On ringing gravitational waves from black holes

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Our new Innovative Area has just started from the last summer.

Synergy between data analysis and theory researches

A: BH binaries

A-01
Testing gravity using gravitational waves

A-02
Gravity and Cosmology

A-03
BH binary formation

B: NS binaries

B-01
Internal structure of NS

B-02
Gamma-ray burst and BH

B-03
r –process elements

C: Supernovae

C-01
SN explosion mechanism

C-02
SN explosion mechanism via ν observation

Physics and astronomy motivated by GW observations

There are many examples of extended models of gravity that would require dedicated analysis of gravitational wave data.

BH quasi-normal mode (QNM)

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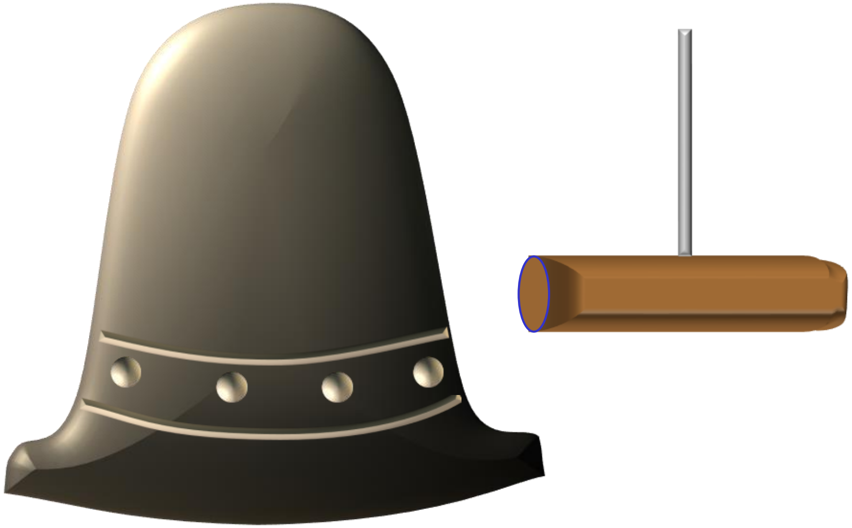
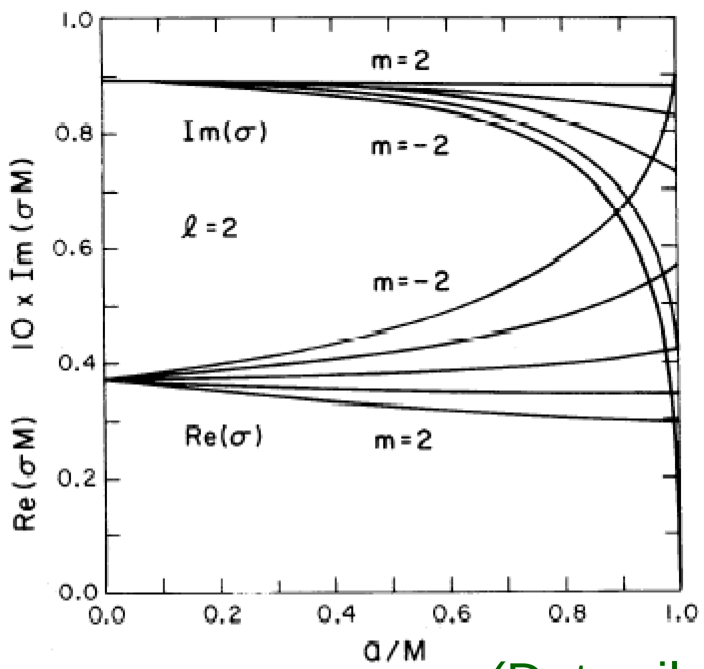




Frequency (f_R) and damping rate (f_I) are determined by the BH mass and spin.

Evidence for the formation of BH

(Detweiler ApJ239 292 (1980))

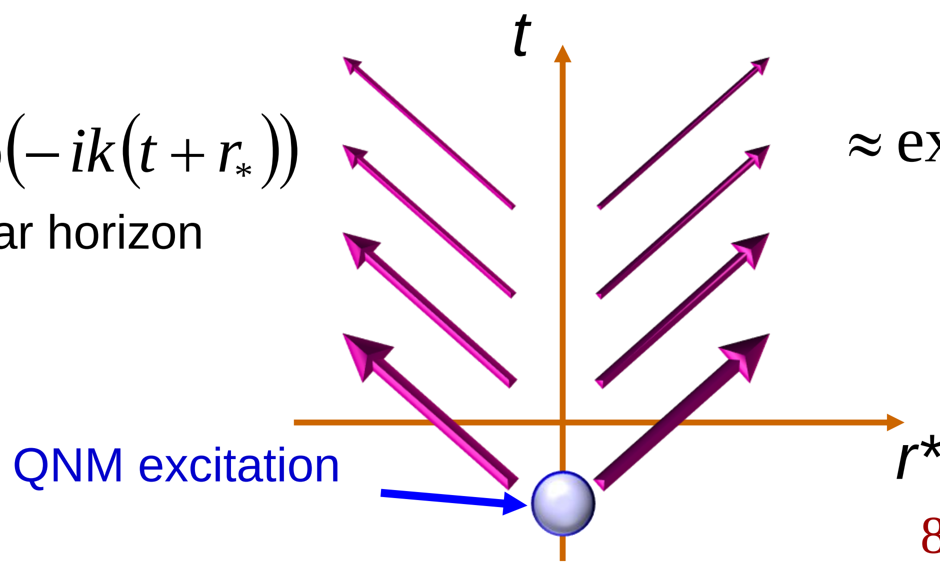
How deeply can we see BH spacetime by observing QNMs?

