Theoretical Aspects of Cosmic Acceleration

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Overview

- Motivations - background, and the problem of cosmic acceleration

- Some possible approaches:
  - The cosmological constant
  - Dynamical dark energy
  - Modified gravity

- What are the theoretical issues facing any dynamical approach? Screening mechanisms - (focus on the Vainshtein mechanism.)

- An example: Galileons

- A few comments.

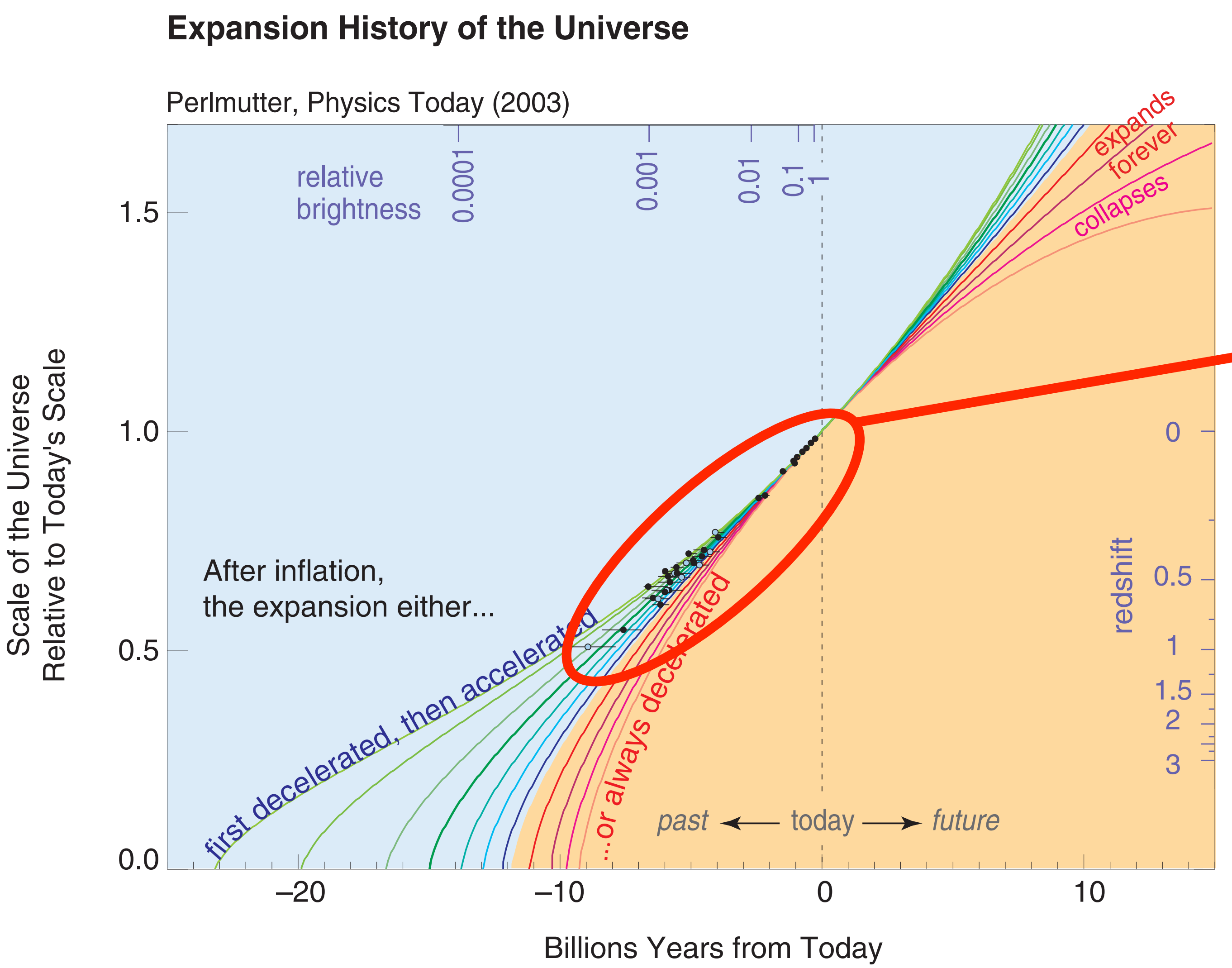
This is a story in progress - no complete answers yet.
Useful (hopefully) reference for a lot of what I'll say is

Beyond the Cosmological Standard Model
Bhuvnesh Jain, Austin Joyce, Justin Khoury and MT

Phys.Rept. **568** 1-98 (2015), *[arXiv:1407.0059]*

The Cosmic Expansion History

What does data tell us about the expansion rate?



We now know, partly through this data, that the universe is not only expanding ...

$$\dot{a} > 0$$

... but is accelerating!!

$$\ddot{a} > 0$$

If we trust GR then $\frac{\ddot{a}}{a} \propto -(\rho + 3p)$

Then we infer that the universe must be dominated by some strange stuff with p<-ρ/3. We call this **dark energy!**

