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Primordial perturbations from hyperinflation

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(Physical Review D 96, 103533)

Inflation

- Phenomenological success

    - Solving problems of big-bang cosmology

      (Flatness problem, Horizon problem, Unwanted relics,…)

    - Providing origin of the structures in the Universe

      almost scale invariant, adiabatic and Gaussian perturbations

      supported by current observations (CMB, LSS)

- Theoretical challenge

    Still nontrivial to embed the single-field slow-roll inflation into more fundamental theory (Review, Baumann & McAllister, `14)

    - Difficult to obtain a flat potential

    - Scalar fields are ubiquitous in fundamental theories

Inflation with negative field-space curvature

- Formulation to analyze perturbations

  Sasaki & Stewart, `96, Gong & Tanaka, `11, Elliston et al, `12

- Examples (without significant effect on perturbation)

  - Inflation with large extra-dimension Kaloper et al, `00

  - Alpha-attractor scenario Kallosh, Linde, Roest, `13, …..

- Examples (with significant effect on perturbation)

  - Geometrical destabilization Renaux-Petel & Turzynski, `15

  - Hyperinflation SM & Mukohyama, `17,  (See also Brown, `17)

Model

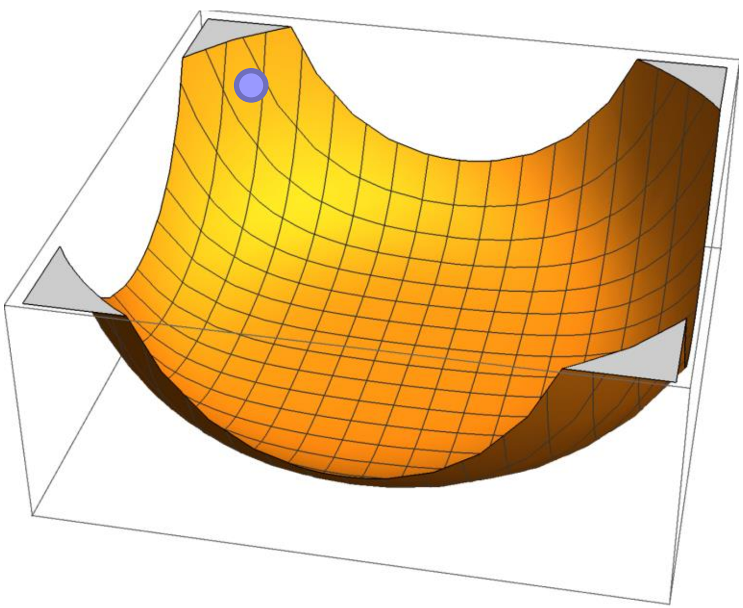
$$S=\int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R-\frac{1}{2}G_{IJ}\nabla_\mu\varphi^I\nabla^\mu\varphi^J-V(\phi)\right]$$

Hyperbolic field-space with curvature scale L

$\varphi^I=(\phi,\chi)$ ϕ : radial direction χ : angular direction

$G_{\phi\phi}=1\,,G_{\chi\chi}=L^2\sinh^2\frac{\phi}{L}\simeq\frac{L^2}{4}e^{2\frac{\phi}{L}}$ (for $\phi\gg L$)

Potential with rotational symmetry, a minimum at $\phi=0$



cf. ``spinflation”

Easson et al, `07

$\Rightarrow$ $\dot\chi=Aa^{-3}e^{-2\frac{\phi}{L}}$

A : integration constant

Background dynamics of scalar-fields

- Basic equations

$$\left[\begin{array}{l} H^2 = \frac{1}{3M_{\rm Pl}^2}\left(\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\frac{L^2}{4}e^{2\frac{\phi}{L}}\dot{\chi}^2 + V(\phi)\right) \\ \ddot{\phi} + 3H\dot{\phi} - \frac{L}{4}e^{2\frac{\phi}{L}}\dot{\chi}^2 + V_{,\phi} = 0 \end{array}\right.$$

with $\dot{\chi} = Aa^{-3}e^{-2\frac{\phi}{L}}$

for ``slow-roll"

- Inflationary attractors

standard inflation

$$\dot{\phi} = -\frac{V_{,\phi}}{3H}$$

$$\dot{\chi} = 0$$

$(V_{,\phi} < 9LH^2)$

hyperinflation

$$\dot{\phi} = -3LH$$

$$\frac{L}{2}e^{\frac{\phi}{L}}\dot{\chi} = hLH$$

with $h \equiv \sqrt{\frac{V_{,\phi}}{LH^2} - 9}$

parametrizing angular velocity

$$\frac{V_{,\phi}}{V} = \frac{3L}{M_{\rm Pl}^2}$$

$(V_{,\phi} > 9LH^2)$

Power-law hyperinflation

- Potential

$$V(\phi) = V_0 \exp\left[\lambda \frac{\phi}{M_{\rm Pl}}\right], \ \lambda > 0 \quad \Rightarrow \quad h = \sqrt{3\lambda \frac{M_{\rm Pl}}{L} - 9} \quad \text{(constant)}$$