$$\approx \exp\left(-ik\left(t+r_*\right)\right)$$

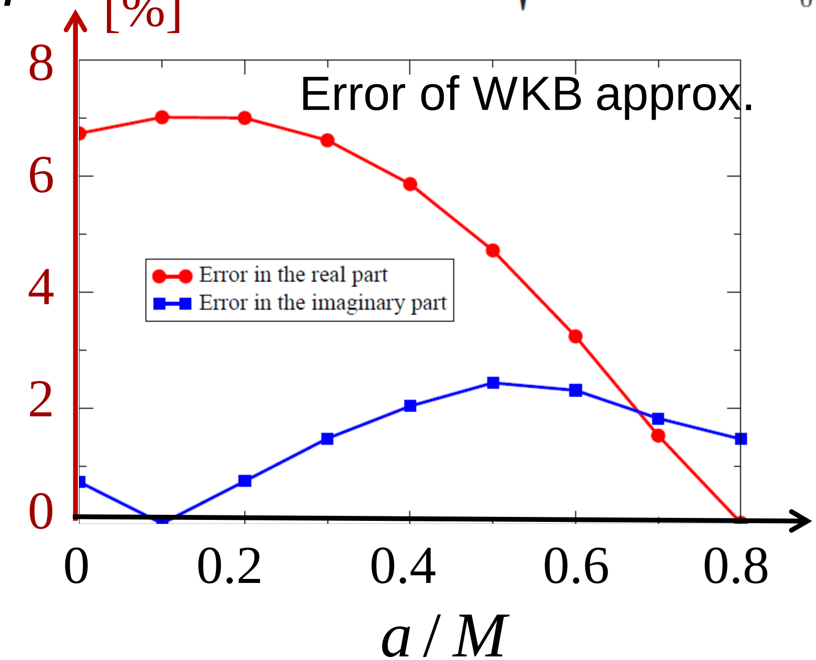
near horizon

$$\approx \exp\left(-i\omega\left(t-r_*\right)\right)$$

near infinity



$$(\omega_r + i\omega_i)^2 = V_{\rm D}(r_0^*) - i\sqrt{-\frac{1}{2}\frac{d^2V_{\rm D}}{dr^{*2}}\Big|_{r^*=r_0^*}}$$

