Limitations of cosmography in extended theories of gravity

Álvaro de la Cruz-Dombriz

Astronomy, Cosmology and Gravitation Centre, ACGC - UNIVERSITY of CAPE TOWN

Collaborators: V. C. Busti, P. K. S. Dunsby, O. Luongo, L. Reverberi and D. Sáez-Gómez

Outline

I. Some rudiments in Cosmography

II. Extended theories of gravity: state-of-the art

III. Limitations of cosmographic approach in extended theories

- Biased results. Spotting ΛCDM ?
- Ruling out and reconstructing higher-order theories

IV. Prospects and Conclusions

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IV. Conclusions and Prospects

✓ Cosmography rudiments (I)

- In order to test GR and the Copernican Principle, a useful tool is to use frameworks able to encompass a large class of models/theories

- Such model independent methods – instead of a case-by-case approach – have been used to infer the **Dark Energy EoS** and reconstruct classes of DE theories

- Cosmography approach just relies on the **Copernican principle** and the expression of the FLRW **scale factor in terms of an auxiliary variable**, such as redshift(s), time, etc.

$$H(z) = \frac{\dot{a}}{a} = H_0 + H_{0z}(z - z_0) + \frac{H_{0zz}}{2}(z - z_0)^2 + \ldots$$

$$q = -\frac{\ddot{a}}{aH^2}\,,\; j = \frac{a^{(3)}}{aH^3}\,,\; s = \frac{a^{(4)}}{aH^4}\,,\; l = \frac{a^{(5)}}{aH^5}\,, \ldots$$

S. Weinberg, *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity*
E. R. Harrison, Nature, 260, 591 (1976).

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or $\quad y = \dfrac{z}{1+z}$ as alternative independent variable [or Padé polynomials, etc.]

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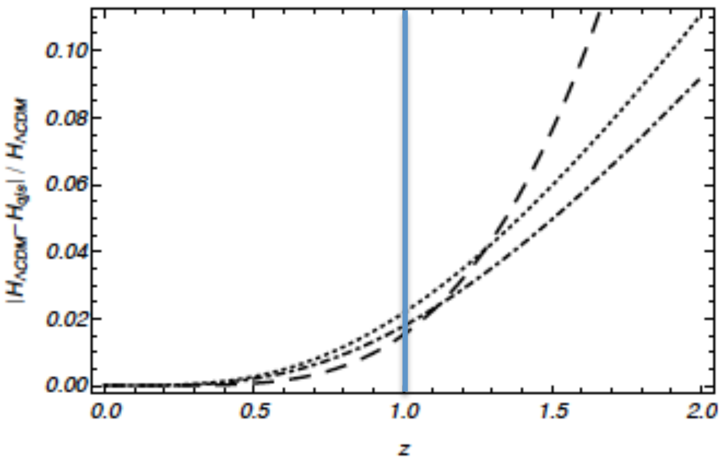
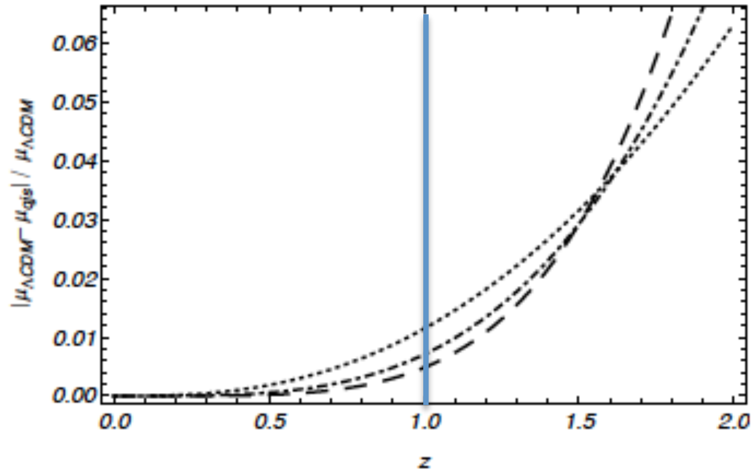
ΛCDM model





S. Capozziello, R. Lazkoz and V. Salzano,
Phys. Rev. D 84 124061 (2011), arXiv:1104.3096 [atro-ph CO]

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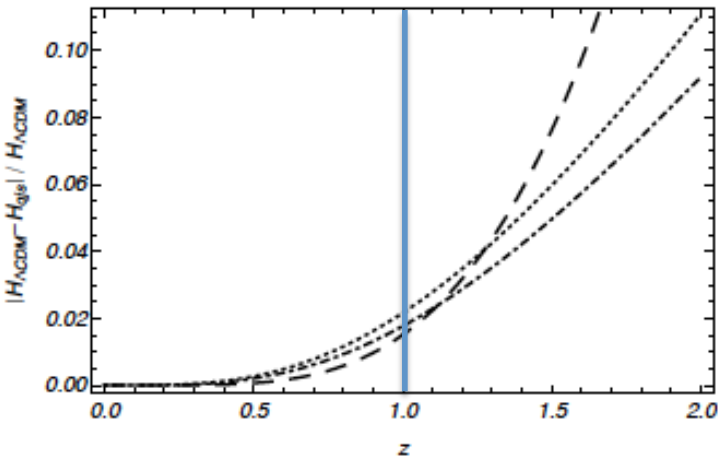
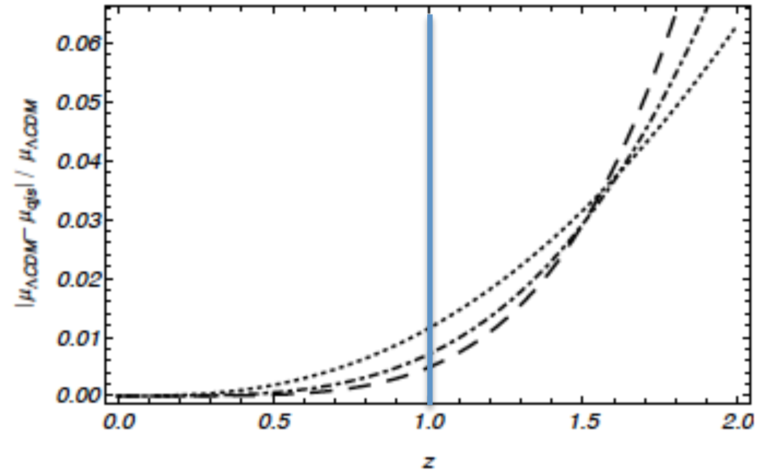
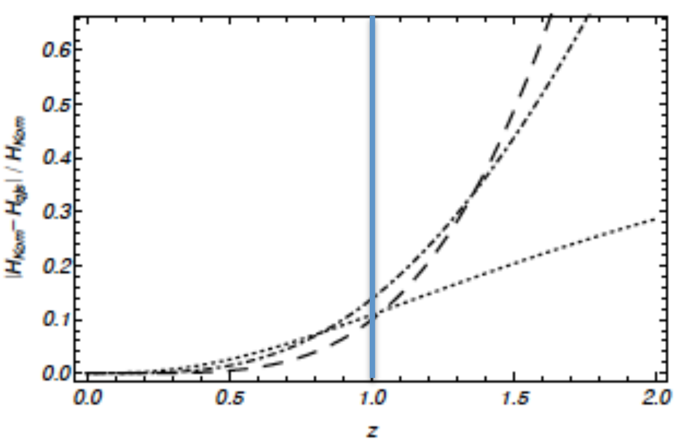


