Covariant EFT of Gravity and Dark Energy

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Kyoto
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in collaboration with Rajeev K. Jain
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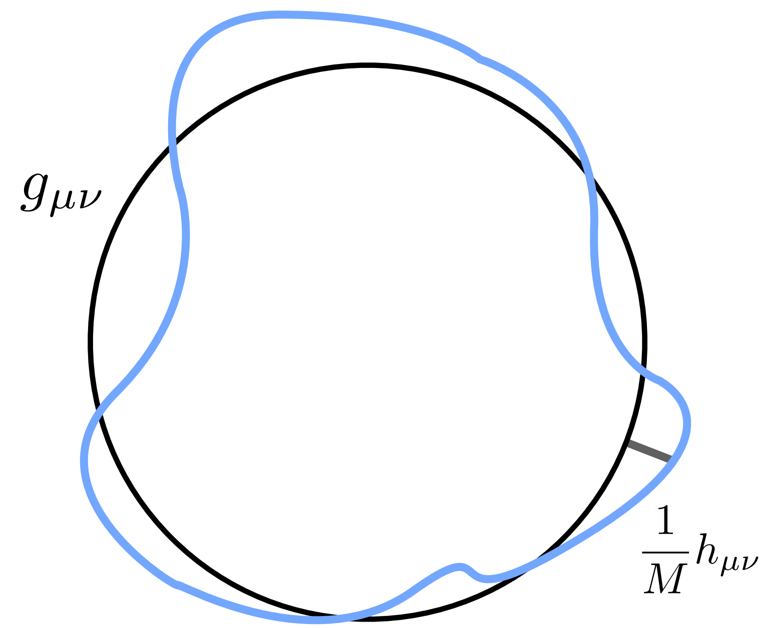
arXiv:1507.07829

Outline of the talk

- Covariant EFT of gravity
- LO quantum corrections
- Curvature expansion
- Cosmological effective action
- Effective Friedmann equations
- Early times: Inflation
- Late times: Dark energy

EFT of Gravity



- The theory of small fluctuations of the metric

$$g_{\mu\nu} \to g_{\mu\nu} + \sqrt{16\pi G} h_{\mu\nu} = g_{\mu\nu} + \frac{1}{M} h_{\mu\nu}$$

- Planck's scale is the characteristic scale of gravity

$$M \equiv \frac{1}{\sqrt{16\pi G}} = \frac{M_{Planck}}{\sqrt{16\pi}}$$

$$M_{Planck} = \frac{1}{\sqrt{G}} = 1.2 \times 10^{19}\,\mathrm{GeV}$$

- Classical theory (CT) is successful over many orders of magnitude

EFT of Gravity

$$S_{eff}[g] = M^2 \left[I_1[g] + \frac{1}{M^2} I_2[g] + \frac{1}{M^4} I_3[g] + ... \right]$$

UV action contains all couplings expressed in terms of the scale M

$$I_1[g] = \int d^4x \sqrt{g} \left[M^2 c_0 - c_1 R \right]$$

$$I_2[g] = \int d^4x \sqrt{g} \left[c_{2,1} R^2 + c_{2,2} \mathrm{Ric}^2 + c_{2,3} \mathrm{Riem}^2 \right]$$

$$I_3[g] = \int d^4x \sqrt{g} \left[c_{3,1} R \Box R + c_{3,2} R_{\mu\nu} \Box R^{\mu\nu} + c_{3,3} R^3 + ... \right]$$

Derivative expansion of the UV action

Covariant EFT of Gravity

$$e^{-\Gamma[g]} = \int_{1PI} \mathcal{D}h_{\mu\nu}\, e^{-S_{eff}[g+\frac{1}{M}h]}$$

$$= \int_{1PI} \mathcal{D}h_{\mu\nu}\, e^{-M^2\left\{I_1[g+\frac{1}{M}h]+\frac{1}{M^2}I_2[g+\frac{1}{M}h]+\ldots\right\}}$$

EFT: saddle point expansion in $\frac{1}{M^2}$

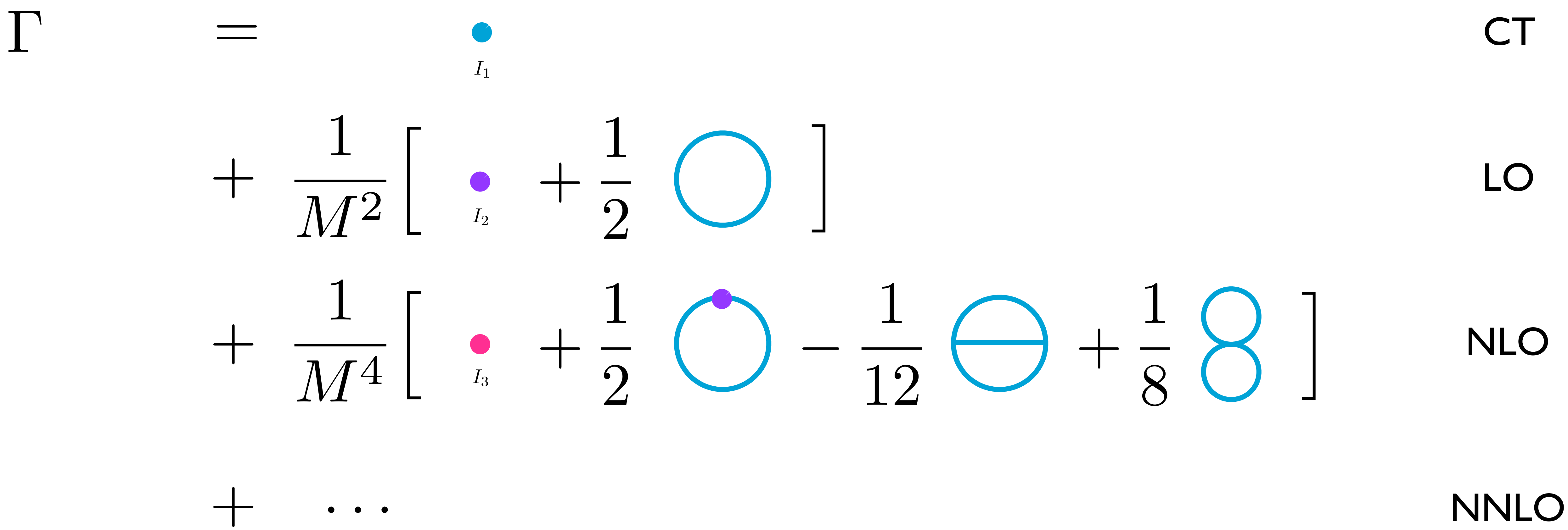
$$\Gamma[g] = I_1[g]$$

$$+ \frac{1}{M^2}\left\{I_2[g] + \frac{1}{2}\mathrm{Tr}\log I_1^{(2)}[g]\right\}$$

$$+ \frac{1}{M^4}\left\{I_3[g] + \frac{1}{2}\mathrm{Tr}\left[\left(I_1^{(2)}[g]\right)^{-1} I_2^{(2)}[g]\right] + \text{2-loops with } I_1[g]\right\}$$

