Hierarchical axion scales for natural inflation & relaxion

Kiwoon Choi

(DSU2015, Kyoto)

KC, H.J. Kim, S. Yun; arXiv:1404.6209

KC, S.H. Im, arXiv:1511.00132

KC, H.J. Kim, arXiv:1511.07201

Center for Theoretical Physics of the Universe

Institute for Basic Science

Outline

- Axion scale hierarchy problem

    * Natural inflation

    * Relaxion mechanism

- Mechanism to generate hierarchical axion scales

    * Alignment

    * Clockwork

- Natural inflation with modulation (axion scale hierarchy)

- UV model for relaxion with the clockwork mechanism

  (next talk by S.H. Im)

- Conclusion

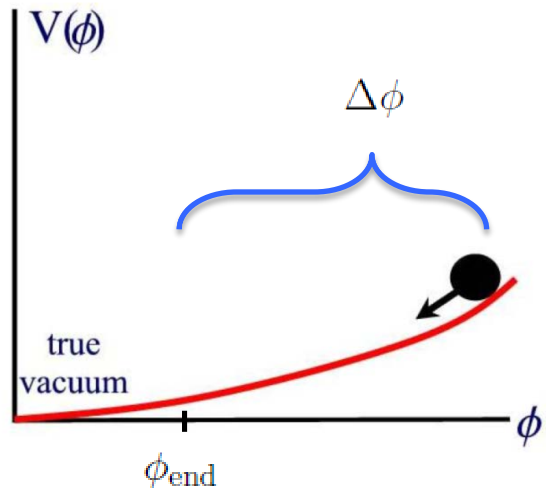
Axion scale hierarchy problem
(in natural inflation & relaxion mechanism)

<u>Natural inflation</u>

* Inflationary early universe
- solves the horizon and flatness problems
- provides a robust mechanism to generate the primordial density perturbation

* Large field (chaotic) inflation
- realize the slow roll inflation with a less constrained form of the inflaton potential and initial conditions
- predict a relatively large tensor perturbation, which might be detectable in the near future

For a large field inflation, one needs to keep the inflaton potential flat over a field range comparable to the Planck scale:



$$\epsilon = \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1$$

$$\eta = M_P^2 \frac{V''}{V} \ll 1$$

$$\left(M_P = \frac{1}{\sqrt{8\pi G_N}} \approx 2\times 10^{18}\,\mathrm{GeV}\right)$$

In view of the UV sensitivity of the scalar field potential, this suggests an approximate shift symmetry along the inflaton direction:

$$\phi \quad \to \quad \phi + \mathrm{constant}$$

In effective QFT, the most natural realization of such shift symmetry is through a periodic pseudo-Nambu-Goldstone boson:

$$\phi \;=\; \phi + 2\pi f_{\rm eff} \qquad (f_{\rm eff} = \text{(effective) decay constant})$$

Natural inflation

Freese, Frieman, Olinto, '90

Inflaton corresponds to a pseudo-Nambu-Goldstone boson with super-Planckian decay constant and the simplest periodic potential:

$$V = \Lambda^4 \left[1 - \cos\left(\frac{\phi}{f_{\rm eff}}\right)\right]$$

$$\Rightarrow \quad \epsilon \sim \eta \sim \frac{1}{N_e} \sim \frac{M_P^2}{f_{\rm eff}^2} \quad \Rightarrow \quad \Delta\phi \sim f_{\rm eff} \sim \sqrt{N_e} M_P \sim 10 M_P$$

inflaton excursion range

Weak Gravity Conjecture

Arkani-Hamed, Motl, Nicolis, Vafa '06; Brown, Cottrell, Shiu, Soler, '15

There should exist an instanton which couples to the corresponding axion with a strength stronger than the gravity:

$$\mathcal{A}(\text{instanton}) \propto \exp\left(-S_{\rm ins} + i\frac{\phi}{f}\right) \quad \text{with} \quad f \lesssim \frac{M_P}{S_{\rm ins}} \sim \frac{g^2}{8\pi^2} M_P \lesssim \frac{M_P}{2\pi}$$

strong-weak coupling duality at $g^2 = 4\pi$

$\Rightarrow$ **Axion scale hierarchy in natural inflation:**

$$\frac{f}{f_{\rm eff}} \lesssim \frac{1}{S_{\rm ins}\sqrt{N_e}} \sim 10^{-2} - 10^{-3}$$

An immediate consequence of this axion scale hierarchy in natural inflation is a subleading modulation of the inflaton potential:

$$V = \Lambda^4 \left[1 - \cos\left(\frac{\phi}{f_{\rm eff}}\right)\right] + \Lambda_{\rm mod}^4 \left[1 - \cos\left(\frac{\phi}{f}\right)\right]$$

Modulation induced by an instanton required by the WGC

$$\Lambda_{\rm mod}^4 \ll \Lambda^4, \quad f_{\rm eff} \sim \sqrt{N_e} M_P \sim 10 M_P, \quad f \lesssim \frac{M_P}{S_{\rm ins}} \lesssim \frac{M_P}{2\pi}$$

Nonperturbative dynamics incorporating a mechanism **(=alignment, exponentiated clockwork,...)** to enlarge the axion decay constant

For the consistency,

$$\frac{f_{\rm eff}}{f} = \text{integer}$$

Relaxion mechanism

* Gauge hierarchy problem:

$$\mathcal{L}_{\rm higgs} = D_\mu H^\dagger D^\mu H - m_H^2|H|^2 - \frac{1}{4}\lambda|H|^4 + y_t H q_3 u_3^c + \ldots$$

$$\Rightarrow \quad \delta m_H^2 = \left[-3y_t^2 + 3\lambda + \frac{9g_2^2 + 3g_1^2}{8} + \ldots\right]\frac{\Lambda_{\rm SM}^2}{16\pi^2}$$

This requires a fine tuning if $\Lambda_{\rm SM}$ >> weak scale.

* Possible solutions:

- New physics to regulate the quadratic divergence near the weak scale

  SUSY, composite Higgs, extra dim, ...

