TIME-DEPENDENT BACKGROUNDS IN SUPERGRAVITY

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with Kengo Maeda

[1] Introduction

- The dynamics of geometry on the basis of unified theory.
- There is viable unified theory at present is supergravity and string theory.
- In order to study the cosmological evolution, we use the time-dependent solution in string theory.

★ Cosmological model, geometrical structure
☆ Black hole in expanding universe
Supersymmetry

How to analyze the evolution of background geometry for the unified theory?

- We will discuss their dynamics in the classical framework of supergravity.

↓

low energy effective theory of string

⇒ We can study the time evolution of metric, the number of preserved SUSY from the viewpoint of general relativity.

- Static solution in higher-dimensional supergravity

- 11-dimensional supergravity
  (Freund & Rubin, Phys. Lett. B 97 (1980) 233)
  (Duff & Nilsson, & Pope, Phys.Lett. 129B (1983) 39)
  (Duff & Stelle, Phys.Lett. 253B (1991) 113)
  (Horava & Witten, Nucl.Phys. B475 (1996) 94)
  (K. Becker & M. Becker, Nucl.Phys. B477 (1996) 155)

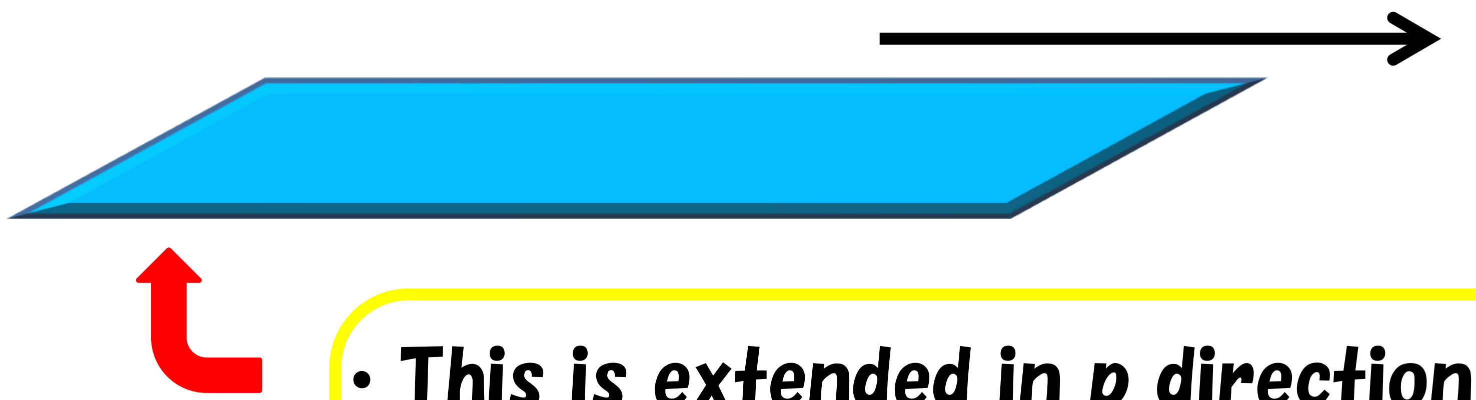
- 10-dimensional supergravity
  (Candelas & Horowitz & Strominger & Witten, Nucl.Phys. B258 (1985) 46)
  (Strominger, Nucl.Phys. B274 (1986) 253)
  (Duff & Lu, Phys.Lett. B273 (1991) 409)
  (Bergshoeff & de Roo & Eyras & Janssen & van der Schaar, Nucl.Phys. B494 (1997) 119)
  (Bergshoeff & Lozano & Ortin, Nucl.Phys. B518 (1998) 363)

✏ What is p-brane?