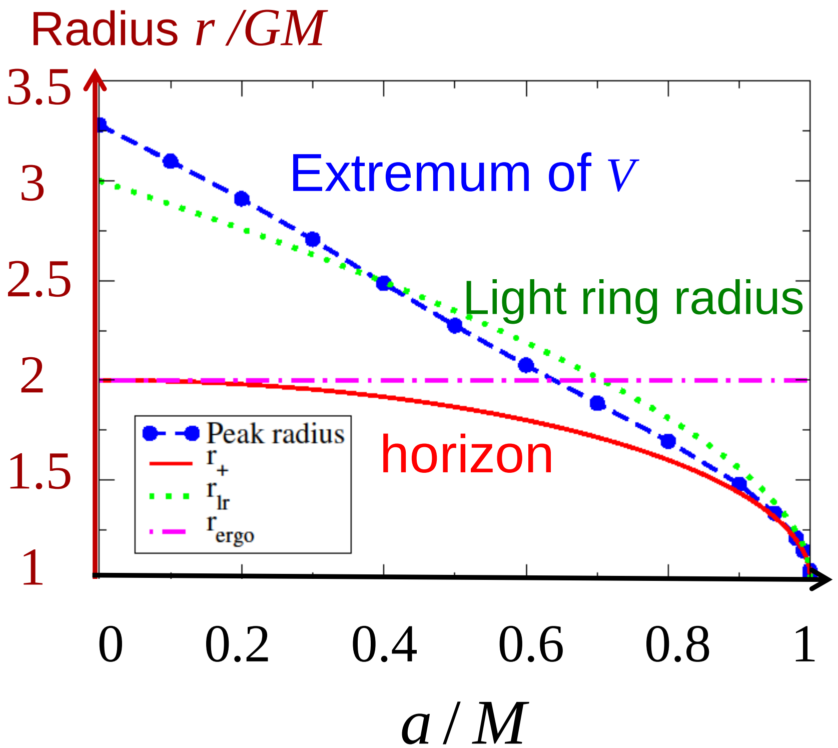


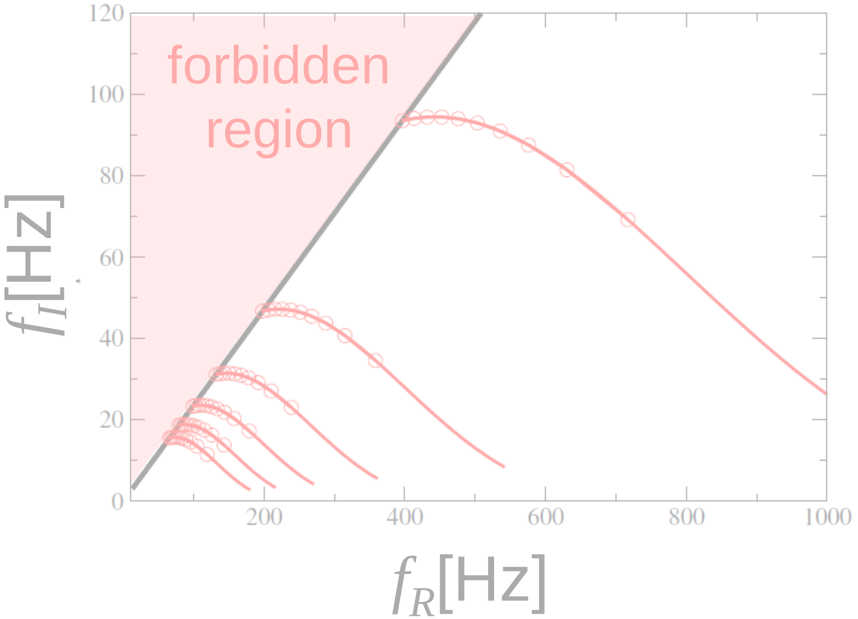
(Nakamura, Nakano, TT arXiv:1601.00356)

- QNM frequencies can be rather accurately obtained by WKB approximation (Schutz &Will, ApJ, 291 (1985))
- But breakdown of WKB approx. is necessary to find a solution connecting in-going and out-going waves.
- The breakdown of WKB approx. occurs at around the extremum of the effective potential V.
- In WKB approx. the behavior of V around the extremum determines the QNMs
- The position of the extremum of V will give an approximate answer to the above question.

(Nakamura et al., Phys.Rev. D93 (2016))

How deeply can we see BH spacetime by observing QNMs?





- Potential maximum that determines QNM frequency is rather close to horizon, especially for rapidly rotating case.

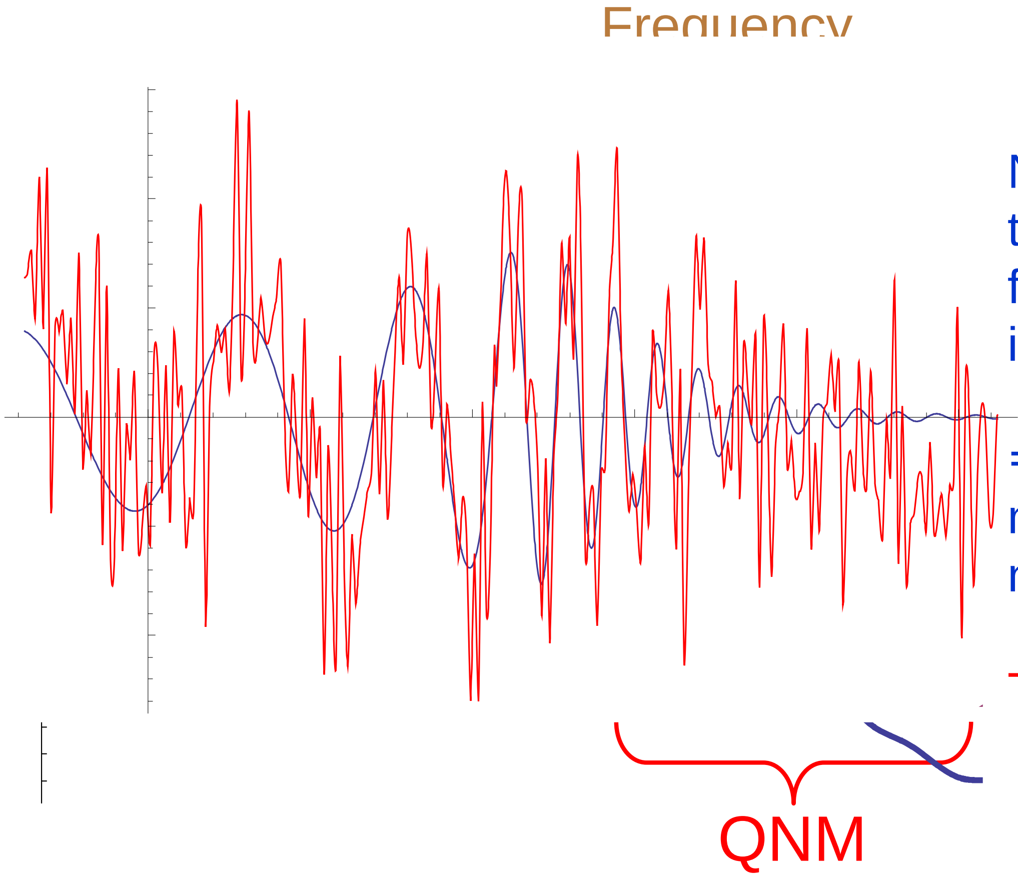
(Nakamura, Nakano, arXiv:1602.02385)

- There is a forbidden region for QNM frequencies in GR
- Black line is corresponding to Schwarzschild case

(Nakano, TT, Nakamura, arXiv:1506.00560)

Mock data challenge

- Many groups have been working on extracting QNMs as a test of performance of advanced data analysis methods.
- But fair comparison of performance has not been done.



