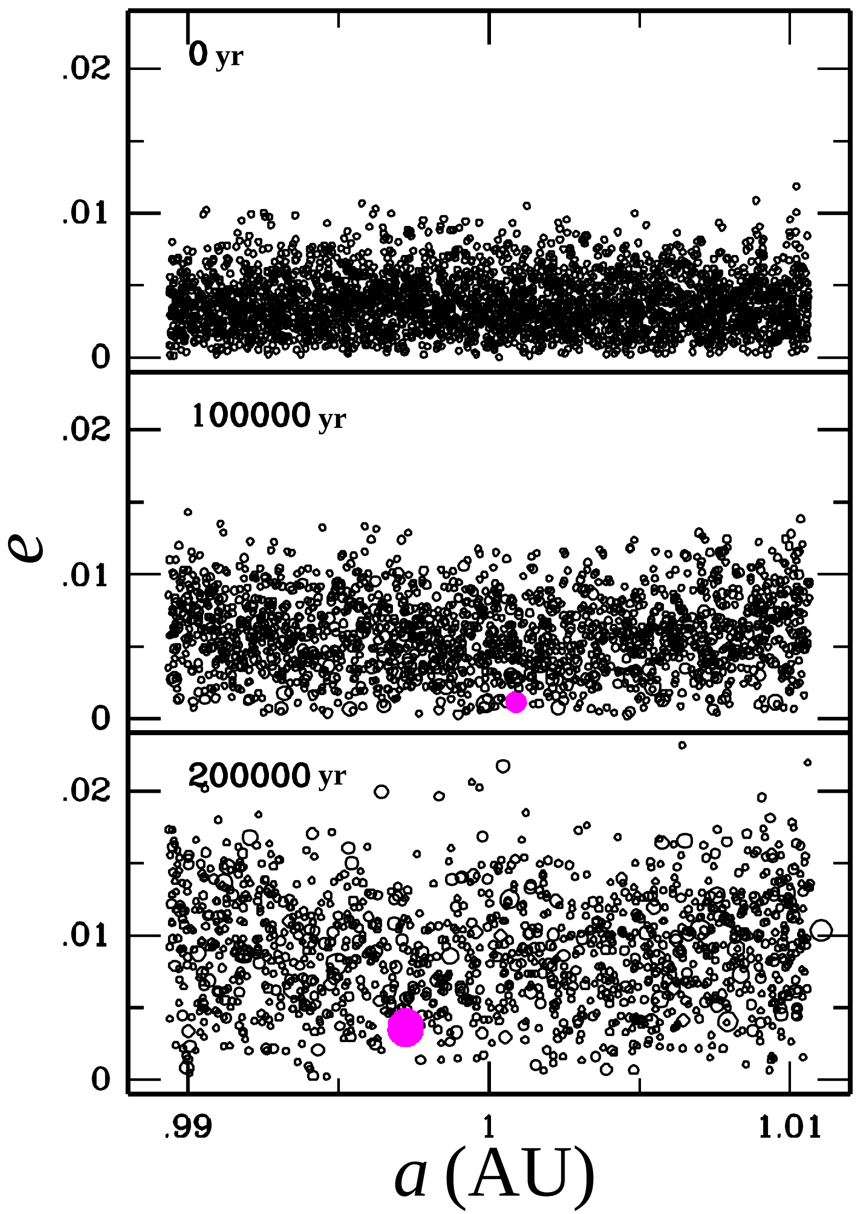
$$\frac{\mathrm{d}M}{\mathrm{d}t} \simeq n\pi R^2 \left(1 + \frac{v_{\rm esc}^2}{v_{\rm rel}^2}\right) v_{\rm rel} m \Rightarrow \frac{1}{M}\frac{\mathrm{d}M}{\mathrm{d}t} \propto M^{1/3} v_{\rm ran}^{-2}$$

$$\left(v_{\rm rel} \simeq v_{\rm ran}, \; n \propto v_{\rm ran}^{-1}, \; v_{\rm esc} \propto M^{1/3}, \; R \propto M^{1/3}, \; v_{\rm rel} < v_{\rm esc}\right)$$

Random velocity controls

- the growth mode
- the growth timescale

Runaway Growth of Planetesimals



(EK & Ida 2000)