- Multiverse

  Anthropic selection for $m_H << \Lambda_{\rm SM}$

- Cosmological relaxation

  Cosmological evolution of a scalar field (=relaxion) to select $m_H << \Lambda_{\rm SM}$

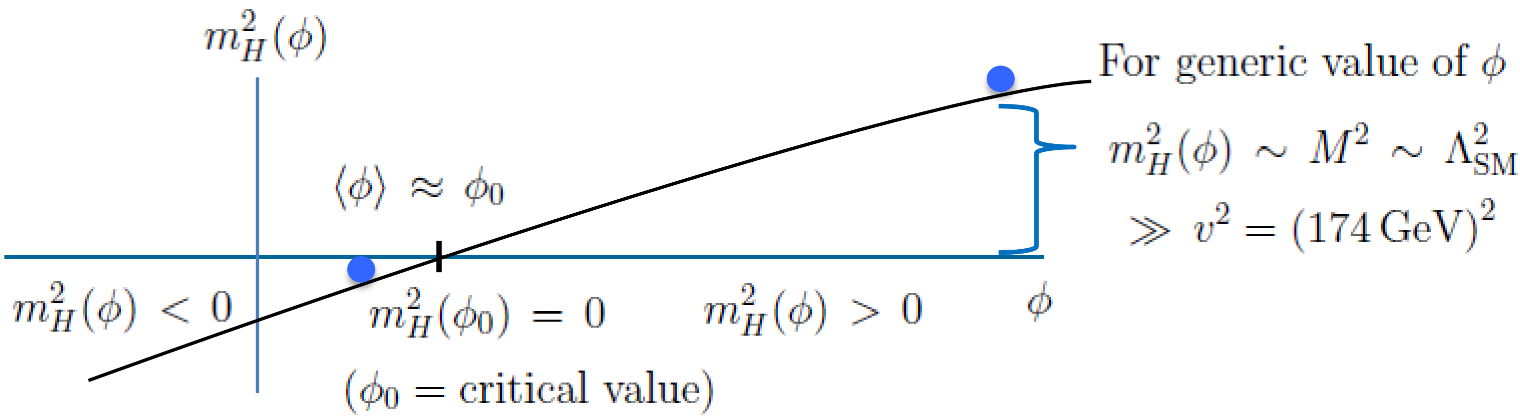
Relaxion mechanism

Graham, Kaplan, Rajendran '15

* Higgs boson mass is a dynamical field depending on the relaxion field ϕ:

$$m_H^2(\phi) = M^2 + g\phi + \ldots$$

$$\left(\text{This can be an approximation for } m_H^2(\phi) = M_1^2 + M_2^2 \cos\left(\frac{\phi}{f_{\rm eff}}\right) \text{ with a large } f_{\rm eff}.\right)$$



* Physical Higgs mass is small as ϕ is stabilized by V(ϕ) at near ϕ_0:

$$v \sim m_H(\langle\phi\rangle) \ll M \quad (\langle\phi\rangle \approx \phi_0)$$

weak scale

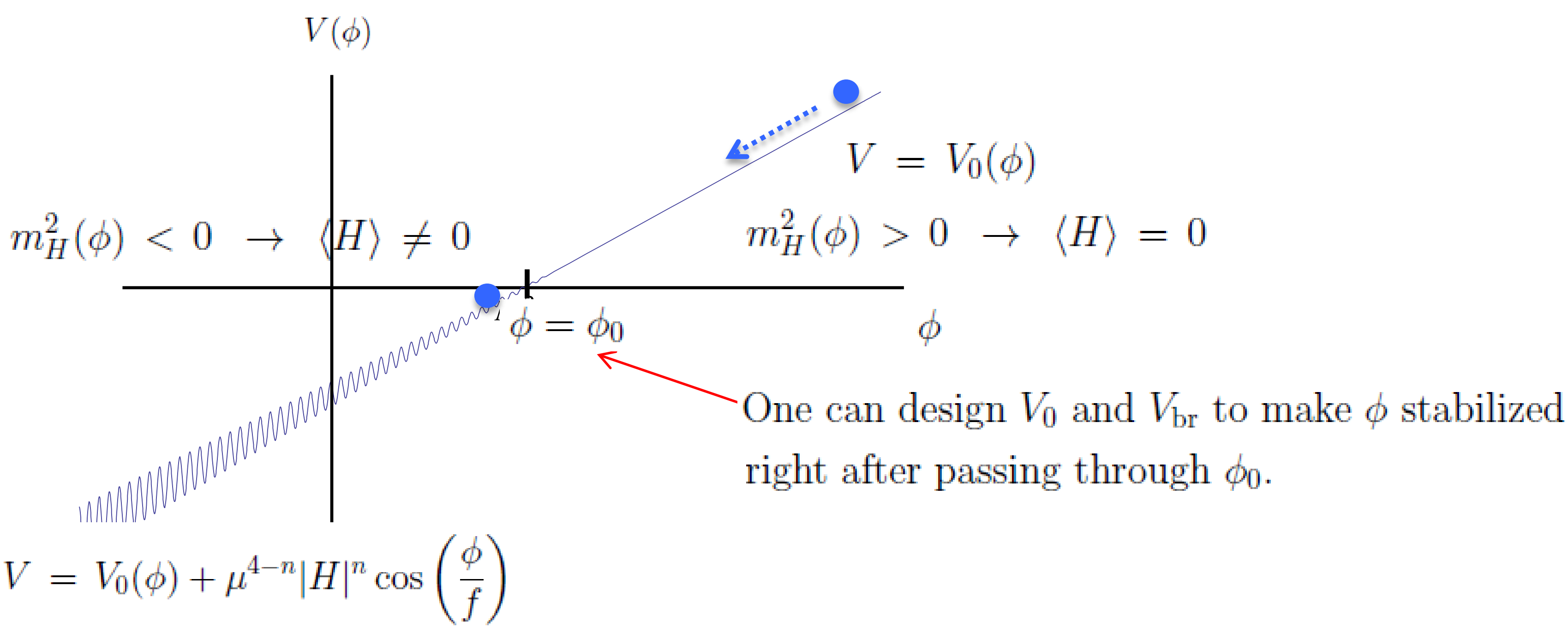
Higgs mass cutoff

Stabilizing ϕ at near the critical value ϕ_0 :

$$V(\phi, H) = V_0(\phi) + V_{\rm br}(\phi, H) + m_H^2(\phi)|H|^2 + \ldots$$

$V_0(\phi) = gM_0^2\phi + \ldots \;\left(V_0(\phi) = M_0^4 \cos\left(\frac{\phi}{f_{\rm eff}}\right) \text{ with a large } f_{\rm eff}.\right)$ enforcing ϕ to slide toward ϕ_0 when $m_H^2(\phi) > 0 \;\rightarrow\; \langle H\rangle = 0$