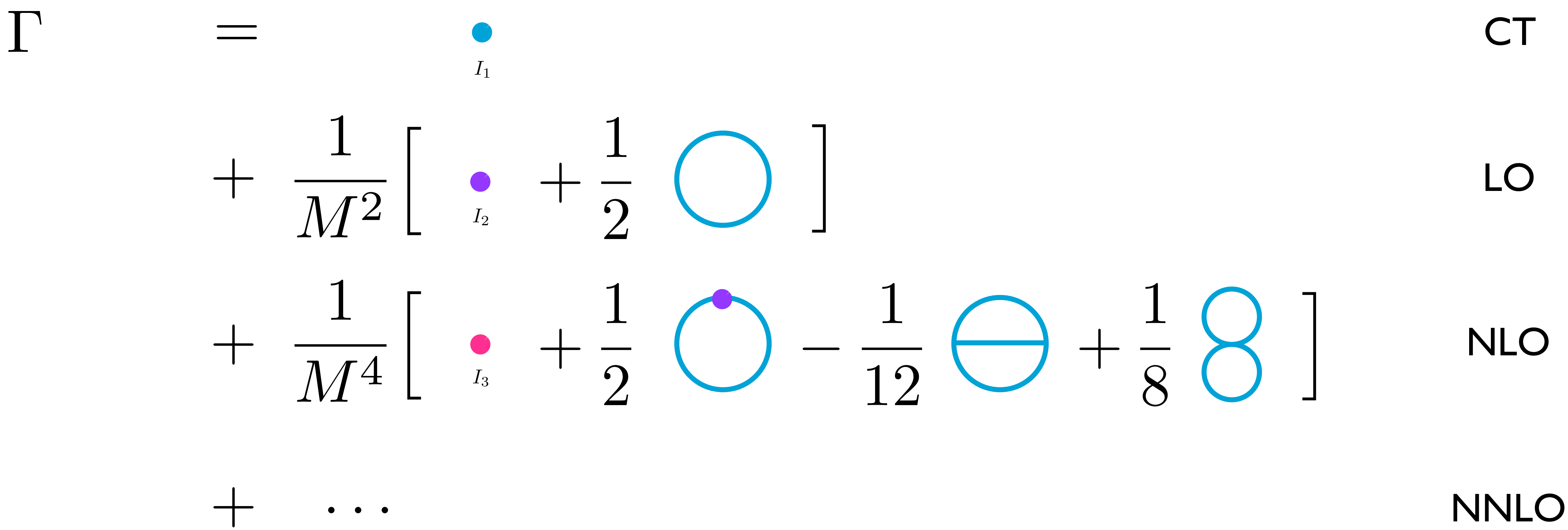
$$+ \ldots$$

Covariant EFT of Gravity



Covariant EFT of Gravity

The EFT recipe in three lines



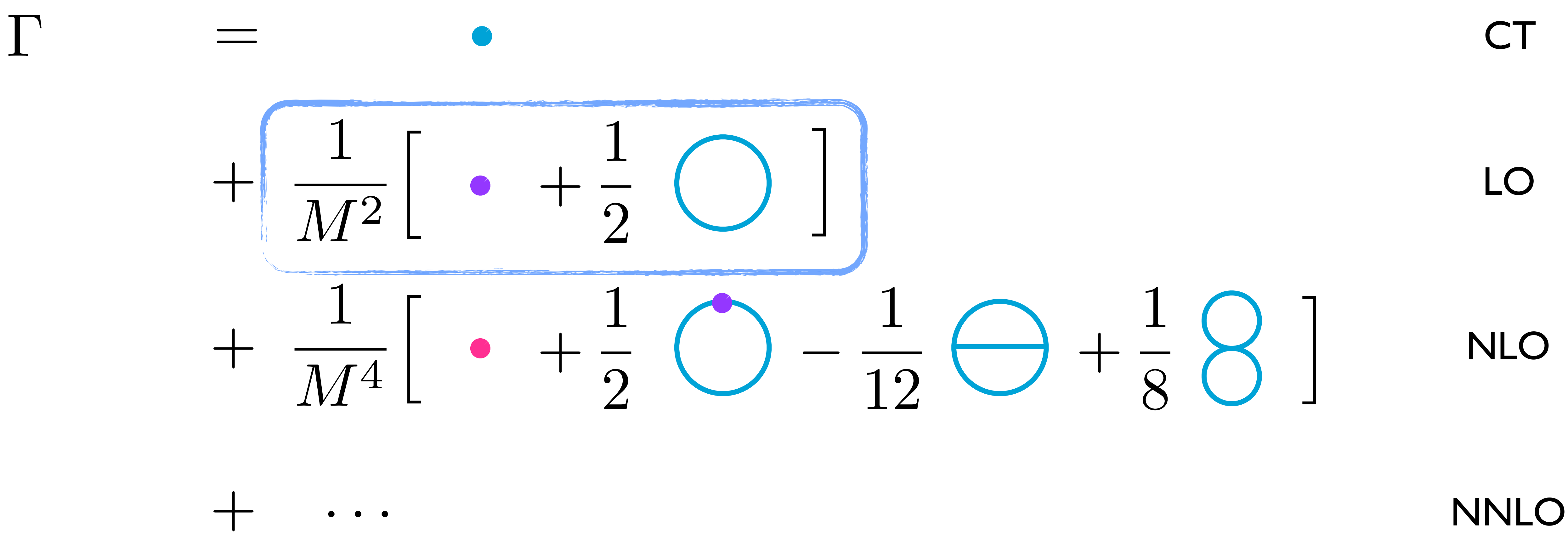
1) the general lagrangian of order E^2 is to be used both at tree level and in loop diagrams

2) the general lagrangian of order $E^{n\geq 4}$ is to be used at tree level and as an insertion in loop diagrams

3) the renormalization program is carried out order by order

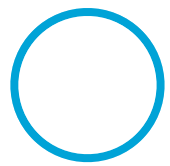
Covariant EFT of Gravity

LOQG: the only QG we will ever observe!



Even if we have a fundamental theory its is generally difficult to compute phenomenological parameters...

Curvature expansion

$+\frac{1}{2}$  $= -\frac{1}{2(4\pi)^{d/2}} \int d^d x \sqrt{g}\, \mathrm{tr}\, \mathcal{R}\, \gamma_i \left(\frac{-\Box}{m^2} \right) \mathcal{R} + \dots$

The finite physical part of the effective action is covariantly encoded in the structure functions which can be computed using the non-local heat kernel expansion

$$\gamma_i \left(\frac{X}{m^2} \right) \equiv \lim_{\Lambda_{UV} \to \infty} \int_{1/\Lambda_{UV}^2}^{\infty} \frac{ds}{s} s^{-d/2+2} \left[f_i(sX) - f_i(0) \right] e^{-sm^2}$$

Non-local heat kernel structure functions

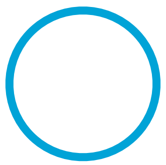
Non-local heat kernel

A. O. Barvinsky and G. A. Vilkovisky, Nucl. Phys. B 282 (1987) 163

I. G. Avramidi, Lect. Notes Phys. M 64 (2000) 1

A. Codello and O. Zanusso, J. Math. Phys. 54 (2013) 013513

Curvature expansion

- $+\frac{1}{2}$ 

$$
= -\frac{1}{2(4\pi)^2}\int d^4x\sqrt{g}\,\mathrm{tr}\Bigg[\mathbf{1}R_{\mu\nu}\gamma_{Ric}\left(\frac{-\Box}{m^2}\right)R^{\mu\nu} + \frac{\mathbf{1}}{120}R\gamma_R\left(\frac{-\Box}{m^2}\right)R
$$

$$
-\frac{1}{6}R\gamma_{RU}\left(\frac{-\Box}{m^2}\right)\mathbf{U} + \frac{1}{2}\mathbf{U}\gamma_U\left(\frac{-\Box}{m^2}\right)\mathbf{U} + \frac{1}{12}\mathbf{\Omega}_{\mu\nu}\gamma_\Omega\left(\frac{-\Box}{m^2}\right)\mathbf{\Omega}^{\mu\nu}\Bigg]
$$

Explicit form for the structure functions

$$
\gamma_{Ric}(u) = \frac{1}{40} + \frac{1}{12u} - \frac{1}{2}\int_0^1 d\xi\,\left[\frac{1}{u} + \xi(1-\xi)\right]^2 \log\left[1 + u\,\xi(1-\xi)\right]
$$

$$
\gamma_R(u) = -\frac{23}{960} - \frac{1}{96u} + \frac{1}{32}\int_0^1 d\xi\,\left\{\frac{2}{u^2} + \frac{4}{u}\left[1+\xi(1-\xi)\right] - 1 + 2\xi(2-\xi)(1-\xi^2)\right\}\log\left[1 + u\,\xi(1-\xi)\right]
$$

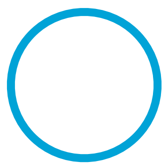
$$
\gamma_{RU}(u) = \frac{1}{12} - \frac{1}{2}\int_0^1 d\xi\,\left[\frac{1}{u} - \frac{1}{2} + \xi(1-\xi)\right]\log\left[1 + u\,\xi(1-\xi)\right]
$$

$$
\gamma_U(u) = -\frac{1}{2}\int_0^1 d\xi\,\log\left[1 + u\,\xi(1-\xi)\right]
$$

$$
\gamma_\Omega(u) = \frac{1}{12} - \frac{1}{2}\int_0^1 d\xi\,\left[\frac{1}{u} + \xi(1-\xi)\right]\log\left[1 + u\,\xi(1-\xi)\right]
$$

$$
u \equiv \frac{-\Box}{m^2}
$$

Curvature expansion

- $+\frac{1}{2}$  $= -\frac{1}{2(4\pi)^2}\int d^4x\sqrt{g}\,\mathrm{tr}\Big[\mathbf{1}R_{\mu\nu}\gamma_{Ric}\Big(\frac{-\Box}{m^2}\Big)R^{\mu\nu} + \frac{\mathbf{1}}{120}R\gamma_R\Big(\frac{-\Box}{m^2}\Big)R$
$-\frac{1}{6}R\gamma_{RU}\Big(\frac{-\Box}{m^2}\Big)\mathbf{U} + \frac{1}{2}\mathbf{U}\gamma_U\Big(\frac{-\Box}{m^2}\Big)\mathbf{U} + \frac{1}{12}\mathbf{\Omega}_{\mu\nu}\gamma_\Omega\Big(\frac{-\Box}{m^2}\Big)\mathbf{\Omega}^{\mu\nu}\Big]$

High energy expansion $u \gg 1$

$$\gamma_{Ric}(u) = -\frac{u}{840} + \frac{u^2}{15120} - \frac{u^3}{166320} + O(u^4)$$

$$\gamma_R(u) = -\frac{u}{336} + \frac{11u^2}{30240} - \frac{19u^3}{332640} + O(u^4)$$

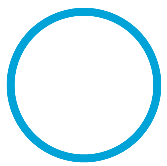
$$\gamma_{RU}(u) = \frac{u}{30} - \frac{u^2}{280} + \frac{u^3}{1890} + O(u^4)$$

$$\gamma_U(u) = -\frac{u}{12} + \frac{u^2}{120} - \frac{u^3}{840} + O(u^4)$$

$$\gamma_\Omega(u) = -\frac{u}{120} + \frac{u^2}{1680} - \frac{u^3}{15120} + O(u^4)$$

$$u \equiv \frac{-\Box}{m^2}$$

Curvature expansion

- $+\frac{1}{2}$  $= -\frac{1}{2(4\pi)^2}\int d^4x\sqrt{g}\,\mathrm{tr}\Big[\mathbf{1}R_{\mu\nu}\gamma_{Ric}\Big(\frac{-\Box}{m^2}\Big)R^{\mu\nu} + \frac{\mathbf{1}}{120}R\gamma_R\Big(\frac{-\Box}{m^2}\Big)R$
$\quad - \frac{1}{6}R\gamma_{RU}\Big(\frac{-\Box}{m^2}\Big)\mathbf{U} + \frac{1}{2}\mathbf{U}\gamma_U\Big(\frac{-\Box}{m^2}\Big)\mathbf{U} + \frac{1}{12}\mathbf{\Omega}_{\mu\nu}\gamma_\Omega\Big(\frac{-\Box}{m^2}\Big)\mathbf{\Omega}^{\mu\nu}\Big]$

