Black Hole fusion in the extreme mass-ratio limit

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Work with **Marina Martínez**

arXiv:1603.00712

and with

Marina Martínez & **Miguel Zilhão**

arXiv:1708.08868

Black Hole fusion

The most complex of all processes governed by $R_{\mu\nu} = 0$

Non-linearity at its most fiendish

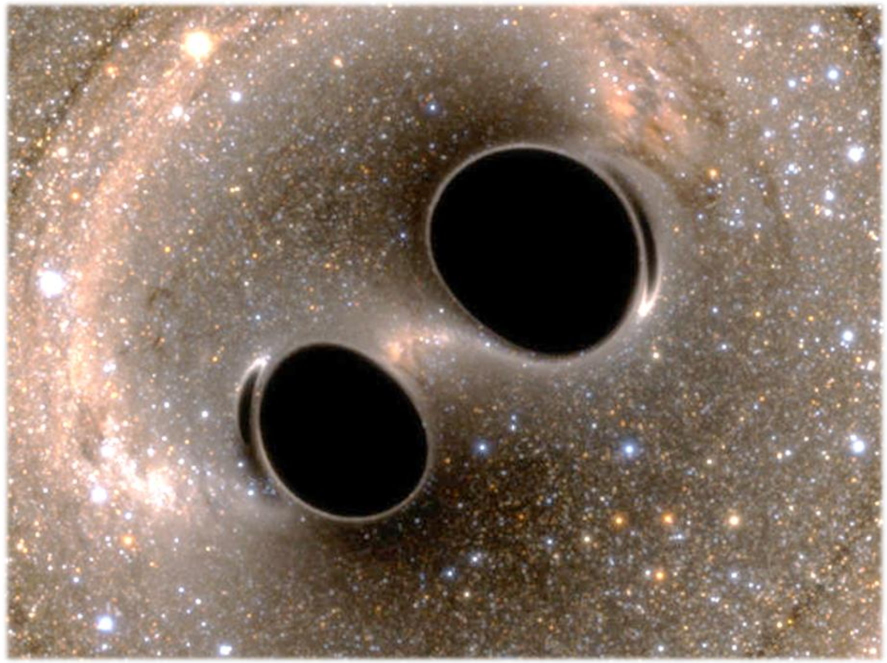
Black Hole fusion

The most complex of all processes governed by $R_{\mu\nu} = 0$

Non-linearity at its most fiendish

or maybe not—not always

This is what we'd *see* (lensing)



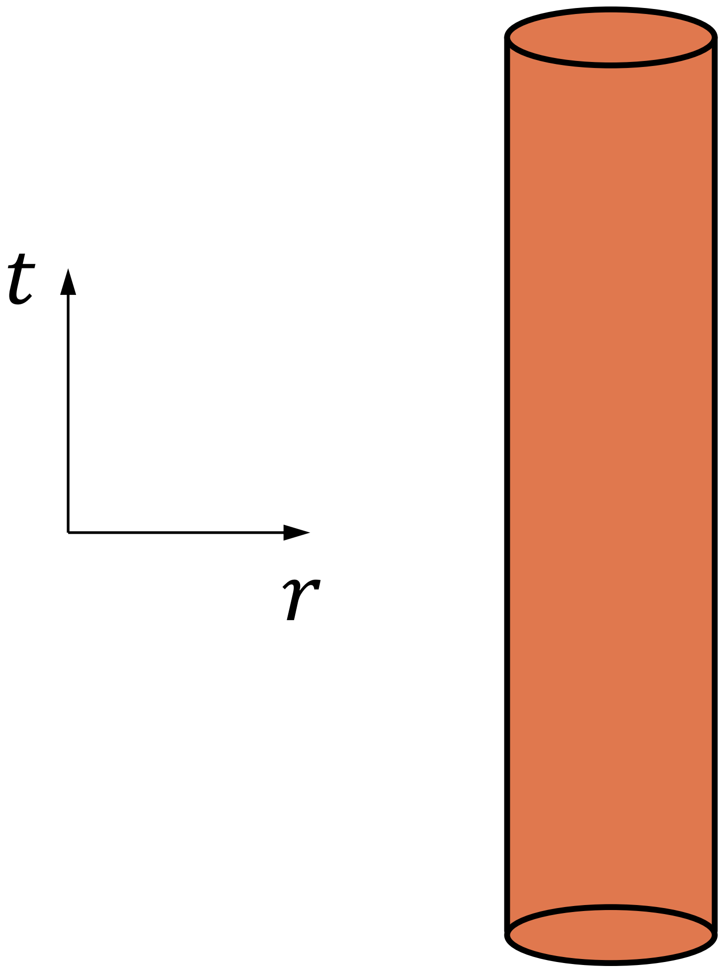
Not a black hole, but its *shadow*

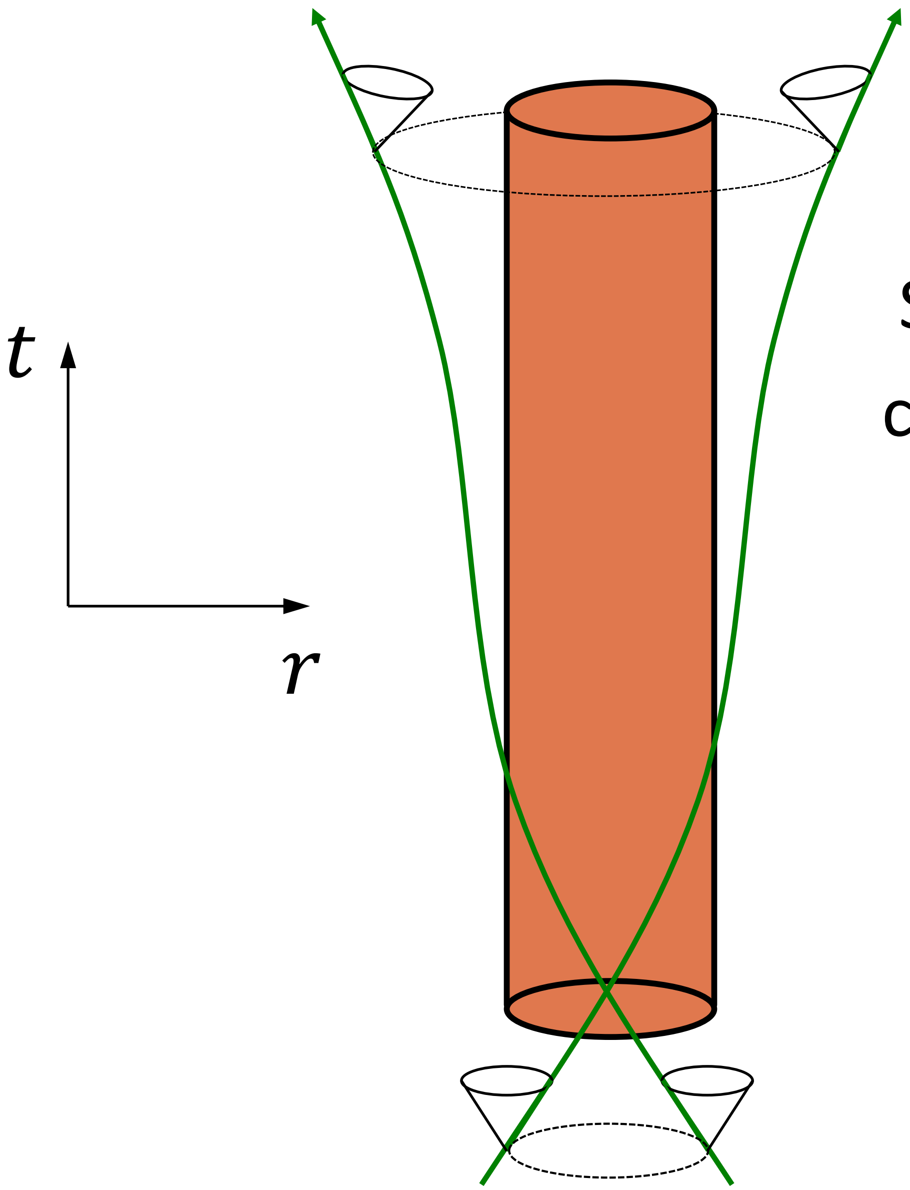
What is a black hole?

Spacetime region from which not even light can escape

Event Horizon

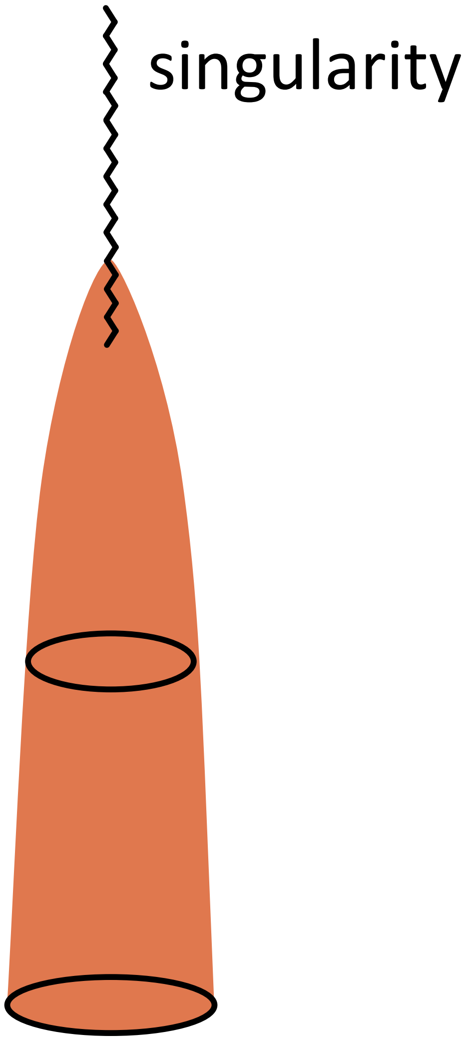
Star

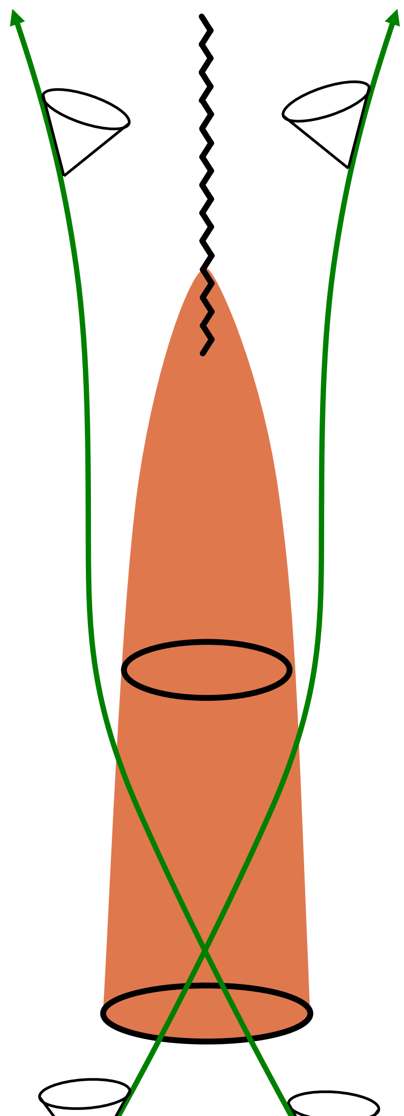


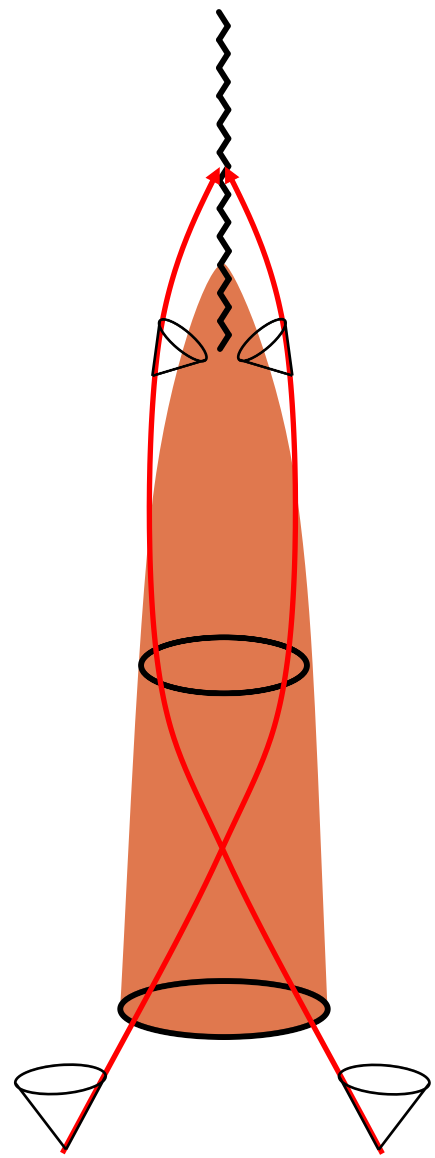


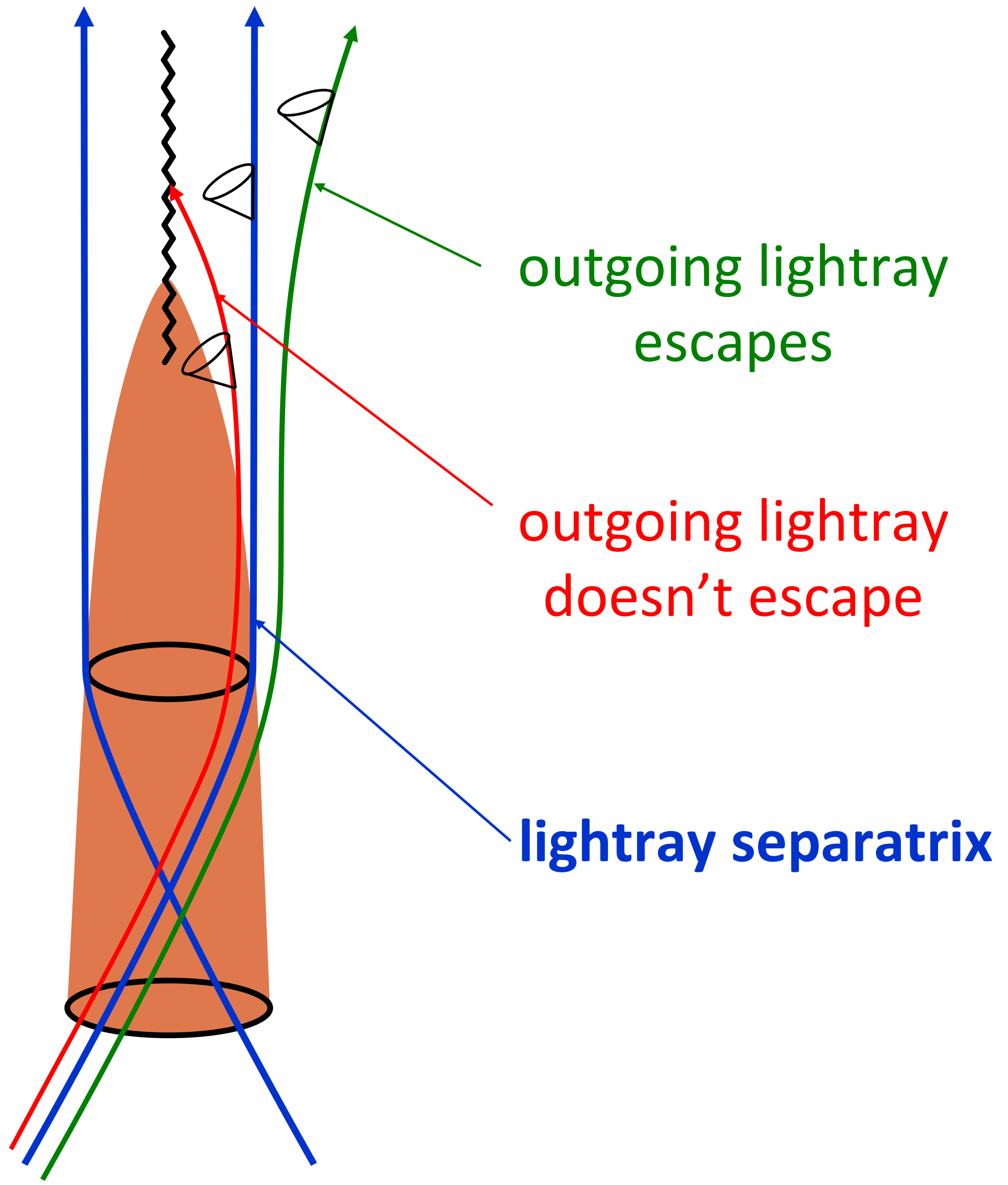
Spherical wavefronts contract, then expand