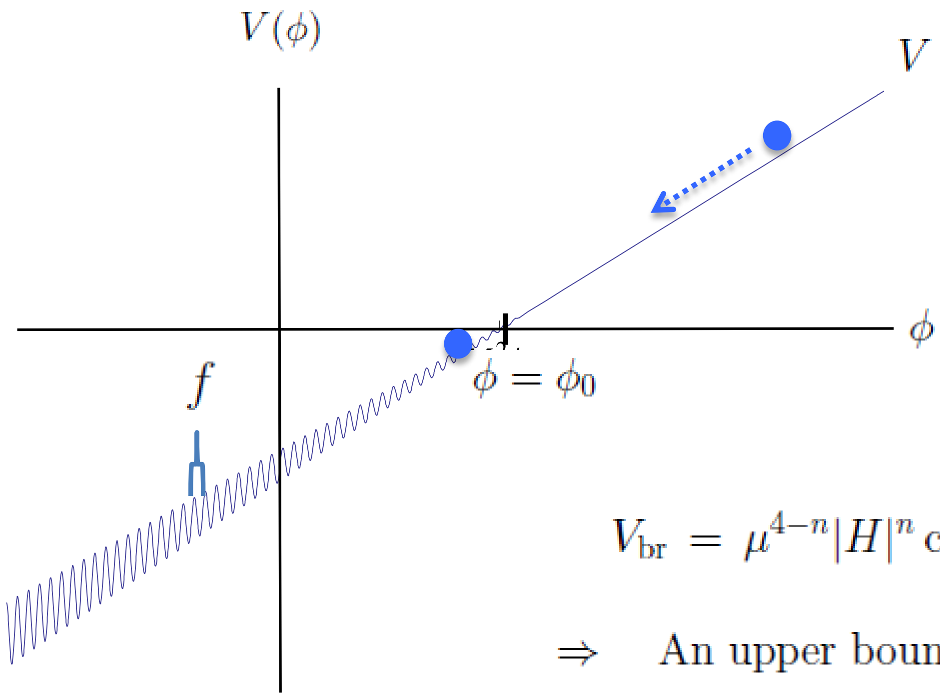
$V_{\rm br} = \mu^{4-n}|H|^n \cos\left(\frac{\phi}{f}\right)$ stabilizing ϕ right after ϕ passes through ϕ_0



Naturalness conditions

* should be stable against power-law-divergent radiative corrections

* no fine tuning of the initial condition for the relaxion cosmology



$$V = V_0(\phi) + m_H^2(\phi)|H|^2 + \ldots$$

$$= (gM_0^2\phi + \ldots) + (M^2 + g\phi + \ldots)|H|^2 + \ldots$$

$$\delta V_0 \sim g\phi\Lambda^2 \gtrsim g\phi M^2$$

$$\Rightarrow \quad M_0^2 \gtrsim M^2$$

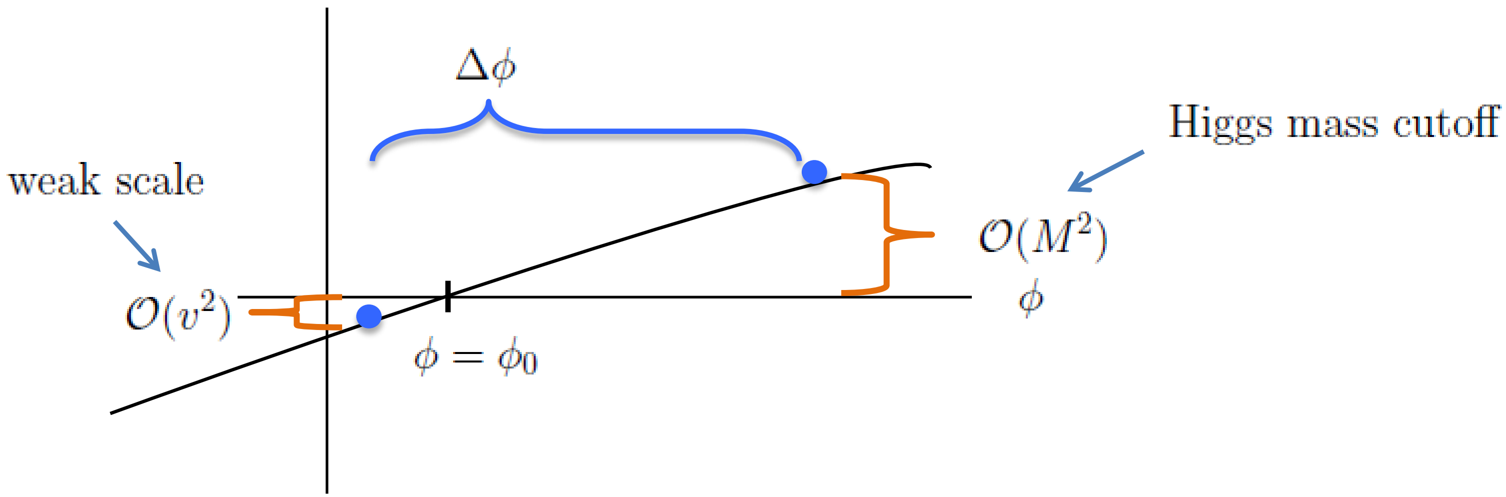
$$V_{\rm br} = \mu^{4-n}|H|^n \cos\left(\frac{\phi}{f}\right) \lesssim v^4 \quad \text{at } |H| = v.$$

