Lecture II: Synthetic Spin-Orbit Coupling for Ultracold Atoms and Majorana fermions

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Outline

- From 1D to 2D synthetic SOC
- Optical Raman lattice schemes for 1D/2D SOCs
- Topological physics for optical Raman lattices
- Experiment realization of 2D SOC and topological bands
- Summary

1D Spin-orbit coupling for cold atoms

Scheme: XJL, M. F. Borunda, X. Liu, and J. Sinova, PRL, 102, 046402 (2009); arXiv: 0808.4137. And some previous related works.

Experiments

^{87}Rb boson: I. Spielman group, 2011
Shuai Chen, Jianwei Pan group, 2012
P. Engels' group, Washington State U.
Y. P. Chen, Puedue U
^{40}K fermion: J. Zhang group, 2012
^{6}Li fermion: M. Zwierlein group, 2012.
^{161}Dy fermion: Lev, 2016; ^{173}Yb fermion: G.-B. Jo, 2016; ....

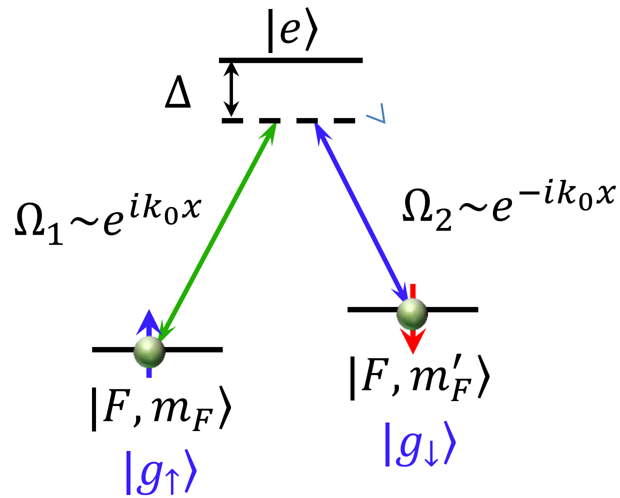
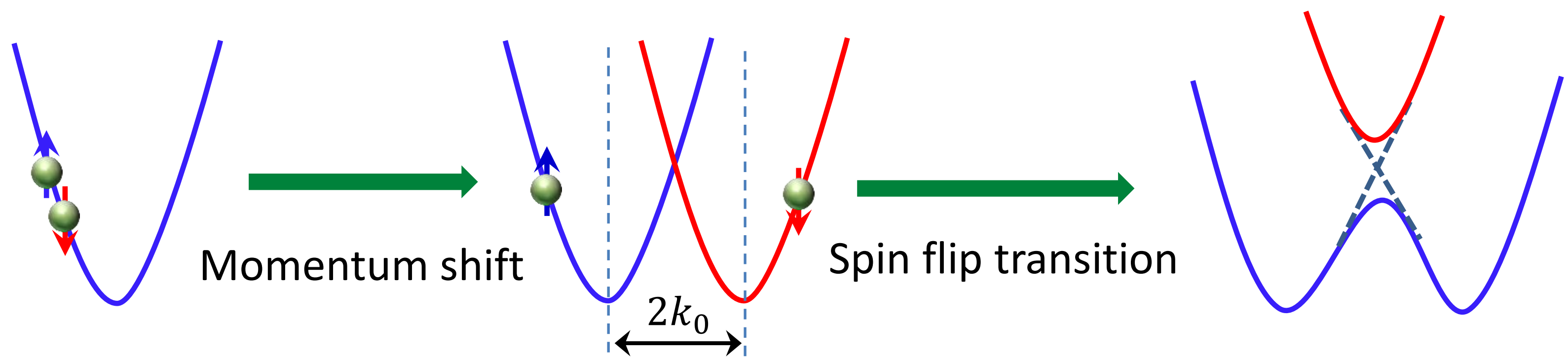


Illustration of 1D SO coupling:



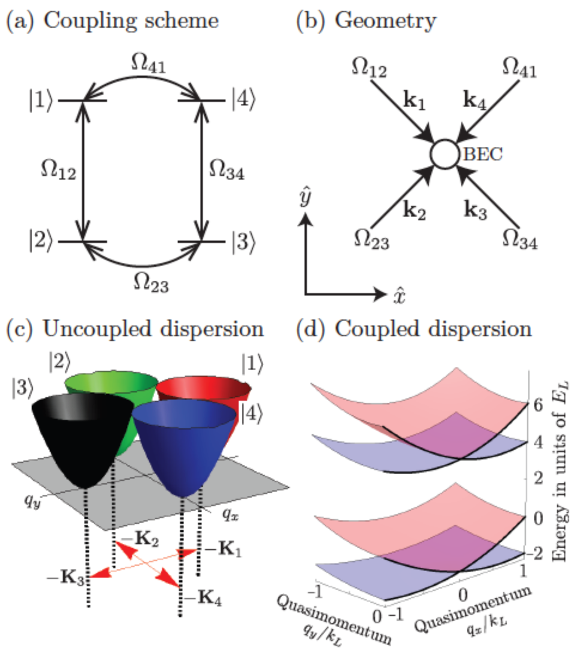
1D spin-orbit coupling plus Zeeman coupling

$$H_0 = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{p_z^2}{2m} + \boxed{\frac{k_0}{m} p_x \sigma_z + \frac{\Omega_R}{2} \sigma_x}$$

However, this is only a 1D SO coupling!

Some proposals of 2D/3D SO Couplings

1. Multi-Raman Couplings

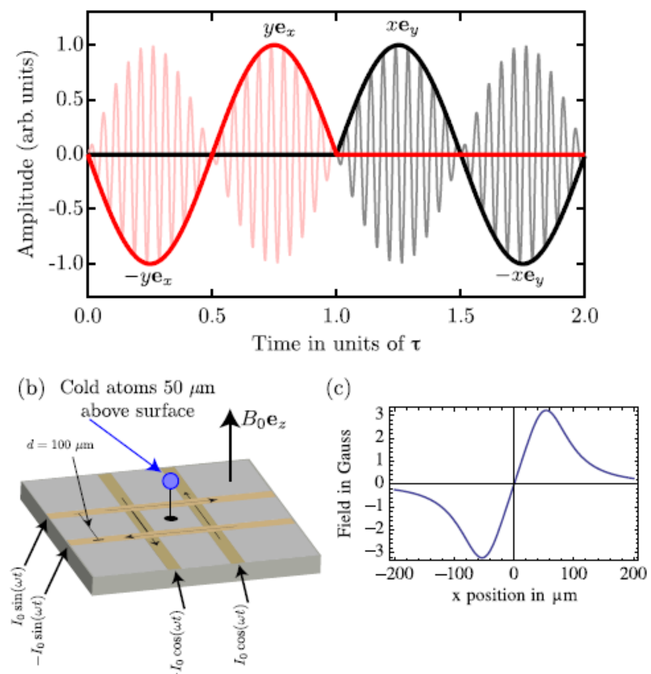


Phys. Rev. Lett. **108**, *235301 (2013)*

Difficulty:

- More lasers, large heating rate;
- Phase lock of the atom-laser coupling.