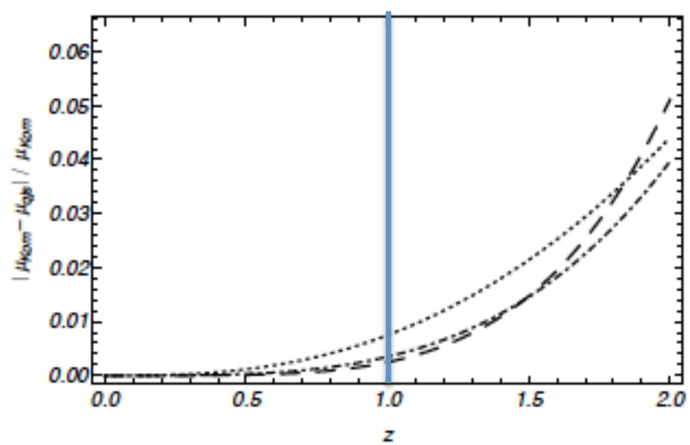


CPL model

$$w(z) = w_0 + w_1 \frac{z}{1+z} \, .$$

S. Capozziello, R. Lazkoz and V. Salzano, Phys. Rev. D 84 124061 (2011), arXiv:1104.3096 [atro-ph CO]





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Extended theories of gravity: a motivation

- The search for a fully consistent **Quantum Gravity theory** is a very active field of research. GR is consistent *if treated* in the frame of quantum effective field theories but it breaks down at Planck scale

J.F. Donoghue and T. Torma, **gr-qc/9405057**

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✓ **Extended theories of gravity**

- must Emulate certain – gravitational – aspects of General Relativity
- must Explain the cosmological evolution in different eras
- are motivated by the cosmological constant (Λ) problem, dark energy, dark matter, singularities…

✓ **Proposals**

- Scalar/Vector-Tensor gravity: **Brans-Dicke theories**, ***f*(*R*) theories,** Horndeski
- Extra dimensions theories: Brane-world theory, String theory
- Massive gravity
- Born-Infeld inspired gravity
- …

The degeneracy problem

✓ *<u>Def.:</u>* several extended gravity theories lead to identical results with either **General Relativity (GR)** or the **Concordance (ΛCDM) Model**

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✓ Consistency tests

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E.g. $$\mathcal{A}_{f(R)} = \frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\,(f(R) + 2\mathcal{L}_m)$$

AdlCD and A. Dobado,
Phys. Rev. D74: 087501, 2006

f(R) model with Robertson-Walker solution the same as ΛCDM solution for dust + Λ.

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[UCT Cosmology group]

- Evolution of geodesics and Raychaudhuri equation
- Importance of averaging and backreaction mechanism
- Evolution of scalar perturbations →
- Black holes properties and thermodynamics
- …
- Dark matter: astrophysical fluxes and (in)direct detection experiments →

[Madrid UCM-Th]

- Model/theory independent tests – fit with data catalogues

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✓ Differences between auxiliary variables: y vs. z

- Mock data generated from a fiducial flat ΛCDM model with redshift distribution Union2.1 catalogue and $\sigma_\mu = 0.15$

$$\Omega_m = 0.3$$
$$H_0 = 73.8 \text{ km s}^{-1} \text{ Mpc}^{-1}$$

- Two sets of parameters and 100 simulations

$$\boldsymbol{\theta_1} = \{H_0, q_0, j_0, s_0\} \qquad \boldsymbol{\theta_2} = \{H_0, q_0, j_0, s_0, l_0\}$$

- How frequent the true cosmographic values fall in 1, 2, 3σ confidence regions

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	$\boldsymbol{\theta}_1$						$\boldsymbol{\theta}_2$					
	y			z			y			z		
	1σ	2σ	3σ	1σ	2σ	3σ	1σ	2σ	3σ	1σ	2σ	3σ
q_0	26	32	42	67	27	6	82	12	6	82	18	0
j_0	10	45	45	64	29	7	93	5	2	88	12	0
s_0	10	67	23	83	15	2	92	7	1	93	6	1
l_0	-	-	-	-	-	-	100	0	0	100	0	0

$$y = \frac{z}{1+z}$$

V. Busti, AdlCD, P. Dunsby, D. Sáez-Gómez
arXiv:1505.5503 [astro-ph.CO], PRD 92 123512 (2015)

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$$y = \frac{z}{1+z}$$

Overestimation of errors

Biased results

Well behaved

V. Busti, AdlCD, P. Dunsby, D. Sáez-Gómez
arXiv:1505.5503 [astro-ph.CO], PRD 92 123512 (2015)

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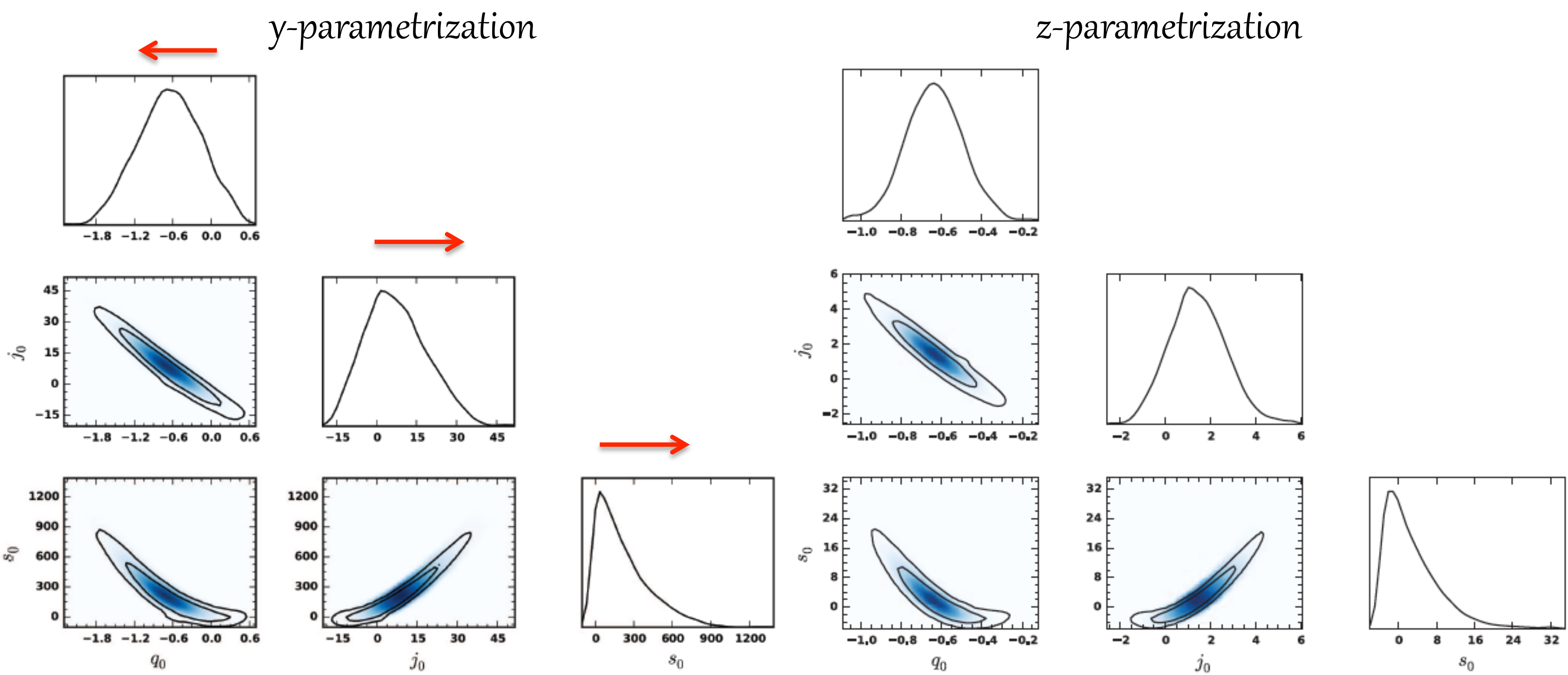


