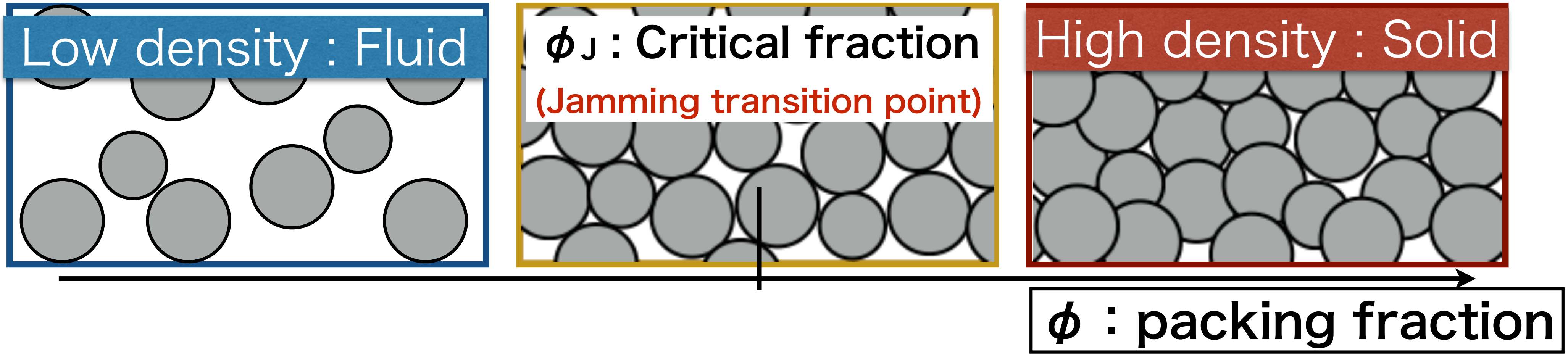
Avalanche contribution to shear modulus of granular materials under oscillatory shear

Effect of friction between particles

Michio Otsuki (Shimane Univ.),
Hisao Hayakawa (Kyoto Univ.)

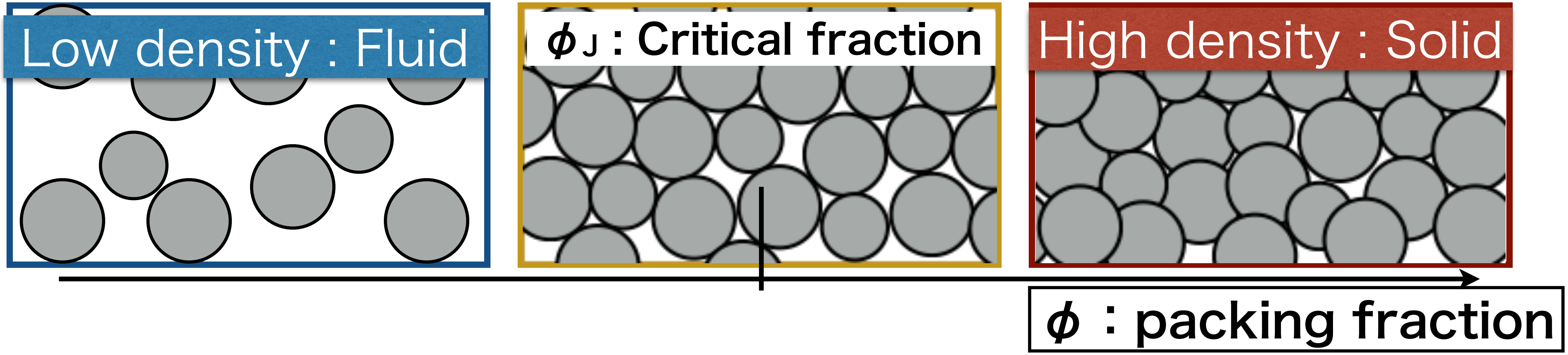
Jamming transition

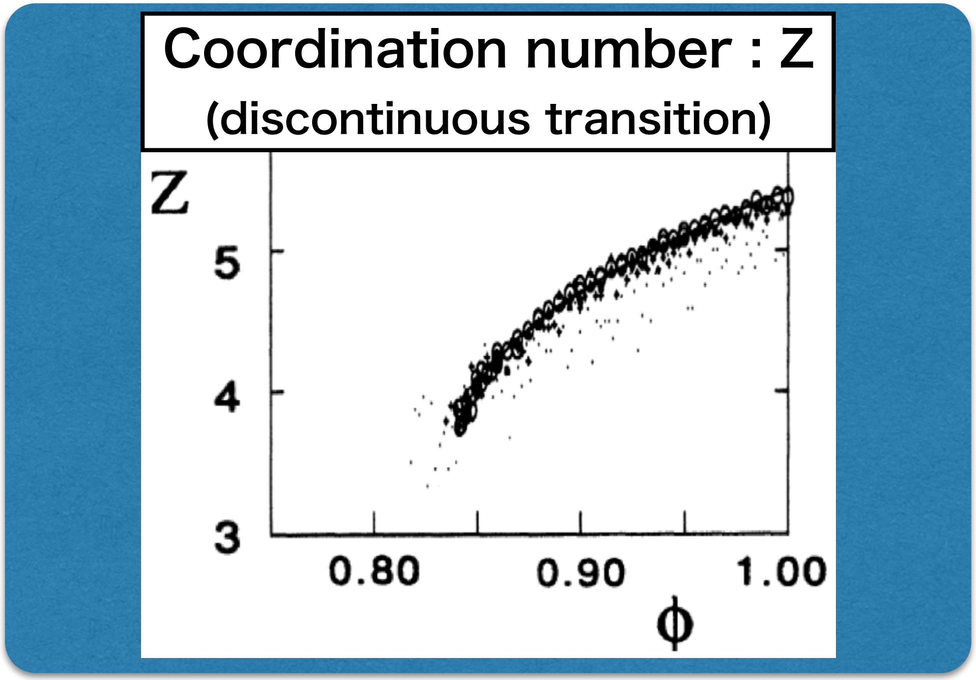


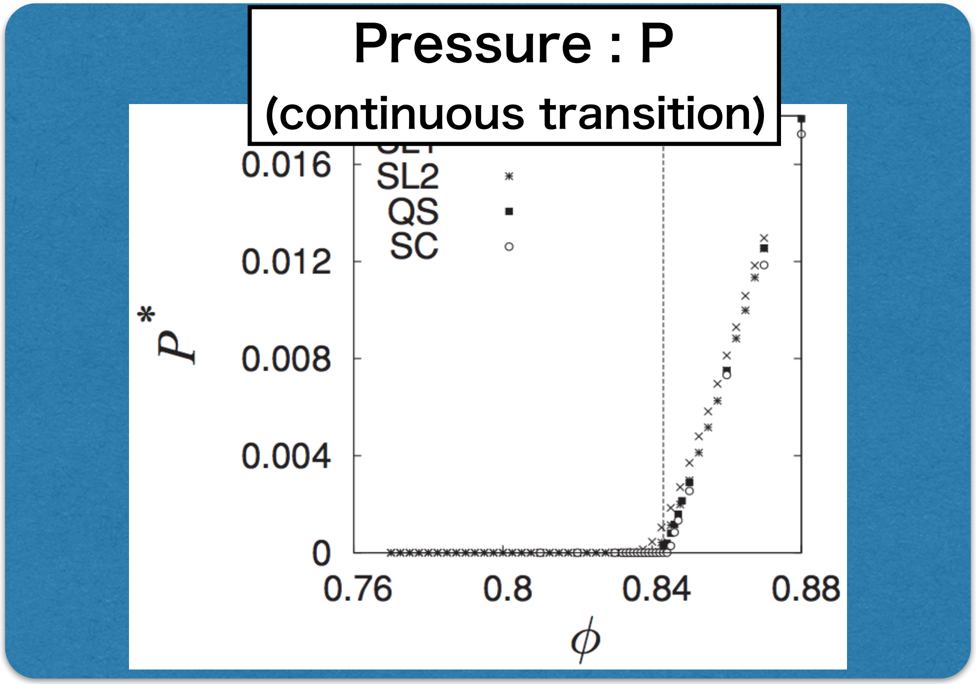
$\phi < \phi_J$ Granular materials flow like fluids.

$\phi > \phi_J$ Granular materials have rigidity like solids.

Critical behaviors : frictionless grains







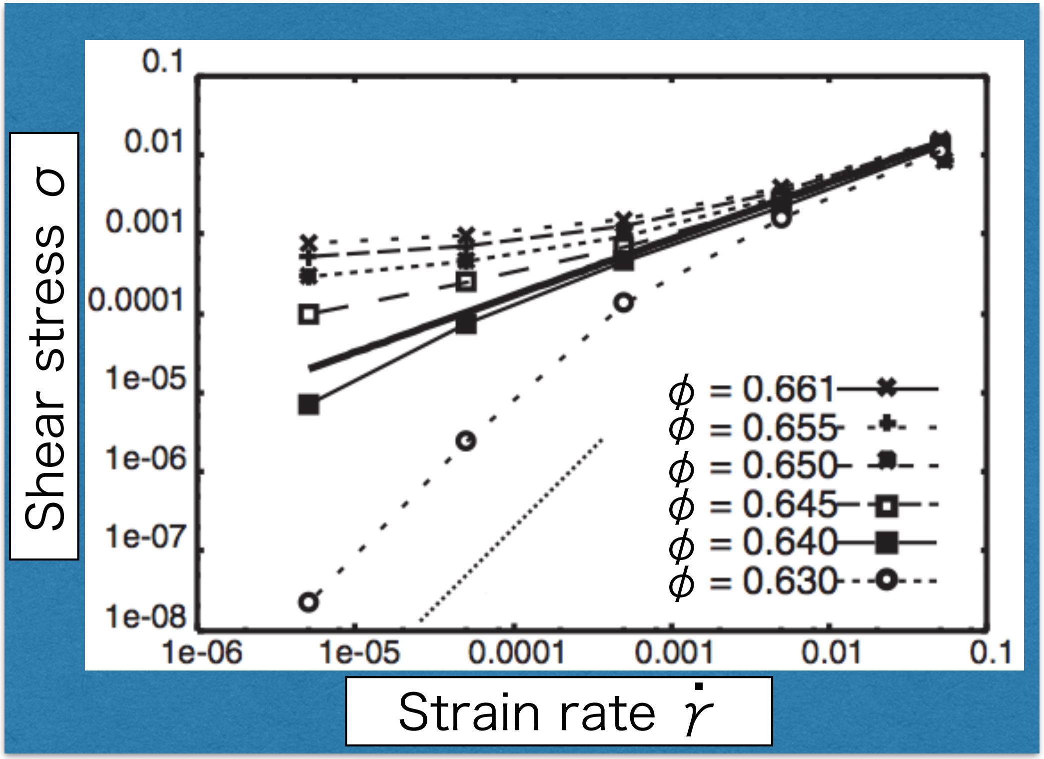
$$Z = \begin{cases} 0, & \phi < \phi_J \\ Z_c + a(\phi - \phi_J)^{1/2}, & \phi > \phi_J \end{cases}$$

$$P = \begin{cases} 0, & \phi < \phi_J \\ b(\phi - \phi_J), & \phi > \phi_J \end{cases}$$

Jamming under steady shear

Frictionless grains

Hatano, Otsuki, Sasa, JPSJ (2007)