(Gary T. Horowitz, Andrew Strominger, Nucl.Phys. B360 (1991) 197-209)

★ Classical membrane solution of Einstein equation

$X^1, X^2, \cdots, X^p$



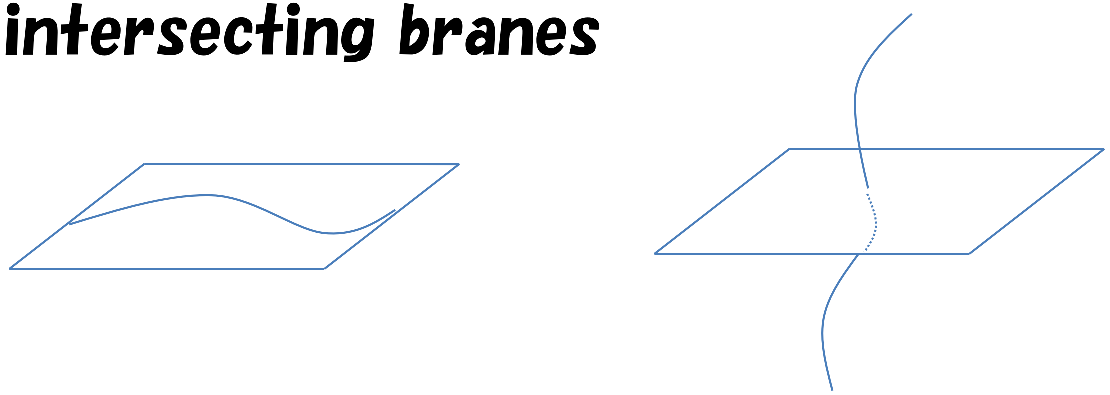
- This is extended in p direction.
- p-brane has p spacelike translational Killing vectors.

- String theory, supergravity theory :
  There are anti-symmetric tensor fields of higher rank.

- (p+1)-form gauge field in D-dimensions:
  There is (p+2)-form field strength.
  ⇒A charged objects (p-dim) couples to (p+1)-form gauge field.

p-(mem)brane

These higher dimensional objects (p-brane) intersect each other in D-dimensions: ⇒ intersecting branes



👓 Time dependent solutions in gravity theory

- 4d theory
  (Kastor & Traschen, Phys.Rev. D47 (1993) 5370-5375)
- Higher dimensional gravity
  (Maki & Shiraishi, Class.Quant.Grav. 10 (1993) 2171-2178)
- String theory
  (Gibbons, Lu, Pope, Phys.Rev.Lett.94 (2005) 131602)
  (Kodama; Uzawa, JHEP 0507 (2005) 061)
  (Binetruy, Sasaki, Uzawa, Phys.Rev.D80 (2009) 026001)
  (Maeda & Nozawa, Phys.Rev. D81 (2010) 044017)
  (Minamitsuji & Ohta & Uzawa, Phys.Rev. D81 (2010) 126005)

♦ Dynamical branes are related to

- brane collision

(Gibbons & Lu & Pope, Phys.Rev.Lett. 94 (2005) 131602)
(Maeda & Minamitsuji & Ohta & Uzawa, Phys. Rev. D82 (2010)046007)
(Uzawa, Phys.Rev. D90 (2014) 025024)

- cosmic Big-Bang of our universe

(Chen, et al., Nucl.Phys. B732 (2006) 118-135)
(Minamitsuji & Ohta & Uzawa, Phys. Rev. D82 (2010)086002))

- black hole in expanding universe

(Maeda & Ohta & Uzawa, JHEP 0906 (2009) 051)
(Maeda & Nozawa, Phys.Rev. D81 (2010) 044017)

◆ Dynamical solution of 4-dim gravity
(D.Kastor & J. Traschen : Phys.Rev. D 47 (1993) 5370)

・4-dim Einstein-Maxwell system + cosmological constant

◇ 4-dimensoinal action : $S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left(R - \frac{1}{4}F_{MN}F^{MN} - 2\Lambda\right)$

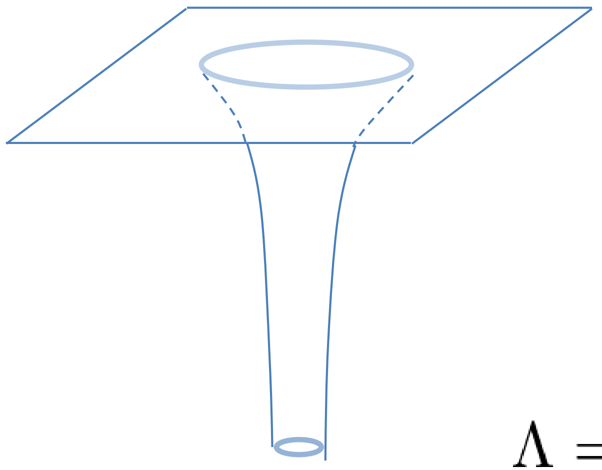
☆ Solution of field equations : $ds^2 = -h^{-2}(t,y)dt^2 + h^2(t,y)\delta_{ij}dy^i dy^j$

$$\begin{aligned} h(t,y) &= at + b + \sum_l \frac{M_l}{|\vec{y} - \vec{y}_l|}, \\ F_2 &= d(h^{-1}) \wedge dt, \\ a &= \pm\sqrt{\frac{\Lambda}{3}} \end{aligned}$$

