# Beyond Einstein


Claudia de Rham

Imperial College London


THE ROYAL SOCIETY

SF

YKIS2018a Symposium

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# Thanks to collaborators

Tate Deskins
(PhD student @ CWRU)

CdR, Deskins, Tolley & Zhou, 1606.08462, RMP

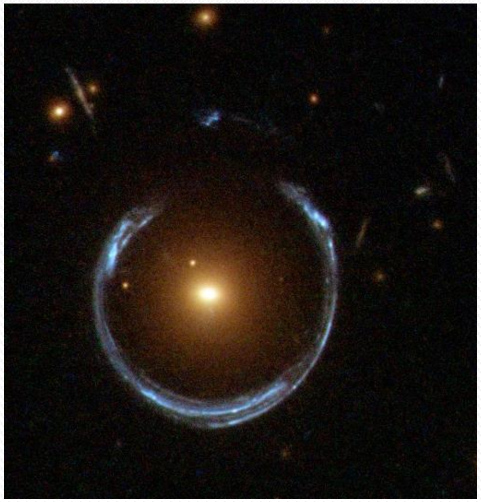
Shuang-Yong Zhou
(@ USTC)

Andrew Tolley
(@ Imperial)

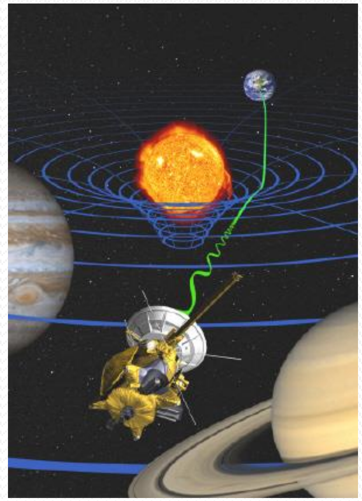
Scott Melville
(PhD student @ Imperial)

CdR, Melville, Tolley, Zhou, 1702.06134 & 1702.08577
CdR, Melville, Tolley, Zhou, 1706.02712 & 18yy.yyyyy
CdR, Melville, Tolley, 1710.09611

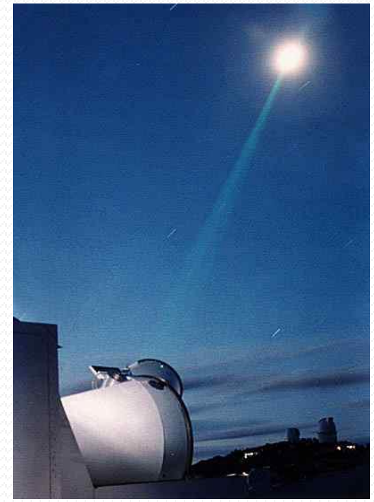
# Strong Evidence for General Relativity



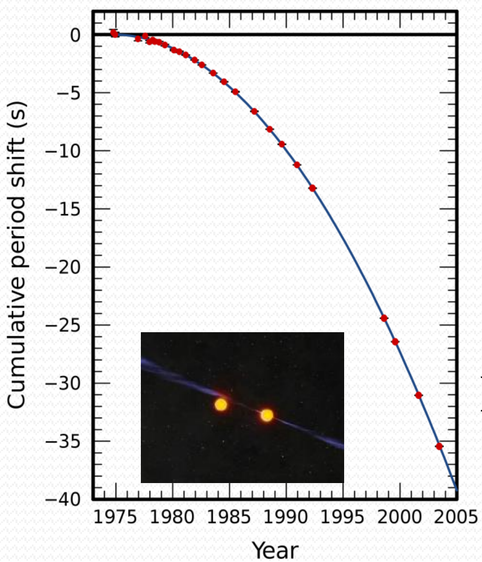
Gravitational Lensing



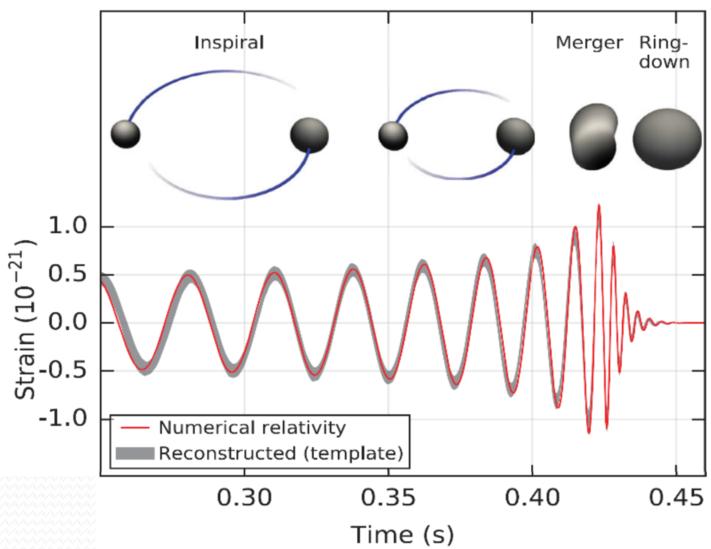
Frame Dragging
(from Earth Rotation)

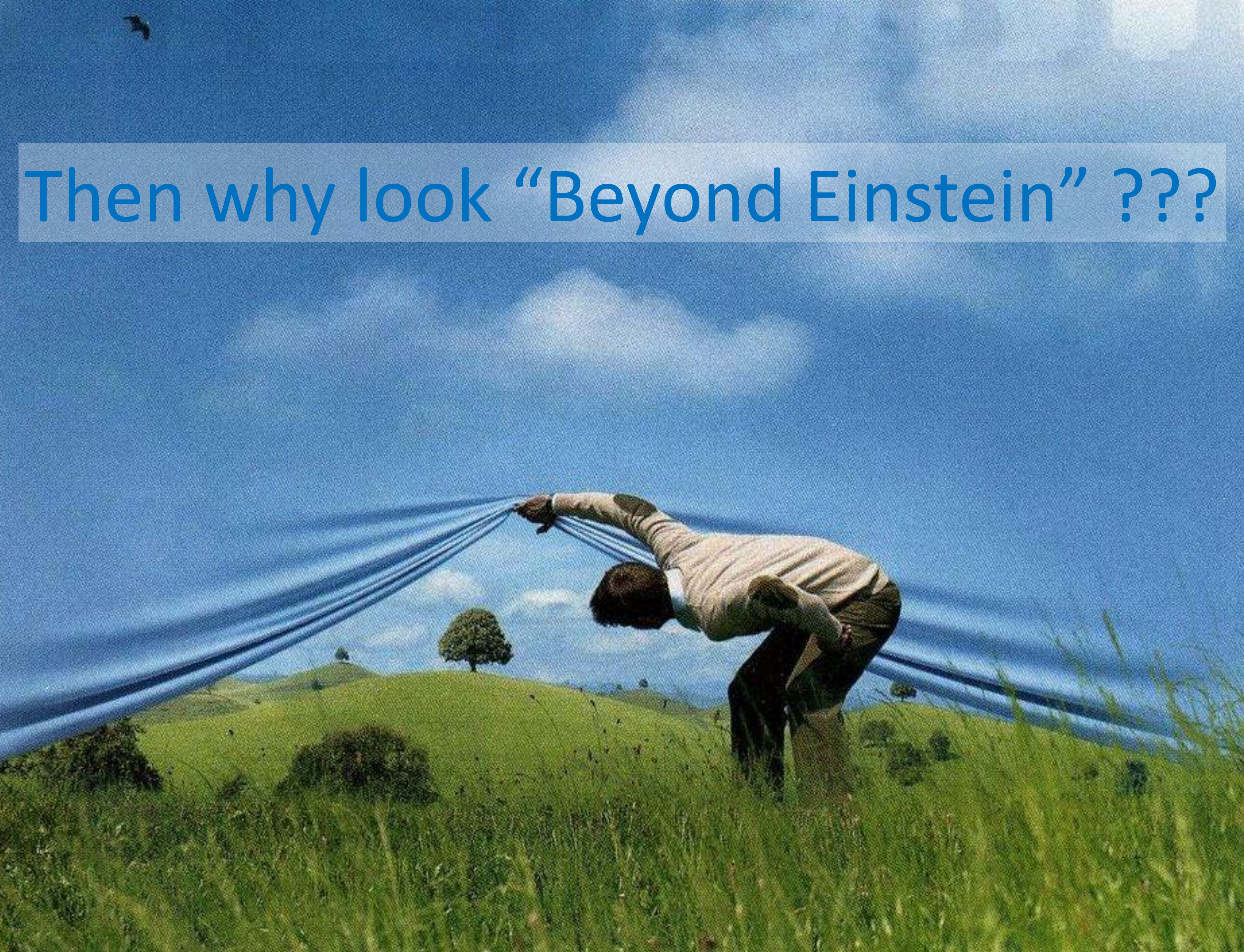


Measure of the advance
of the Perihelion

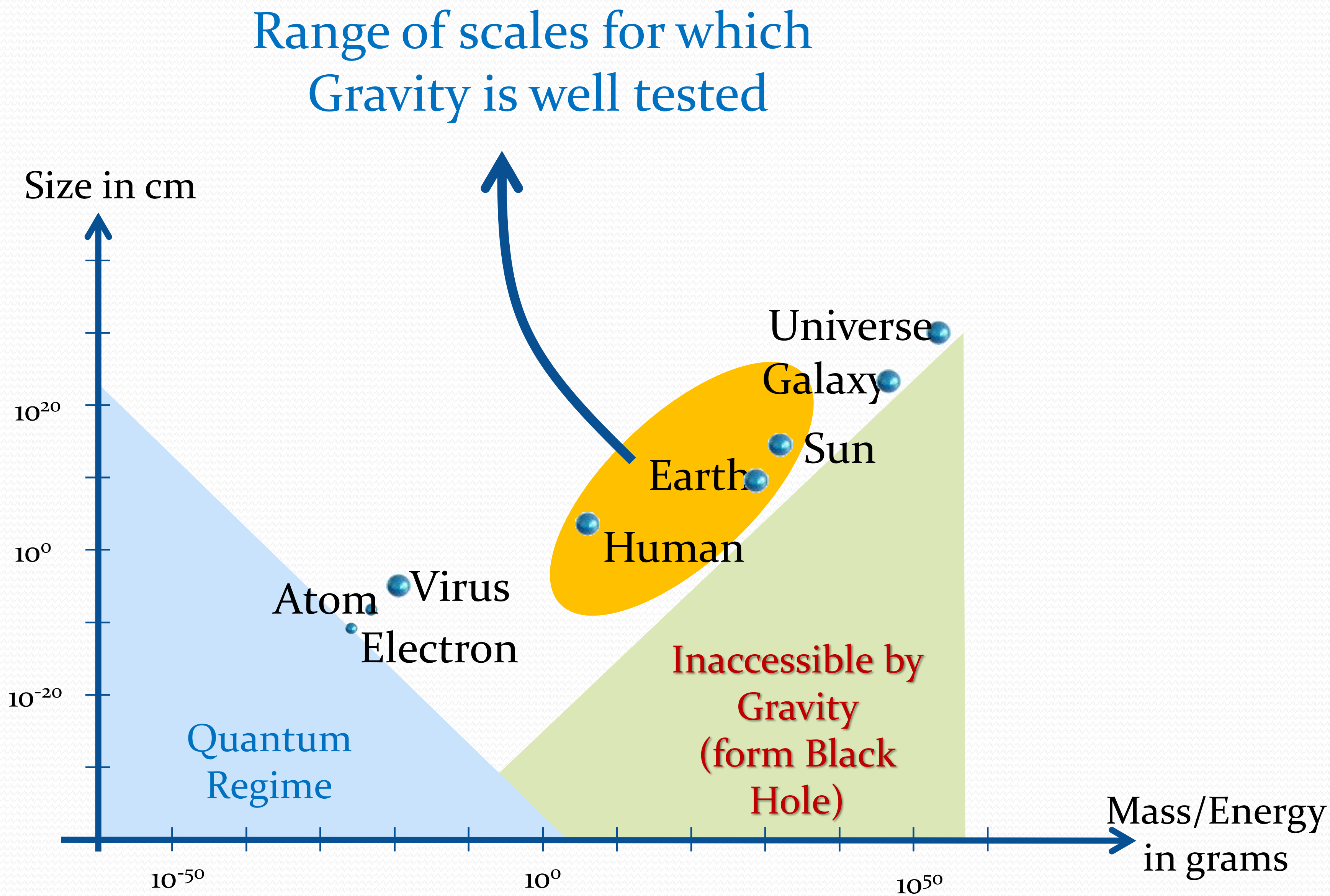



Binary Pulsar
spin-down




Then why look “Beyond Einstein” ???