One simulation, $\theta_1 = \{H_0, q_0, j_0, s_0\}$

✓ For ΛCDM $\boxed{j_0 = 1}$

V. Busti, AdlCD, P. Dunsby, D. Sáez-Gómez
arXiv:1505.5503 [astro-ph.CO], PRD 92 123512 (2015)

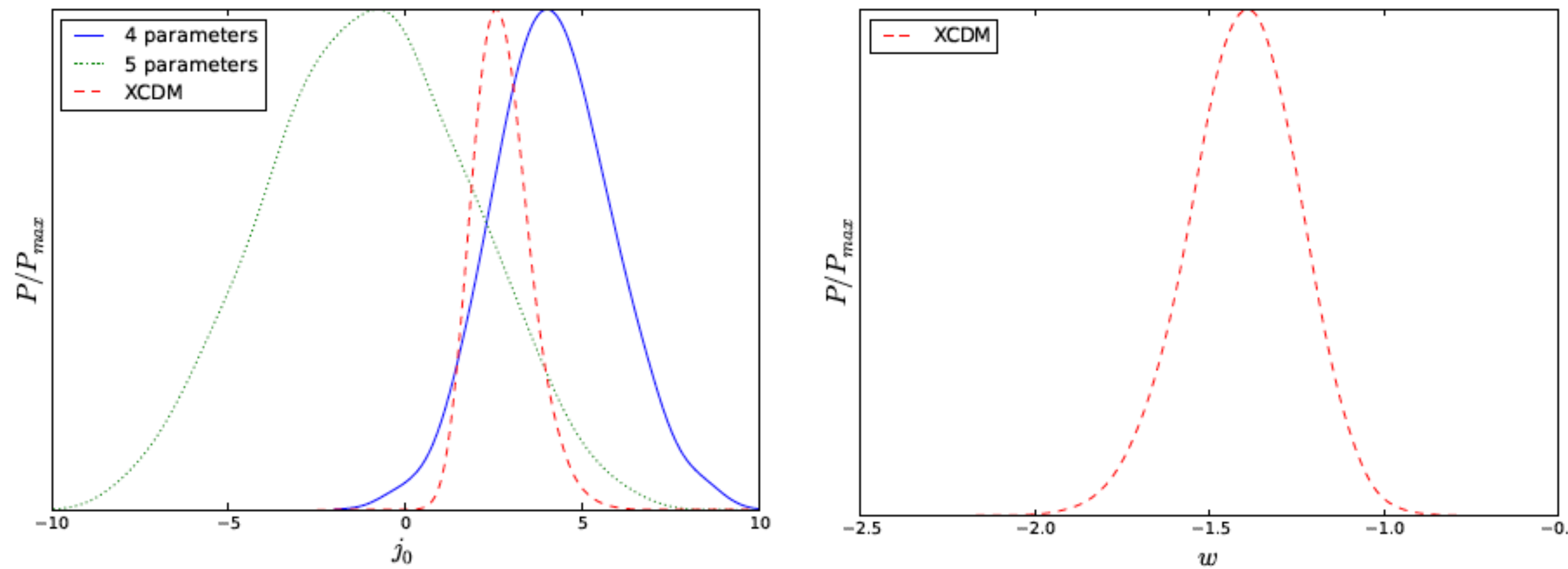
✓ Is Cosmography able to spot the correct *X*CDM model?

- Mock realizations of data for a flat *X*CDM $\Omega_m = 0.3 \quad w = -1.3 \quad j_0 = 1.945$

- Constraints for $\boldsymbol{\theta}_1$ (fourth order) , $\boldsymbol{\theta}_2$ (fifth order) and direct constraint of parameters

V. Busti, AdlCD, P. Dunsby, D. Sáez-Gómez

arXiv:1505.5503 [astro-ph.CO] PRD 92 123512 (2015)



Fitting to the model spots deviations from ΛCDM with less effort

Some evidence of $j_0 \neq 1$ when considering $\boldsymbol{\theta}_1$, but dissapears assuming $\boldsymbol{\theta}_2$!!!

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- Cosmography as a tool to reconstruct DE models

Capozziello *et al.*, Bamba *et al*. Astrophys. Space Sci. 342, 155 (2012)

- Nonetheless in theories with higher derivatives, the appearance of extra parameters apart from the cosmographic ones, imposes some limitations in the method

E.g. 1: K-essence

$$\mathcal{S} = \int \mathrm{d}x^4 \sqrt{-g} \left[\frac{1}{2}R - \frac{1}{2}\omega(\phi)\partial_\mu\phi\partial^\mu\phi - V(\phi) + \mathcal{L}_m \right]$$

when scalar potential and kinetic term expanded around z=0 and expressed in terms of the cosmographic parameters

$$\frac{V_0}{H_0^2} = 2 - q_0 - \frac{3\Omega_m}{2} \, ,$$
$$\frac{V_{z0}}{H_0^2} = 4 + 3q_0 - j_0 - \frac{9\Omega_m}{2} \, ,$$
$$\frac{V_{2z0}}{H_0^2} = 4 + 8q_0 + j_0(4+q_0) + s_0 - 9\Omega_m$$
$$\frac{V_{3z0}}{H_0^2} = j_0^2 - l_0 - q_0 j_0(7+3q_0) - s_0(7+3q_0) - 9\Omega_m$$

$$\omega_0 = 2(1+q_0) - 3\Omega_m \, ,$$
$$\omega_{z0} = -4 - 4q_0^2 + 6q_0(\Omega_m - 1) + 2j_0 + 3\Omega_m \, ,$$
$$\omega_{2z0} = 12 - 8q_0\left[-3 - 4q_0 - 2q_0^2 + 3(1+q_0)\Omega_m\right] - 2j_0(8 + 7q_0 - 3\Omega_m) - 2s_0 - 6\Omega_m$$
$$\omega_{3z0} = -14j_0^2 + 2l_0 - 24q_0\left[5 + 10q_0 + 2q_0^2(5+2q_0)\right] + 22q_0 s_0 + 2j_0\left[60 + q_0(105 + 59q_0 - 39\Omega_m) - 27\Omega_m\right] + 6\Omega_m\left[6q_0(3 + 6q_0 + 2q_0^2) - s_0\right] + 6(-8 + 5s_0 + 3\Omega_m) \, .$$

