~~[illegible]~~

Add Features

- Subsystems, Subalgebra codes
- Only Errasure Error
- RT as a general property of QEC

An Example $\quad |\psi\rangle = \sum_{i=1}^{2} C_i |i\rangle$

$$|\tilde{\psi}\rangle = \sum_{i=1} C_i |\tilde{i}\rangle$$

$$|\tilde{0}\rangle = \frac{1}{\sqrt{3}}\left(|000\rangle + |111\rangle + |222\rangle\right)$$

$$|\tilde{1}\rangle = \frac{1}{\sqrt{3}}\left(|012\rangle + |120\rangle + |201\rangle\right)$$

$$|\tilde{2}\rangle = \frac{1}{\sqrt{3}}\left(|021\rangle + |102\rangle + |210\rangle\right)$$

Symetric

- Entangled $\Rightarrow \rho_1 = \rho_2 = \rho_3 = \frac{I}{3}$

Claim : $\exists U_{12}$ s.e.. $U_{12}^{\dagger} |\tilde{i}\rangle = |i\rangle_1 |\chi\rangle_{23}$

$$\left(|\chi\rangle = \frac{1}{\sqrt{3}}\left(|00\rangle + |11\rangle + |22\rangle\right) \right)$$

$$U_{12}^{\dagger} |\tilde{\psi}\rangle = |\psi\rangle_1 |\chi\rangle_{23}$$

Now, say $\quad \tilde{O}|\tilde{i}\rangle = \sum_j (O)_{ji} |\tilde{j}\rangle$

$$\tilde{O}^{\dagger}|\tilde{i}\rangle = \sum_j (O)^*_{ij} |\tilde{j}\rangle$$

D.H.2

$$O_{12} = U_{12} O_1 U_{12}^{+}$$

$$O_{12} |\varphi\rangle = \tilde{O} |\tilde{\varphi}\rangle$$

$$O_{12}^{+} |\tilde{\varphi}\rangle = \tilde{O}^{+} |\varphi\rangle$$

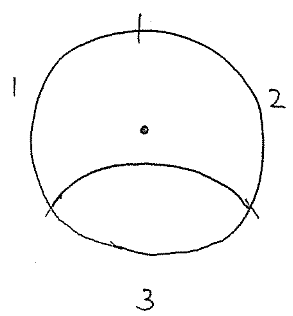
$$\tilde{\rho} = \sum_{ij} \tilde{\rho}_{ij} |i\rangle\langle j|$$

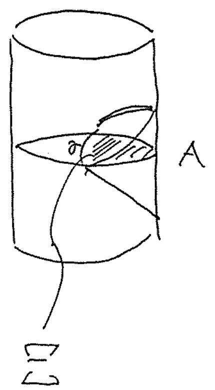
Finally, $\tilde{\rho} = U_{12} \left(\rho_1 \otimes |\chi\rangle\langle\chi| \right) U_{12}^{+}$

$$S(\tilde{\rho}_3) = \log 3$$

B.H.がある時の bulk中の correctionといいでしょう.

$$S(\tilde{\rho}_{12}) = S(\tilde{\rho}) + \log 3$$





$$\partial \Xi = \gamma \cup A$$

~~$\mathcal{E}_A = D$~~

$$\mathcal{E}_A = D(\Xi)$$

・QGでは

$$S_A = \frac{\langle \mathrm{Area}(\gamma) \rangle}{4G} + S_{bulk}(\Xi) \qquad \text{(FLM)}$$

Standard QEC　　　Say $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_{\bar{A}}$

$\mathcal{H}_{code} \subset \mathcal{H}$　　Some subsystem

Moreover: $|\tilde{i}\rangle$ is a basis for H_{code}　$(|R| \equiv \dim H_{code})$

$$|\Phi\rangle = \frac{1}{\sqrt{|R|}} \sum_i |i\rangle_R |\tilde{i}\rangle_{A\bar{A}}$$

D.H.3
THM

The following are equivalent

1) Pick $\mathcal{H}_A = \mathcal{H}_{A_1} \otimes \mathcal{H}_{A_2}$ $\quad |A_1| = |R|$ Then $\exists U_A$, $|\chi\rangle_{A_2\bar{A}}$

s.t. $|\tilde{i}\rangle = U_A\left(|i\rangle_{A_1} |\chi\rangle_{A_2}\right)$ ← (1)が正しいと(2)、(3)、(4)は正しい.