Low energy expansion $u \ll 1$

$$\gamma_{Ric}(u) = \frac{23}{450} - \frac{1}{60}\log u + \frac{5}{18u} - \frac{\log u}{6u} + \frac{1}{4u^2} - \frac{\log u}{2u^2} + O\Big(\frac{1}{u^3}\Big)$$

$$\gamma_R(u) = \frac{1}{1800} - \frac{1}{120}\log u - \frac{2}{9u} + \frac{\log u}{12u} + \frac{1}{8u^2} + \frac{\log u}{4u^2} + O\Big(\frac{1}{u^3}\Big)$$

$$\gamma_{RU}(u) = -\frac{5}{18} + \frac{1}{6}\log u + \frac{1}{u} - \frac{1}{2u^2} - \frac{\log u}{u^2} + O\Big(\frac{1}{u^3}\Big)$$

$$\gamma_U(u) = 1 - \frac{1}{2}\log u - \frac{1}{u} - \frac{\log u}{u} - \frac{1}{2u^2} + \frac{\log u}{u^2} + O\Big(\frac{1}{u^3}\Big)$$

$$\gamma_\Omega(u) = \frac{2}{9} - \frac{1}{12}\log u + \frac{1}{2u} - \frac{\log u}{2u} - \frac{3}{4u^2} - \frac{\log u}{2u^2} + O\Big(\frac{1}{u^3}\Big)$$

$$u \equiv \frac{-\Box}{m^2}$$

LO effective action to $\mathcal{R}^2$

$$\Gamma[g] = \frac{1}{16\pi G}\int d^4x\sqrt{g}\,(2\Lambda - R) + \frac{1}{2\lambda}\int d^4x\sqrt{g}\,C^2 + \frac{1}{\xi}\int d^4x\sqrt{g}\,R^2$$
$$+ \int d^4x\sqrt{g}\,C_{\mu\nu\alpha\beta}\,\mathcal{G}\left(\frac{-\Box}{m^2}\right)C^{\mu\nu\alpha\beta} + \int d^4x\sqrt{g}\,R\,\mathcal{F}\left(\frac{-\Box}{m^2}\right)R + O(\mathcal{R}^3)$$

Graviton contributions:

$$\mathcal{G}_2(u) = -\frac{1}{2(4\pi)^2}\Big(5\gamma_{Ric}(u) + 3\gamma_U(u) - 12\gamma_\Omega(u)$$
$$- 4\gamma_{Ric}(u) - \gamma_U(u) + 4\gamma_\Omega(u)\Big)$$

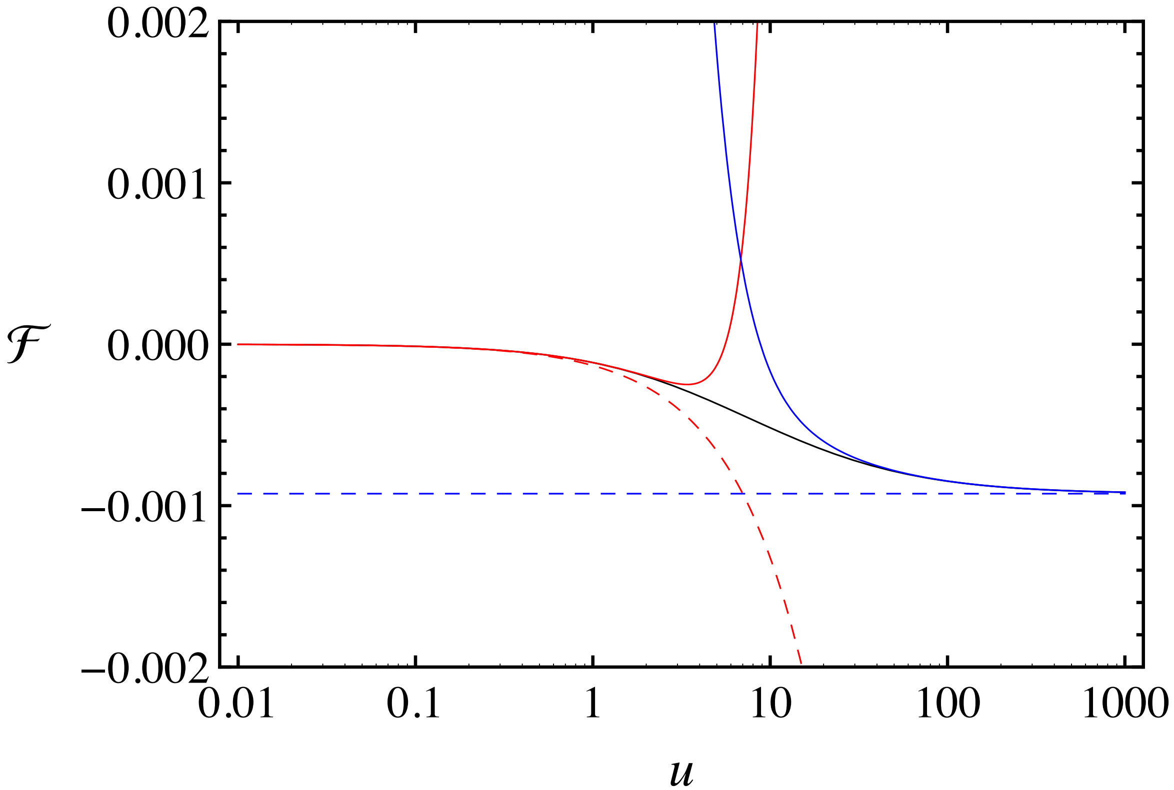
$$\mathcal{F}_2(u) = -\frac{1}{2(4\pi)^2}\left(\frac{10}{3}\gamma_{Ric}(u) + 10\gamma_R(u) + 6\gamma_{RU}(u) + 4\gamma_U(u) - 2\gamma_\Omega(u)\right.$$
$$\left. -\frac{8}{3}\gamma_{Ric}(u) - 8\gamma_R(u) + 2\gamma_{RU}(u) - \frac{2}{3}\gamma_U(u) + \frac{2}{3}\gamma_\Omega(u)\right)$$

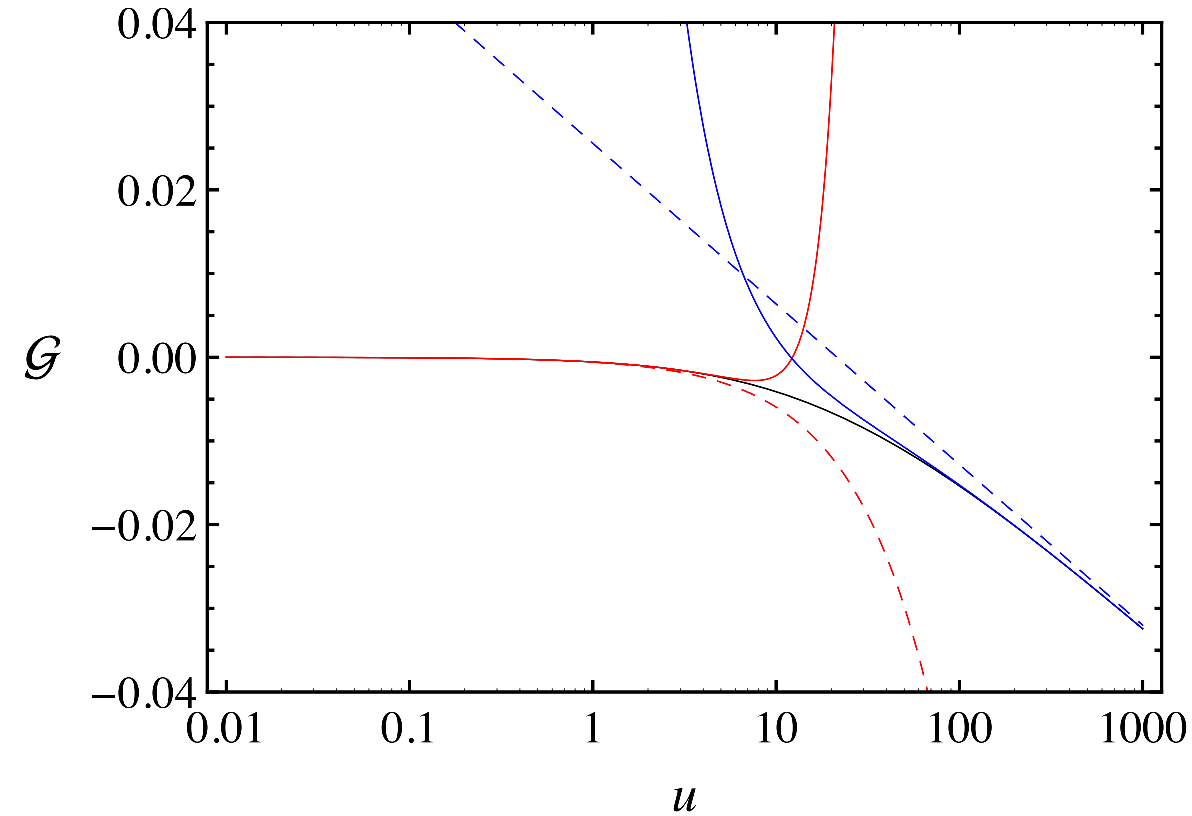
Matter contributions:

$$\mathcal{G}_0(u), \mathcal{F}_0(u), \mathcal{G}_{1/2}(u), \mathcal{F}_{1/2}(u), \mathcal{G}_1(u), \mathcal{F}_1(u)$$