2. Gradient magnetic field pulse



Phys. Rev. Lett. **111**, *125301 (2013)*

Phys. Rev. A **85**, *043605 (2012)*

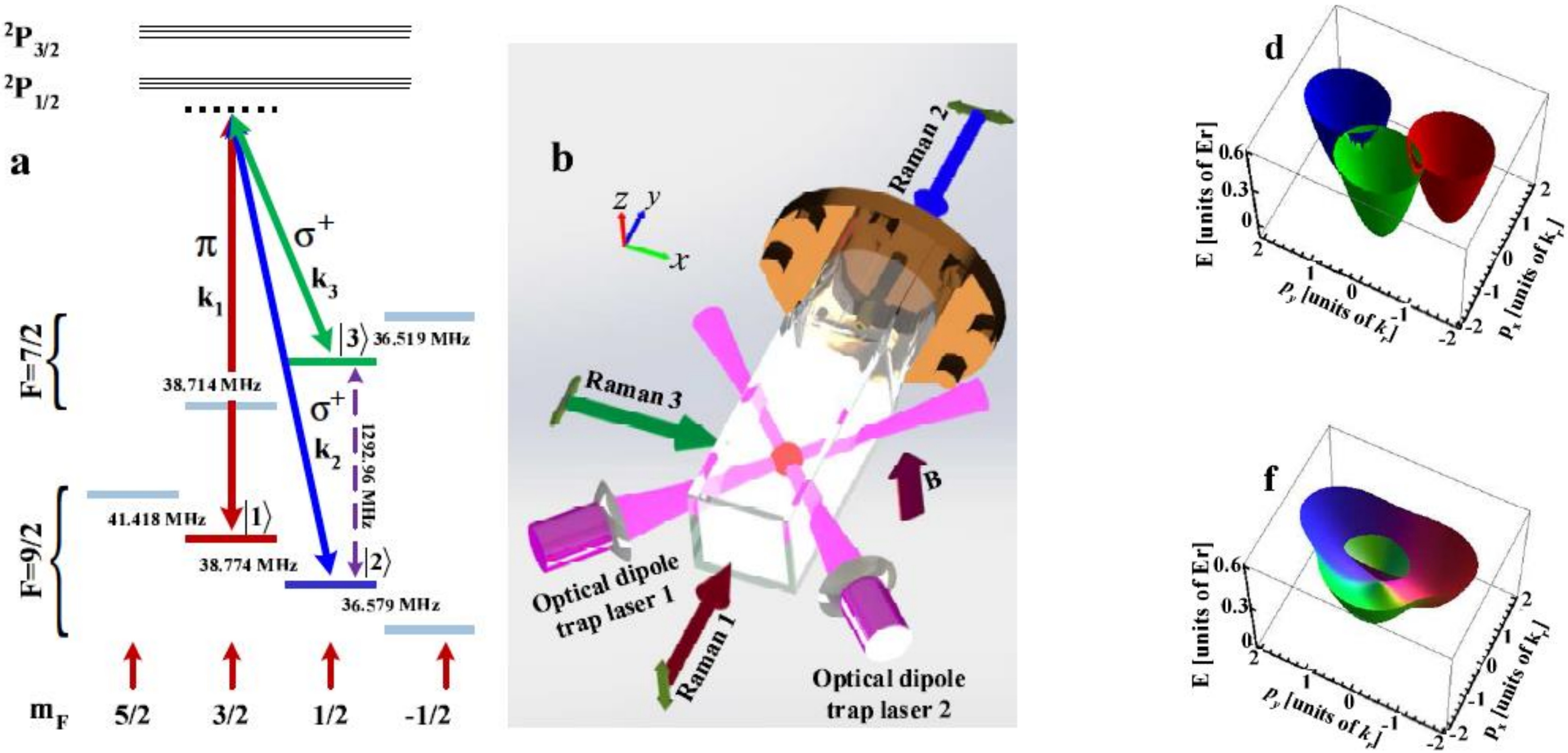
Difficulty:

Fast switch of the magnetic fields

Possible solution: atom chip

3. An illustration of 2D SOC with a tripod system

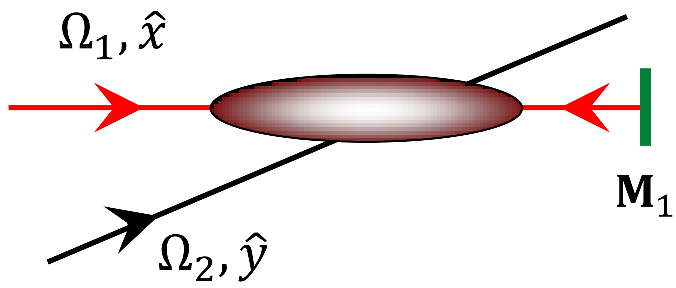
RF spectrum measurement of a 2D SOC band structure: J. Zhang group: L. Huang, et.al., Nature Phys. 12, 540 (2016).