$\Rightarrow$ An upper bound on gM_0^2 to stabilize ϕ by $V_{\rm br}$

$\Rightarrow$ **The slope parameter g should be small:** $\displaystyle \frac{g}{f} \lesssim \left(\frac{V_{\rm br}}{v^4}\right)\left(\frac{v^2}{M^2}\right)\left(\frac{v^2}{f^2}\right)$

The slope parameter g should be small: $\frac{g}{f} \lesssim \left(\frac{V_{\rm br}}{v^4}\right)\left(\frac{v^2}{M^2}\right)\left(\frac{v^2}{f^2}\right)$

$$m_H^2(\phi) = M^2 + g\phi + \ldots$$



$\Rightarrow$ There should be a large excursion of ϕ to scan m_H^2 from $\mathcal{O}(M^2)$ to $\mathcal{O}(v^2)$:

$$\frac{f}{\Delta\phi} \lesssim \left(\frac{V_{\rm br}}{v^4}\right)\left(\frac{v^4}{M^4}\right) \quad \text{for} \quad M \gg v \quad \text{and} \quad V_{\rm br} \lesssim v^4$$

Back reaction potential from QCD: $\frac{V_{\rm br}}{v^4} \sim 10^{-12}$

Back reaction potential from hidden color: $\frac{V_{\rm br}}{v^4} \sim \mathcal{O}(10^{-1} - 10^{-3})$

Small slope parameter g is technically natural, i.e. stable against radiative corrections, as it corresponds to tiny breaking of the shift symmetry:

$$\phi \rightarrow \phi + \text{constant}$$

However the resulting axion scale hierarchy is so large, therefore calls for an explanation.

Axion scale hierarchy in relaxion mechanism:

$$\frac{f}{f_{\text{eff}}} \lesssim \left(\frac{V_{\text{br}}}{v^4}\right)\left(\frac{v^4}{M^4}\right) \quad \text{for} \quad M \gg v \;\; \text{and} \;\; V_{\text{br}} \lesssim v^4 \quad (\Delta\phi \sim f_{\text{eff}})$$

$$f = \text{relaxion scale in } V_{\text{br}}(\phi).$$

$$f_{\text{eff}} = \text{relaxion scale in } V_0(\phi) \text{ and } m_H^2(\phi).$$

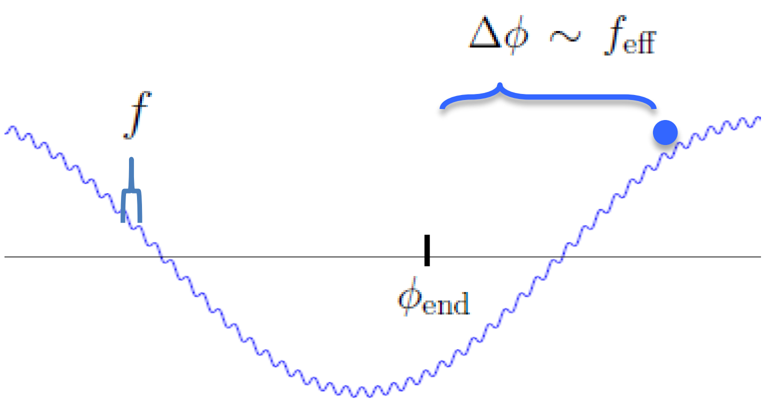
$$\text{Back reaction potential from QCD:} \quad \frac{V_{\text{br}}}{v^4} \sim 10^{-12}$$

$$\text{Back reaction potential from hidden color:} \quad \frac{V_{\text{br}}}{v^4} \sim \mathcal{O}(10^{-1} - 10^{-3})$$

Q: What is the origin of this big axion scale hierarchy?

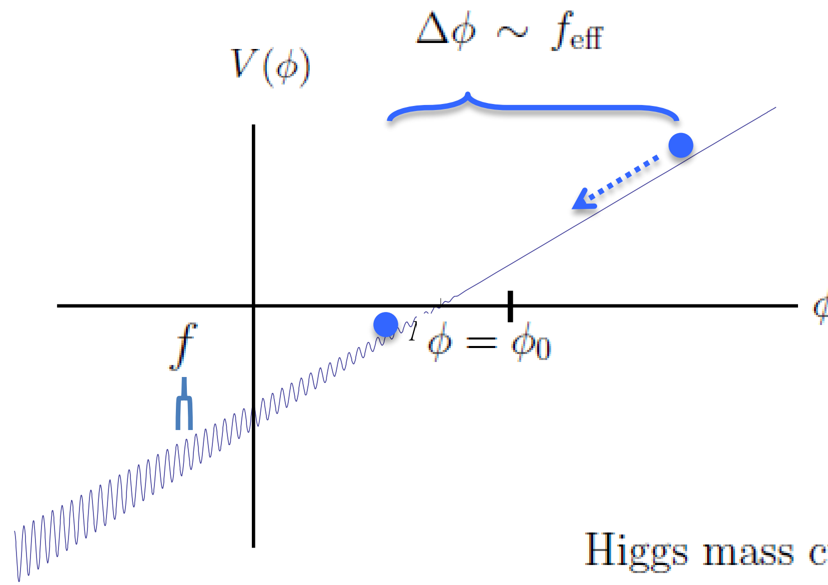
Axion scale hierarchy in natural inflation & relaxion mechanism

Natural inflation



$$\frac{f}{f_{\rm eff}} \lesssim \frac{1}{S_{\rm ins}\sqrt{N_e}} \sim 10^{-2} - 10^{-3}$$

