Testing Maximal Supersymmetry in Cosmology

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Based on work with **Ferrara, Roest, Carrasco, Wrase, Yamada**
Achúcarro, Wang, Welling, Arkami, Vardanyan

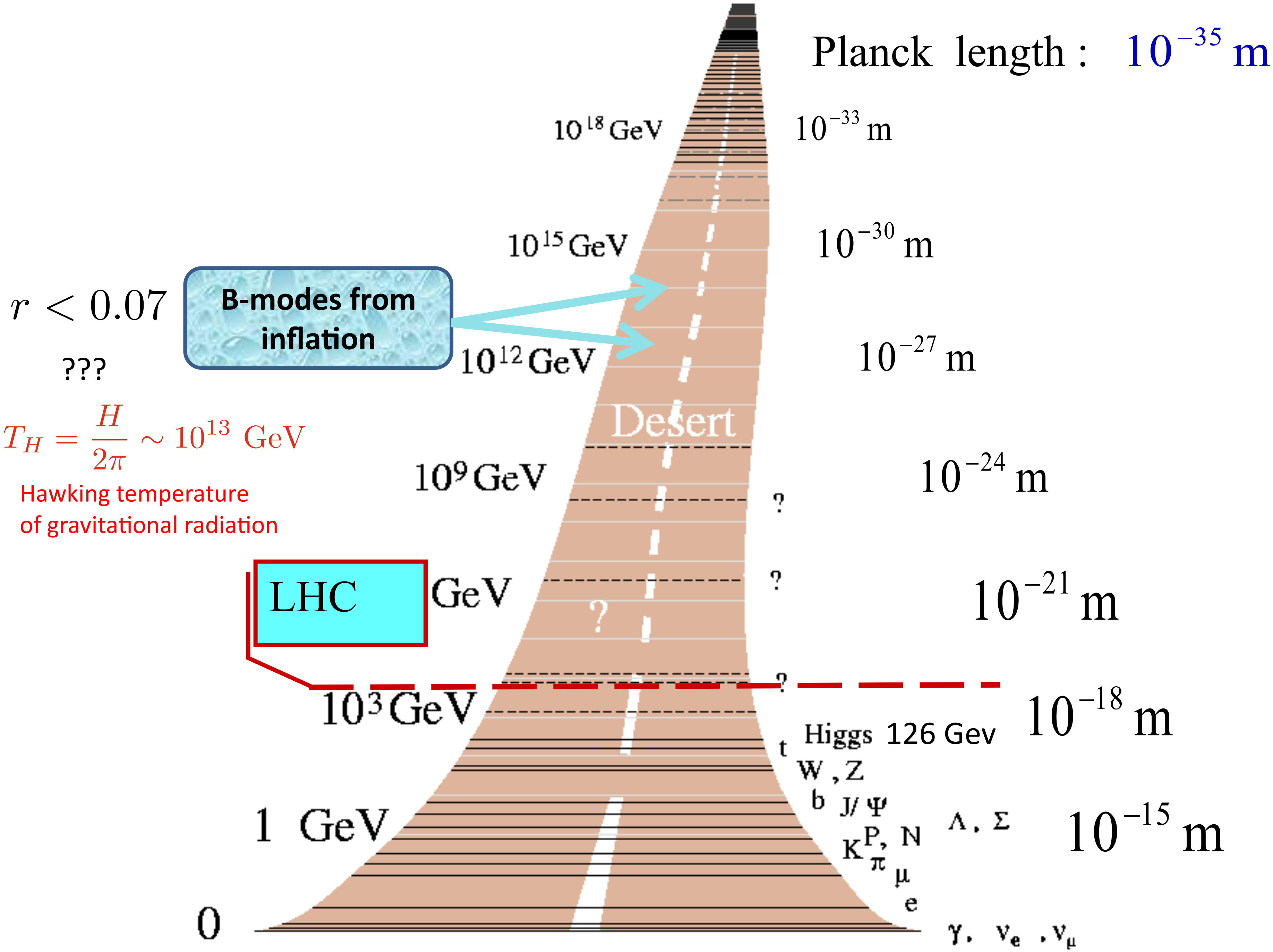
We discuss fundamental theories with maximal supersymmetry: M-theory, Superstring theory and ***maximally extended supergravity*** *and their symmetries. Some of these symmetries play an important role in improved ultraviolet behaviour of extended supergravities.*

When ***maximal supersymmetry is spontaneously broken to minimal supersymmetry*** *we deduce phenomenological models interesting for observational cosmology. These models, called* ***alpha-attractor models****, are in good agreement with CMB and LSS observations, and provide targets for future satellite missions designed for detection of primordial gravitational waves*

Introduction

Part I, formal. M-theory d=11, superstring theory d=10, maximal supergravity d=4

Part II, cosmology. From the sky to the fundamental physics



Planck length : 10^{-35} m
10^{18} GeV
10^{-33} m
10^{15} GeV
10^{-30} m
$r < 0.07$
B-modes from inflation
???
10^{12} GeV
10^{-27} m
$T_H = \frac{H}{2\pi} \sim 10^{13}$ GeV
Hawking temperature of gravitational radiation
Desert
10^9 GeV
10^{-24} m
LHC
GeV
10^{-21} m
10^3 GeV
10^{-18} m
Higgs 126 Gev
t
W , Z
b J/ Ψ
Λ , Σ
1 GeV
10^{-15} m
K P, N
π
μ
e
0
γ , ν_e , ν_μ

The energy scale of inflation

$$V^{1/4} \sim 1.04 \times 10^{16}\ \mathrm{GeV}\ \left(\frac{r}{0.01}\right)^{1/4}$$ GUT

The energy of inflationary perturbations

$$H = \frac{1}{M_{Pl}}\sqrt{V/3} \sim 2.6 \times 10^{13}\mathrm{GeV}\left(\frac{r}{0.01}\right)^{\frac{1}{2}}$$

If primordial gravitational waves are detected

$$r \approx 10^{-2} \qquad H \approx 2.6 \times 10^{13}\mathrm{GeV}$$

$$r \approx 10^{-3} \qquad H \approx 0.8 \times 10^{13}\mathrm{GeV}$$

we will probe energies billion times higher than the energies probed at LHC

Fundamental idea following the discovery of General Relativity: local supersymmetry

Einstein's dream of unifying electromagnetism and gravity was realized starting with extended **$\mathcal{N}$=2 supergravity**. The model does so by adding **two real gravitino** to the photon and the graviton. The first breakthrough into finiteness of quantum supergravity occurred via this unification: an explicit calculation of **photon—photon scattering** which was known to be divergent in the coupled Maxwell—Einstein system yielded a dramatic result : the new diagrams involving gravitinos cancelled the divergences found previously, 1976.

More such cancellation were found later in higher $\mathcal{N}$.

LHC did not discover low-energy $\mathcal{N}$=1 supersymmetry yet, nor gave evidence of extra dimensions.

However, the idea of a **maximal supersymmetry, spontaneously broken to minimal supersymmetry, can be tested in cosmology**

$\mathcal{N}$=8 in d=4 supergravity,

M-theory , $\mathcal{N}$=1 in d=11

Superstring theory, $\mathcal{N}$=2 in d=10

B-mode targets

Hidden symmetries

Short summary

- B-mode detection, if it takes place, will probe energies at about 10^{13} GeV, billion times higher than the energies probed at LHC

- Whereas LIGO discovery of gravitational waves confirms General Relativity, a discovery of primordial gravitational waves will confirm our understanding of Quantum Gravity, up to energies of inflation, since we describe inflationary perturbations using both General Relativity and Quantum field Theory

- The range of B-mode space detectors $10^{-3} < r < 10^{-2}$ is particularly interesting since it has targets from the fundamental physics: string theory, M-theory, maximal supergravity

Seven values scanning the range between 10^{-3} and 10^{-2}

$$r \approx 3\alpha \, \frac{4}{N^2} \qquad n_s \approx 1 - \frac{2}{N}$$

α-attractor models

Example
$n_s \approx 0.963$
N=55 e-foldings

3α= 7,6,5,4,3,2,1

Starobinsky and Higgs,
α=1

CMB-S4
Next Generation CMB Experiment

Concept Definition Task force (CDT)

Working from CMB-S4 Science Book, earlier documents, and new simulation work, the NSF & DOE-sponsored Concept Definition Task force submitted its report in October 2017

https://cmb-s4.org/CMB-S4workshops/index.php/File:CMBS4_CDT_final.pdf

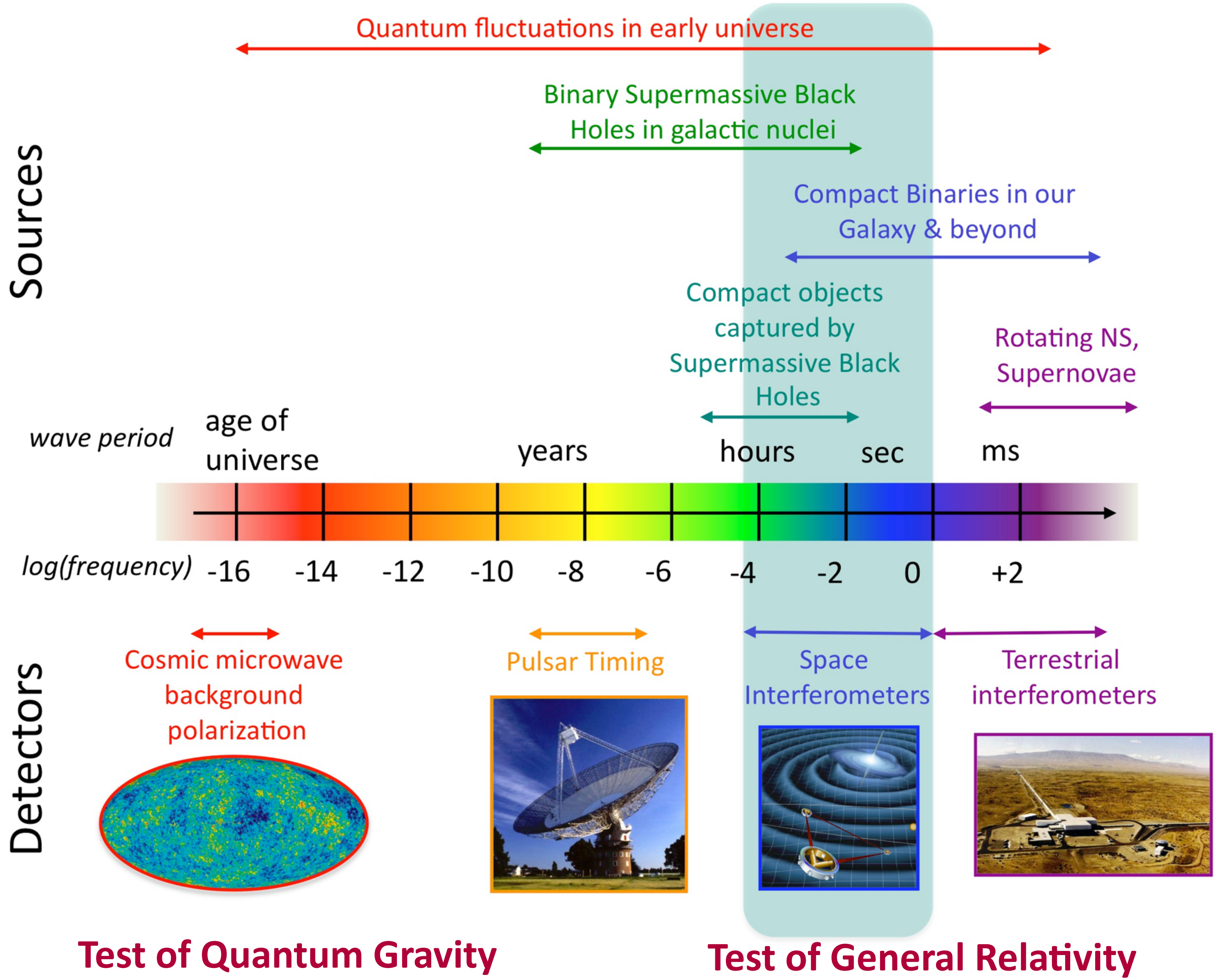
From the Executive Summary of the CDT Report