☆ **4-dim geometry :**

$$ds^2 = -\left(at + b + \frac{M}{r}\right)^{-2} dt^2 + \left(at + b + \frac{M}{r}\right)^2 (dr^2 + r^2 d\Omega^2)\,,$$

de Sitter space



$M_l = 0, \quad \text{or} \quad r \to \infty$ **: de Sitter solution**

$$ds^2 = -d\tau^2 + e^{2a\tau}\delta_{ij}dy^i dy^j\,, \qquad \tau = a^{-1}\ln(at + b)$$

⇕

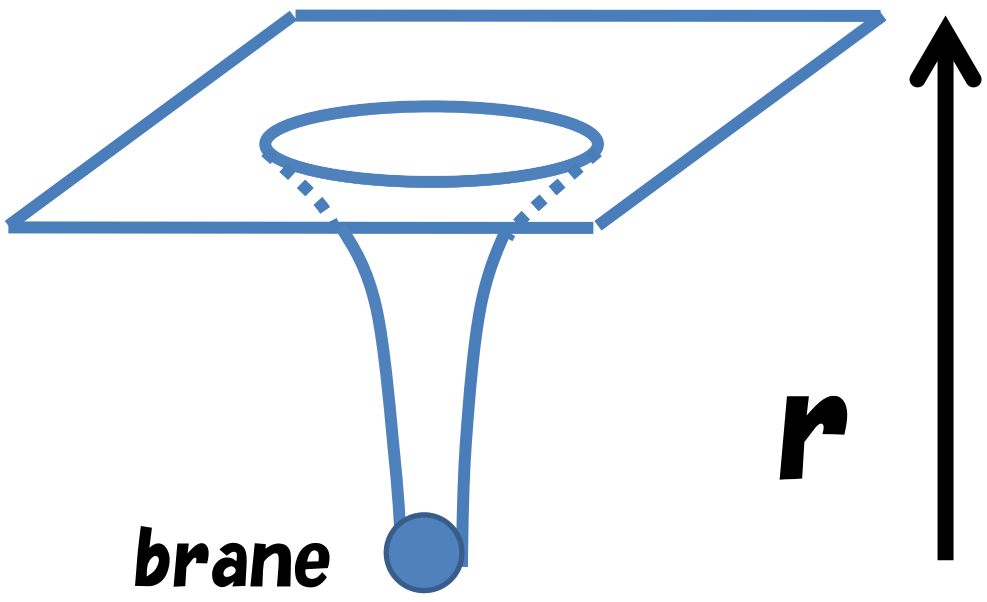
$\Lambda = 0, \quad \text{or} \quad r \to 0$ **: Reissner & Nordström solution**

$$ds^2 = -\left(b + \frac{M}{r}\right)^{-2} dt^2 + \left(b + \frac{M}{r}\right)^2 (dr^2 + r^2 d\Omega^2)\,,$$

- **Time dependent Dp-brane solution**

(G.W. Gibbons, H. Lu, C.N. Pope Phys.Rev.Lett.94 (2005) 131602)
(P. Binetruy, M. Sasaki, K. Uzawa, Phys.Rev. D 80 (2009) 026001)

$$
\begin{aligned}
ds^2 &= h^{-(D-p-3)/(D-2)}\eta_{\mu\nu}dx^\mu dx^\nu + h^{(p+1)/(D-2)}(dr^2 + r^2 d\Omega^2_{D-p-2}),\\
h(t,r) &= c_1 t + c_2 + M r^{-D+p+3}, \quad F_{p+2} = d(h^{-1}) \wedge dt \wedge dx^1 \wedge \cdots \wedge dx^p,\\
e^\phi &= h^{c/2}, \quad c^2 = 4 - 2(p+1)(D-p-3)(D-2)^{-1}
\end{aligned}
$$



