Naturalness, supersymmetry and dark matter

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PRD90 5 055008 (2014)

conventional knowledge:

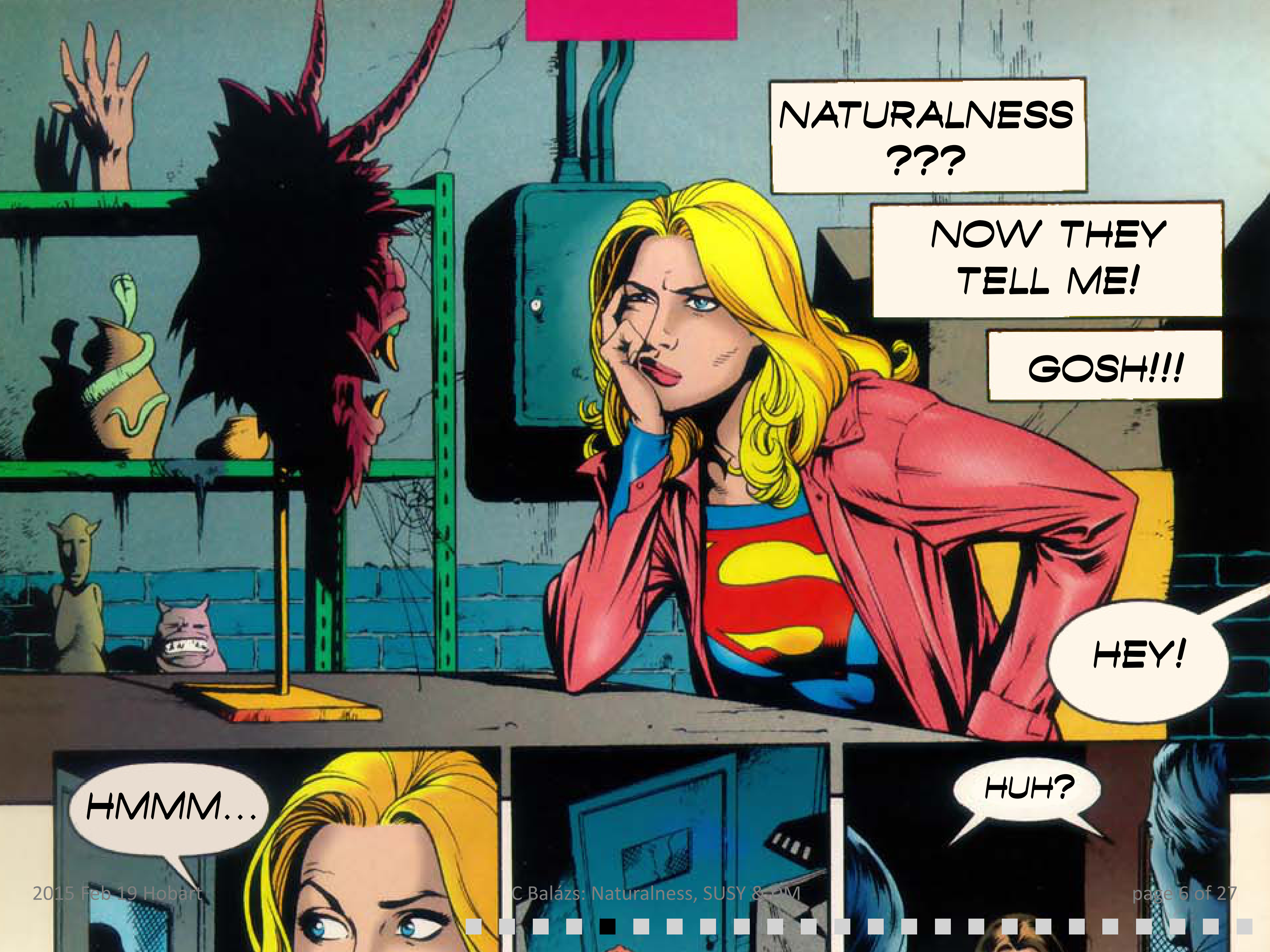
new physics models with O(TeV) dark matter possess unacceptably high electroweak fine tuning example:

the Minimal Supersymmetric Standard Model is pushed by the LHC to parameter regions which are fine tuned at worse than percent level this talk:

electroweak fine tuning is overestimated by the conventional measures

O(TeV) dark matter can be natural

outline


NATURALNESS ???
NOW THEY TELL ME!
GOSH!!!
HEY!
HMMM...
HUH?