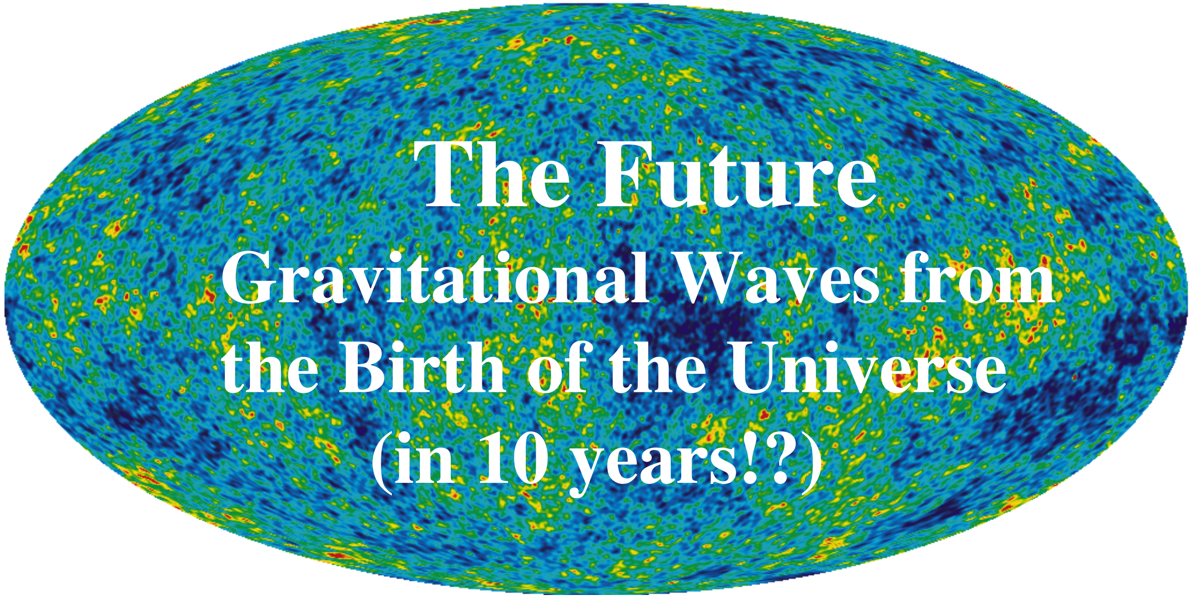
- The first goal and requirement for CMB-S4 is to measure the imprint of primordial gravitational waves on the CMB polarization anisotropy, quantified by the tensor-to-scalar ratio r. Specifically, CMB-S4 will be designed to provide a detection of $r \geq 0.003$. In the absence of a signal, CMB-S4 will be designed to constrain $r < 0.001$ at the 95% confidence level, nearly two orders of magnitude more stringent than current constraints. This will test many of the simplest models of inflation, including those based on symmetry principles, that occur at high energy and large inflaton field range. The r requirements have been translated into measurement requirements consistent with projecting out foregrounds and other contamination as detailed in Appendix A.

LIGO detected GW from binary black holes and neutron stars, with the wavelength of thousands of kilometers

But the primordial GW affecting the CMB have wavelengths of billions of light-years!!!

The Gravitational Wave Spectrum





L. Page, talk at the Breakthrough Prize Symposium, December 4, 2017 at Stanford

Primordial gravitational waves would be a direct connection between gravitation and quantum mechanical processes

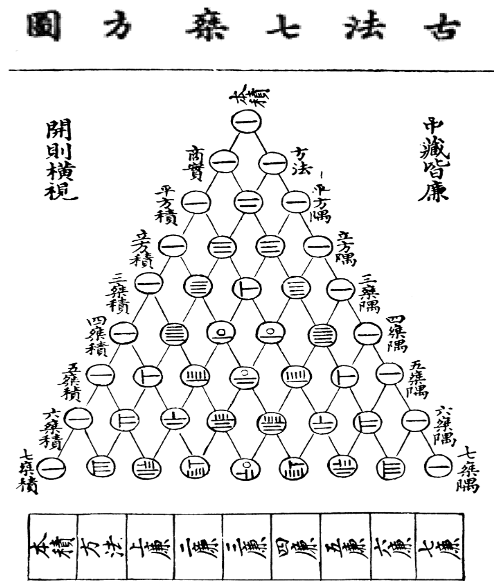
....a test of cosmology

....and a link between Einstein and Bohr that has eluded physics for 100 years

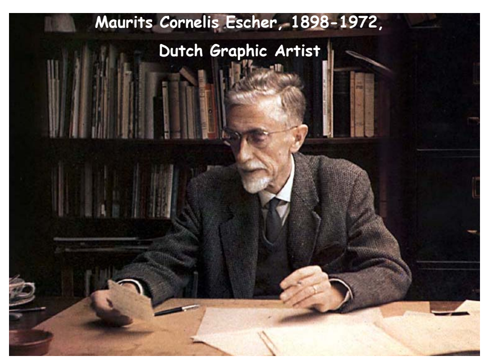
$40 Million Grant Establishes Simons Observatory, a New Investigation into the Formation of the Early Universe

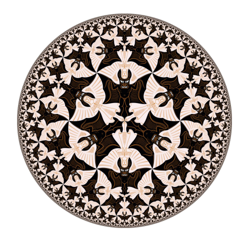
Part I, formal. M-theory d=11, superstring theory d=10, maximal supergravity d=4

Yanghui triangle,
13th century



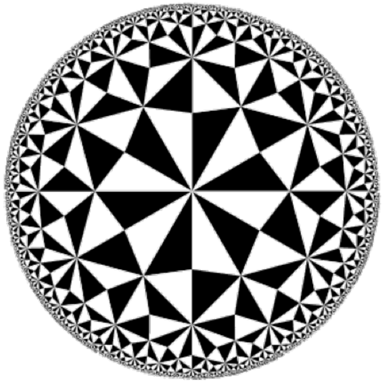
Hyperbolic disk and half-plane





$$ds^2 = \frac{dx^2 + dy^2}{(1 - x^2 - y^2)^2}$$

http://mathworld.wolfram.com/PoincareHyperbolicDisk.html



$$ds^2 = 3\alpha \frac{dZ\, d\bar{Z}}{(1 - Z\bar{Z})^2}$$

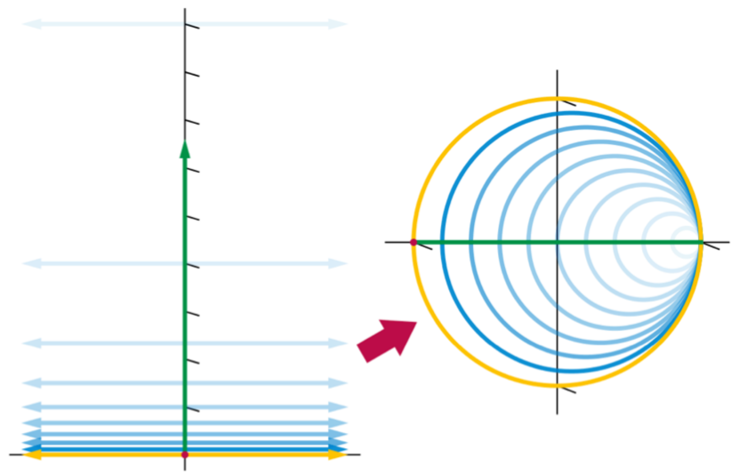
For a unit size Poincare disk:

$$r \sim 10^{-3} \qquad \alpha = \frac{1}{3}$$

Next CMB satellite mission target

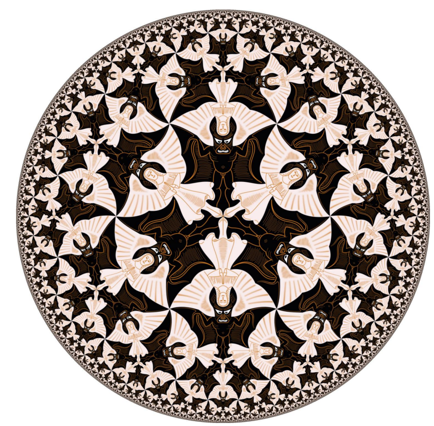
From the hyperbolic disk to a half-plane (the Cayley transform)

$$T = \frac{1+Z}{1-Z} \qquad Z = \frac{T-1}{T+1}$$



$SL(2,\mathbb{R})$

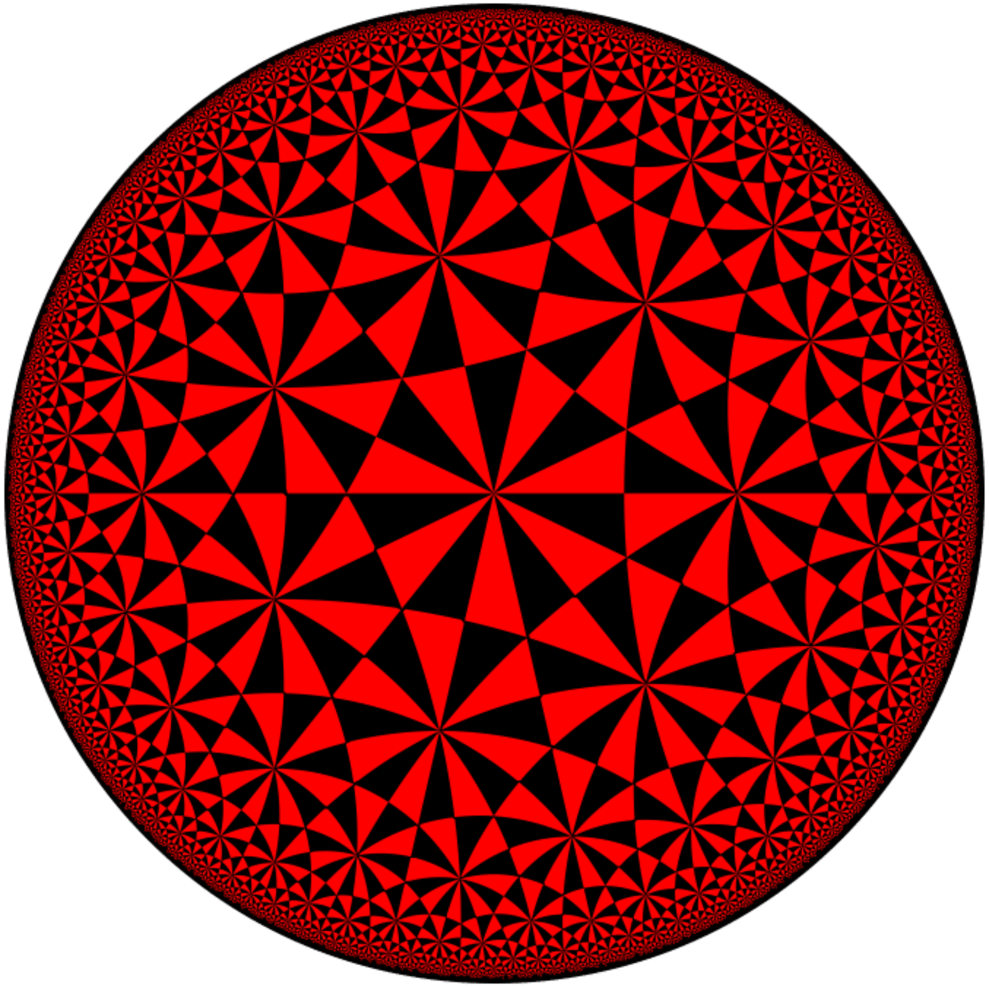
$SU(1,1)$



$$ds^2 = 3\alpha \frac{\partial T \partial \bar{T}}{(T+\bar{T})^2}$$

$$ds^2 = 3\alpha \frac{dZ\, d\bar{Z}}{(1 - Z\bar{Z})^2}$$

Möbius transformations applied to hyperbolic tilings



- **Maximal supersymmetry and B-modes**
- M-theory in d=11
- Superstring theory in d=10
- $\mathcal{N}$=8 supergravity in d=4