Numerical relativity simulation tells that the evolution of frequency and amplitude in GR is rather simple and smooth.

⇒We construct waveform with modified f_R and f_I with Gaussian noise.

Then, can we extract f_R and f_I?

Matched filtering

$$\left(s\,|\,h\right)\approx\int\frac{df}{S_n\left(f\right)}s\left(f\right)h^*\left(f\right)$$

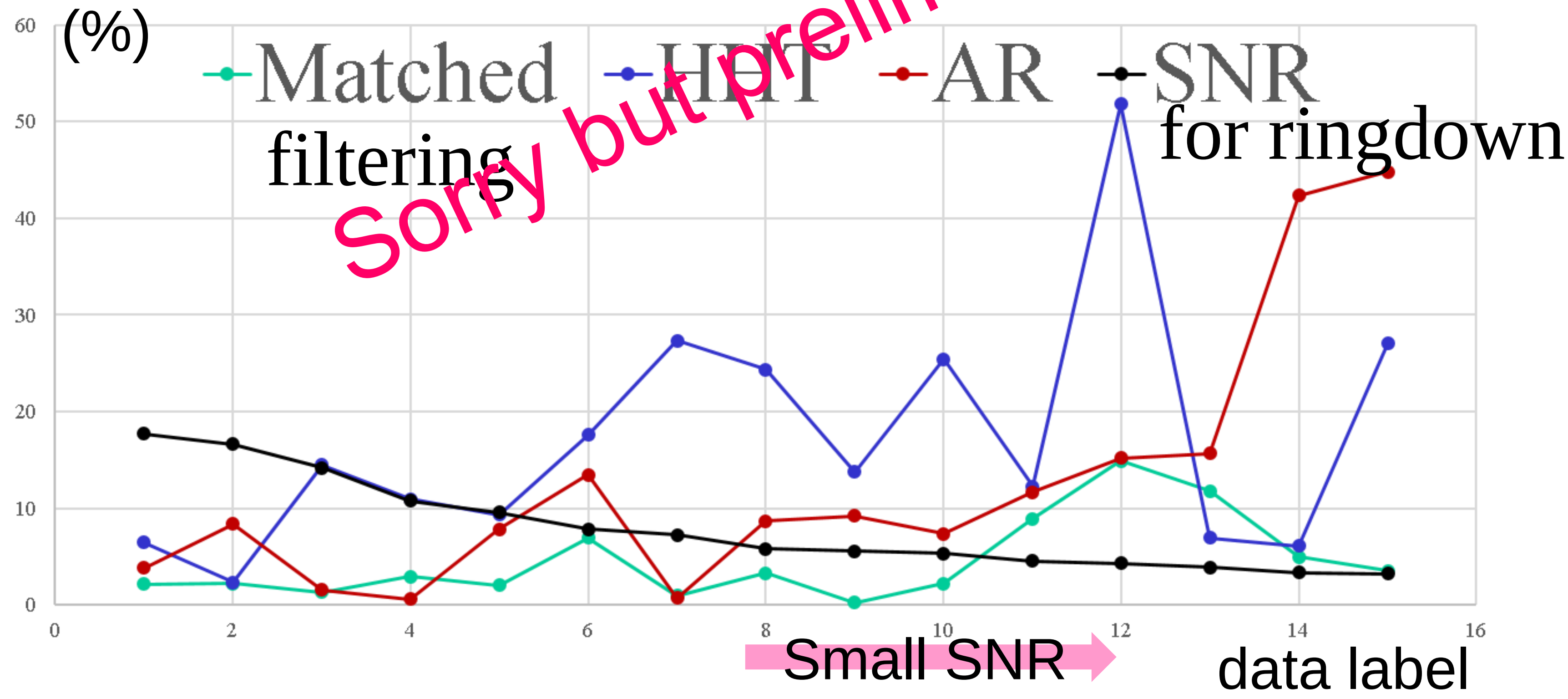
s: data
h: template with parameter $\boldsymbol{\theta}$
$S_n(f):=2\int_{-\infty}^{+\infty}d\tau\langle n(t)n(t+\tau)\rangle e^{2\pi i f\tau}$
n: noise

Find parameter $\boldsymbol{\theta}$ that realizes maximum $(s|h)$.

- Matched filtering is the optimal one among linear filtering methods for Gaussian noise.
- However, it is not generally guaranteed to be optimal.
- Also, the estimate of QNM frequency based on matched filtering might be systematically biased depending on the assumed wave form.

Challenge results(1)