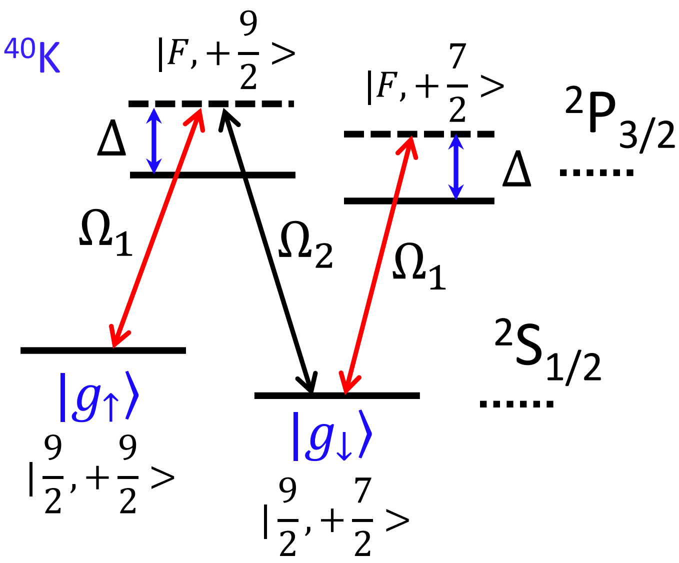
Our study: we have considered to realize 2D SOC and topological band for a degenerate gas in an optical lattice.

Optical Raman lattice: 1D spin-orbit coupling

1D model for spin-1/2 atoms



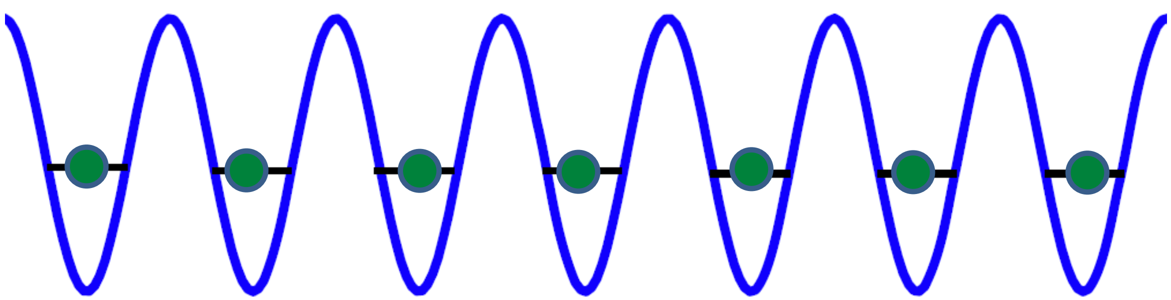
candidate: ^{40}K



Generated 1D lattice and Raman coupling potential:

$M(x) \propto \Omega_2^* \Omega_1$

$V_{\mathrm{latt}}(x) \propto |\Omega_1|^2$



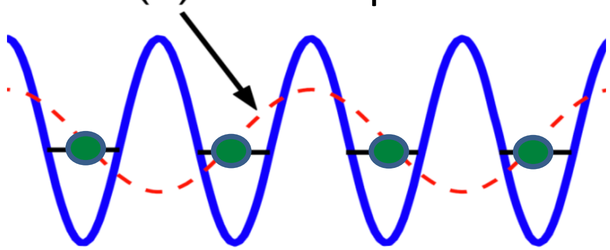
Key features:

- $M(x)$: anti-symmetric with respect to each lattice-site.
- $M(x)$ has half periodicity relative to the lattice.

XJL, Z.-X. Liu, M. Cheng, PRL, 110, 076401 (2013)

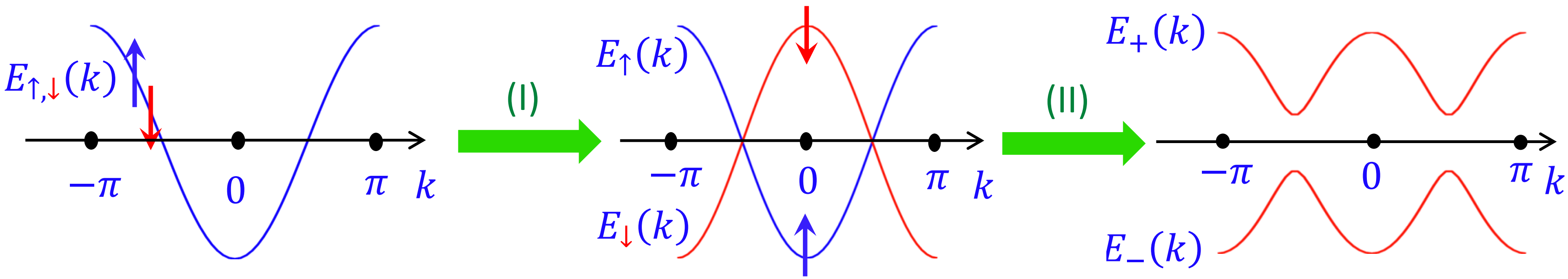
Band structure due to the Raman-lattice configuration