2016, Ferrara and RK

Scalars are coordinates of the coset space in $\mathcal{N}$=8 supergravity in d=4

$$\frac{G}{H} = \frac{E_{7(7)}}{SU(8)}$$

$$E_{7(7)}(\mathbb{R}) \supset [SL(2,\mathbb{R})]^7$$

Geometries with discreet number of unit size Poincaré disks are possible when consistent reduction of supersymmetry is performed. Upon identification of their moduli one finds

$$ds^2 = k\frac{dT d\bar{T}}{(T+\bar{T})^2}, \qquad k = 1, 2, 3, 4, 5, 6, 7$$

= 3 α

At least one disk and no more than seven

N=55 e-foldings

$$r \approx \{1.3,\ 2.6,\ 3.9,\ 5.2,\ 6.5,\ 7.8,\ 9.1\} \times 10^{-3}$$

$$n_s \approx 0.963$$

M theory on a 7-manifold with G_2 holonomy

Betti numbers

$$(b_0, b_1, b_2, b_3) = (1, 0, 0, 7)$$

This theory is identified with the maximal rank reduction on the seven torus and leads directly to d=4 $\mathcal{N} = 1$ 'curious supergravity' where 7 complex scalars are coordinates of the coset space

$$\left[\frac{SL(2,\mathbb{R})}{SO(2)}\right]^7$$

$$\begin{array}{ll} g_{\mu\nu} & \to b_0 = 1 \\ A_\mu & \to b_1 = 0 \\ \mathcal{A} & \to b_1 + b_3 = 7 \end{array} \qquad \begin{array}{ll} \psi_\mu & \to b_0 + b_1 = 1 \\ \chi & \to b_2 + b_3 = 7 \end{array} \qquad \begin{array}{ll} A_{\mu\nu\rho} & \to b_0 = 1 \\ A_{\mu\nu} & \to b_1 = 0 \\ A_\mu & \to b_2 = 0 \\ A & \to b_3 = 7 \end{array}$$

7 scalars, 7 spin 1/2 fields and 7 pseudoscalars

$$\tau_i = \mathcal{A}_i + iA_i, \;\; \chi_i$$

The corresponding Kähler geometry is the seven-disk manifold

$$K = -\sum_{i=1}^{7} \ln(-i(\tau_i - \bar{\tau}_i))$$

Compactification of string theory in d=10 to d=4 $\mathcal{N}$=1 supergravity

$$\int d^4x\sqrt{-g}\, e^{-\phi}(\mathcal{L}_1 + \mathcal{L}_2) \ .$$

$$\mathcal{L}_1 = R + g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho} \ ,$$

$$\mathcal{L}_2 = \frac{1}{8}\mathrm{tr}(\partial_\mu M^{-1}\partial^\mu M) \ .$$

Here M is a symmetric $O(6,6)$ matrix

$$M = \begin{pmatrix} G^{-1} & -G^{-1}B \\ BG^{-1} & G - BG^{-1}B \end{pmatrix}$$

where $G_{\alpha\beta}$ and $B_{\alpha\beta}$ are the internal space metric and a 2-form, $\alpha, \beta = 1, \ldots, 6$. Together they represent the 36 coordinates of the coset space $\frac{SO(6,6)}{SO(6)\times SO(6)}$, we recover the moduli space of the six-torus T_6 in string theory. We would like now to perform the truncation of the 6-torus to three T_2

six-torus T_6 in string theory

$$T_2 \times T_2 \times T_2 \subset T_6$$

reduction $SO(6,6) \supset [SO(2,2)]^3$

coset representative

$$\frac{SO(6,6)}{SO(6)\times SO(6)} \to \Big[\frac{SO(2,2)}{SO(2)\times SO(2)}\Big]^3$$

$$G_{(IJ)} = (g_{11}, g_{22}, g_{12}; g_{33}, g_{44}, g_{34}; g_{55}, g_{66}, g_{56}) \qquad 9$$

$$B_{[IJ]} = (b_{12} \equiv b_1, b_{34} \equiv b_2, b_{56} \equiv b_3) \qquad 3$$

$\frac{SO(2,2)}{SO(2)\times SO(2)}$ is isomorphic to $\frac{SL(2,\mathbb{R})}{SO(2)} \times \frac{SL(2,\mathbb{R})}{SO(2)}$

$$g_1 \equiv g_{11}g_{22} - g_{12}^2\,, \qquad g_2 \equiv g_{33}g_{44} - g_{34}^2\,, \qquad g_3 \equiv g_{55}g_{66} - g_{56}^2$$

Kähler moduli

$$t_1 = b_1 + i\sqrt{g}_1\,, \qquad t_2 = b_2 + i\sqrt{g}_2\,, \qquad t_3 = b_3 + i\sqrt{g}_3$$

complex structure moduli

$$u_1 = \frac{g_{12}}{g_{22}} + i\frac{\sqrt{g}_1}{g_{22}}\,, \qquad u_2 = \frac{g_{34}}{g_{44}} + i\frac{\sqrt{g}_2}{g_{44}}\,, \qquad u_3 = \frac{g_{56}}{g_{66}} + i\frac{\sqrt{g}_3}{g_{66}}$$

Add axion-dilaton, axion from $H_{\mu\nu\lambda}$

$$s = a + ie^{\phi}$$

2

7 Poincaré disk geometry of the unit radius each

Maximal $\mathcal{N} = 8$ supergravity

DeWit, Freedman (1977); Cremmer, Julia, Scherk (1978);
Cremmer, Julia (1978,1979); De Wit, Nicolai (1982)

- Theory has $2^8 = 256$ massless states.
- Multiplicity of states, vs. helicity, from coefficients in binomial expansion of $(x+y)^8$ – 8^{th} row of Pascal's triangle

$$\mathcal{N} = 8: \quad 1 \leftrightarrow 8 \leftrightarrow 28 \leftrightarrow 56 \leftrightarrow 70 \leftrightarrow 56 \leftrightarrow 28 \leftrightarrow 8 \leftrightarrow 1$$

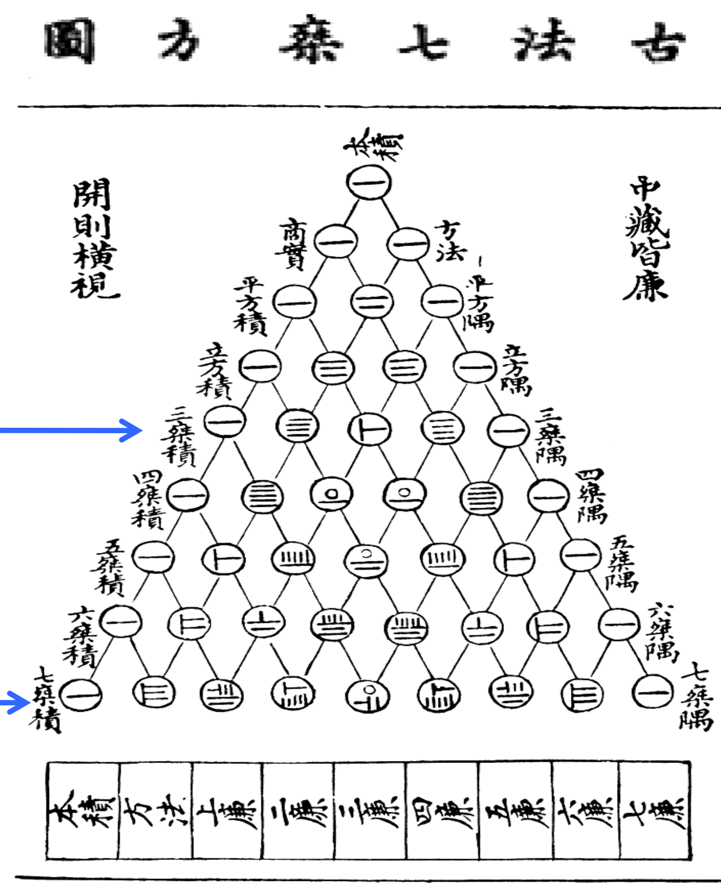
$$\text{helicity}: \quad -2 \quad -\tfrac{3}{2} \quad -1 \quad -\tfrac{1}{2} \quad 0 \quad \tfrac{1}{2} \quad 1 \quad \tfrac{3}{2} \quad 2$$

$$h^- \quad \psi_i^- \quad v_{ij}^- \quad \chi_{ijk}^- \quad s_{ijkl} \quad \chi_{ijk}^+ \quad v_{ij}^+ \quad \psi_i^+ \quad h^+$$

SUSY charges Q_a, a=1,2,…,8 shift helicity by 1/2 $\longleftrightarrow$

- Ungauged theory, in flat spacetime

Yanghui triangle
13th century

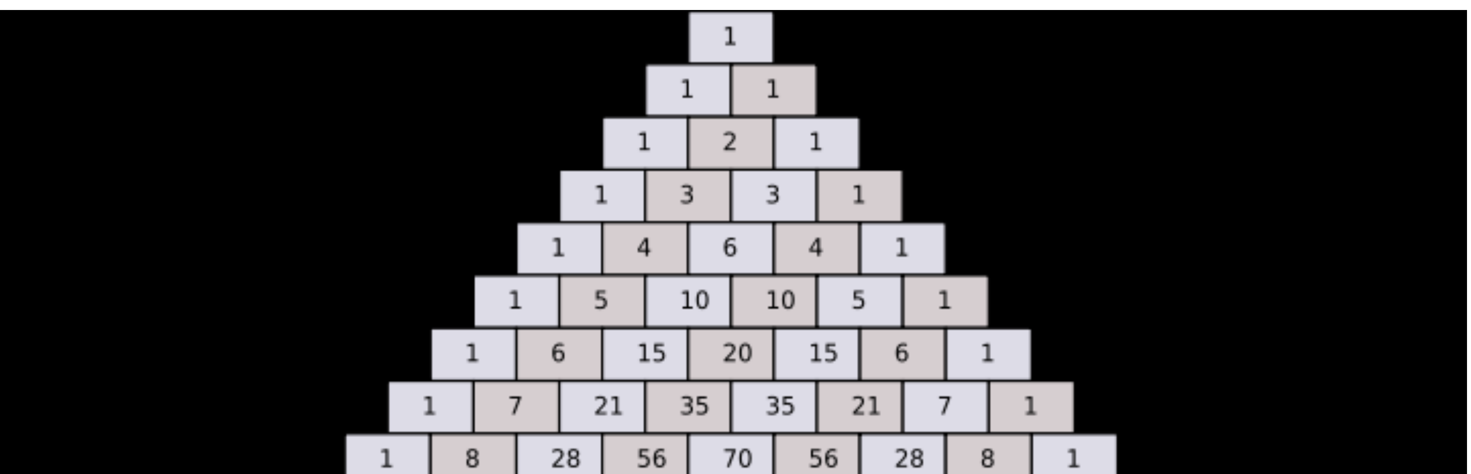


$(x+y)^4$ N=4 SYM

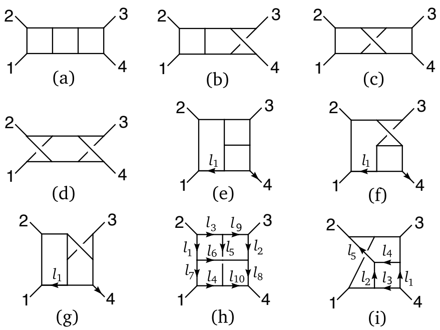
$(x+y)^8$ N=8 SG

Pascal's triangle determines the coefficients which arise in binomial expansions.

Each number in the triangle is the sum of the two directly above it.