- **Asymptotically Time dependent vacuum spacetime**

↕

- **Static spacetime at $r \to 0$**

For trivial or vanishing dilaton, we find

(P. Binetruy, M. Sasaki, K. Uzawa, Phys.Rev. D 80 (2009) 026001)

$$ds^2 = h^{-(D-p-3)/(D-2)}(x,y)\eta_{\mu\nu}(\mathrm{X})dx^\mu dx^\nu + h^{(p+1)/(D-2)}(x,y)w_{ij}(\mathrm{Y})dy^i dy^j \,,$$

$$R_{ij}(\mathrm{Y}) = -\frac{\lambda(D-p-3)(p+1)}{2(D-2)} w_{ij}(\mathrm{Y})$$

$$h(x,y) = \frac{\lambda}{2} x^\mu x_\mu + c_\mu x^\mu + c + h_1(y)$$

$$-t^2 + (x^1)^2 + (x^2)^2 + \cdots \,,$$

★ Cosmology:

(Binetruy, Sasaki, Uzawa, Phys.Rev.D80:026001,2009)
(Maeda, Ohta, Uzawa, JHEP 0906:051,2009)

We assume an isotropic and homogeneous three space in the four-dimensional spacetime.

Note that the time dependence in the metric comes from only one brane even if we consider several branes.

- Solutions in the original higher-dimensional theory (10D or 11D).

- For each case, the scale factor of 4-dimensional universe is given by $a(\tau) \propto \tau^{\lambda}$, where τ denotes the cosmic time.

- Since the three-dimensional spatial space of our universe stays in the transverse space to the brane, D-dimensional theory gives the fastest expansion of our universe.

- The power of the scale factor becomes

$$\lambda = (p+1)/(D+p-1) < 1 \text{ for } D>2.$$

It is impossible to find the cosmological model that our universe is accelerating expansion .

● The characteristics of M-brane :

- Classical solution of 11-dim SUGRA
- Static limit of M-brane : Black brane
- M-brane on time-dependent background
  ⇒ Black hole in expanding universe

(Maeda & Ohta & Uzawa, JHEP 0906 (2009) 051)
(Maeda & Nozawa, Phys.Rev. D81 (2010) 044017)

★Outline my talk

* Property of static M-brane

* Preserved supersymmetry

* SUSY breaking

* Summary and comments

[2] Property of static M-brane

(Duff & Stelle, Phys.Lett. B253 (1991) 113-118)
(Gibbons & Townsend, Phys. Rev. Lett., 71, 3754 (1993))
(Güven, Phys.Lett. B276 (1992) 49-55)

◆ Background

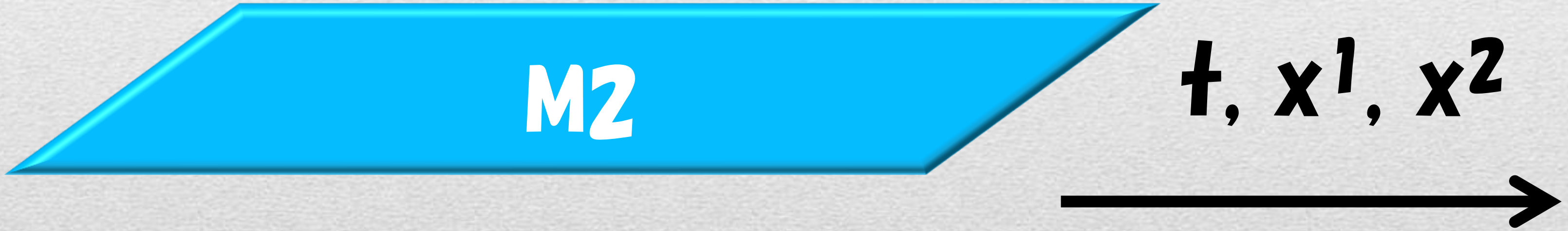
(1) The background has gravity, 4-form field strength, gravitino.
⇒ 11-dimensional supergravity

(2) Classical solutions.
ex) M2-brane, M5-brane (½ SUSY), pp-wave (Maximal SUSY)

✿ M2-brane

(Duff & Stelle, Phys.Lett. B253 (1991) 113-118)
(Güven, Phys.Lett. B276 (1992) 49-55)