LO effective action to $\mathcal{R}^2$

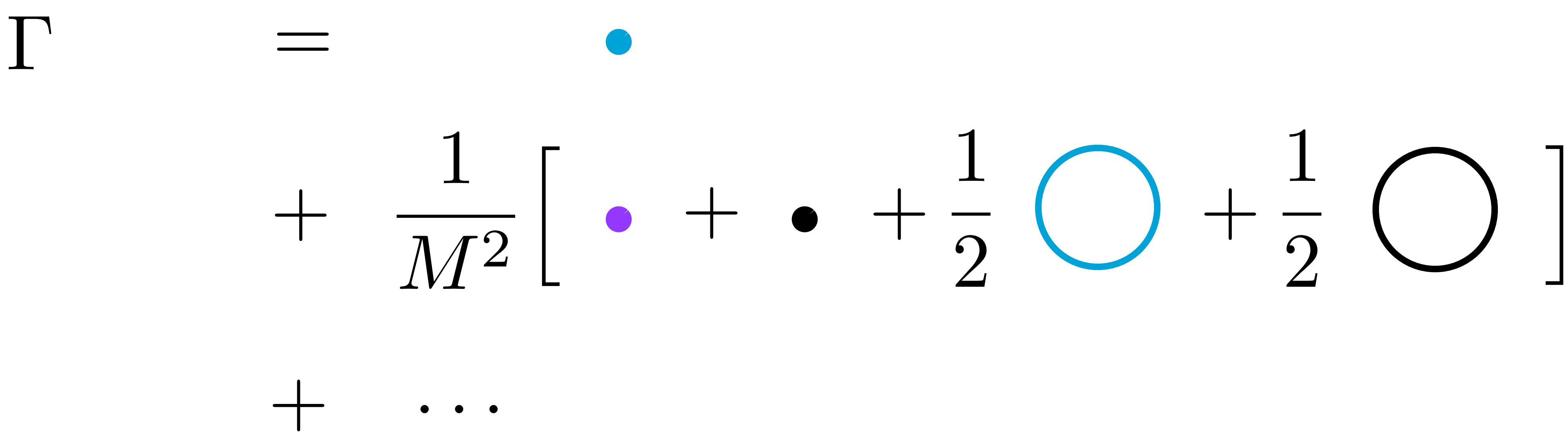
$$\Gamma[g] = \frac{1}{16\pi G}\int d^4x\sqrt{g}\,(2\Lambda - R) + \frac{1}{2\lambda}\int d^4x\sqrt{g}\,C^2 + \frac{1}{\xi}\int d^4x\sqrt{g}\,R^2$$
$$+\int d^4x\sqrt{g}\,C_{\mu\nu\alpha\beta}\,\mathcal{G}\left(\frac{-\Box}{m^2}\right)C^{\mu\nu\alpha\beta} + \int d^4x\sqrt{g}\,R\,\mathcal{F}\left(\frac{-\Box}{m^2}\right)R + O(\mathcal{R}^3)$$