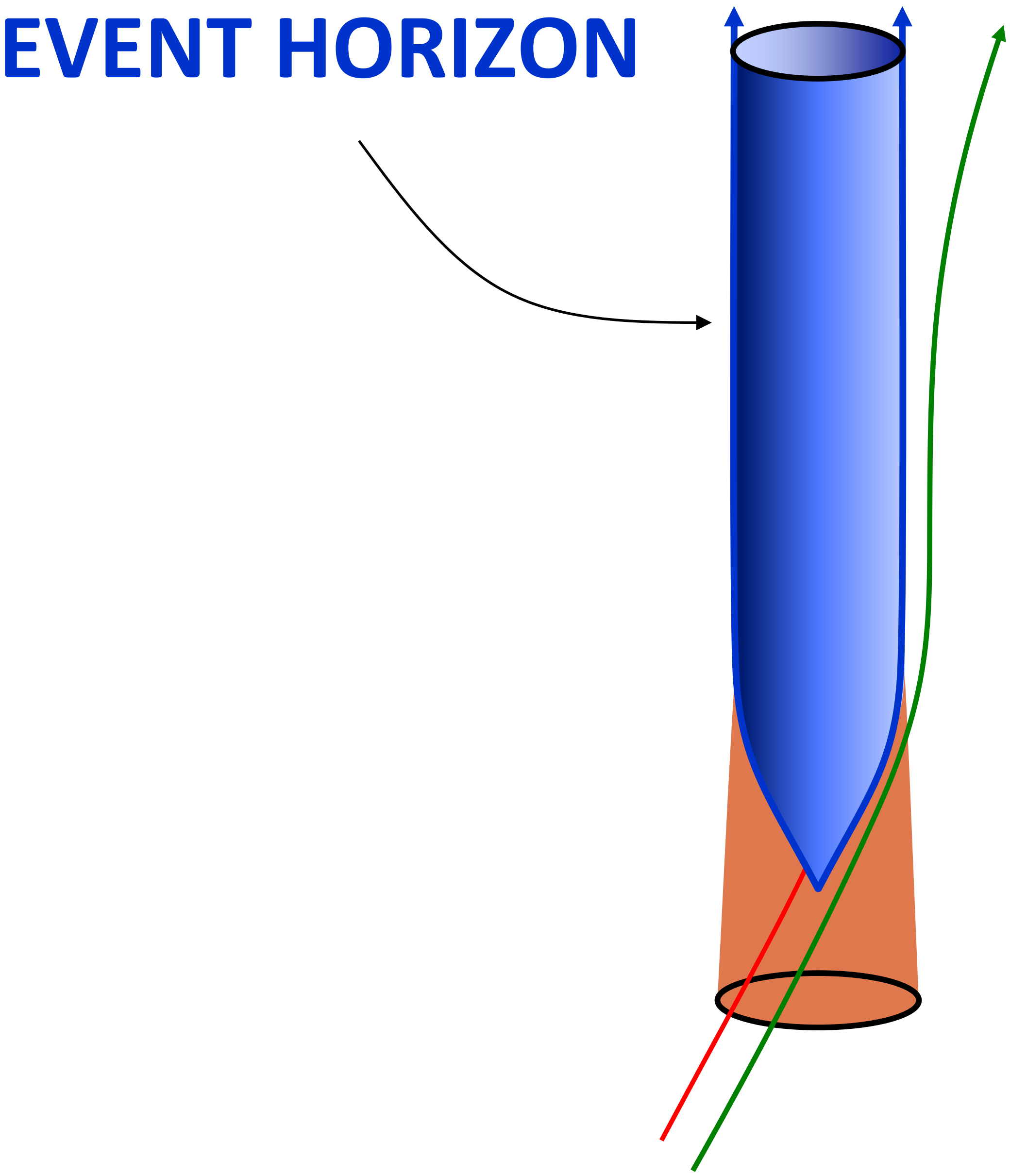
Collapsed Star



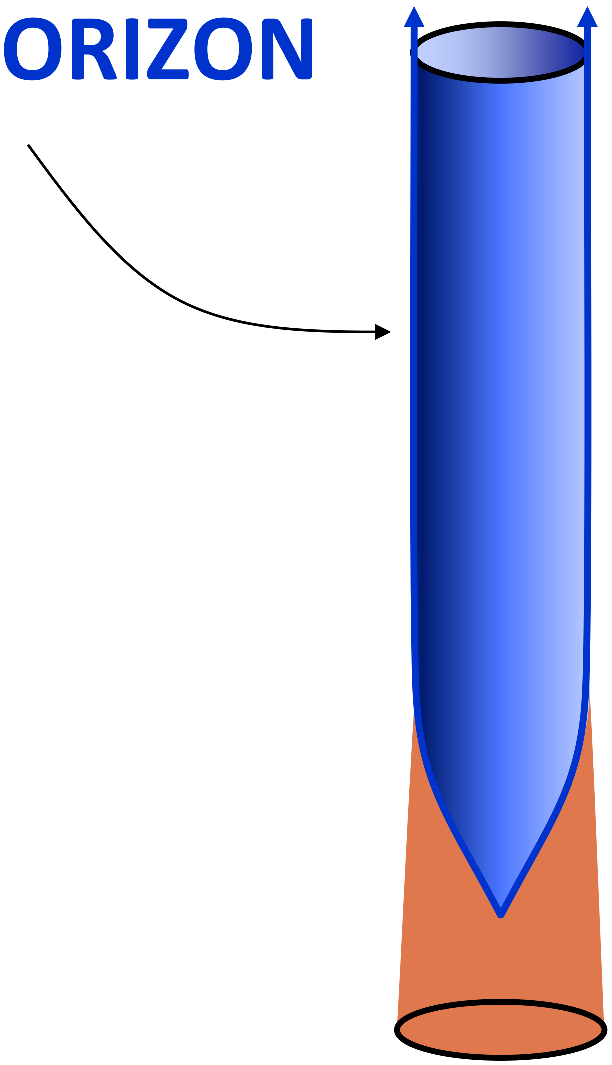





outgoing lightray escapes
outgoing lightray doesn't escape
lightray separatrix


EVENT HORIZON

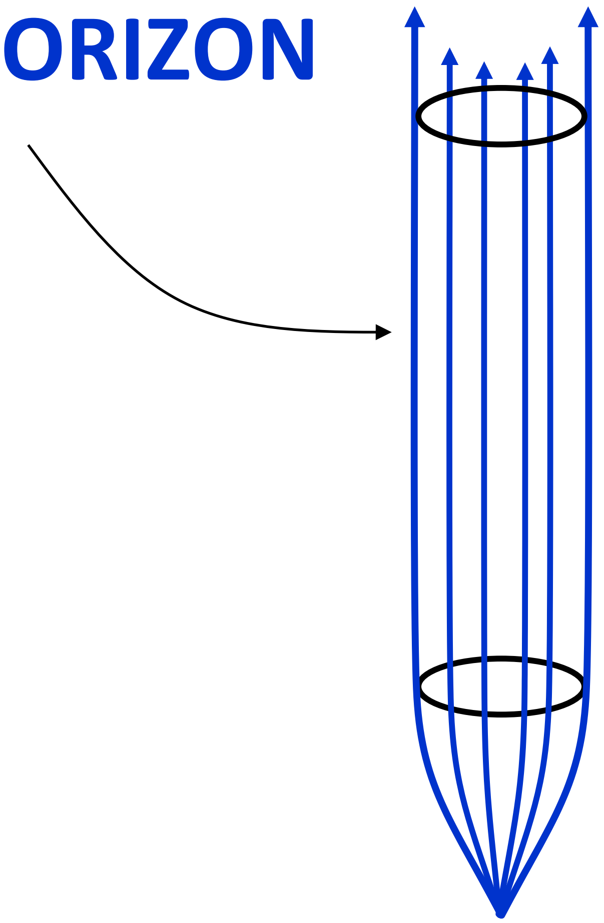
EVENT HORIZON



Null hypersurface

3-dimensional in
4-dimensional spacetime

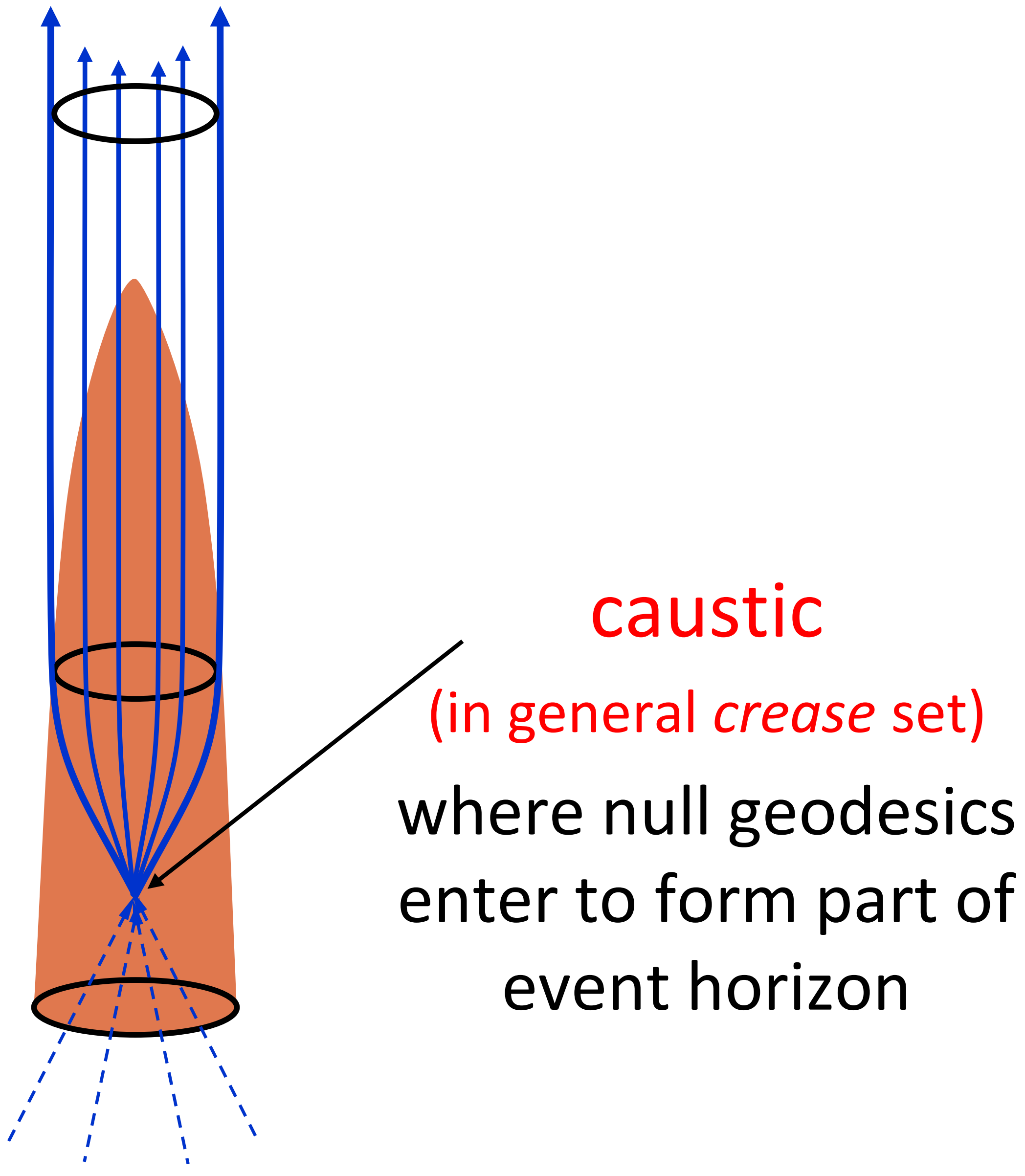
EVENT HORIZON



Null hypersurface

<u>made of null geodesics</u>

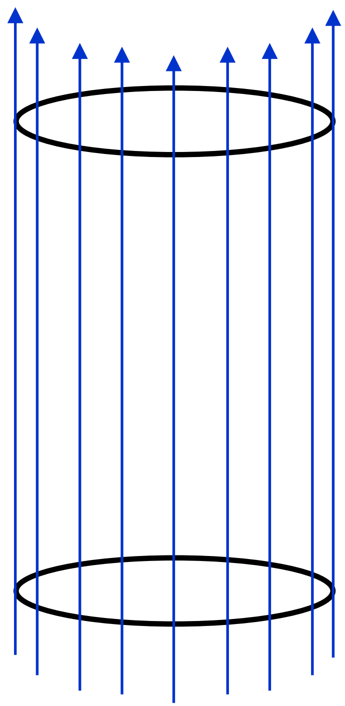
(light rays)


caustic
(in general *crease* set)
where null geodesics enter to form part of event horizon