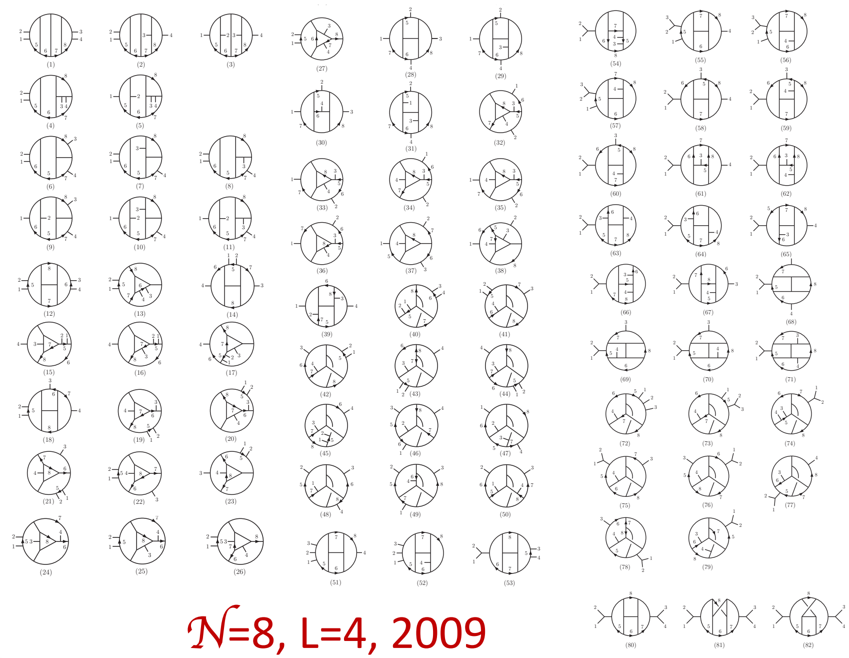


N=8 and N=5 d=4 supergravity are UV finite at L=1,2,3,4



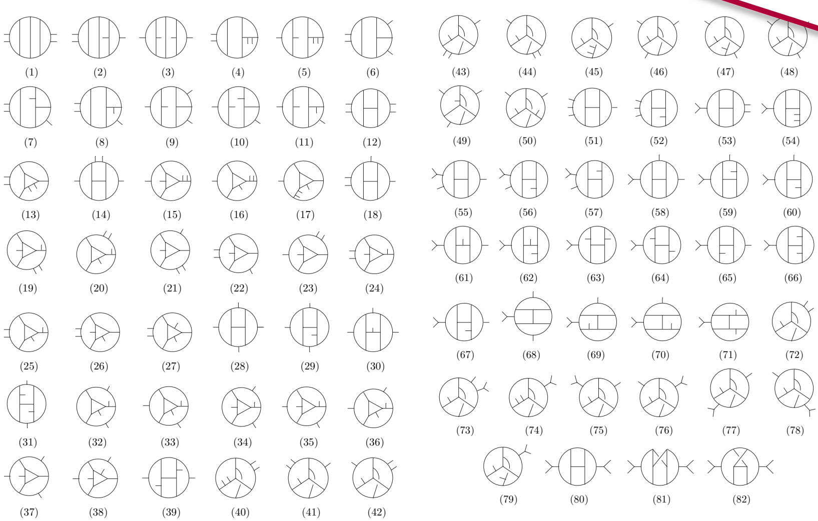
$\mathcal{N}$=8, L=3, 2007

UV div. in 9 diagrams cancel



$\mathcal{N}$=8, L=4, 2009

UV div. in 82 diagrams cancel



$\mathcal{N}$=5, L=4, 2014

UV div. in 82 diagrams cancel

$$E_{7(7)}$$

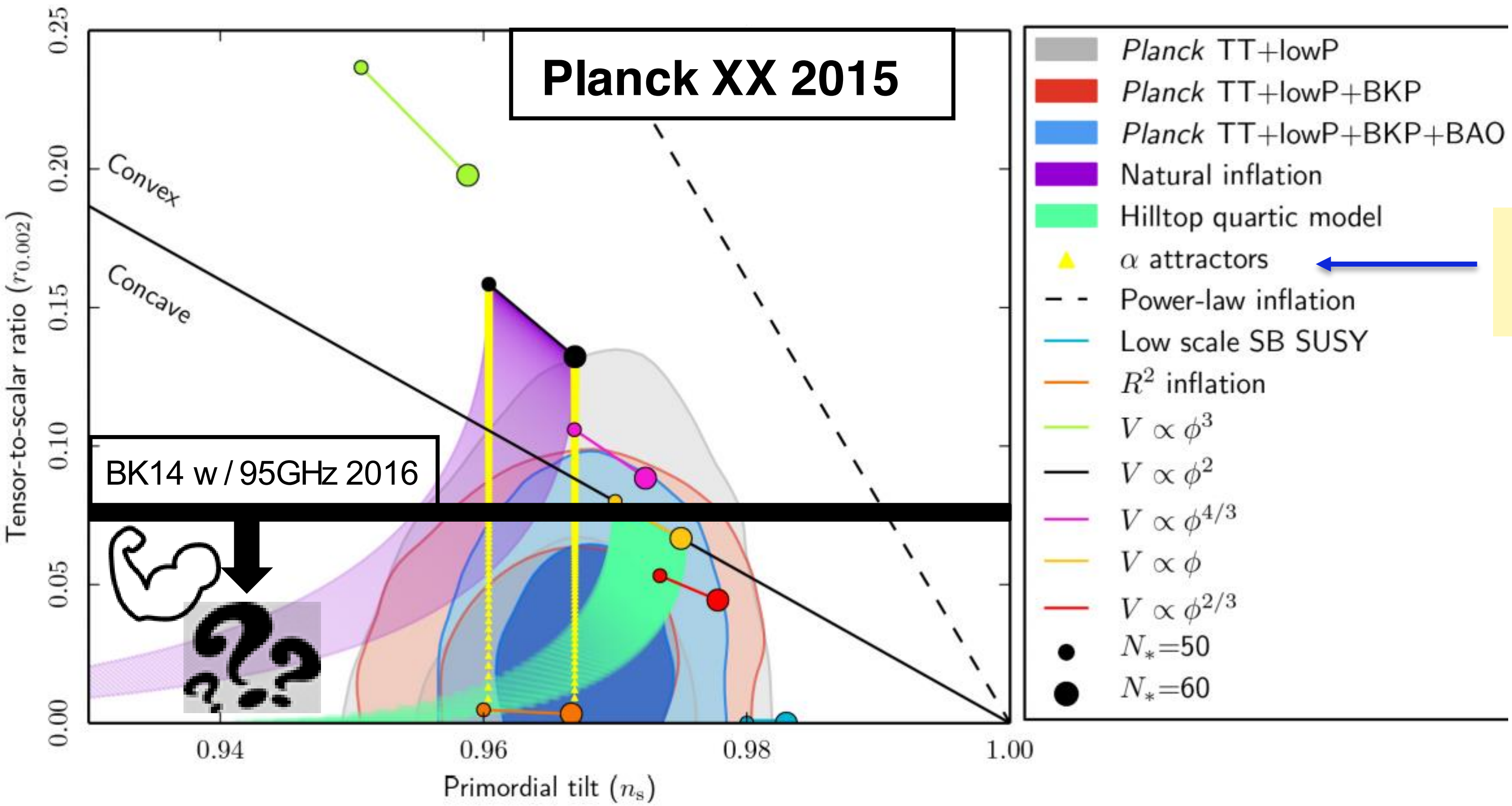
Explains!

$$E_{7(7)}(\mathbb{R}) \supset [SL(2,\mathbb{R})]^7$$

7 disks story in cosmology

No explanation known! Our current project

Part II, Cosmology: from the sky to the fundamental physics



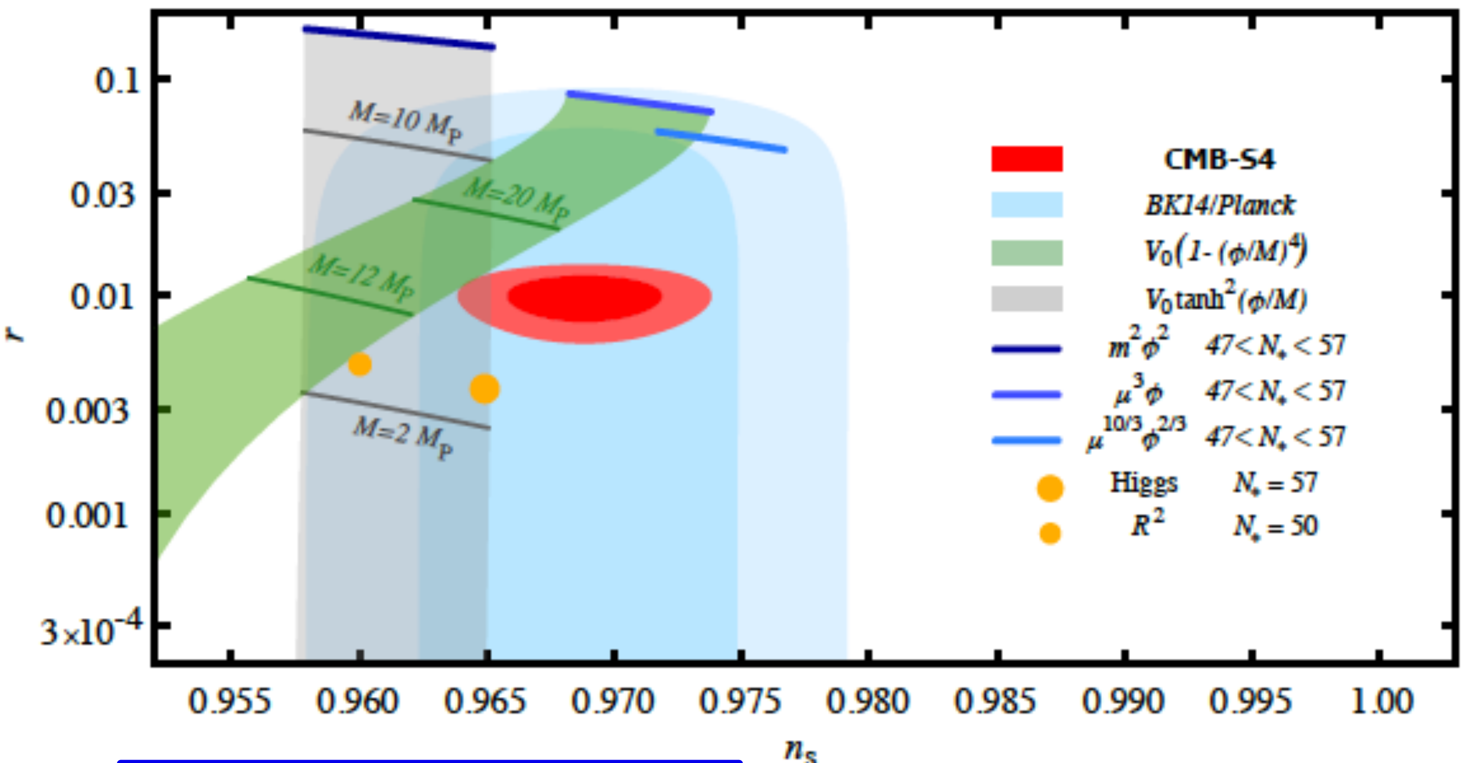
RK, Linde, Roest, 2013

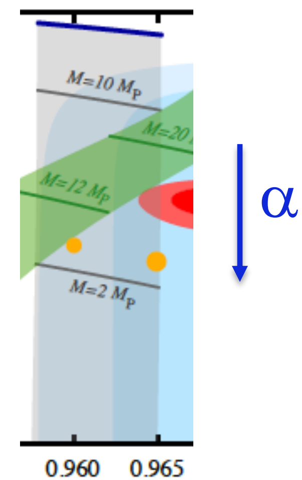
If B-modes are discovered soon with r > 10^{-2}
natural inflation models, axion monodromy models, α-attractor models,…, will be validated
No need to worry about log scale r