# Why look "Beyond Einstein" ???



# Why look “Beyond Einstein” ???

Open questions and puzzles of Cosmology…

inflaton or its alternative

Hierarchy Problem

Dark Matter

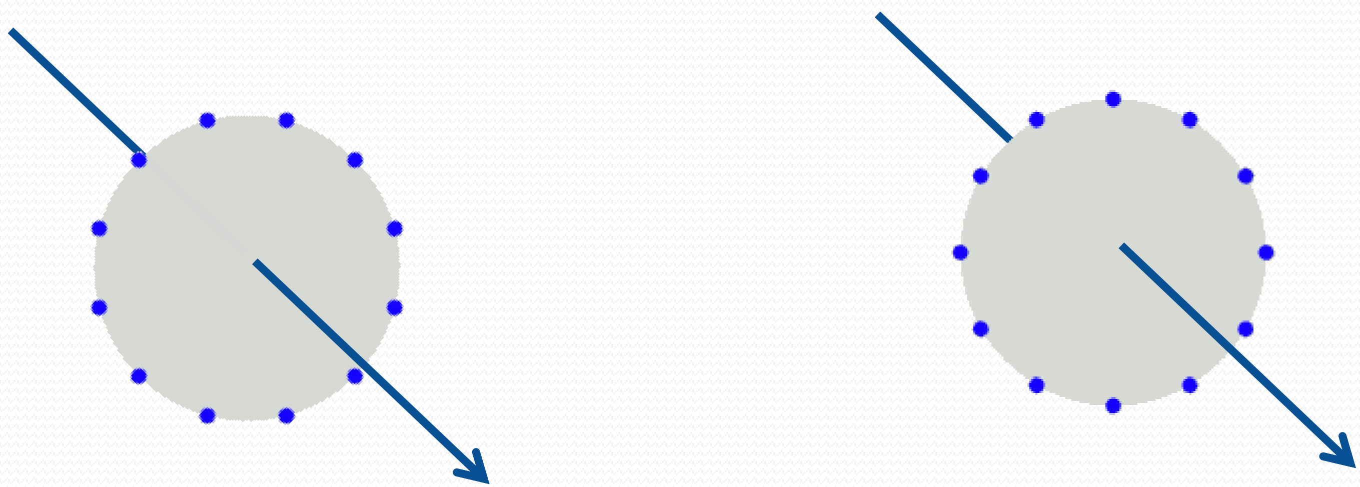
Dark Energy

CC problem

# General Relativity

$$S = \int \sqrt{-g}\frac{M_{\rm Pl}^2}{2}\ R$$

- **GR: 2** polarizations



# Massive Gravity

$$S = \int \sqrt{-g}\frac{M_{\rm Pl}^2}{2}\left(R - \text{Mass Term}\right)$$

- The notion of mass requires a ***reference*** !

# Massive Gravity

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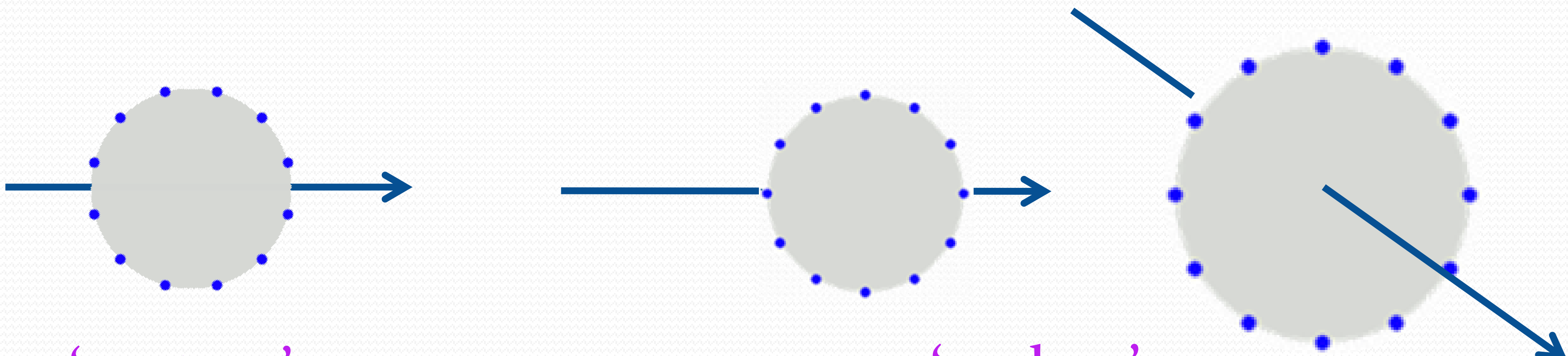
- The notion of mass requires a ***reference*** !
- Generates new dof

$$2 + 4 = 6$$

GR ↵ (2)     (4) ↳ Loss of 4 sym

# Gravitational Waves

- **GR: 2** polarizations
- In principle GW could have 4 other polarizations



2 'vectors'

2 'scalars'

Potential `new degrees of freedom'

# Fierz-Pauli Massive Gravity

$$\mathcal{U}_{\rm FP} = h_{\mu\nu}^2 - h^2$$

- Mass term for the fluctuations around flat space-time

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

Fierz & Pauli, Proc.Roy.Soc.Lond.A **173**, 211 (1939)

# Fierz-Pauli Massive Gravity

$$\mathcal{U}_{\rm FP} = h_{\mu\nu}^2 - h^2$$

- Mass term for the fluctuations around flat space-time

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

- Transforms under a change of coordinate

$$x^\mu \;\to\; x^\mu + \partial^\mu \xi$$

$$h_{\mu\nu} \;\to\; h_{\mu\nu} + 2\partial_\mu\partial_\nu\xi + \partial_\mu\partial_\alpha\xi\partial_\nu\partial^\alpha\xi$$

Typically involves some higher derivatives which leads to a ghost

Deffayet & Rombouts, 2005; Creminelli et. al. 2005

# Massive Gravity

$$S = \int \sqrt{-g}\,\frac{M_{\rm Pl}^2}{2}\,(R - \text{Mass Term})$$

- The notion of mass requires a ***reference*** !

- Generates new dof

$$2 + 4 = 6 = 5 + 1$$

Boulware & Deser, PRD6, 3368 (1972)

# Massive Gravity

While it is true that most model of massive gravity suffer from ghost pathologies, there is a special class of theory for which the mode is fully absent


CdR & Gabadadze, 2010
CdR, Gabadadze & Tolley, 2011

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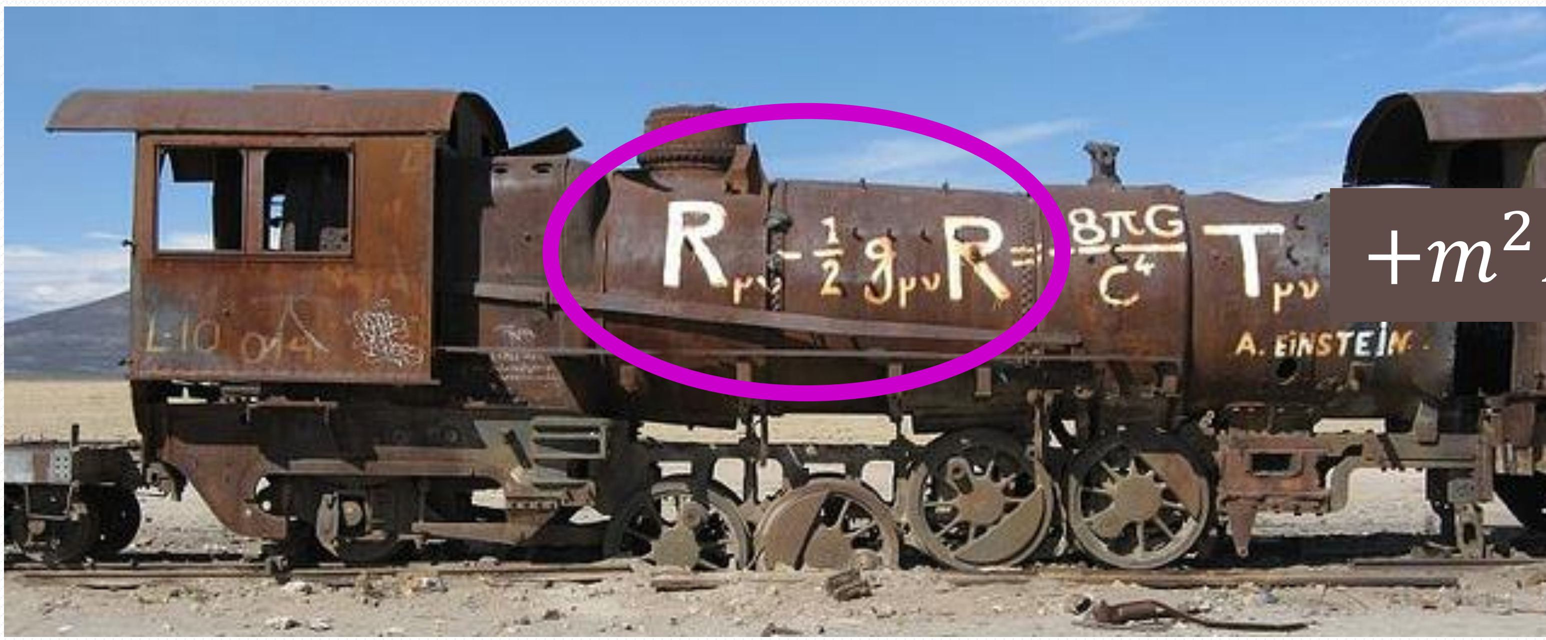



$+m^2 M_{\mu\nu}$

Kinetic term has to be identical as in GR

With Andrew Matas & Tolley, 2013, 2015, 2015, 2015

# Massive Gravity

While it is true that most model of massive gravity suffer from ghost pathologies, there is a special class of theory for which the mode is fully absent


$+m^2 M_{\mu\nu}$

Matter coupling has to be identical as in GR

# Massive Gravity

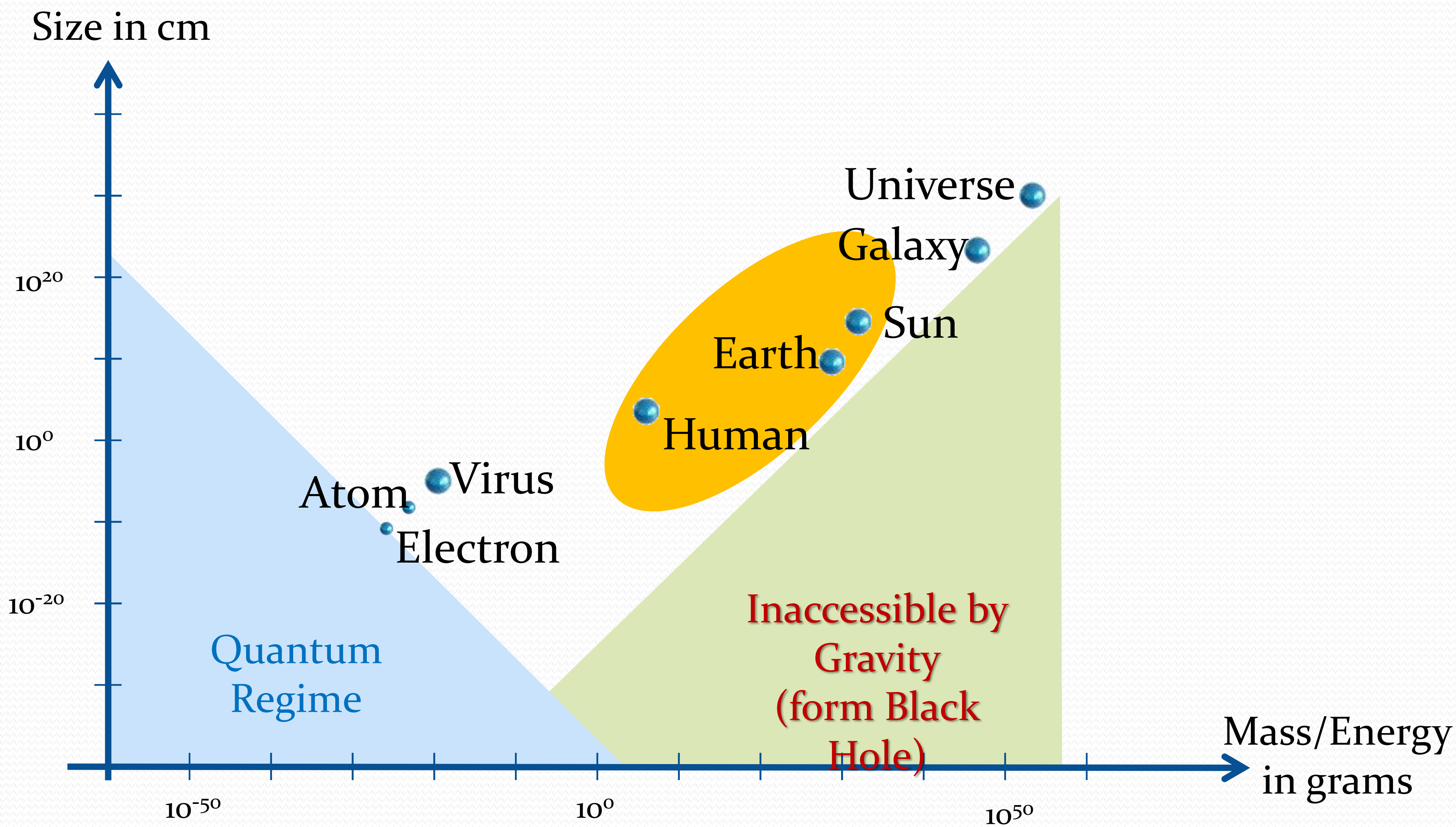
While it is true that most model of massive gravity suffer from ghost pathologies, there is a special class of theory for which the mode is fully absent

Can we test such a theory ???