✏ matter (bosonic) :
gravity, 4-form field strength



	0	1	2	3	4	5	6	7	8	9	10
M2	○	○	○								
x^M	t	x^1	x^2	r	z^1	z^2	z^3	z^4	z^5	z^6	z^7

Static M2-brane solution :

(1+2)-dim worldvolume spacetime

$$ds^2 = \left(1 + \frac{M}{r^6}\right)^{-2/3} \eta_{\mu\nu}(\mathrm{X}) dx^\mu dx^\nu$$

$$+ \left(1 + \frac{M}{r^6}\right)^{1/3} \left(dr^2 + r^2 d\Omega_{(7)}\right)$$

8-dim transverse space to brane

For static background, $AdS_4 \times S^7$, the background has the full supersymmetry.

(Freund & Rubin, Phys. Lett. B 97 (1980) 233)

✍ The dynamical D3-brane solution preserves ¼ SUSY in the conifold background.

(H. Kodama & K. Uzawa, JHEP 0507 061 (2005))

☆ Question

Do supersymmetries preserve in the dynamical M-brane background?

How to obtain the SUSY solutions …

Dynamical case :

- pp-wave

(Matthias Blau, et al.,
JHEP 0201 (2002) 047)

(M. Sakaguchi, K. Yoshida,
JHEP 0311 (2003) 030)

- D3-brane

(H. Kodama & K. Uzawa,
JHEP 0507 061 (2005))

[3] Preserved supersymmetry (11d SUGRA)

The 11-dimensional action is invariant under local SUSY transformations :

$e^A{}_M$: graviton Ψ_M : gravitino,
A_{MNP} : 3-form gauge potential

$$\delta e^A{}_M = \bar{\varepsilon}\, \Gamma^A\, \Psi_M \,,$$

$$\delta A_{MNP} = -3\, \bar{\varepsilon}\, \Gamma_{[MN}\, \Psi_{P]} \,,$$

$$\begin{aligned}\delta \Psi_M &= D_M\, \varepsilon \\ &= \left[\nabla_M + \frac{1}{12 \cdot 4!} \left(\Gamma_M\, F_{MNPQ} \Gamma^{MNPQ} - 12 F_{MNPQ} \Gamma^{NPQ} \right) \right] \varepsilon\end{aligned}$$

★Supersymmetry in dynamical M2-brane

- The only fermionic field is the gravitino Ψ_M.

- Supersymmetric configuration is a nontrivial solution to the Killing spinor equation :

$$
\begin{aligned}
&\delta\Psi_M = 0\,, \\
&\Rightarrow \quad \left[\nabla_M + \frac{1}{12\cdot 4!}\left(\Gamma_M\, F_{MNPQ}\Gamma^{MNPQ}\right.\right. \\
&\qquad\qquad \left.\left. -12F_{MNPQ}\Gamma^{NPQ}\right)\right]\varepsilon = 0
\end{aligned}
$$

☆ Ansatz for fields

- 11-dim metric

(1+2)-dim worldvolume spacetime

$$ds^2 = g_{MN} dx^M dx^N = h^{-2/3}(x,r)\eta_{\mu\nu}(\mathrm{X})dx^\mu dx^\nu$$

$$+ h^{1/3}(x,r)\left(dr^2 + r^2 u_{ab}(\mathrm{Z})dz^a dz^a\right)$$

8-dim transverse space to brane

- 4-form field strength & gravitino

$$F_{r\mu\nu\rho} = \pm h^{-2}\partial_r h\, \varepsilon_{\mu\nu\rho}\,, \quad \Psi_M = 0$$