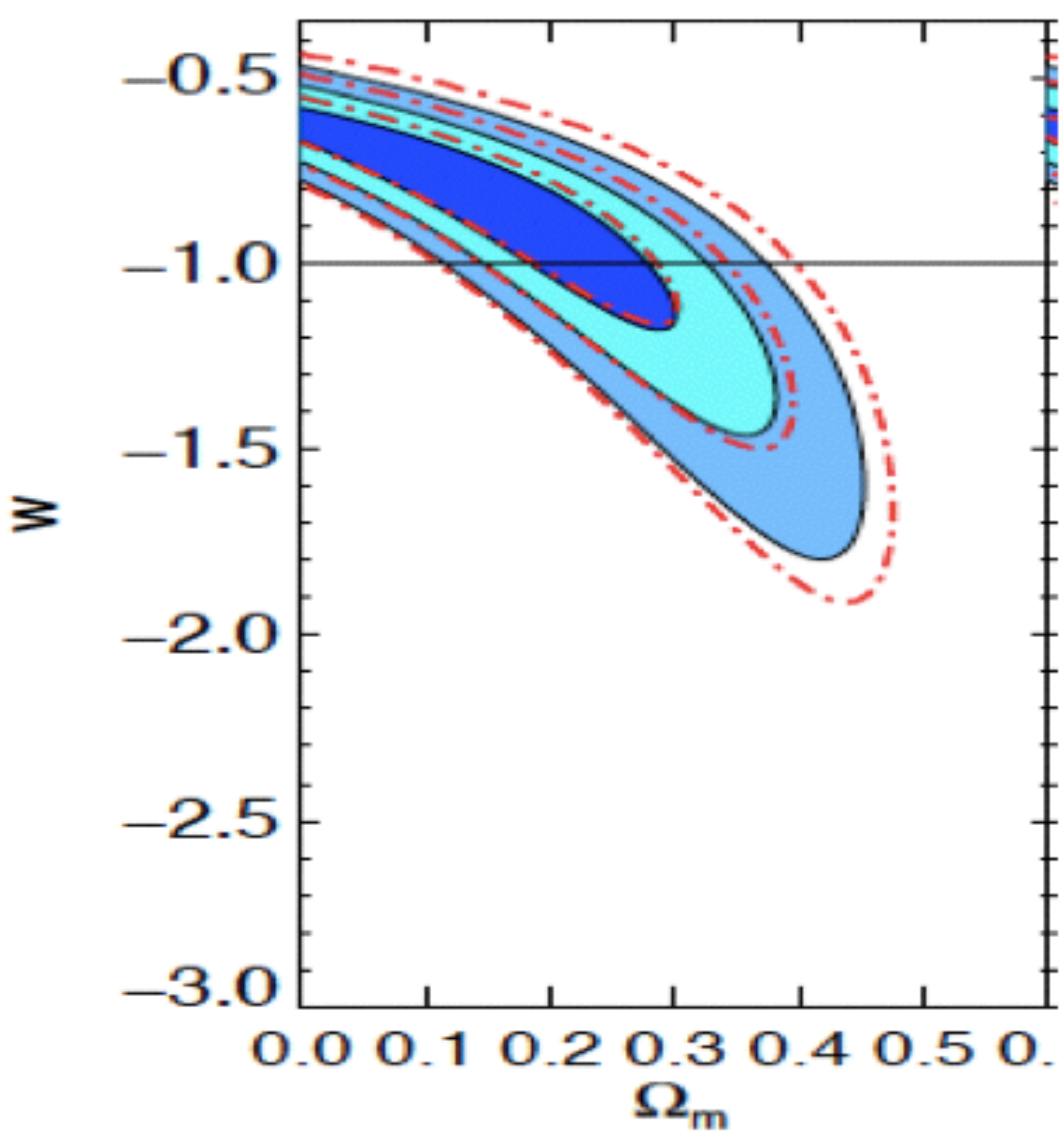
Cosmic Acceleration

$$\frac{\ddot{a}}{a} \propto -(\rho + 3p)$$

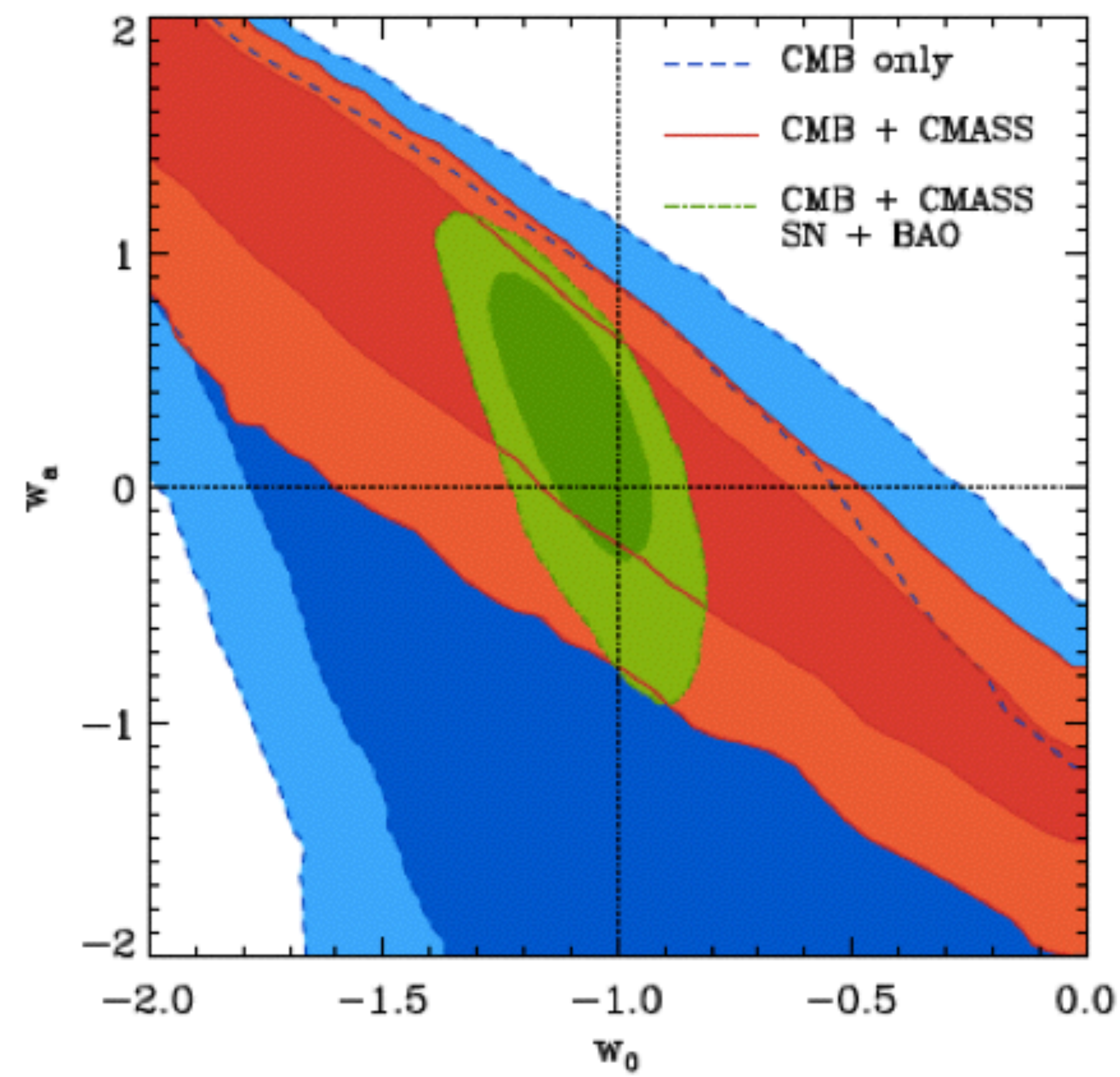
So, writing p=wρ, accelerating expansion means p<-ρ/3 or

w<-1/3

Conley et al. 2011



Sanchez et al. 2012



The Cosmological Constant

Vacuum is full of virtual particles carrying energy. Equivalence principle (Lorentz-Invariance) gives

$$\langle T_{\mu\nu} \rangle \sim -\langle \rho \rangle g_{\mu\nu}$$

A constant vacuum energy! How big? Quick & dirty estimate of size only by modeling SM fields as collection of independent harmonic oscillators and then summing over zero-point energies.

$$\langle \rho \rangle \sim \int_0^{\Lambda_{\rm UV}} \frac{d^3k}{(2\pi)^3} \frac{1}{2} \hbar E_k \sim \int_0^{\Lambda_{\rm UV}} dk\ k^2 \sqrt{k^2 + m^2} \sim \Lambda_{\rm UV}^4$$