$+m^2 M_{\mu\nu}$

Only 2-parameters + mass scale

# Observational Tests



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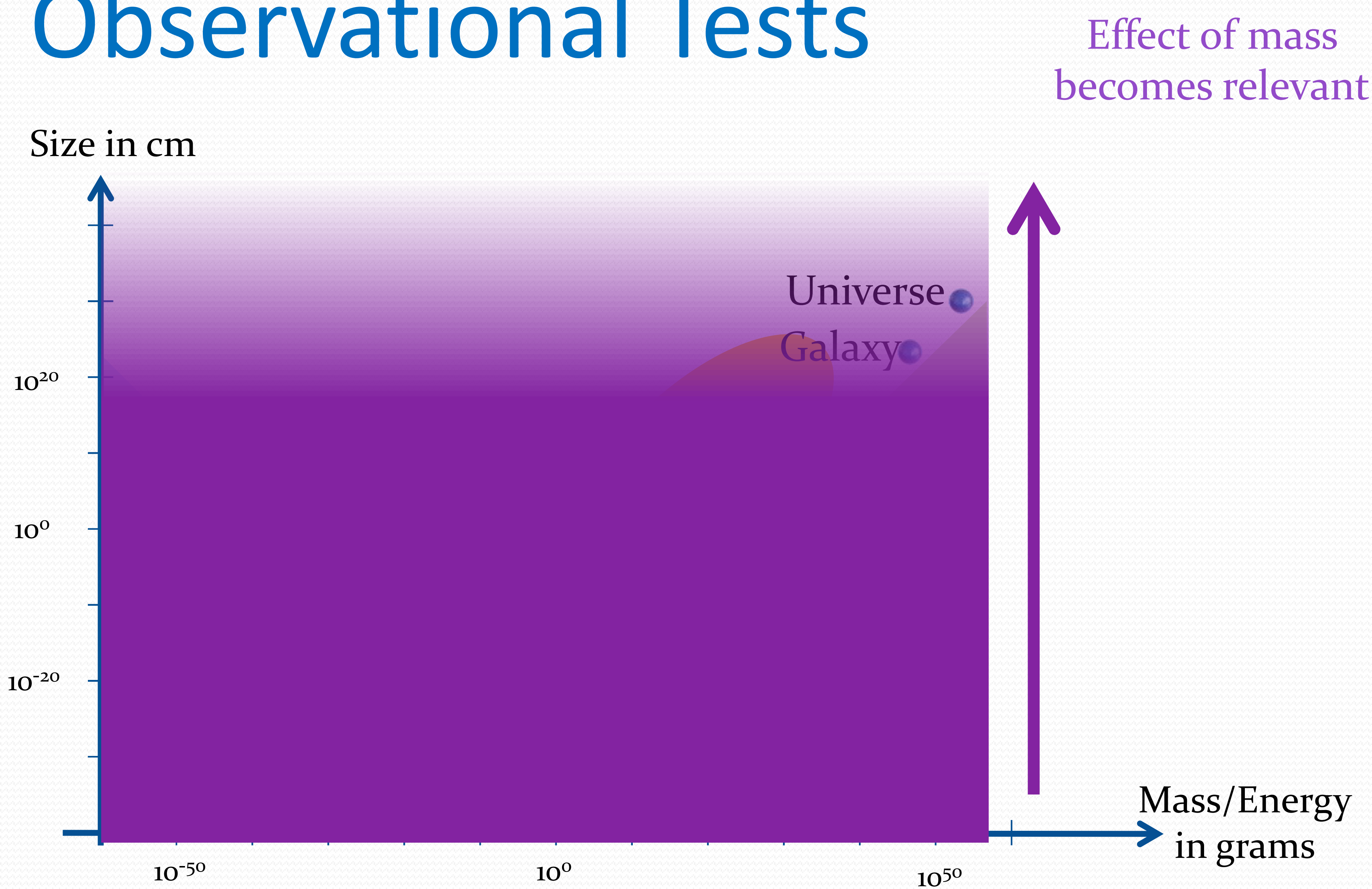
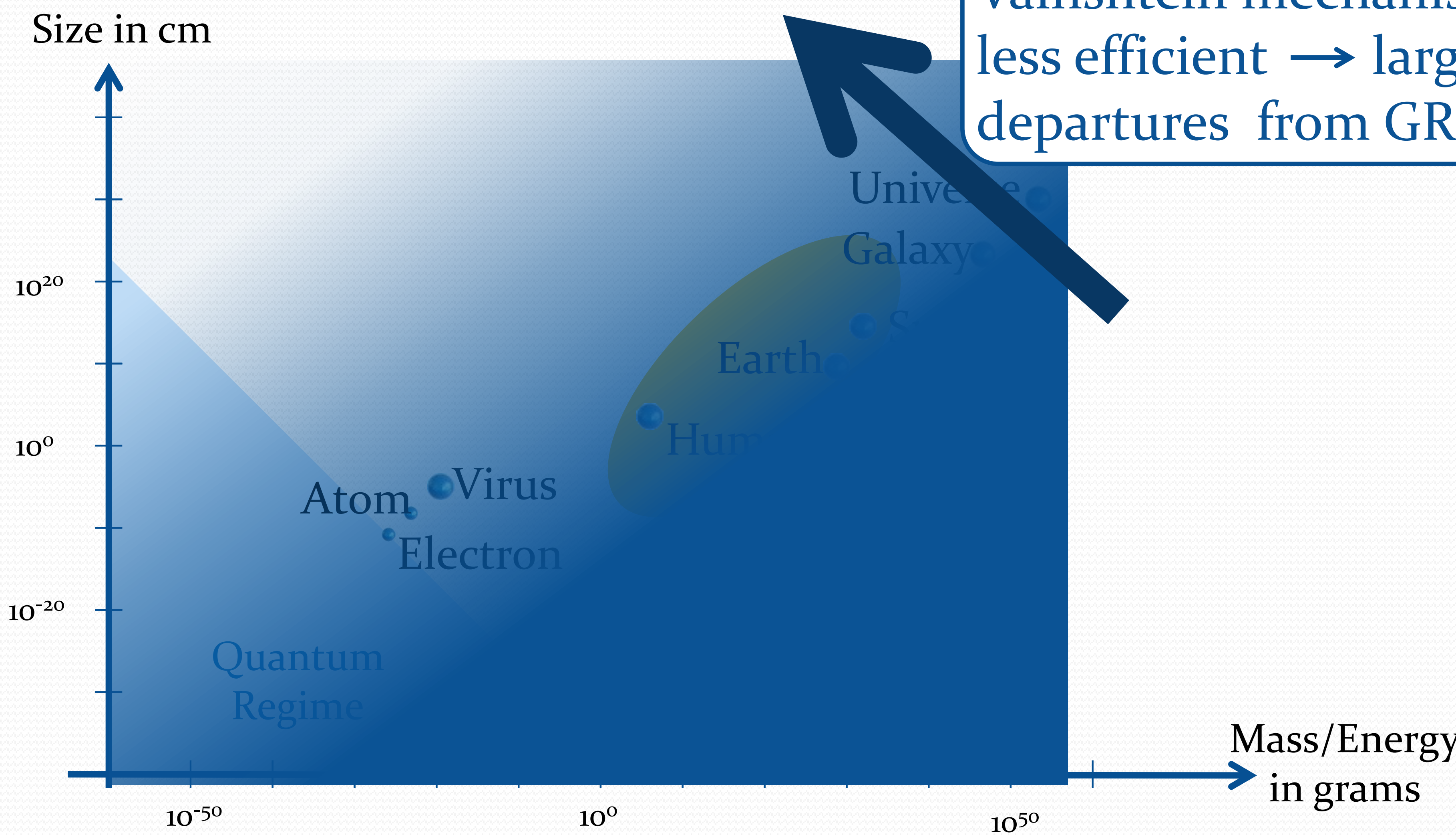


# Observational Tests

Vainshtein mechanism less efficient ⟶ larger departures from GR



# How light is gravity ???

**Dispersion Relation**

| $m_g$ (eV) | $\lambda_g$ (km) | |
|---|---|---|
| $10^{-22}$ | $10^{11}$ | aLIGO bound |
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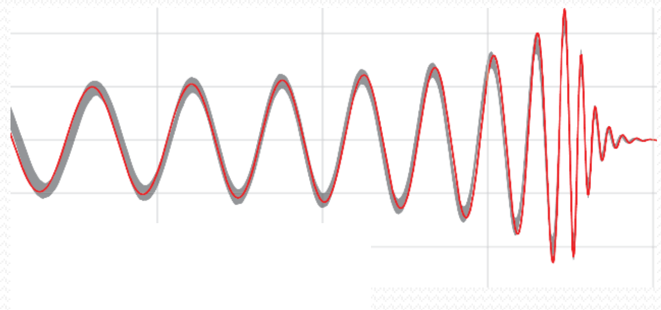

**Fifth Force**

| $m_g$ (eV) | $\lambda_g$ (km) | |
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**Yukawa**

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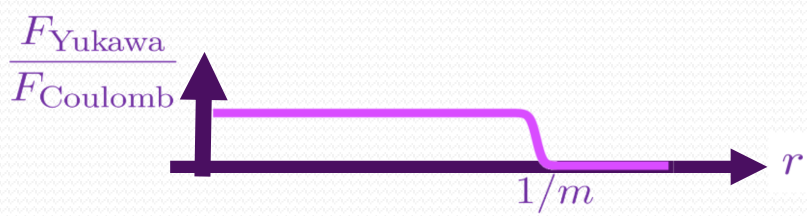


CdR, Deskins, Tolley, Zhou, 1606.08462, RMP

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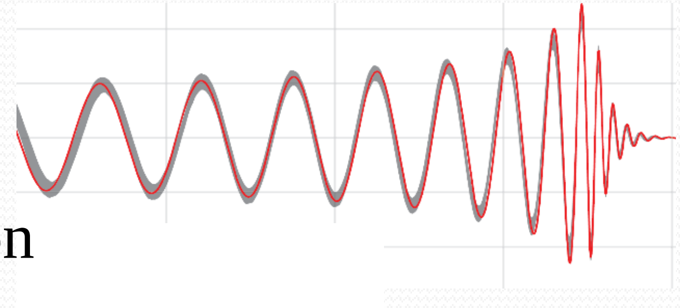
Cleanest
(least model dependent)

Only for models
that carry a helicity-0 mode
(ie. For Local and Lorentz-invariant models)

CdR, Deskins, Tolley, Zhou, 1606.08462, RMP

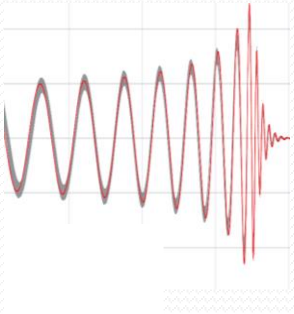
# Direct detection of GWs



Constraints modifications of the dispersion relation

$$E^2 = \mathbf{k}^2 + m_g^2$$

Generic for the helicity-2 modes of any Lorentz invariant model of massive gravity (including DGP at the level of spectral representation)



GW signal would be more squeezed than in GR

matched filtering technique allows to determine the signal duration when emitted $\Delta\tau_e$ very accurately which can be compared with the signal duration when observed $\Delta\tau_a$.

$$\Delta t = \Delta\tau_a - \Delta\tau_e(1+z)$$

Will 1998

# Direct detection of GWs

modifications of the dispersion relation put a bound on the graviton mass

$$m_g \lesssim 4 \times 10^{-22}\text{eV} \left( f\Delta t \frac{f}{100\text{Hz}} \frac{200\text{Mpc}}{D} \right)^{1/2}$$

Phase distortion $f\Delta t$ can be measured up to $1/\rho$ ($\rho$: the signal to noise ratio)

For GW150914,

$$D \sim 400\text{Mpc}, \; f \sim 100\text{Hz}, \; \rho \sim 23 \quad \Rightarrow \quad m_g \lesssim 10^{-22}\text{eV}$$

For GW151226, $\rho$ is smaller and the BHs are lighter so $f$ is larger $\longrightarrow$ not as competitive

Will 1998
Abbott et al., 2016

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For GW170817 & GRB170817A

$$\Delta c = |c_\gamma - c_{\mathrm{GW}}| < 10^{-15} \quad \Rightarrow \quad m_g \lesssim 10^{-21}\mathrm{eV}$$