self-gravity of planetesimals dominant for random velocity

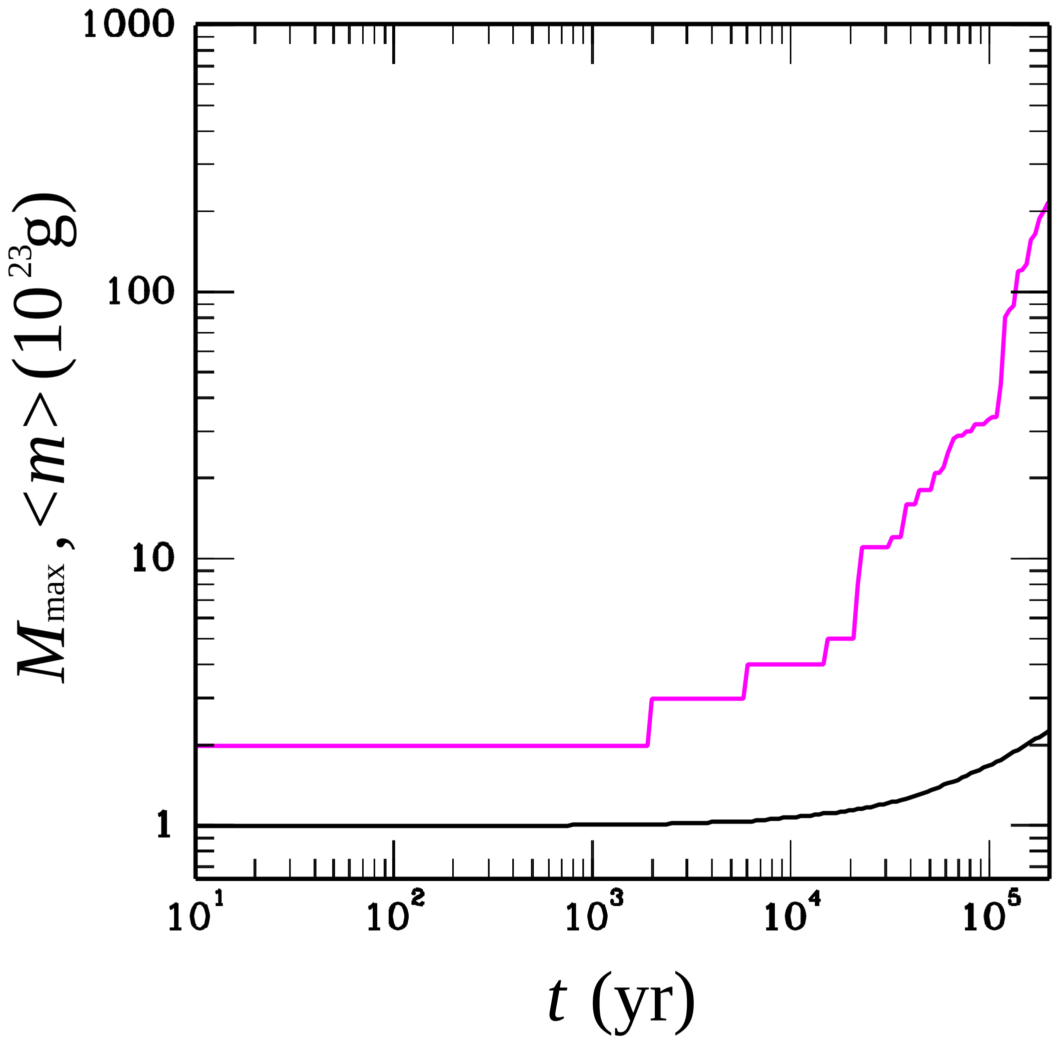
$$v_{\rm ran} \neq f(M)$$

$$\Downarrow$$

$$\frac{1}{M}\frac{{\rm d}M}{{\rm d}t} \propto M^{1/3} v_{\rm ran}^{-2} \propto M^{1/3}$$

runaway growth!

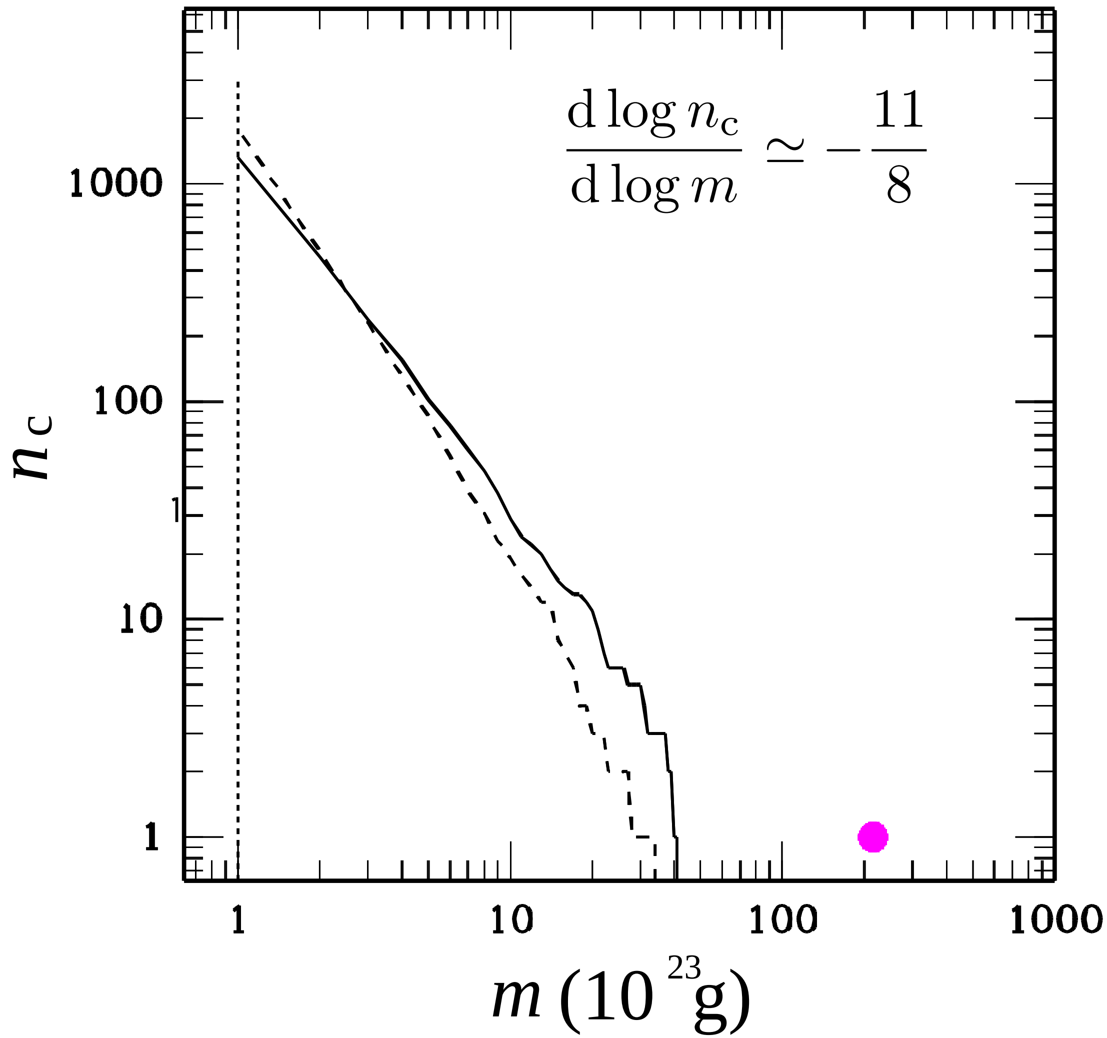
Runaway Growth of Planetesimals



solid: $M_{\rm max}$, dashed: $\langle m \rangle$

(EK & Ida 2000)

