

Long-wavelength perturbations around homogeneous **but anisotropic** spacetimes

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Going **beyond** δN

text book for δN

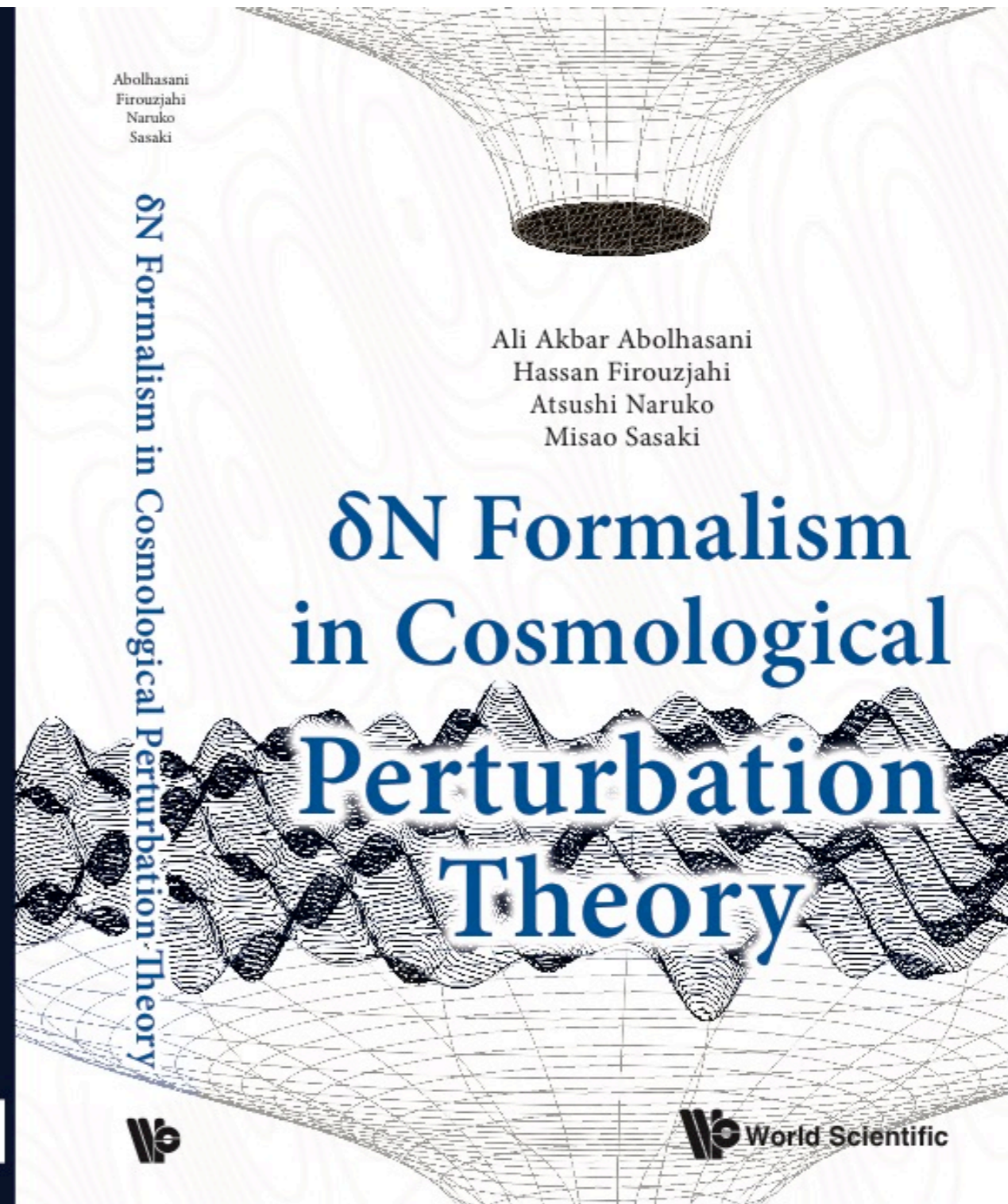
δN Formalism in Cosmological Perturbation Theory

Early Universe cosmology is an active area of research and cosmic inflation is a pillar of modern cosmology. Among predictions of inflation, observationally the most important one is the generation of cosmological perturbations from quantum vacuum fluctuations that source all inhomogeneous structures in the Universe, not to mention the large-scale structures such as clusters of galaxies.

Cosmological perturbation theory is the basic tool to study the perturbations generated from inflation. There are a few different approaches to primordial cosmological perturbations. In the conventional approach one perturbs the field equations and after quantizing the perturbations by the use of the corresponding action, one calculates the power spectrum of cosmological observables. This approach extends to higher order perturbations such as bispectrum etc., but the analysis becomes increasingly difficult.