Most conservative estimate of cutoff: ~ 1 TeV. Gives

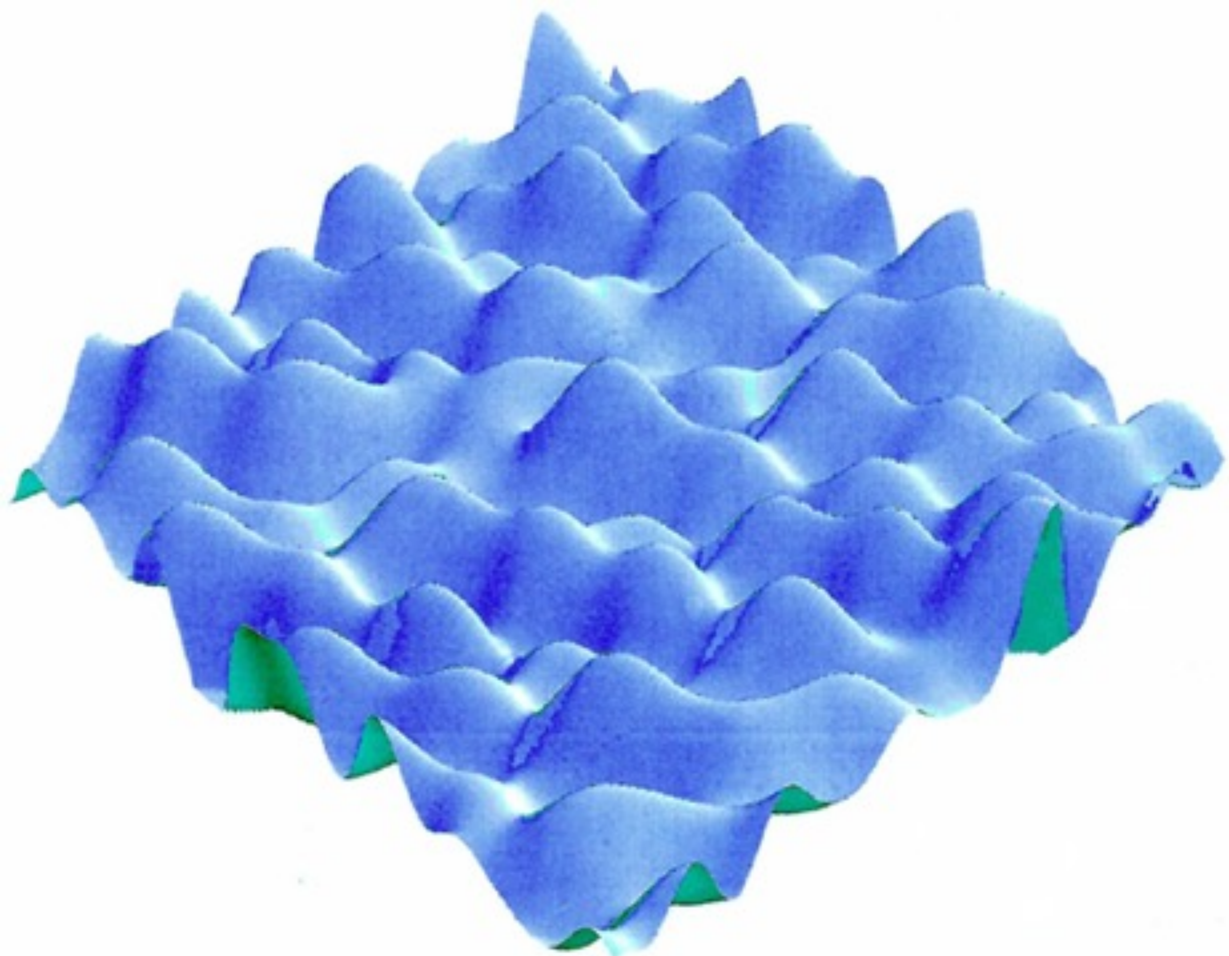
$$\Lambda_{\rm theory} \sim ({\rm TeV})^4 \sim\ 10^{-60}\ M_{\rm Pl}^4 << \Lambda_{\rm obs.} \sim M_{\rm Pl}^2 H_0^2 \sim 10^{-60} ({\rm TeV})^4 \sim 10^{-120}\ M_{\rm Pl}^4$$

An enormous, and entirely unsolved problem in fundamental physics, made more pressing by the discovery of acceleration!

At this stage, fair to say we are almost completely stuck! - No known dynamical mechanism, and a no-go theorem (Weinberg) to be overcome.

Lambda, the Landscape & the Multiverse

Anthropics provide a logical possibility to explain this, but a necessary (not sufficient) requirement is a way to realize and populate many values. The string landscape, with eternal inflation, may provide a way to do this.

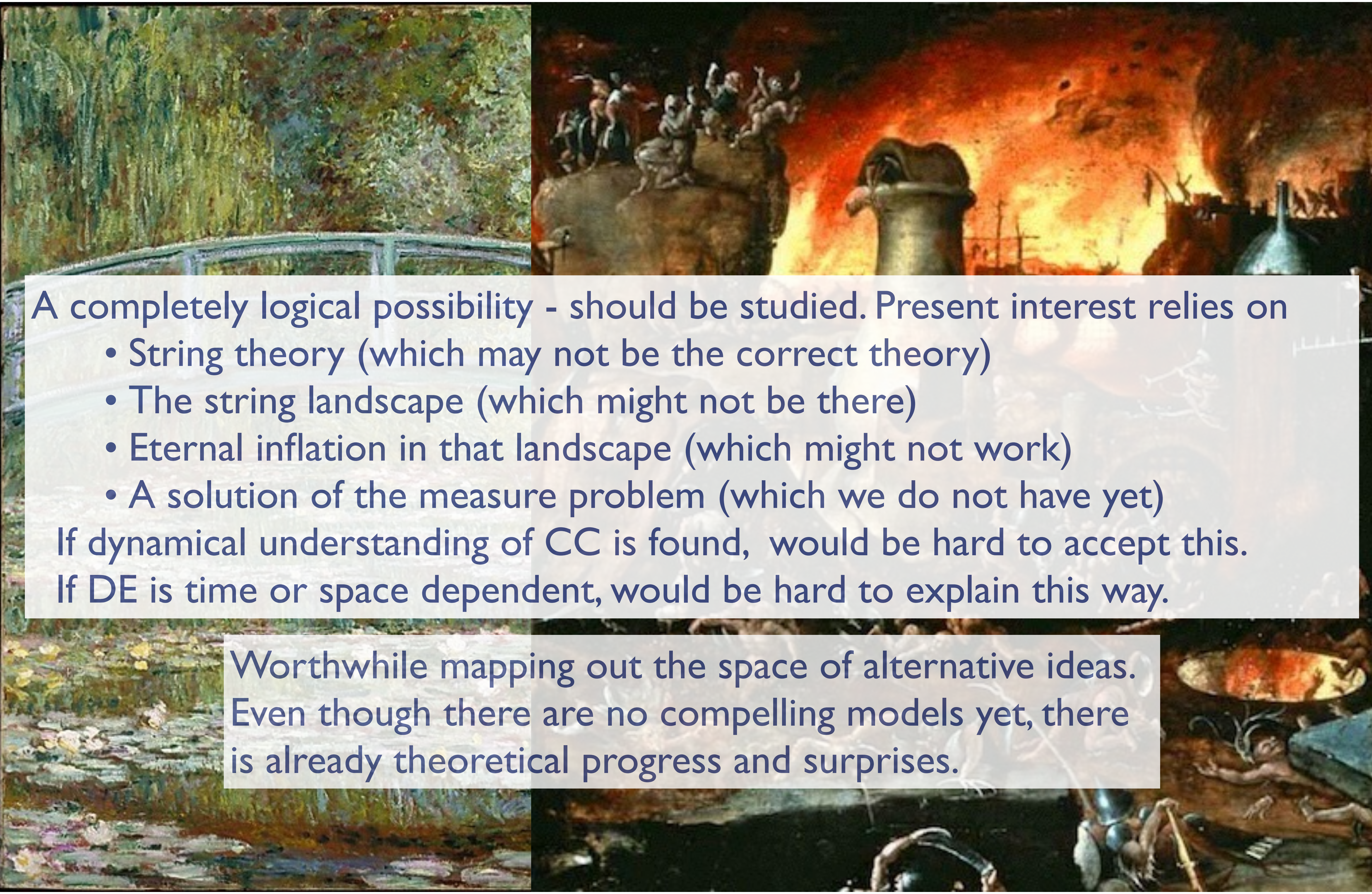


[Image: SLIM FILMS. Looking for Life in the Multiverse, A. Jenkins & G. Perez, Scientific American, December 2009]

An important step is understanding how to compute probabilities in such a spacetime

No currently accepted answer, but quite a bit of serious work going on. Too early to know if can make sense of this.

How to Think of This



A completely logical possibility - should be studied. Present interest relies on

- String theory (which may not be the correct theory)
- The string landscape (which might not be there)
- Eternal inflation in that landscape (which might not work)
- A solution of the measure problem (which we do not have yet)

If dynamical understanding of CC is found, would be hard to accept this.

If DE is time or space dependent, would be hard to explain this way.