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For LISA, could have

$$\rho \sim 10^3$$

$$D \sim 3\mathrm{Gpc} \quad \Longrightarrow \quad m_g \lesssim 10^{-26}\mathrm{eV}$$

$$f \sim 10^{-3}\mathrm{Hz}$$

Will 1998

# Indirect Gravitational Wave Detection

Pulsar Timing Arrays could in principle detect $\eta$HZ GWs

would put a bound $$m_g \lesssim f \sim 10^{-23}\mathrm{eV}$$

Lee et al., 2010

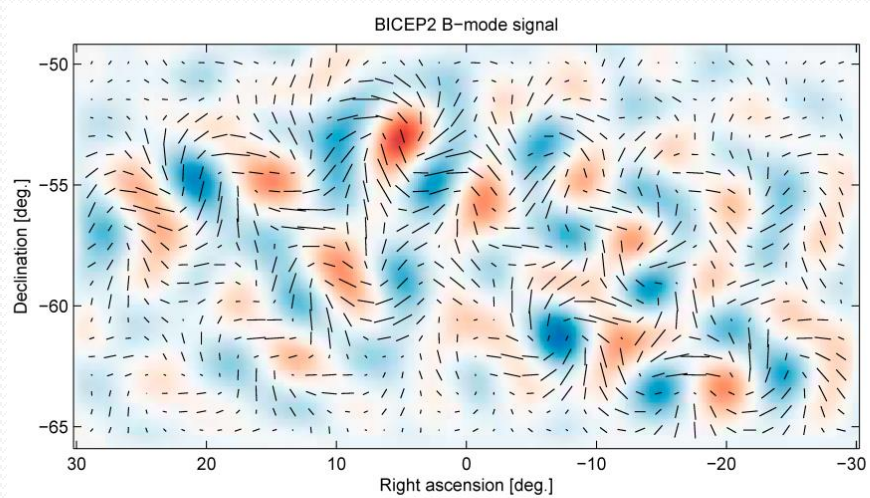
Binary Pulsar Radiation

expect a correction of order $m^2/f^2$ to the power emitted by the tensor modes

$$m_g \lesssim \frac{10^{-1}}{(\text{few hours})} \sim 10^{-20}\mathrm{eV}$$

Finn and Sutton, 2002

# Bounds from Primordial Gravitational Waves




*if ever detected…*

would imply the graviton is effectively massless at the time of recombination

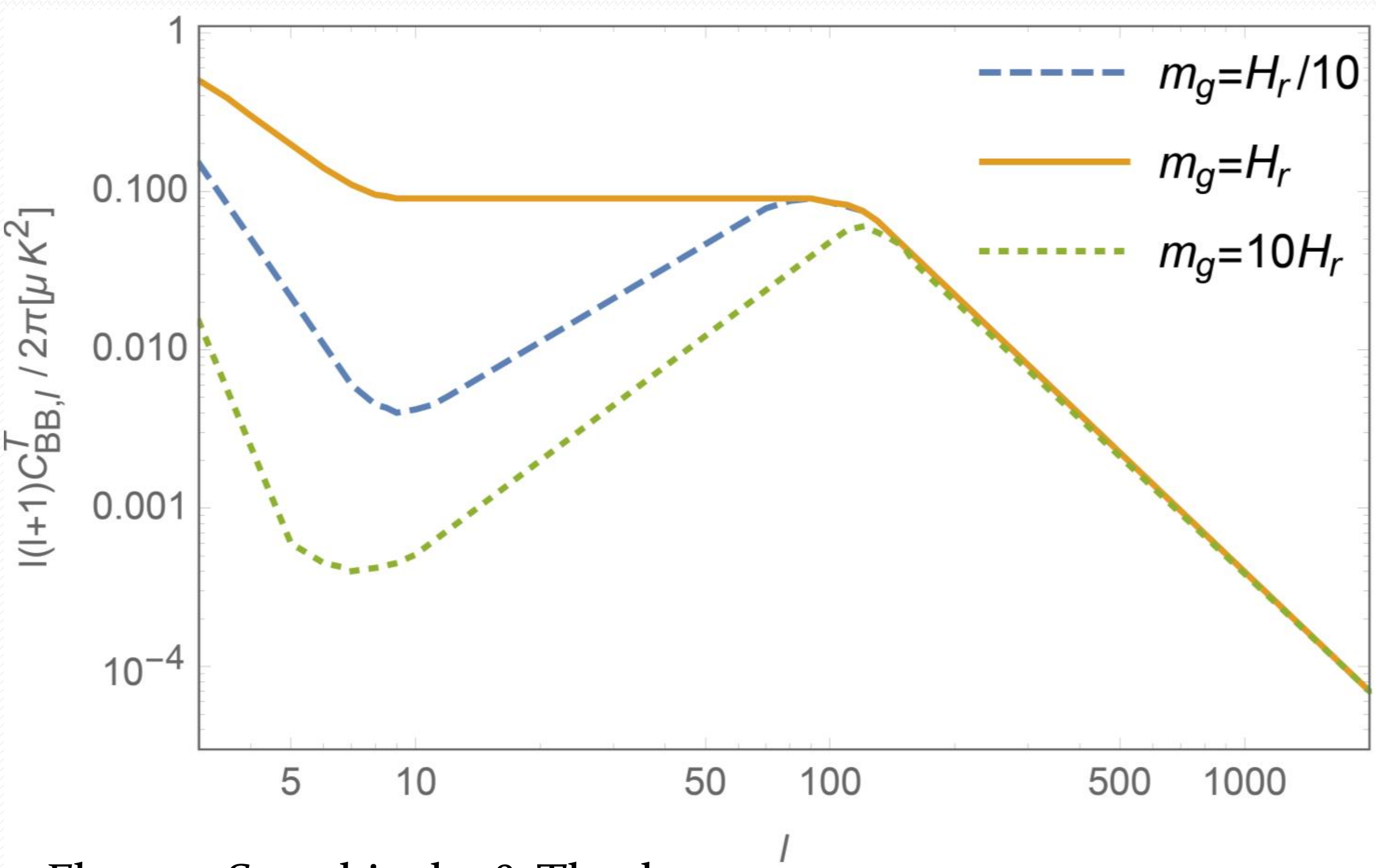
$$m_{\text{eff}} \ll 10^{-29}\text{eV}$$

Dubovsky, Flauger, Starobinsky & Tkachev, 2010
Fasiello & Ribeiro, 2015, (for bi-gravity)
Lin&Ishak, 2016 (Testing gravity using tensor perturbations)

# Bounds from Primordial Gravitational Waves

Modification to the tensor mode evolution

$$\mathcal{D}_q''(\tau) + 2\frac{a'}{a}\mathcal{D}_q'(\tau) + \left(q^2 + a^2 m_g^2\right)\mathcal{D}_q(\tau) = J_q(\tau)$$

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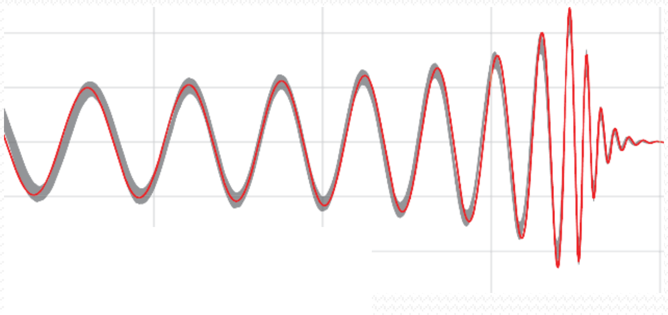
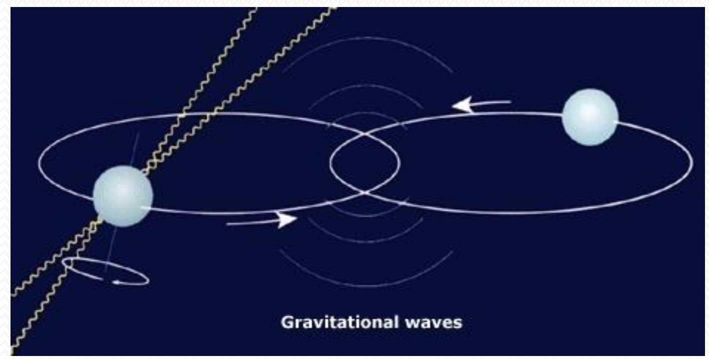
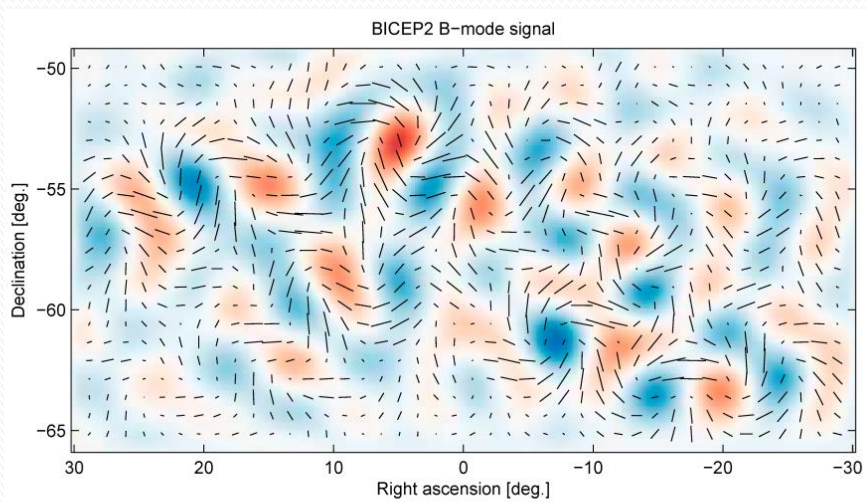
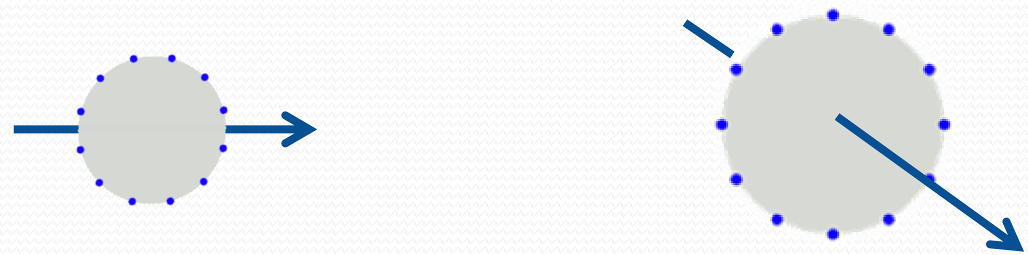




# Scalar and Vector modes of the graviton

In a Lorentz invariant theory, a massive graviton also carries a helicity-0 and 2 helicity-1 modes.



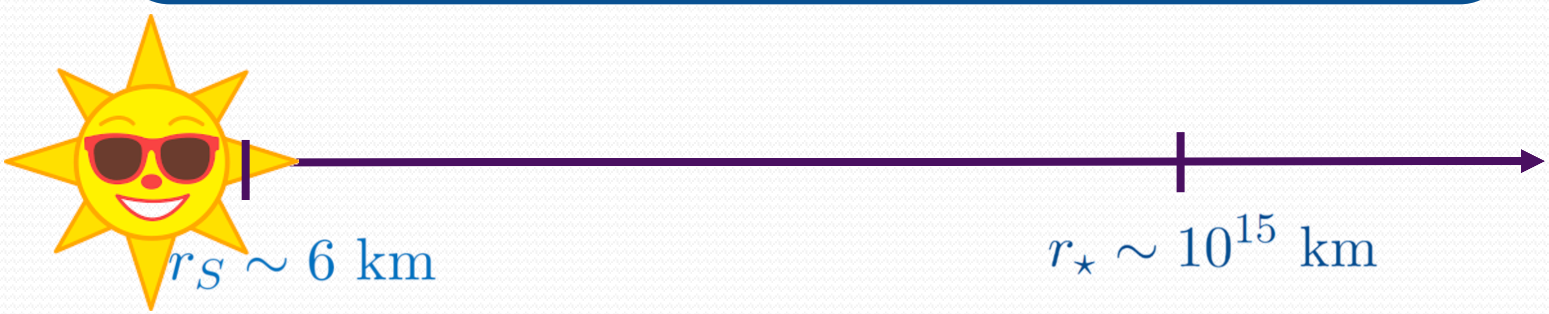
Helicity-0 mode propagates an additional gravitational force that can be very well tested (particularly in the Solar System)

Screened via a Vainshtein mechanism

# Vainshtein mechanism

- Well understood for Static & Spherically Symmetric configurations $e.g. \quad T = -M_\oplus \, \delta^{(3)}(r)$
- Force mediated by the helicity-o mode $\phi'(r)$

$$\frac{\phi'(r)}{r} + \frac{1}{M_{\rm Pl} m^2} \left( \frac{\phi'(r)}{r} \right)^2 = \frac{M_\oplus}{4\pi M_{\rm Pl} r^3}$$




Vainshtein radius: $r_*^3 = \frac{1}{M_{\rm Pl} m^2} \frac{M_\oplus}{M_{\rm Pl}}$

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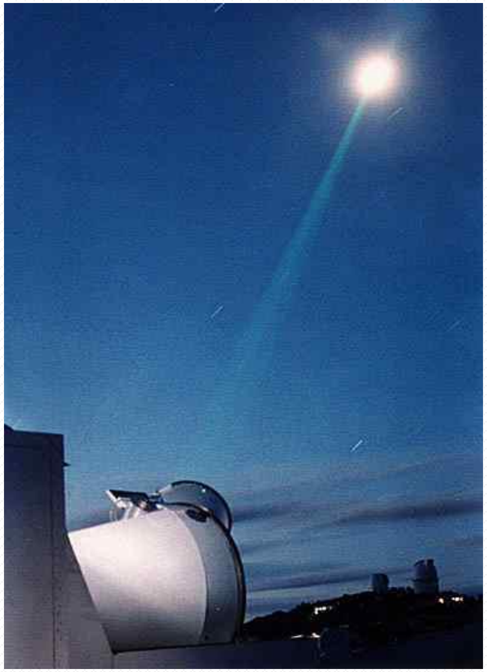
Vainshtein radius:

$$r_*^3 = \frac{1}{M_{\rm Pl} m^2}\frac{M_{\oplus}}{M_{\rm Pl}}$$

$$\text{for} \quad r \gg r_* , \quad \phi'(r) \sim \frac{M_{\oplus}}{M_{\rm Pl}}\frac{1}{r^2}$$

$$\text{for} \quad r \ll r_* , \quad \phi'(r) \sim \frac{M_{\oplus}}{M_{\rm Pl}}\frac{1}{r_*^{3/2}\sqrt{r}}$$

# Lunar Laser Ranging bounds



For DGP, (cubic Galileon)

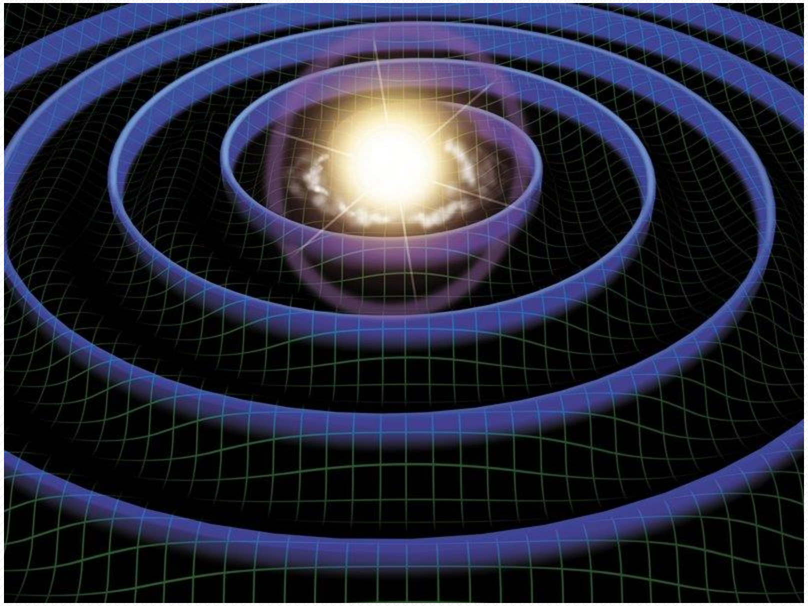
$$m_g < \delta\phi \left(\frac{r_{S,\oplus}}{a^3}\right)^{1/2} \qquad m_g \lesssim 10^{-32}\mathrm{eV}$$

For hard mass graviton, (~ quartic Galileon)

$$m_g < \delta\phi^{3/4} \left(\frac{r_{S,\oplus}}{a^3}\right)^{1/2} \qquad m_g \lesssim 10^{-30}\mathrm{eV}$$

# Radiation into the scalar mode of the graviton

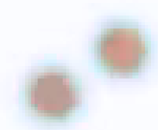
The existence of a scalar mode means new channels of radiation



Monopole & dipole exist but are suppressed by conservation of energy & momentum.