- Slow-roll parameter

$$\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{3}{2}\lambda \frac{L}{M_{\rm Pl}} \qquad \left(= \frac{3L}{2}\left(\frac{V_{,\phi}}{V}\right)\right)$$ for general potential

- Condition for hyperinflaion

$$1 \gg \epsilon > \epsilon_{\rm crit} \quad \Rightarrow \quad \boxed{\frac{2M_{\rm Pl}}{3L} \gg \lambda > \frac{3L}{M_{\rm Pl}}}$$

For $M_{\rm Pl} \gg L$, we can obtain inflation from steeper potential !!

cf. $\sqrt{2} > \lambda > 0$ for standard power-law inflation

Basic equations for linear perturbations

- Perturbation (spatially-flat gauge, $h_{ij} = a(t)^2 \delta_{ij}$)

$$\phi = \bar{\phi} + \delta\phi\,, \quad \chi = \bar{\chi} + \delta\chi\,,$$

- Canonical variables

$$u_\phi \equiv a\delta\phi\,, \quad u_\chi \equiv a\sqrt{G_{\chi\chi}}\delta\chi\,, \quad \text{with} \quad G_{\chi\chi} = \frac{L^2}{4} e^{2\frac{\phi}{L}}$$

- Equations of motion (conformal time $\tau \simeq -\frac{1}{aH}$)

$$u''_\phi + \frac{2h}{\tau} u'_\chi - \frac{4h}{\tau^2} u_\chi - \frac{2(h^2+1)}{\tau^2} u_\phi + k^2 u_\phi = 0$$

$$u''_\chi - \frac{2h}{\tau} u'_\phi - \frac{2}{\tau^2} u_\chi - \frac{2h}{\tau^2} u_\phi + k^2 u_\chi = 0$$

Coupling depending on h $\quad h = \sqrt{\frac{V_{,\phi}}{LH^2} - 9}$

Behavior of perturbations in asymptotic regions

- Asymptotic solutions on subhorizon scales $(|k\tau| \gg 1)$

$$u_\chi = C_1 e^{ik\tau + ih\log|k\tau|} + C_2 e^{ik\tau - ih\log|k\tau|} + C_3 e^{-ik\tau + ih\log|k\tau|} + C_4 e^{-ik\tau - ih\log|k\tau|},$$

$$u_\phi = iC_1 e^{ik\tau + ih\log|k\tau|} - iC_2 e^{ik\tau - ih\log|k\tau|} + iC_3 e^{-ik\tau + ih\log|k\tau|} - iC_4 e^{-ik\tau - ih\log|k\tau|}$$