$$u \equiv \frac{-\Box}{m^2}$$

Adding matter

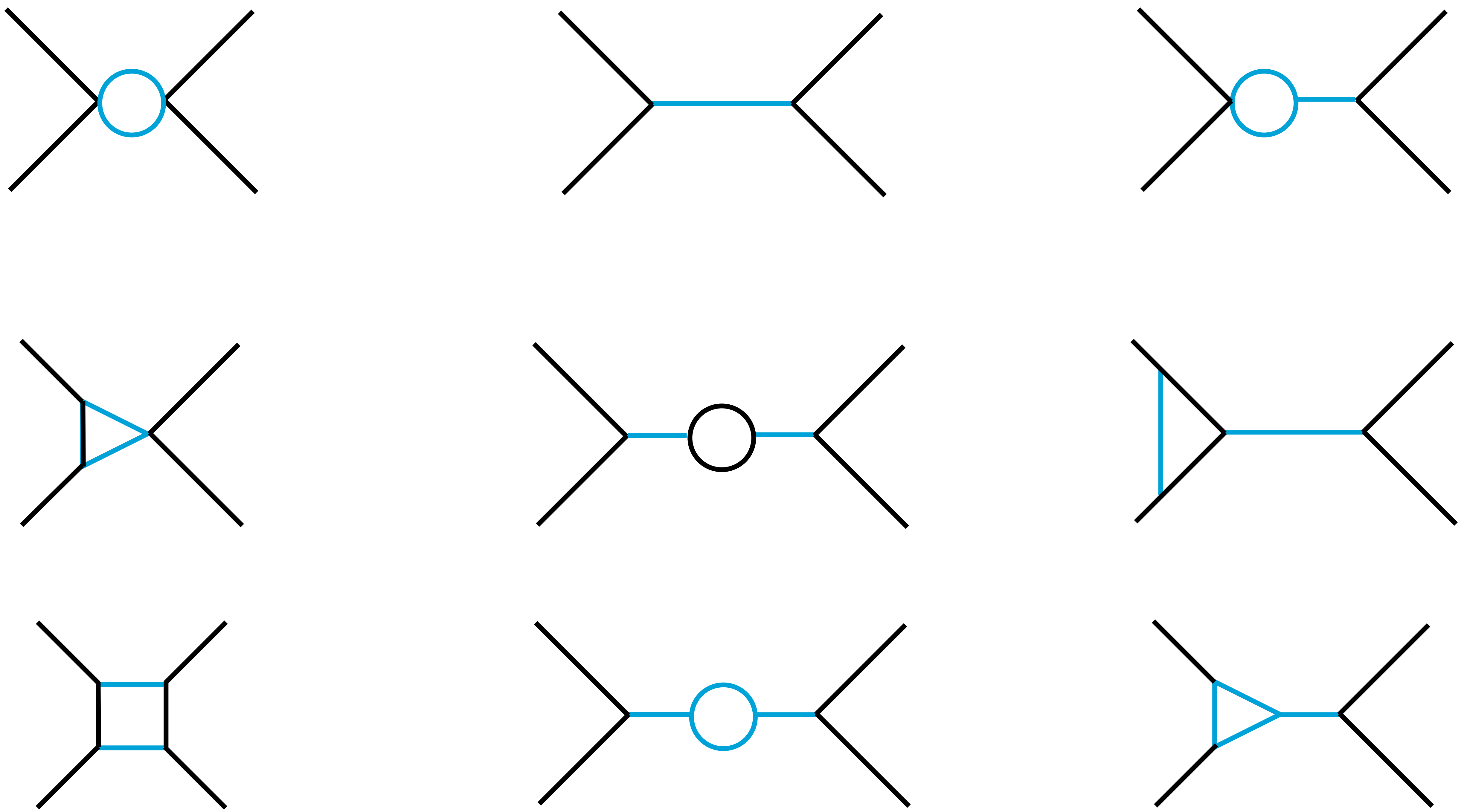


CT

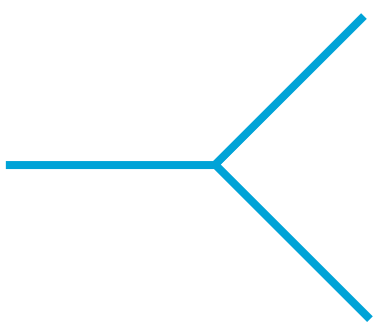
LO

NLO

Corrections to Newton's potential



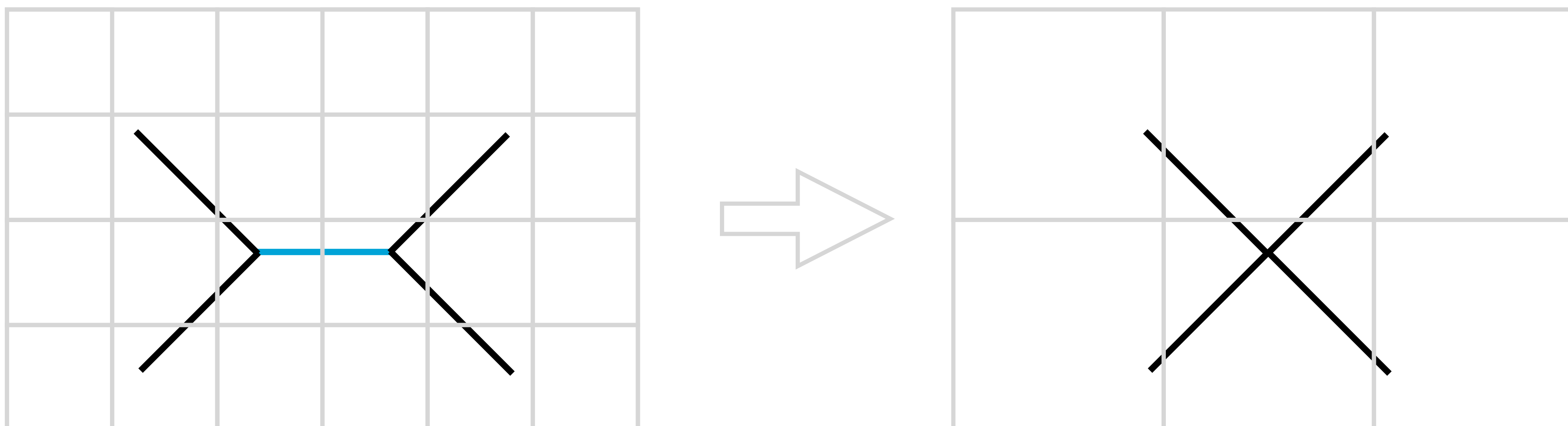
Corrections to Newton's interaction



$$
\begin{aligned}
= \; & \frac{i\kappa}{2}\Bigg(P_{\alpha\beta,\gamma\delta}\left[k^\mu k^\nu + (k-q)^\mu (k-q)^\nu + q^\mu q^\nu - \frac{3}{2}\eta^{\mu\nu}q^2\right] \\
& + 2q_\lambda q_\sigma \left[I^{\lambda\sigma,}{}_{\alpha\beta} I^{\mu\nu,}{}_{\gamma\delta} + I^{\lambda\sigma,}{}_{\gamma\delta} I^{\mu\nu,}{}_{\alpha\beta} - I^{\lambda\mu,}{}_{\alpha\beta} I^{\sigma\nu,}{}_{\gamma\delta} - I^{\sigma\nu,}{}_{\alpha\beta} I^{\lambda\mu,}{}_{\gamma\delta}\right] \\
& + \Big[q_\lambda q^\mu \left(\eta_{\alpha\beta} I^{\lambda\nu,}{}_{\gamma\delta} + \eta_{\gamma\delta} I^{\lambda\nu,}{}_{\alpha\beta}\right) + q_\lambda q^\nu \left(\eta_{\alpha\beta} I^{\lambda\mu,}{}_{\gamma\delta} + \eta_{\gamma\delta} I^{\lambda\mu,}{}_{\alpha\beta}\right) \\
& - q^2 \left(\eta_{\alpha\beta} I^{\mu\nu,}{}_{\gamma\delta} + \eta_{\gamma\delta} I^{\mu\nu,}{}_{\alpha\beta}\right) - \eta^{\mu\nu} q^\lambda q^\sigma \left(\eta_{\alpha\beta} I_{\gamma\delta,\lambda\sigma} + \eta_{\gamma\delta} I_{\alpha\beta,\lambda\sigma}\right)\Big] \\
& + \Big[2q^\lambda \Big(I^{\sigma\nu,}{}_{\alpha\beta} I_{\gamma\delta,\lambda\sigma}(k-q)^\mu + I^{\sigma\mu,}{}_{\alpha\beta} I_{\gamma\delta,\lambda\sigma}(k-q)^\nu \\
& - I^{\sigma\nu,}{}_{\gamma\delta} I_{\alpha\beta,\lambda\sigma} k^\mu - I^{\sigma\mu,}{}_{\gamma\delta} I_{\alpha\beta,\lambda\sigma} k^\nu\Big) \\
& + q^2 \left(I^{\sigma\mu,}{}_{\alpha\beta} I_{\gamma\delta,\sigma}{}^{\nu} + I_{\alpha\beta,\sigma}{}^{\nu} I^{\sigma\mu,}{}_{\alpha\delta}\right) + \eta^{\mu\nu} q^\lambda q_\sigma \left(I_{\alpha\beta,\lambda\rho} I^{\rho\sigma,}{}_{\gamma\delta} + I_{\gamma\delta,\lambda\rho} I^{\rho\sigma,}{}_{\alpha\beta}\right)\Big] \\
& + \Big\{\left(k^2 + (k-q)^2\right)\left(I^{\sigma\mu,}{}_{\alpha\beta} I_{\gamma\delta,\sigma}{}^{\nu} + I^{\sigma\nu,}{}_{\alpha\beta} I_{\gamma\delta,\sigma}{}^{\mu} - \frac{1}{2}\eta^{\mu\nu} P_{\alpha\beta,\gamma\delta}\right) \\
& - \left(k^2 \eta_{\gamma\delta} I^{\mu\nu,}{}_{\alpha\beta} + (k-q)^2 \eta_{\alpha\beta} I^{\mu\nu,}{}_{\gamma\delta}\right)\Big\}\Bigg)
\end{aligned}
$$

the truth behind Feynman diagrams...

Corrections to Newton's interaction



$$V = -\frac{GMm}{r}\left[1 + 3\frac{G(M+m)}{c^2 r} + \frac{41}{10\pi}\frac{G\hbar}{c^3 r^2} + \cdots\right]$$

Leading quantum corrections to Newton's potential

J.F. Donoghue, Phys. Rev. Lett. 72, 2996 (1994)

Cosmological effective action

FRW metric

$$ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2$$

Weyl tensor vanishes

$$C_{\alpha\beta\gamma\delta} = 0$$

Euclidean to Lorentzian

$$\Gamma[g] = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,(R - 2\Lambda) + \xi\int d^4x\sqrt{-g}\,R^2 + \int d^4x\sqrt{-g}\,RF(\Box)R$$

A predictive framework for cosmology

$$C_{\alpha\beta\gamma\delta} = 0$$

Cosmological effective action

$$\Gamma[g] = \frac{1}{16\pi G} \int d^4x \sqrt{-g}\,(R - 2\Lambda) + \xi \int d^4x \sqrt{-g}\, R^2 + \int d^4x \sqrt{-g}\, R F(\Box) R$$

$C_{\alpha\beta\gamma\delta} = 0$

Cosmological effective action

$$\Gamma[g] = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,(R - 2\Lambda) + \xi\int d^4x\sqrt{-g}\,R^2 + \int d^4x\sqrt{-g}\,RF(\Box)R$$

$$F(\Box) = \alpha\log\frac{-\Box}{m^2}$$

$$+\,\beta\frac{m^2}{-\Box}$$

$\alpha,\ \beta,\ \gamma,\ \delta$

are calculable constants depending on effective gravitons and matter content

$$+\,\gamma\frac{m^2}{-\Box}\log\frac{-\Box}{m^2}$$

$$+\,\delta\frac{m^4}{(-\Box)^2}$$

$$+\,...$$

$C_{\alpha\beta\gamma\delta} = 0$

Cosmological effective action

$$\Gamma[g] = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,(R-2\Lambda) + \xi\int d^4x\sqrt{-g}\,R^2 + \int d^4x\sqrt{-g}\,RF(\Box)R$$

$$F(\Box) = \alpha\log\frac{-\Box}{m^2}$$

Leading logs

J. F. Donoghue and B. K. El-Menoufi, Phys. Rev. D 89, 104062 (2014)

$$+\,\beta\frac{m^2}{-\Box}$$

Non-local cosmology

S. Deser and R. P. Woodard, Phys. Rev. Lett. 99, 111301 (2007)

$\alpha,\,\beta,\,\gamma,\,\delta$

are calculable constants depending on effective gravitons and matter content

$$+\,\gamma\frac{m^2}{-\Box}\log\frac{-\Box}{m^2}$$

$$+\,\delta\frac{m^4}{(-\Box)^2}$$

Non-local gravity and dark energy

M. Maggiore and M. Mancarella, Phys. Rev. D 90, 023005 (2014).

$$+\,...$$

Effective non-local cosmology

A. C. and K. J. Jain

Effective Friedmann equations (local)

Modified Einstein's equations

$$G_{\mu\nu} + \Delta G_{\mu\nu} = 8\pi G \, T_{\mu\nu}$$

Local correction (covariant form)

$$-\frac{\xi}{16\pi G}\Delta G_{\mu\nu}^{R^2} = 2\left(R_{\mu\nu} - \frac{1}{4}g_{\mu\nu}R\right)R - 2\left(\nabla_\mu\nabla_\nu - g_{\mu\nu}\Box\right)R$$

Local correction (FRW form)

$$H^2 + 96\pi\xi G\left[2H\ddot{H} + 6H^2\dot{H} - \dot{H}^2\right] = \frac{1}{3}\Lambda + \frac{8\pi G}{3}\rho$$

Effective Friedmann equations (local)

Jordan frame Starobinsky gravity

$$R \qquad\qquad R^2 \qquad\qquad \Lambda \qquad \rho$$

$$H^2 + 96\pi\xi G\left[2H\ddot{H} + 6H^2\dot{H} - \dot{H}^2\right] = \frac{1}{3}\Lambda + \frac{8\pi G}{3}\rho$$