Quadrupole emitted by helicity-0 mode is suppressed by Vainshtein mechanism
(best understood in a Galileon approximation)

Work with Furqan Dar, Tate Deskins,
John Tom Giblin & Andrew Tolley



Contours of $\dot{\phi}^2$

For the cubic Galileon:
Power still in the quadrupole as in GR
Corrections to GR are very suppressed

# Galileon Quadrupole emission

$$P_{\text{Quadrupole}} \sim \frac{(\Omega_P \bar{r})^3}{(\Omega_P r_\star)^{3/2}} \frac{\mathcal{M}^2}{M_{\text{Pl}}^2} \Omega_P^2$$

$$r_*^3 = \frac{1}{M_{\text{Pl}} m^2} \frac{M_{\text{Binary}}}{M_{\text{Pl}}}$$

For the Hulse-Taylor Pulsar $m_g \lesssim 10^{-27} \text{eV}$

- For the Cubic Galileon, higher multipoles are suppressed by additional powers of velocity

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For the Hulse-Taylor Pulsar $m_g \lesssim 10^{-27}\,{\rm eV}$

- For the Cubic Galileon, higher multipoles are suppressed by additional powers of velocity
- Massive gravity and stable self-accelerating models always include *at least* a *quartic Galileon*
- In the Quartic Galileon, the angular direction is *not screened as much* as the others → many multipoles contribute to the power with the same magnitude…

→ Multipole expansion breaks down

# How light is gravity ???

**Dispersion Relation**

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**Yukawa**

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# Massive Gravity is one in many theories considered

- There has recently been an explosion of models that can play important roles for cosmology

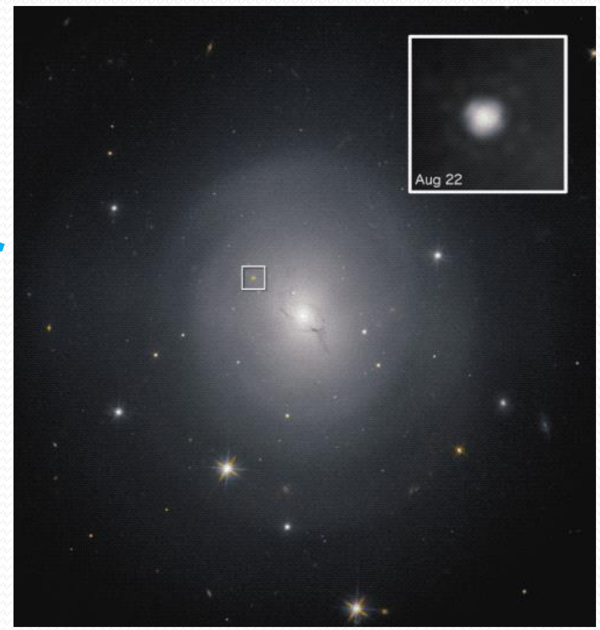
  (eg. DBI, K-inflation, G-inflation, gauge inflation, ghost inflation, Axion Monodromy, Chromo-Natural Inflation, f(R), Chameleon, Symmetron, ghost condensate, Galileon, generalized galileon, Horndeski, beyond Horndeski, beyond beyond Horndeski, Fab4, beyond Fab4, EST, DHOST, K-essence, DGP, cascading gravity, massive gravity, minimal massive gravity, bi-gravity, multi-gravity, mass-varying massive gravity, f(R) massive gravity, mass-varying massive gravity, quasi-dilaton, extended quasi-dilaton, superfuid dark matter, Proca dark energy, generalized Proca, beyond generalized Proca, gauge field dark energy, Galileon genesis, extended Galileon genesis, SLED, mimetic gravity, unimodular gravity, dipolar dark matter, …, …, … )

# Setting different EFTs apart

- We could simply wait for observations to tell them apart

(eg. DBI, K-inflation, G-inflation, gauge inflation, ghost inflation, Axion Monodromy, Chromo-Natural Inflation, f(R), Chameleon, Symmetron, ghost condensate, Galileon, ~~generalized galileon~~, ~~Horndeski~~, beyond Horndeski, beyond beyond Horndeski, ~~Fab4~~, beyond Fab4, EST, DHOST, K-essence, DGP, cascading gravity, massive gravity, minimal massive gravity, bi-gravity, multi-gravity, mass-varying massive gravity, f(R) massive gravity, mass-varying massive gravity, quasi-dilaton, extended quasi-dilaton, superfuid dark matter, Proca dark energy, generalized Proca, beyond generalized Proca, gauge field dark energy, Galileon genesis, extended Galileon genesis, SLED, mimetic gravity, unimodular gravity, dipolar dark matter, ..., ... )

GW&GBR 170817



# Setting different EFTs apart

- We could simply wait for observations to tell them apart
  Already doing well !

In parallel, we can question their theoretical consistency

Do these theories:

1. preserve perturbative unitarity ?
2. have any chance of ever admitting
   a standard Wilsonian UV completion ?
3. … 4. … causal, well-posedeness, caustics, …

# “Standard” UV completion – should we care ???

- By “standard” UV completion, mean

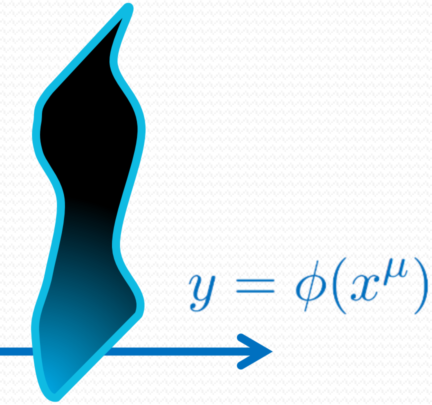
Unitary,

Lorentz-invariant,

Local (to some extend),

Analytic

- Analyticity is implied by causality
- The absence of such a UV completion would have profound consequences for our understanding of UV physics




# The example of DBI / anti DBI

$$\mathcal{L}_{\mathrm{DBI}} \sim -\sqrt{1+(\partial\phi)^2}$$

Model relevant for inflation

$$(\partial\phi)^2 = -\dot{\phi}^2 \to -1$$

Model that naturally emerges as probe brane in extra dimension

No obstructions to standard UV completion (known so far)

$$\mathcal{L}_{\overline{\mathrm{DBI}}} \sim \sqrt{1-(\partial\phi)^2}$$

Model relevant for dark energy with screening in dense environments

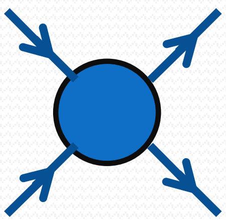
$$(\partial\phi)^2 = \phi'(r)^2 \to 1$$

Model that naturally emerges as probe brane in extra *time* dimension…

Known obstructions to standard UV completion

# $2 \rightarrow 2$ Scattering Amplitude

For a low energy EFT described by a massive Lorentz invariant scalar field



Mandelstam variables:
$s$: center of mass energy[2]
$t$: momentum transfer
$u = 4m^2 - s - t$

$$|\text{initial state}\rangle \longrightarrow |\text{final state}\rangle = \hat{S}\,|\text{initial state}\rangle$$

$$\hat{S} = 1 + i\hat{T}$$

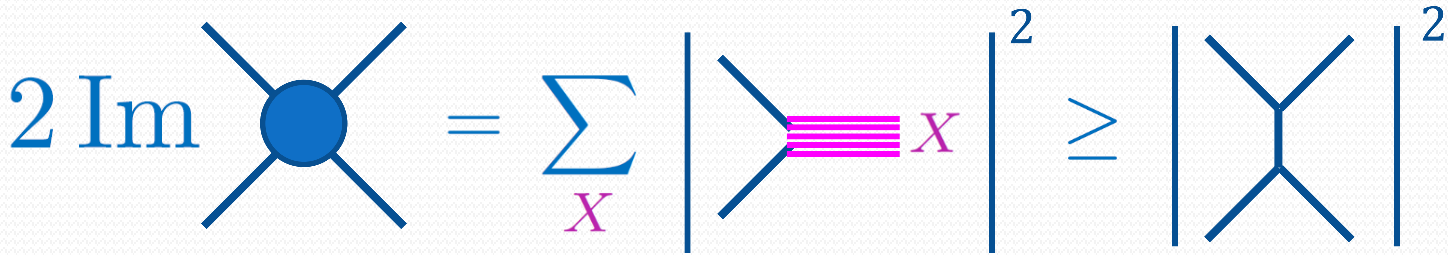
Scattering amplitude $\mathcal{A} = \langle \text{final} \,|\, \hat{T} \,|\, \text{initial} \rangle$

# Optical theorem:

$$\mathcal{A} = \langle \text{final} \,|\, \hat{T} \,|\, \text{initial} \rangle$$

$$\sigma(s) = \frac{\text{Im}\mathcal{A}(s,0)}{\sqrt{s(s-4m^2)}} > 0$$

Physical scattering for $s \geq 4\, m^2$
In the forward scattering limit, ie. $t = 0$




# Analyticity (implied by causality) & locality imply:

$$B''(s) \sim \int_{4m^2}^{\infty} \mathrm{d}\mu \frac{\text{Im}A(\mu)}{(\mu - s)^3}$$

($B$: pole subtracted amplitude)

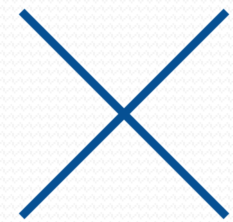
$$B''(s)\big|_{s=0} > 0$$

Adams et. al. 2006

# Positivity bounds for $P(X)$

eg. $P(X)$ model $\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + \frac{c}{\Lambda^4}(\partial\phi)^4 + \cdots$

$$\mathcal{A}^{\text{tree}}_{2\to 2} = \frac{c}{\Lambda^4}\left(s^2 + t^2 + u^2 - 4m^2\right)$$



Positivity bounds requires: $c > 0$

No $P(X)$ model with $c \leq 0$ can ever have an analytic Wilsonian UV completion

(explains the conclusions on DBI / opposite sign DBI model)

Adams et. al. 2006

# Setting different EFTs apart

- There has recently been an explosion of models that can play important roles for cosmology

(eg. DBI, K-inflation, G-inflation, gauge inflation, ~~ghost inflation~~, Axion Monodromy, Chromo-Natural Inflation, f(R), Chameleon, Symmetron, ghost condensate, Galileon, generalized galileon, Horndeski, beyond Horndeski, beyond beyond Horndeski, Fab4, beyond Fab4, EST, DHOST, ~~K-essence~~, DGP, cascading gravity, massive gravity, minimal massive gravity, bi-gravity, multi-gravity, mass-varying massive gravity, f(R) massive gravity, mass-varying massive gravity, quasi-dilaton, extended quasi-dilaton, ~~superfuid dark matter~~, Proca dark energy, generalized Proca, beyond generalized Proca, gauge field dark energy, Galileon genesis, extended Galileon genesis, SLED, ~~mimetic gravity~~, unimodular gravity, ~~dipolar dark matter~~, ..., ... )