Otherwise, we switch to log r to see
$10^{-3} < r < 10^{-2}$

Alpha-Attractors and B-mode Targets

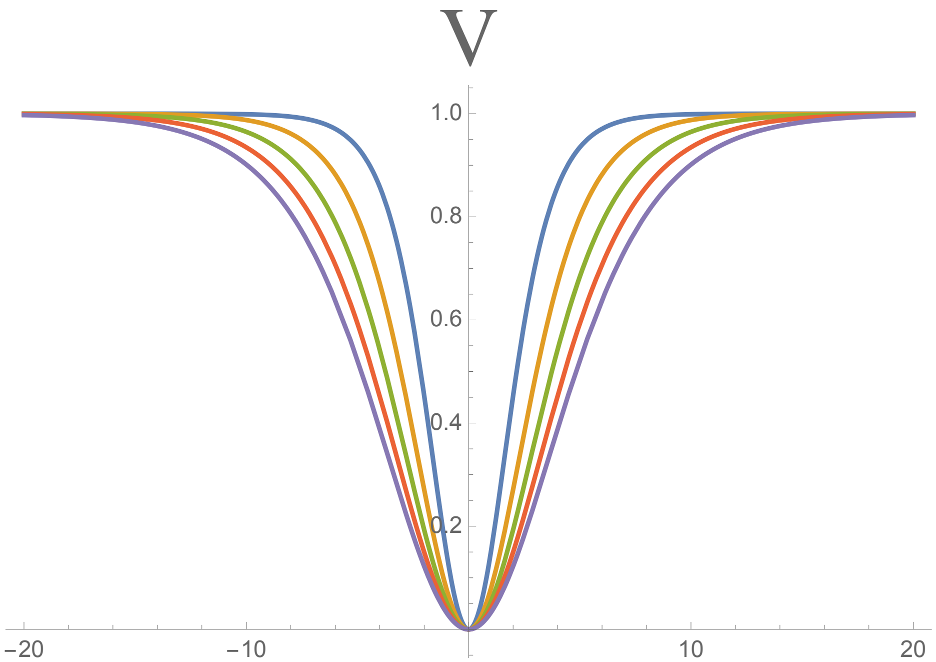
CMB-S4

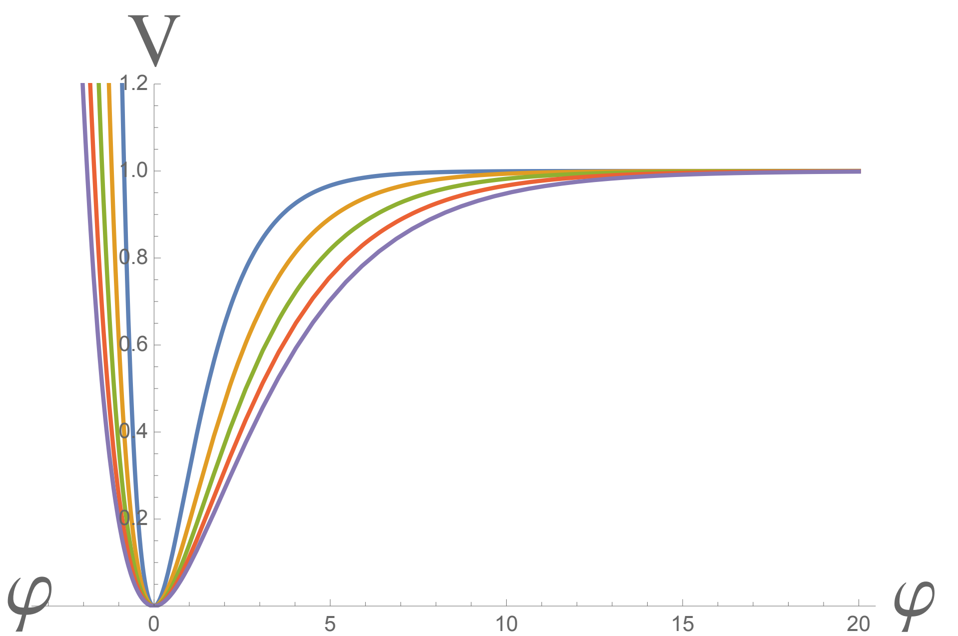




$$\boxed{n_s = 1 - \frac{2}{N}\,, \qquad r = \alpha\,\frac{12}{N^2}}$$

Plateau potentials of α-attractors





$$\frac{1}{2}R - \frac{1}{2}\partial\varphi^2 \ \ -\alpha\mu^2\Big(\tanh\frac{\varphi}{\sqrt{6\alpha}}\Big)^2$$

$$\frac{1}{2}R - \frac{1}{2}\partial\varphi^2 \ \ -\alpha\mu^2\Big(1 - e^{-\sqrt{\frac{2}{3\alpha}}\varphi}\Big)^2$$

Simplest T-model in canonical variables Simplest E-model

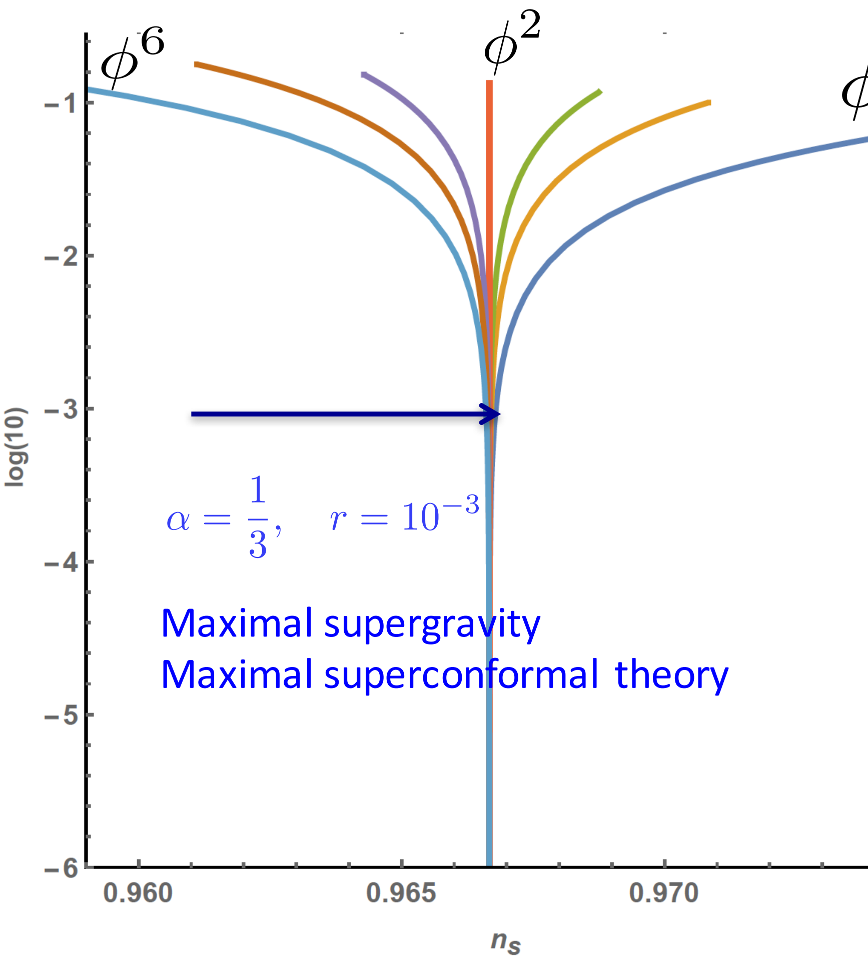
$$\frac{1}{2}R - 3\alpha\frac{\partial Z\partial\bar{Z}}{(1-Z\bar{Z})^2} - V_0 Z\bar{Z}$$

$$\frac{1}{2}R - 3\alpha\frac{\partial T\partial\bar{T}}{(T+\bar{T})^2} - V_0(T-1)^2$$

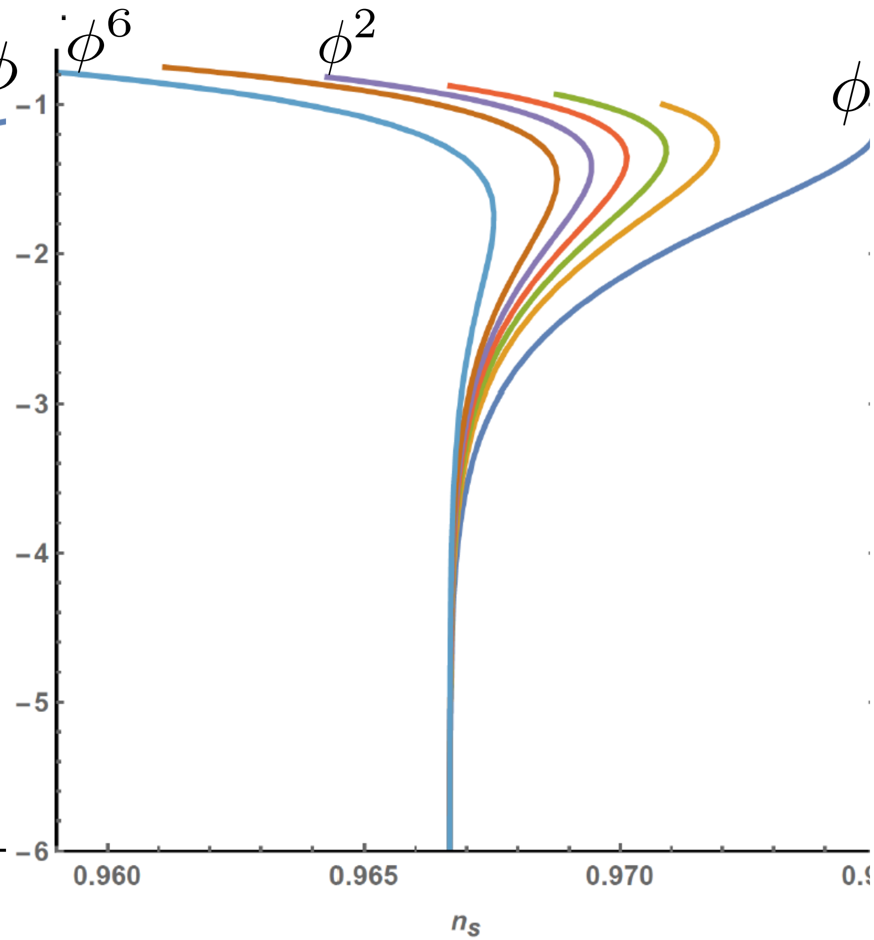
In geometric variables

α-attractors

Simple Fanned T-models



Simple Fanned E-models



$$\left(\tanh\frac{\varphi}{\sqrt{6\alpha}}\right)^{2n}$$

$$\left(1 - e^{\sqrt{\frac{2}{3\alpha}}\varphi}\right)^{2n}$$

Starobinsky and Higgs, α=1, n=1

What is the meaning of α-attractors?