2) $\rho_{R\bar{A}}(\phi) = \rho_R(\phi) \otimes \rho_{\bar{A}}(\phi)$

3) $\forall \tilde{O}$ ating within $\mathcal{H}_{code}$, $\exists O_A$ on $\mathcal{H}_A$ s.t. $\forall |\tilde{\varphi}\rangle \in \mathcal{H}_{code}$

$$O_A |\tilde{\varphi}\rangle = \tilde{O} |\tilde{\varphi}\rangle$$
$$O_A^\dagger |\tilde{\varphi}\rangle = \tilde{O}^\dagger |\tilde{\varphi}\rangle$$

π: projection op

zero

4) $\forall X_{\bar{A}}$ on $\mathcal{H}_{\bar{A}}$, $\pi_{code} X_{\bar{A}} \pi_{code} \propto I_{code} \oplus O_{code\perp}$

1) ⇒ 2) : $\quad U_A^\dagger |\phi\rangle = \frac{1}{\sqrt{|R|}} \left(\sum_i |i\rangle_R |i\rangle_{A_1}\right) \otimes |\chi\rangle_{A_2\bar{A}}$

$$\Rightarrow \rho_{R\bar{A}} = \rho_R \otimes \rho_{\bar{A}}$$

2) ⇒ 1) : Note : $|\phi\rangle$ is a prification of $\rho_{R\bar{A}} = \rho_R \otimes \rho_{\bar{A}}$

$$|\phi'\rangle = \frac{1}{\sqrt{|R|}} \sum_i |i\rangle \sum_{a_1} \sqrt{C_a}\, |a\rangle_{\bar{A}} |i\rangle_{A_1} |a\rangle_{A_2}$$

eigenstate of $\rho_{\bar{A}}$

Any two prification differs by base U_A

$$\Rightarrow |\phi\rangle = U_A |\phi'\rangle = \frac{1}{\sqrt{|R|}} \sum_i |i\rangle_R U_A \Big(|i\rangle_A \underbrace{\sum_a \sqrt{C_a}\, |a\rangle_{A_2} |a\rangle_{\bar{A}}}_{\equiv |\chi\rangle}\Big)$$

D.H. 4

1) ⇒ 3) : $\mathcal{O}_A \equiv U_A \, \mathcal{O}_{A_1} U_A^\dagger$

3) ⇒ 4) : Not 4) ⇒ Not 3)

$\pi \, X_{\bar{A}} \, \pi \not\propto I_{code}$

Then $\exists \, \tilde{\mathcal{O}}_x$ on X_{code} s.t. $[\pi_{code} X_{\bar{A}} \pi_{code}, O_x] \neq 0$

$\Rightarrow \exists \, |\tilde{\varphi}\rangle, |\hat{\phi}\rangle \in X_{code}$ s.t. $\langle\tilde{\varphi}| [O_x, \pi X_{\bar{A}} \pi] |\hat{\phi}\rangle$

$= \langle\tilde{\varphi}| [O_x, X_{\bar{A}}] |\hat{\phi}\rangle \neq 0$

4) ⇒ 2) : $\pi X_{\bar{A}} \pi = \lambda I_{code}$, $\forall X_{\bar{A}}$

note : $\langle\phi| \pi X_A \pi |\phi\rangle = \langle\phi| X_{\bar{A}} |\phi\rangle = \lambda$

$\forall \mathcal{O}_R$ $\quad \langle\phi| \mathcal{O}_R X_{\bar{A}} |\phi\rangle = \langle\phi| O_R \pi X_{\bar{A}} \pi |\phi\rangle$

$= \lambda \langle\phi| O_R |\phi\rangle$

$= \lambda \langle\phi| X_{\bar{A}} |\phi\rangle \langle\phi| O_R |\phi\rangle$

⇒ 2) ✓

$\Rightarrow \langle\varphi| \mathcal{O}_A X_{\bar{A}} |\varphi\rangle = \langle\varphi| \mathcal{O}_A |\varphi\rangle \langle\varphi| X_{\bar{A}} |\varphi\rangle \quad \forall \mathcal{O}_A, X_{\bar{A}}$