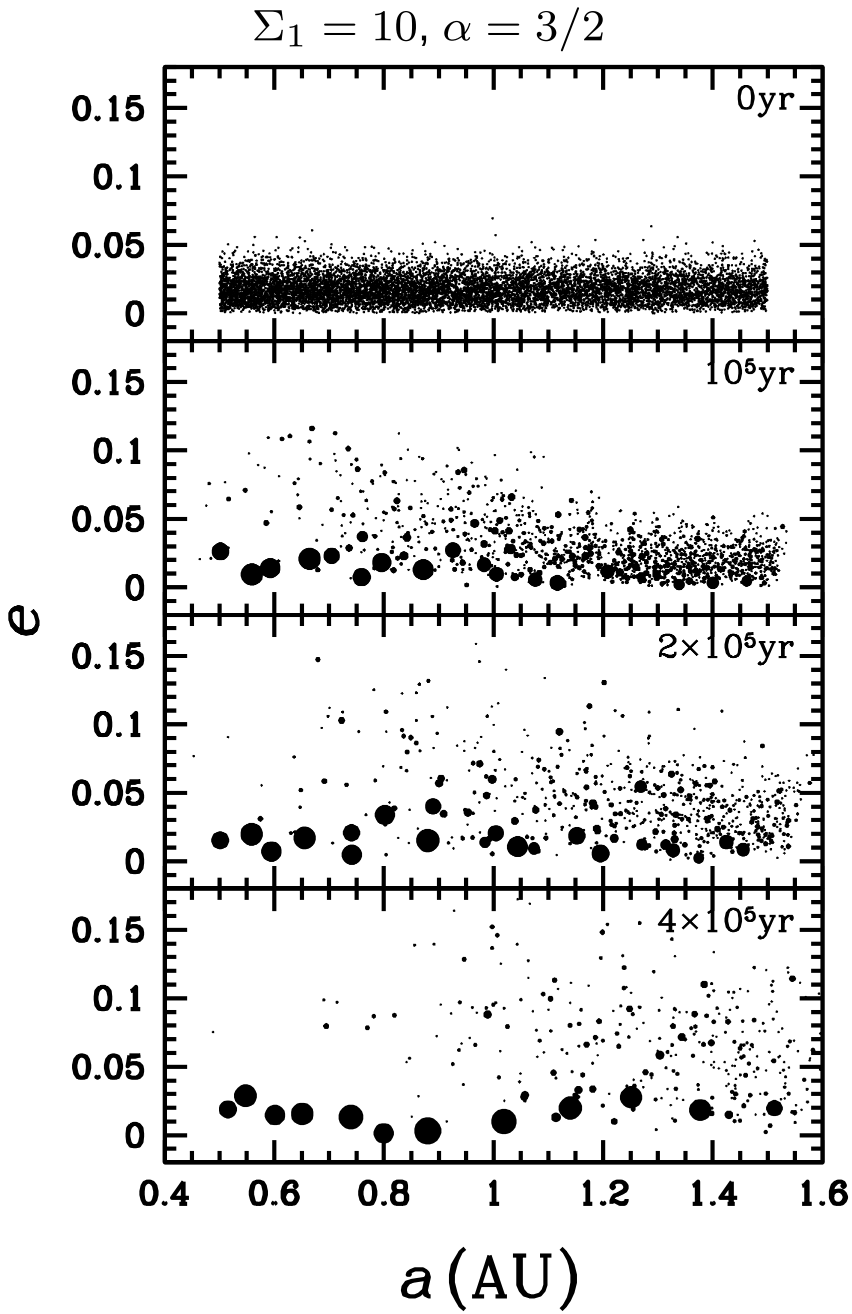
Runaway Growth of Planetesimals



dotted: 0 yr, dashed: 10^5 yr, solid: 2×10^5 yr

(EK & Ida 2000)

Oligarchic Growth of Protoplanets



(EK & Ida 2002)

Slowdown of runaway

scattering of planetesimals by a protoplanet with $M \gtrsim 100m$

$$v_{\rm ran} \propto r_{\rm H} \propto M^{1/3}$$

$$\Downarrow$$

$$\frac{1}{M}\frac{{\rm d}M}{{\rm d}t} \propto M^{1/3} v_{\rm ran}^{-2} \propto M^{-1/3}$$

orderly growth!

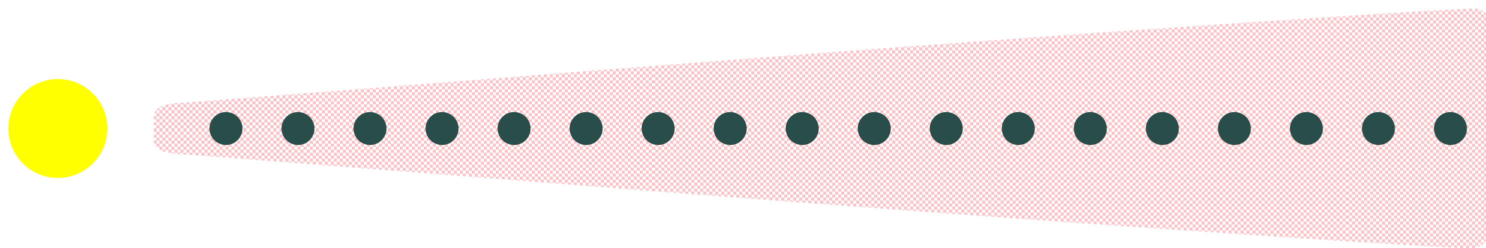
(Ida & Makino 1993)

Orbital repulsion

orbital separation: $b \simeq 10 r_{\rm H}$

(EK & Ida 1998)

Protoplanets



Assumptions

- no radial migration
- 100% accretion efficiency

Isolation mass

$$M_{\rm iso} \simeq 2\pi a b \Sigma_{\rm solid} = 0.16 \left(\frac{b}{10 r_{\rm H}}\right)^{3/2} \left(\frac{\Sigma_1}{10}\right)^{3/2} \left(\frac{a}{1\,{\rm AU}}\right)^{(3/2)(2-\alpha)} M_\oplus$$