Worthwhile mapping out the space of alternative ideas. Even though there are no compelling models yet, there is already theoretical progress and surprises.

Dynamical Dark Energy

Once we allow dark energy to be dynamical, we are imagining that is is some kind of honest-to-goodness mass-energy component of the universe.

It isn't enough for a theorist to model matter as a perfect fluid with energy density ρ and pressure p (at least it shouldn't be enough at this stage!)

$$T_{\mu\nu} = (\rho + p)U_\mu U_\nu + p g_{\mu\nu}$$

Our only known way of describing such things, at a fundamental level is through quantum field theory, with a Lagrangian. e.g.

$$S_m = \int d^4x \, L_m[\phi, g_{\mu\nu}]$$

$$L_m = \frac{1}{2} g^{\mu\nu} \left(\partial_\mu \phi\right) \partial_\nu \phi - V(\phi)$$

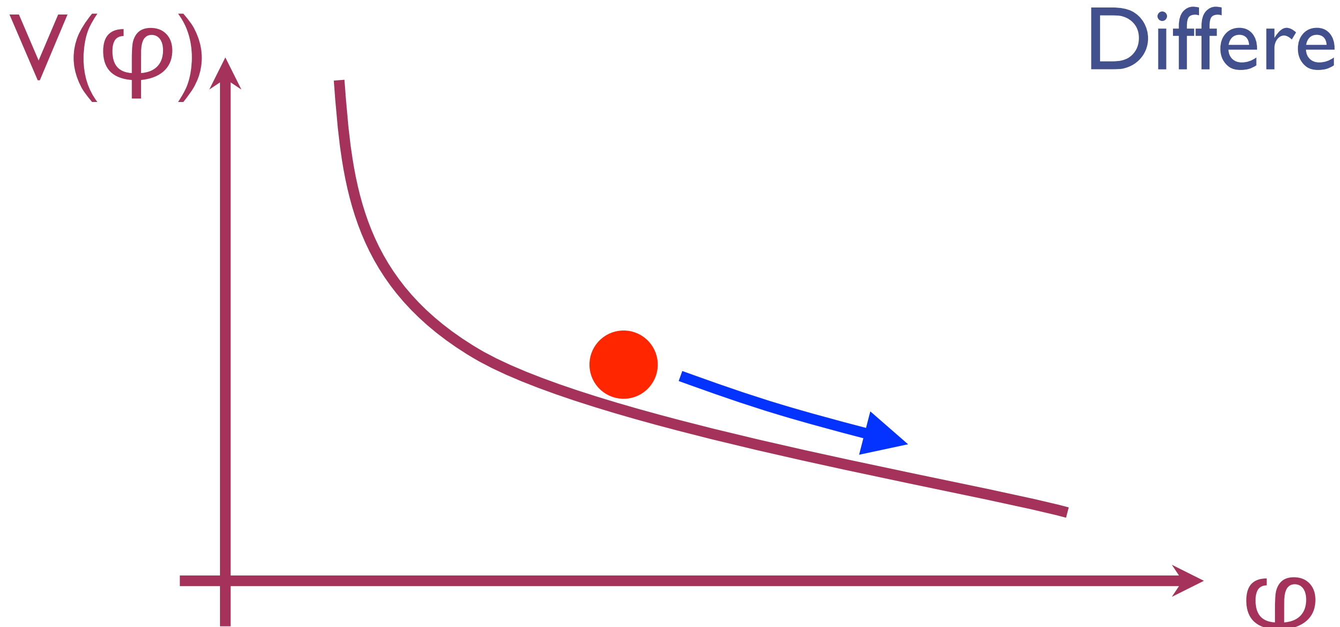
$$T_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}} \frac{\delta S_m}{\delta g^{\mu\nu}}$$

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Dynamical Dark Energy

Maybe there's some principle that sets vacuum energy to zero. Then dark energy might be inflation at the other end of time.

Use scalar fields to source Einstein's equation - *Quintessence*.



Difference: no minimum or reheating

$$L = \frac{1}{2}(\partial_\mu \phi)\partial^\mu \phi - V(\phi)$$

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}(\nabla\phi)^2 + V(\phi)$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0$$

Small slope $\rho_\phi \approx V(\phi) \approx constant$

$$w = -\left[\frac{2V(\phi) - \dot{\phi}^2}{2V(\phi) + \dot{\phi}^2}\right]$$

Are we Being Fooled by Gravity?

We don't *really* measure w - we infer it from the Hubble plot via

$$w_{eff} = -\frac{1}{1-\Omega_m}\left(1+\frac{2}{3}\frac{\dot{H}}{H^2}\right)$$

Maybe, if gravity is modified, can infer value not directly related to energy sources (or perhaps without them!)

One example - Brans-Dicke theories

$$S_{BD} = \int d^4x\sqrt{-g}\left[\phi R - \frac{\omega}{\phi}\left(\partial_\mu\phi\right)\partial^\mu\phi - 2V(\phi)\right] + \int d^4x\sqrt{-g}L_m(\psi_i, g)$$

$\omega > 40000$ (Signal timing measurements from Cassini)

As proof of principle, can show that (with difficulty) can measure $w<-1$, even though no energy conditions violated.

[Carroll, De Felice & M.T., (2005)]

Modifying Gravity

Maybe cosmic acceleration is *entirely* due to corrections to GR!

One thing to understand is: what degrees of freedom does the metric $g_{\mu\nu}$ contain in general?

$g_{\mu\nu}$

The graviton: a spin 2 particle

Scalar fields: spin 0 particles

$h_{\mu\nu}$

A_μ

A vector field: a spin 1 particle

ϕ

We're familiar with this.

These are less familiar.

GR pins vector A_μ and scalar ϕ fields, making non-dynamical, and leaving only familiar graviton $h_{\mu\nu}$

Almost any other action will free some of them up

e.g., f(R) models [Carroll, Duvvuri, M.T. & Turner, (2003)]

More interesting things also possible - massive gravity - see later

A common Language - EFT

How do theorists think about all this? In fact, whether dark energy or modified gravity, ultimately, around a background, it consists of a set of interacting fields in a Lagrangian. The Lagrangian contains 3 types of terms:

- Kinetic Terms: e.g.

$$\partial_\mu\phi\partial^\mu\phi \qquad F_{\mu\nu}F^{\mu\nu} \qquad i\bar{\psi}\gamma^\mu\partial_\mu\psi \qquad h_{\mu\nu}\mathcal{E}^{\mu\nu;\alpha\beta}h_{\alpha\beta} \qquad K(\partial_\mu\phi\partial^\mu\phi)$$

- Self Interactions (a potential)

$$V(\phi) \qquad m^2\phi^2 \qquad \lambda\phi^4 \qquad m\bar{\psi}\psi \qquad m^2h_{\mu\nu}h^{\mu\nu} \qquad m^2h^\mu{}_\mu h^\nu{}_\nu$$

- Interactions with other fields (such as matter, baryonic or dark)

$$\Phi\bar{\psi}\psi \qquad A^\mu A_\mu\Phi^\dagger\Phi \qquad e^{-\beta\phi/M_p}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi \qquad (h^\mu{}_\mu)^2\phi^2 \qquad \frac{1}{M_p}\pi T^\mu{}_\mu$$

Depending on the background, such terms might have functions in front of them that depend on time and/or space.

Many of the concerns of theorists can be expressed in this language

e.g. Weak Coupling

When we write down a classical theory, described by one of our Lagrangians, we are usually implicitly assuming that the effects of higher order operators are small, and therefore mostly ignorable. This needs us to work below the strong coupling scale of the theory, so that quantum corrections, computed in perturbation theory, are small. We therefore need.