The realized Hamiltonian: $H = \frac{p_x^2}{2m} + V_0 \cos^2 k_0 x + M_0 \cos k_0 x\, \sigma_x + \frac{\delta}{2}\sigma_z$



Effects of the Raman coupling:

(I) $\frac{\pi}{a}$ momentum transfer; (II) SO Coupling.

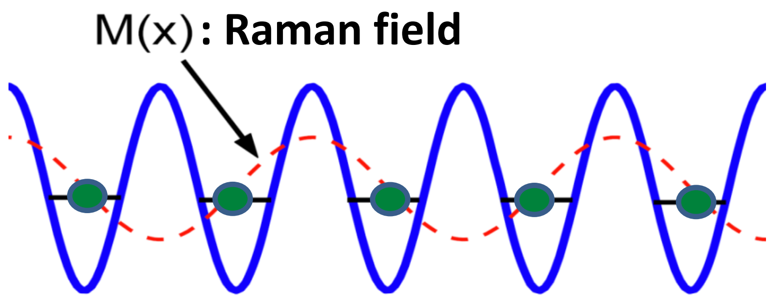


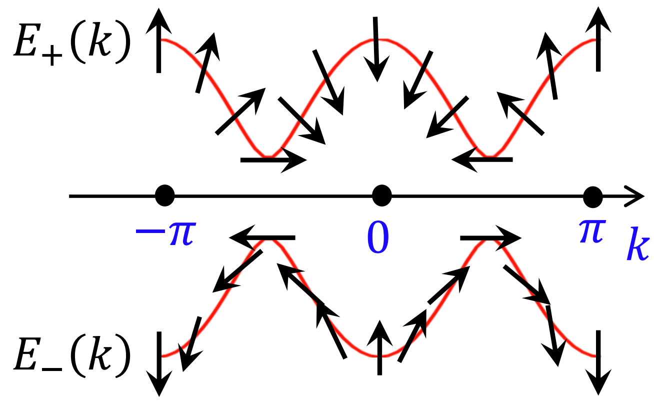
Tight-binding model with spin-orbit coupled hopping ($\Gamma_z = \frac{\delta}{2}$):

$$H = -t_s \sum_{<i,j>} (\hat{c}^\dagger_{i\uparrow}\hat{c}_{j\uparrow} - \hat{c}^\dagger_{i\downarrow}\hat{c}_{j\downarrow}) + \sum_i \Gamma_z (\hat{n}_{i\uparrow} - \hat{n}_{i\downarrow})$$
$$+\big[\sum_j t^{(0)}_{\rm so}(\hat{c}^\dagger_{j\uparrow}\hat{c}_{j+1\downarrow} - \hat{c}^\dagger_{j\uparrow}\hat{c}_{j-1\downarrow}) + {\rm H.c.}\big].$$

XJL, Z.-X. Liu, M. Cheng, PRL, 110, 076401 (2013).

Symmetry protected topological state: AIII class and Z invariant (chiral symmetry)





Topology: classified by integer winding numbers: Z

Discussions

1) Fractional charge, 1/4-spin states; topological classification: $Z \to Z_4$ (with interaction);
XJL, Z.-X. Liu, M. Cheng, PRL, 110, 076401 (2013).