Growth time

$$t_{\rm grow} \simeq 3.2 \times 10^5 \left(\frac{f_{\rm gas}}{240}\right)^{-2/5} \left(\frac{b}{10 r_{\rm H}}\right)^{1/10} \left(\frac{\Sigma_1}{10}\right)^{-9/10} \left(\frac{a}{1\,{\rm AU}}\right)^{(9\alpha+16)/10} {\rm yr}$$

(EK & Ida 2002, 2012)

Isolation Mass of Protoplanets

Standard protosolar disk

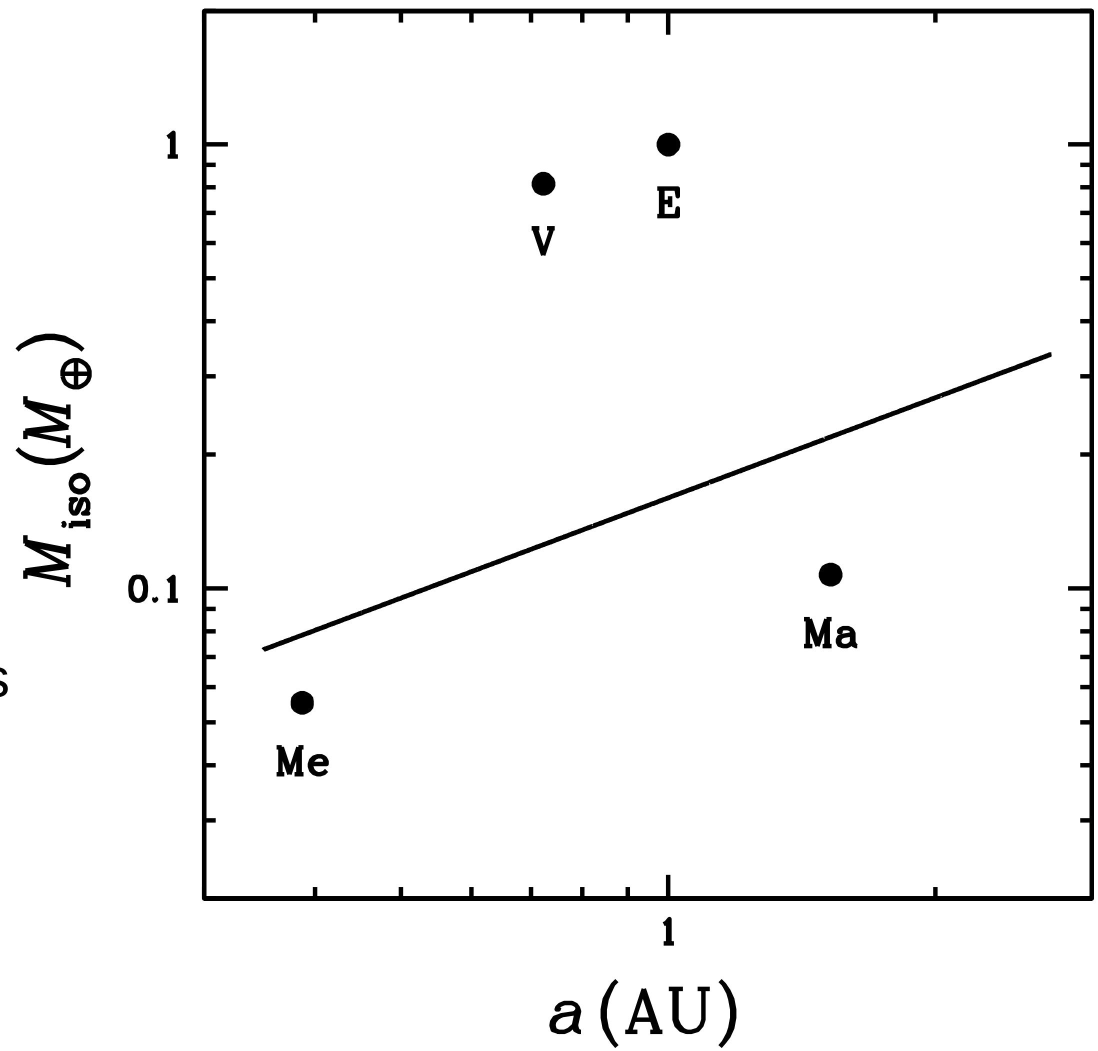
$$\Sigma_1 = 10, \ \alpha = 3/2$$

Terrestrial planet zone

$$M_{\rm iso} \simeq 0.1 M_\oplus$$

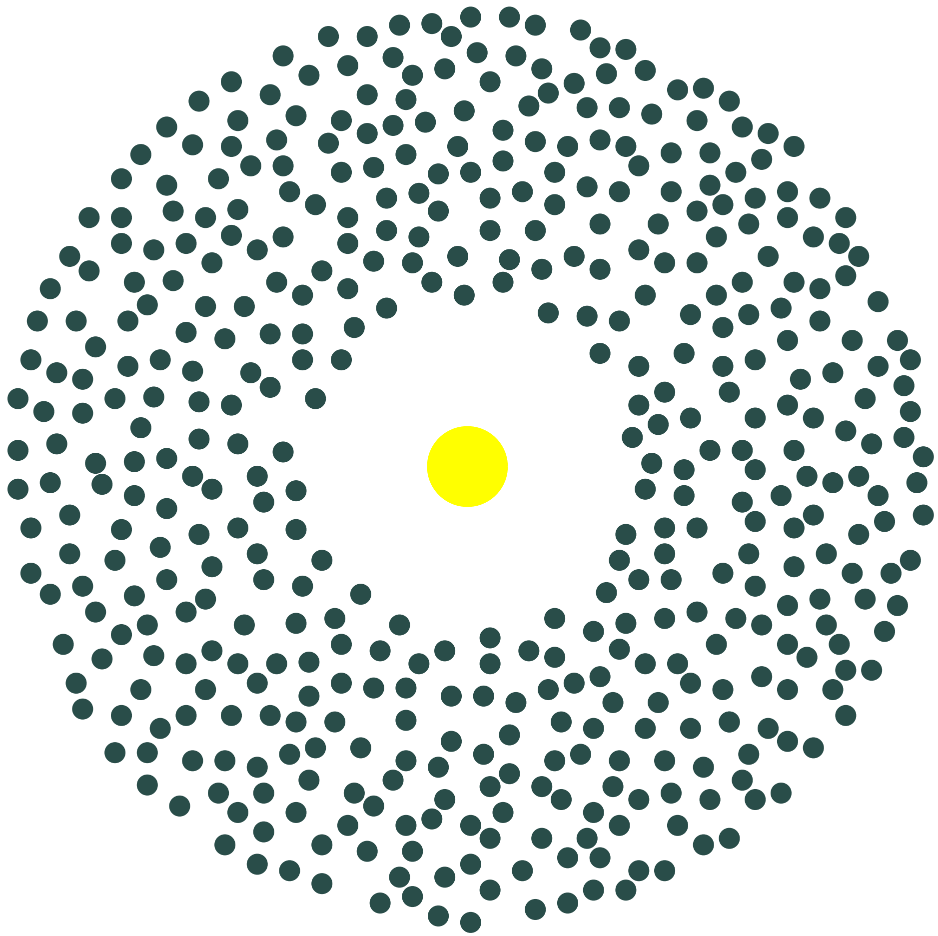