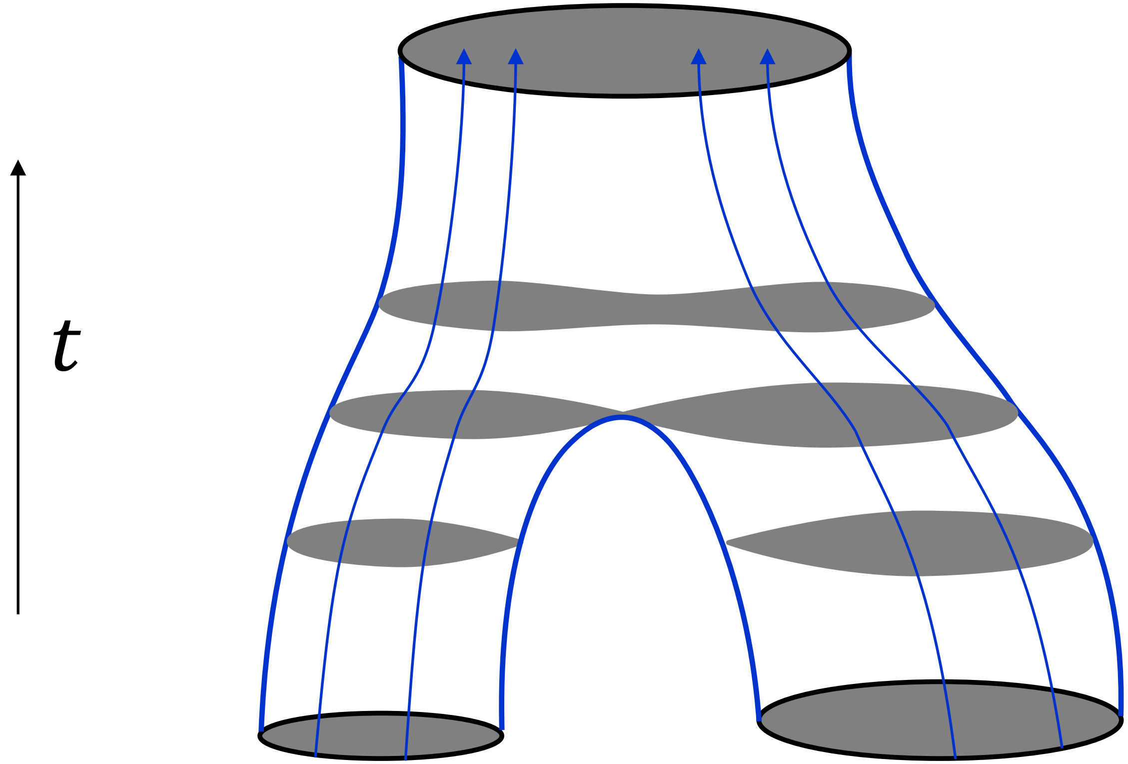
Event horizon

found by

tracing a family of light rays in a given spacetime

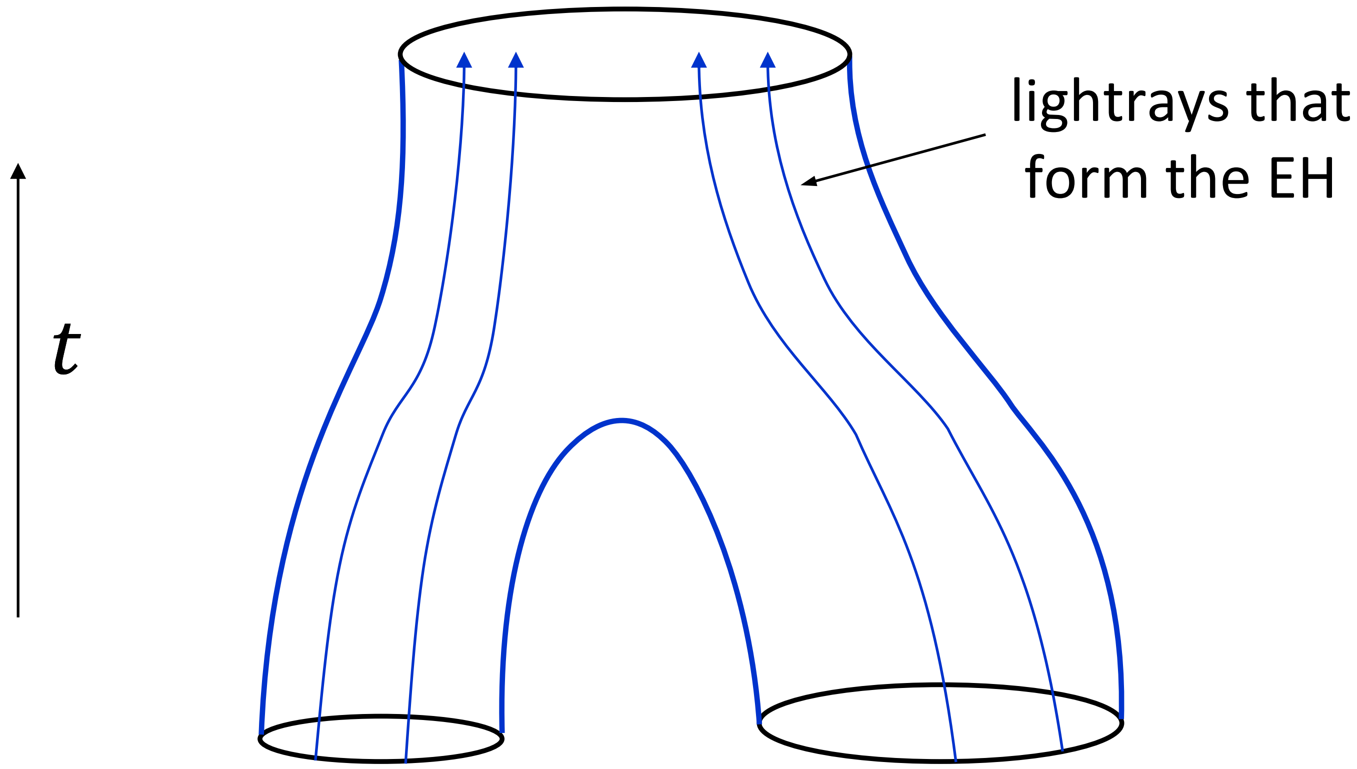


Event horizon of binary black hole fusion

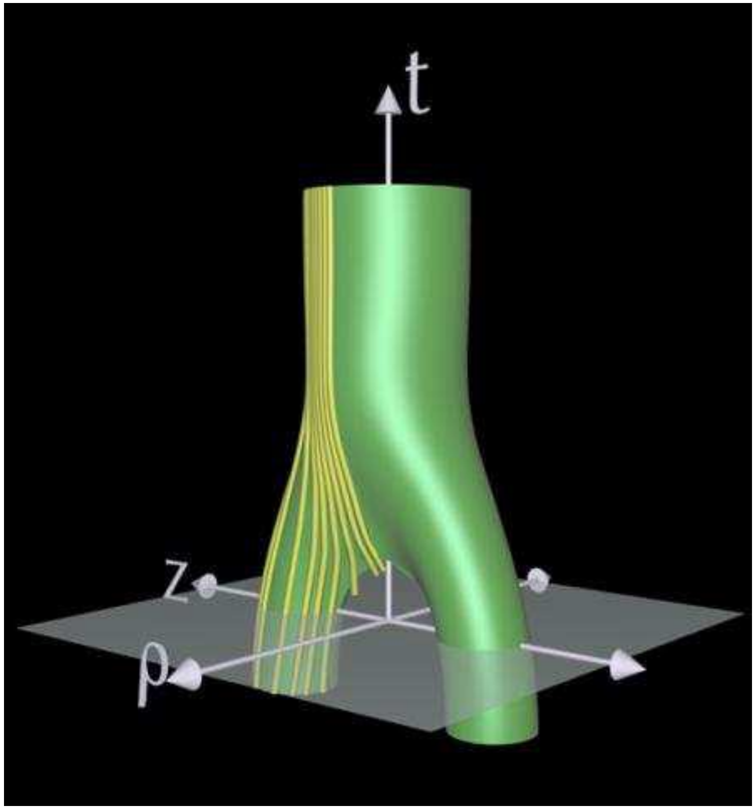


Event horizon of binary black hole fusion

“pants” surface



Event horizon of binary black hole fusion



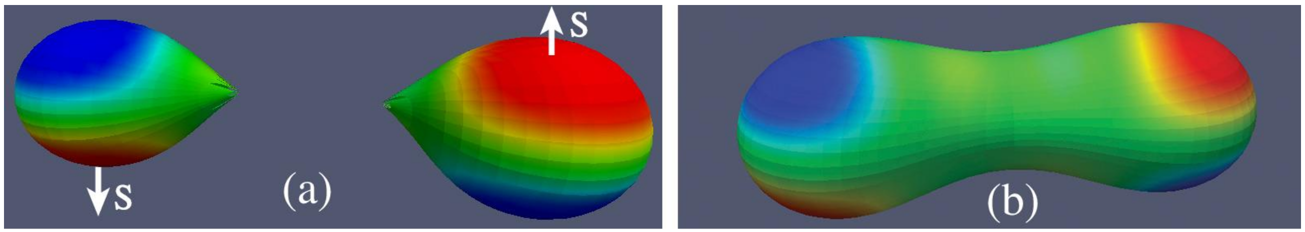
head-on
(axisymmetric)

equal masses

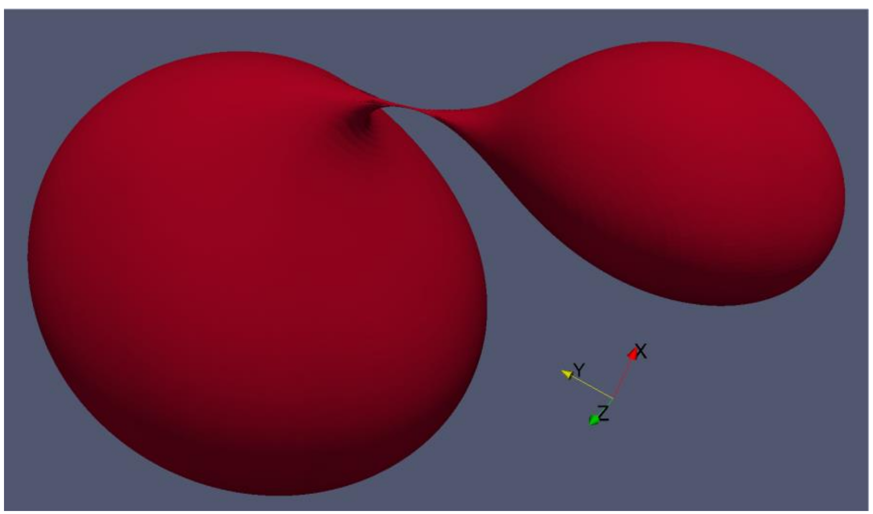
Cover of *Science,* November 10, 1995

Binary Black Hole ***Grand Challenge*** *Alliance* (Matzner et al)

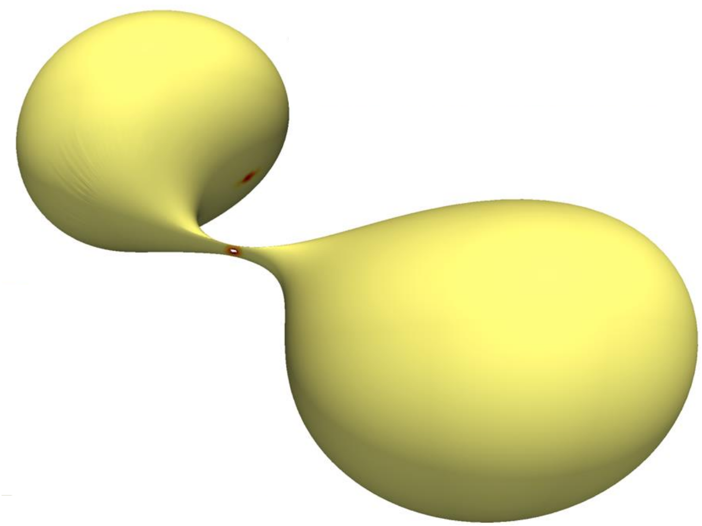
Spatial sections of event horizon of binary black hole fusion



Owen et al, Phys.Rev.Lett. 106 (2011) 151101



Cohen et al, Phys.Rev. D85 (2012) 024031



Bohn et al, Phys.Rev. D94 (2016) 064009

Surely the fusion of horizons can only be captured with supercomputers

Surely the fusion of horizons can only be captured with supercomputers

or so it'd seem

$\exists$ limiting (but realistic) instance where horizon fusion can be described **exactly**

It involves only elementary ideas and techniques

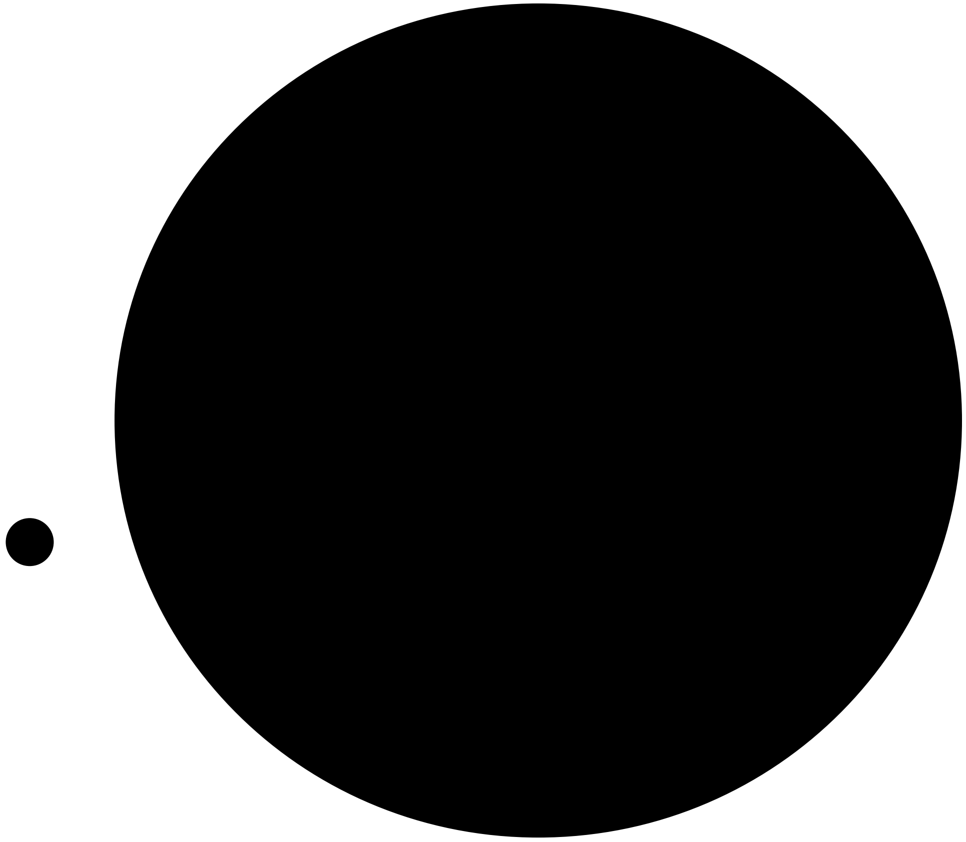
Equivalence Principle (1907)

Schwarzschild solution & Null geodesics (1916)

Kerr solution (1964)

Notion of Event Horizon (1950s/1960s)