Bunch-Davies vacuum ⇒ $C_1 = C_2 = 0\,, \quad C_3 = C_4 = \dfrac{1}{\sqrt{2k}}$

- Asymptotic solutions on superhorizon scales $(|k\tau| \ll 1)$

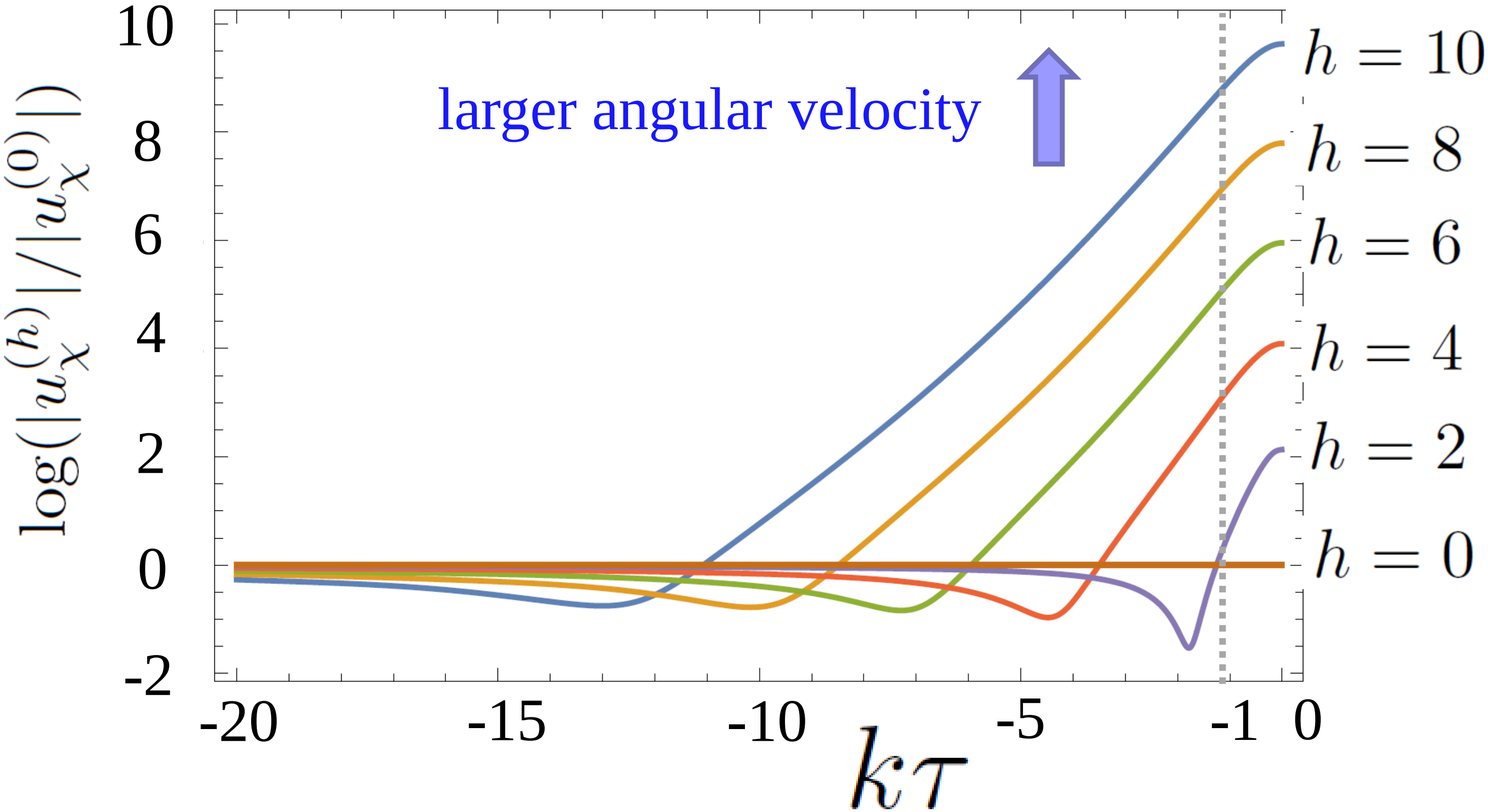
$$u_\chi = \frac{c_1}{(-\tau)} + c_2(-\tau)^2 + c_3(-\tau)^{\frac{1}{2}+\frac{1}{2}\sqrt{9-8h^2}} + c_4(-\tau)^{\frac{1}{2}-\frac{1}{2}\sqrt{9-8h^2}},$$

$$u_\phi = -\frac{3}{h}\frac{c_1}{(-\tau)} + \frac{\sqrt{9-8h^2}-3}{4h} c_3(-\tau)^{\frac{1}{2}+\frac{1}{2}\sqrt{9-8h^2}} - \frac{\sqrt{9-8h^2}+3}{4h} c_4(-\tau)^{\frac{1}{2}-\frac{1}{2}\sqrt{9-8h^2}}$$

(Adiabatic mode, constant shift in χ , two heavy modes)

For the concrete value of c_1 , we need numerical calculations !!

Time evolution of perturbations



Instability starts at

$$k = h|\tau|$$

$$\frac{|u_\chi^{(h)}|}{|u_\chi^{(0)}|} \sim e^{p+qh} \quad \text{with} \quad p = 0.395\,, \quad q = 0.924$$