- The dimensionless quantities determining how higher order operators, with dimensionful couplings (irrelevant operators) affect the lower order physics be <<1 (or at least <1)

$$\frac{E}{\Lambda} << 1 \qquad \text{(Energy << cutoff)}$$

But be careful - this is tricky! Remember that our kinetic terms, couplings and potentials all can have background-dependent functions in front of them, and even if the original parameters are small, these may make them large - the ***strong coupling problem***! You can no longer trust the theory!

e.g. Ghost-Free

The Kinetic terms in the Lagrangian, around a given background, tell us, in a sense, whether the particles associated with the theory carry positive energy or not.

- Remember the Kinetic Terms: e.g.

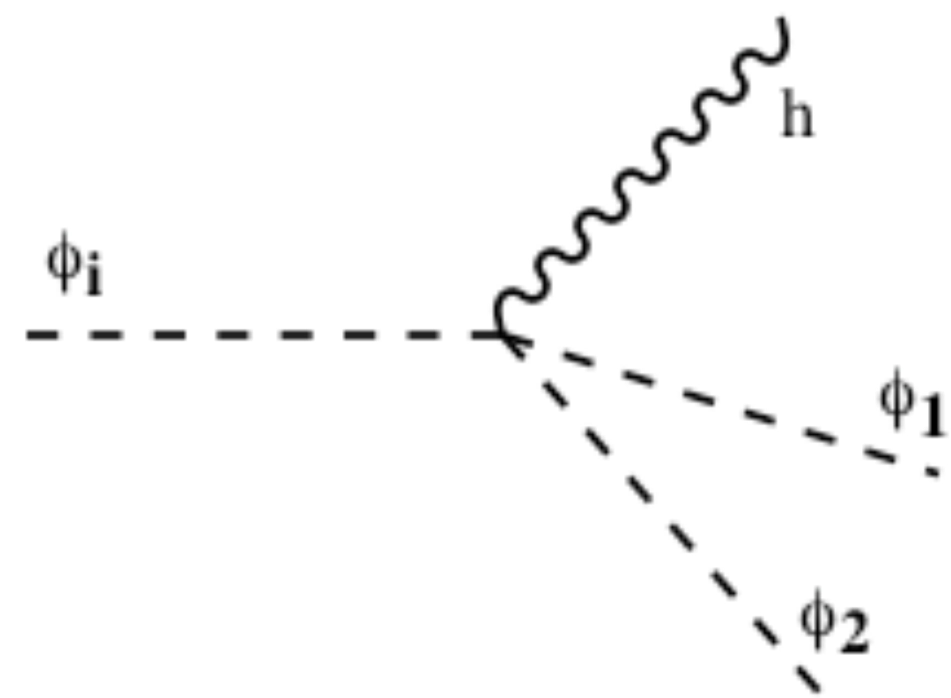
$$-\frac{f(\chi)}{2}K(\partial_\mu\partial^\mu\phi) \rightarrow F(t,x)\frac{1}{2}\dot{\phi}^2 - G(t,x)(\nabla\phi)^2$$

This sets the sign of the KE

- If the KE is negative then the theory has ***ghosts***! This can be catastrophic!

If we were to take these seriously, they'd have negative energy!!

- Ordinary particles could decay into heavier particles plus ghosts
- Vacuum could fragment



(Carroll, Hoffman & M.T.,(2003); Cline, Jeon & Moore. (2004))

e.g. Superluminality ...

Crucial ingredient of Lorentz-invariant QFT: *microcausality*. Commutator of 2 local operators vanishes for spacelike separated points as operator statement

$$[\mathcal{O}_1(x), \mathcal{O}_2(y)] = 0 \; ; \qquad \text{when} \qquad (x-y)^2 > 0$$

Turns out, even if have superluminality, under right circumstances can still have a well-behaved theory, as far as causality is concerned. e.g.

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + \frac{1}{\Lambda^3}\partial^2\phi(\partial\phi)^2 + \frac{1}{\Lambda^4}(\partial\phi)^4$$

- Expand about a background: $\phi = \bar{\phi} + \varphi$
- Causal structure set by effective metric

$$\mathcal{L} = -\frac{1}{2}G^{\mu\nu}(x, \bar{\phi}, \partial\bar{\phi}, \partial^2\bar{\phi}, \ldots)\partial_\mu\varphi\partial_\nu\varphi + \cdots$$

- If G globally hyperbolic, theory is perfectly causal, but *may* have directions in which perturbations propagate outside lightcone used to define theory. May or may not be a problem for the theory - remains to be seen.

But: there can still be worries here, such as analyticity of the S-matrix, …

… & (a little something for the aficionados) Analyticity

Theory may not have a Lorentz-Invariant UV completion! Sometimes can see from 2 to 2 scattering amplitude - related to superluminality: can think of propagation in G as sequence of scattering processes with background field

- Focus on 4-point amplitude $\mathcal{A}(s,t)$ expressed as fn of Mandelstam variables.

- Won't provide details here, but can use analyticity properties of this, with a little complex analysis gymnastics, plus the optical theorem to show

$$\left.\frac{\partial^2}{\partial s^2}\mathcal{A}(s,0)\right|_{s=0} = \frac{4}{\pi}\int_{s_*}^{\infty} ds \frac{\mathrm{Im}\mathcal{A}(s,0)}{s^3} \geq 0$$

So, in forward limit, amplitude must have +ve s^2 part. True for *any* L-I theory described by an S-matrix. Violation implies violation of L-I in the theory.

- There exist other consistency relations. In general can conclude

May have to have a non-Wilsonian, non-LI UV completion of the theory. Might be very hard!!

A Toy Example

Consider a simple and benign-looking model, that is clearly LI

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + \frac{\alpha}{4\Lambda^4}(\partial\phi)^4$$

Can compute 2 to 2 scattering amplitude in field theory

$$\mathcal{A}_{2\to 2}(s,t) = \frac{\alpha}{2\Lambda^4}(s^2+t^2+u^2) = \frac{\alpha}{\Lambda^4}(s^2+t^2-st)$$

Take the forward limit t = 0:

$$\mathcal{A}_{2\to 2}(s,0) = \frac{\alpha}{\Lambda^4}s^2$$

So are not free to choose alpha<0 in a Lorentz-invariant theory with an analytic S-matrix. Note also that, in this theory alpha<0 is naively interesting because it exhibits screening. It *also* exhibits superluminality for that choice: Circumstantial evidence for connection between superluminality and analyticity - but not a proof.

The Need for Screening in the EFT

Look at the general EFT of a scalar field conformally coupled to matter

$$\mathcal{L} = -\frac{1}{2}Z^{\mu\nu}(\phi, \partial\phi, \ldots)\partial_\mu\phi\partial_\nu\phi - V(\phi) + g(\phi)T^\mu_\mu$$