Final formation stage

- large planets:
  impacts among protoplanets
- small planets:
  leftover protoplanets



(EK & Ida 2000)

Question



What is the final state of disk evolution?

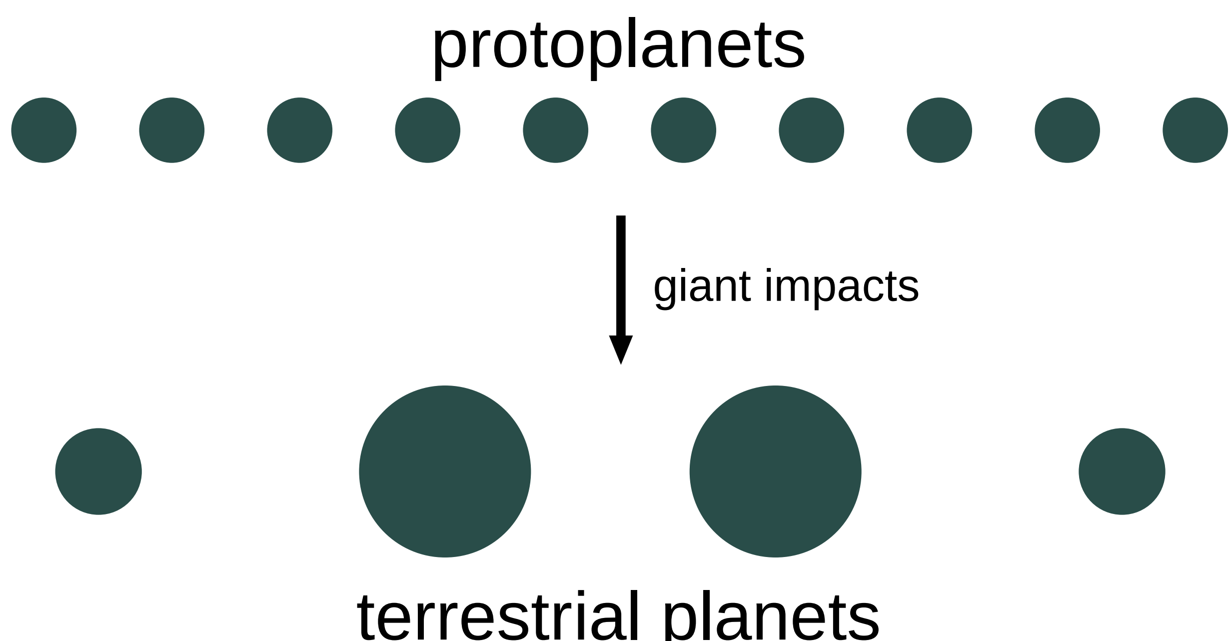
Protoplanets to Terrestrial Planets

Giant Impacts among Protoplanets

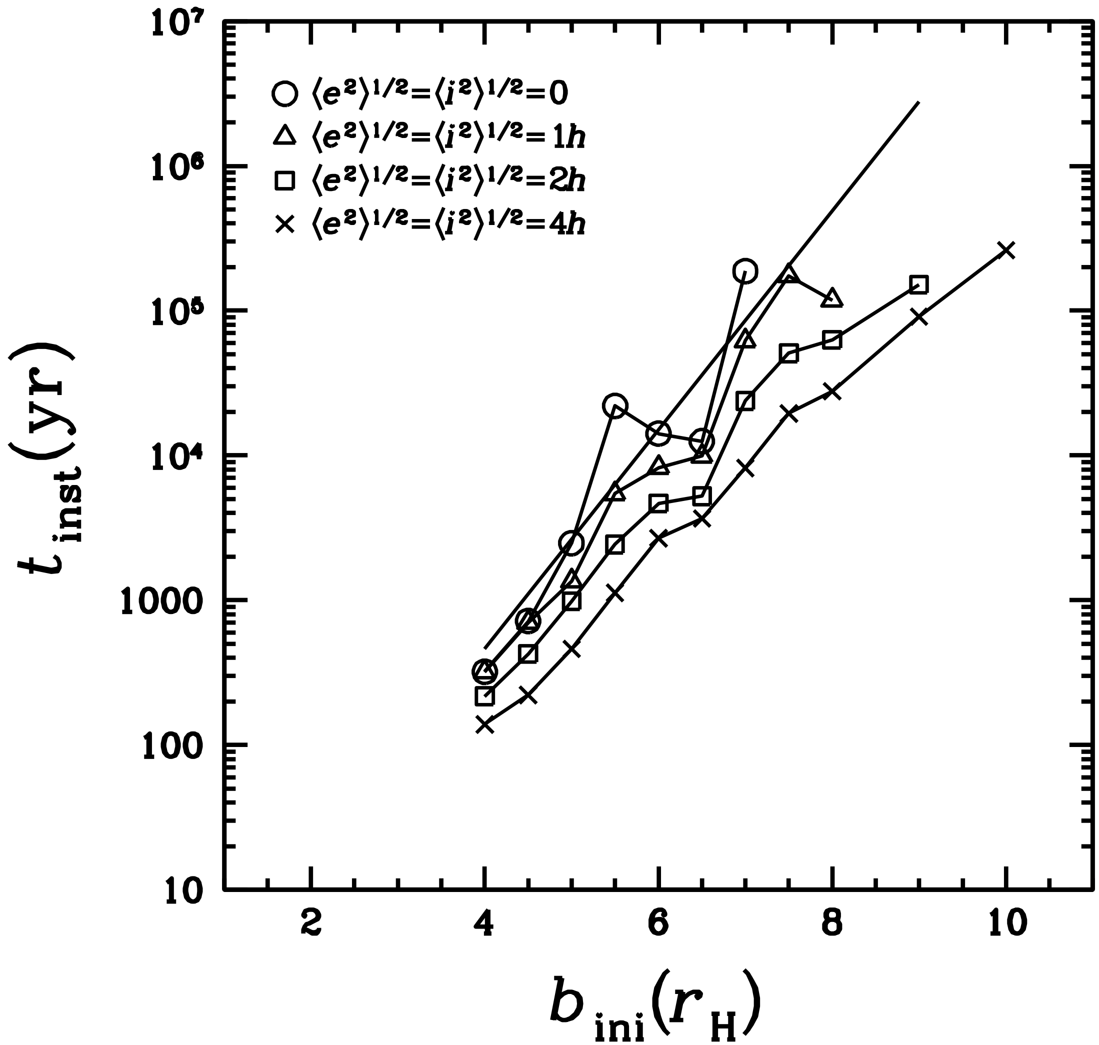
- Protoplanets gravitationally perturb each other to become orbitally unstable after gas dispersal ($t_{\rm gas} \lesssim 10^7\,{\rm yr}$)