LETS ...
... GET ...
... NATURAL


DESTROY YOU!!!
NO!
HELP!
HA-HA-HA...


PREPARE ...
... TO ...
... DIE!!!

the electroweak naturalness problem

physical phenomena characterized by disparate (energy or length) scales are separated:
their governing laws can be understood largely independently from each other

when viewed as an effective theory
the Standard Model Higgs mass receives quantum corrections from the Planck scale

$m_H = 125\ \mathrm{GeV} \Rightarrow$ the Standard Model is unnatural

What is natural?

naturalness is quantified by a fine-tuning measure

Barbieri-Ellis-Giudice measure

the most used, misused and abused electroweak fine tuning measure

$$\frac{\partial(electroweak\ observable)}{\partial(theory\ parameters)}$$

measures the “sensitivity” of the “weak scale” to the change in the “theory parameter(s)”

Barbieri-Ellis-Giudice measure: example

Constrained Minimal Supersymmetric Standard Model

theory parameters: $M_0, M_{1/2}, A_0, \tan\beta, sign(\mu)$

dominant fine tuning term: $\frac{\partial m_Z}{\partial \mu}$

Barbieri-Ellis-Giudice measure: example

electroweak symmetry breaking condition:

$$\frac{m_Z^2}{2} = \frac{M_{H_d}^2 - M_{H_u}^2 \tan^2\beta}{\tan^2\beta - 1} - \mu^2$$

fixes μ by connecting it to m_Z

this justifies $\frac{\partial m_Z}{\partial \mu}$

Bayesian evidence

$$\mathcal{E} = \int \mathcal{L}(\mu)\, \pi(\mu)\, d\mu$$

for a theory with a single parameter μ

$\mathcal{E}$ quantifies the plausibility of the theory the theory predicts $m_Z(\mu)$

so $m_Z(\mu)$ is invertible

$$m_Z(\mu) \;\Rightarrow\; \mu(m_Z)$$

write $\mathcal{E}$ as an integral over m_Z

$$\mathcal{E} = \int \mathcal{L}(m_Z)\, \pi(m_Z) \frac{d\mu}{dm_Z} dm_Z$$

$$\mathcal{E} = \int \mathcal{L}(m_Z)\, \pi(m_Z) \left(\frac{dm_Z}{d\mu}\right)^{-1} dm_Z$$

Allanach, Hooper JHEP 0810:071 (2008)

Bayesian evidence ~ inverse of fine-tuning measure!

$$\mathcal{E} = \int \mathcal{L}(m_Z)\, \pi(m_Z) \left(\frac{dm_Z}{d\mu}\right)^{-1} dm_Z$$

fixing μ using m_Z automatically induces $\frac{dm_Z}{d\mu}$ in $\mathcal{E}$

$\mathcal{E}$ ~ plausibility that the theory correctly predicts m_Z

this is Occam's razor at work!

MSSM

trade $\{m_Z, m_t, tan\beta\}$
for $\{\ \mu,\ y_t,\ B_0\}$

$$naturalness\ prior = \begin{vmatrix} \frac{\partial m_Z}{\partial \mu} & \frac{\partial m_t}{\partial \mu} & \frac{\partial tan\beta}{\partial \mu} \\ \frac{\partial m_Z}{\partial y_t} & \frac{\partial m_t}{\partial y_t} & \frac{\partial tan\beta}{\partial y_t} \\ \frac{\partial m_Z}{\partial B_0} & \frac{\partial m_t}{\partial B_0} & \frac{\partial tan\beta}{\partial B_0} \end{vmatrix}$$

Cabrera, Casas, deAustri JHEP 0903:075,2009

Barbieri-Ellis-Giudice EW FT measure:

$$\frac{\partial(electroweak\ observable)}{\partial(theory\ parameters)}$$

- evidence = integral over parameters
  observables can be used to eliminate parameters
- evidence ratios have clear normalization scale
- Observables: subjective choice!
- Parameters: subjective choice!
- Form: depends on prior!
- Expression: determinant!
- What does $\frac{\partial m_Z}{\partial \mu}$ have to do with $\Delta m_H, \mu, m_{\tilde{t}}, m_{\tilde{g}}, \ldots$?
- Beyond MSSM, NMSSM, SUSY: evidence can be defined!