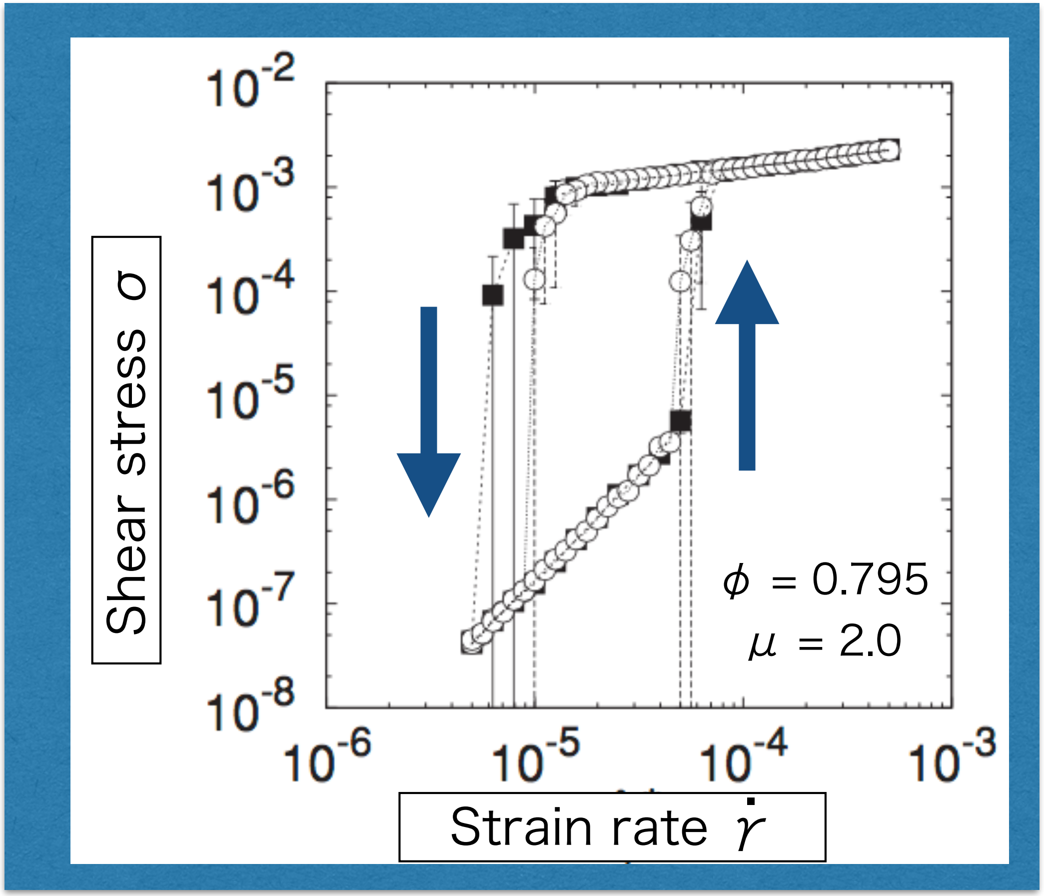


$\phi < \phi_J$	$\sigma \propto \dot{\gamma}^2$
$\phi > \phi_J$	$\sigma = \text{const.}$

Continuous transition

Frictional grains

Otsuki, Hayakawa, PRE (2011)

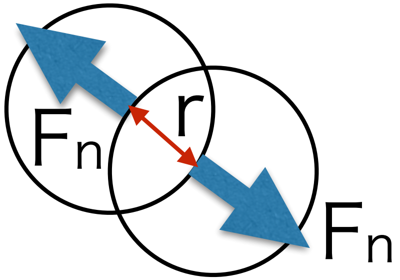


Strong shear thickening and hysteresis

(They disappear in frictionless limit.)

Discontinuous transition

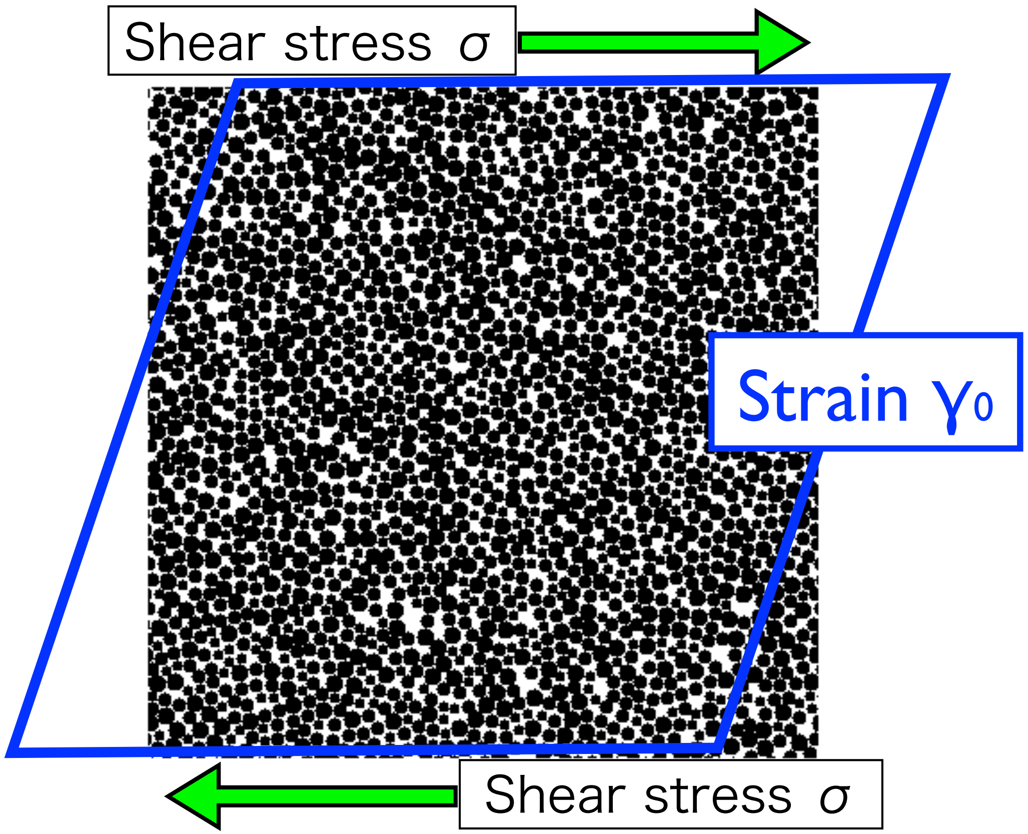
Shear modulus : frictionless grains



Elastic interaction force

$$F_n \propto r$$

compression length : r

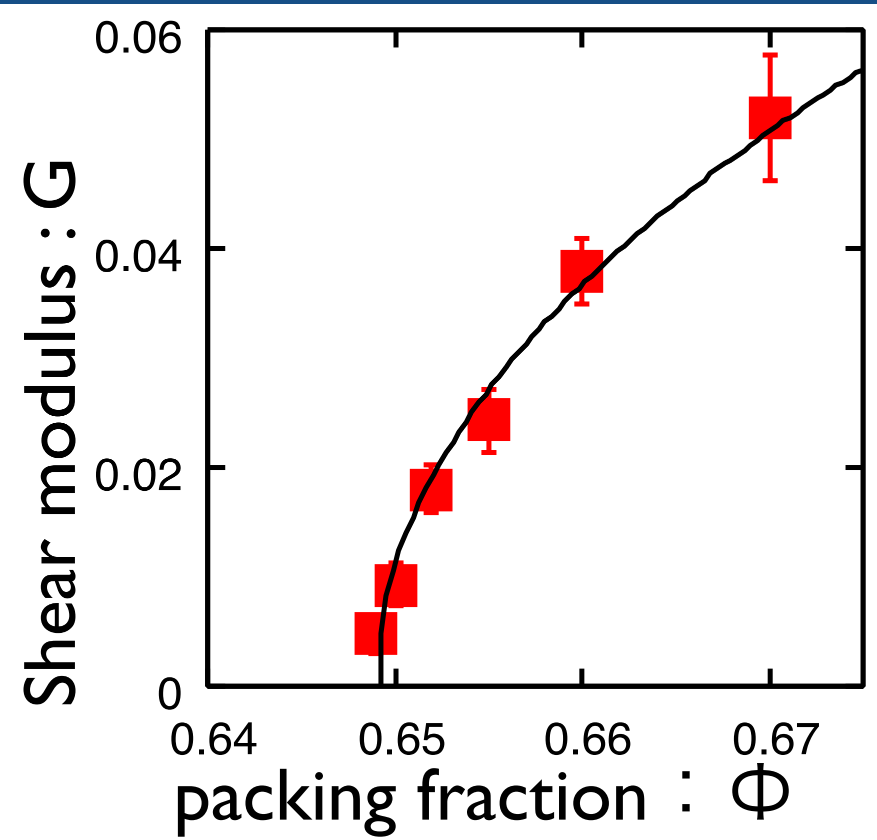


Shear modulus : $G = \sigma / \gamma_0$

$$G \propto (\phi - \phi_J)^{1/2}$$

Linear response

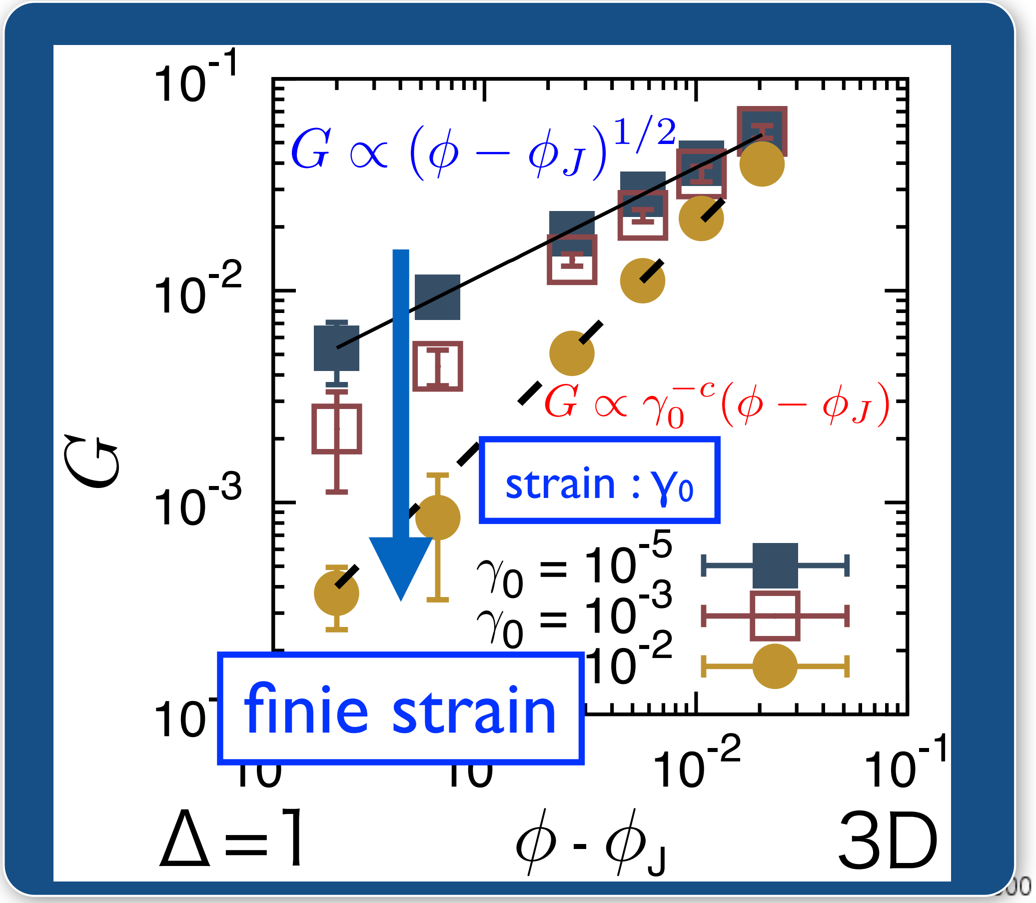
C. S. O'Hern et al., PRE 68, 011306 (2003)



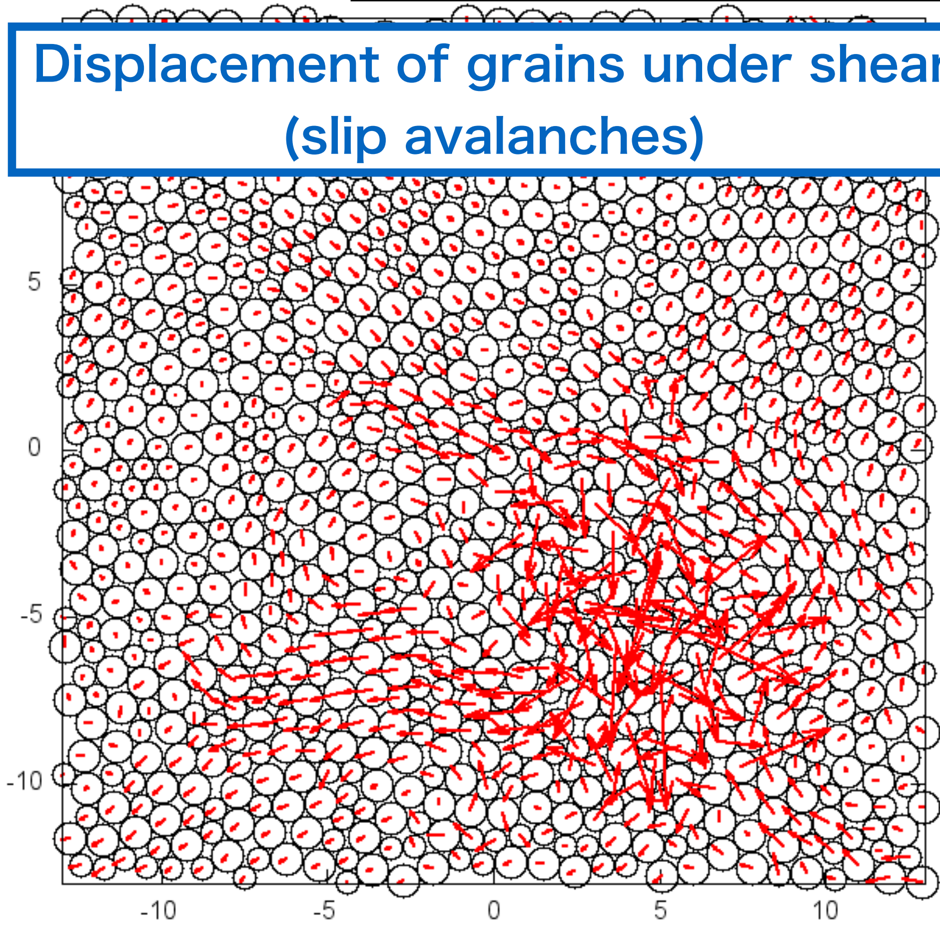
[extensively small strain $\gamma_0 = 10^{-5}$]