$$\log t_{\rm inst} \simeq c_1(b/r_{\rm H}) + c_2$$

(e.g., Chambers+ 1996; Yoshinaga, EK & Makino 1999)



Timescale of Orbital Instability



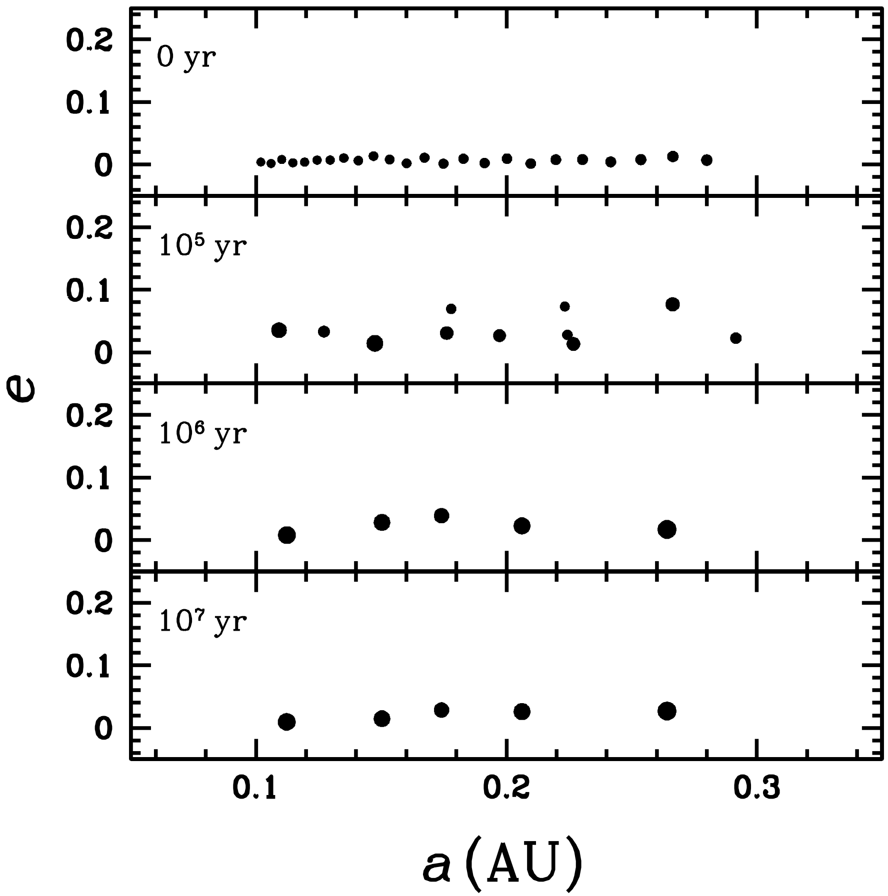
$$\log t_{\rm inst} \simeq c_1(b_{\rm ini}/r_{\rm H}) + c_2$$

(Yoshinaga, EK & Makino 1999)