NMSSM

$$\text{trade } \{\ln m_Z^2, \ \ln\lambda, \ln tan\beta\}$$
$$\text{for } \{\ln m_S^2, \ln\lambda_0, \quad \ln\kappa_0\}$$

$$naturalness\ prior = \begin{vmatrix} \frac{\partial \ln m_Z^2}{\partial \ln m_S^2} & \frac{\partial \ln\lambda}{\partial \ln m_S^2} & \frac{\partial \ln tan\beta}{\partial \ln m_S^2} \\ \frac{\partial \ln m_Z^2}{\partial \ln\lambda_0} & \frac{\partial \ln\lambda}{\partial \ln\lambda_0} & \frac{\partial \ln tan\beta}{\partial \ln\lambda_0} \\ \frac{\partial \ln m_Z^2}{\partial \ln\kappa_0} & \frac{\partial \ln\lambda}{\partial \ln\kappa_0} & \frac{\partial \ln tan\beta}{\partial \ln\kappa_0} \end{vmatrix}$$

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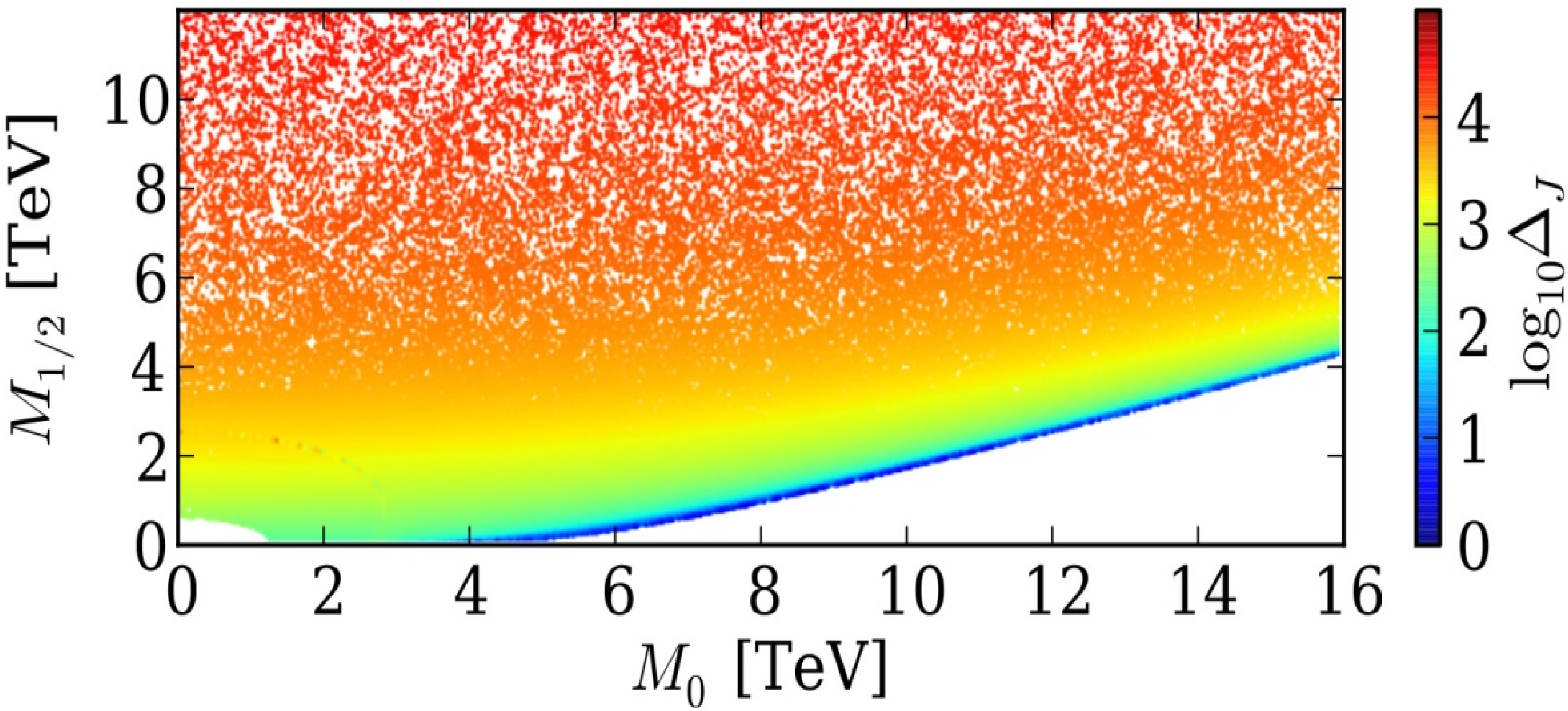
naturalness prior map in the CMSSM