Shear modulus under finite strain

MO and H. Hayakawa, PRE (2014)



Displacement of grains under shear (slip avalanches)



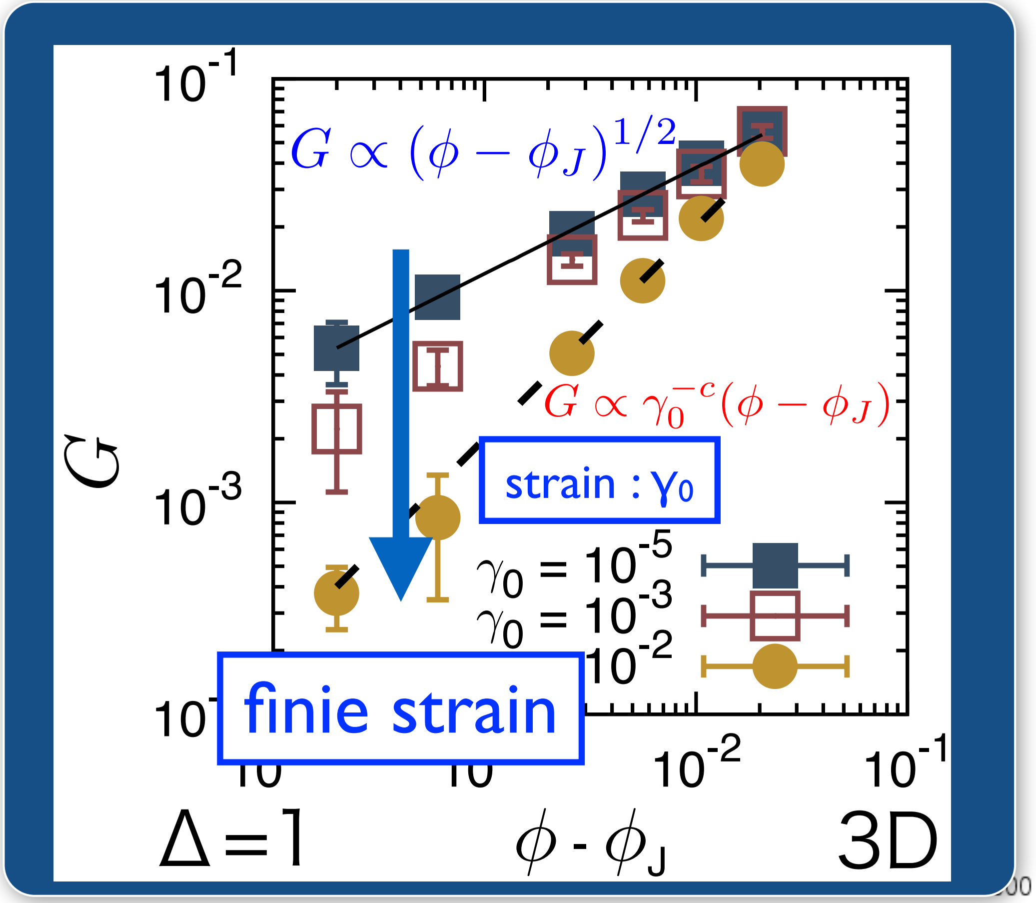
Infinitesimal strain : $G \propto (\phi - \phi_J)^{1/2}$ Linear response

Finite strain : $G \propto \gamma_0^{-c}(\phi - \phi_J)$ Nonlinear response

Origin of different scaling laws : slip avalanches

Shear modulus under finite strain

MO and H. Hayakawa, PRE (2014)



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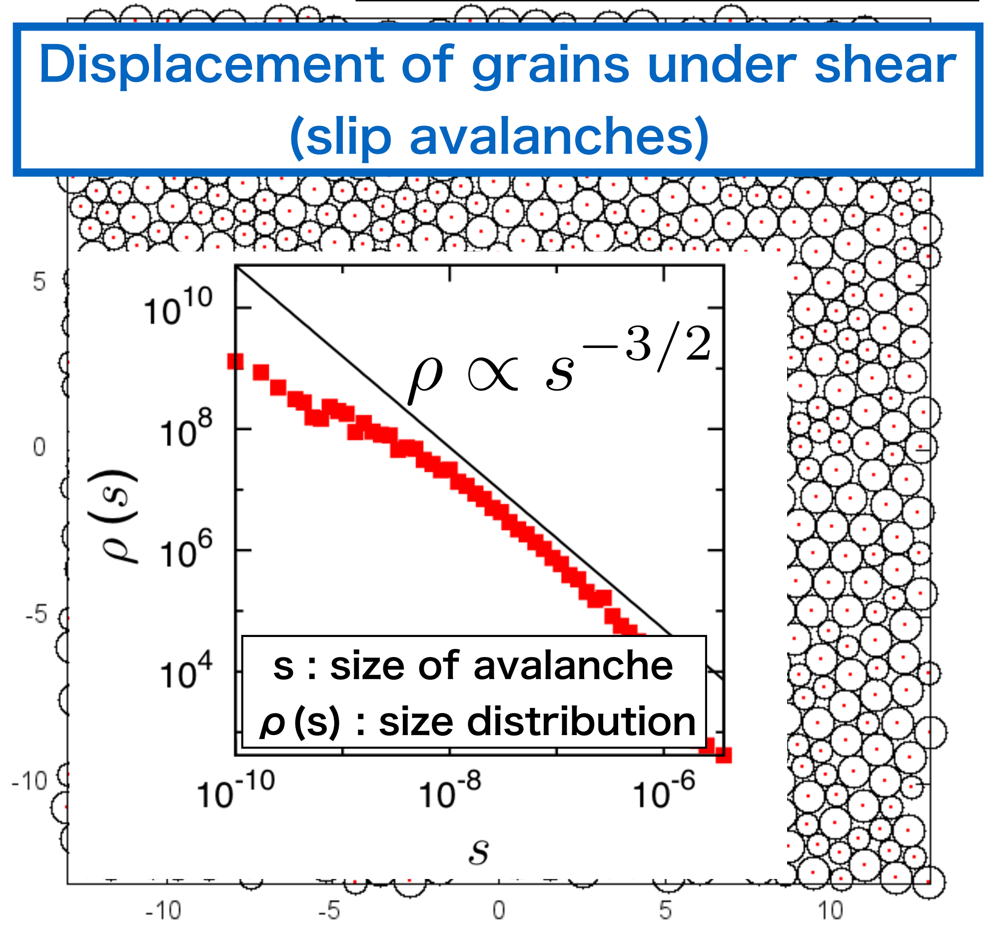

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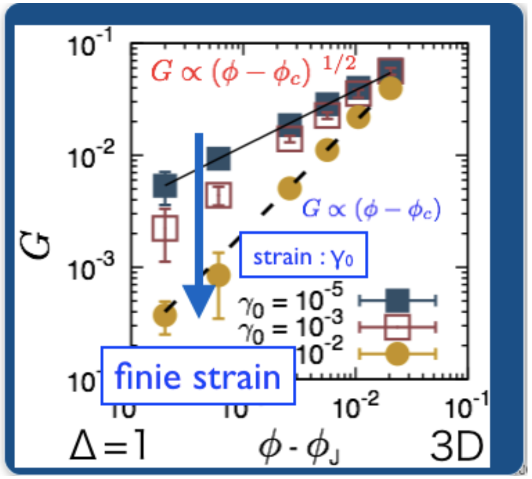
Origin of different scaling laws : slip avalanches

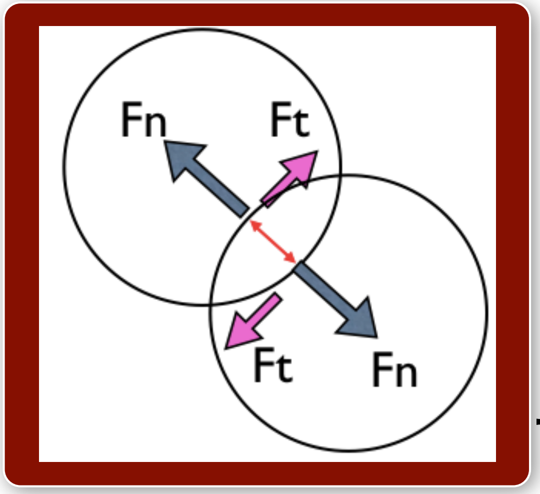
Purpose : shear modulus of frictional grains



Shear modulus of frictionless grains

Recent studies : including nonlinear elasticity

Otsuki and Hayakawa, PRE (2014)
Coulais, Seguin, and Dauchot, PRL (2014),
Goodrich, Liu, Sethna, arXiv : 1510.03469
Nakayama, Yoshino, Zhamponi, arXiv:1512.06544
Boschan, Vagberg, Somfai, Tighe, arXiv : 1601.00068



Shear modulus of frictional grains

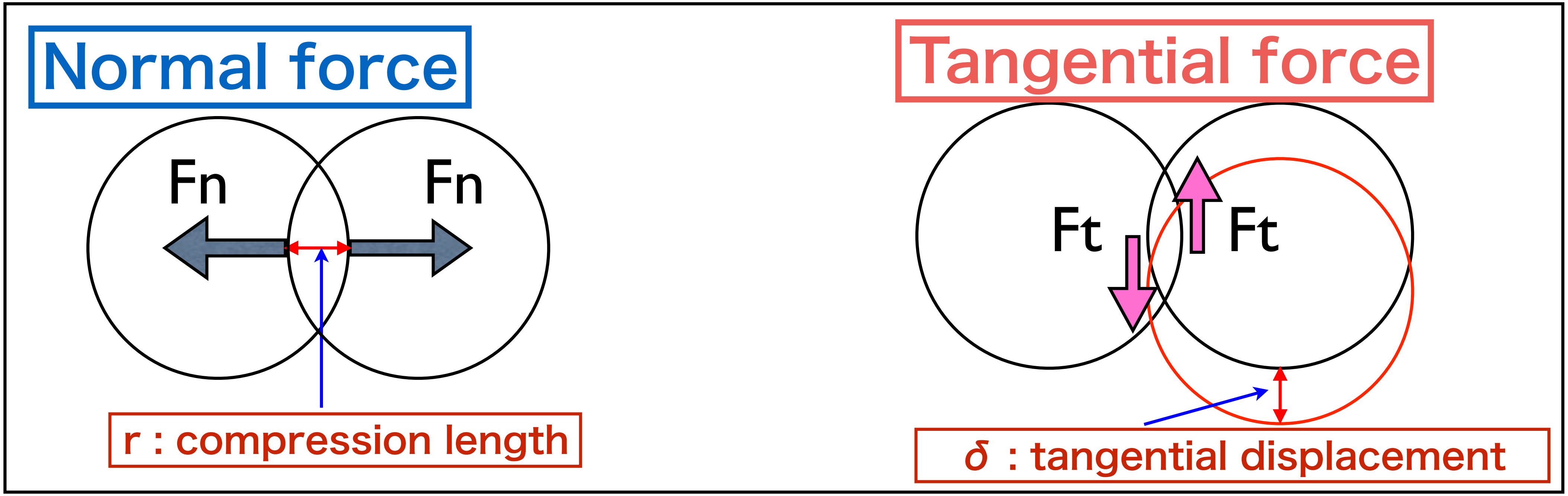
Linear elasticity :

Somfai, van Hecke, Ellenbroek, Shundyak, van Saarloos, PRE (2007)
Magnanimo, La Ragione, Jenkins, Wang, Makse, EPL 81, 34006 (2000)

There is no studies on non-linear elasticity.

We numerically study the effect of friction on G.