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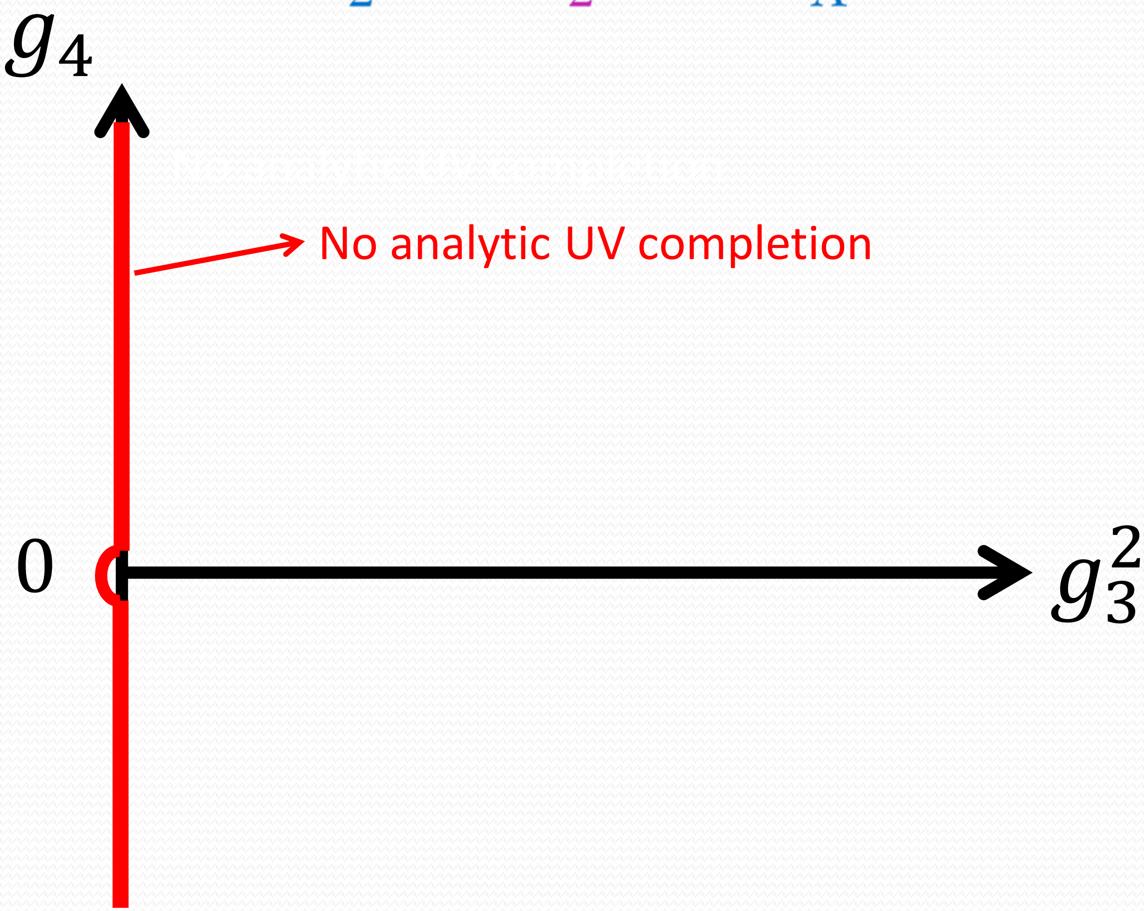
# Positivity bounds for massive Galileon (in forward limit)

$$\mathcal{L}_{\rm mGal} = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{g_3}{\Lambda^3}(\partial\phi)^2\Box\phi - \frac{g_4}{\Lambda^6}(\partial\phi)^2\left((\Box\phi)^2 - (\partial_\mu\partial_\nu\phi)^2\right)$$

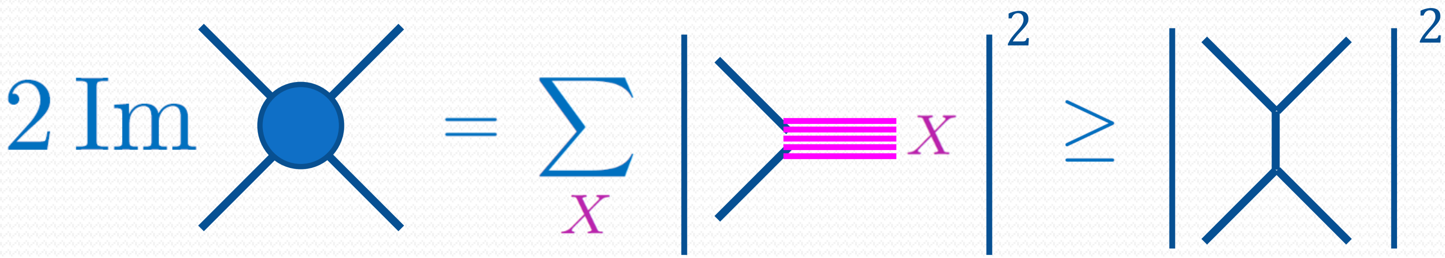
$$+\mathcal{L}_{\text{higher derivatives}}$$




CdR, Melville, Tolley, Zhou, 1702.08577

# Optical theorem

$$\sigma(s) = \frac{\mathrm{Im}\mathcal{A}(s,0)}{\sqrt{s(s-4m^2)}} > 0$$




The optical theorem carries an infinite more information than just $\sigma > 0$

$$\mathcal{A}(s,t) = 16\pi\sqrt{\frac{s}{s-4m^2}}\sum_{\ell=0}^{\infty}(2\ell+1)P_\ell(\cos\theta)a_\ell(s)$$

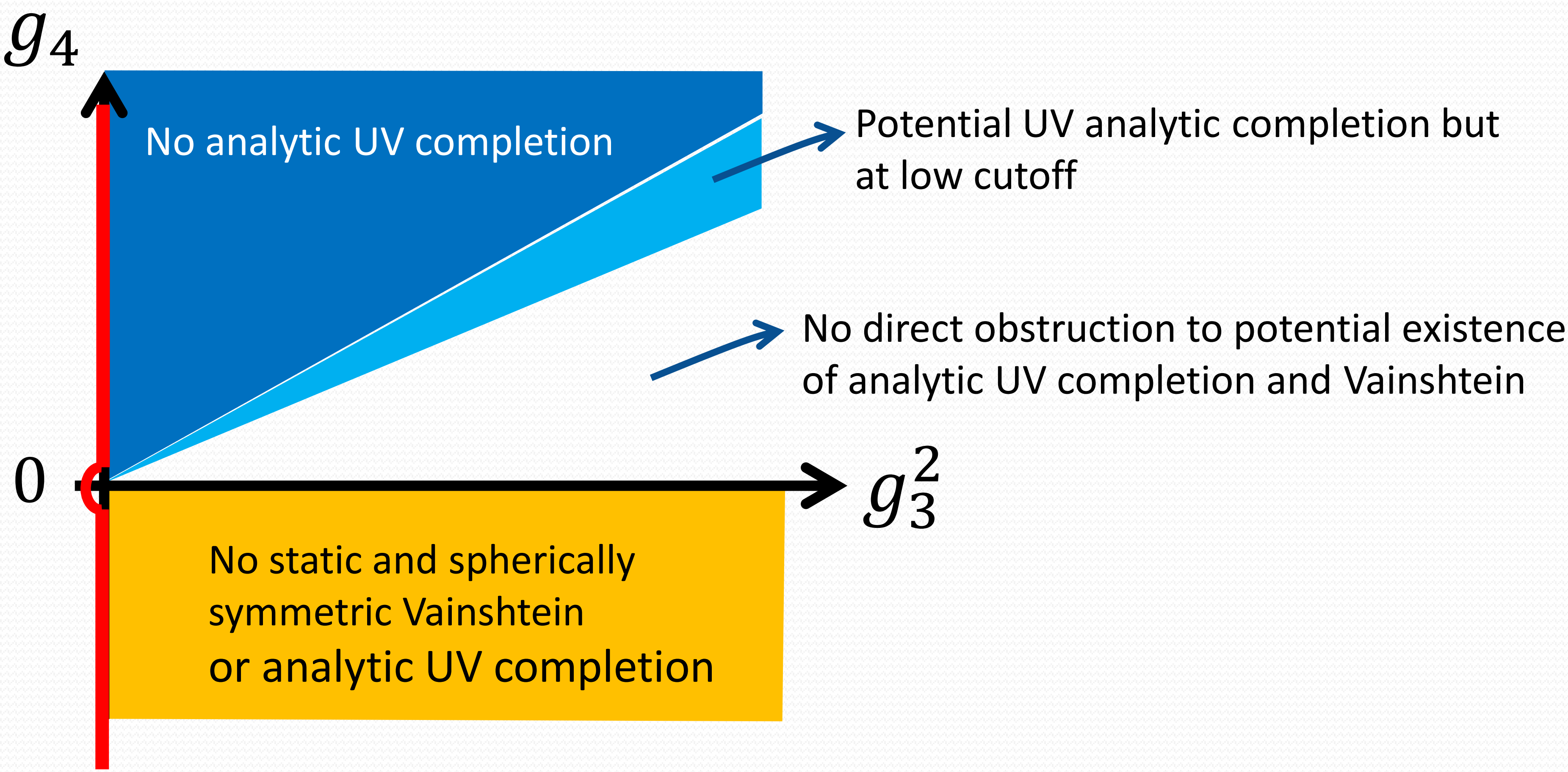
$$\mathrm{Im}\, a_\ell(s) = |a_\ell(s)|^2 + \cdots$$

$$\frac{\partial^n}{\partial t^n}\mathrm{Im}\mathcal{A}(s,t) > 0$$

$$0 \leq |a_\ell(s)^2| \leq \mathrm{Im}\, a_\ell(s) \leq 1 \quad \text{for} \quad s \geq 4m^2$$

Explicit form of these bounds is derived in 1702.08577 CdR, Melville, Tolley, Zhou

# Positivity bounds for massive Galileon (beyond forward limit)

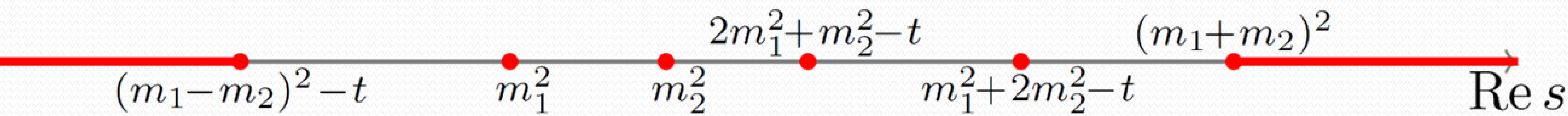



CdR, Melville, Tolley, Zhou, 1702.08577

# Extensions

1. **Multi-fields** (multiple scalars with different mass eigenstates)

   *eg.* 2 scalar field with mass $m_1 \leq m_2$,




**no issue extending** the positivity bound away from the forward scattering limit to the whole region $0 \leq t < 4m_1^2 \leq 4m_2^2$ so long as the poles and branchcuts remain separated $m_2^2 < 4m_1^2$.

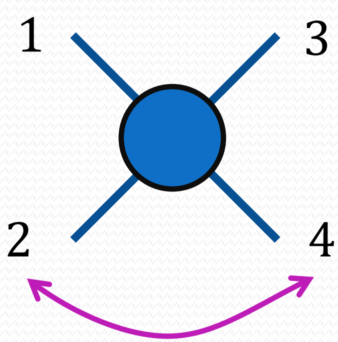
CdR, Melville, Tolley, Zhou, 1706.02712

# Higher spins

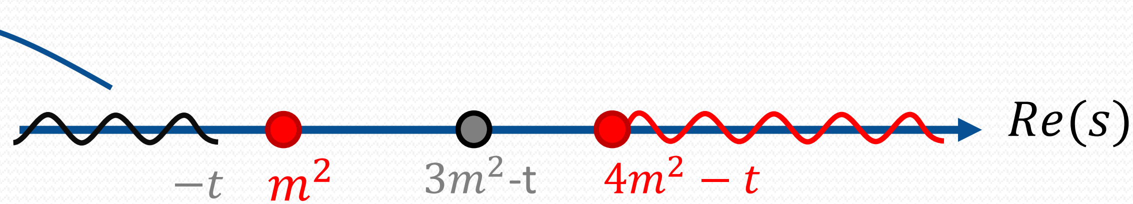
The lowest order bound applies at $t = 0$ for all spins

Away from the forward scattering limit $t \neq 0$,
the $s \leftrightarrow u = 4m^2 - s - t$ crossing symmetry is highly non-trivial

A definite helicity mode transforms non-trivially under crossing




No obvious positivity properties in the 2nd branchcut in helicity formalism



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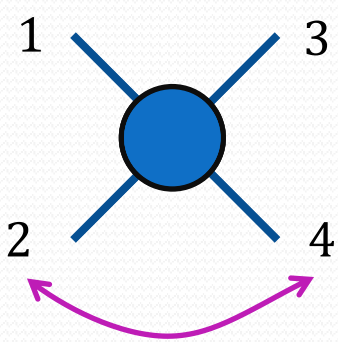
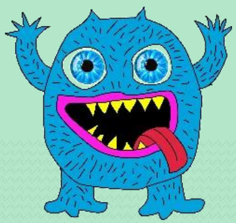