Start with the simplest chaotic inflation model

$$\frac{1}{\sqrt{-g}}\mathcal{L} = \frac{1}{2}R - \frac{1}{2}\partial\phi^2 - \frac{1}{2}m^2\phi^2$$

Modify its kinetic term

$$\frac{1}{\sqrt{-g}}\mathcal{L} = \frac{1}{2}R - \frac{1}{2}\frac{\partial\phi^2}{(1-\frac{\phi^2}{6\alpha})^2} - \frac{1}{2}m^2\phi^2$$

Switch to canonical variables $\phi = \sqrt{6\alpha}\,\tanh\frac{\varphi}{\sqrt{6\alpha}}$

The potential becomes

$$V = 3\alpha\, m^2 \tanh^2\frac{\varphi}{\sqrt{6\alpha}}$$

The essence of α-attractors

$$\frac{1}{2}R - \frac{3}{4}\alpha\left(\frac{\partial t}{t}\right)^2 - V(t)$$

Suppose inflation takes place near the pole at t = 0, and V(0) > 0, V'(0) >0, and V has a minimum nearby. Then in canonical variables

$$\frac{1}{2}R - \frac{1}{2}(\partial\varphi)^2 - V_0(1 - e^{-\sqrt{\frac{2}{3\alpha}}\varphi} + ...)$$

Then in the leading approximation in 1/N, for any non-singular V

$$n_s = 1 - \frac{2}{N}, \qquad r = \alpha\frac{12}{N^2}$$

Complex scalar fields in supergravity and string theory

$$Z(t,\vec{x})\,,\bar{Z}(t,\vec{x})$$

are coordinates of some geometric space: **MODULI SPACE**

$$ds^2 = g_{Z\bar{Z}}\,dZ\,d\bar{Z}$$

The metric of the moduli space is defined by a second derivative of the Kahler potential

$$g_{Z\bar{Z}} = \partial_Z \partial_{\bar{Z}} K(Z,\bar{Z})$$

The curvature of the MODULI SPACE, Kahler curvature for our models is

$$\mathcal{R}_{\text{Kähler}} = -g^{-1}_{Z\bar{Z}}\partial_Z\partial_{\bar{Z}}\log g_{Z\bar{Z}} = -\frac{2}{3\alpha}$$

Meaning of the measurement of the curvature of the 3d space

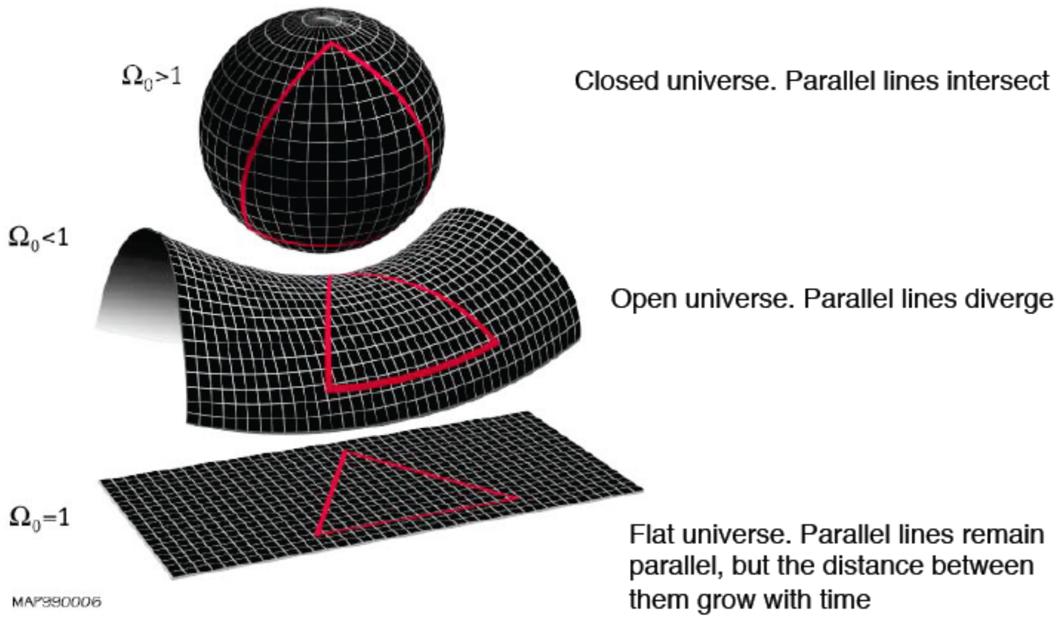
k=+1, k=-1, k=0

Spatial curvature parameter

$$ds^2 = -dt^2 + a(t)^2 \gamma_{ij} dx^i dx^j$$

$$\Omega_K = -0.0004 \pm 0.00036$$

Closed, open or flat universe



In the context of new supergravity cosmological models, measuring r means measuring the curvature of the hyperbolic geometry of the moduli space

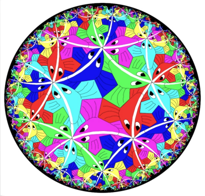
$$\boxed{n_s = 1 - \frac{2}{N}\,, \qquad r = \alpha\,\frac{12}{N^2}}$$

$$R_K = -\frac{2}{3\alpha}$$

scalar fields are coordinates of the Kahler geometry

Decreasing r, decreasing α, increasing curvature R_K

$$3\alpha = R^2_{\rm Escher} \approx 10^3 r$$



Hyperbolic geometry of a Poincaré disk

Anti-D3 Brane Induced Geometric Inflation:

Model Building Paradise

RK, Linde, Roest, Yamada, 2017

$\mathcal{G}$ **Kahler function**

Cremmer, Ferrara, Girardello, Julia, Scherk, van Nieuwenhuizen, Van Proeyen, from 1978

We are interested in anti-D3 brane interaction with Calabi-Yau moduli T_i. In supergravity we expect some interaction between the nilpotent superfield S, **representing KKLT type anti-D3 brane,** and Calabi-Yau moduli T_i

$$\mathcal{G}(T^i, \bar{T}^i; S, \bar{S})$$

$$\mathcal{G} \equiv K + \log W + \log \bar{W} \, , \qquad \mathbf{V} = e^{\mathcal{G}}(\mathcal{G}^{\alpha\bar{\beta}}\mathcal{G}_\alpha \mathcal{G}_{\bar{\beta}} - 3)$$

simple relation between the potential and the nilpotent field geometry

$$\mathcal{G}^{S\bar{S}}(T_i, \bar{T}_i) = \frac{\mathbf{V}(T_i, \bar{T}_i) + 3|m_{3/2}|^2}{|m_{3/2}|^2}$$

From the sky to fundamental physics

7-disk cosmological model

3α=7 example

Explaining disk merger: talk by Yamada

1. Start with M-theory, or String theory, or $\mathcal{N}$=8 supergravity

2. Perform a consistent truncation to $\mathcal{N}$=1 supergravity in d=4 with a 7-disk manifold

$$\mathcal{G} = \log W_0^2 - \frac{1}{2}\sum_{i=1}^{7} \log \frac{(1 - Z_i \overline{Z}_i)^2}{(1 - Z_i^2)(1 - \overline{Z}_i^2)} + S + \overline{S} + \mathcal{G}_{S\overline{S}} S\overline{S},$$

$$\mathcal{G}^{S\overline{S}} = \frac{1}{W_0^2}(3W_0^2 + \mathbf{V}).$$

corresponding to the **merger of seven disks** of unit size

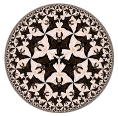
The scalar potential defining geometry is

$$\mathbf{V} = \Lambda + \frac{m^2}{7}\sum_i |Z_i|^2 + \frac{M^2}{7^2}\sum_{1\le i\le j\le 7}\Big((Z_i + \overline{Z}_i) - (Z_j + \overline{Z}_j)\Big)^2,$$

During inflation $\mathbf{V}(\varphi) = \Lambda + m^2 \tanh^2 \frac{\varphi}{\sqrt{14}},$

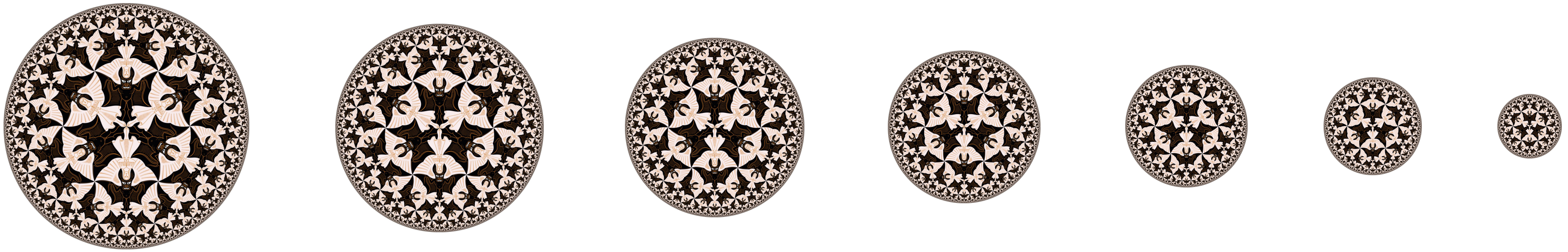
De Sitter exit

$r \approx 10^{-2}$

Based on CMB data on the value of the tilt of the spectrum n_s as a function of N we deduced that hyperbolic geometry of a Poincaré disk  suggests a way to explain the experimental formula

$$\boxed{n_s \approx 1 - \frac{2}{N}}$$

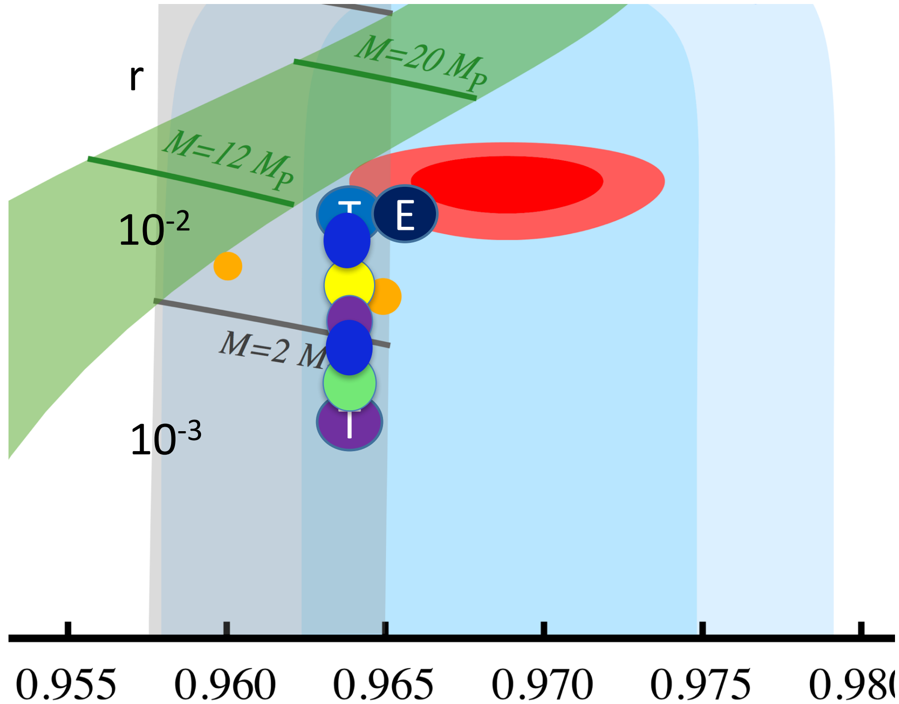
Using a consistent reduction from maximal $\mathcal{N}$=8 supersymmetry theories: M-theory in d=11, String theory in d=10, maximal supergravity in d=4, to the minimal $\mathcal{N}$=1 supersymmetry we have deduced the favorite models with hyperbolic geometry with $R^2_{\text{Escher}} = 3\alpha = 7,6,5,4,3,2,1$



$r \approx 0.9 \times 10^{-2}$ $r \approx 1.3 \times 10^{-3}$

B-mode targets from disks merger

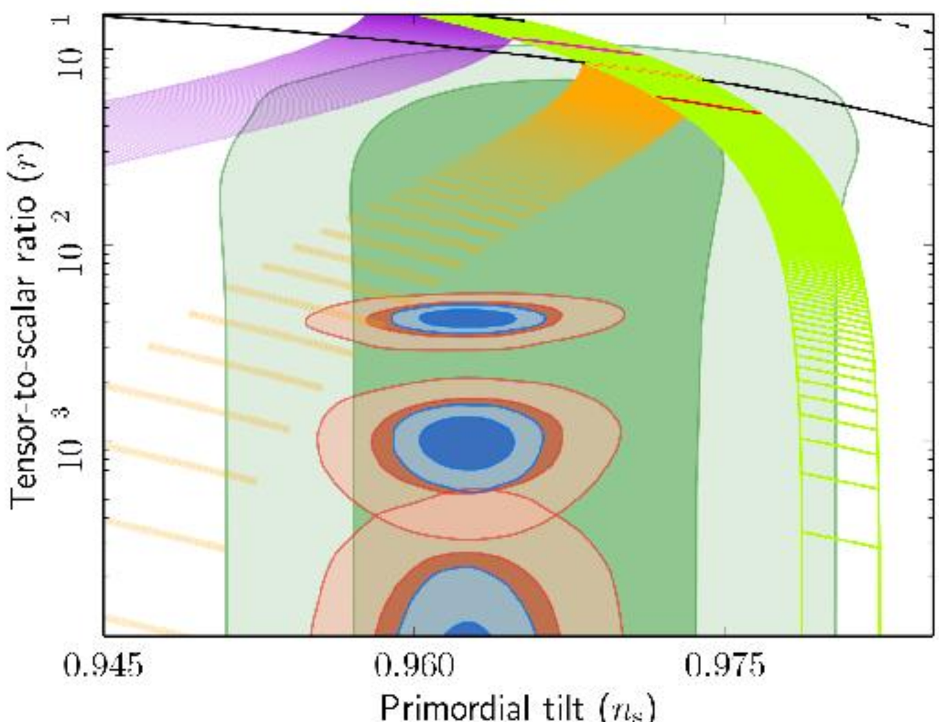
In contrast with $\mathcal{N}$=1 supersymmetry models where 3α is arbitrary