11-dimensional gamma matrices satisfying

$$\Gamma^M \Gamma^N + \Gamma^N \Gamma^M = 2g^{MN} \,,$$

$$\Gamma^\mu = h^{1/3} \gamma^\mu \,, \quad \Gamma^r = h^{-1/6} \, \gamma^r \,, \quad \Gamma^a = r^{-1} h^{-1/6} \, \gamma^a$$

and we define

$$\gamma_{(3)} = \gamma_0 \, \gamma_1 \, \gamma_2$$

☆ Killing spinor equation

$$\bar{\nabla}_\mu \varepsilon = \left[\partial_\mu + \frac{1}{6}\partial_\nu \ln h \gamma^\nu{}_\mu - \frac{1}{6}h^{-3/2}\partial_r h \gamma^\mu \gamma^r \left(1 \pm \gamma_{(3)}\right)\right]\varepsilon\,,$$

$$\bar{\nabla}_r \varepsilon = \left[\partial_r - \frac{1}{12}h^{-1/2}\partial_\nu h \gamma^\nu \gamma^r + \frac{1}{6}\chi h^{-1}\partial_r h \gamma_{(3)}\right]\varepsilon\,,$$

$$\bar{\nabla}_a \varepsilon = \left[{}^{\mathrm{Z}}\nabla_a - \frac{r}{12}h^{-1/2}\partial_\nu h \gamma^\nu \gamma_a - \frac{r}{12}h^{-1}\partial_r h \gamma^r \gamma_a \left(1 \pm \gamma_{(3)}\right)\right]\varepsilon$$

• If the function h(x, r) is included in the spinor $\varepsilon = h^{-1/6}\ \varepsilon_0$ (ε_0 : constant Killing spinor), we find …

- Solution for dynamical background

$$\partial_\mu h \gamma^\mu \varepsilon = 0\,, \quad \left(1 \pm \gamma_{(3)}\right) \varepsilon = 0$$

(i) Induced effective mass for the spinor field

$$\sim h^{-1} \partial_\mu h$$

(ii) This mass scale diverges at the naked singularity where the function h vanishes.

(iii) In the region with a large warp factor, the SUSY breaking becomes negligible.

Dynamical M2-brane solution :

(1+2)-dim worldvolume spacetime

$$ds^2 = \left(c_\mu x^\mu + c + \frac{M}{r^6}\right)^{-2/3} \eta_{\mu\nu}(\mathrm{X}) dx^\mu dx^\nu$$

$$+ \left(c_\mu x^\mu + c + \frac{M}{r^6}\right)^{1/3} \left(dr^2 + r^2 d\Omega_{(7)}^2\right)$$

8-dim transverse space to brane

(1) $c_\mu \neq 0$: ¼ SUSY

(2) $M=0$, $c_\mu \neq 0$: ½ SUSY

✻ The behavior of background

(i) Asymptotic region ($r \to \infty$):
Kasner or plane wave spacetime

$\Rightarrow$ Time dependent vacuum spacetime

(ii) Near horizon limit :