at late-time

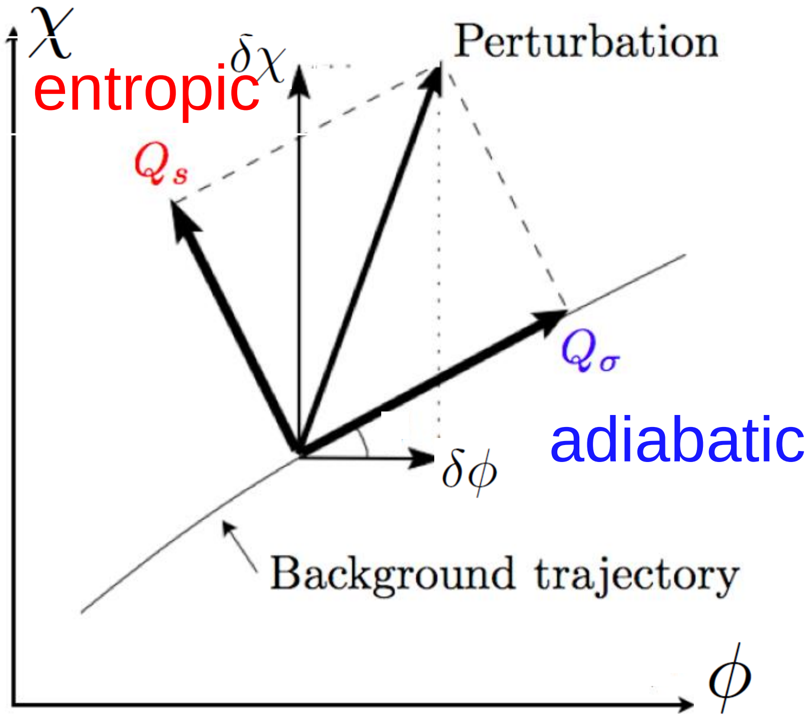
Curvature perturbation

- Curvature perturbation

$$h_{ij} = a^2(1-2\psi)\delta_{ij}\,, \quad T^0{}_i = \partial_i q \qquad\qquad \dot\sigma \equiv \sqrt{\dot\phi^2 + G_{\chi\chi}\dot\chi^2}$$

$$\Rightarrow \quad \mathcal{R} \equiv \psi - \frac{H}{\rho+p}\delta q = \frac{H}{\dot\sigma}Q_\sigma = \frac{H}{\dot\phi}\delta\phi$$

- Super-Hubble evolution of $\mathcal{R}$ in multi-field inflation



Gordon, Wands, Bassett, Maartens `01

$$\dot{\mathcal{R}} \simeq -2\frac{H}{\dot\sigma^2}V_{,s}Q_s = 0$$

For hyperinflation

$$u_\phi = -\frac{3}{h}u_\chi$$

Observational constraints

- Power spectrum

Exponential enhancement in h !!

$$\mathcal{P}_{\mathcal{R}} = \frac{H^2}{\dot{\phi}^2}\mathcal{P}_{\delta_\phi} = \frac{1}{(2\pi)^2}\frac{1}{2M_{\rm Pl}^2}\frac{H^2}{\epsilon}\boxed{\frac{h^2+9}{h^2}e^{2p+2qh}}$$

- Spectrum index

with $p = 0.395\,, \quad q = 0.924$

$$n_s - 1 \equiv \frac{d\ln\mathcal{P}_{\mathcal{R}}}{d\ln k} \simeq -2\epsilon + (qh-1)\eta \qquad \eta \equiv \frac{\dot{\epsilon}}{H\epsilon}$$

cf. Planck constraint $\quad n_s = 0.9655 \pm 0.0062 \quad (68\%\ \text{C. L.})$

➡ Deviation from exponential potential must be small !!

- Tensor-to-scalar ratio

$$r \equiv \frac{\mathscr{P}_T}{\mathscr{P}_{\mathscr{R}}} = 16\varepsilon\frac{h^2}{h^2+9}e^{-2p-2qh}$$

➡ GW detection will reject hyperinflation with large h !!

Summary

- We have studied hyperinflation with action

$$S = \int d^4x\sqrt{-g}\left[-\frac{1}{2}(\nabla_\mu\phi)^2 - \frac{1}{2}L^2\sinh^2\frac{\phi}{L}(\nabla_\mu\chi)^2 - V(\phi)\right]$$

(See also, Brown, `17)

- We have quantified the deviation from de Sitter spacetime

$$\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{3L}{2}\left(\frac{V_{,\phi}}{V}\right), \quad \eta \equiv \frac{\dot{\epsilon}}{H\epsilon} \simeq 3L\left(\frac{V_{,\phi}}{V} - \frac{V_{,\phi\phi}}{V_{,\phi}}\right)$$

Inflation from potentials steeper than usual for $M_{\rm Pl} \gg L$!!

- We have calculated the power spectrum of $\mathscr{R}$

$$\mathcal{R} = \frac{H}{\dot{\phi}}\delta\phi\,, \quad \mathcal{P}_{\mathcal{R}} = \frac{1}{(2\pi)^2}\frac{1}{2M_{\rm Pl}^2}\frac{H^2}{\epsilon}\frac{h^2+9}{h^2}e^{2p+2qh}\,,$$

$$p = 0.395\,, \quad q = 0.924$$

$$n_s - 1 \simeq -2\epsilon + (qh-1)\eta$$

Potentials deviating from exponential are strongly constrained !!

Thank you very much !!