2D model of frictional grains



Normal force

$$F_n = kr - \eta\dot{r}$$

Tangential force

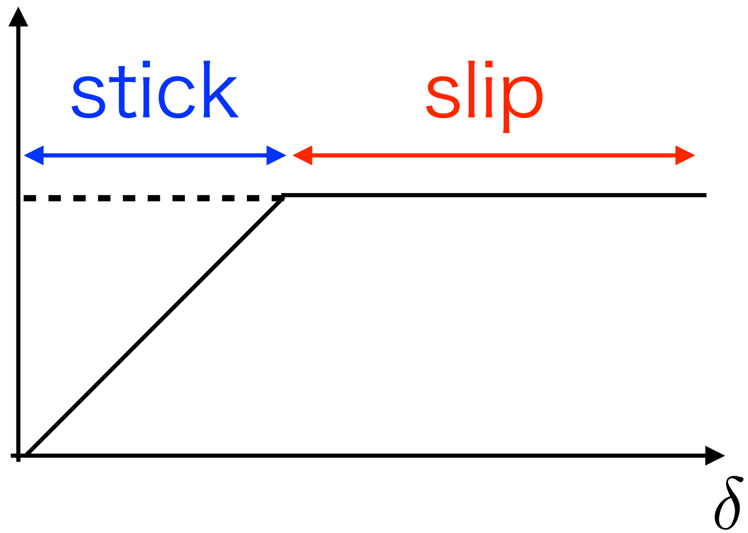
$$|F_t| = k_t\delta, \quad F_t < \mu F_n$$

$$|F_t| = \mu F_n, \quad \text{otherwise}$$

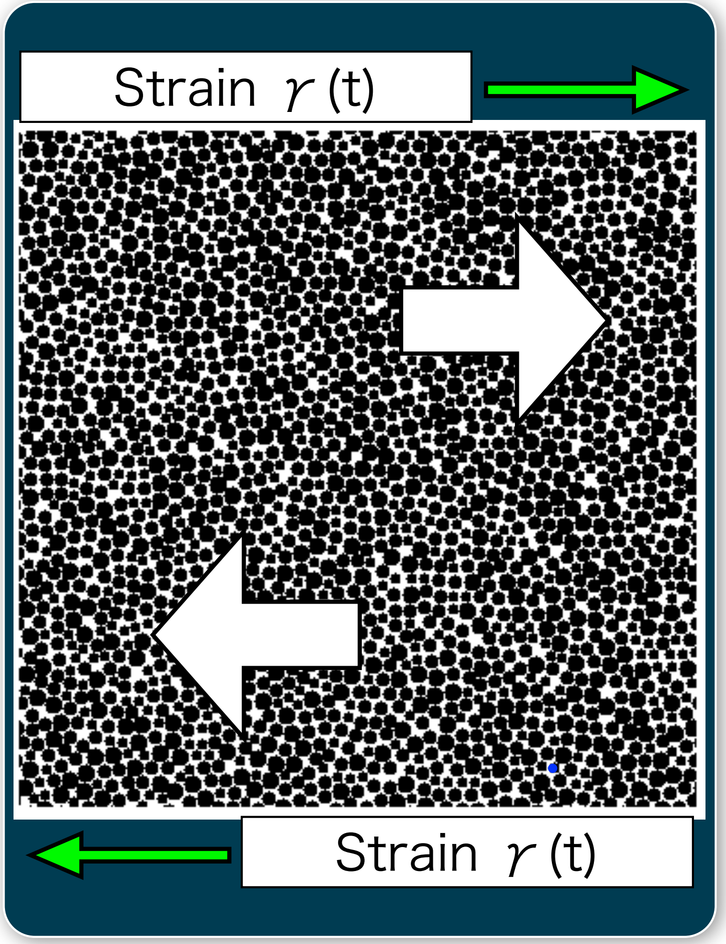
μ : friction coefficient

μ = 0 : frictionless

μ > 0 : frictional



Oscillatory shear



- Oscillatory shear strain :

$$\gamma(t) = \gamma_0(1 - \cos\omega t)$$

- Frequency : ω

Quasi-static limit : $\omega \to 0$

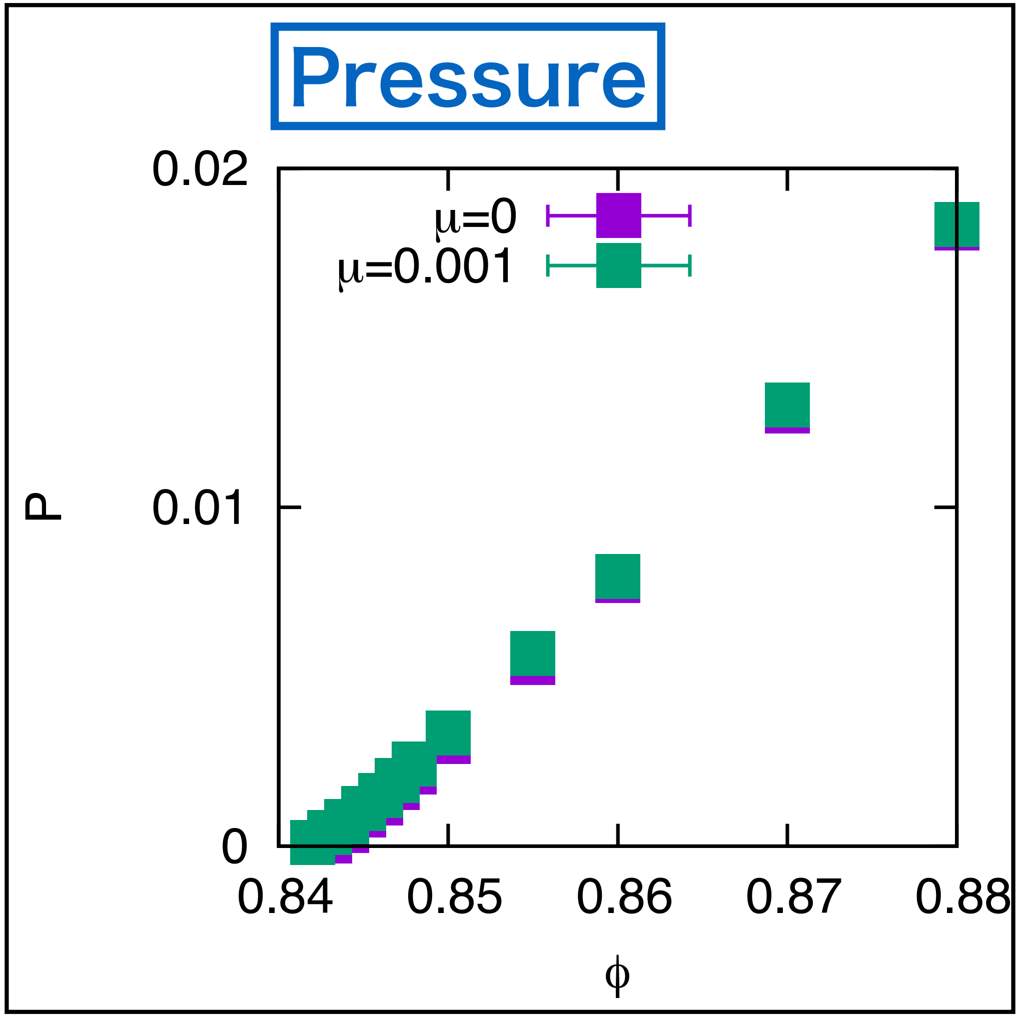
- **Strain amplitude : γ_0**

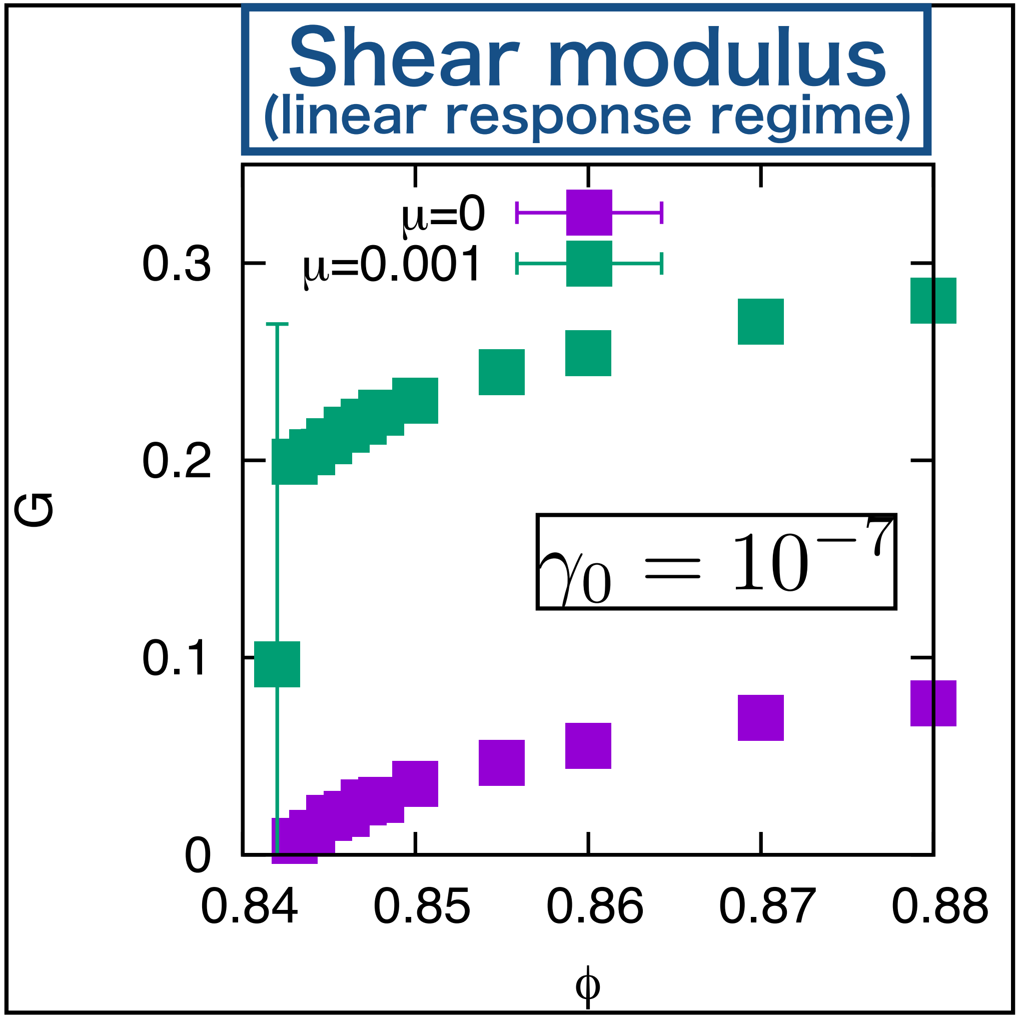
- Shear stress : $\sigma(t)$

Shear modulus (storage modulus)

$$G(\gamma_0, \phi) = \frac{\omega}{\pi}\int_0^{2\pi/\omega} dt \frac{\sigma(t)\cos(\omega t)}{\gamma_0}$$

Effect of friction





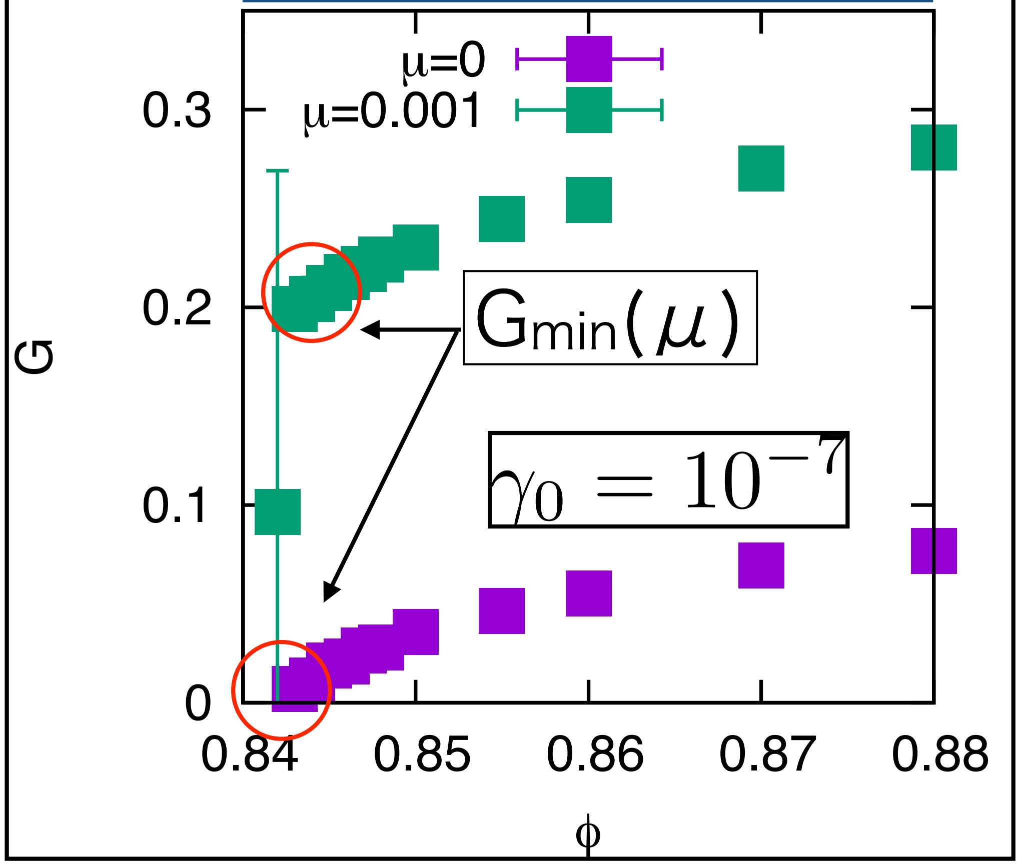
c.f. Somfai et al., PRE (2007)