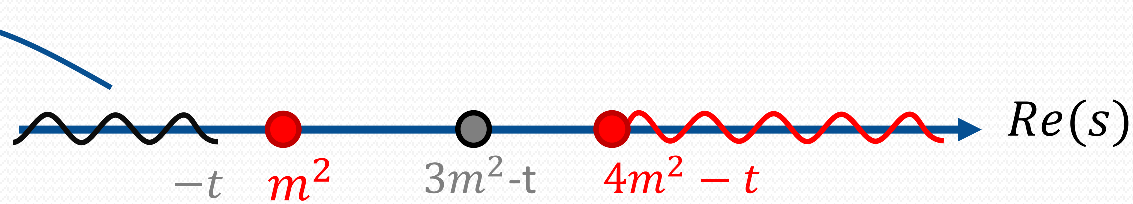

$$\mathcal{H}_{\lambda_1\lambda_2\mu_1\mu_2}(s,t) = (-1)^{\sigma} e^{i\pi(\mu_1-\lambda_1)}$$
$$\cdot \sum_{\lambda'_1\lambda'_2\mu'_1\mu'_2} d^{S_1}_{\lambda'_1\lambda_1}(\pi-\chi) d^{S_2}_{\lambda'_2\lambda_2}(\chi) d^{S_1}_{\mu'_1\mu_1}(\chi-\pi) d^{S_2}_{\mu'_2\mu_2}(-\chi) \mathcal{H}_{\lambda_1\mu_2,\mu_1\lambda_2}(u,t)$$



d: Wigner matrices

$$\sin\chi = \frac{-2m\sqrt{t}}{\sqrt{(s-4m^2)(u-4m^2)}}$$

No obvious positivity properties in the 2nd branchcut in helicity formalism



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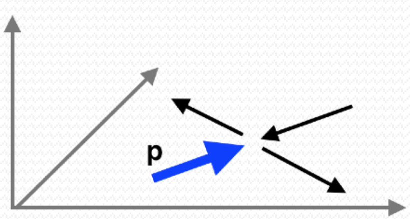
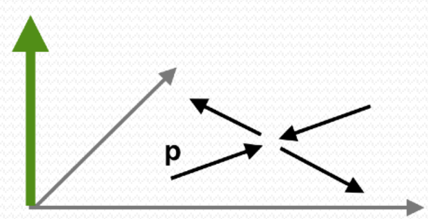
A definite helicity mode transforms non-trivially under crossing

Need to work instead in the transversity formalism
(i.e. spin projections orthogonal to the scattering plane)
Only makes sense for $2 \rightarrow 2$

Helicity

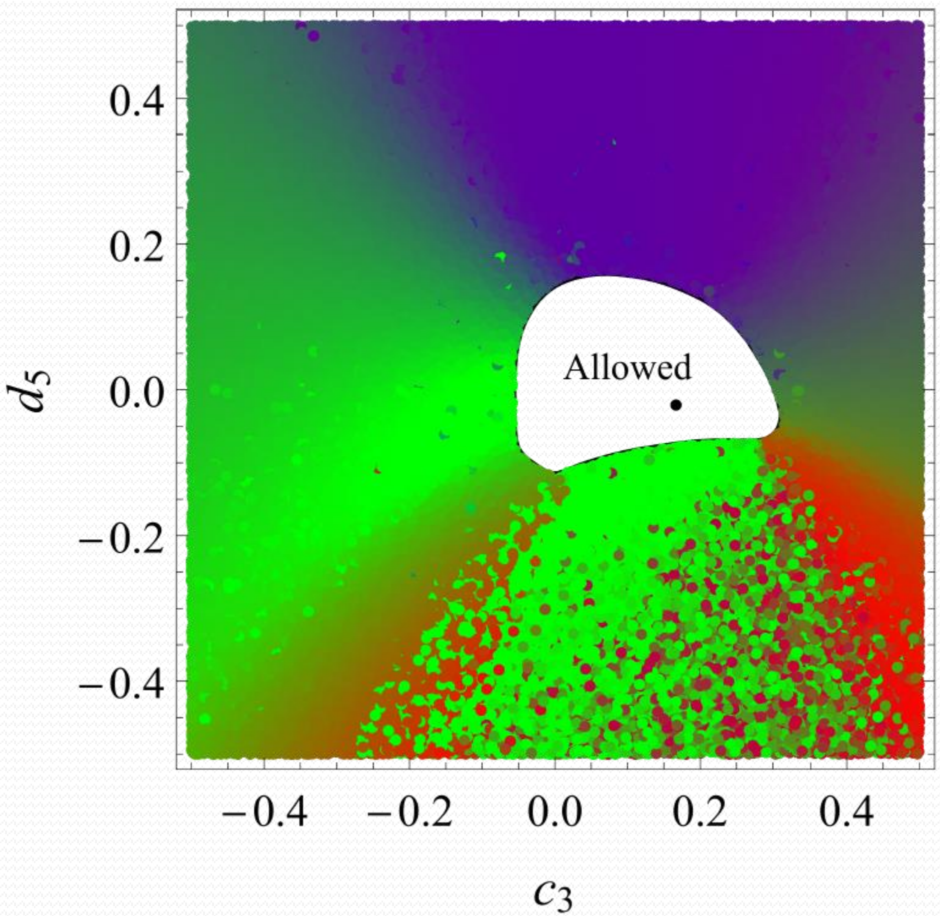



Transversity




CdR, Melville, Tolley, Zhou, 1706.02712 → Derived an infinite number of positivity bounds valid for any spin, applicable to any EFT

# Eg. Constraints on Massive Gravity from UV completion (basic bound in forward limit)




$$\mathcal{L} = \mathcal{L}_{\text{Ghost-free MG}}(c_3, d_5)$$

2-parameter family for Ghost-free massive gravity

Has no ghost and a strong coupling scale $\Lambda^3 = M_{Pl} m^2$

Cheung & Remmen, JHEP 1604 (2016)

# Massive gravity from an EFT viewpoint

$$\mathcal{L} = \mathcal{L}_{\text{Ghost−free MG}}(c_3, d_5) + \Delta c\,[h^3] + \Delta d\,[h^4] + \cdots$$

has no ghost and a strong coupling scale $\Lambda^3 = M_{Pl} m^2$

A priori, from a naïve EFT point of view, there is "*nothing wrong*" with considering other operators that would lead to "ghost" at a scale

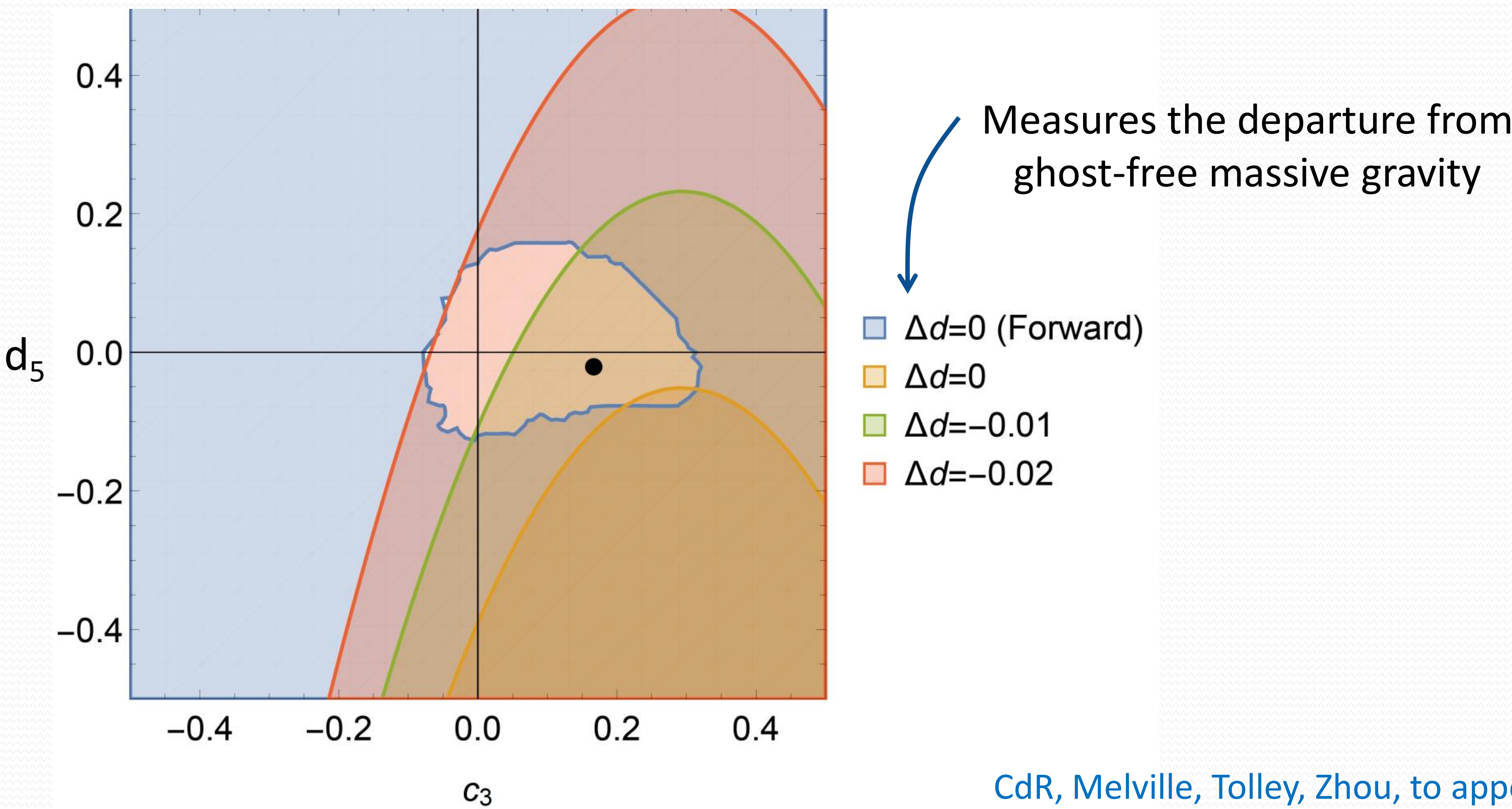
$\Lambda^5 = M_{Pl} m^4$ (for $\Delta c$) or $\Lambda^4 = M_{Pl} m^3$ (for $\Delta d$)

It just means that the cutoff of the EFT is lower

Are these parameters constrained by the positivity bounds ?

# Constraints beyond forward limit

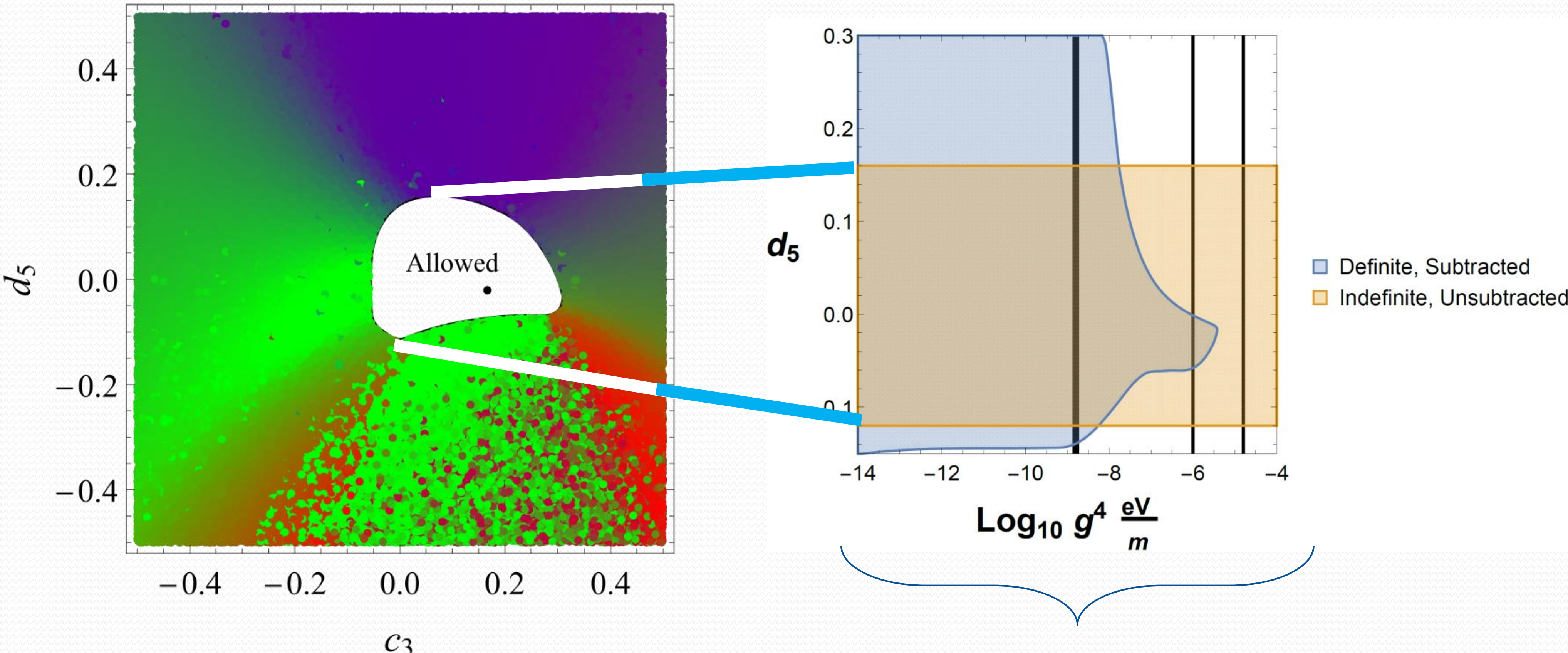
$$\mathcal{L} = \mathcal{L}_{\text{Ghost-free MG}}(c_3, d_5) + \cancel{\Delta c}\,[h^3] + \Delta d\,[h^4] + \cdots$$