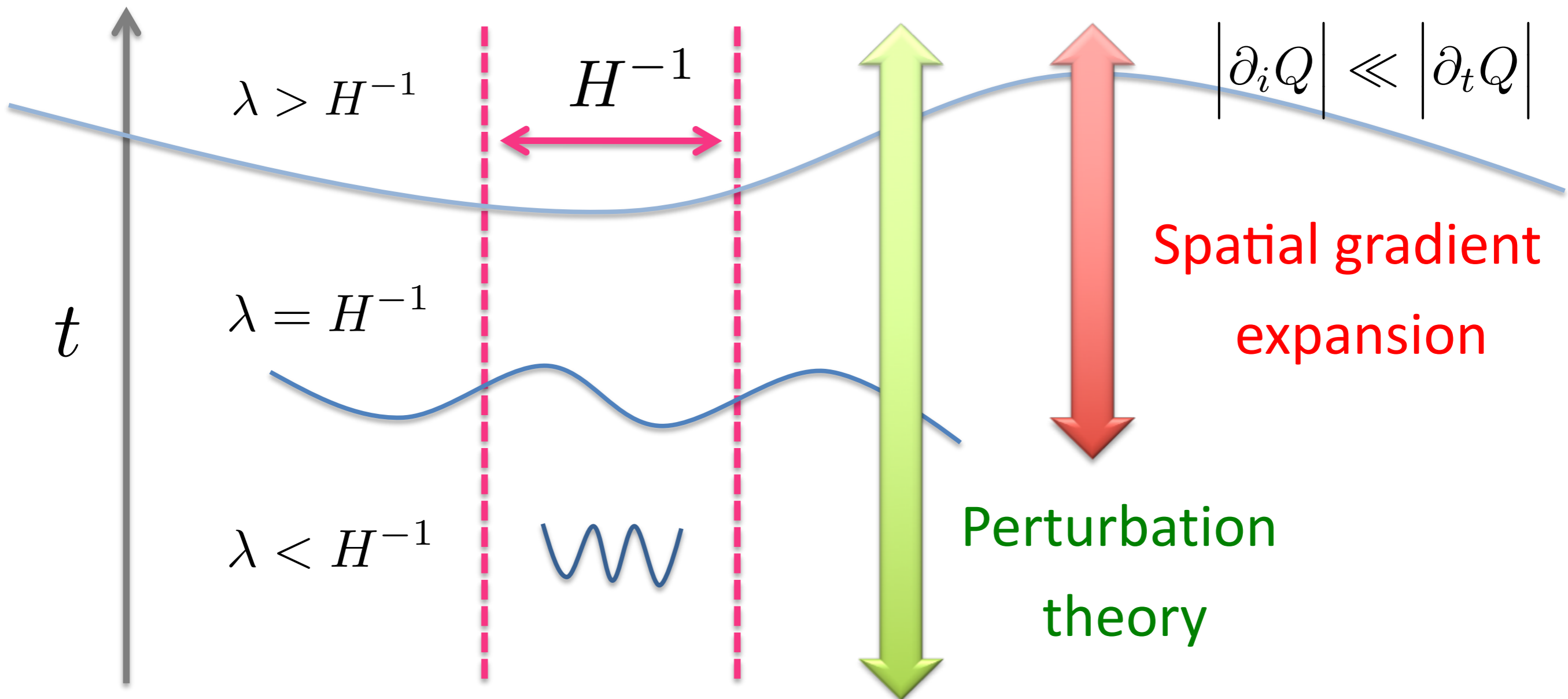
The delta N formalism, the topic of this book, is an alternative approach. The novelty of this approach is that, under the condition that the scale of interest is very large so that the spatial derivatives may be ignored in the dynamics, it can be applied to all orders in perturbation theory and has a rigorous foundation in general relativity. Thanks to the fact that one can evaluate perturbations with only the knowledge of background solutions, it is proved to be much easier than the conventional approach in evaluating higher order effects in many cases.

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What is δN ??

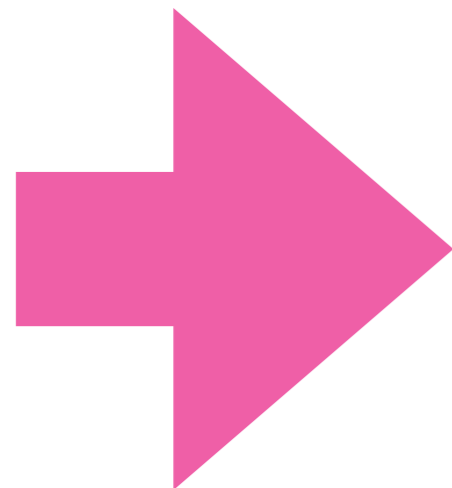
✓ evolution of fluctuations during inflation



δN and the next ??