Pressure : Weak dependence on μ

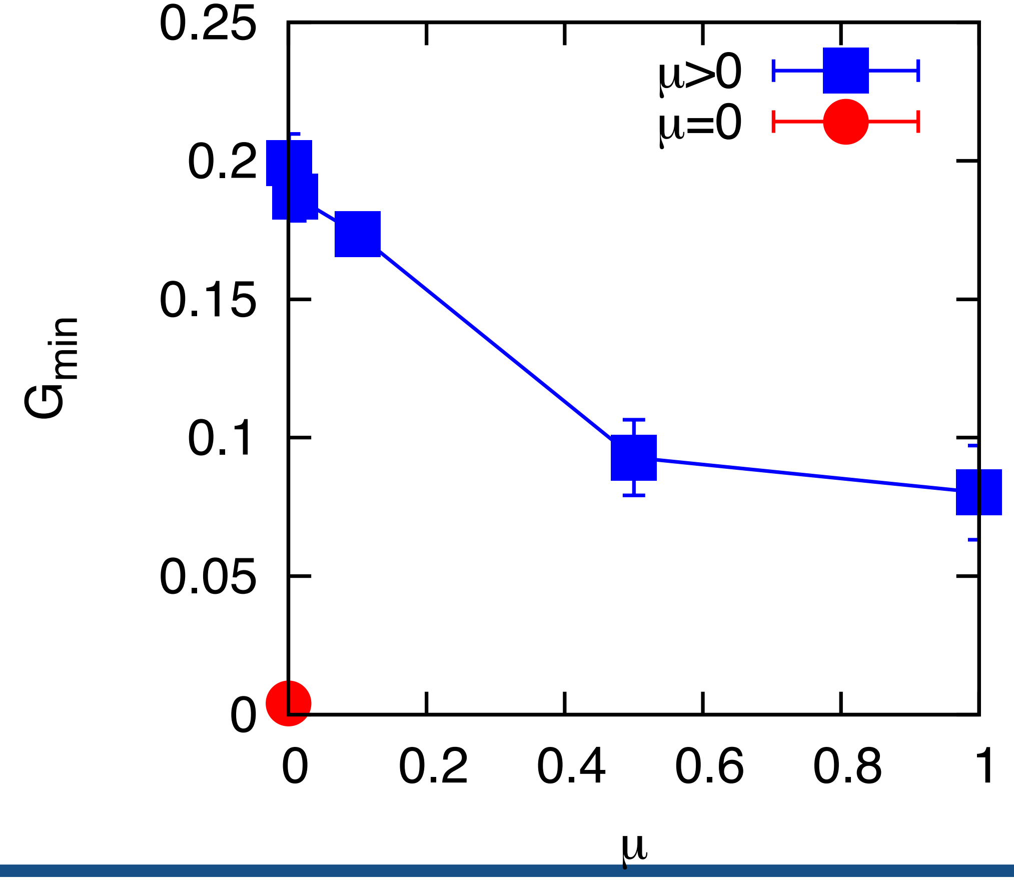
Shear modulus : Strong dependence on μ
Discontinuous change at ϕJ

μ-dependence of minimum shear modulus

Shear modulus
(linear response regime)



Minimum shear modulus



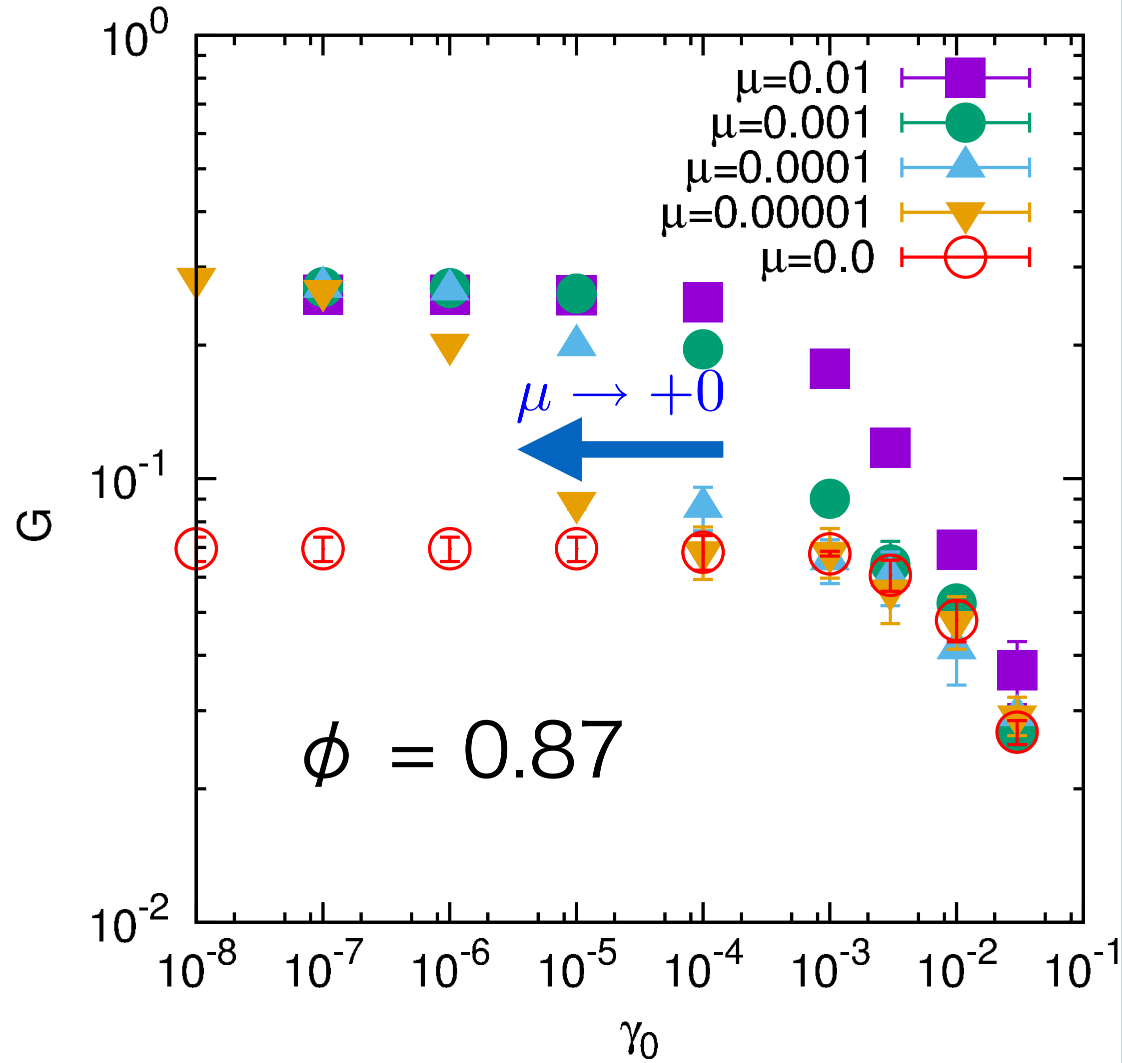
$\mu = 0$ $\quad G_{min} = 0$

$\mu > 0$ $\quad G_{min} > 0$ (even in the limit $\mu \to +0$)

G for $\mu>0$ is completely different from G for $\mu=0$?

Origin ?

γ_0-dependence of shear modulus



γ_0-dependence :

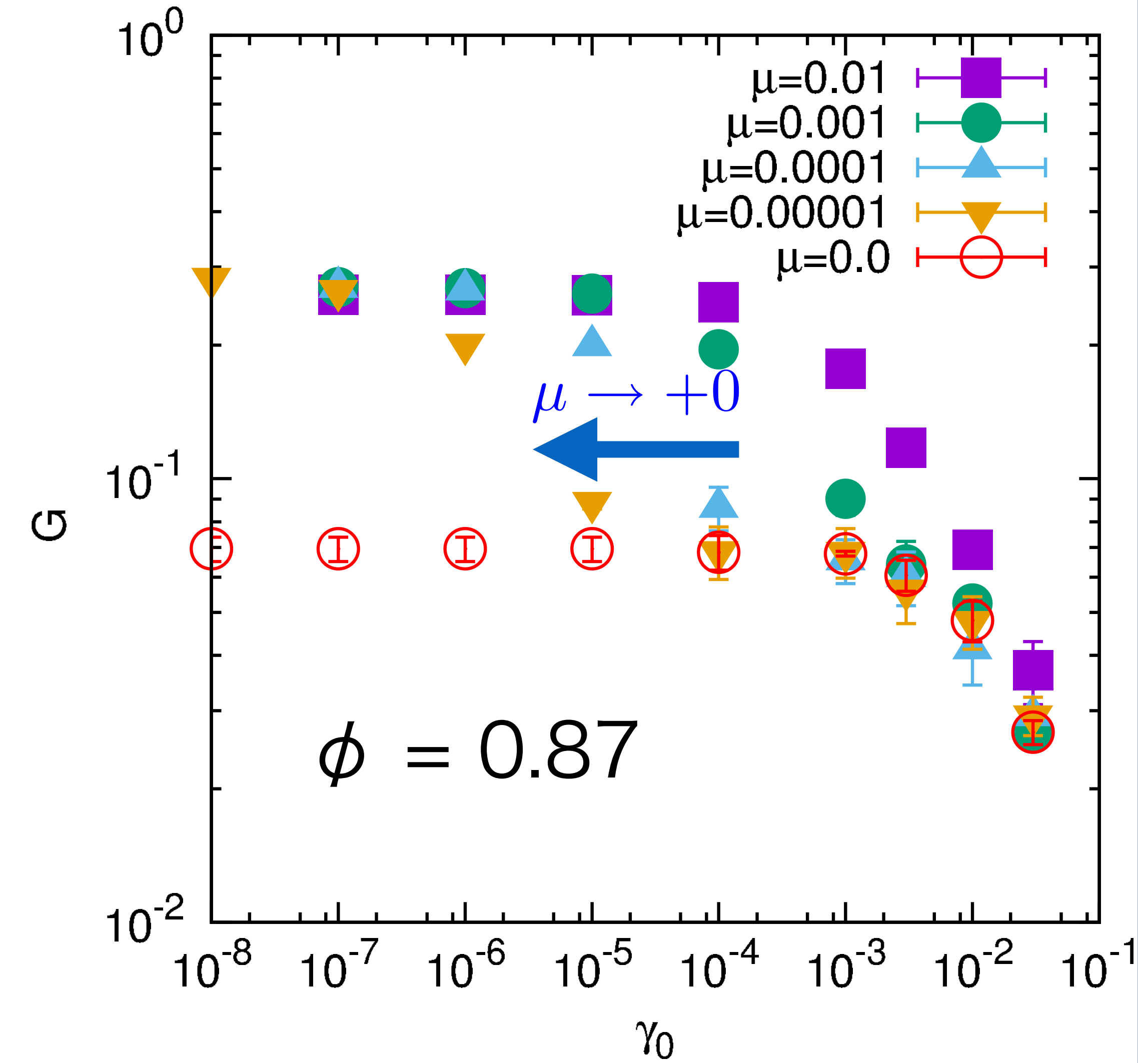
- For small γ_0, G is constant.
  (Linear response regime)
- As γ_0 increases, G decreases.
  (Nonlinear response regime)

Linear response regime :

- G for $\mu>0$ does not depend on μ.
- G for $\mu>0$ differs from G for $\mu=0$.
- As $\mu \to +0$, the linear response regime shrinks.

$$\lim_{\mu\to+0}\lim_{\gamma_0\to 0} G(\mu,\gamma_0,\phi) \neq G(\mu=0,0,\phi)$$

γ_0-dependence of shear modulus



γ_0-dependence :

- For small γ_0, G is constant.
  (Linear response regime)
- As γ_0 increases, G decreases.
  (Nonlinear response regime)

Linear response regime :