Relaxion



Higgs mass cutoff

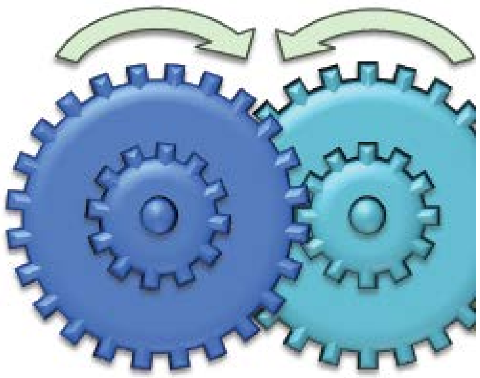
$$\frac{f}{f_{\rm eff}} \lesssim \mathcal{O}\left(10^{-1} - 10^{-12}\right) \times \left(\frac{v^4}{M^4}\right) \quad \text{for } M \gg v$$

weak scale

Clockwork mechanism to generate hierarchical axion scales

KC, Im, arXiv:1511.00132; Kaplan, Rattazzi, arXiv:1511.01827

$$\frac{\phi_i}{f_i} = \text{rotation of each wheel}$$



Single rotation of the 1st wheel induces a multiple rotation of the 2nd wheel, leading to a longer collective rotation.

$$V_{\rm cw} = -\Lambda^4 \cos\left(\frac{\phi_1}{f_1} + n\frac{\phi_2}{f_2}\right)$$

$$\phi_H = \text{frozen heavy axion} \propto \frac{\phi_1}{f_1} + n\frac{\phi_2}{f_2} = 0$$

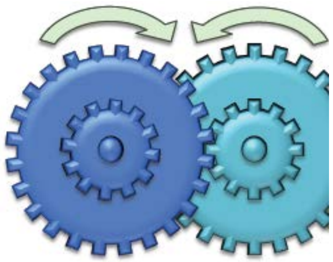
$$\phi = \text{relaxion describing the collective rotation}$$

$$\Rightarrow \quad \frac{\phi_1}{f_1} = \frac{\phi}{f} \equiv n\frac{\phi}{f_{\rm eff}}, \quad \frac{\phi_2}{f_2} = -\frac{\phi}{f_{\rm eff}} \quad \left(f_{\rm eff} = \sqrt{n^2 f_1^2 + f_2^2} \equiv nf\right)$$

(This particular form of axion mass-mixing has been applied recently for large field inflation: KC, Kim, Yun, '14; Tye, Wong, '14; Ben-Dayan, Pedro, Westphal, '14; Harigaya, Ibe, '14; Bai, Stefanek, '15; de la Fuente, Saraswat, Sundrum, '15; …)

Clockwork mechanism for hierarchical axion scales

$$
\begin{aligned}
V &= V_{\rm cw} + V_1 + V_2 \\
V_{\rm cw} &= -\Lambda^4 \cos\left(\frac{\phi_1}{f_1} + n\frac{\phi_2}{f_2}\right) \\
V_1 &= -m_1^4 \cos\left(\frac{\phi_1}{f_1} + \delta_1\right) \\
V_2 &= -m_2^4 \cos\left(\frac{\phi_2}{f_2} + \delta_2\right)
\end{aligned}
$$



$$(m_i^4 \ll \Lambda^4)$$

$$\Rightarrow \quad \frac{\phi_1}{f_1} = \frac{\phi}{f} \equiv n\frac{\phi}{f_{\rm eff}}, \quad \frac{\phi_2}{f_2} = -\frac{\phi}{f_{\rm eff}} \quad \left(f_{\rm eff} = \sqrt{n^2 f_1^2 + f_2^2} \equiv nf\right)$$

$$V_{\rm eff} = -m_1^4 \cos\left(\frac{\phi}{f} + \delta_1\right) - m_2^4 \cos\left(\frac{\phi}{f_{\rm eff}} + \delta_2\right)$$

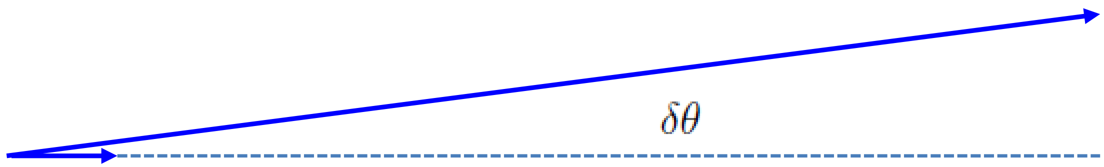
There are two axion scales in the effective theory, which are split by an integer n:

$$f_{\rm eff} = nf : \; f \sim f_1 \sim f_2$$

We need a big hierarchy: $\dfrac{f_{\rm eff}}{f} \gg 1$

* Alignment of two axion couplings: Kim, Nilles, Peloso '05

$$\vec{q} = \left(\frac{n}{f_2}, \frac{1}{f_1}\right)$$

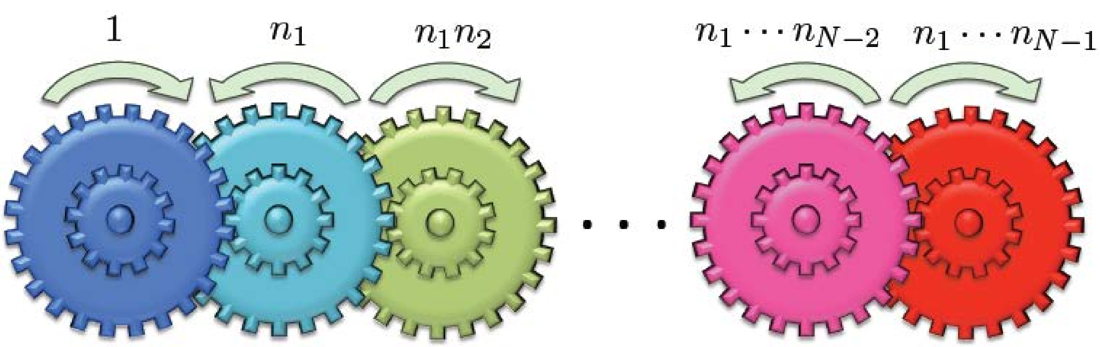


$(f_1 \approx f_2)$

$$\vec{p} = \left(\frac{1}{f_2}, 0\right)$$

$$\frac{f_{\rm eff}}{f} = n \gg 1 \quad \left(\delta\theta \sim \frac{1}{n} \ll 1\right)$$