V. Busti, AdlCD, P. Dunsby, D. Sáez-Gómez
arXiv:1505.5503 [astro-ph.CO], PRD 92 123512 (2015)

✓ It requires assumption on the model today $\Omega_m \approx 2/3(1+q_0)$

✓ Thus, a one-to-one correspondence between cosmographic parameters and the model emerges

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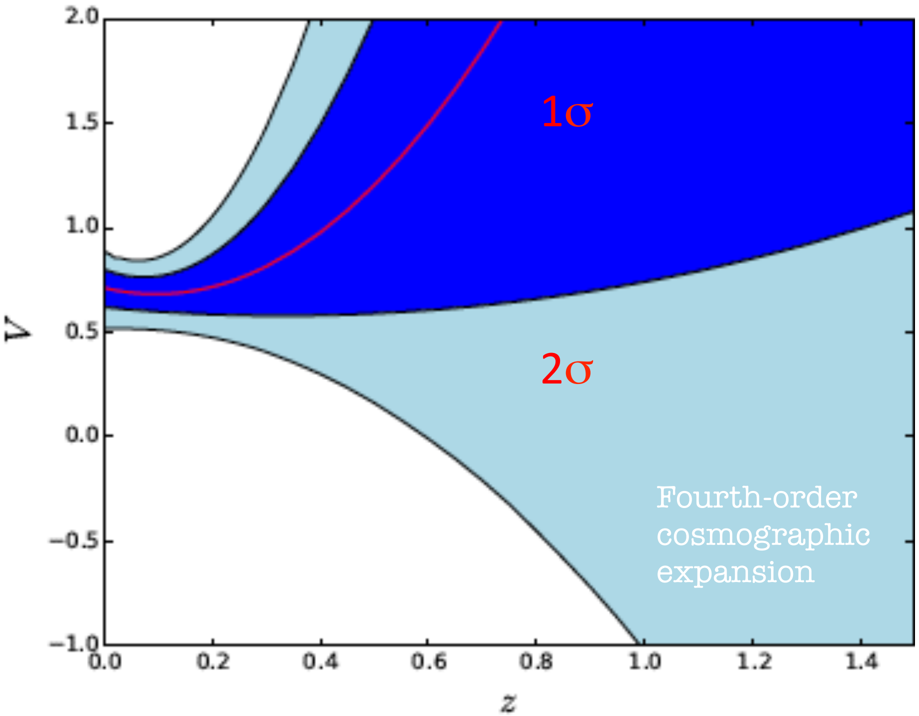
✓ Generic realization of ΛCDM



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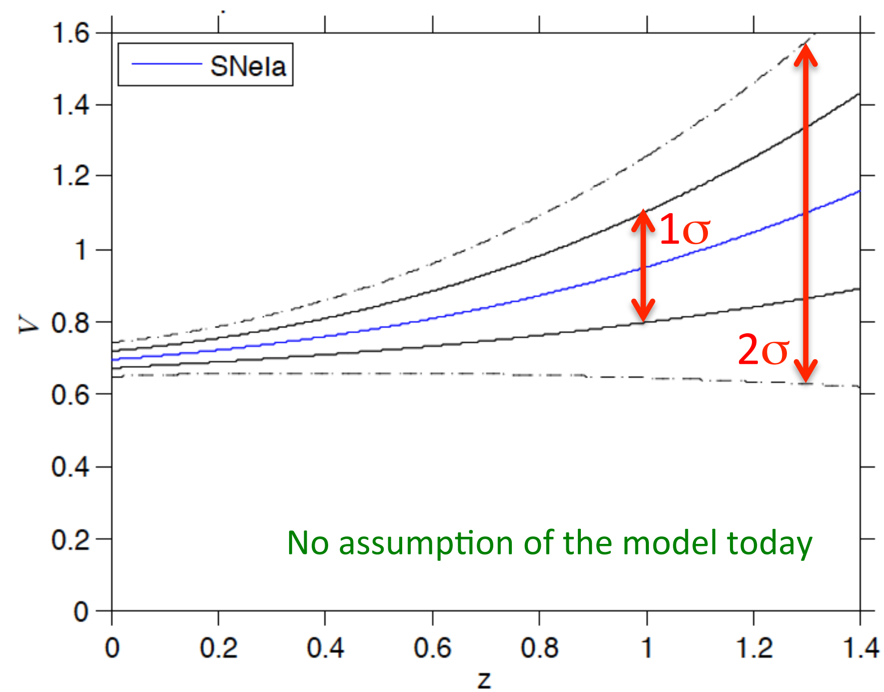
✓ Gaussian processes