Extreme-Mass-Ratio (EMR) merger



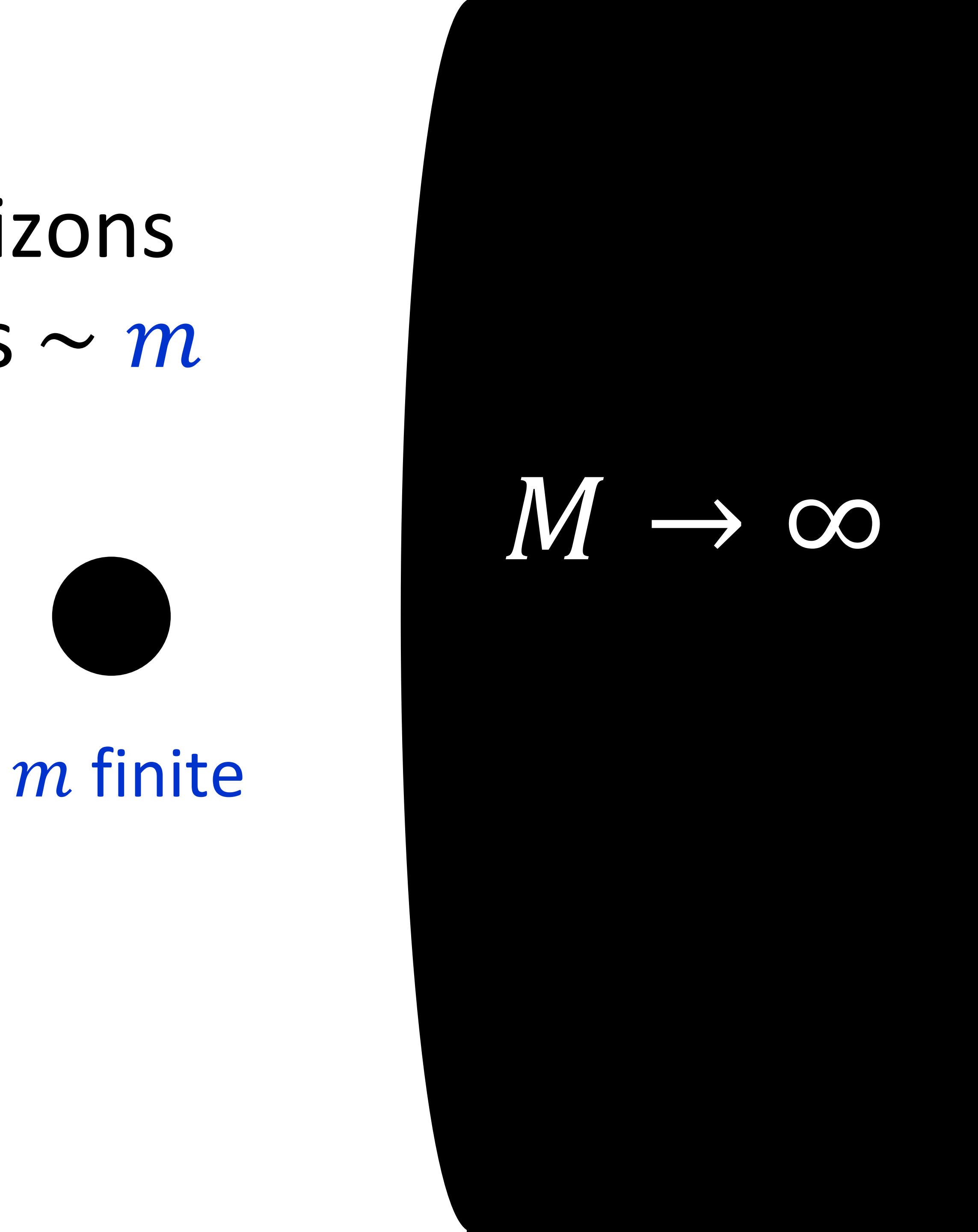
$m \ll M$

$m \ll M$

most often taken as

M sets the scale for the radiation emitted

Fusion of horizons involves scales $\sim m$




gravitational
Dude, Where are the waves???

Gravitational waves?

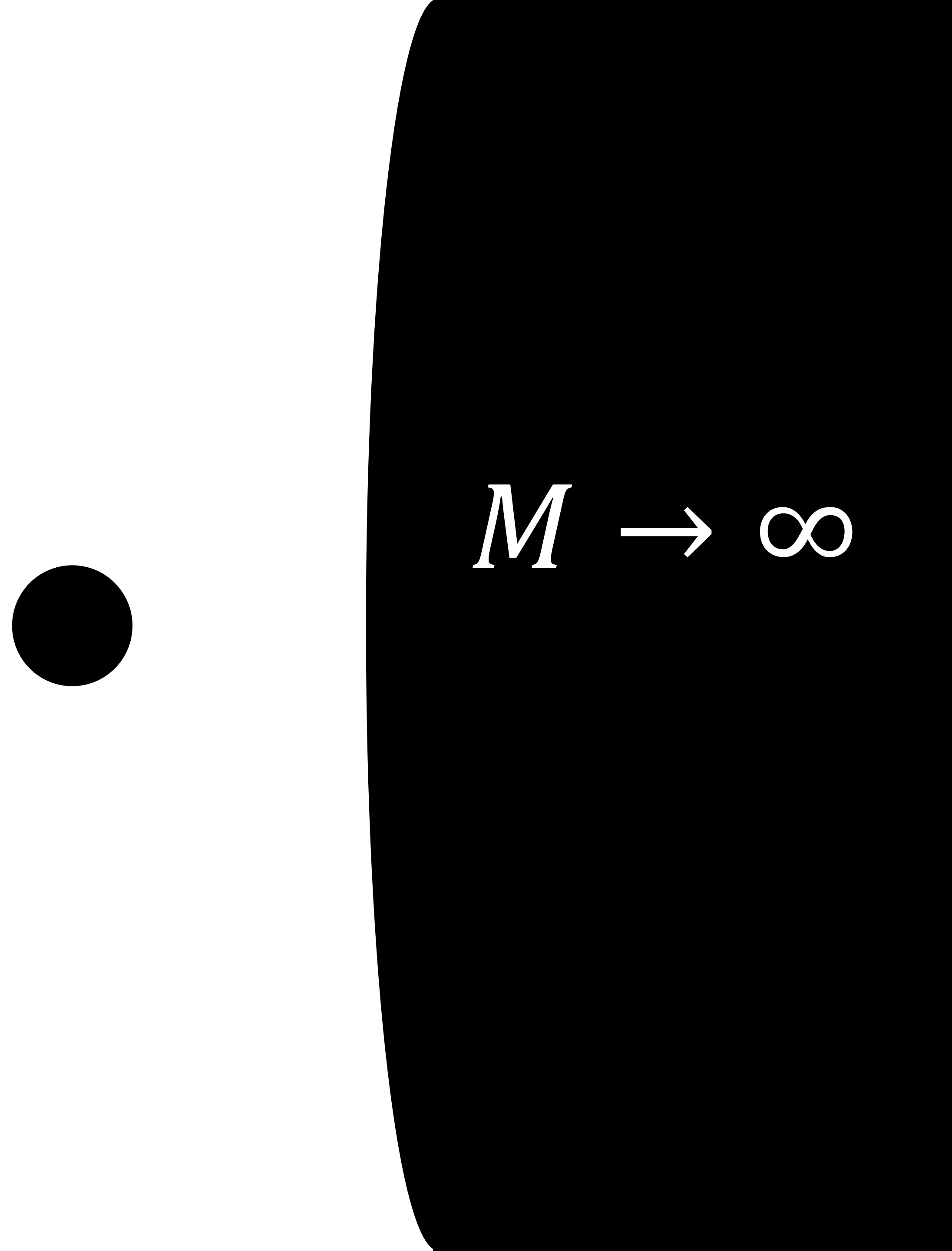
When $M \to \infty$ the radiation zone is pushed out to infinity

No gravitational waves in this region

Gravitational waves?

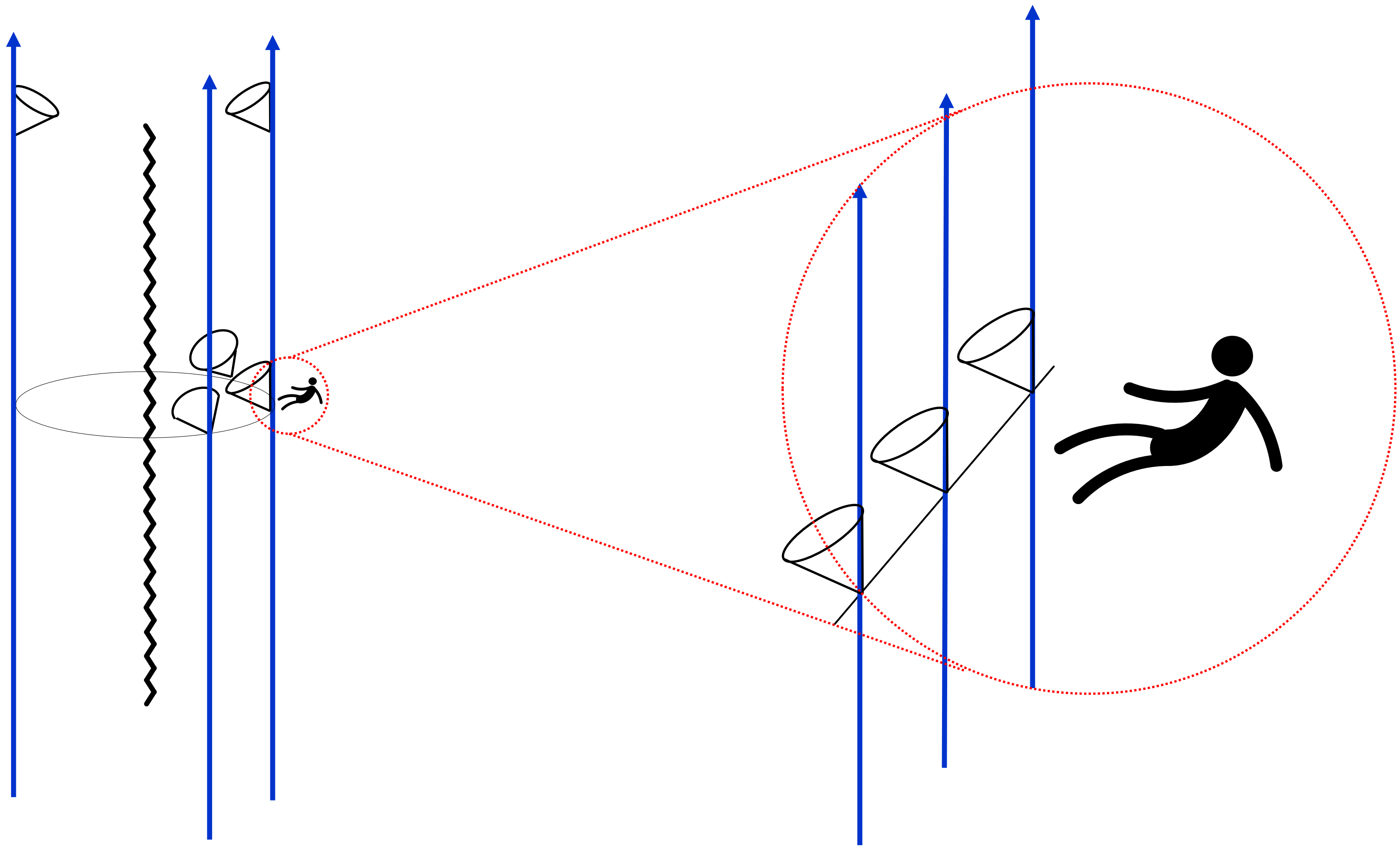
GWs will reappear if we introduce corrections for finite small $\frac{m}{M}$

matched asymptotic expansion to Hamerly+Chen 2010
Hussain+Booth 2017


$M \to \infty$

$$M \to \infty$$

Very large black hole / Very close to the horizon



Very close to a Black Hole

Horizon well approximated
by null plane
in Minkowski space

This follows from
the Equivalence Principle

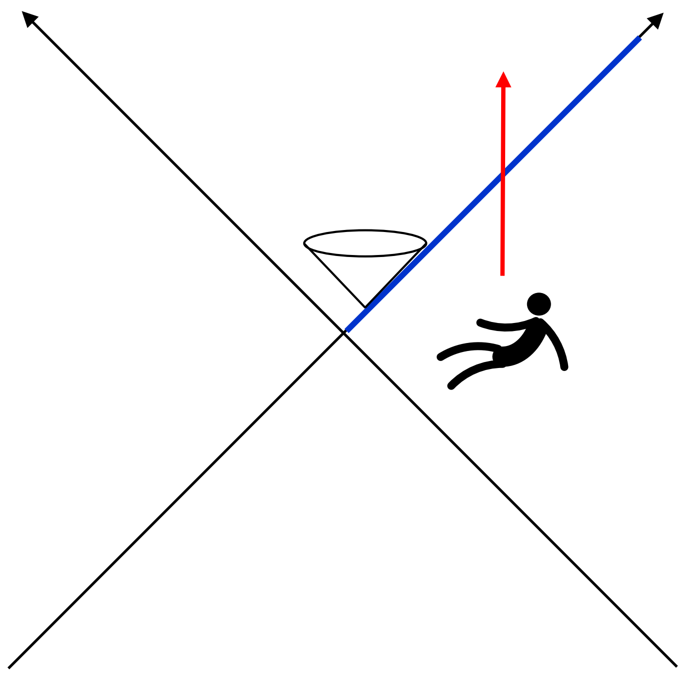
At short enough scales, geometry is
equivalent to flat Minkowski space

Curvature effects become small,
but horizon remains

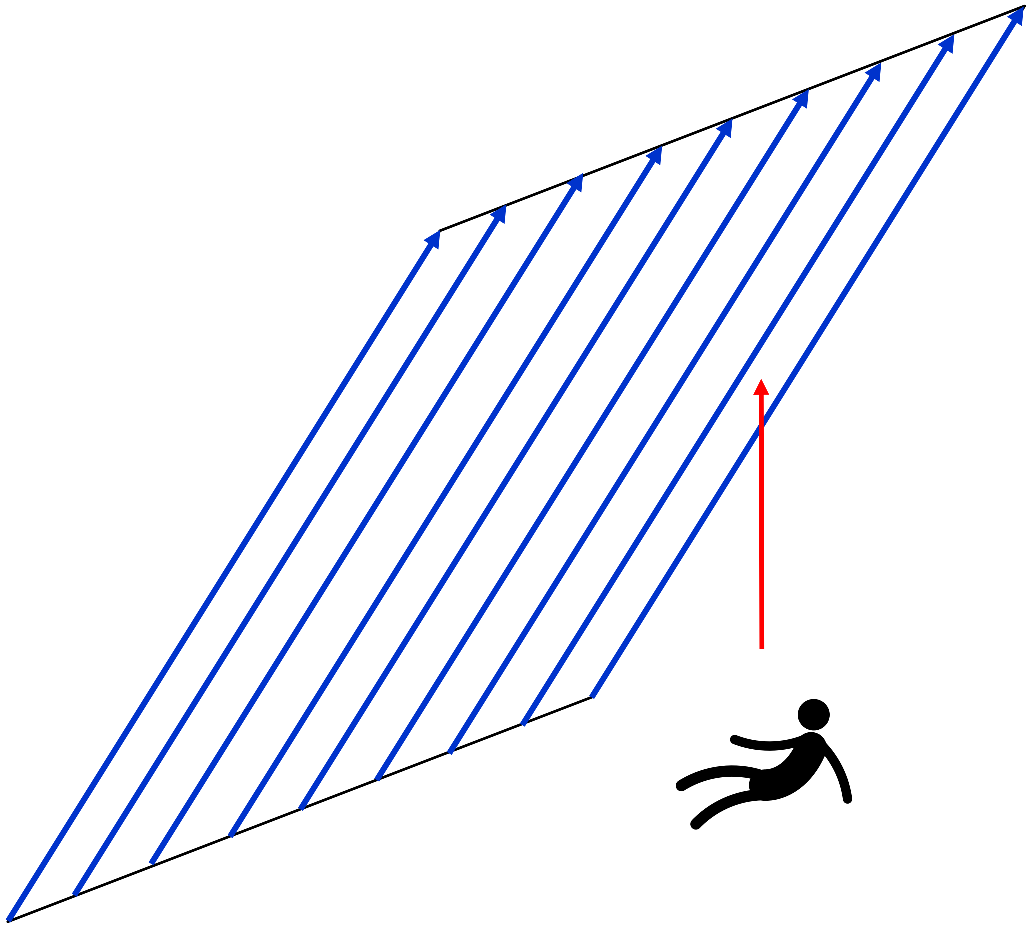
Locally gravity is equivalent to acceleration