$$A_0 = -2.5\ \text{TeV}, tan\beta = 10$$



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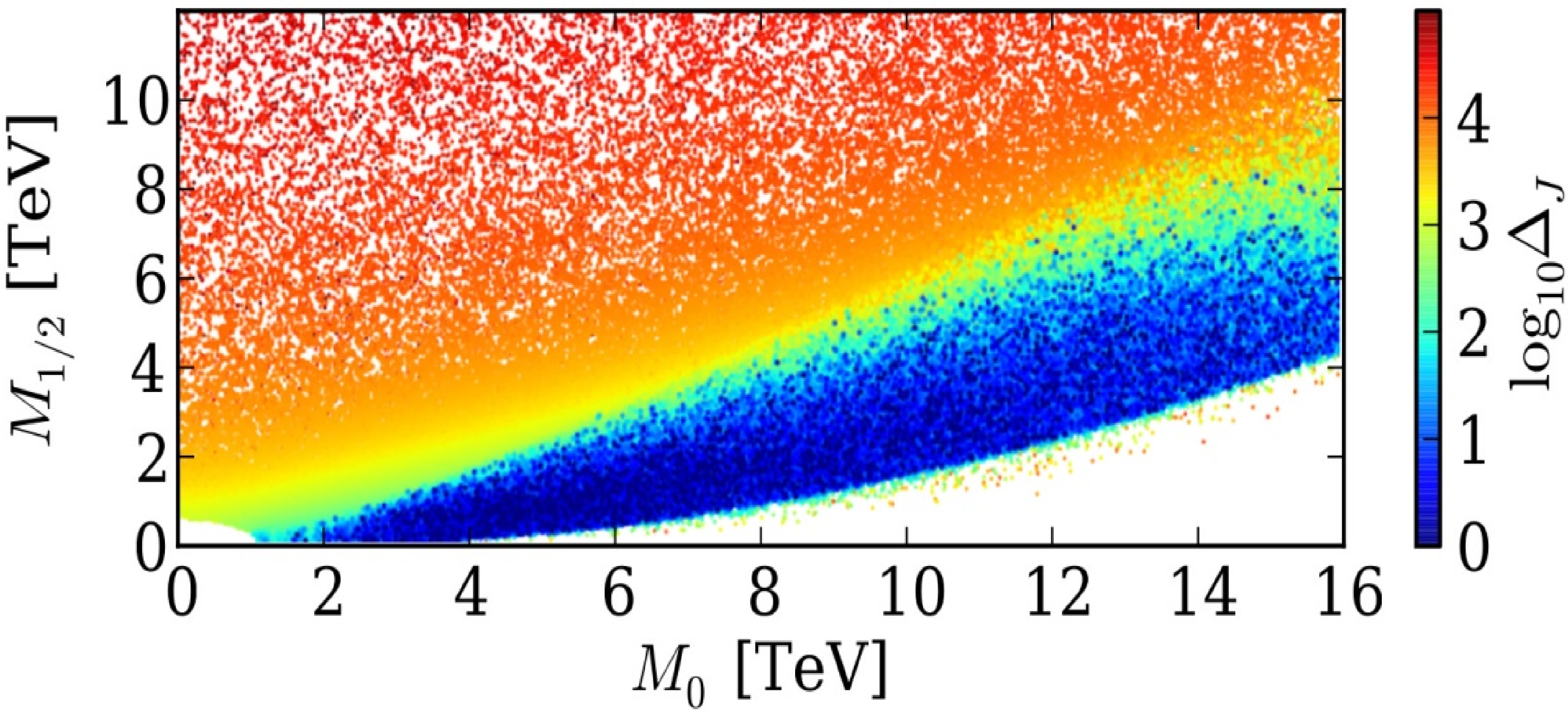
naturalness prior map in the CNMSSM