R. Nair, S. Jhingan and D. Jain, JCAP 01 (2014) 005 [arXiv:1306.0606]

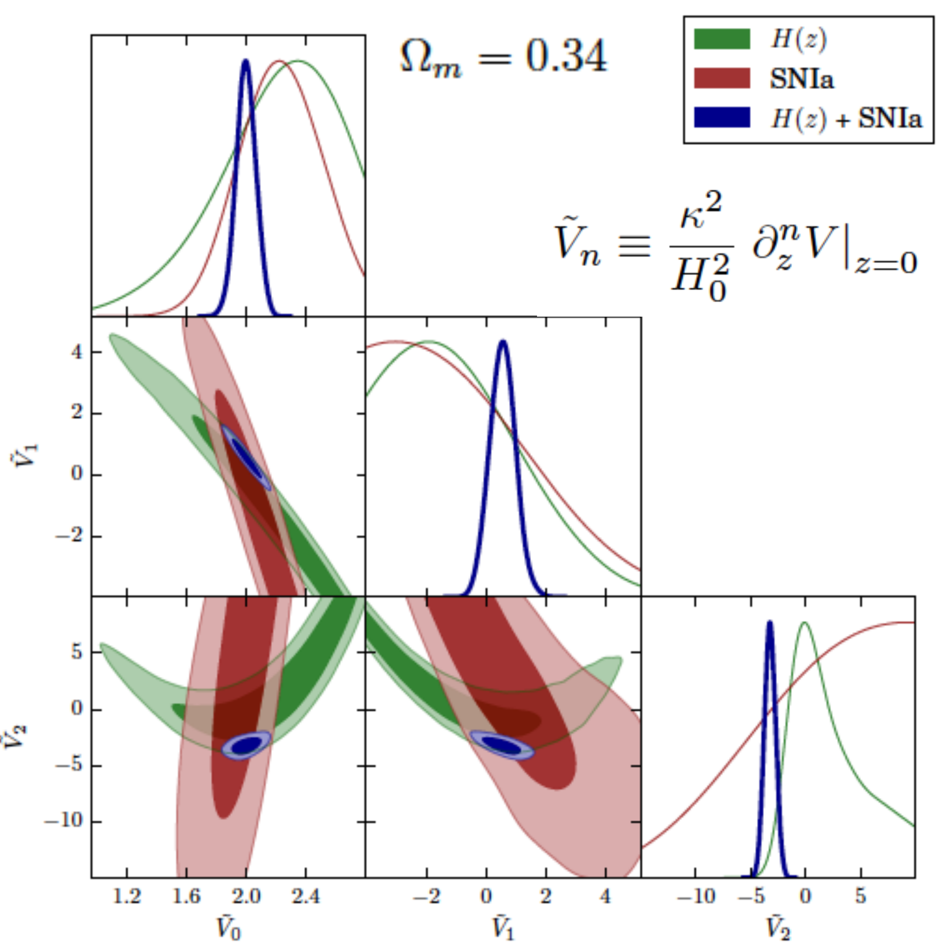
E.g. 1: K-essence

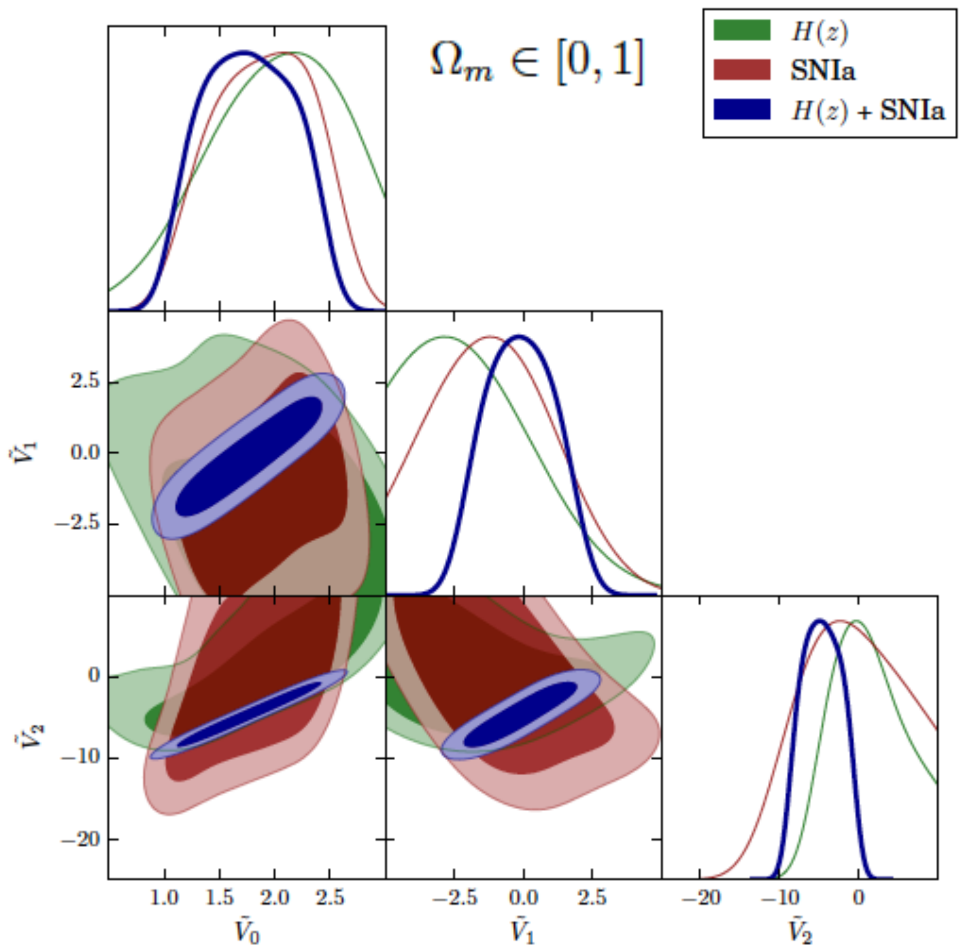
SNIa Union 2.1 data and/or H(z) data

O. Farooq and B. Ratra,
Astroph. J. Lett. 776:L7 (2013) arXiv:1301.5243

PRELIMINARY

(Calculations by L. Reverberi, ACGC – Cape Town)





E.g. 2: $f(R)$ theories

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{1}{2}f(R) + \mathcal{L}_m\right]$$

Higher-order derivatives theories

$$\frac{f_0}{6H_0^2} = -\alpha q_0 + \Omega_m + 6\beta(2 + q_0 - j_0) \ ,$$

$$\frac{f_{z0}}{6H_0^2} = \alpha(2 + q_0 - j_0) \ ,$$

$$\frac{f_{2z0}}{6H_0^2} = 6\beta(2 + q_0 - j_0)^2 + \alpha[2 + 4q_0 + (2 + q_0)j_0 + s_0]$$

$$\left.\frac{\partial f}{\partial R}\right|_{R=R_0} = \alpha, \quad \left.\frac{\partial^2 f}{\partial R^2}\right|_{R=R_0} = \frac{\beta}{H_0^2}$$

Two extra parameters

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Two extra parameters

✓ $f(R)$-derivatives ⟺ cosmographic parameters one-to-one correspondence must be abandoned

✓ Sensible priors for α and β are required

$$\alpha = \left.\frac{\partial f}{\partial R}\right|_{R=R_0} = 1 \qquad \beta = \left.\frac{\partial^2 f}{\partial R^2}\right|_{R=R_0} = 0$$

A. Aviles *et al.*, Phys. Rev. D 87, no. 4, 044012 (2013)
Phys. Rev. D 87, no. 6, 064025 (2013)

However, whenever these values are assumed, an instability occurs

L. Pogosian and A. Silvestri, Phys. Rev. D 77, 023503 (2008)
Phys. Rev. D 81, 049901 (2010)

✓ Cosmological values $\alpha \neq 1$ and $\beta \neq 0$ may still produce viable cosmological models

E.g. 2: *f*(R) theories

$$f(R) = R + aR^2 + bR^3 \qquad \alpha = 2.81 \quad \beta = 0.06$$

✓ Mock data generated from the given *f*(R) model [exact background evolution]