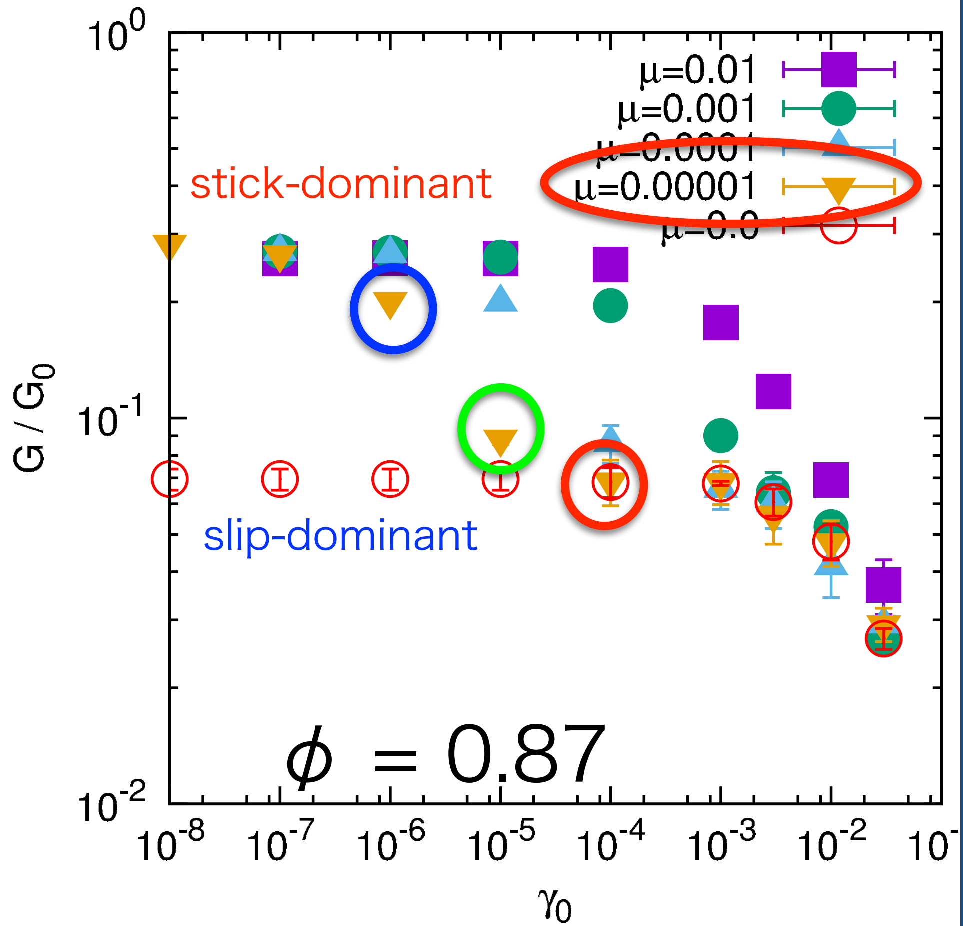
$$\lim_{\mu\to+0}\lim_{\gamma_0\to 0} G(\mu,\gamma_0,\phi) \neq G(\mu=0,0,\phi)$$

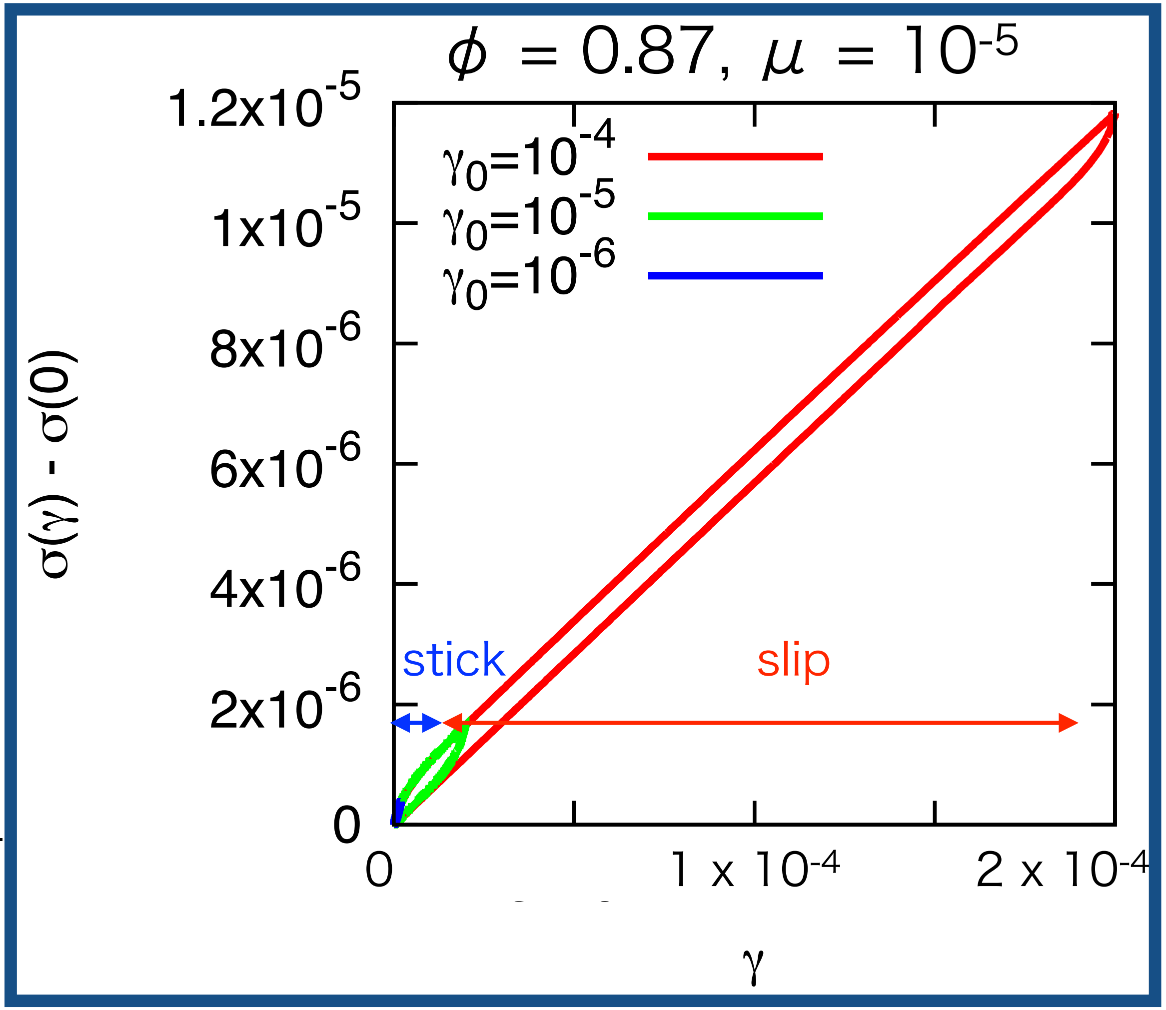
Nonlinear response regime :

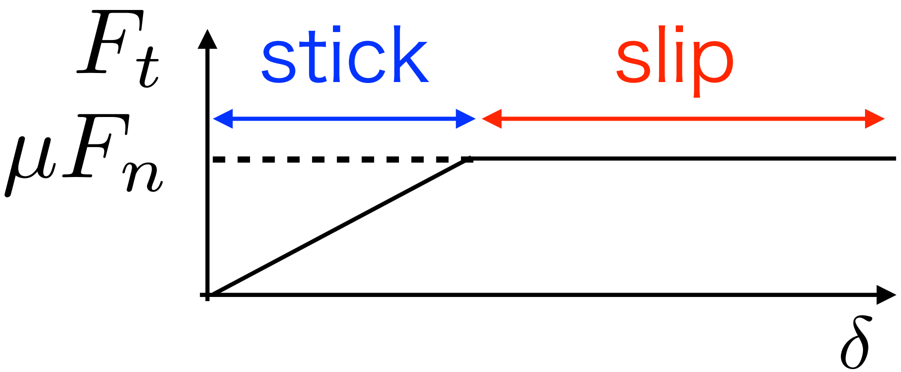
- G converges to G for μ=0 as $\mu \to 0$.
- G has a plateau as $\mu \to +0$.

$$\lim_{\gamma_0\to 0}\lim_{\mu\to+0} G(\mu,\gamma_0,\phi) = G(\mu=0,0,\phi)$$

γ_0-dependence of σ - γ relation





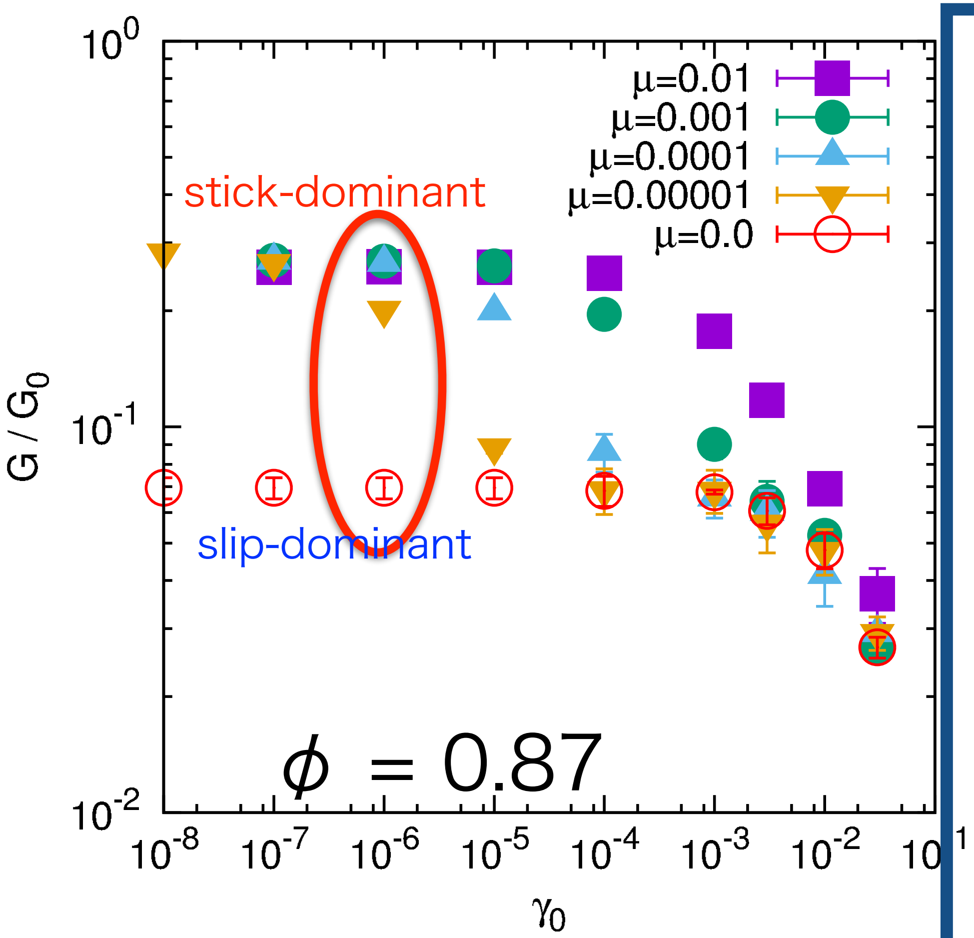


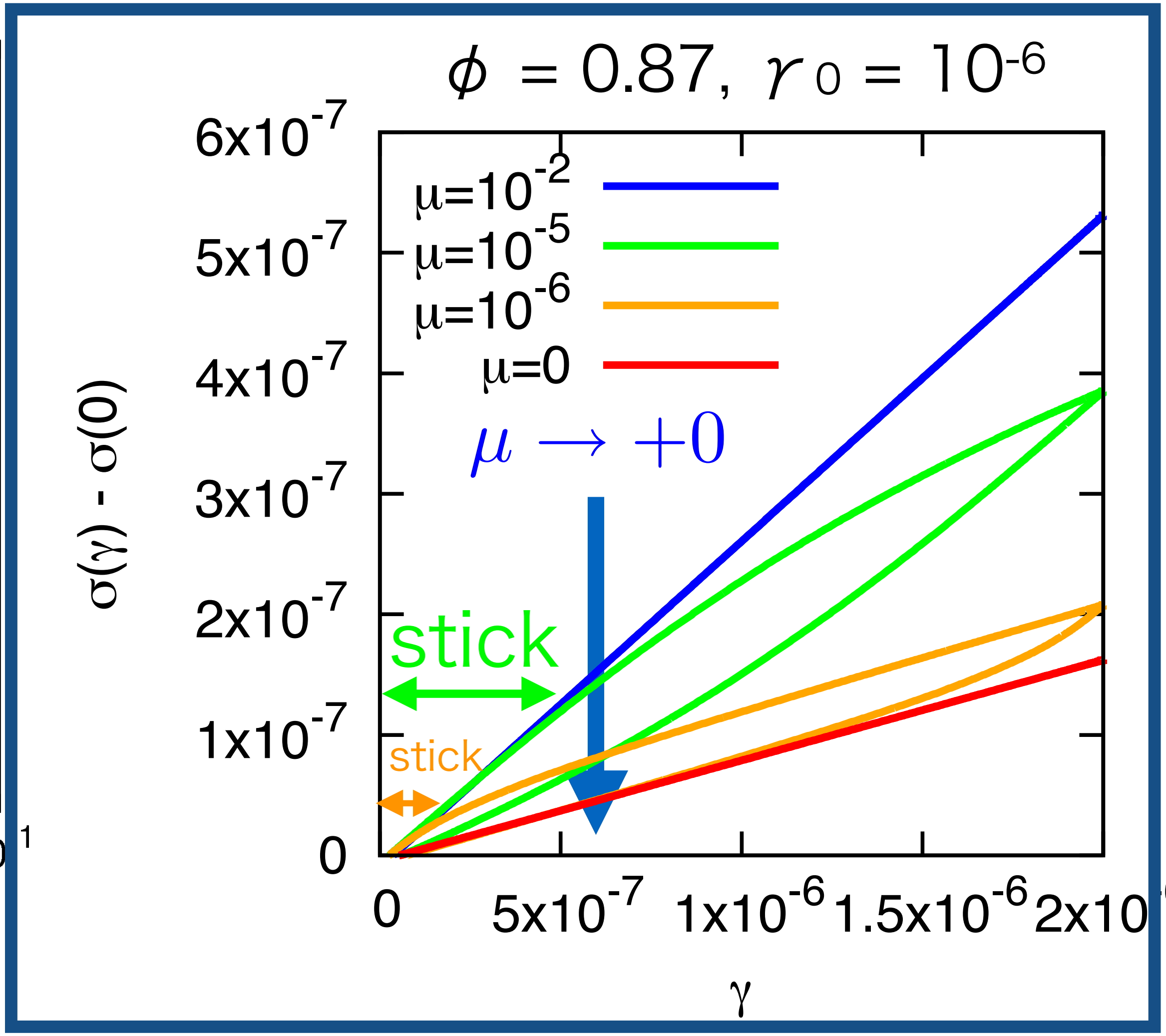
$\gamma_0 = 10^{-6}$: Linear (stick region)