Higgs $N_* = 57$

R^2 $N_* = 50$

Seven new targets



σ(n_s) = 0.0014

Improvement factor
CORE to Planck 2015
3.4

COrE : Cosmic Origin Explorer

forecast regions for (n_s,r) for CORE (blue) and LiteBIRD (red)

Axions in α-attractors

More on axions in Yamada's talk

A disk or a half-plane variables are complex scalars: the real part is an inflaton, the imaginary part is an axion

Most models in the past were designed to have a heavy axion, so that we have a single field inflation

Choice of the shift symmetry of the Kahler potential,
slightly broken by the superpotential

Möbius transformations

- Translation of the imaginary part: $T \to T - ib$,
- Dilatation of the entire plane: $T \to a^2 T$,
- Inversion : $T \to 1/T$,
- Reflection of the imaginary part: $T \to \overline{T}$.

$$T = \exp\left(\sqrt{\frac{2}{3\alpha}}\,\varphi\right) + i\chi$$

Depending which subgroup symmetry is manifest, one can either have a shift symmetry

for the axion

$$K = -3\alpha \log(T + \overline{T})$$

or for the inflaton

$$K = -\frac{3\alpha}{2} \log\left[\frac{(T+\overline{T})^2}{4(T\overline{T})}\right]$$

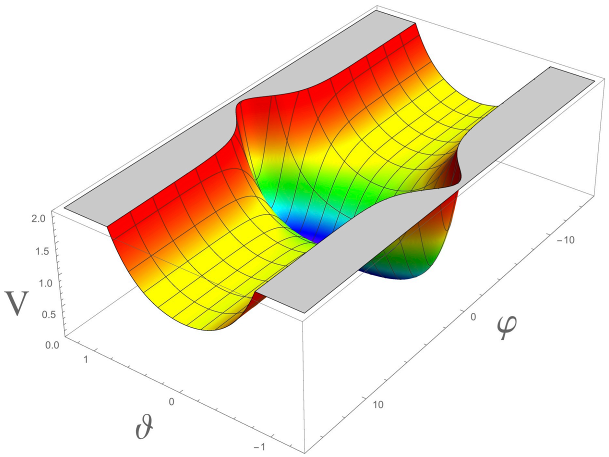
In our earlier cosmological **α-attractor** models, **axion is stabilized** to provide a single field inflation in supersymmetric models where the scalar is complex.

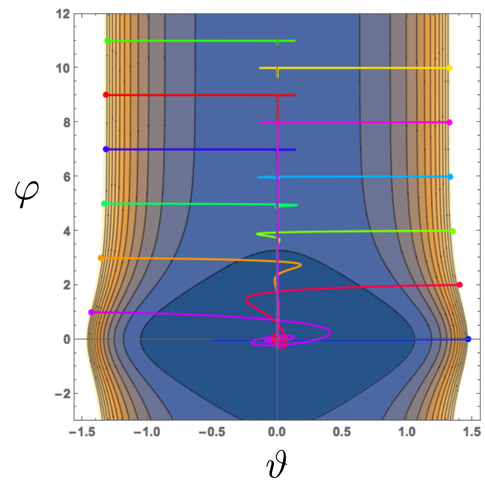
$$Z = \tanh \frac{\Phi}{\sqrt{6\alpha}} \qquad \Phi \equiv \varphi + i\vartheta$$

Kahler potential has an inflaton shift symmetry

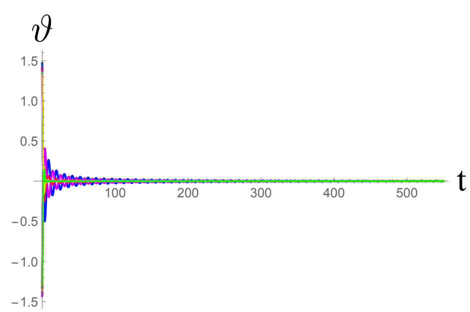
$$K_{\mathbb{D}} = -\frac{3\alpha}{2} \log \left[\frac{(1 - Z\bar{Z})^2}{(1 - Z^2)(1 - \bar{Z}^2)} \right] + S\bar{S}$$

$$W_{\mathbb{D}} = A(Z) + S\,B(Z) \ .$$





Time evolution of scalars on a contour plot of the potential



The field ϑ moves quickly towards the minimum, at the bottom of the dS valley. Then after a short stage of oscillations, the field ϑ vanishes and remains in all cases at the bottom of the dS valley.

α-attractors with a non-stabilized angular direction

$\alpha = 1/3$

$$K = -\ln(1 - Z\bar{Z} - S\bar{S})$$

$$W = \mu\, S\, Z$$

A projection of the Escher disk of the radius $3\alpha = 1$ on the quadratic inflationary potential

Multi-field α-attractors

Achúcarro, RK, Linde, Dong-Gang Wang, Welling arXiv:1711.09478

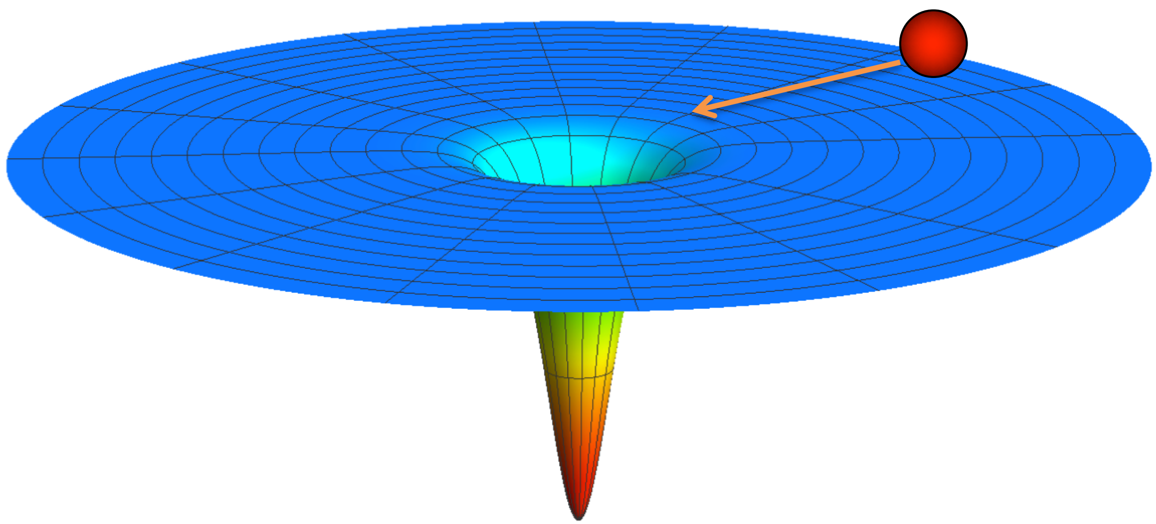
Our original expectation was that if we consider models with a very light non-stabilized axion, our predictions for n_s and r and non-Gaussianity will become very different from our universal α-attractor models with stabilized heavy axion. The model we looked at was $\alpha=1/3$ model which we knew can easily have light axions.

$$K = -\ln(1 - Z\bar{Z} - S\bar{S})$$

α = 1/3

$$W = \mu\, S\, Z$$

This is the same potential as at the previous slide, using a canonical radial variable



$$Z = e^{i\vartheta}\ \tanh\frac{\varphi}{\sqrt{2}}$$

ϑ- independent potential

$$V = \mu^2 \tanh^2\frac{\varphi}{\sqrt{2}}$$

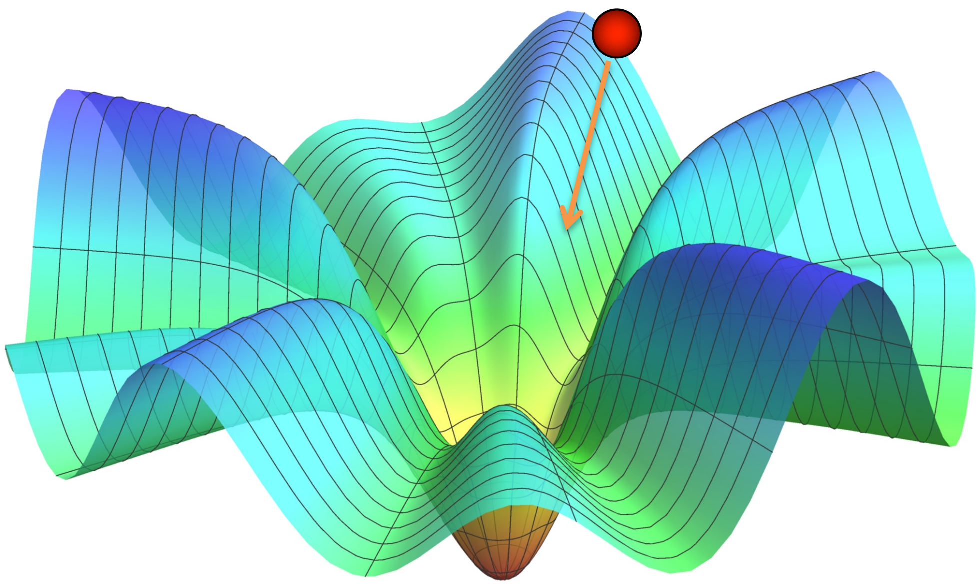
Multifield α-attractors

New model with axion shift symmetry in the geometry, broken by the potential

$$\mathcal{G} = \ln|W_0|^2 - \ln(1 - Z\bar{Z}) + S + \bar{S} + G_{S\bar{S}}(Z,\bar{Z})S\bar{S} \qquad G_{S\bar{S}}(Z,\bar{Z}) = \frac{|W_0|^2}{(1-Z\bar{Z})\Big(|F_S|^2 + V(Z,\bar{Z})\Big) + 2|W_0|^2 Z\bar{Z}}$$

$$Z = e^{i\theta}\tanh\frac{\varphi}{\sqrt{2}} \qquad g^{-1}\mathcal{L} = \frac{1}{2}(\partial\varphi)^2 + \frac{1}{4}\underline{\sinh^2(\sqrt{2}\varphi)}(\partial\theta)^2 - V(\varphi,\theta)$$

One could expect a very complicated cosmological evolution and model-dependent predictions



Surprize! $\textit{rolling on the ridge}$ with almost constant θ

θ does not seem to move because physical distance in angular direction during inflation is exponentially large, proportional to $\sinh\sqrt{2}\varphi \sim e^{\sqrt{2}\varphi}$

[Computing observables in curved multifield models of inflation—A guide (with code) to the transport method](#) Dias, Frazer, Seery, 2015

Puzzling numerical result: Universal prediction for α-attractor models with light axions

$$n_s = 1 - \frac{2}{N} \quad \text{and} \quad r = \frac{12\alpha}{N^2}$$

Very small non-Gaussianity

Why? <u>We did not expect it at all</u>. A miraculously simple result, after all the struggle with powerful but complicated methods.