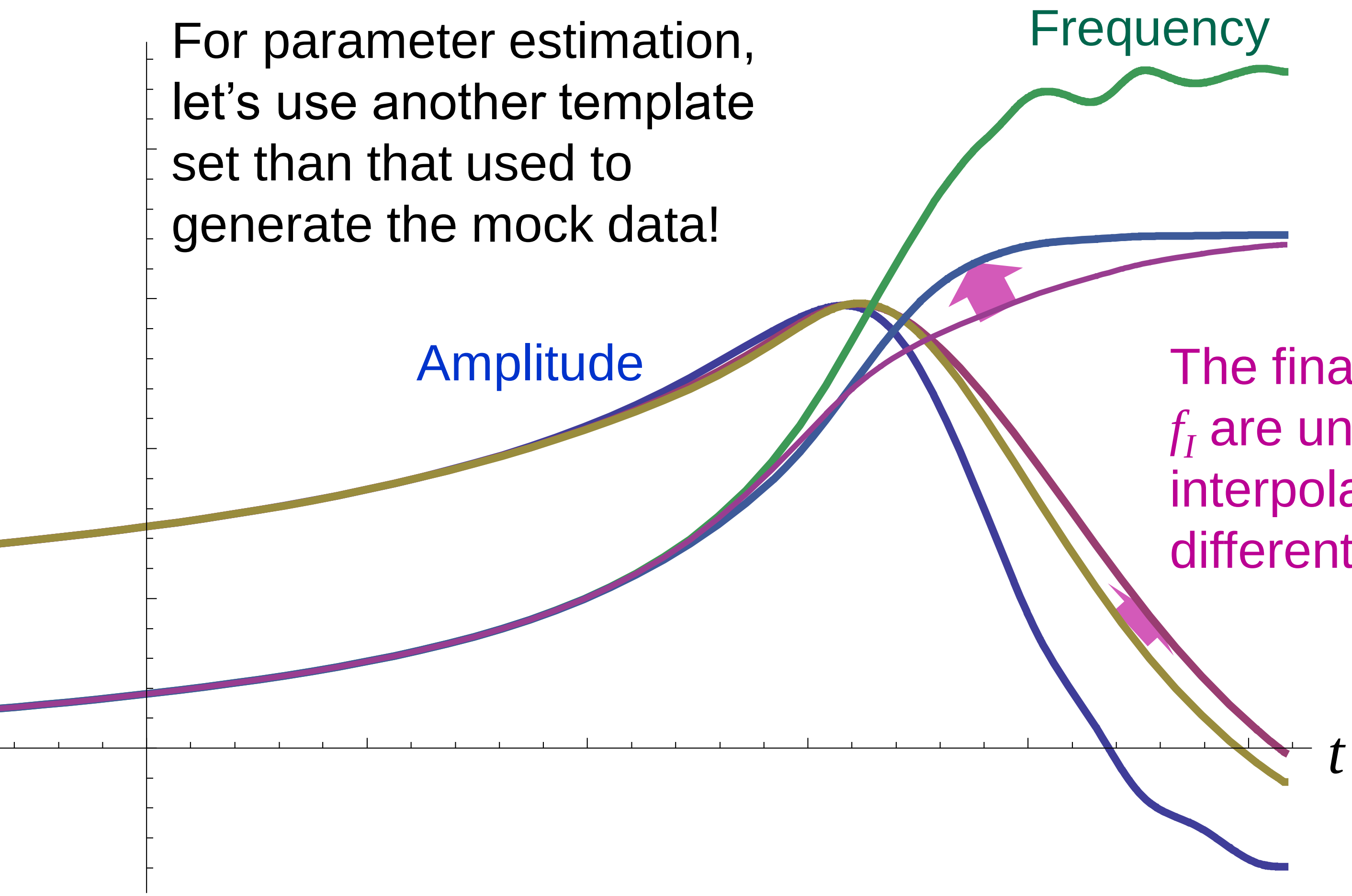
Error in the estimate of f_R



The reference matched filtering shows the best performance on average but here it is a little cheating, since we used the modified ring-down wave form that is used to generate the mock data, which is unknown in reality.

Is matched filtering by using the same templates for mock data generation and filtering so cheating?

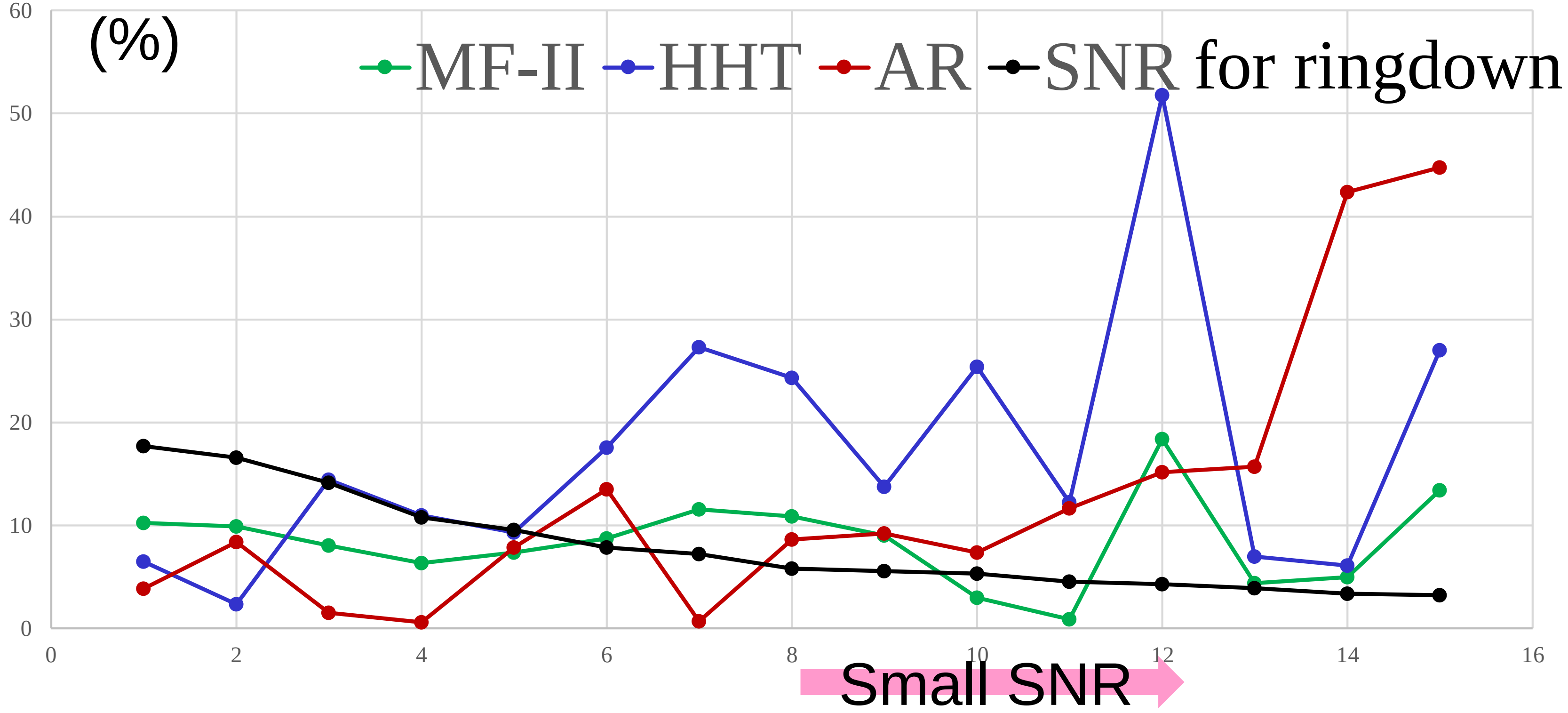
For parameter estimation, let’s use another template set than that used to generate the mock data!