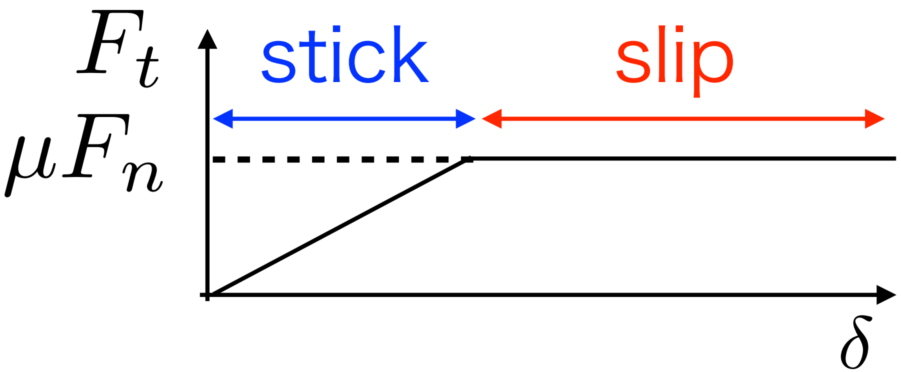
$\gamma_0 = 10^{-5}$: Nonlinear with loop (slip region)

$\gamma_0 = 10^{-4}$: Nonlinear, another linear region (origin of a plateau region in G - γ_0 plot)

μ-dependence of σ-γ relation





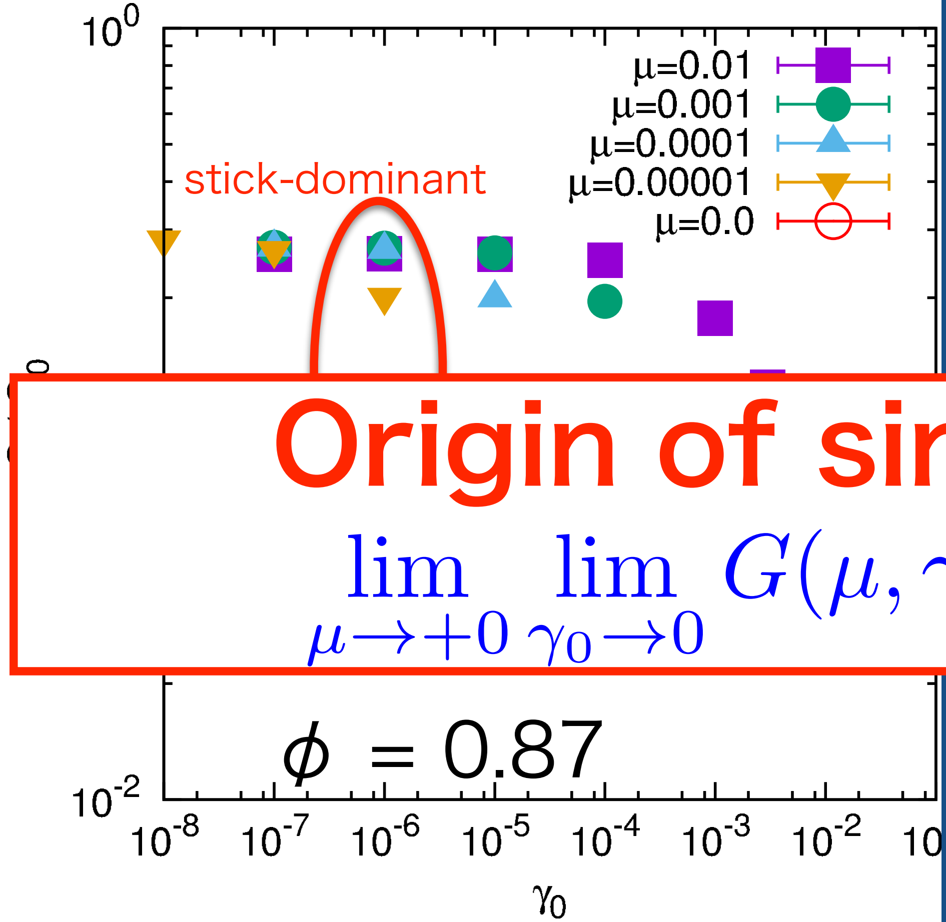


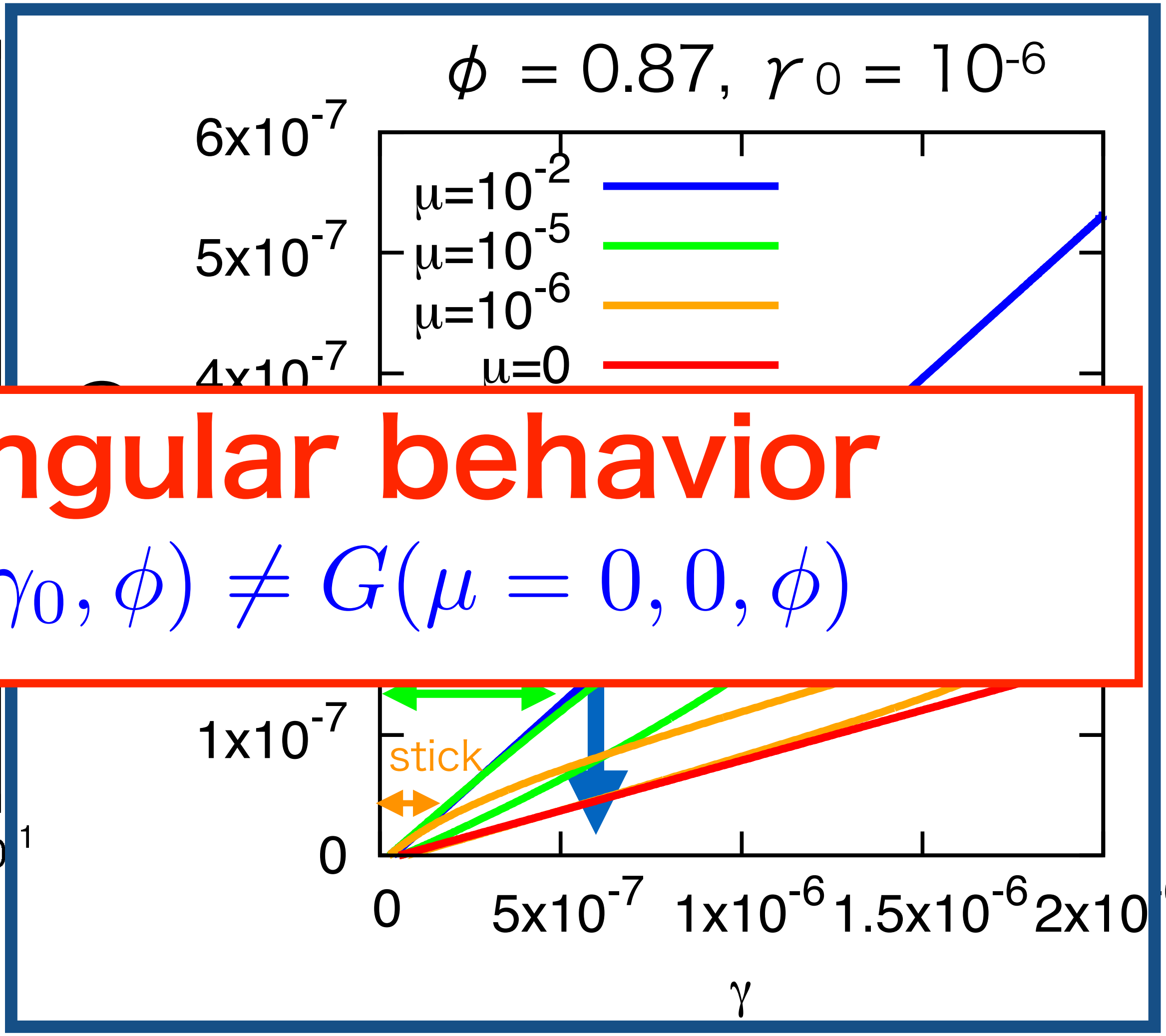
μ = 10^{-2} : Linear response

μ = 10^{-5} : Nonlinear response with loop

μ = 10^{-6} : Shrink of linear response regime, another linear region ([the angle] = [G for μ=0])

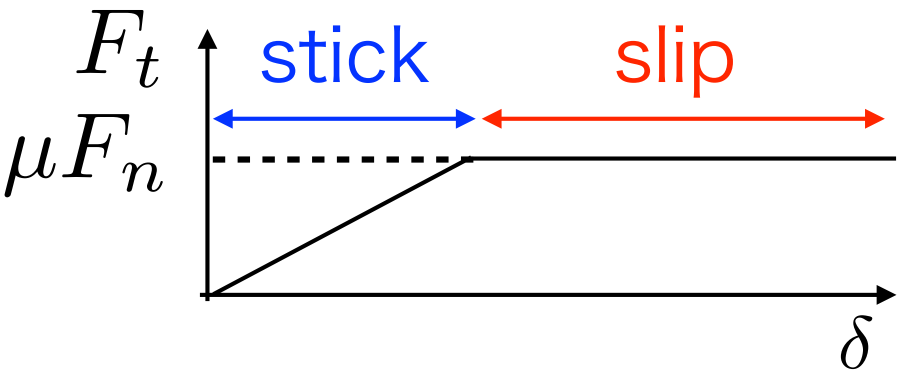
μ-dependence of σ-γ relation





Origin of singular behavior

$$\lim_{\mu \to +0} \lim_{\gamma_0 \to 0} G(\mu, \gamma_0, \phi) \neq G(\mu = 0, 0, \phi)$$