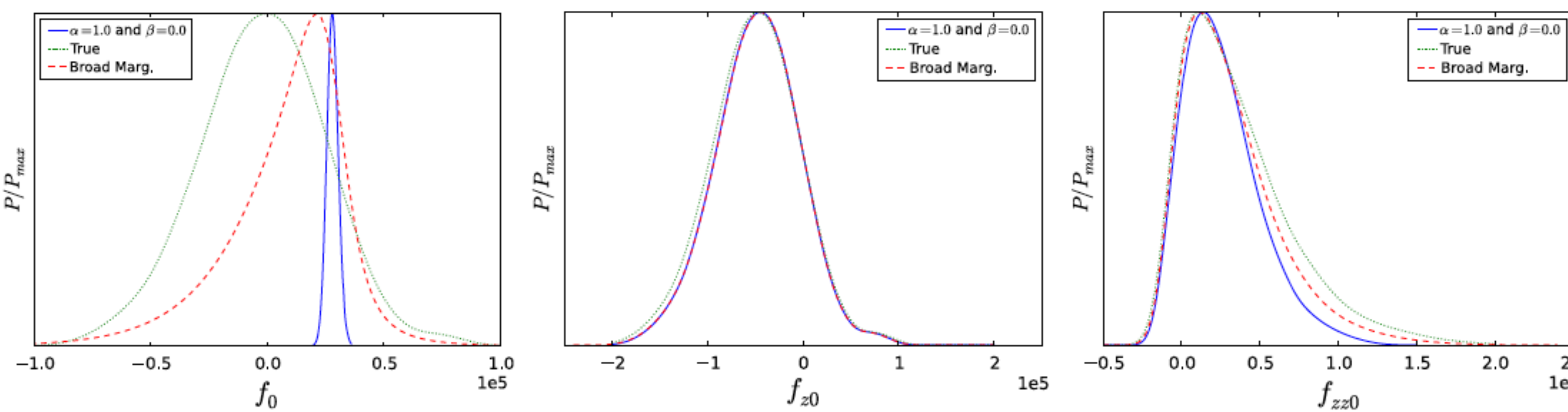
✓ Simulations with three different hypotheses:

True values of $\{\alpha, \beta\}$, $\{\alpha = 1, \beta = 0\}$, broad marginalization **(**$\alpha \sim N(1, 0.05)$ and $\beta \sim N(0.07, 0.05)$**)**

$\{f_0, f_{z0}, f_{zz0}\}$

V. Busti, AdlCD, P. Dunsby, D. Sáez-Gómez

arXiv:1505.5503 [astro-ph.CO], PRD 92 123512 (2015)



- The probability of f_0 is highly dependent on the choice of $\{\alpha, \beta\}$ which may even lead to rule out the true values
- The errors are very large for every case leading to a completely degenerated fit, such that a wide range of completely different and viable *f(R)* models - e.g., W. Hu & I. Sawicki PRD 76 064004 - lie in the 1σ region

E.g. 2: *f*(*R*) theories

SNIa Union 2.1 data + H(z) data

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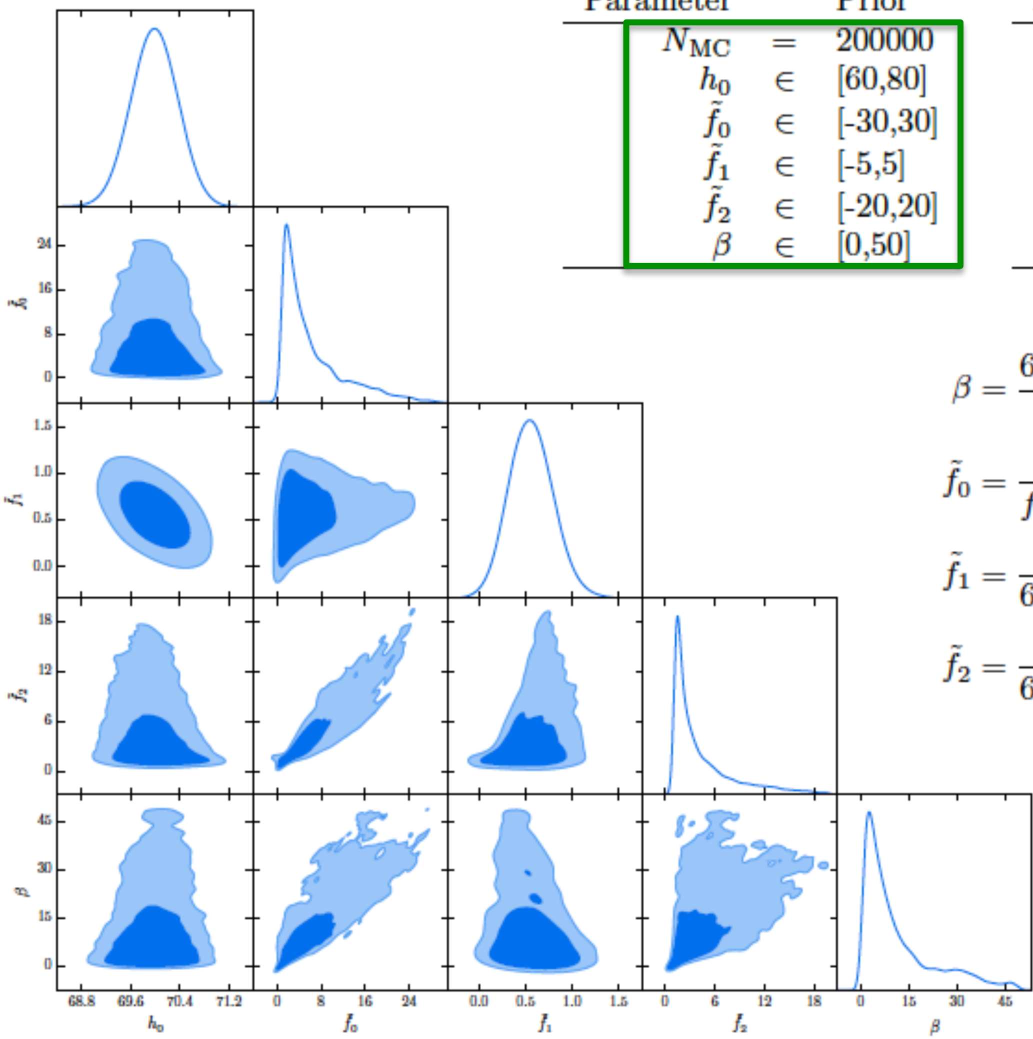

Parameter		Prior
$N_{\rm MC}$	$=$	200000
h_0	$\in$	[60,80]
$\tilde{f}_0$	$\in$	[-30,30]
$\tilde{f}_1$	$\in$	[-5,5]
$\tilde{f}_2$	$\in$	[-20,20]
β	$\in$	[0,50]

Parameter	Best fit
h_0	69.9616
$\tilde{f}_0$	3.55976
$\tilde{f}_1$	0.618366
$\tilde{f}_2$	2.62293
β	4.84324
χ^2	580.23

Parameter	95% limits
h_0	$69.98^{+0.74}_{-0.75}$
$\tilde{f}_0$	$6.3^{+13}_{-6.4}$
$\tilde{f}_1$	$0.56^{+0.50}_{-0.45}$
$\tilde{f}_2$	$4.3^{+8.9}_{-3.8}$
β	11^{+25}_{-12}

$$\beta = \frac{6H_0^2 f_{RR,0}}{f_{R,0}}$$

$$\tilde{f}_0 = \frac{1}{f_{R,0}}\left[\frac{f_0}{6H_0^2} - \Omega_{m,0}\right] = -q_0 + \beta(2 + q_0 - j_0)$$

$$\tilde{f}_1 = \frac{f_0'}{6H_0^2 f_{R,0}} = 2 + q_0 - j_0$$

$$\tilde{f}_2 = \frac{f_0''}{6H_0^2 f_{R,0}} = 2 + 4q_0 + j_0(2 + q_0) + s_0 + \beta(2 + q_0 - j_0)^2$$