The final values of f_R and f_I are unchanged, but the interpolation is done differently.

Challenge results(2)

Error in the estimate of f_R



AR method shows rather good performance except for low SNR case.

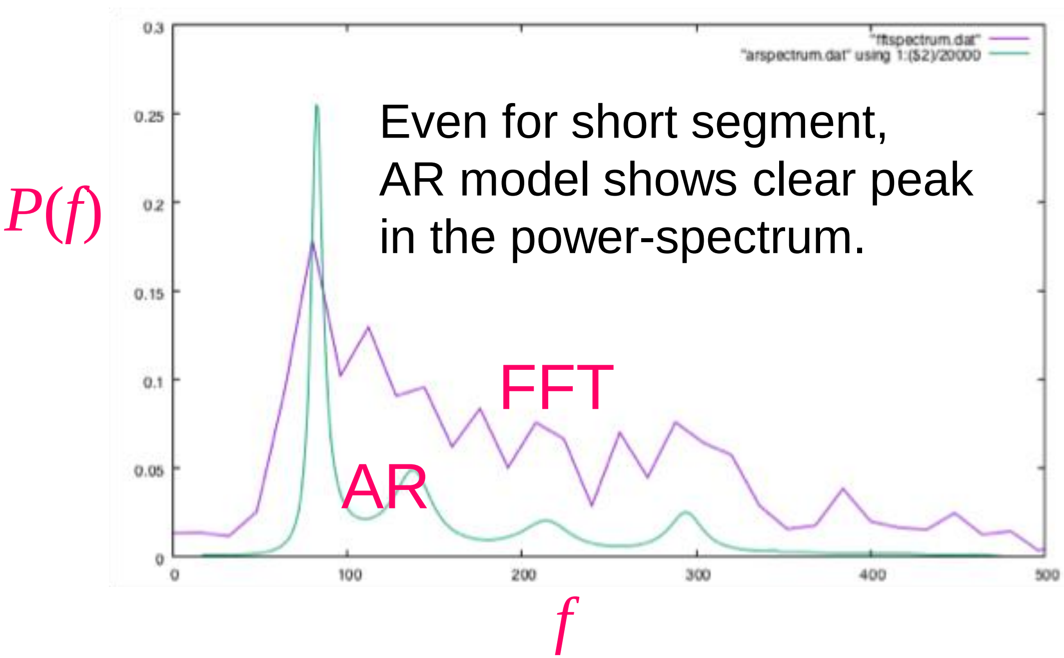
S.Yamamoto, H.Shinkai (OIT)

Fitting data in time sequence with linear func.

$$
\begin{aligned}
x_n &= a_1 x_{n-1} + a_2 x_{n-2} + \cdots + a_M x_{n-M} + \varepsilon \\
&= \sum_{j=1}^{M} a_j x_{n-j} + \varepsilon
\end{aligned}
$$