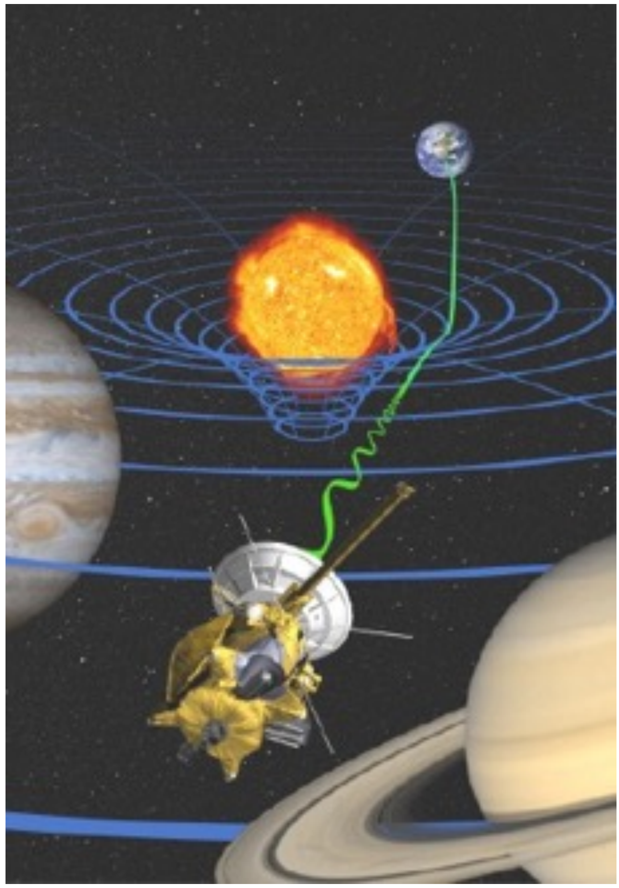
Specialize to a point source $T^\mu_\mu \to -\mathcal{M}\delta^3(\vec{x})$ and expand $\phi = \bar{\phi} + \varphi$

$$Z(\bar{\phi})\left(\ddot{\varphi} - c_s^2(\bar{\phi})\nabla^2\varphi\right) + m^2(\bar{\phi})\varphi = g(\bar{\phi})\mathcal{M}\delta^3(\vec{x})$$

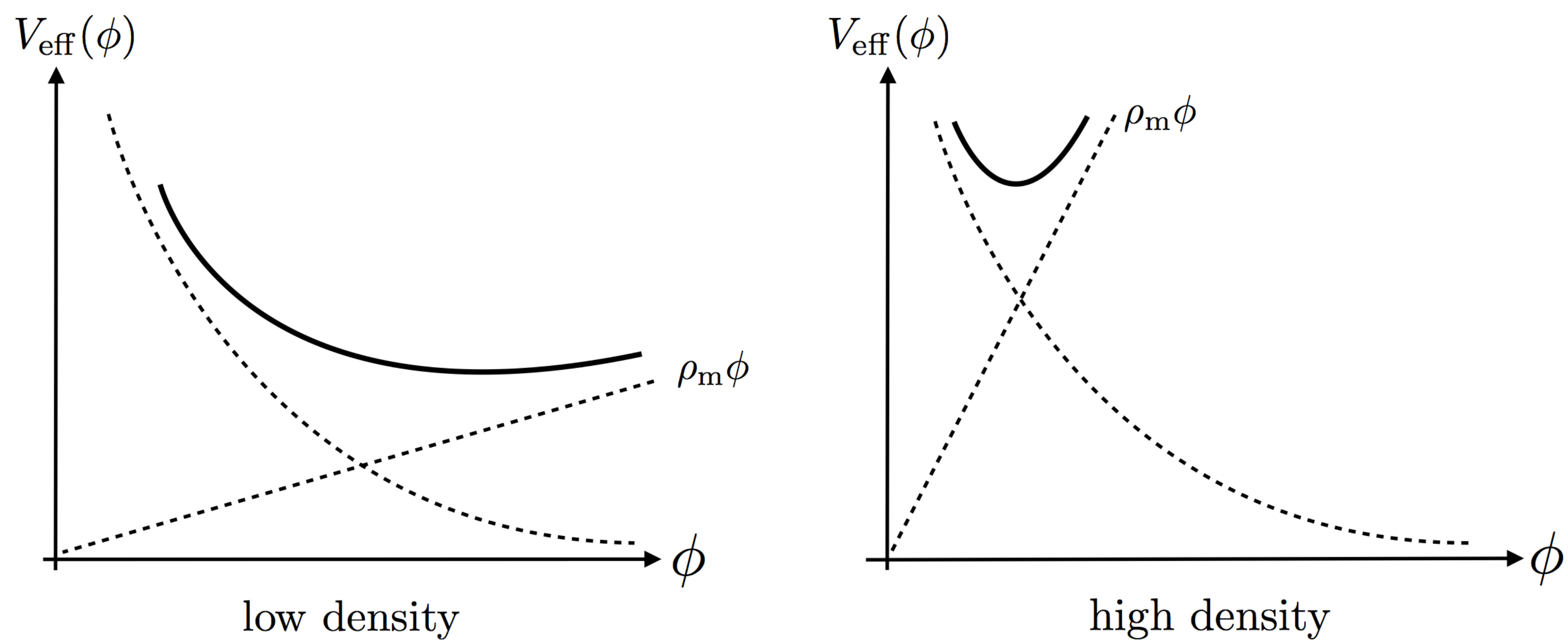
Expect background value set by other quantities; e.g. density or Newtonian potential. Neglecting spatial variation over scales of interest, static potential is

$$V(r) = -\frac{g^2(\bar{\phi})}{Z(\bar{\phi})c_s^2(\bar{\phi})}\frac{e^{-\frac{m(\bar{\phi})}{\sqrt{Z(\bar{\phi})}c_s(\bar{\phi})}r}}{4\pi r}\mathcal{M}$$

So, for light scalar, parameters O(1), have gravitational-strength long range force, ruled out by local tests of GR! If we want workable model need to make this sufficiently weak in local environment, while allowing for significant deviations from GR on cosmological scales!



- There exist several versions, depending on parts of the Lagrangian used
  - Vainshtein: Uses the kinetic terms to make coupling to matter weaker than gravity around massive sources.
  - Chameleon: Uses coupling to matter to give scalar large mass in regions of high density
  - Symmetron: Uses coupling to give scalar small VEV in regions of low density, lowering coupling to matter



General limitation of chameleon (& symmetron) - and any mechanism with screening condition set by local Newtonian potential: range of scalar-mediated force on cosmological scales is bounded. So have negligible effect on linear scales today, and so deviation from LCDM is negligible.

So here I'll focus on the Vainshtein Mechanism

Massive gravity

Quite recent concrete suggestion - consider massive gravity

- Fierz and Pauli showed how to write down a linearized version of this, but...

$$\propto m^2(h^2 - h_{\mu\nu}h^{\mu\nu})$$

- ... thought all nonlinear completions exhibited the "Boulware-Deser ghost".

Over last few years a counterexample has been found. This is a very new, and potentially exciting development!

[de Rham, Gabadadze, Tolley (2011]

$$\mathcal{L} = M_P^2\sqrt{-g}(R + 2m^2\mathcal{U}(g, f)) + \mathcal{L}_m$$

Proven to be ghost free, and investigations of the resulting cosmology - acceleration, degravitation, ... are underway, both in the full theory and in its decoupling limit - galileons!
(Also a limit of DGP)

[Hassan & Rosen(2011)]

Focus on Galileons

In a limit yields novel and fascinating 4d EFT that many of us have been studying. Symmetry: $\pi(x) \to \pi(x) + c + b_\mu x^\mu$

Relevant field referred to as the *Galileon*

(Nicolis, Rattazzi, & Trincherini 2009)

$$\mathcal{L}_1 = \pi \qquad \mathcal{L}_2 = (\partial\pi)^2 \qquad \mathcal{L}_3 = (\partial\pi)^2 \Box\pi$$

$$\mathcal{L}_{n+1} = n\eta^{\mu_1\nu_1\mu_2\nu_2\cdots\mu_n\nu_n} \left(\partial_{\mu_1}\pi\partial_{\nu_1}\pi\partial_{\mu_2}\partial_{\nu_2}\pi\cdots\partial_{\mu_n}\partial_{\nu_n}\pi\right)$$