<u>Entropies</u> | $\tilde{\rho} = U_A \left(\rho_A \otimes |\chi\rangle\langle\bar{\chi}|_{A,\bar{A}} \right) U_A^\dagger$

$\Rightarrow S(\tilde{\rho}_A) = \underbrace{S(\tilde{\rho})}_{\text{Quantum correction}} + S(\mathrm{tr}_A |\chi\rangle\langle\chi|)$

$S(\tilde{\rho}_{\bar{A}}) = S(\mathrm{tr}_{\bar{A}} |\chi\rangle\langle\chi|)$

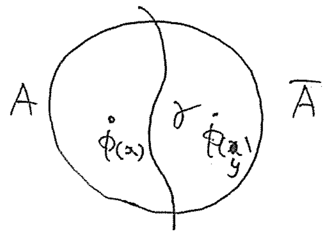
complement regionでは Quantum correction 0

⇒ 問題

D.H.5

~~Properties~~ Problems

1)



$\langle \phi(x)\phi(y)\rangle = 0$ であるべきなのに 0 にならない!

2) $S(\tilde{\rho}_{\bar{A}})$ has no quantum correction

Say $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_{\bar{A}}$, $\mathcal{H}_{code} = \mathcal{H}_a \otimes \mathcal{H}_{\bar{a}}$

$|\tilde{i}\rangle$: a basis for $\mathcal{H}_{a}$

$|\tilde{j}\rangle$: a basis for $\mathcal{H}_{\bar{a}}$

$|\phi\rangle \propto \sum_{ij} |i\rangle_R |j\rangle_{\bar{R}}$ ~~$|ij\rangle$~~ $|\tilde{ij}\rangle_{A\bar{A}}$

THM1 1) Say $\mathcal{H}_A = \mathcal{H}_{A_1} \otimes \mathcal{H}_{A_2}$ Then $\exists U_A$, $|\chi_i\rangle_{A_2\bar{A}}$ (orthogonal)

s.t. $|\tilde{ij}\rangle = U_A(|i\rangle_{A_1} |j\rangle_{A_2\bar{A}})$

2) $\rho_{R\bar{R}A}(\phi) = \rho_R(\phi) \otimes \rho_{\bar{R}\bar{A}}(\phi)$

3) $\forall \tilde{O}_a$ on $\mathcal{H}_a$, $\exists O_A$ s.t. $\forall |\tilde{\Psi}\rangle \in \mathcal{H}_{code}$

$$O_A |\tilde{\Psi}\rangle = \tilde{O}_A |\tilde{\Psi}\rangle$$
$$O_A^{\dagger} |\tilde{\Psi}\rangle = \tilde{O}_A^{*} |\tilde{\Psi}\rangle$$

4) $\forall X_{\bar{A}}$ on $\mathcal{H}_{\bar{A}}$, $\Pi X_{\bar{A}} \Pi = (I_a \otimes I_{\bar{a}})_{code} \oplus O_{code\perp}$

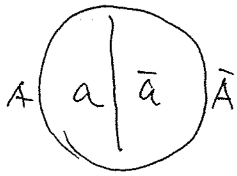
Now, say 1-4) hold basis for $a \to A$, and $\bar{a} \to \bar{A}$

Then $\Rightarrow$ $|\tilde{ij}\rangle = U_A U_{\bar{A}} (|i\rangle_{A_1} |j\rangle_{\bar{A}_1} \cdot |\chi\rangle_{A_2\bar{A}_2})$

D.H. 6

$\boxed{\text{Entropies}}$

$$\tilde{\rho} = U_A U_{\bar{A}} \left(\rho_{A_1 \bar{A}_1} \otimes |\chi\rangle\langle\chi|_{A_2 \bar{A}_2} \right) U_A^\dagger U_{\bar{A}}^\dagger$$



$$S(\tilde{\rho}_A) = S(\tilde{\rho}_a) + S(\mathrm{tr}_{\bar{A}_2} |\psi\rangle\langle\psi|)$$

Area term

$$S(\tilde{\rho}_{\bar{A}}) = S(\tilde{\rho}_{\bar{a}}) + S(\mathrm{tr}_{\bar{A}_2} |\chi\rangle\langle\chi|)$$

~~+loop~~

Spin 1 loop correction