Locally black hole horizon is equivalent to acceleration horizon

Falling into very large bh =
crossing a null plane in Minkowski space

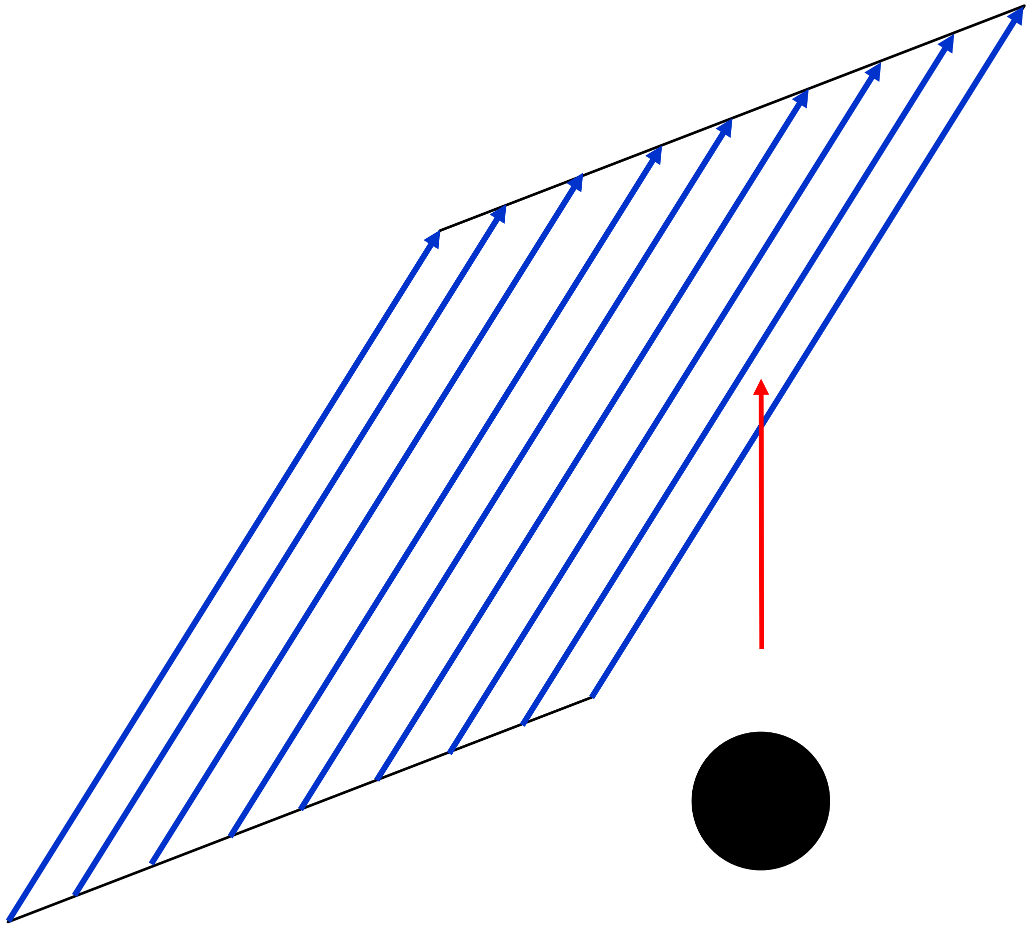


Object falling into a Large Black Hole



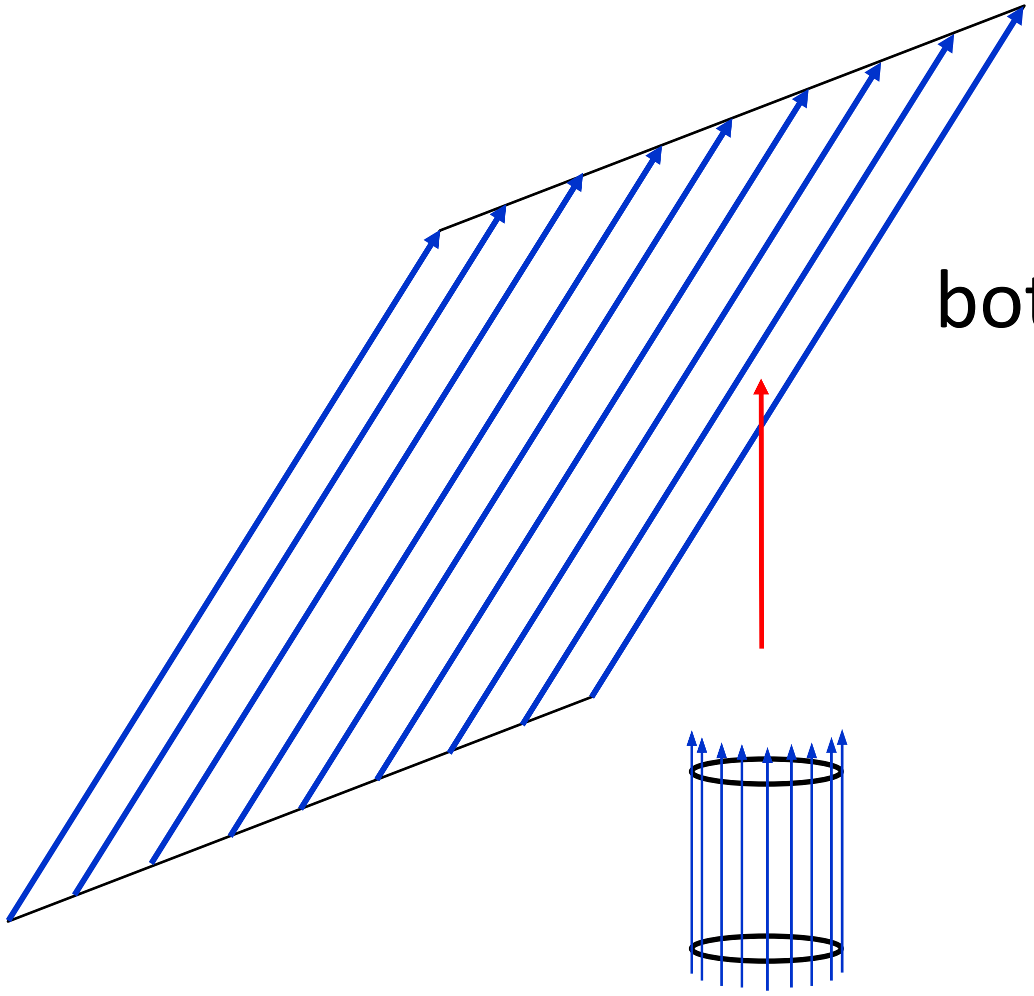
in rest frame of infalling object

Small Black Hole falling into a Large Black Hole



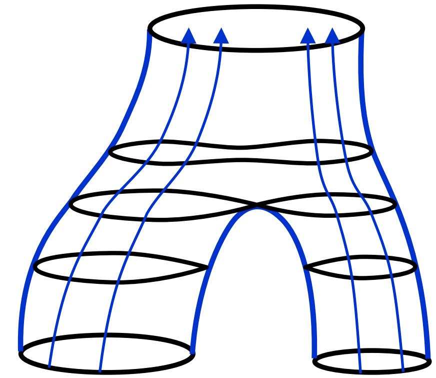
in rest frame of
small black hole

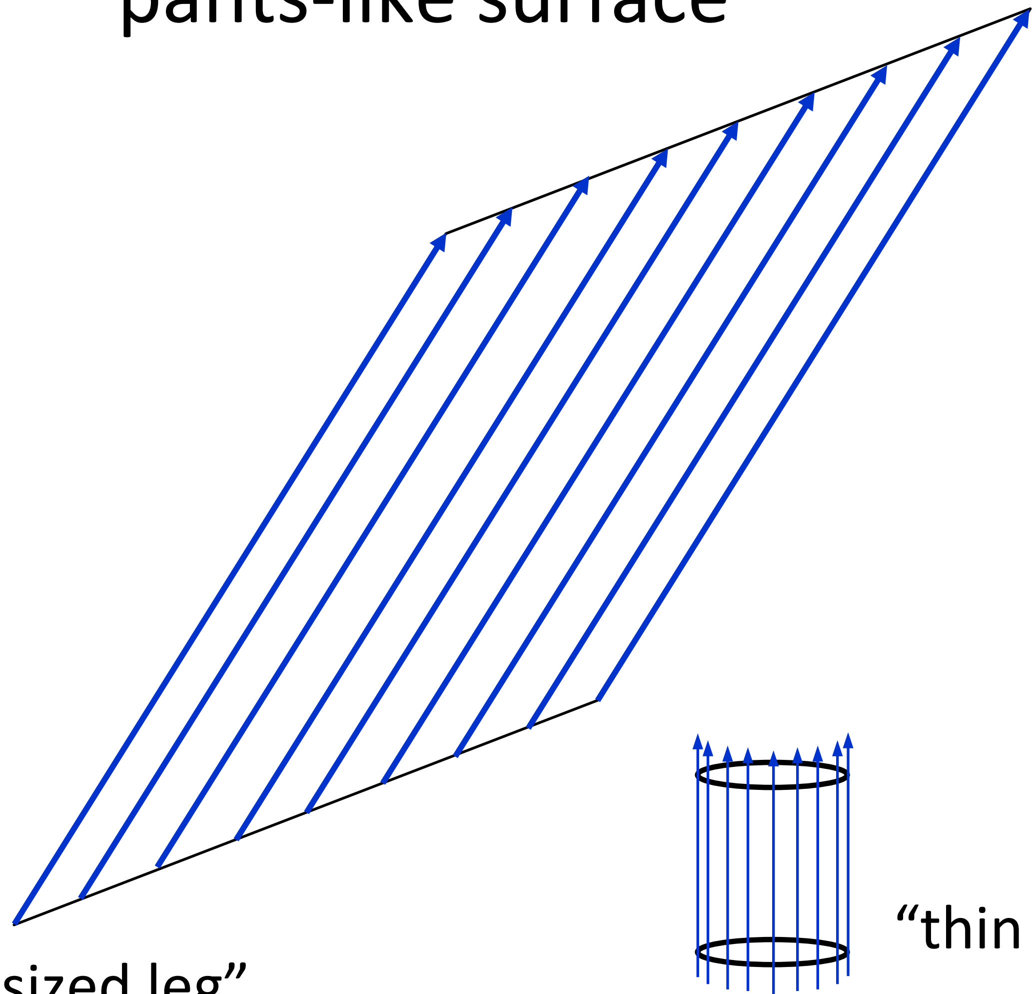
Small Black Hole falling into a Large Black Hole



both are made of
lightrays

Lightrays must merge to form a pants-like surface

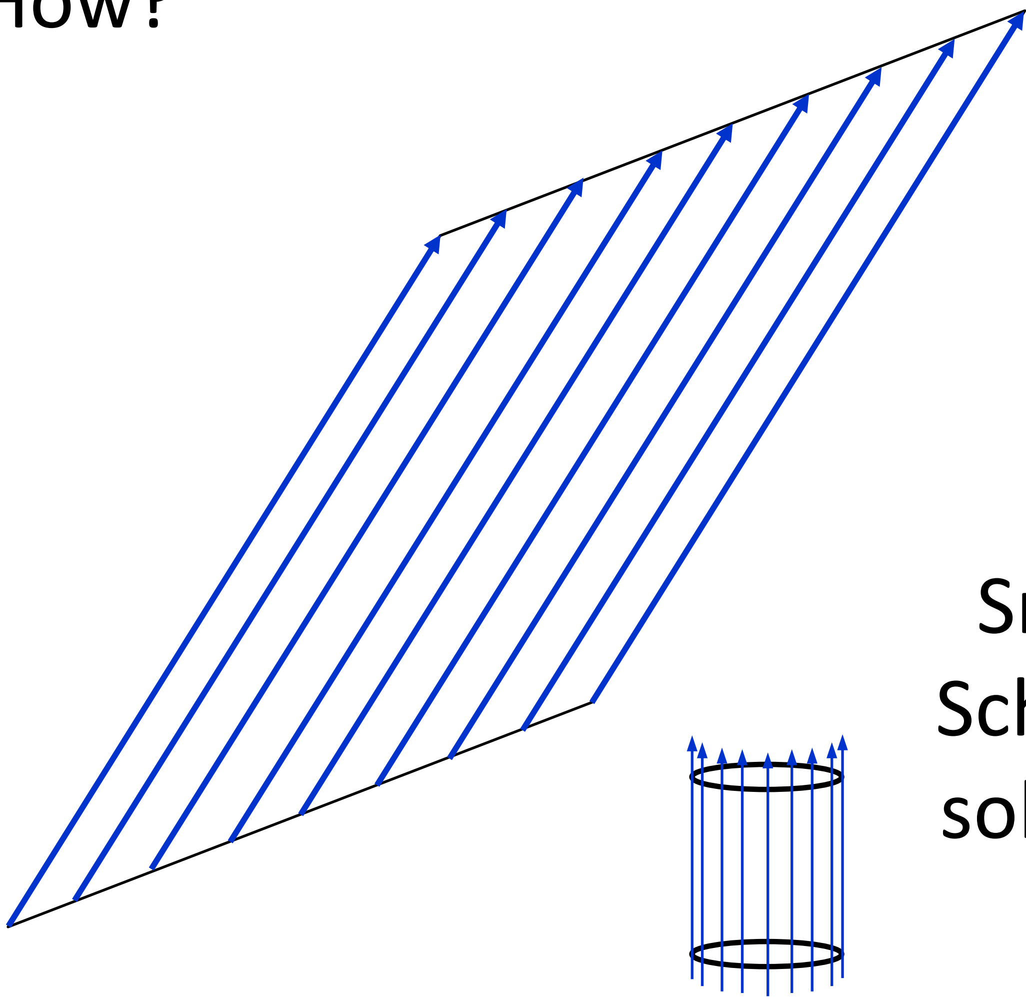




"oversized leg"

"thin leg"

How?