There is a separation of scales

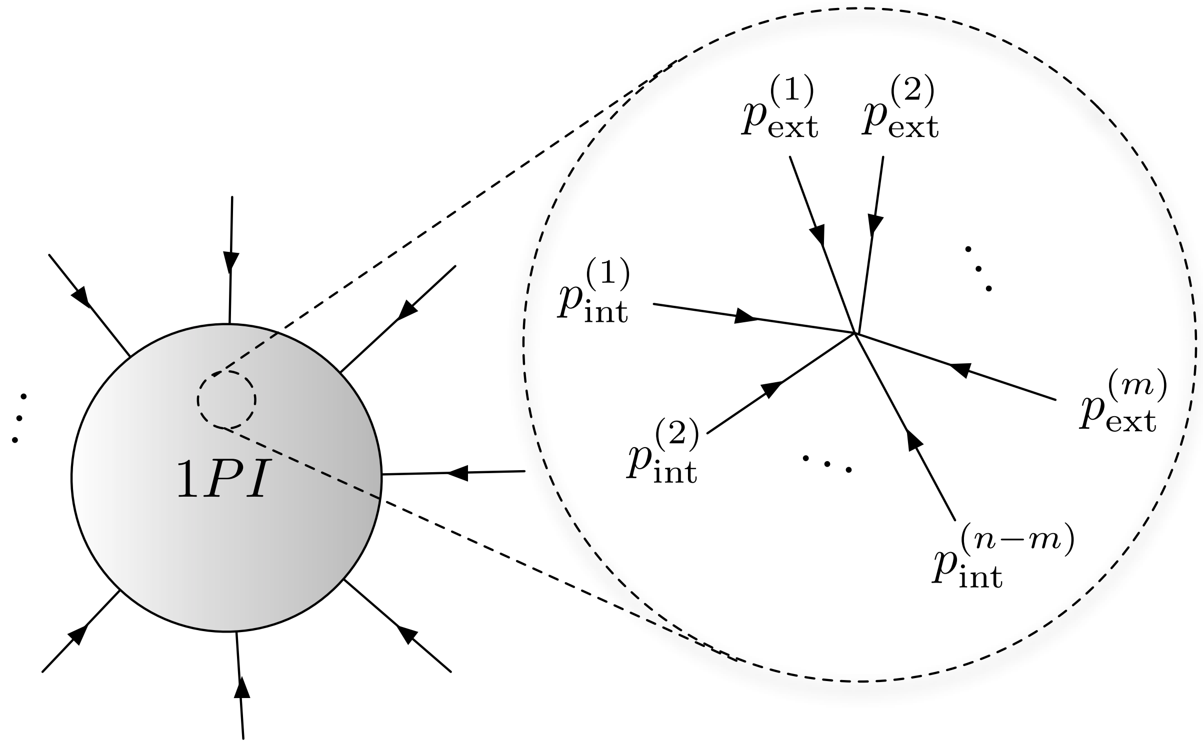
- Allows for classical field configurations with order one nonlinearities, but quantum effects under control.
- So can study non-linear classical solutions.
- Some of these very important (Vainshtein screening)

We now understand that there are many variations on this that share its attractive properties (probe brane construction; coset construction)

Nonrenormalization!

Amazingly terms of galilean form are nonrenormalized (c.f SUSY theories).
Possibly useful for particle physics & cosmology. We'll see.

Expand quantum effective action for the classical field about expectation value



The n-point contribution contains at least 2n powers of external momenta: cannot renormalize Galilean term with only 2n-2 derivatives.
Can show, just by computing Feynman diagrams, that at all loops in perturbation theory, for any number of fields, terms of the galilean form cannot receive new contributions.

[Luty, Porrati, Ratazzi (2003); Nicolis, Rattazzi (2004); Hinterbichler, M.T., Wesley, (2010)]

Can even add a mass term and remains technically natural

The Vainshtein Effect

Consider, for example, the cubic galileon, coupled to matter

$$\mathcal{L} = -3(\partial\pi)^2 - \frac{1}{\Lambda^3}(\partial\pi)^2\Box\pi + \frac{1}{M_{Pl}}\pi T$$

Now look at spherical solutions around a point mass

$$\pi(r) = \begin{cases} \sim \Lambda^3 R_V^{3/2}\sqrt{r} + const. & r \ll R_V \\ \sim \Lambda^3 R_V^3 \frac{1}{r} & r \gg R_V \end{cases} \qquad R_V \equiv \frac{1}{\Lambda}\left(\frac{M}{M_{Pl}}\right)^{1/3}$$

Looking at a test particle, strength of this force, compared to gravity, is then

$$\frac{F_\pi}{F_{\text{Newton}}} = \frac{\pi'(r)/M_{Pl}}{M/(M_{Pl}^2 r^2)} = \begin{cases} \sim \left(\frac{r}{R_V}\right)^{3/2} & R \ll R_V \\ \sim 1 & R \gg R_V \end{cases}$$

So forces much smaller than gravitational strength within the Vainshtein radius - hence safe from 5th force tests.

The Vainshtein Effect

Suppose we want to know the the field that a source generates within the Vainshtein radius of some large body (like the sun, or earth)

Perturbing the field and the source

$$\pi = \pi_0 + \varphi, \quad T = T_0 + \delta T,$$

yields

$$\mathcal{L} = -3(\partial\varphi)^2 + \frac{2}{\Lambda^3}\underbrace{\left(\partial_\mu\partial_\nu\pi_0 - \eta_{\mu\nu}\Box\pi_0\right)}_{\sim\left(\frac{R_v}{r}\right)^{3/2}}\partial^\mu\varphi\partial^\nu\varphi - \frac{1}{\Lambda^3}(\partial\varphi)^2\Box\varphi + \frac{1}{M_4}\varphi\delta T$$

Thus, if we canonically normalize the kinetic term of the perturbations, we raise the effective strong coupling scale, and, more importantly, heavily suppress the coupling to matter!

Regimes of Validity

The usual quantum regime of a theory

$$r \ll \frac{1}{\Lambda}$$

$$\alpha_{cl} \sim \left(\frac{R_V}{r}\right)^{3/2} \gg 1$$

$$\alpha_q \sim \frac{1}{(r\Lambda)^2} \gg 1$$

$$\frac{1}{\Lambda} \ll r \ll R_V$$

$$\alpha_{cl} \sim \left(\frac{R_V}{r}\right)^{3/2} \gg 1$$

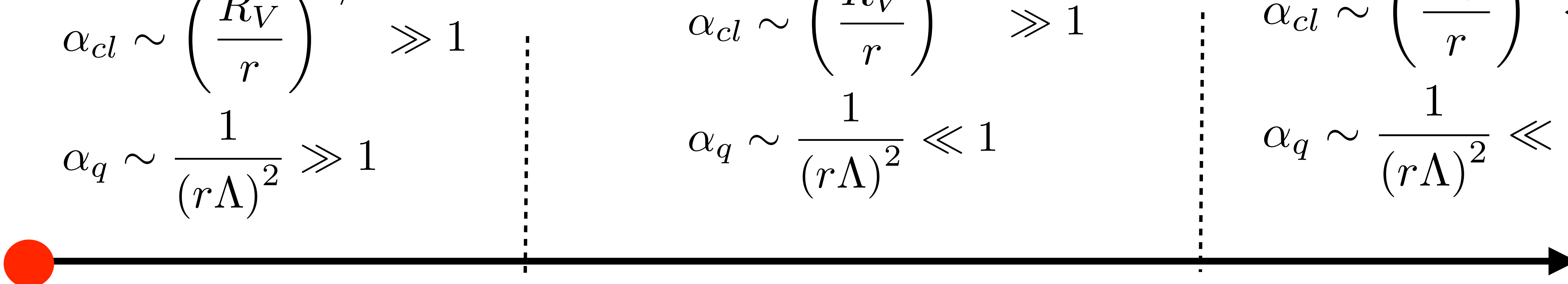
$$\alpha_q \sim \frac{1}{(r\Lambda)^2} \ll 1$$

The usual linear, classical regime of a theory

$$r \gg R_V$$

$$\alpha_{cl} \sim \left(\frac{R_V}{r}\right)^{3} \ll 1$$

$$\alpha_q \sim \frac{1}{(r\Lambda)^2} \ll 1$$



r

$$r \sim \frac{1}{\Lambda}$$

$$r \sim R_V$$

A new classical regime, with order one nonlinearities

New Perspective - the Coset Construction

- For those of you who are more mathematically inclined, there is a nice story here that may have implications for, among other things, better understanding the nonrenormalization theorems.