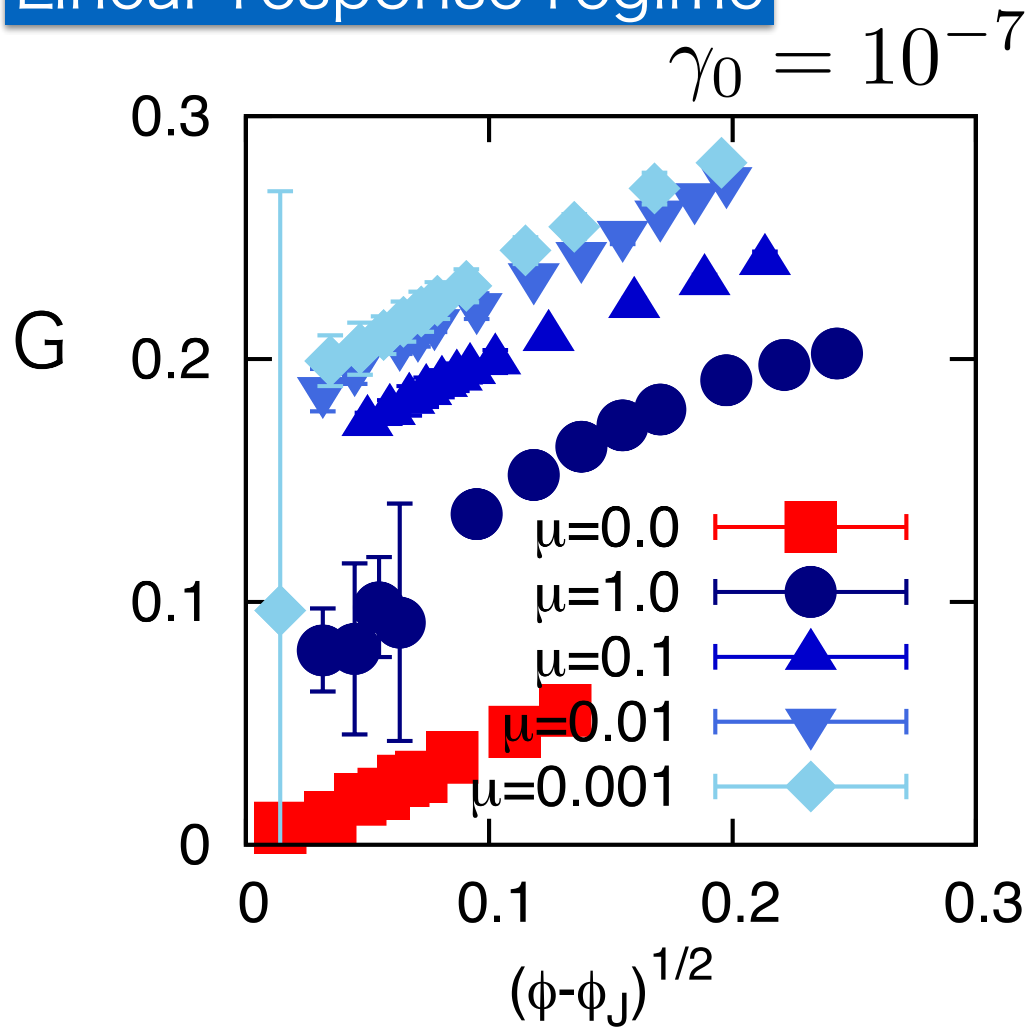
$\mu = 10^{-2}$: Linear response

$\mu = 10^{-2}$: Nonlinear response with loop

$\mu = 10^{-2}$: Shrink of linear response regime, another linear region ([the angle] = G for μ=0)

Scaling of G

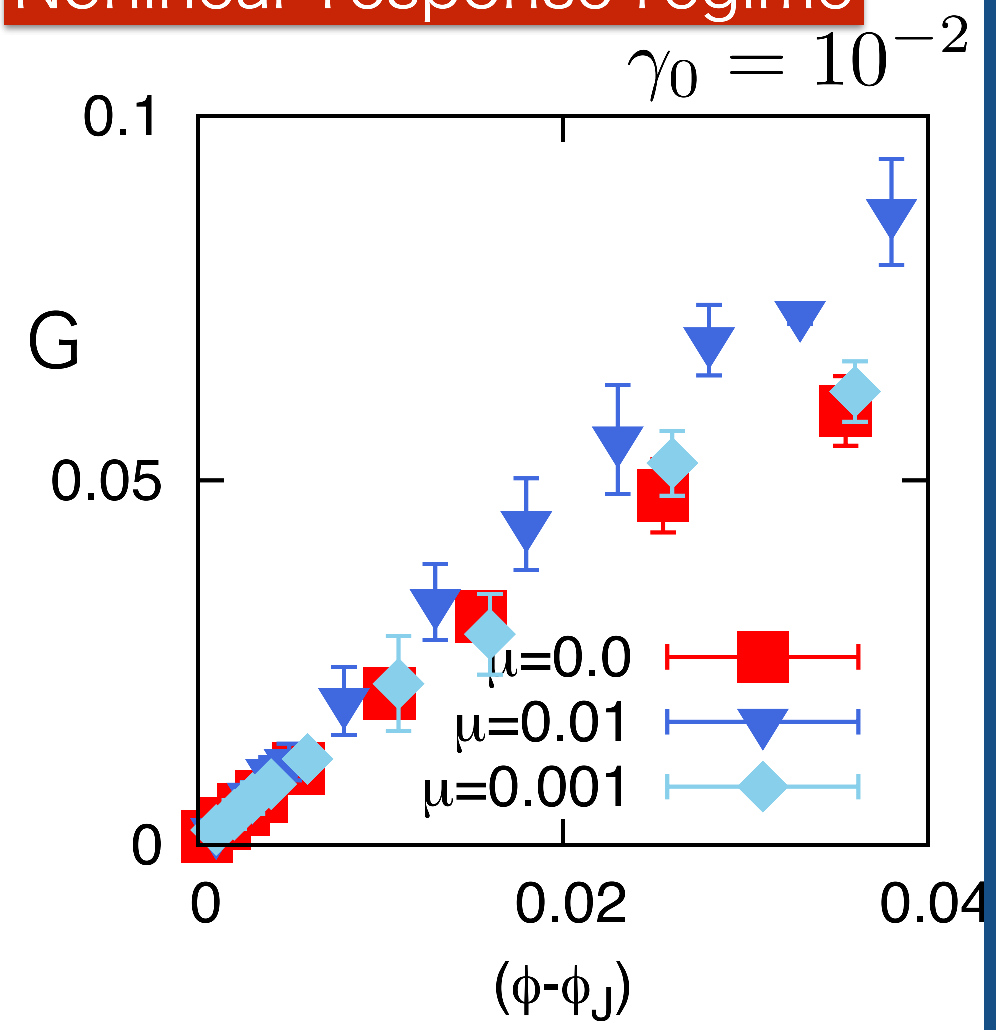
Linear response regime



$$G = G_0(\mu) + A(\phi - \phi_J)^{1/2}$$

Somfai, et al., PRE (2007)

Noninear response regime

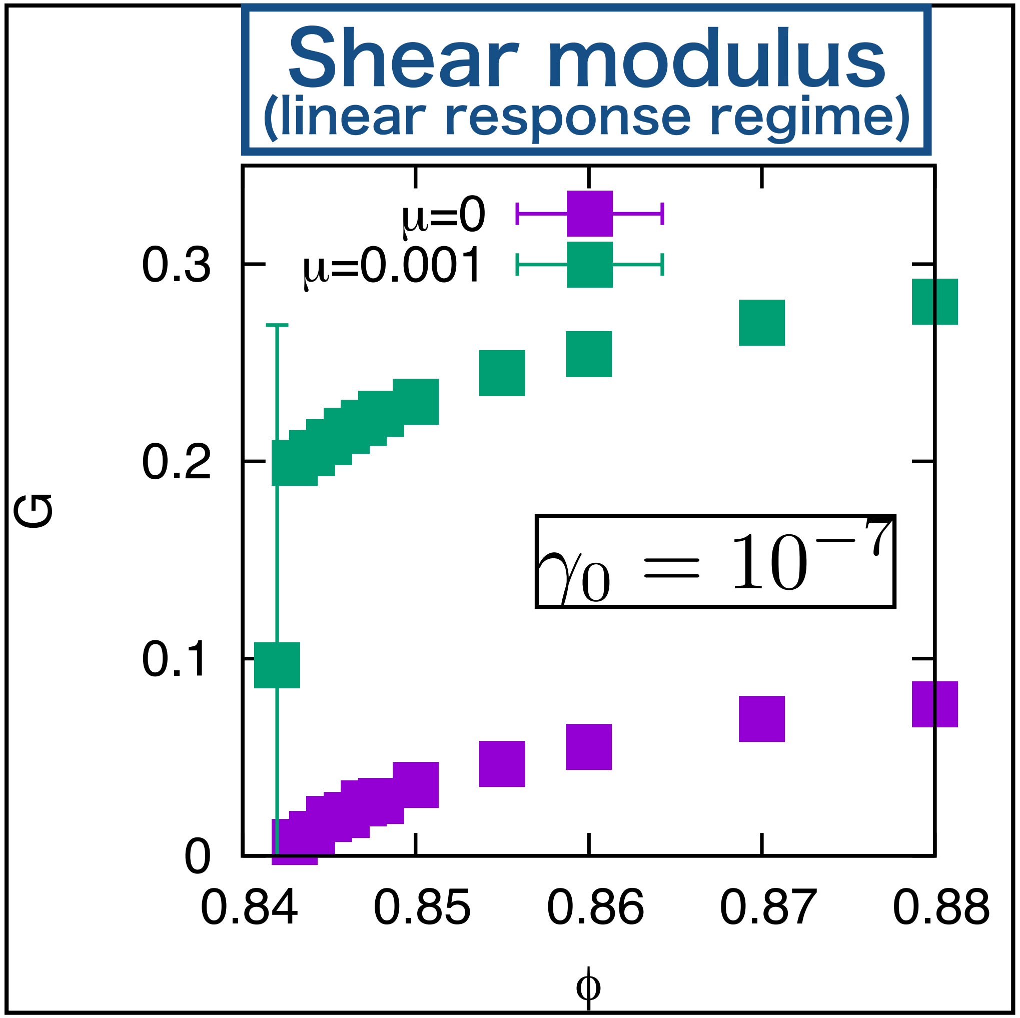


$$G \propto \gamma_0^{-c}(\phi - \phi_J)$$

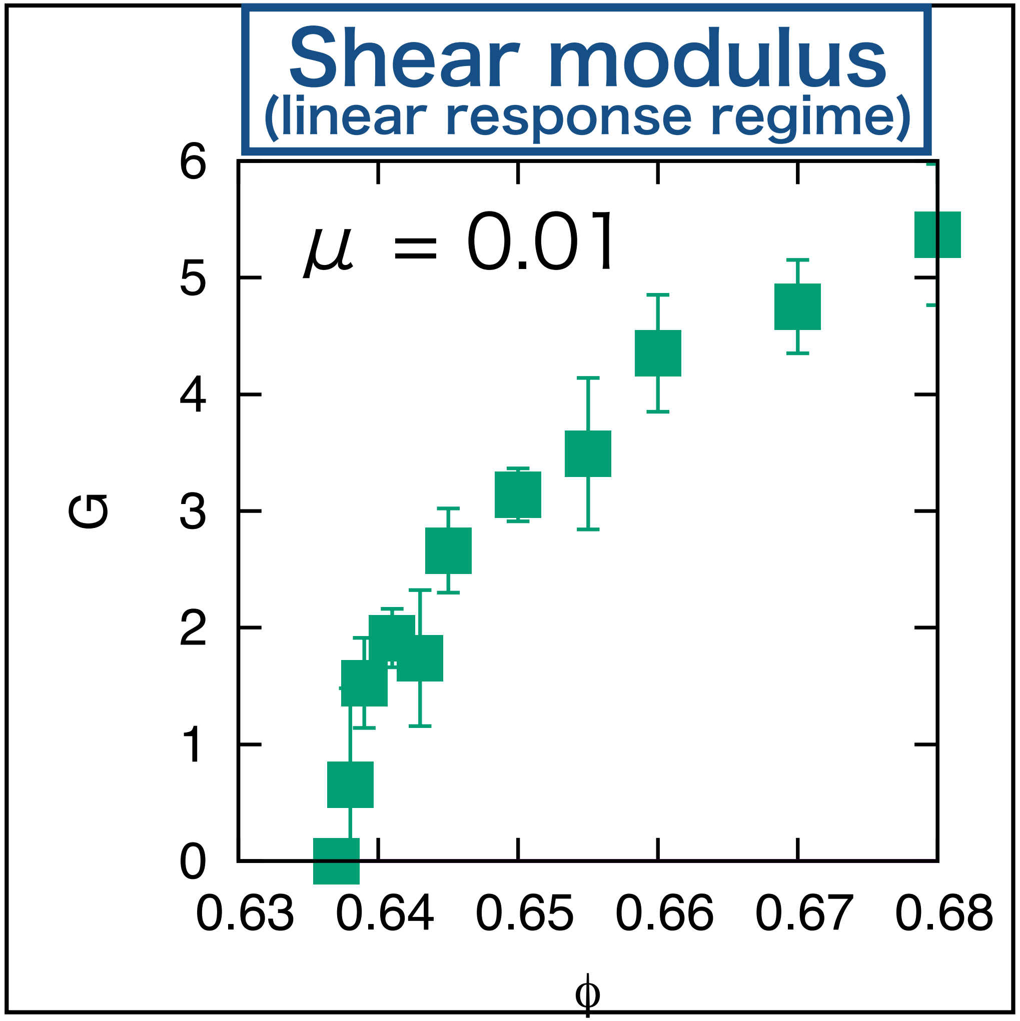
consistent with the result of μ=0.

Discussion : Dependence on dimension

2D



3D **(Preliminary result)**



Continuous transition in 3D ?

Summary

- Purpose : shear modulus of frictional granular materials.
- Infinitesimal friction changes the shear modulus in linear response regime.
- The shear modulus for $\mu \to +0$ in nonlinear response regime is consistent with that of frictionless grains.