EH is a family of lightrays in spacetime

Small black hole: Schwarzschild/Kerr solution with finite mass m

To find the pants surface:

Trace a family of null geodesics in the

Schwarzschild/Kerr solution

that approach a null plane at infinity

All the equations you need to solve

(for Schwarzschild)

$$t_q(r) = \int \frac{r^3 dr}{(r-1)\sqrt{r(r^3 - q^2(r-1))}}$$

$2m = 1$

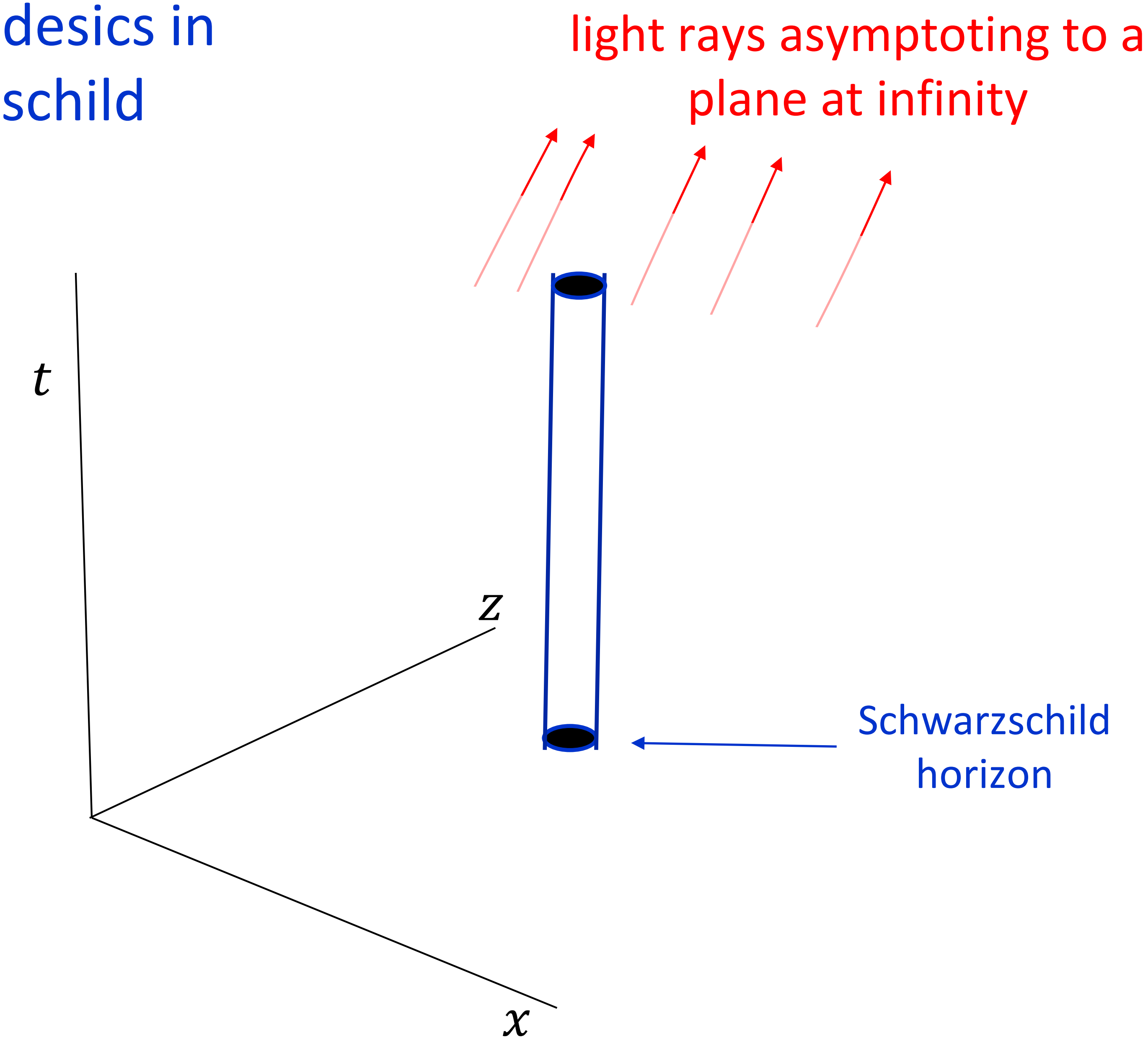
$$\phi_q(r) = \int \frac{q dr}{\sqrt{r(r^3 - q^2(r-1))}}$$

q = impact parameter of lightrays at infinity

with appropriate final conditions:

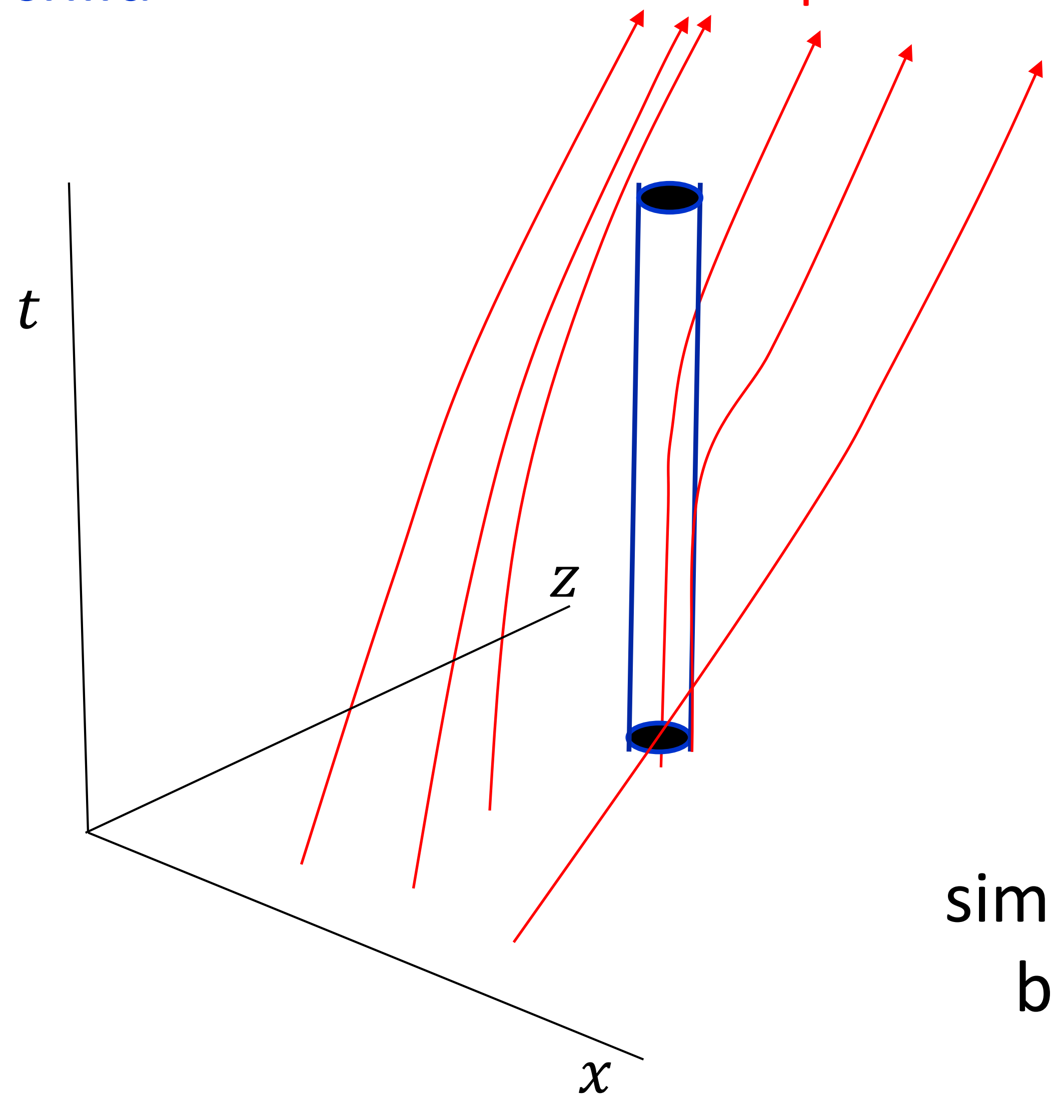
null plane at infinity

Null geodesics in Schwarzschild solution



Null geodesics in Schwarzschild solution

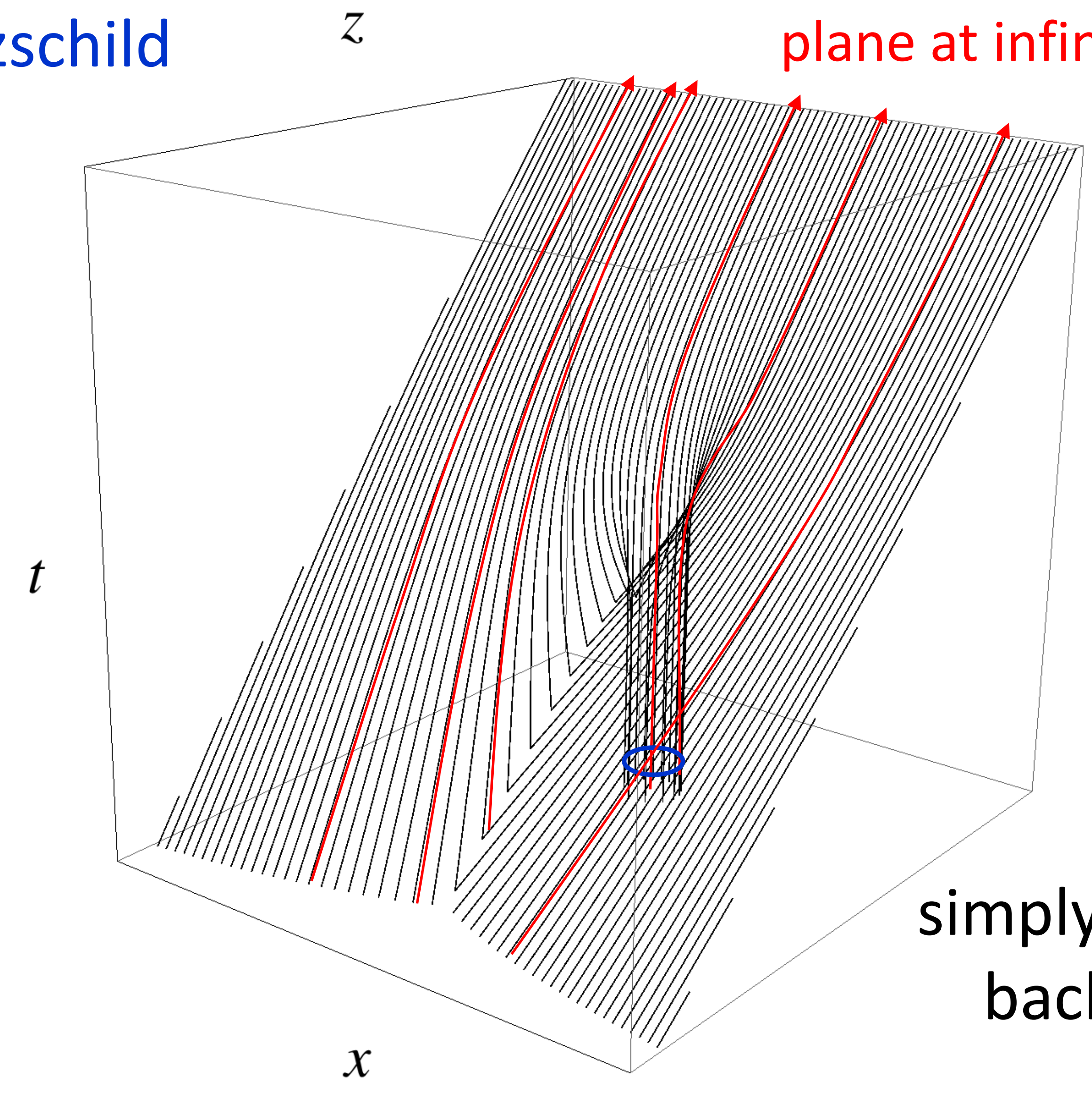
light rays asymptoting to a plane at infinity



simply, integrate back in time

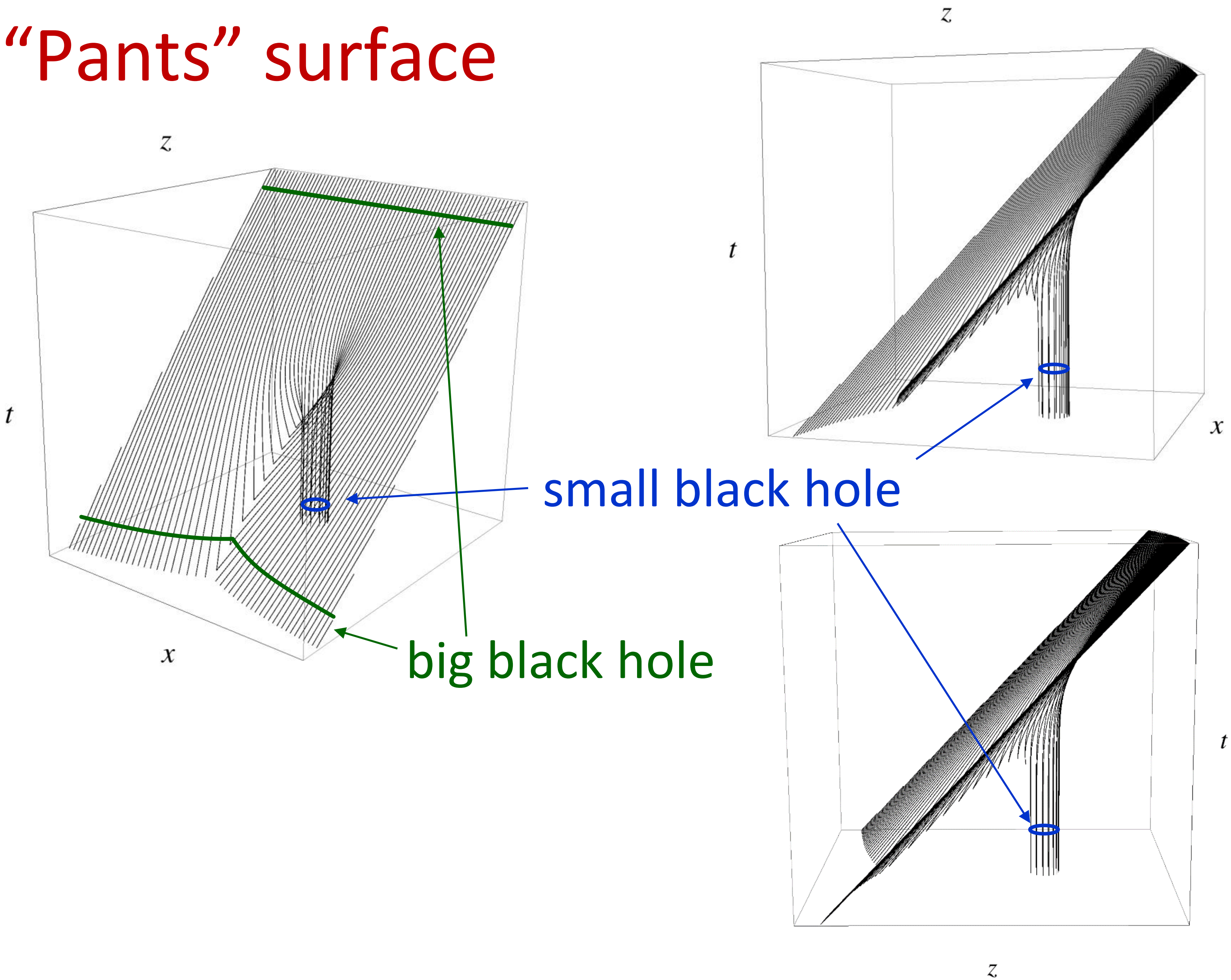
Null geodesics in Schwarzschild solution