PRELIMINARY

(Calculations by L. Reverberi, ACGC – Cape Town)

Outline

Conclusions on Cosmographic Approach

- Cosmography results do depend upon both the chosen auxiliary variable (redshifts z or y) and the expansion order. Parameter y is highly disfavoured

- Reliability of cosmography to spot ΛCDM around close-enough XCDM competitors remains very limited with results once again depending upon the expansion order

- For extended theories of gravity, the method provides a sort of clear picture for theories with no higher-order derivatives, although not competitive - larger errors – when compared to other mehods

- For extended theories with higher derivatives in either geometrical - like $f(R)$ - or matter sector - like Galileons -, there are extra free parameters requiring priors and marginalisation. Large errors emerge

- Other neglected limitations: spatial cuvature (Clarkson 2011), lensings effects (Wald 1998, Bacon 2014) and local gravitational redshift (Wojtak 2015) may lead to extra scatter in Hubble diagrams

Is there any hope for Cosmography?

1. Clear definition of - auxiliary variables (redshifts, Padé polynomials, etc.)
   - testing against mock data

2. Establish a trade-off between number of data points, number of cosmological parameters and Bayesian evidence, so criteria can be provided

3. Motivated priors over extra parameters : stability conditions, absence of ghosts, behaviour of perturbations vs. blind tests

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NRF SAAO SOUTH AFRICAN ASTRONOMICAL OBSERVATORY

NASSP

S·A·L·T

http://www.acgc.uct.ac.za

Backup slides

$$H_{z0}/H_0 = 1 + q_0, \qquad H_{zz0}/H_0 = -q_0^2 + j_0,$$

$$H_{3z0}/H_0 = 3q_0^2(1 + q_0) - j_0(3 + 4q_0) - s_0,$$

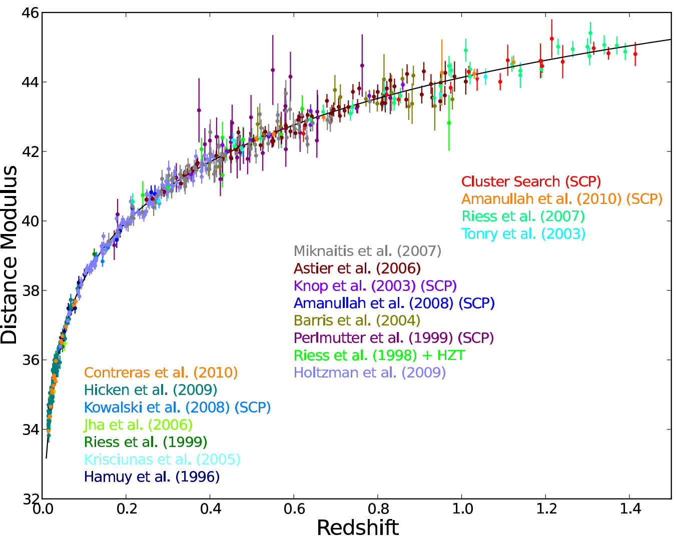
$$\begin{aligned} H_{4z0}/H_0 = &-3q_0^2(4 + 8q_0 + 5q_0^2) \\ &+ j_0(12 + 32q_0 + 25q_0^2 - 4j_0) \\ &+ s_0(8 + 7q_0) + l_0. \end{aligned}$$

Cosmography

- Hence we can construct observable magnitudes by starting from such expansions, and then compare with the observational data.

Luminosity distance

$$H_0 d_L(z) = z + \frac{1-q_0}{2} z^2 + \frac{1}{6}\left(-1 - j_0 + q_0 + 3q_0^2\right) z^3$$
$$+ \frac{1}{24}\left[2 + 5j_0(1+2q_0) - q_0(2 + 15q_0(1+q_0)) + s_0\right] z^4$$
$$+ \frac{1}{120}\left\{-6 + 10j_0^2 - l_0 - j_0[27 + 5q_0(22 + 21q_0)] + 3q_0[2 + q_0(27\right.$$
$$\left. + 5q_0(11 + 7q_0)) - 5s_0] - 11s_0\right\} z^5$$
$$+ \frac{1}{720}\left\{m_0 + l_0(19 + 21q_0) - 10j_0^2(19\right.$$
$$+ 28q_0) - 3q_0[8 + q_0(168 + 5q_0(104 + 7q_0(19 + 9q_0)) - 70s_0) - 95s_0]$$
$$\left. + j_0[168 + 5q_0(208 + 21q_0(19 + 12q_0)) - 35s_0] + 8(3 + 13s_0)\right\} z^6 + \mathcal{O}(z^7)$$



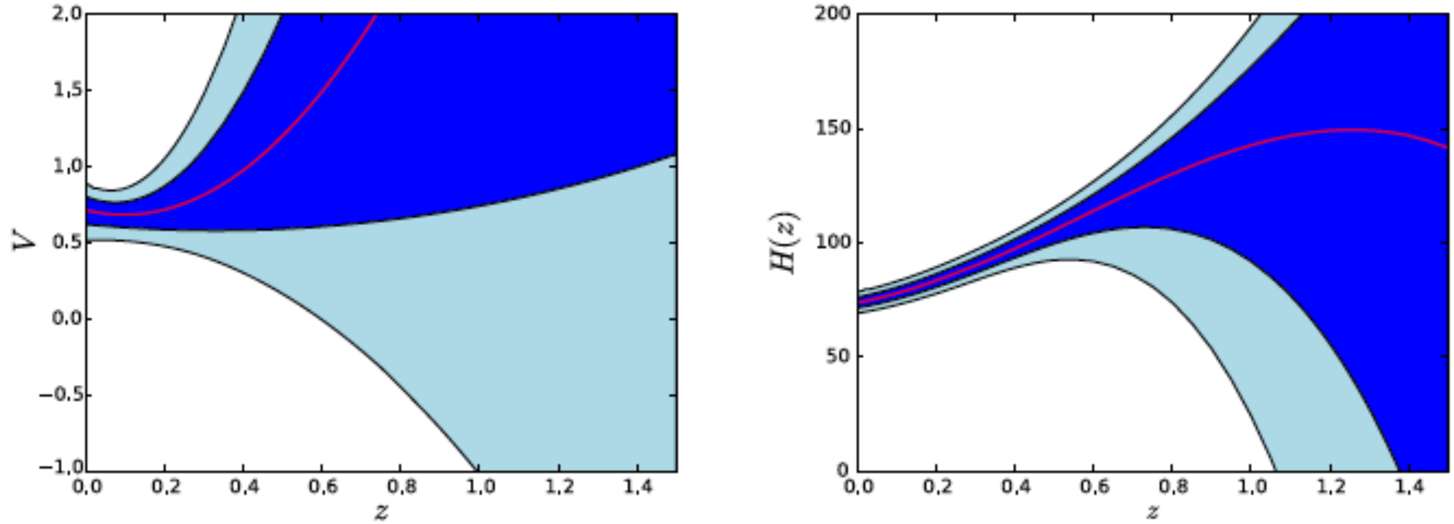
LCDM model and cosmographic parameters $\frac{H^2}{H_0^2} = \Omega_m(1+z)^3 + (1-\Omega_m)$