Example Runs

$\Sigma_1 = 10$, $\alpha = 3/2$, $b = 10 r_{\rm H}$

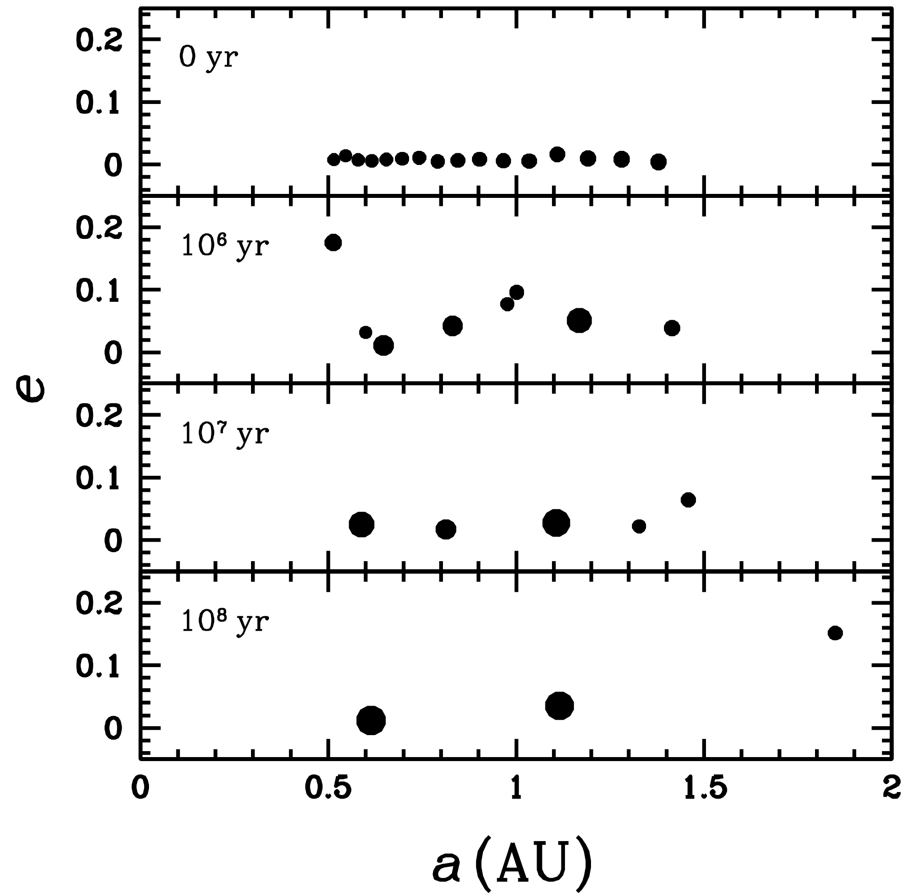
$r = 0.1\text{-}0.3\,\mathrm{au}$



$N:\ 24 \rightarrow 5$

compact, dynamically cold

$r = 0.5\text{-}1.5\,\mathrm{au}$



$N:\ 16 \rightarrow 3$

sparse, dynamically hot

System Parameters

Mass Distribution

- most massive: $M_1/M_{\rm tot}$ (0.51)
- dispersion: $\sigma_M/\bar{M}$ (0.85)

Orbital Structure

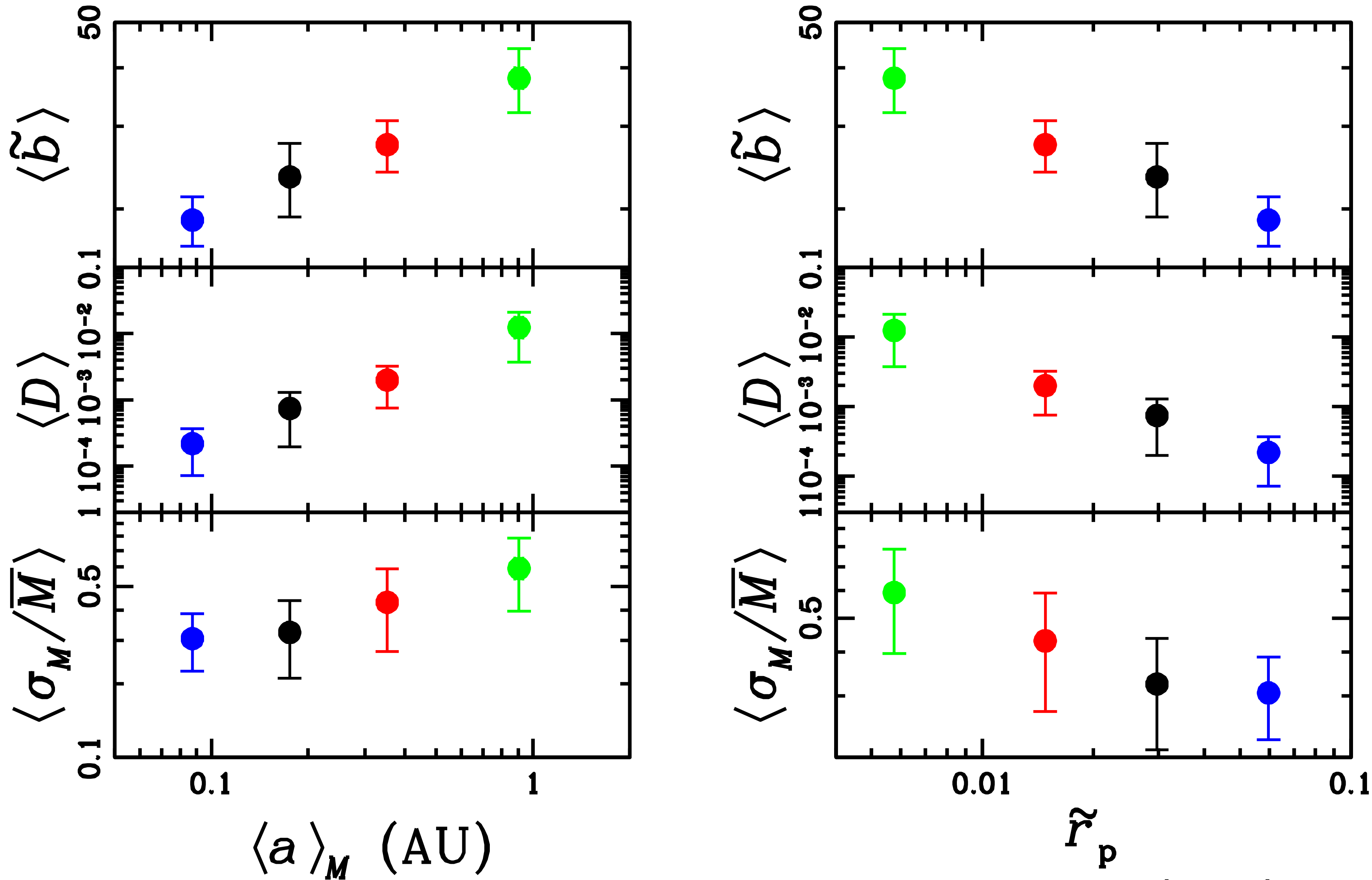
- mass-weighted orbital elements: $\langle a\rangle_M$, $\langle e\rangle_M$, $\langle i\rangle_M$ ($0.90\,{\rm au}$, 0.022, 0.034)
- mean orbital separation: $\tilde{b} = b/r_{\rm H}$ (43)
- mean eccentricity: $\tilde{e} = ea/r_{\rm H}$ (10)
- angular momentum deficit (AMD): (0.0018)

$$D = \frac{\sum_j M_j\sqrt{a_j}\left(1-\sqrt{1-e_j^2}\cos i_j\right)}{\sum_j M_j\sqrt{a_j}} \simeq \frac{\sum_j M_j(e_j^2+i_j^2)/2}{\sum_j M_j}$$ (Hill's approximation)