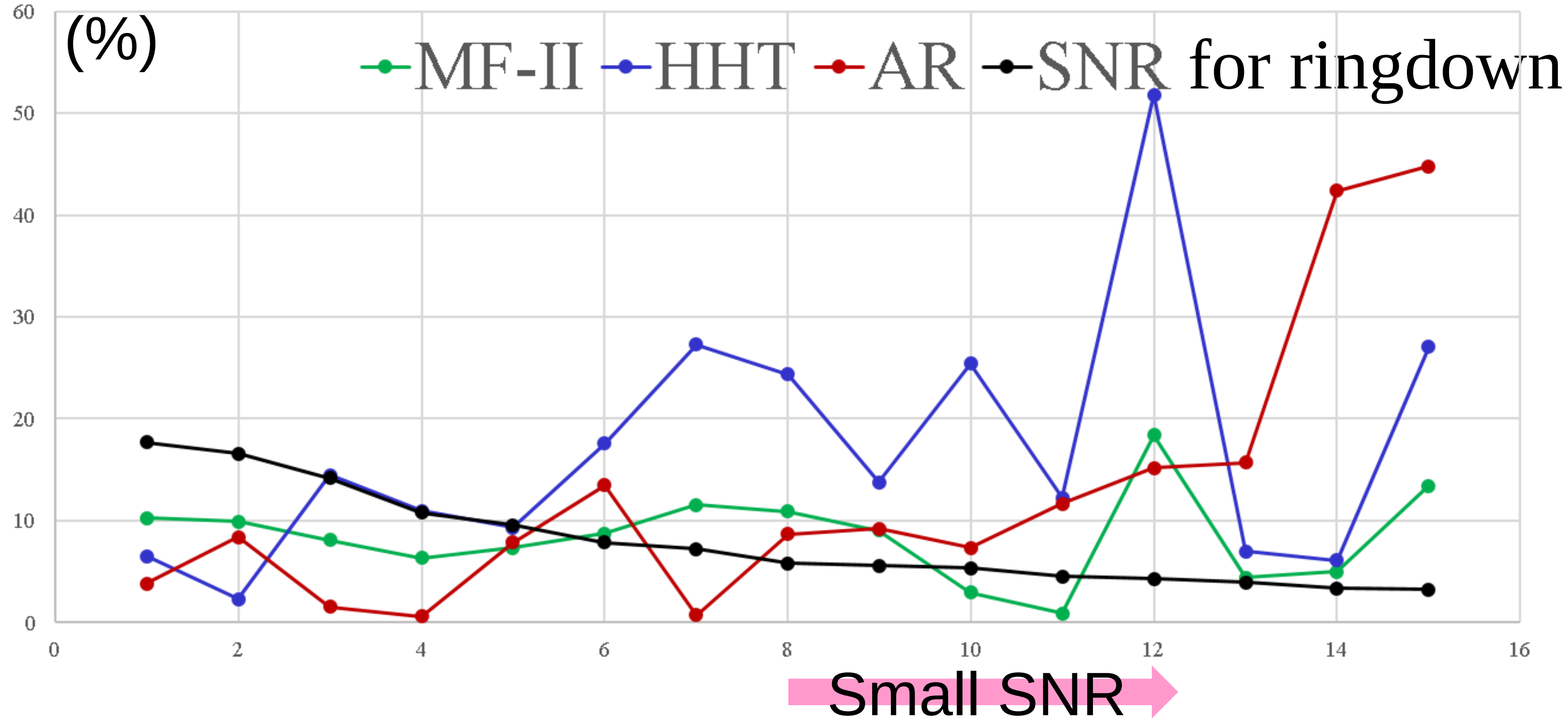
- find a_j ,ε
- re-construct wave signal using the fitted function
- apply FFT to the re-constructed wave.

- The order M was fixed at 20〜30.



Challenge results(2)

Error in the estimate of f_R

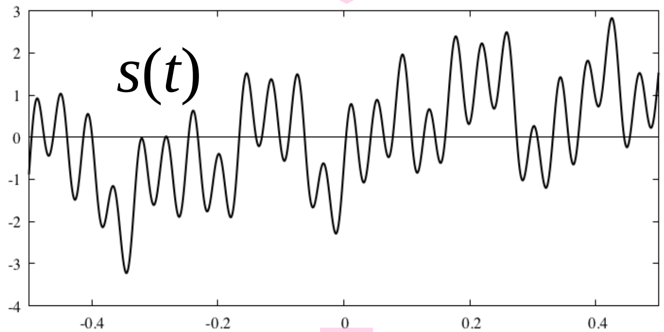


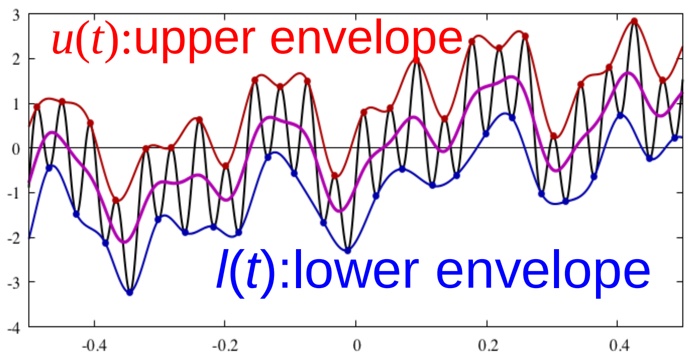
HHT method may work better for small SNR, contrary to our naïve expectation, although the number of samples is still too small.