$\longrightarrow$ $\boxed{j_0 = 1}$

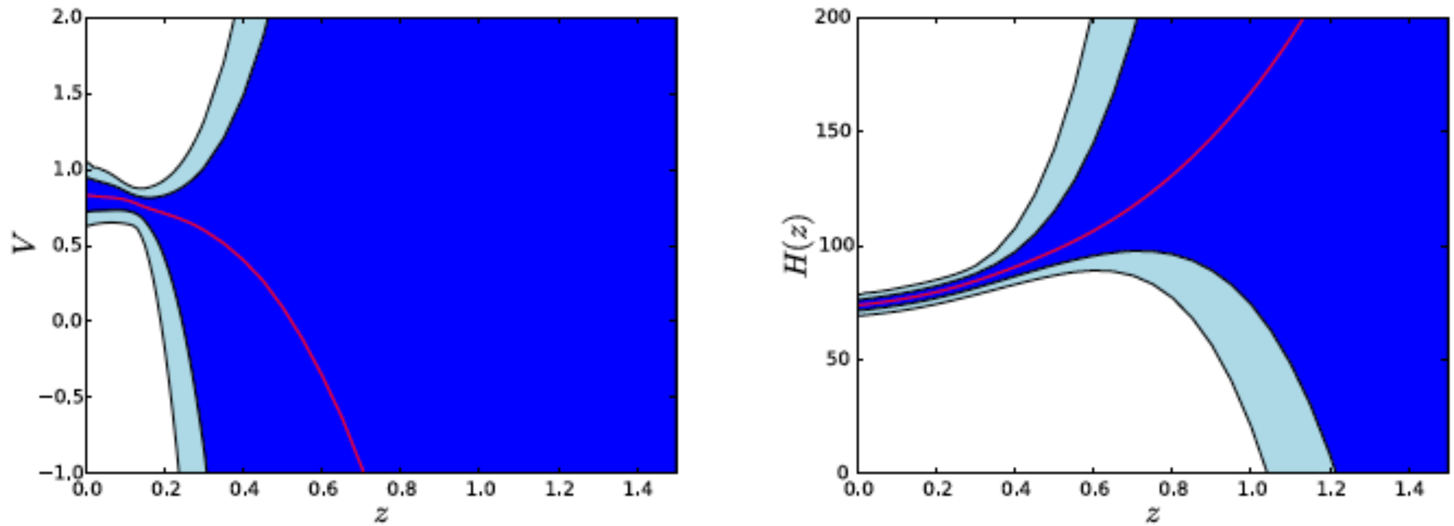
E.g. 1: K-essence

$$\mathcal{S} = \int \mathrm{d}x^4 \sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}\omega(\phi)\partial_\mu\phi\partial^\mu\phi - V(\phi) + \mathcal{L}_m\right]$$

Fourth-order cosmographic expansion



Fifth-order cosmographic expansion



Higher-order derivatives theories

E.g. 2: $f(R)$ theories

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{1}{2}f(R) + \mathcal{L}_m\right]$$

$$\frac{f_0}{6H_0^2} = -\alpha q_0 + \Omega_m + 6\beta(2 + q_0 - j_0) \ ,$$

$$\frac{f_{z0}}{6H_0^2} = \alpha(2 + q_0 - j_0) \ ,$$

$$\frac{f_{2z0}}{6H_0^2} = 6\beta(2 + q_0 - j_0)^2 + \alpha[2 + 4q_0 + (2 + q_0)j_0 + s_0]$$

$$\left.\frac{\partial f}{\partial R}\right|_{R=R_0} = \alpha, \quad \left.\frac{\partial^2 f}{\partial R^2}\right|_{R=R_0} = \frac{\beta}{H_0^2}$$

Two extra parameters

$$\Omega_m \in [0,1]$$

E.g. 3: Galileons theories

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{1}{2}R + \frac{1}{2}(c_1\phi + c_2\partial_\mu\phi\partial^\mu\phi + c_3\partial_\mu\phi\partial^\mu\phi\Box\phi) + \mathcal{L}_m\right]$$

$$\frac{c_1}{6H_0^2}\phi_0 = -1 - \alpha_G - \beta_G + \Omega_m \ ,$$

$$\frac{\phi_{zz0}}{2\phi_{z0}} = -\frac{1}{\beta_G} + q_0 - 3\frac{\alpha_G}{\beta_G} - 2\left[1 + \frac{1}{4}(1 + q_0)\right] ,$$

$$\alpha_G \equiv c_2\phi_{0z}^2/6$$

$$\beta_G \equiv c_3H_0^2\phi_{0z}^3$$

Are *viable* modified gravities viable?

Where is the problem?

They do contain a singularity what makes the cosmological evolution singular.

Scalar-tensor picture: $$S = \int d^4x\sqrt{-g}\left[\phi\ R - V(\phi) + 2\kappa^2\mathcal{L}_m\right]$$

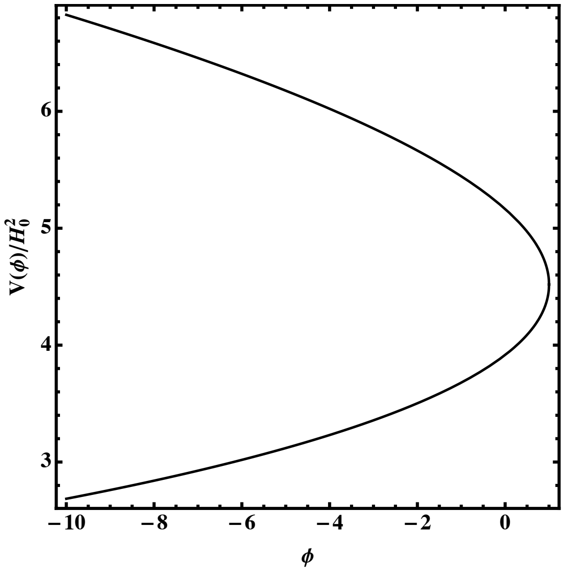
$$R = V'(\phi) \quad \rightarrow \quad \phi = \phi(R)\ ,\ \Rightarrow \quad f(R) = \phi(R)R - V(\phi(R))$$

An exact case: *n=1*

$$V(\phi) = cH_0^2\,\frac{b + (1-\phi) \pm \sqrt{b(1-\phi)}}{d}\ .$$

$$\dot{H} \propto V'(\phi) \xrightarrow{\phi\,\rightarrow\,1} \infty$$

A sudden singularity!!



S. A. Appleby, R. A. Battye and A. A. Starobinsky, JCAP 1006, 005 (2010), arXiv:0909.1737
DSG, Class. Quant. Grav. **30** 095008 (2013), arXiv:1207.5472.