Pure Starobinsky gravity is exactly solvable

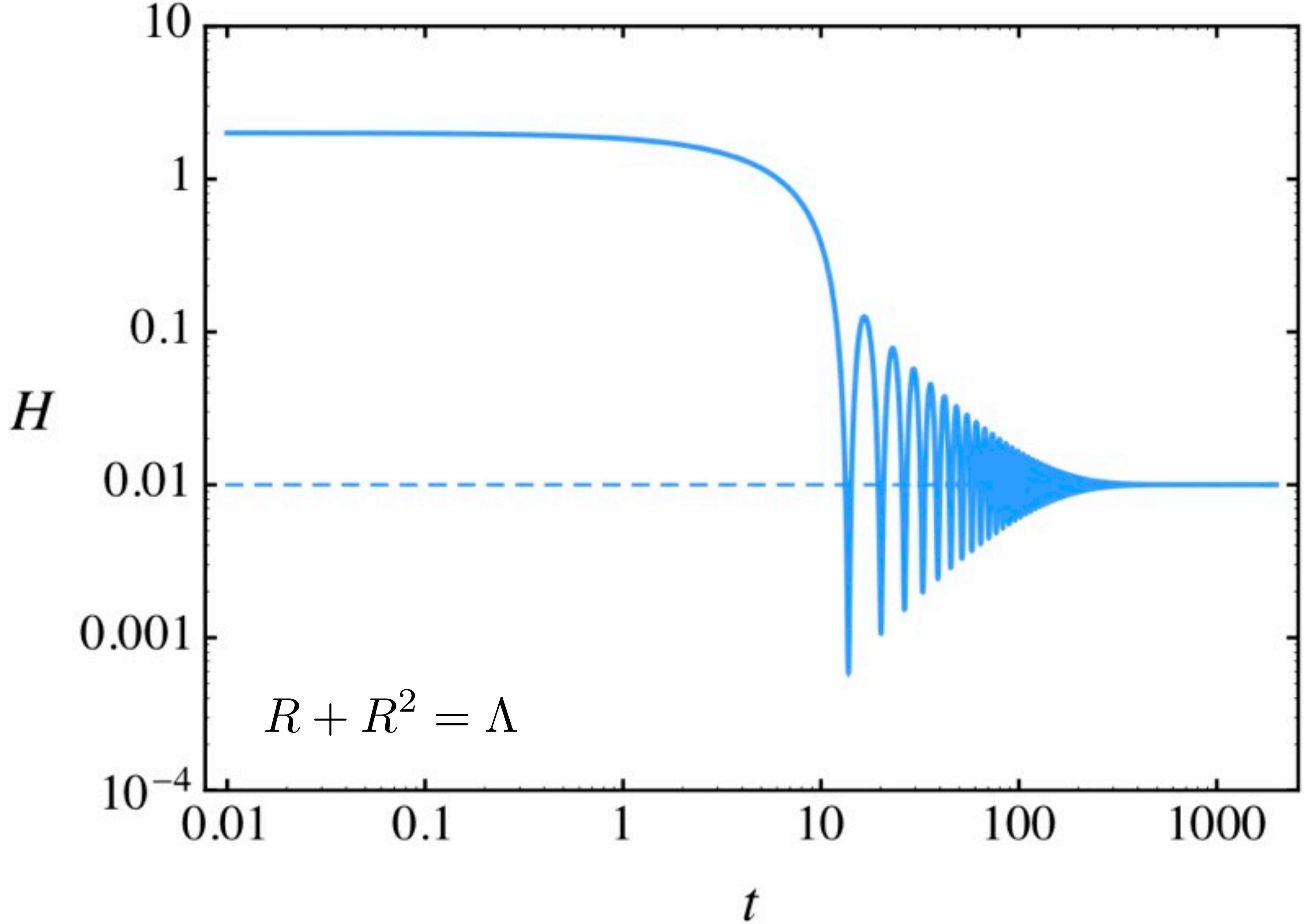
$$2H\ddot{H} + 6H^2\dot{H} - \dot{H}^2 = 0$$

$$H = 0$$ inflation

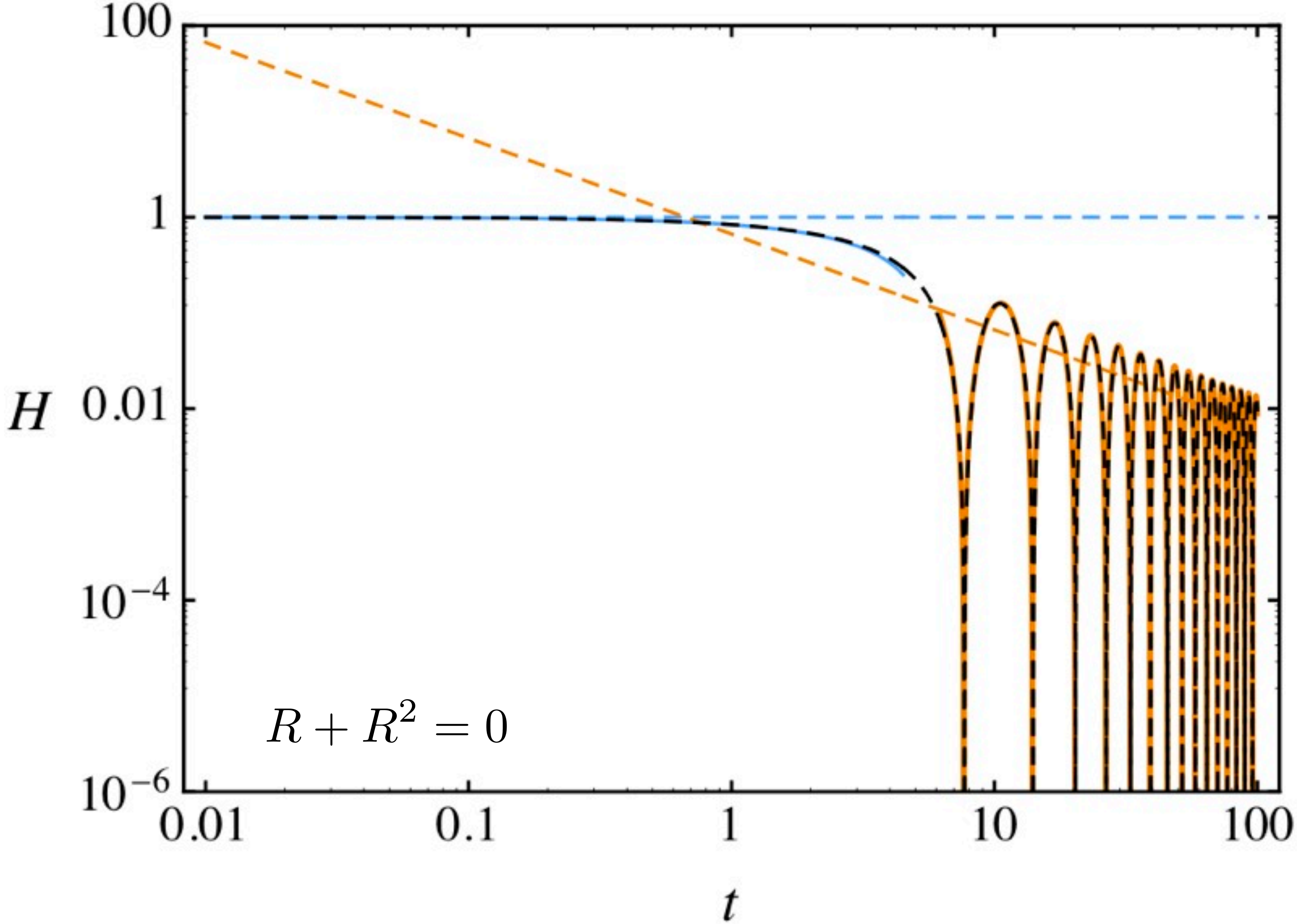
$$t + C_2 = \int \frac{dH}{C_1\sqrt{H} - 2H^2}$$