Ohara, Sakai, Takahashi

Empirical Mode decomposition

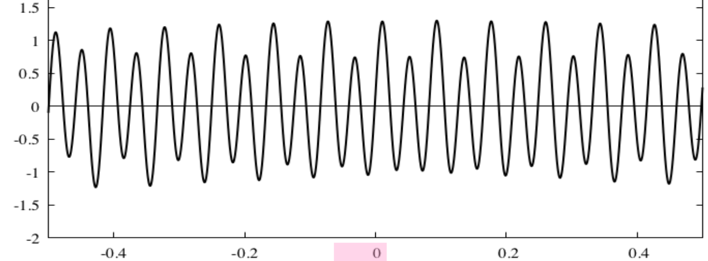
We drop high and low frequency modes by filtering the data [f_L, f_H]





$m(t) := (u(t) + l(t))/2$

$s(t) \longrightarrow s(t) - m(t)$



Iterate this process until $m(t)$ becomes sufficiently small

Hilbert-Spectral Analysis

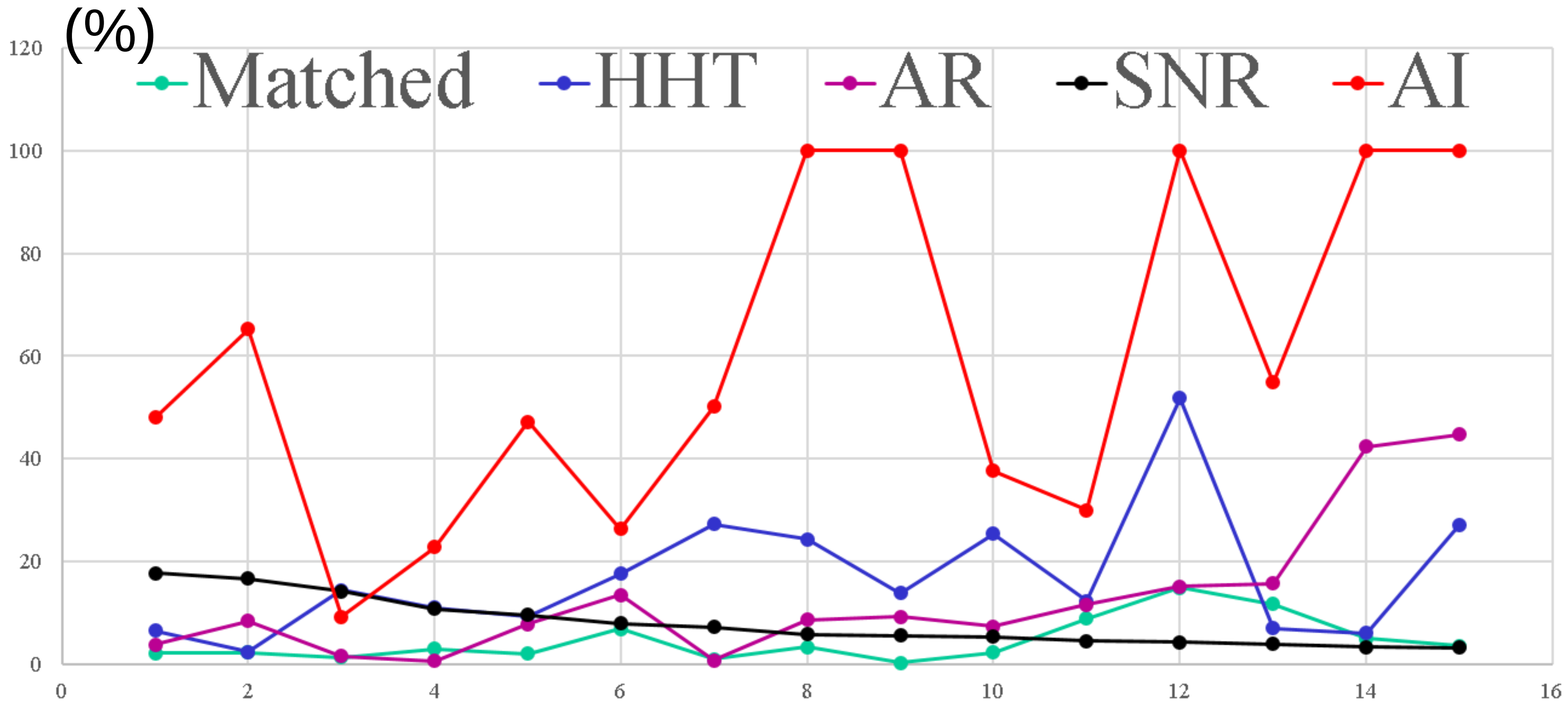
$$v(t) = \frac{1}{\pi} P \int_{\infty}^{\infty} \frac{s(\tau)}{t-\tau} d\tau$$

$$a(t)e^{i\theta(t)} = s(t) + iv(t)$$

We extract f_R (from $\theta(t)$) and f_I (from $a(t)$).

The choice of initial filtering band [f_L, f_H] is a little ad hoc.

Use of AI



Currently, the performance of our AI approach is not good, but it is still under development.

Summary

- Our group is planning to develop systematic tests of modified gravity by using gravitational wave data.
- Today we focused on the extraction of black hole ring-down frequency.
- Matched filtering analysis gives a good estimate but it can be biased.
- There have been already many works in extracting QNM frequencies, but impressive improvement of the estimation accuracy has not been actually achieved in our group.
- However, the performance of alternative methods can be further improved.
- The goal would be to find a method *s.t.*
  - the accuracy is better than the matched filtering with appropriate modified waveform being used
  - the systematic bias is small enough.