* Exponentiation with more (N > 2) axions: KC, Kim, Yun '14



Exponentially many rotation of the last wheel

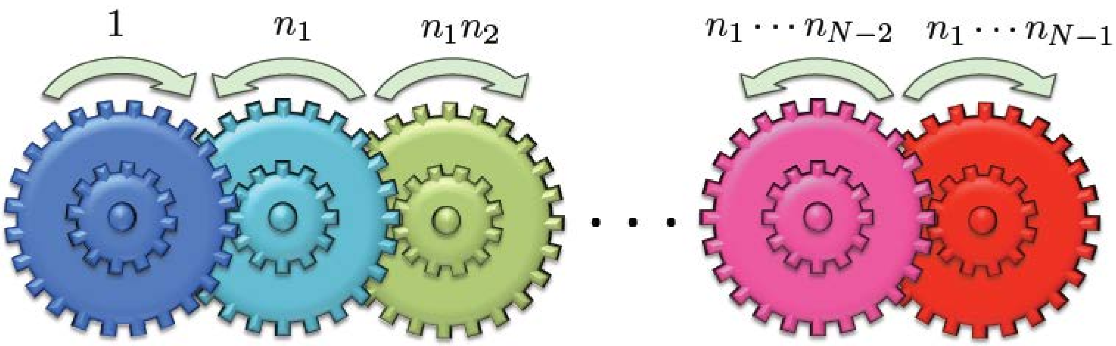
$\Rightarrow$ Exponentially long field space for the collective rotation

(= Relaxion ϕ with an exponentially large effective decay constant)

$$\frac{f_{\rm eff}}{f} = n_1 n_2 ... n_{N-1} \sim e^N \quad \text{with} \quad N \gg 1$$

Exponentiated clockwork mechanism for relaxion

KC, Im, arXiv:1511.00132; Kaplan, Rattazzi, arXiv:1511.01827



$$V_{\rm cw} = -\sum_{i=1}^{N-1} \Lambda_i^4 \cos\left(\frac{\phi_i}{f_i} + n_i \frac{\phi_{i+1}}{f_{i+1}}\right)$$

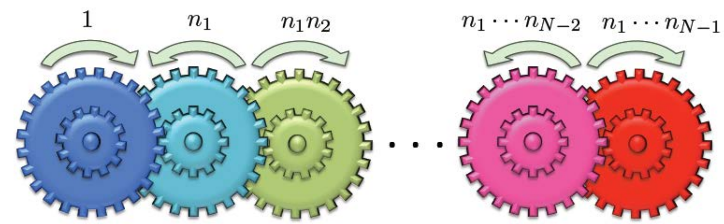
$$V_0 = -m_0^4 \cos\left(\frac{\phi_N}{f_N} + \delta_N\right)$$

$$m_H^2 = M_1^2 + M_2^2 \cos\left(\frac{\phi_N}{f_N} + \tilde{\delta}_N\right)$$

$$V_{\rm br} = \mu^{4-n} |H|^n \cos\left(\frac{\phi_1}{f_1} + \delta_1\right)$$

Effective potential of the collective rotation angle (=relaxion):

$$\frac{\phi_1}{f_1} = \frac{\phi}{f}, \qquad \frac{\phi_N}{f_N} = (-1)^{N-1}\frac{\phi}{f_{\rm eff}}$$



$$\begin{aligned} V_{\rm eff} &= -m_0^4 \cos\left(\frac{\phi}{f_{\rm eff}} + (-1)^{N-1}\delta_N\right) \\ &+ \left(M_1^2 + M_2^2 \cos\left(\frac{\phi}{f_{\rm eff}} + (-1)^{N-1}\tilde{\delta}_N\right)\right)|H|^2 \\ &- \mu^{4-n}|H|^n \cos\left(\frac{\phi}{f} + \delta_1\right) \end{aligned}$$

with an exponential hierarchy between axion scales:

$$f \sim f_i$$

$$f_{\rm eff} = \sqrt{\sum_{i=1}^{N}\left(\prod_{j=i}^{N-1} n_j^2\right) f_i^2} = n_1 n_2 \ldots n_{N-1} f \sim e^N f$$

Observable consequence of modulation in natural inflation

KC, Kim, ArXiv:1511.07201

Natural inflation compatible with the WGC generically involves a small modulation of the inflaton potential:

$$V = \Lambda^4 \left[1 - \cos\left(\frac{\phi}{f_{\rm eff}}\right)\right] + \Lambda_{\rm mod}^4 \left[1 - \cos\left(\frac{\phi}{f}\right)\right]$$

(Previous works on natural inflation with modulation:
Kobayashi, Takahashi, '10; Abe, Kobayashi, Otsuka, '15; Kappl, Nilles, Winkler, '15)

In the presence of modulation, one needs a new parametrization of the scalar curvature power spectrum: Flauger et al, '09; Flauger and Pajer, '10

$$\mathcal{P}_{\mathcal{R}}(k) = \mathcal{P}_{\mathcal{R}}^{(0)}(k)\left[1 + \delta n_s \cos\left(\frac{\phi_k}{f}\right)\right] \quad \left(\mathcal{P}_{\mathcal{R}}^{(0)} = A_*\left(\frac{k}{k_*}\right)^{n_s - 1 + \frac{1}{2}\alpha\ln(k/k_*) + \ldots}\right)$$

$$\phi_k = \text{inflaton value when the CMB scale } k \text{ exits the horizon}$$