- Since the galilean symmetry in nonlinearly realized, can use the coset construction to build the effective theory. (We've recently shown that one can do this for massive gravity also!)

[Goon, Hinterbichler, Joyce & M.T., arxiv:1412.6098 [hep-th])

- Galileons are Wess-Zumino terms! In d dimensions are d-form potentials for (d+1)-forms which are non-trivial co-cycles in Lie algebra cohomology of full symmetry group relative to unbroken one. Slightly different stories for DBI and conformal Galileons.

[Goon, Hinterbichler, Joyce & M.T., arxiv:1203.3191 [hep-th])

The Vainshtein Effect is Very Effective!

Fix r_c to make solutions cosmologically interesting - 4000 Mpc $=10^{10}$ ly

$$r^* = \left(\frac{2GM}{c^2} r_c^2\right)^{1/3}$$

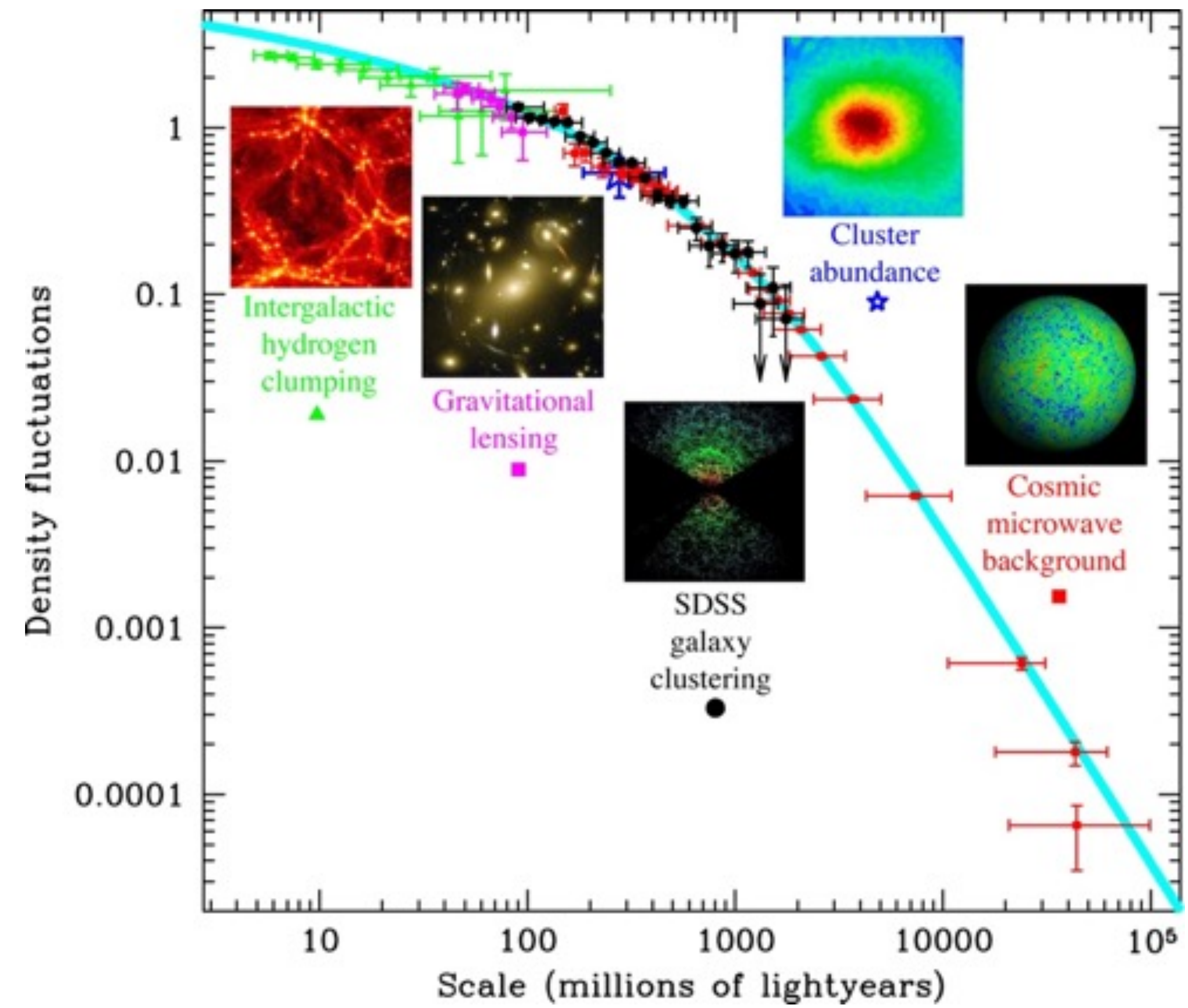
→ ~0.1 kpc = 10^7 AU

→ ~Mpc ~ 30 galactic radii

→ ~10 Mpc ~ 10 virial radii

Can look for signals in, e.g., cosmology

- Weak gravitational lensing
- CMB lensing and the ISW effect
- Redshift space galaxy power spectra
- Combining lensing and dynamical cross-correlations
- The halos of galaxies and galaxy clusters



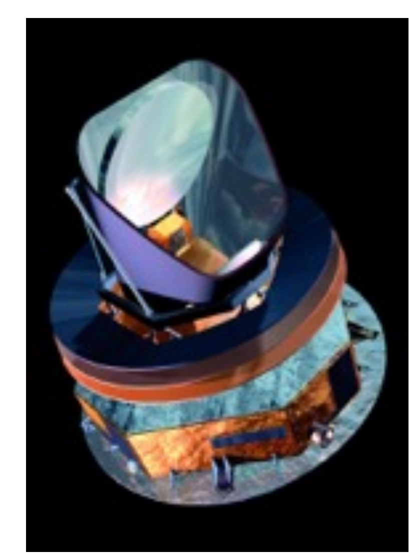
- Very broadly: Gravity is behind the expansion history of the universe

- But it is also behind how matter clumps up - potentially different.

- This could help distinguish a CC from dark energy from other possibilities

DARK ENERGY SURVEY

Large Synoptic Survey Telescope



These Theories are Difficult

Summary

- Cosmic acceleration: one of our deepest problems
- Questions thrown up by the data need to find a home in fundamental physics, and many theorists are hard at work on this. Requires particle physicists and cosmologists to work together.
- We still seem far from a solution in my opinion, but some very interesting ideas have been put forward in last few years.
- Many ideas (and a lot of ugly ones) being ruled out or tightly constrained by these measurements. And fascinating new theoretical ideas are emerging (even without acceleration)
- Serious models only need apply - theoretical consistency is a crucial question. We need (i) models in which the right questions can be asked and (ii) A thorough investigation of the answers. (*Beware of theorists' ideas of likelihood.*)

Thank You!