- ✓ based on the **leading order** approximation of GE method
- ✓ a way to evaluate **the (conserved) curvature perturbation** around **homogeneous & isotropic** universe
just by solving BG eqs. not (involved ?) perturbation eqs.

$$\mathcal{R}_c \Big|_{\text{final}} = \delta N$$



1. the **next order** in gradient expansion ??
2. breaking **homogeneity** ??
3. breaking **isotropy** ??

① next order in GE

✓ We have already investigated **the next order** in GE.

Beyond δN formalism

PTEP 2013 (2013)
arXiv:1210.6525

Atsushi Naruko^{1,*}, Yu-ichi Takamizu², and Misao Sasaki²

- ◆ gauge choice : **uniform N (e-folding)** gauge (slicing)
Sasaki & Tanaka [1998]
- ◆ non-linear gauge transformation :
sol. in the **N gauge** \rightarrow sol. in the **comoving gauge**
- ◆ proper definition of non-linear curvature perturbation :
R = [perturbation of “a”] + **[perturbation of GW]**

② breaking homogeneity

✓ maybe interesting... analysis involved ? any application ?

③ breaking isotropy

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motivation -why anisotropic BG-

Gibbons & Hawking 1977, Wald 1983

- ✓ **cosmic no-hair** theorem/conjecture for inflationary universe
 - Λ (c.c.) + (homogeneous) matter with energy conditions
 - universe will be **isotropized** & evolve towards **de Sitter**
= **anisotropy will disappear** (even if exist initially)
- ✓ **symmetry during inflation**
 - **time-translation symmetry** in de Sitter \leftrightarrow **scale inv. Ps**
$$ds^2 = -dt^2 + e^{2Ht} d\vec{x}^2 \quad [t \rightarrow t + \lambda \ \& \ x \rightarrow e^{-H\lambda} x]$$
 - ns -1 = $O(\epsilon)$ implies the **breaking** of time-tr. symmetry !!
 - what about **spatial rotational symmetry** ?? **$O(\epsilon)$** ???

anisotropic inflation

✓ *inflation with anisotropic hair* Watanabe, Kanno, Soda [2009]

$$\mathcal{L} = R + \mathcal{L}^\phi + f(\phi) F_{\mu\nu}^2 + \vec{A}_{BG}$$

- $f(\phi)$ breaks **conformal inv.** & **no instability** in pert.
- predict **(statistically) anisotropic** power spectrum

$$P(\vec{k}) = P(k) \left[1 + g_* (\vec{k} \cdot \vec{v}_P) \right] \quad \text{vP : preferred direction}$$

$$\rightarrow g_* = 0.002^{+0.016}_{-0.016} \quad \text{Kim \& Komatsu [2013]}$$

✓ *implication for **symmetry** during inflation* AN + [2015]

→ breaking of rotational sym. = $10^{-8} \mathbf{O}(\epsilon)$ \leftrightarrow $\mathbf{O}(\epsilon)$
for time-sym.

GE in anisotropic BG

✓ let us focus on **linear perturbations** in a simple setup !!

Bianchi I : $ds^2 = -dt^2 + e^{2\alpha(t)} \left[e^{-4\beta(t)} dx^2 + e^{2\beta(t)} (dy^2 + dz^2) \right]$

$\delta g_{\mu\nu} dx^\mu dx^\nu = -2A dx^2 + (B_x dx + B_{,a} dx^a) dt$

$+ (dx \ dx^a) \begin{pmatrix} 2C - 4D & E_{,a} \\ E_{,a} & (2C + 2D)\delta_{ab} + F_{,ab} \end{pmatrix} \begin{pmatrix} dx \\ dx^a \end{pmatrix}$

δ (BG eqs) ^{only} = pert. eqs

almost all

in the N gauge

[C = const. & B=0]

summary

- ✓ We have considered possible extensions of δN formalism.
- ✓ First, we have investigated the next leading order in gradient expansion method around isotopic case.
- ✓ Next, we have investigated gradient expansion method around homogeneous but anisotropic background.
- ✓ By carefully studying linear perturbations on that background, we confirm that the nature of perturbations on large scales can be completely captured by the background physics while there would be seemingly non-trivial one.

**Thank you
for your attention !!**