For natural inflation with modulation KC, Kim, ArXiv:1511.07201

$$V = \Lambda^4 \left[1 - \cos\left(\frac{\phi}{f_{\rm eff}}\right)\right] + \Lambda^4_{\rm mod}\left[1 - \cos\left(\frac{\phi}{f}\right)\right]$$

$$\mathcal{P}_{\mathcal{R}}(k) = \mathcal{P}^{(0)}_{\mathcal{R}}(k)\left[1 + \delta n_s \cos\left(\frac{\phi_k}{f}\right)\right]$$

$$\delta n_s = \frac{3b\sqrt{2\pi\gamma\coth\frac{\pi}{2\gamma}}}{\sqrt{(1+\frac{3}{2}\gamma^2/f^2_{\rm eff})^2 + (3\gamma)^2}}$$

$$\left(b = \frac{\Lambda^4_{\rm mod}}{\Lambda^4}\left(\frac{f_{\rm eff}}{f\sin(\phi_*/f_{\rm eff})}\right) \ll 1, \quad \gamma = \frac{f_{\rm eff} f}{M_P^2}\tan\left(\frac{\phi_*}{2f_{\rm eff}}\right)\right)$$

$$\frac{\cos\left(\phi_k/2f_{\rm eff}\right)}{\cos\left(\phi_*/2f_{\rm eff}\right)} \approx \left(\frac{k}{k_*}\right)^{M_P^2/2f^2_{\rm eff}}$$

In which case can the effect of modulation be described by the standard form of the curvature power spectrum?

$$V = \Lambda^4 \left[1 - \cos\left(\frac{\phi}{f_{\rm eff}}\right)\right] + \Lambda_{\rm mod}^4 \left[1 - \cos\left(\frac{\phi}{f}\right)\right]$$

$$\frac{f}{M_P} \gg \frac{M_P}{f_{\rm eff}} \ln\left(\frac{k_{\rm max}}{k_{\rm min}}\right) \quad \Rightarrow \quad \mathcal{P}_\mathcal{R} \approx \mathcal{P}_\mathcal{R}^{(0)} = A_* \left(\frac{k}{k_*}\right)^{n_s - 1 + \frac{1}{2}\alpha \ln(k/k_*) + \dots}$$

$$\left(\ln\left(\frac{k_{\rm max}}{k_{\rm min}}\right) \approx 7 - 8\right)$$

However, in our case

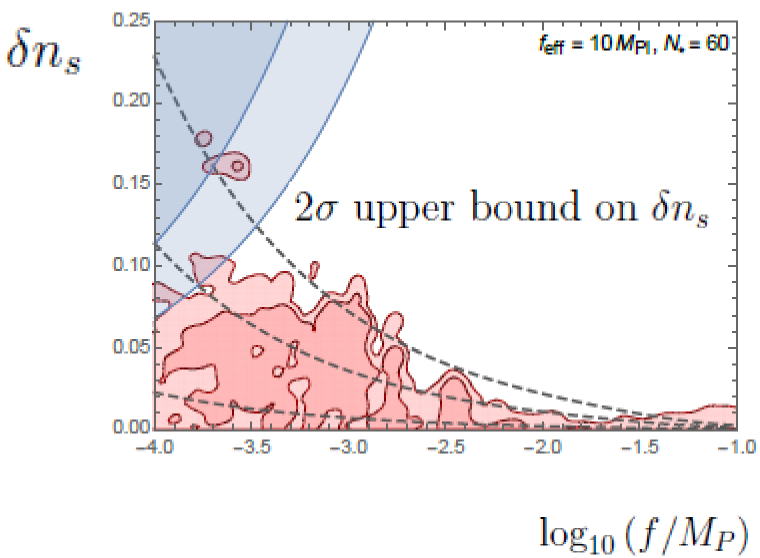
$$\frac{f}{M_P} \lesssim \frac{1}{S_{\rm ins}} \lesssim \frac{1}{2\pi}, \qquad \frac{M_P}{f_{\rm eff}} \sim \frac{1}{\sqrt{N_e}} \sim \frac{1}{\ln\left(k_{\rm max}/k_{\rm min}\right)}$$

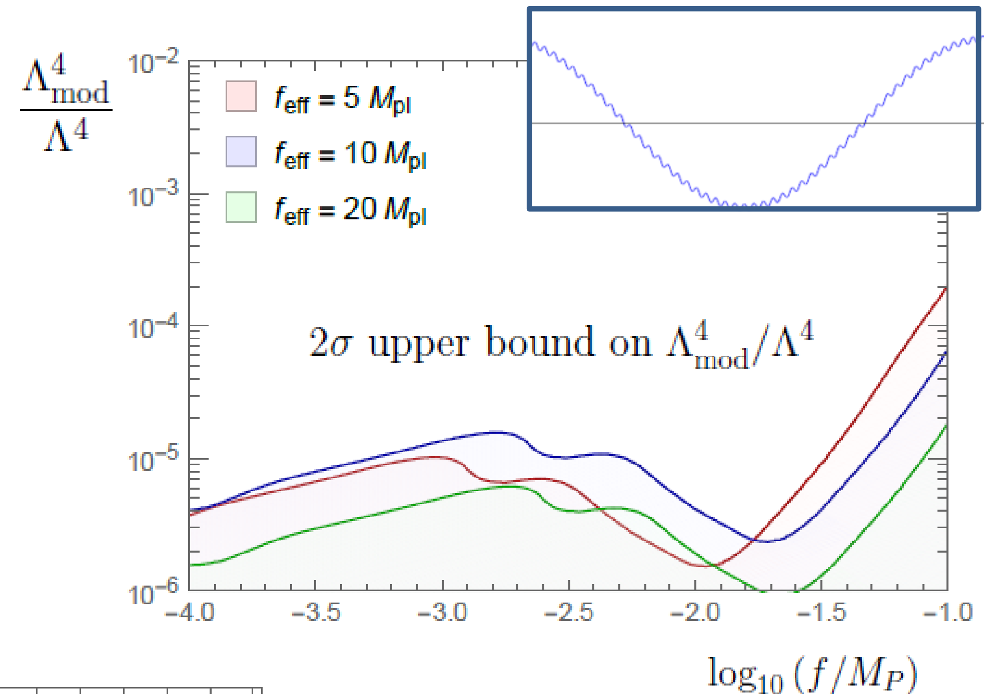
therefore we need to use the parametrization