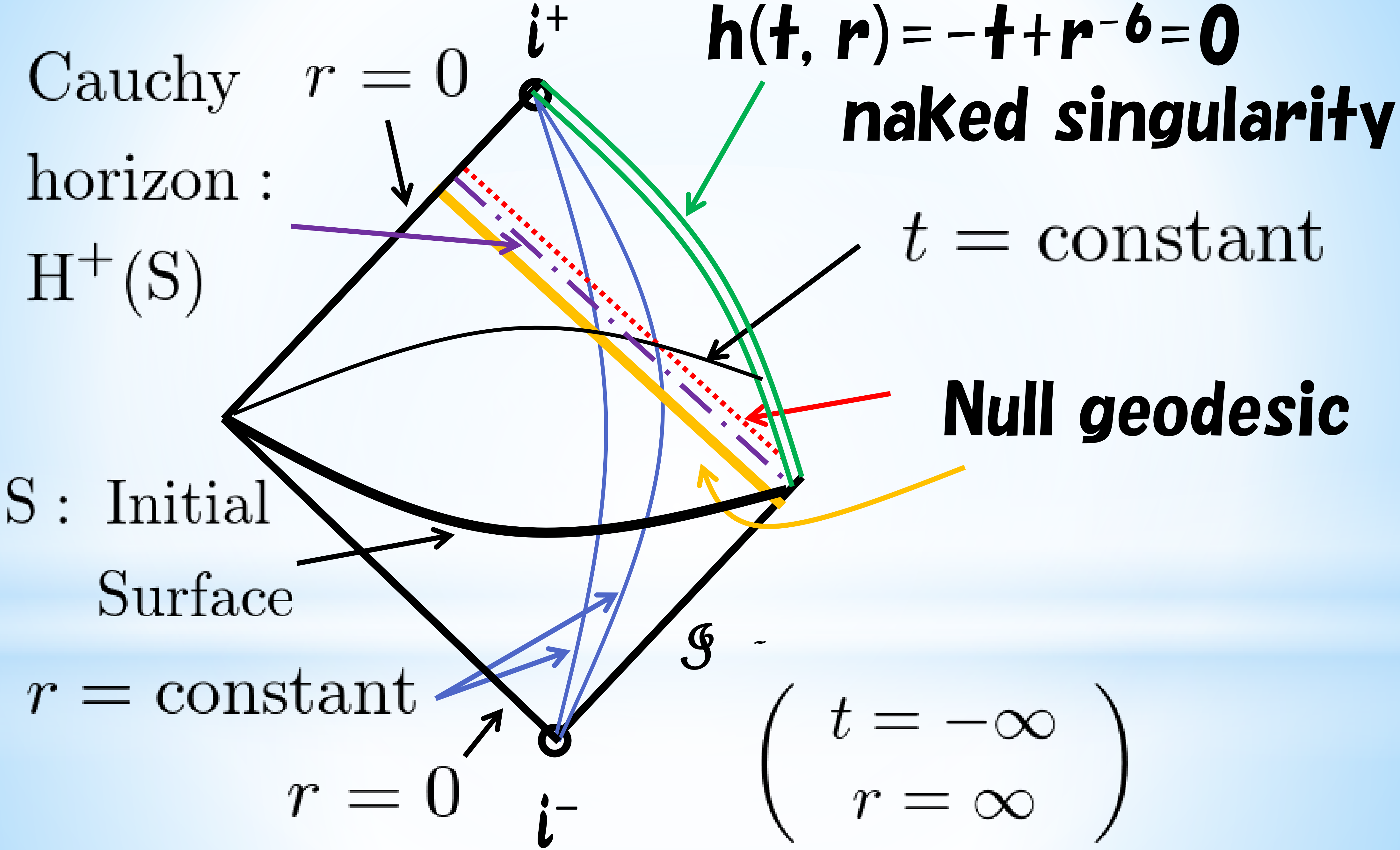
$$t \to t/\xi, \quad r \to \xi r, \quad \xi \to 0$$

$\Rightarrow$ $AdS_4 \times S^7$, static spacetime

💣 It is of great significance to understand the cosmological backgrounds profoundly.

☠ There is a naked singularity in the dynamical brane background.
(Maeda & Ohta & Uzawa, JHEP 0906 (2009) 051)

☹ Dynamical M-brane background gives violation of cosmic censorship.
(K.Maeda & K. Uzawa, Phys. Rev. D93 (2016) no.4, 044003)



Cauchy horizon : H+(S)
r = 0
i+
h(t, r) = −t + r^−6 = 0
naked singularity
t = constant
Null geodesic
S : Initial Surface
r = constant
r = 0
i−
𝔍
t = −∞
r = ∞

[4] SUSY breaking

(1) SUSY solution: $h=h(\tau, x^i, r)$, $\tau / \tau_0=(ct)^{2/3}$

$$-\left(c\,t+c_i x^i+\frac{M}{r^6}\right)^{-\frac{2}{3}} dt^2+\cdots$$

$$=-\left[1+\left(\frac{\tau}{\tau_0}\right)^{-\frac{3}{2}}\left(c_i x^i+\frac{M}{r^6}\right)\right]^{-\frac{2}{3}} d\tau^2+\cdots$$

(2) As time increases (for $c_i x^i \ll M/r^6$),

$$1+\left(\frac{\tau}{\tau_0}\right)^{-\frac{3}{2}}\left(c_i x^i+\frac{M}{r^6}\right) \rightarrow 1+\left(\frac{\tau}{\tau_0}\right)^{-\frac{3}{2}} \frac{M}{r^6}$$

(3) $h(\tau, x^i, r)$ (SUSY) → $h(\tau, r)$ (Non SUSY)

[3] Summary and comments

(1) The dynamical M2-brane background preserved the ¼ supersymmetry. For vanishing M2-brane charge, we also find ½ SUSY solution.

(2) The solutions of field equations cannot give a homogeneous expansion at constant r unless supersymmetries are completely broken.

(3) Although the solution itself is by no means realistic, its interesting behavior suggests a possibility that the Universe preserved originally SUSY and began to evolve toward a Universe without SUSY.