light rays asymptoting to a plane at infinity

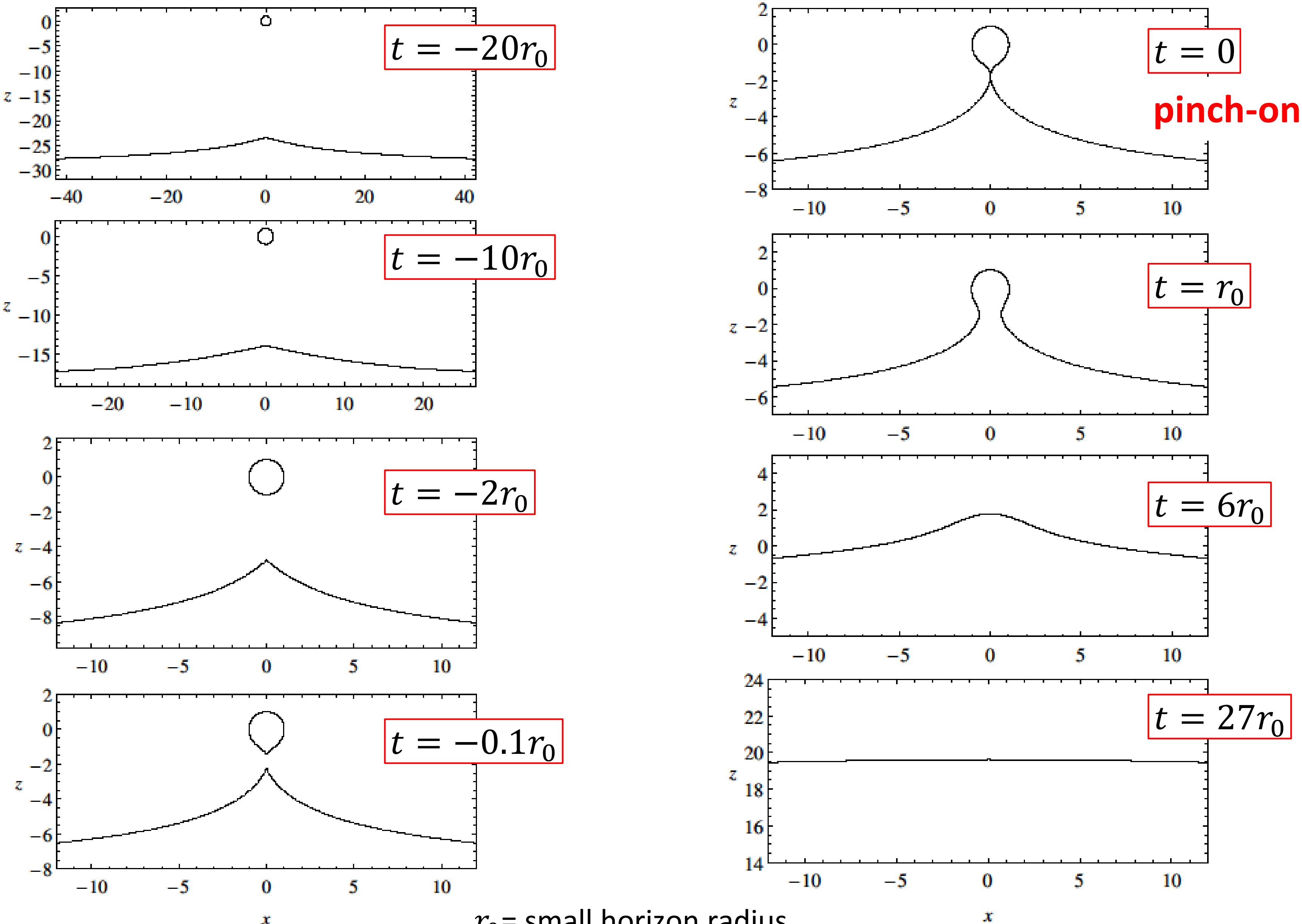


simply, integrate back in time

“Pants” surface



Sequence of constant-time slices



r_0= small horizon radius

Preferred time-slicing

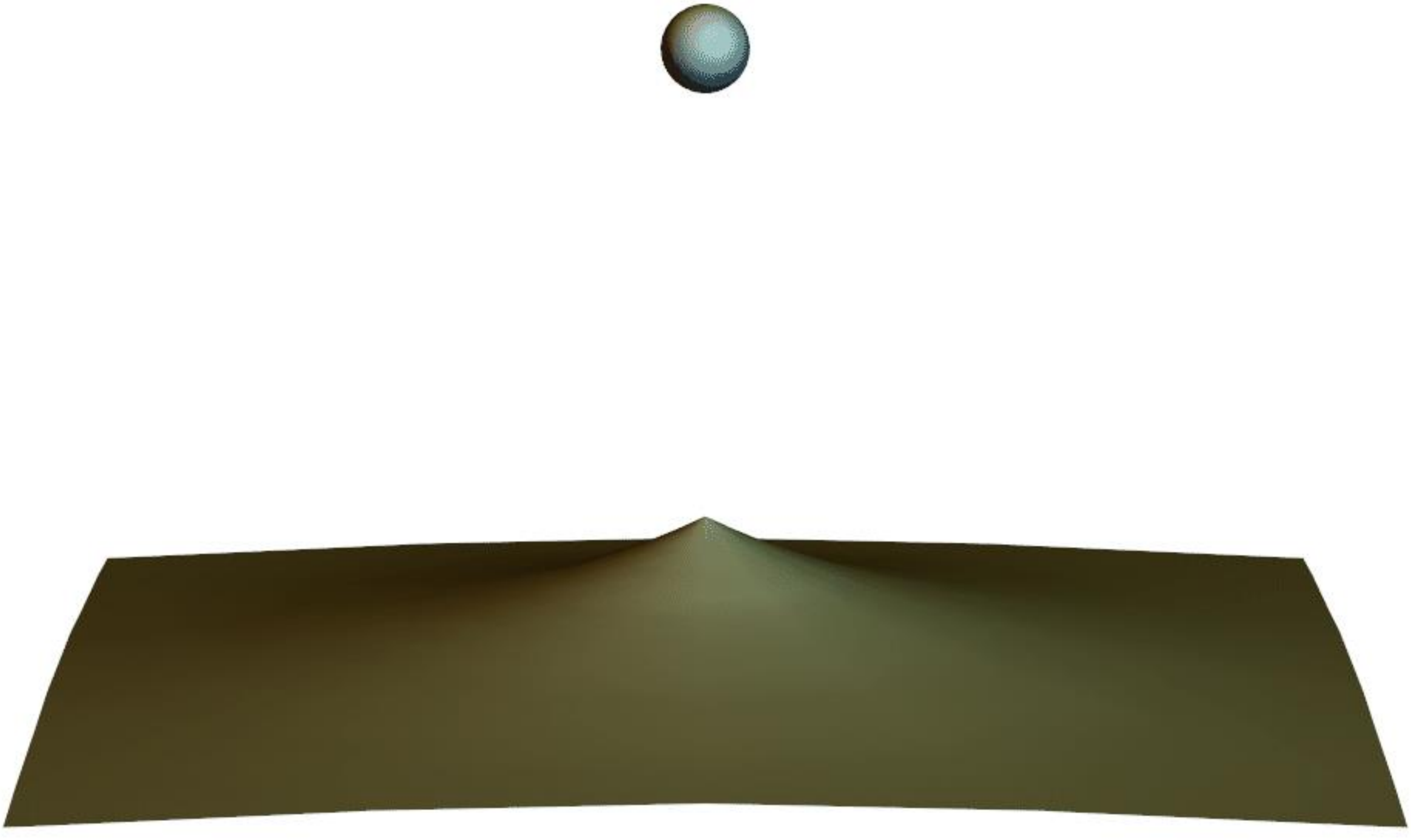
∃ timelike Killing vector

Schwarzschild time

Rest-frame of small black hole

is well defined

made with *Mathematica* in a laptop computer



The full monty

The ultimate description of EMR mergers

Arbitrary spins of either black hole

Arbitrary relative orientations of the spins

Arbitrary infall trajectories

Arbitrary relative velocities

in EMR limit $\frac{m}{M} \to 0$

Rotation and motion

Large black hole rotation

Relative motion in infall

Just a boost

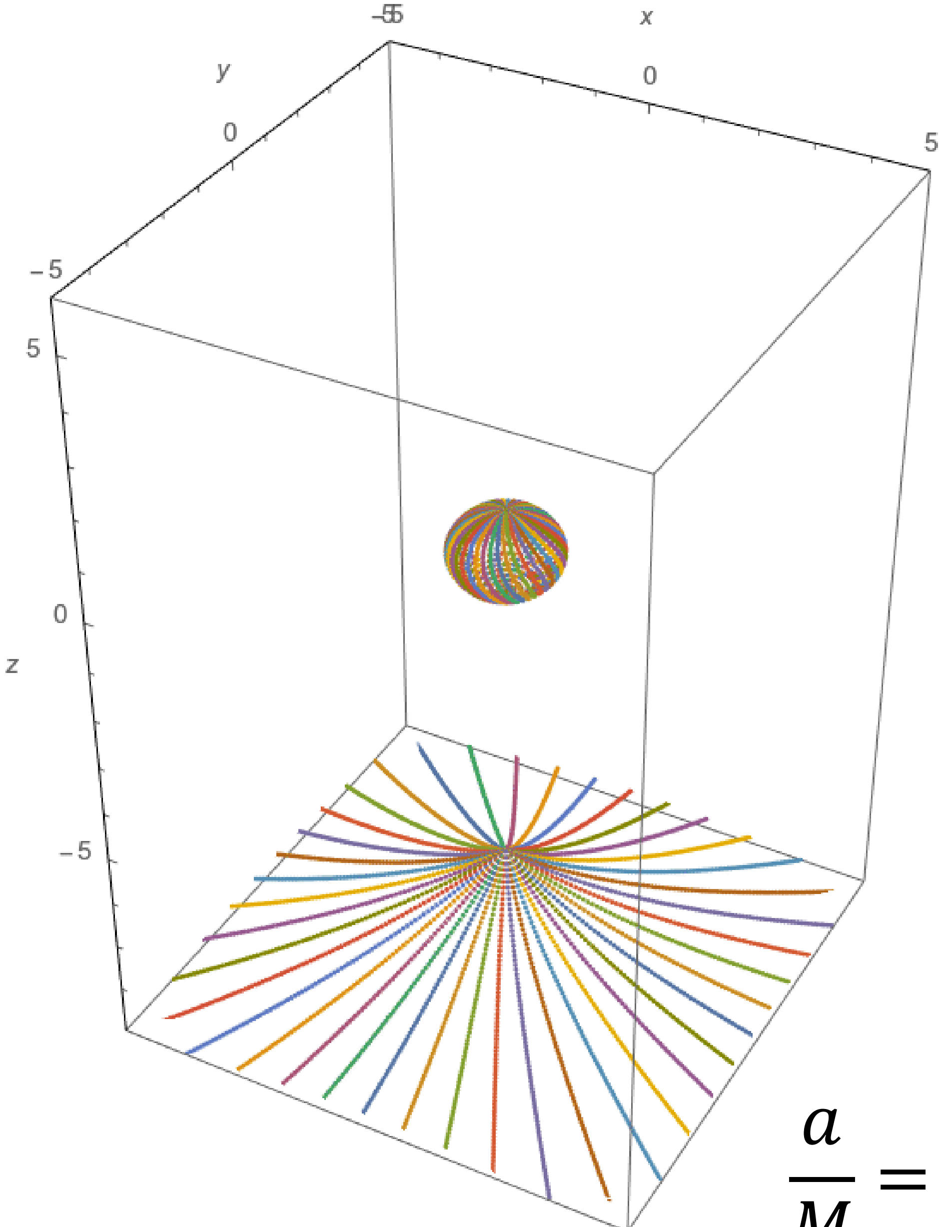
Equivalent to a rotation of the surface

Small black hole rotation

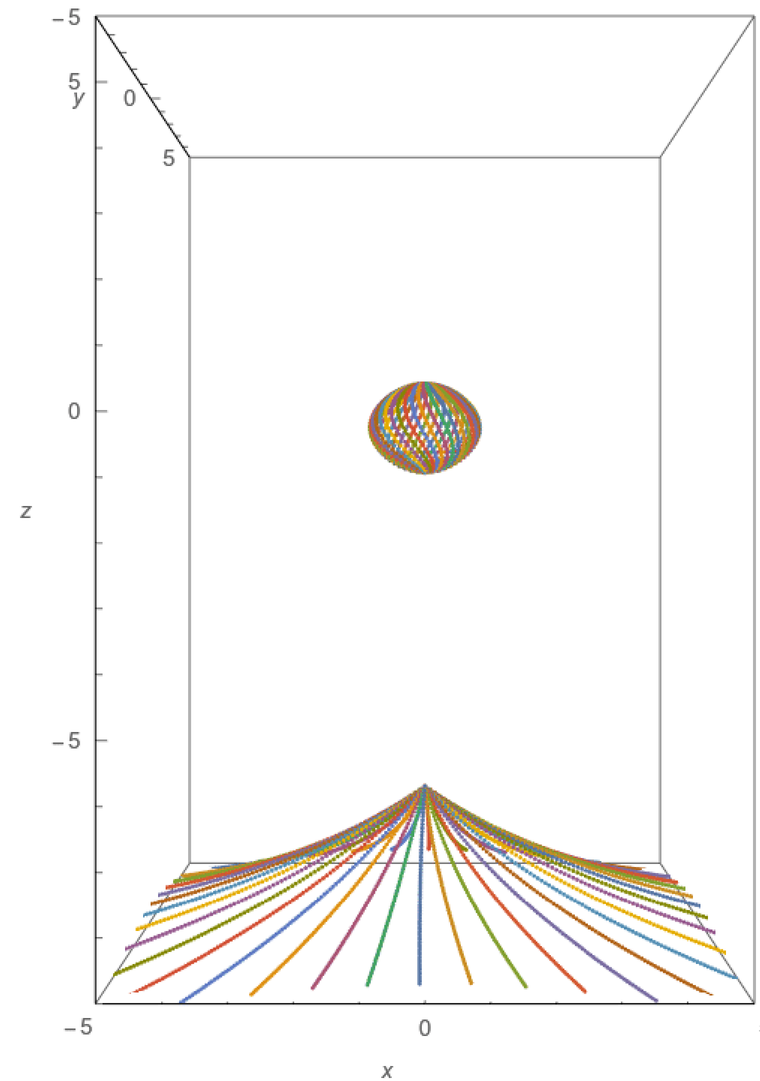
Change Schwarzschild $\rightarrow$ Kerr

Fusion of **any EMR Black Hole binary in the Universe**

to leading order in $\frac{m}{M} \ll 1$



$$\frac{a}{M} = 0.8$$



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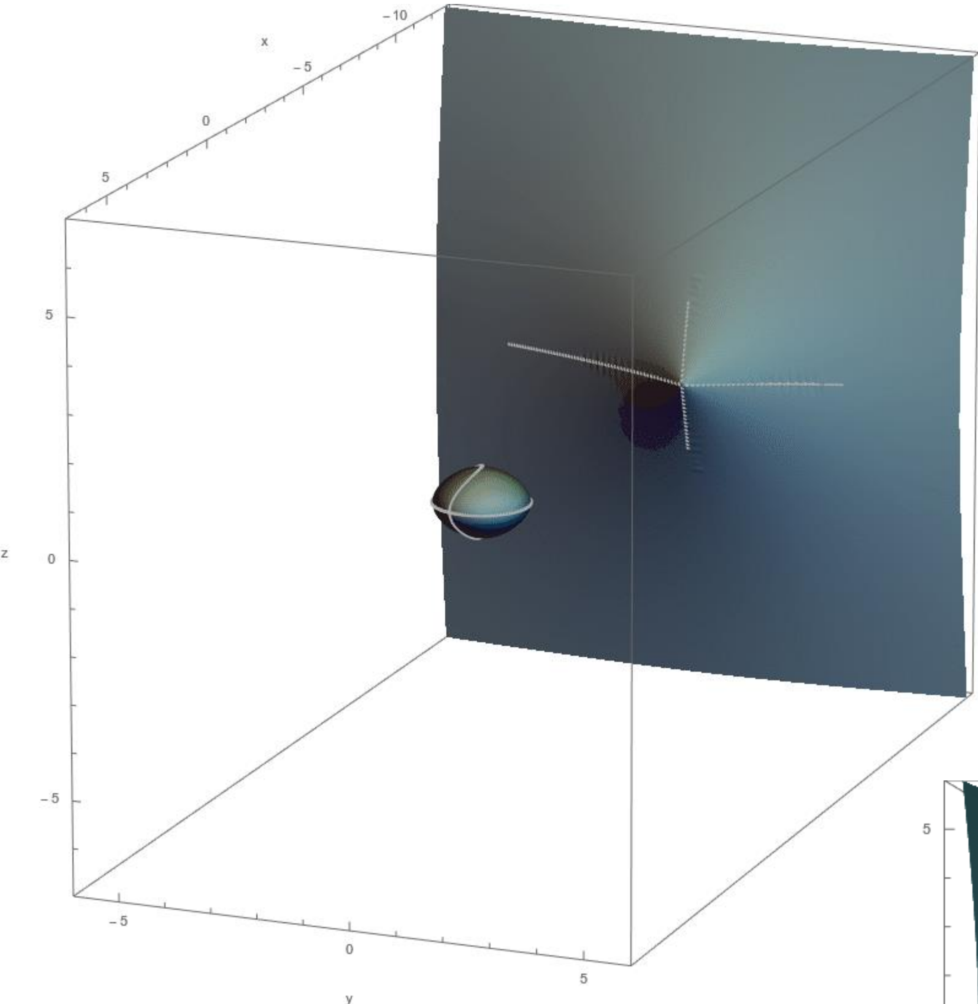
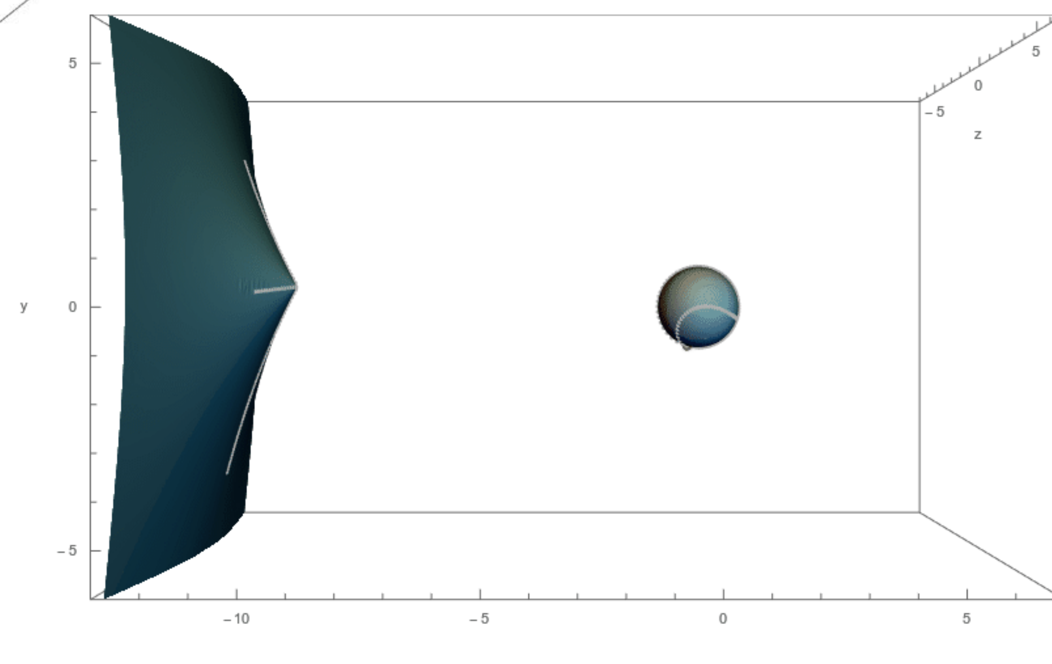
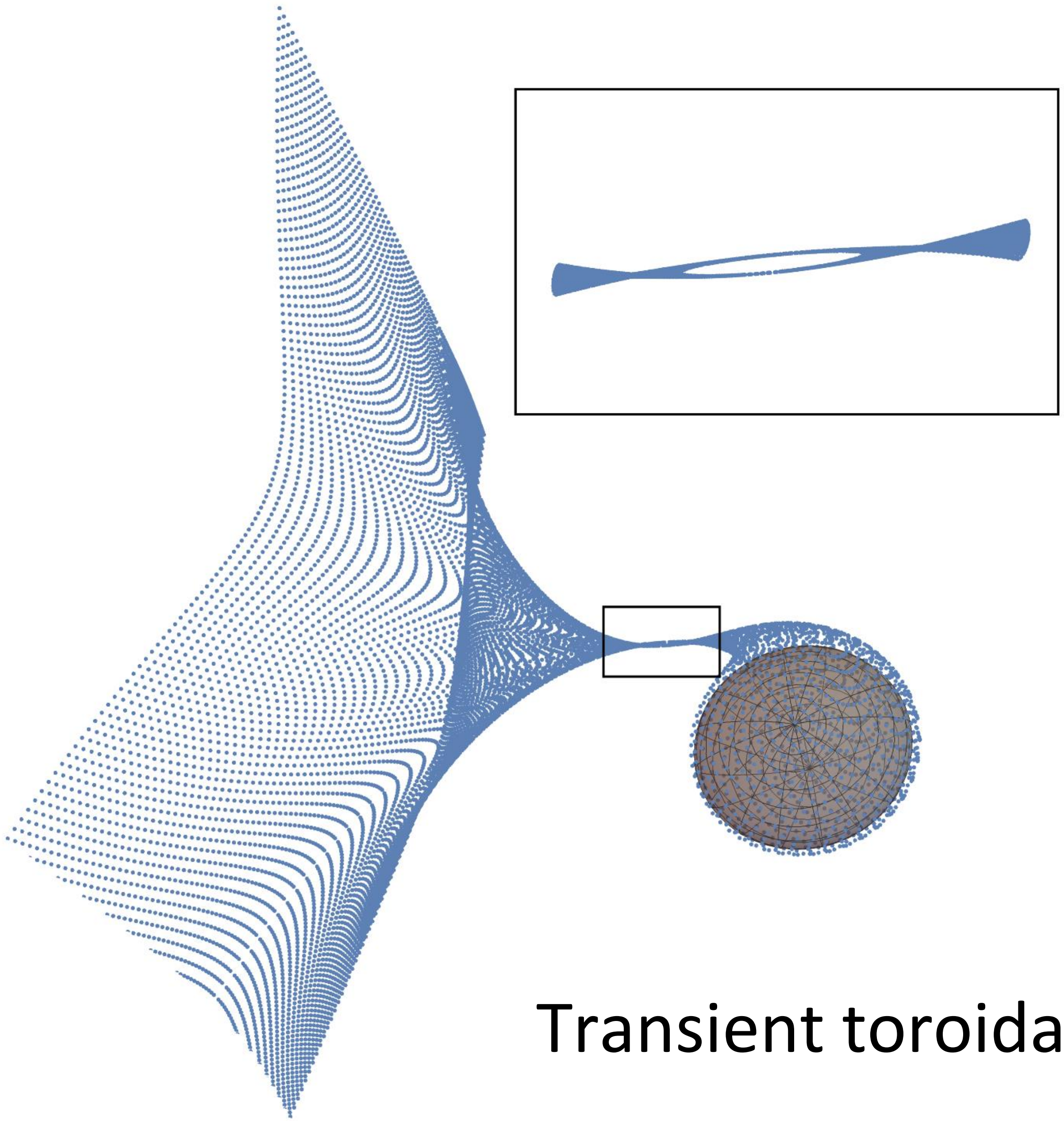


$$\frac{a}{M} = 0.9$$

view from above





Transient toroidal topology

Complete characterization of fusion

Precise quantitative results for:

Crease set and caustics

Area increase

Relaxation time

Dependence on spin and relative angles

Universal critical behavior at axisymmetric pinch

Final remarks

Simple, accurate, generic description of a process that is happening all over the Universe

Can we *observe* this?

Maybe not

Then, what is it good for?

Fusion of Black Hole Event Horizons
is a signature phenomenon of
General Relativity

Equivalence Principle allows to
capture and *understand* it easily in
a (realistic) limit

Exact construction

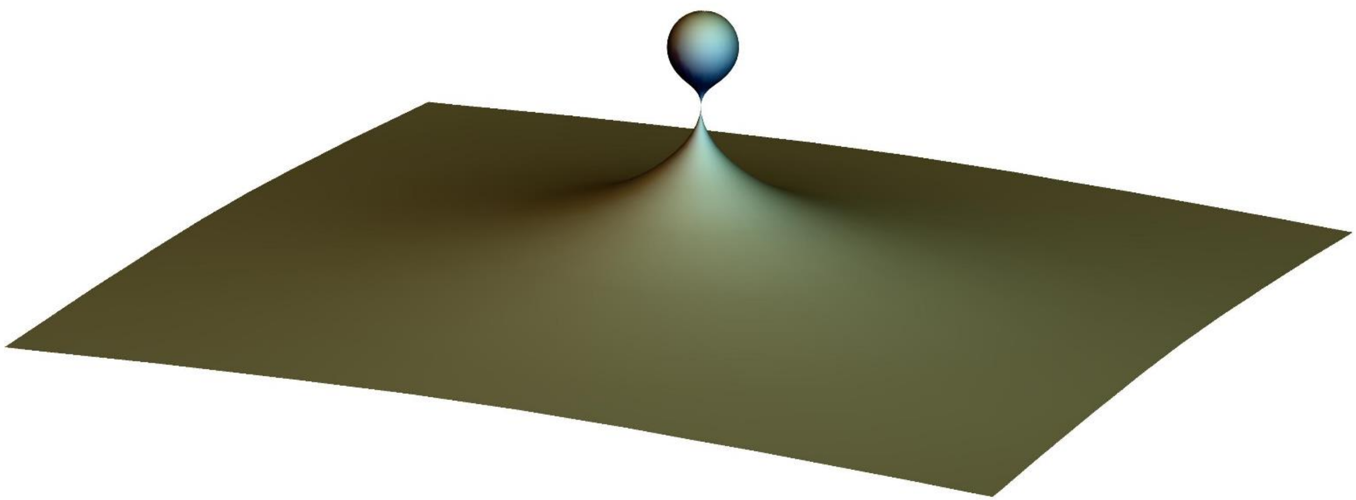
Benchmark for detailed numerical studies

First step in expansion in $\frac{m}{M} \ll 1$
to incorporate gravitational waves
(matched asymptotic expansion)

Equivalence Principle magic

Get 2 black holes

out of a geometry with only 1



This could have been done (at least) 50 years ago!

Gravitational waves?

Quasinormal vibrations

wavelength $\sim M$: become constant

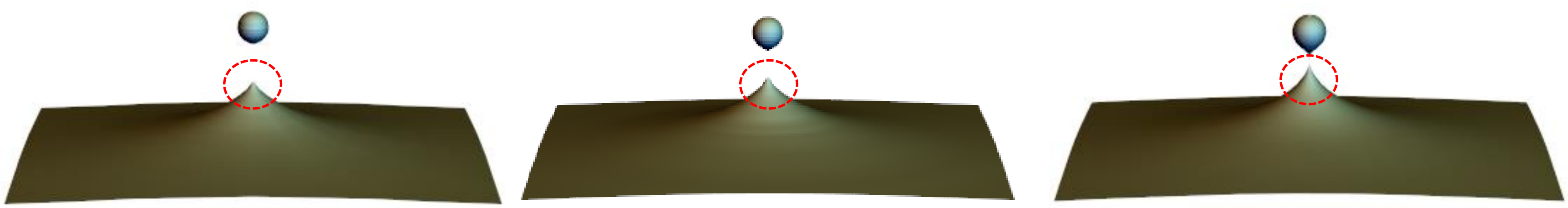
wavelength $\sim m$: $\ell \sim \frac{M}{m} \gg 1$

localized near photon orbit at distance $\sim M \to \infty$

No gravitational waves in this region

Pinch-on: Criticality

Opening angles of cones $\sim |t|^{1/2}$

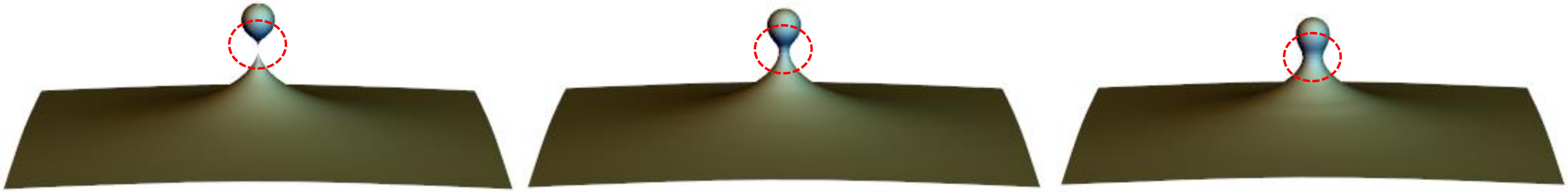


∃ simple local model for pinch

valid for **all** axisymmetric mergers

Pinch-on: Criticality

Throat growth $\sim t$



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