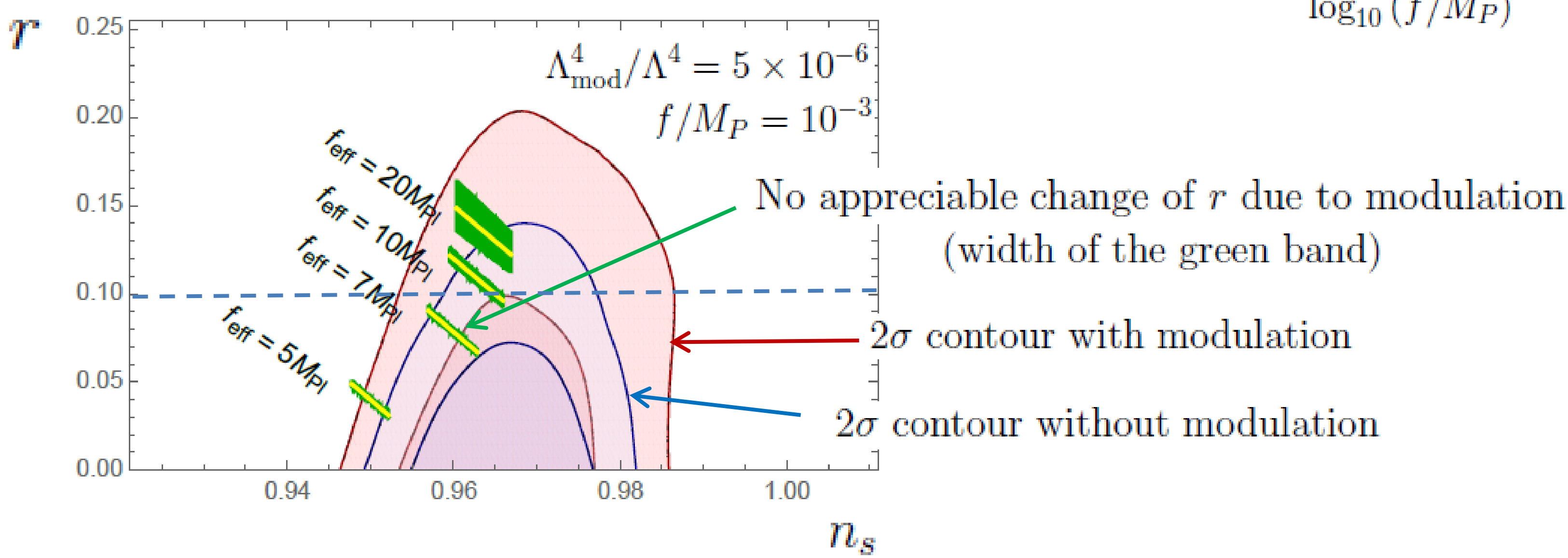
$$\mathcal{P}_\mathcal{R}(k) = \mathcal{P}_\mathcal{R}^{(0)}(k) \left[1 + \delta n_s \cos\left(\frac{\phi_k}{f}\right)\right]$$

CMB (Planck) constraints on the modulation

KC, Kim, ArXiv:1511.07201







UV model of the exponentiated clockwork for relaxion

Supersymmetric model with (next talk for the details)

* Higgs mass cutoff = $m_{\rm SUSY} \gg$ weak scale
* N global $U(1)$ symmetries spontaneously broken by $\langle \Phi_i \rangle \sim f_i \sim \sqrt{m_{\rm SUSY} M_P}$

$$V(\Phi_i) = -m_{\rm SUSY}^2 |\Phi_i|^2 + \frac{|\Phi_i|^6}{M_P^2} \quad \Rightarrow \quad N \text{ axions : } \langle \Phi_i \rangle = f_i e^{i\phi_i/f_i} \quad \left(f_i \sim \sqrt{m_{\rm SUSY} M_P} \right)$$

* Hidden sector gaugino condensation by $G_{\rm hidden} = \prod_{i=1}^{N-1} SU(p_i)$ with a confinement scale $\Lambda_i \gg m_{\rm SUSY}$

$$\Rightarrow \quad V_{\rm cw} = -\sum_i m_{\rm SUSY} \Lambda_i^3 \cos\left(\frac{1}{p_i} \left(\frac{\phi_i}{f_i} + n_i \frac{\phi_{i+1}}{f_{i+1}} \right) \right)$$

* $\Delta K = \dfrac{X_{N-1}^2 X_N^{2*}}{M_P^2} + {\rm h.c} \quad \Rightarrow \quad V_0(\phi)$
* $\Delta W = \left(\dfrac{X_{N-1}^2}{M_P} + \dfrac{X_N^2}{M_P} \right) H_u H_d \quad \Rightarrow \quad m_H^2(\phi)$

* Hidden color which confines around the weak scale with hidden-colored matter fields $(N, N^c) + (L, L^c)$: $\quad W_{\rm br} = \dfrac{X_1^2 L L^c}{M_P} + \kappa_u H_u L N^c + \kappa_d H_d L^c N \quad \Rightarrow \quad V_{\rm br}(\phi)$

Conclusion

- Two of the major applications of axion-like field for cosmology and particle physics, i.e. **natural inflation and relaxion mechanism**, requires a hierarchy structure of the involved axion scales. (Typically a big hierarchy for the relaxion case!)

- We propose a scheme (= **exponentiated clockwork**) to generate an exponential hierarchy between the effective axion scales within an EFT of multiple axions, where all fundamental axion scales are comparable to each other and well below the Planck scale.

- Axion scale hierarchy in natural inflation, suggested by the weak gravity conjecture, implies that generically there is a small modulation of the inflaton potential, which can lead to an interesting consequence in the primordial density perturbation.

- Our scheme for hierarchical relaxion scales can have a natural UV completion in SUSY models with $m_{SUSY} >>$ weak scale.