Then we used Sasaki's δN formalism, and everything became clear:

Salopek, Bond;
Sasaki, Stewart, Starobinsky;
Sasaki, Tanaka;
Lee, Sasaki, Stewart, Tanaka, Yokoyama

curvature perturbation at the end of inflation

$$\zeta = \delta N = \frac{\partial N}{\partial \varphi}\delta\varphi + \frac{\partial N}{\partial \theta}\delta\theta = \frac{\sqrt{2}e^{\sqrt{2}\varphi}}{B}\delta\varphi + \left(C_\theta - \frac{B_\theta}{B^2}e^{\sqrt{2}\varphi}\right)\delta\theta$$

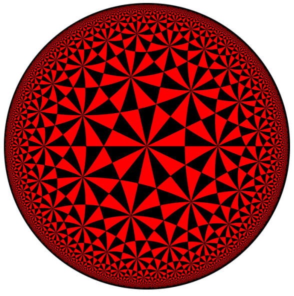
As we see here, $\frac{\partial N}{\partial \varphi}$ and $\frac{\partial N}{\partial \theta}$ can be comparable to each other. However, one should keep in mind that θ field is non-canonical, thus to estimate the field fluctuation amplitudes at horizon-exit, one should consider the canonically normalized ones: $\delta\varphi$ and $\frac{1}{\sqrt{2}}\sinh(\sqrt{2}\varphi)\delta\theta$. Approximately in the large-φ region we have the following relation

$$\delta\varphi \simeq \frac{e^{\sqrt{2}\varphi}}{2\sqrt{2}}\delta\theta \simeq \frac{H}{2\pi}$$

$$\frac{1}{2}(\partial\varphi)^2 + \frac{1}{4}\sinh^2(\sqrt{2}\varphi)(\partial\theta)^2$$

the field fluctuation $\delta\theta$ is exponentially suppressed.

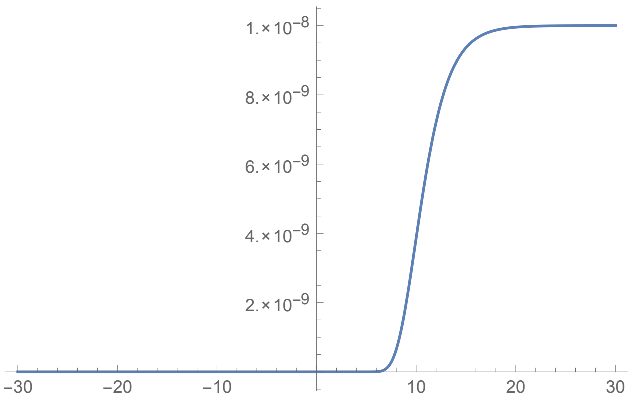
Thus <u>only the radial direction contributes to perturbations</u>, which is why we got the universal result of single-field α-attractor models

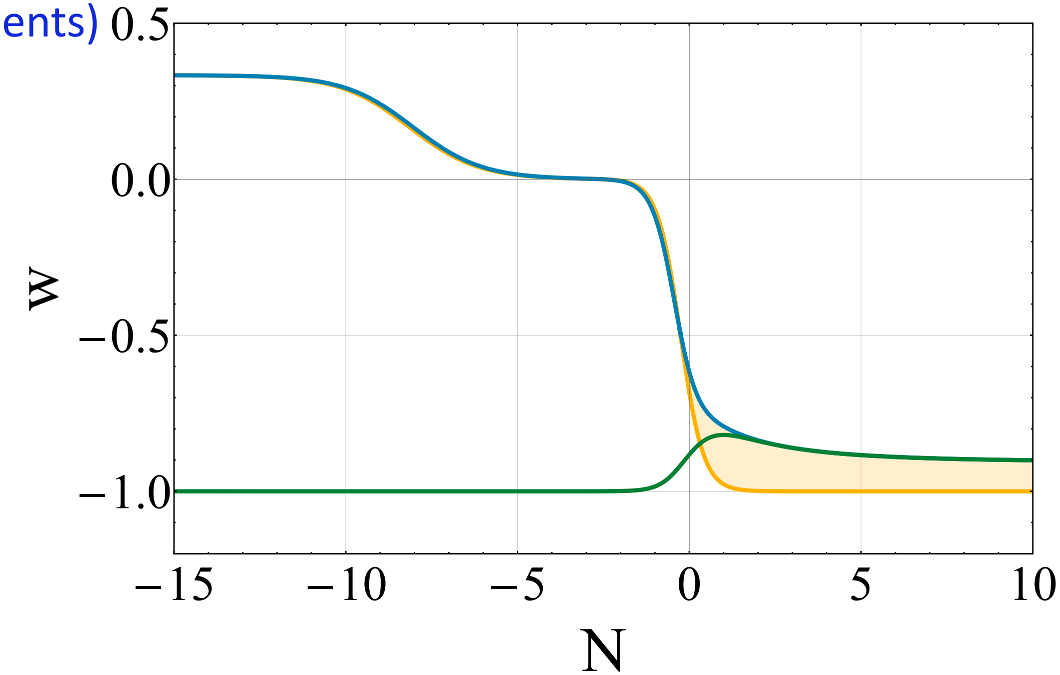


Dark Energy with α-attractors : w= -1, in most cases

Dec 2017, Arkami, RK, Linde, Vardanyan

A simple quintessential inflation 2-shoulder α-attractor model (requires large exponents)





$$r = 4\,\frac{3\alpha}{N^2}$$

$$w_\infty = -1 + \frac{2}{3}\,\frac{1}{3\alpha}$$

$$3\alpha = 7 \qquad r \approx 10^{-2} \qquad w_\infty \approx -0.9$$

LiteBird?

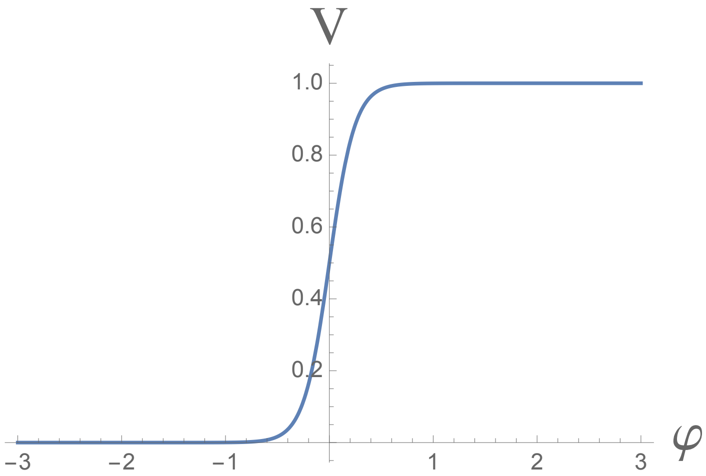
Euclid?

Quintessential α-attractor model with linear potential:

$$V(\phi) = \gamma\phi + \Lambda$$

In canonical variables:

$$V(\varphi) = \Lambda + \gamma\sqrt{6\alpha}(\tanh\frac{\varphi}{\sqrt{6\alpha}} + 1) \approx \Lambda + 2\gamma\sqrt{6\alpha}\, e^{\sqrt{\frac{2}{3\alpha}}\varphi}$$



Very simple potential, predictions for w depend on efficiency of reheating. Requires $\alpha = 10^{-2}$.

Thus there is nothing simpler than the cosmological constant, but if the data show that **w** is different from **-1**, we can account for it without modifying GR.

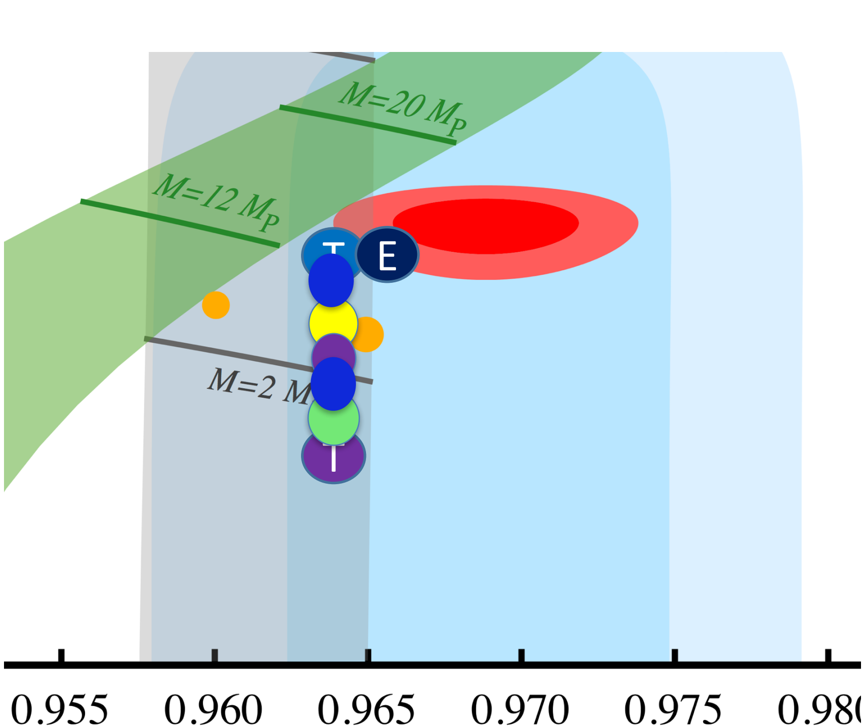
In quintessential α-attractors with gravitational preheating and a long stage of kinetic energy dominance, inflation must be longer than in the conventional α-attractors with a long stage of oscillations at about

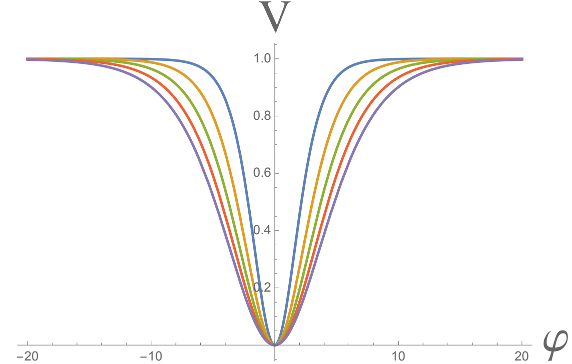
$$\Delta N \sim \frac{1}{6} \ln \left(\frac{\rho_{\rm end}}{\rho_{\rm reh}} \right)$$

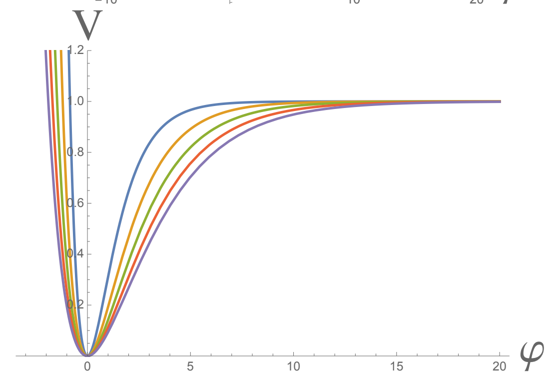
The required number of e-folds N in the quintessential α-attractor models can be greater than in the conventional α-attractors, or in the Starobinsky model, by

$$\Delta N \sim 10$$

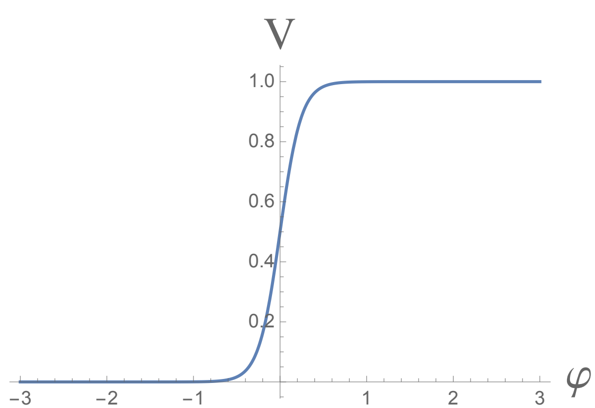
As a result, the value of $\mathbf{n_s}$ in the quintessential α-attractors with gravitational preheating is typically greater than in more traditional models by about 0.006 or so. This number coincides with one standard deviation in the Planck results. Thus by a more precise determination of $\mathbf{n_s}$ to be achieved in the future, we may be able to distinguish between the quintessential α-attractors and conventional models with a cosmological constant, even if we cannot tell the difference between **w** and **-1**. This emphasizes importance of precise measurement of $\mathbf{n_s}$.







Quintessential inflation allows to increase the number of e-foldings N, which slightly increases $\mathbf{n_s}$ for α-attractor models. With better precision on spectral index $\mathbf{n_s}$ we may differentiate in the future between inflation ending at the minimum of the potential, and the one ending at a second plateau, even if the equation of state there is **w = -1**



Looking forward for the new data