$$H = \frac{1}{2t}$$ radiation

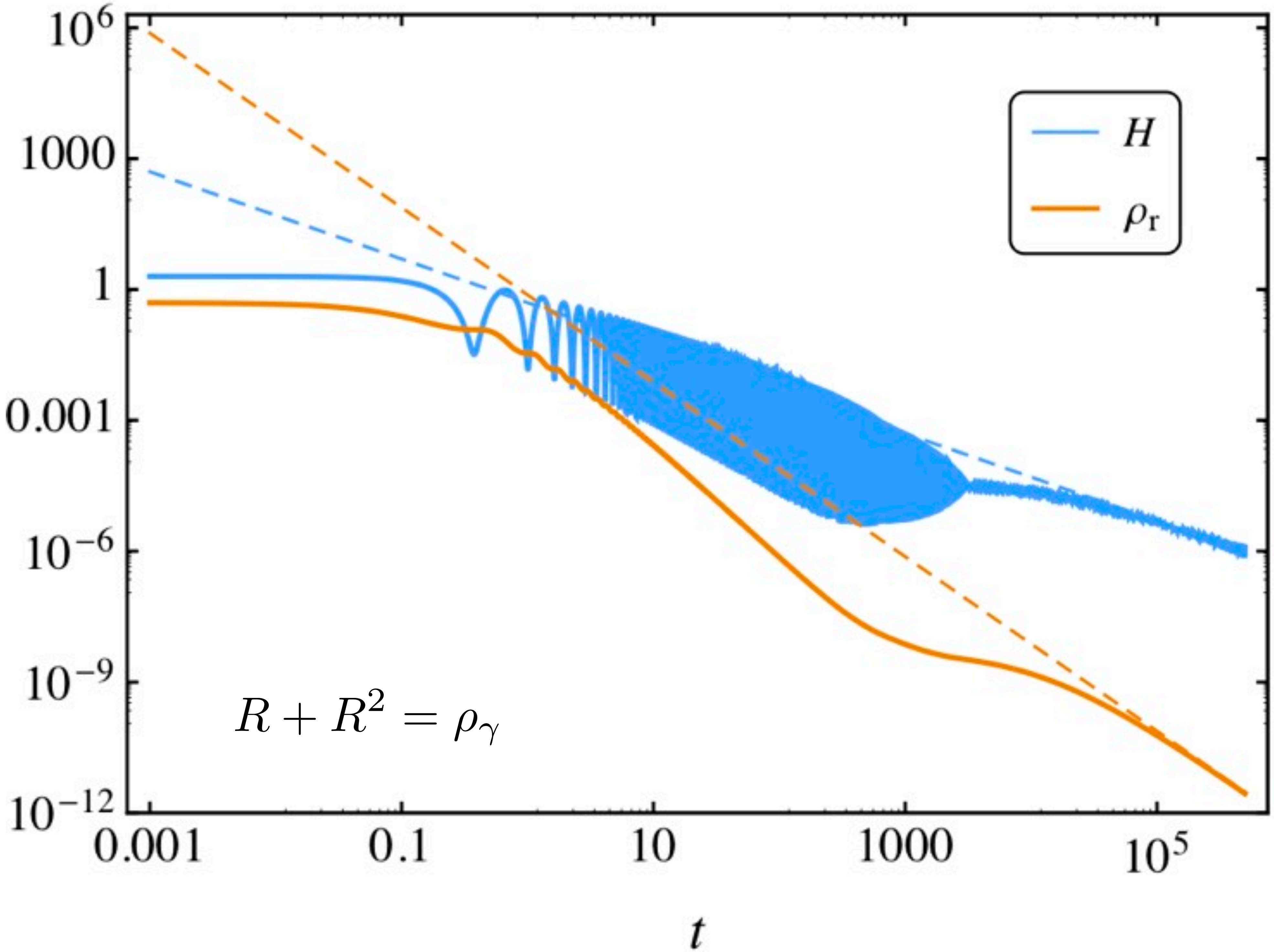
Early times: Inflation



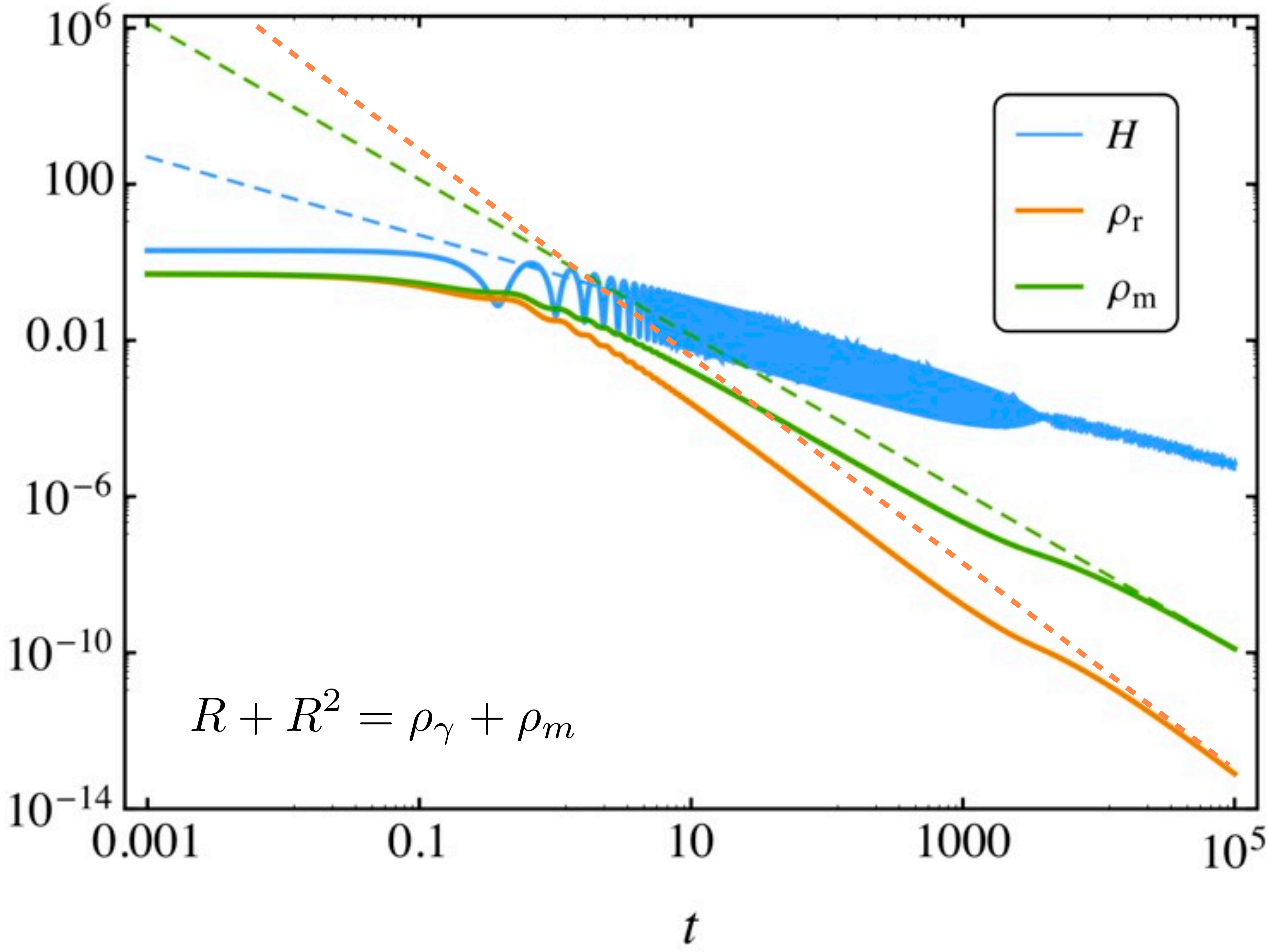
Early times: Inflation



Early times: Inflation



Early times: Inflation



Effective Friedmann equations (non-local)

Modified Einstein's equations

$$G_{\mu\nu} + \Delta G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

Non local correction (covariant form)

$$\frac{1}{16\pi G\beta m^2}\Delta G_{\mu\nu}^{R\frac{1}{-\Box}R} = -2G_{\mu\nu}U + 2g_{\mu\nu}R + 2\nabla_\mu\nabla_\nu U + \nabla_\mu U\nabla_\nu U - \frac{1}{2}g_{\mu\nu}\nabla^\alpha U\nabla_\alpha U$$

$$-\Box U = R$$

Non local correction (FRW form)

$$H^2 - 16\pi G\beta m^2\left(\frac{1}{6}\dot{U}^2 - 2H\dot{U} - 2H^2 U\right) = \frac{8\pi G}{3}\rho$$

$$\ddot{U} + 3H\dot{U} = 6\left(2H^2 + \dot{H}\right)$$

Effective Friedmann equations (non-local)

$$\frac{1}{16\pi G \delta m^4}\Delta G_{\mu\nu}^{R\frac{1}{\Box^2}R} = -2G_{\mu\nu}S + 2g_{\mu\nu}U + 2\nabla_\mu\nabla_\nu S + 2\nabla_\mu U\nabla_\nu S - g_{\mu\nu}\nabla^\alpha U\nabla_\alpha S + \frac{1}{2}g_{\mu\nu}U^2$$

$$-\Box U = R$$

$$-\Box S = U$$

$$H^2 - 32\pi G\delta m^4\left(2H^2 S + H\dot{S} + \frac{1}{2}\dot{H}S - \frac{1}{6}\dot{U}\dot{S}\right) = \frac{8\pi G}{3}\rho$$

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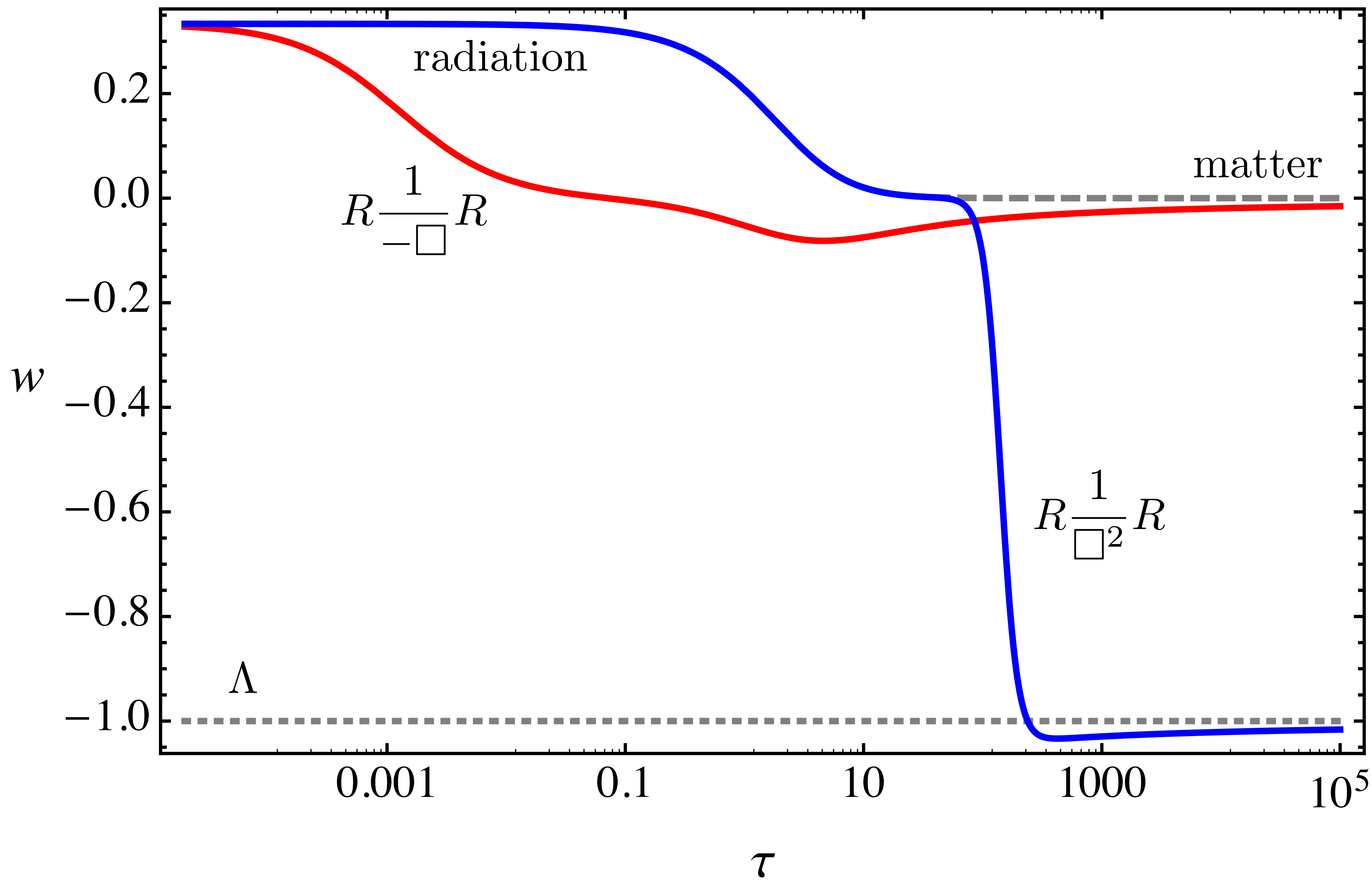
$$\ddot{S} + 3H\dot{S} = U$$