(solar system terrestrial planets)

System Radius Dependence

$\Sigma_1 = 10$, $\alpha = 2$, $b = 10 r_{\mathrm{H}}$, $\langle e^2 \rangle^{1/2} = 0.01$, $r =$ 0.05-0.15, 0.1-0.3, 0.2-0.6, 0.5-1.5 au



(EK+ in prep.)

$$\mathrm{d} \log \langle \tilde{b} \rangle / \mathrm{d} \log \tilde{r}_{\mathrm{p}} \simeq -0.3$$

Orbital Architecture by Giant Impacts

Key Parameter

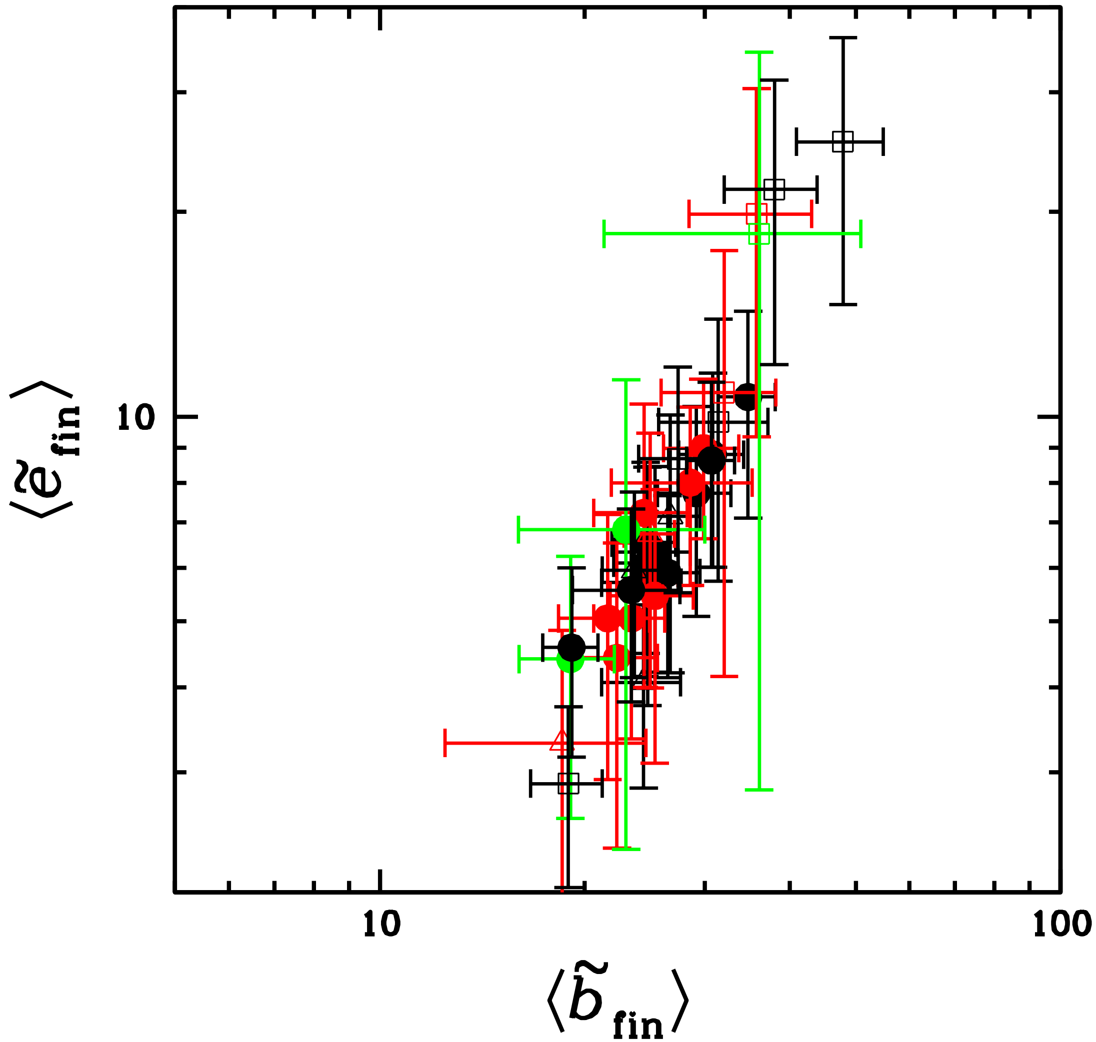
- physical to Hill radius ratio: $\tilde{r}_{\rm p} = r_{\rm p}/r_{\rm H} = \left(\frac{9M_*}{4\pi\rho}\right)^{1/3}\left(\frac{1}{a}\right)$

Large $\tilde{r}_{\rm p}$ Effects

- relatively weak scattering and effective collisions $\rightarrow$
  smaller e, less mobility $\rightarrow$
  local accretion $\rightarrow$
  dynamically cold compact system

Final Configuration

$\Sigma_1 = 10, 30, 100$, $\alpha = 3/2\text{-}5/2$, $b = 5\text{-}15 r_{\rm H}$, $r = 0.05\text{-}0.15, 0.1\text{-}0.3, 0.2\text{-}0.6, 0.5\text{-}1.5\,{\rm au}$



$\langle \tilde{e}_{\rm fin} \rangle$ increases with $\langle \tilde{b}_{\rm fin} \rangle$ (with decreasing $\tilde{r}_{\rm p}$)

$$\mathrm{d}\log\langle \tilde{e} \rangle / \mathrm{d}\log\langle \tilde{b} \rangle \simeq 2$$

(EK+ in prep.)

Summary

Planetesimal System

- Dissipative self-gravitating particle disk

Planetesimal Dynamics

- Viscous stirring
- Dynamical friction
- Orbital repulsion

Planetesimal Accretion

- Runaway growth of planetesimals
- Oligarchic growth of protoplanets
- Giant impacts