$$A_0 = -2.5\ \mathrm{TeV}, tan\beta = 10$$



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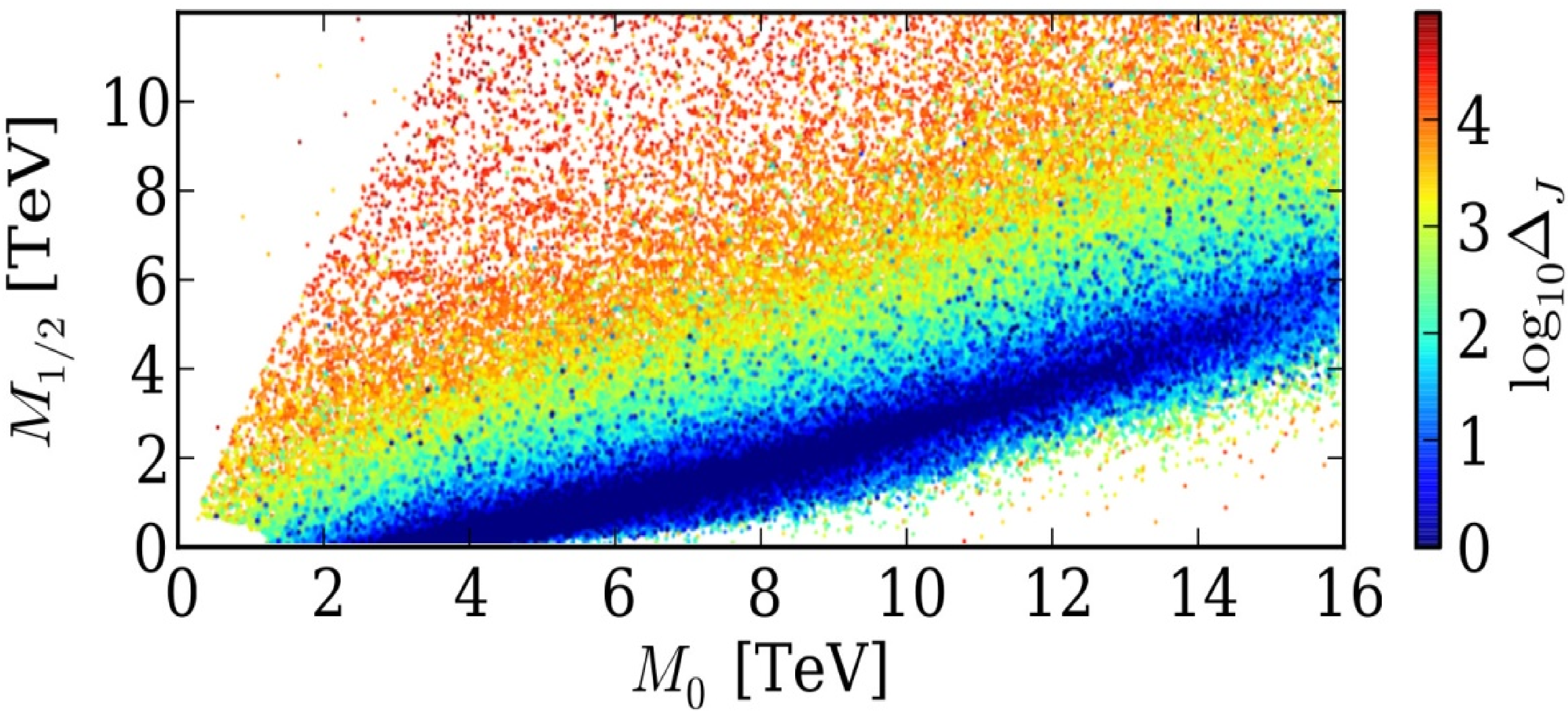
naturalness prior map in NMSSM-11