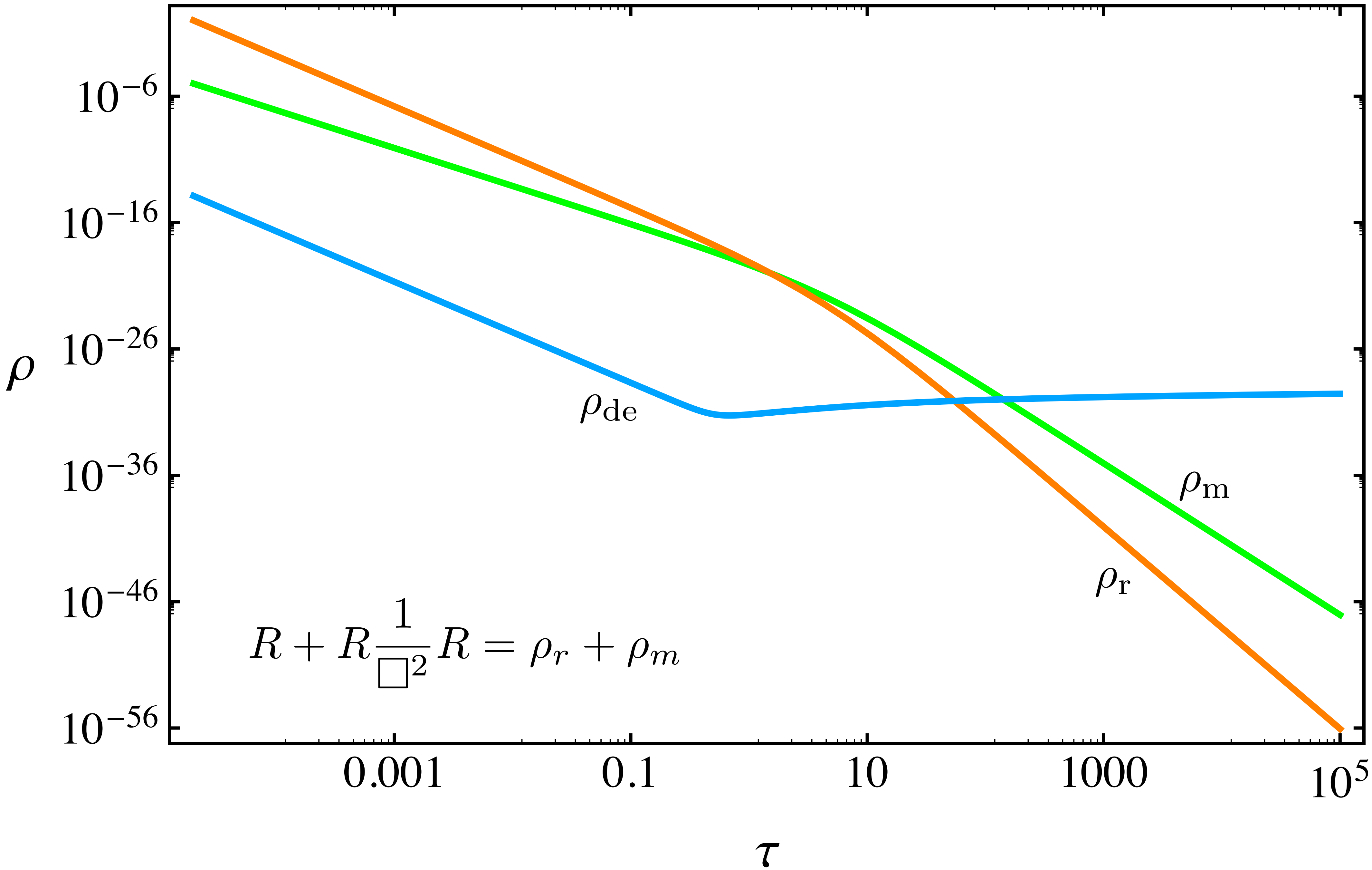
$$H^2 = \frac{8\pi G}{3}(\rho + \rho_{DE})$$

Late times: Dark energy



$$\dot{\rho}_{\rm tot} + 3H(1+w)\rho_{\rm tot} = 0 \qquad\qquad w = -1 - \frac{1}{3H}\frac{\dot{\rho}_{\rm tot}}{\rho_{\rm tot}}$$

Late times: Dark energy



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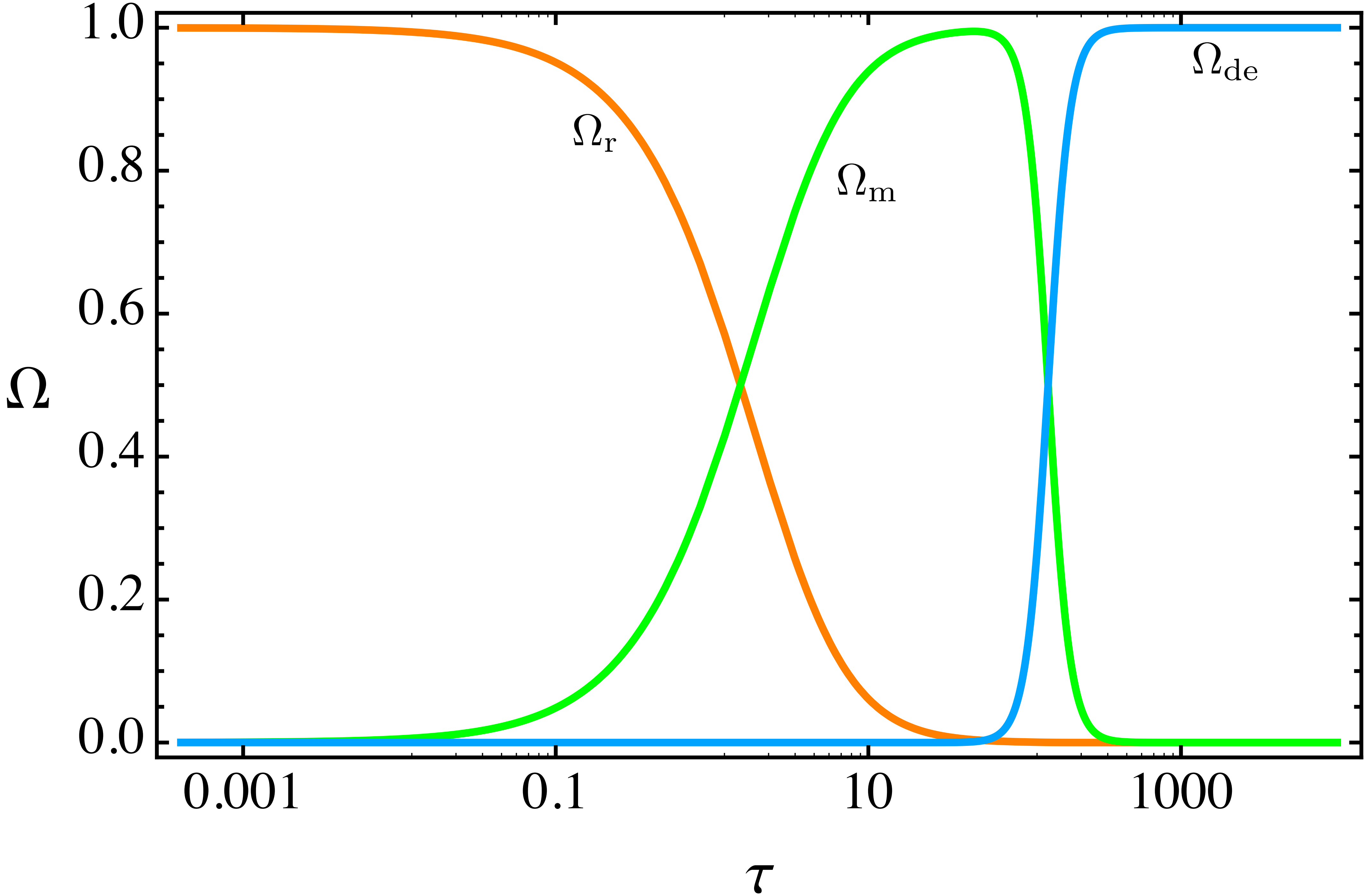

Conclusions and Outlook

- Consistent cosmological history
- Compute all LO terms
- The role of the Conformal anomaly
- Apply to stars/black holes
- Connection with high energy quantum gravity
- Falsify!

Thank you