CdR, Melville, Tolley, Zhou, to appear

# Improved positivity bounds

$$B^{(2,0)}(0) > \frac{4}{\pi}\int_{4m^2}^{\epsilon^2\Lambda^2} \mathrm{d}\mu \frac{\mathrm{Im}\, A(\mu,t)}{(\mu-2m^2)^3} = \frac{4}{\pi}\int_{4m^2}^{\epsilon^2\Lambda^2} \mathrm{d}\mu \sqrt{1-\frac{4m^2}{\mu^2}}\,\frac{\mu\sigma_{\mathrm{total}}(\mu)}{(\mu-2m^2)^3}\,.$$




Effectively measures the scale of the cutoff

CdR, Melville, Tolley, 1710.09611

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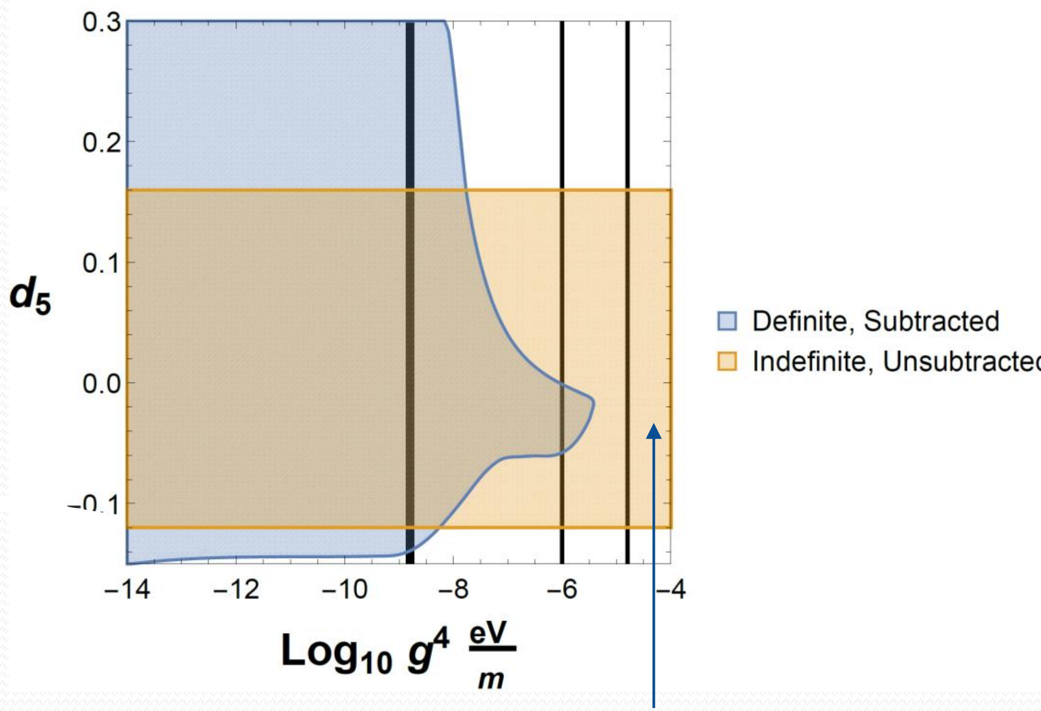
$$B^{(2,0)}(0) > \frac{4}{\pi}\int_{4m^2}^{\epsilon^2\Lambda^2} \mathrm{d}\mu \frac{\mathrm{Im}\, A(\mu,t)}{(\mu-2m^2)^3} = \frac{4}{\pi}\int_{4m^2}^{\epsilon^2\Lambda^2} \mathrm{d}\mu \sqrt{1-\frac{4m^2}{\mu^2}}\,\frac{\mu\sigma_{\mathrm{total}}(\mu)}{(\mu-2m^2)^3}\,.$$




Bellazzini, Riva, Serra, Sgarlata 1710.0253

Assuming a large enough g, ***the improved positivity*** bounds can rule out the allowed parameter space

CdR, Melville, Tolley, 1710.09611: the **improved positivity bounds** should be seen as a **constrain on the value of the cutoff !**

# Summary

- Cosmology has motivated the (re)development of entire new classes of scalar EFTs
- Observations already put strong constraints on some of these models, and particularly on the (effective) graviton mass
- (perturbative) unitarity & analyticity can allow for a better segregation
- Framework not only serves modified gravity but the whole set of EFTs used in cosmology for the description of
  - inflation, (including gauge field inflation, etc…)
  - pre big-bang/bouncing cosmology/other alternatives to inflation
  - dark energy
  - potential framework to tackle the CC problem
  - CFT's
  - …

# How light is gravity ???

**Yukawa**

| $m_g$ (eV) | $\lambda_g$ (km) | |
|---|---|---|
| $10^{-23}$ | $10^{12}$ | Solar System tests |
| $10^{-29}$ | $10^{19}$ | Bound clusters |

**Dispersion Relation**

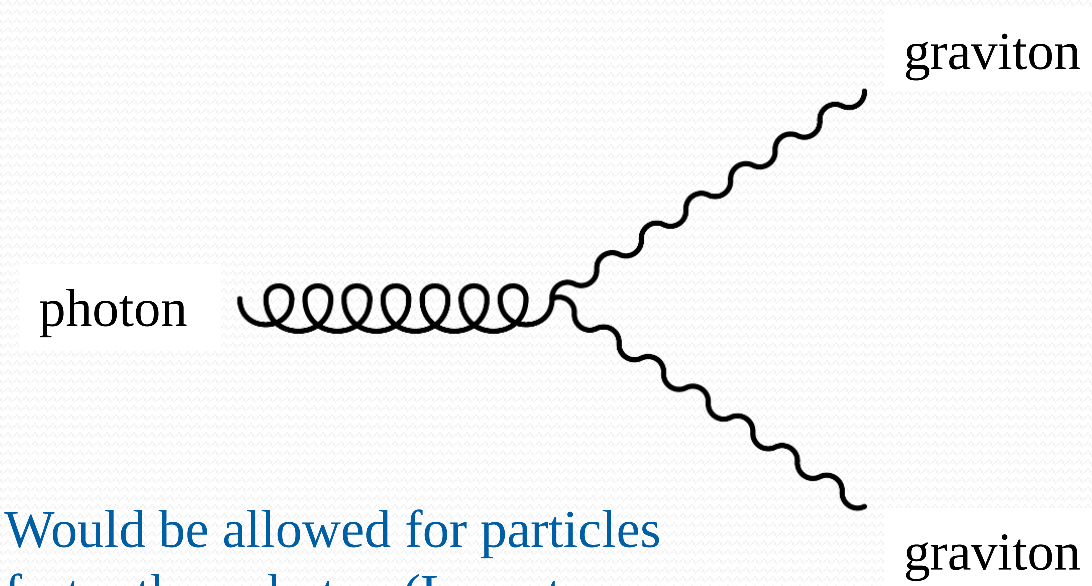
| $m_g$ (eV) | $\lambda_g$ (km) | |
|---|---|---|
| $10^{-22}$ | $10^{11}$ | aLIGO bound |
| $10^{-20}$ | $10^{9}$ | Pulsar timing |
| $10^{-30}$ | $10^{20}$ | B–mode's in CMB |

**Fifth Force**

| $m_g$ (eV) | $\lambda_g$ (km) | |
|---|---|---|
| $10^{-32}$ | $10^{22}$ | Lunar Laser Ranging |
| $10^{-27}$ | $10^{17}$ | Binary pulsar |
| $10^{-32}$ | $10^{22}$ | Structure formation |

# Cherenkov Radiation

Particles traveling faster than GWs could decay into GWs




Forbidden process in
Lorentz invariant models
(if the photon is massless)

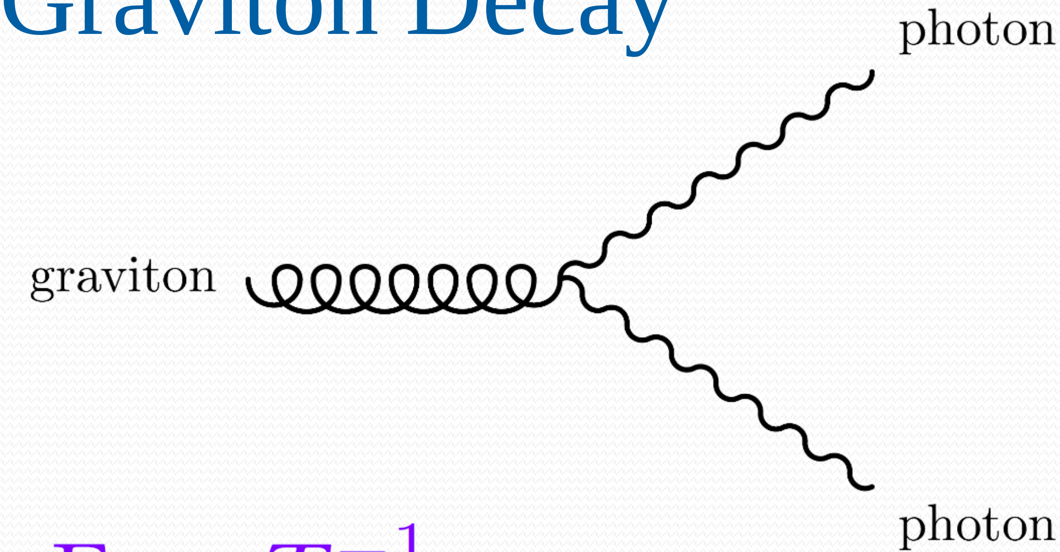
Would be allowed for particles faster than photon (Lorentz violating models)

eg. Blas, Ivanov, Sawicki, Sibiryakov1602.04188

Can be used to put bounds on the difference of speeds
but those translate into very weak bounds on the graviton mass

# Graviton Decay

If the graviton has a mass:




aLIGO direct detection: $\Gamma \ll T^{-1}_{\mathrm{GW\ travel\ time}}$

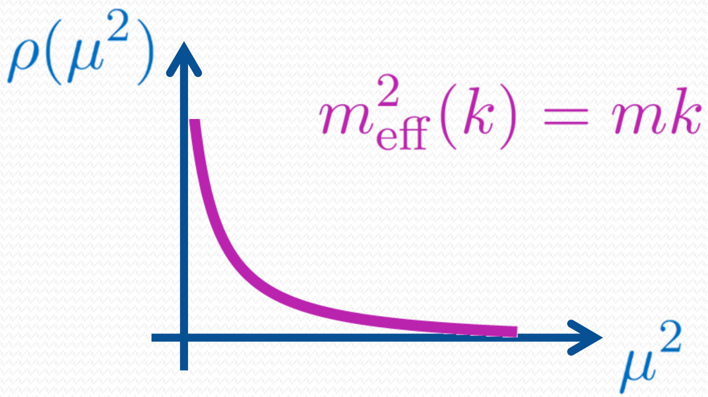
Very weak bound…

Constraints from cosmology: $\Gamma \ll H_{\mathrm{today}}$

$$\mathrm{Im}[m_g^2] \ll H_{\mathrm{today}} \sqrt{\mathrm{Re}[m_g^2]}$$

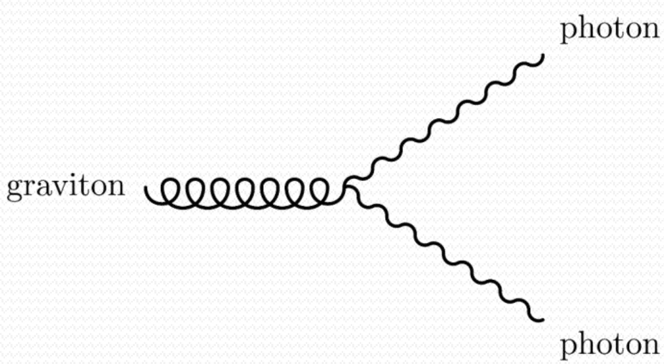
# Graviton Decay

If the graviton is a resonance (eg. in DGP, Cascading Gravity,…)



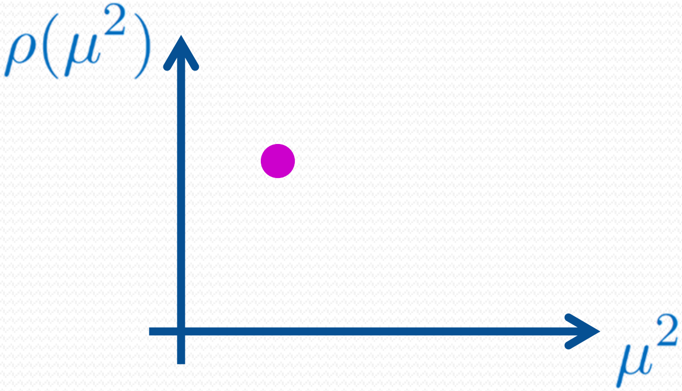

The graviton already has a finite lifetime even without taking into account its possible decay into photons

$$m \lesssim H_{\text{today}}$$

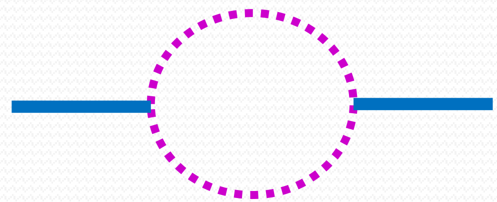


# Graviton Decay

For a hard mass graviton At tree-level, $\mathrm{Im}[m_g^2] = \Gamma = 0$




loop-effect on graviton self-energy



$$\Gamma \sim N \frac{m_g^3}{M_{\mathrm{Pl}}^2}$$

$N$: total number of light particles that may exist
(photon + axion, hidden sector not subject to SM constraints,…)

$$m_g \lesssim 10^7 \mathrm{eV} \times N^{-1/3}$$