$$A_0 = -2.5\ \text{TeV}, tan\beta = 10$$



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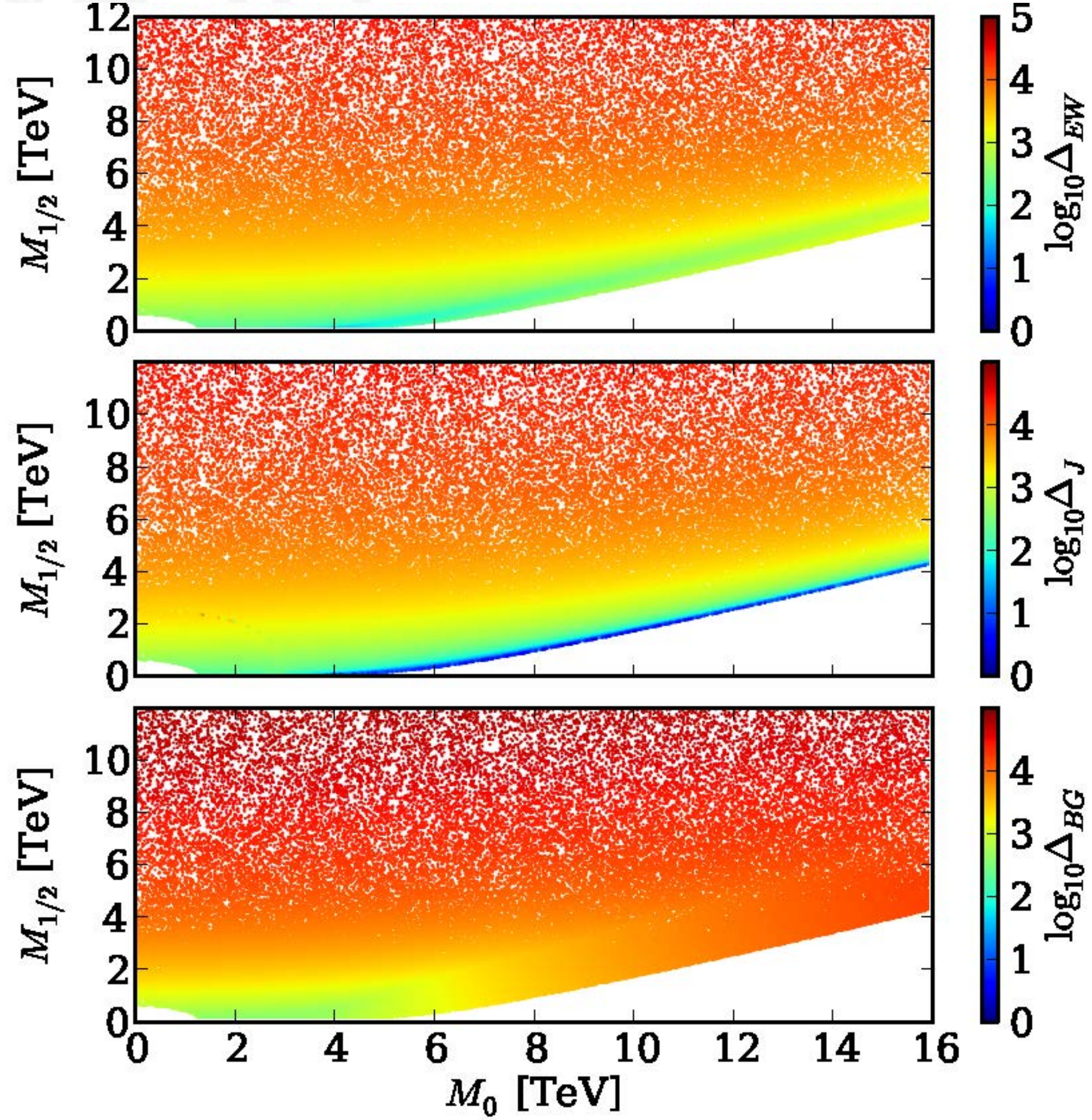
naturalness prior agrees with EW FT measure



conclusions

naturalness is a robust principle

it can be quantified within the Bayesian framework

fine tuning is measured by the naturalness prior

according to the naturalness prior O(TeV) dark matter can be perfectly natural