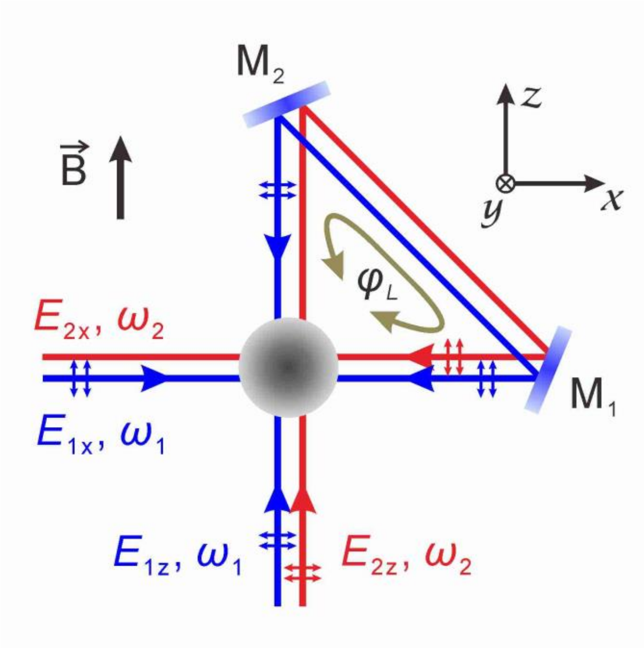
2) BDI class topological superconductivity/superfludity with s-wave pairing;
He, Wu, Choy, XJL, Tanaka, Law, Nat. Comm. 5, 3232 (2014).

3) Topological superradiant phase by putting in the cavity;
Pan, XJL, Zhang, Yi, and Guo, PRL 115, 045303 (2015).

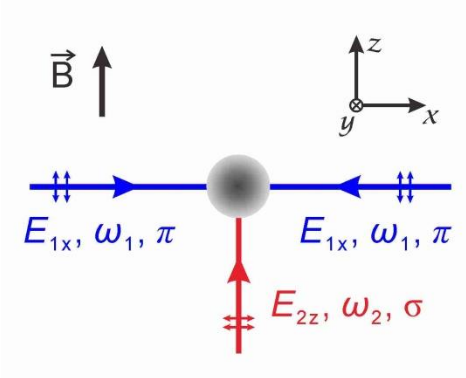
4) Hidden nonsymmorphic symmetry and band degeneracy;
H. Chen, XJL, and X. C. Xie, PRA 93, 053610 (2016).

2D SOC: Experimental scheme

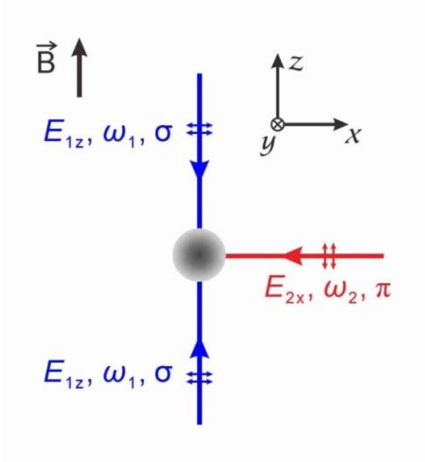
candidate: ^{87}Rb bosons



1) Raman coupling (I)



2) Raman coupling (II)



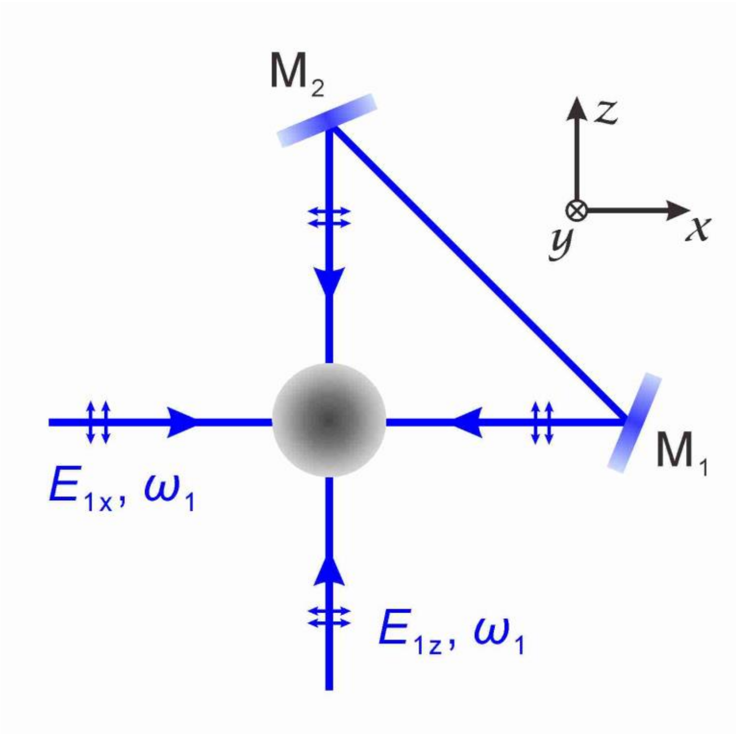
The phase of the lasers go through the loop:

Light ω_1 $\qquad \varphi_L = k_0 L \qquad \delta\varphi_L = \frac{\delta\omega}{c} L$

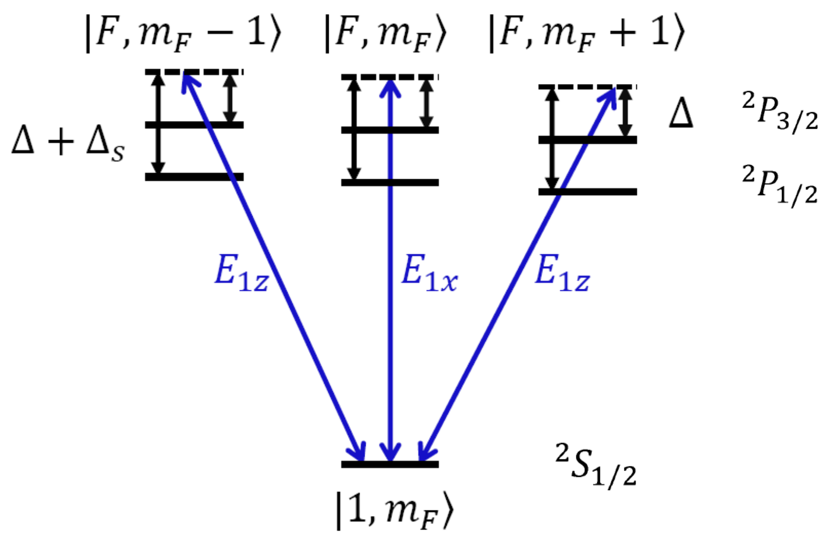
Light ω_2 $\qquad \varphi_L + \delta\varphi_L = k_0 L + \frac{\delta\omega}{c} L \qquad \delta\omega = \omega_2 - \omega_1$

Proposal: *XJL etal @PKU.* Experiment: *S. Chen & J.-W. Pan etal @USTC*

1. Generation of 2D blue-detuned square lattice



Polarization: in the x-z plane,
No interference between x and z direction



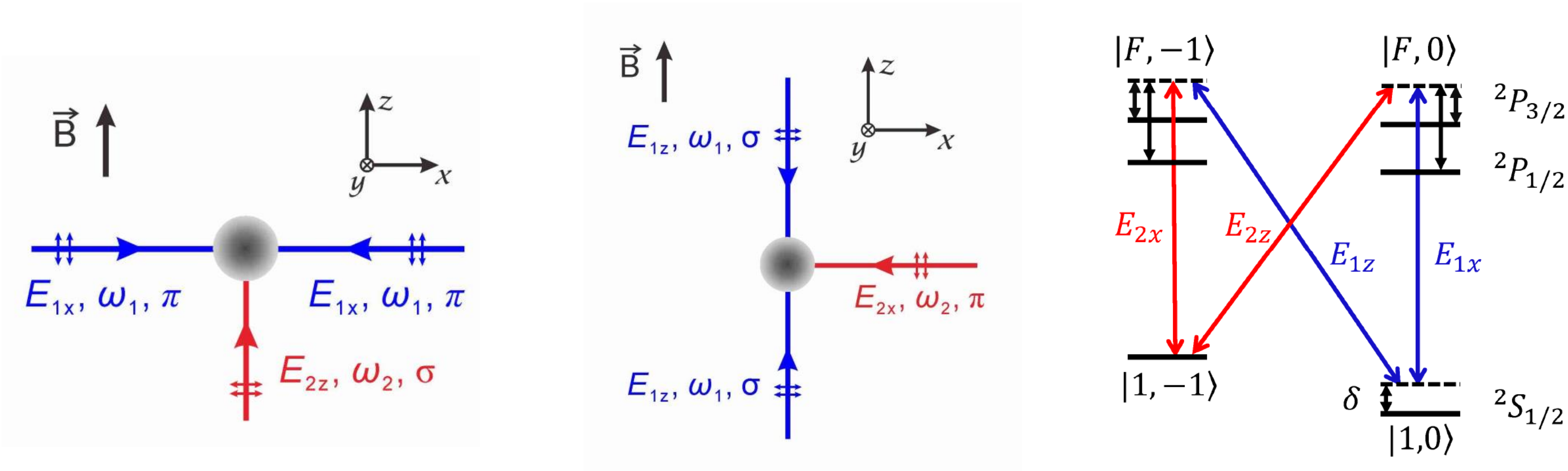
Optical lattice potential:

$$V_{Latt}(x,y) = V_{0x}\cos^2(k_0 x + \frac{\varphi_{1x} - \varphi_{1z} - \varphi_L}{2}) + V_{0z}\cos^2(k_0 z + \frac{\varphi_{1z} - \varphi_{1x} - \varphi_L}{2})$$

which is spin-independent:

$$V_{0x,0z} = \frac{3\Delta + 2\Delta_s}{3\Delta(\Delta + \Delta_s)}\alpha_{D_1}^2 |E_{1x,1z}^{(0)}|^2$$

2. Generation of two Raman coupling potentials:



E_{1x} and E_{2z} generate one Raman coupling

$$M_1 = M_{0x}\cos(k_0 x + \frac{\varphi_{1x} - \varphi_{1z} - \varphi_L}{2})e^{i(k_0 z + \frac{\varphi_{1z} - \varphi_{1x} - \varphi_L}{2}) + i(\varphi_2 - \varphi_{1z})}$$

E_{1z} and E_{2x} generate another Raman coupling

$$M_2 = M_{0y}\cos(k_0 z + \frac{\varphi_{1z} - \varphi_{1x} - \varphi_L}{2})e^{-i(k_0 x + \frac{\varphi_{1x} - \varphi_{1z} - \varphi_L}{2}) + i(\varphi_2 - \varphi_{1x}) + i\delta\varphi_L}$$

The realized effective Hamiltonian

The effective Hamiltonian can be write as ($m_z = \delta/2$: two-photon detuning):

$$H = \frac{p^2}{2m} + V_{\mathrm{latt}}(x, z) + m_z\sigma_z + (M_x - M_y \cos\delta\varphi_L)\sigma_x + M_y \sin\delta\varphi_L \sigma_y$$

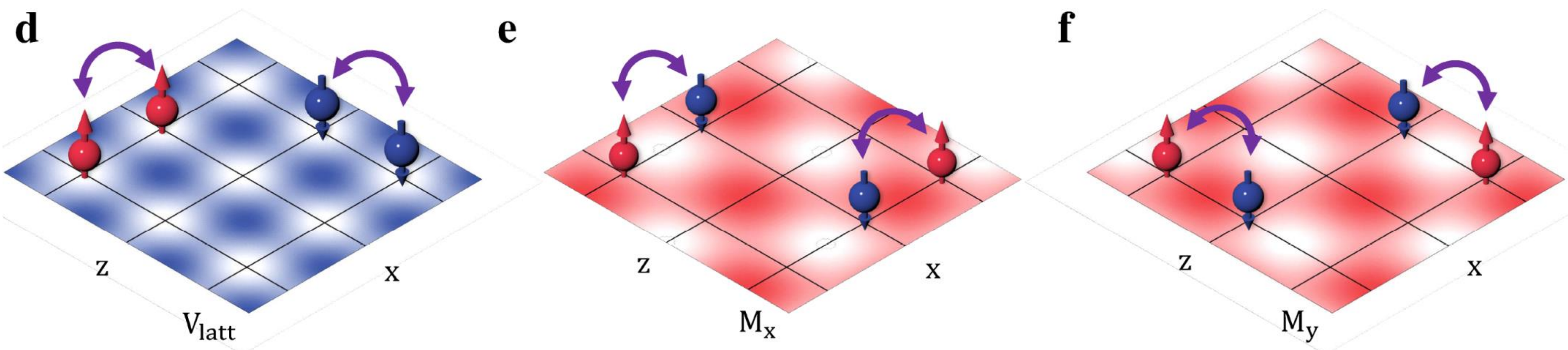
The Raman coupling potentials:

$$M_x = M_0 \cos k_0 x \sin k_0 z \qquad\qquad M_y = M_0 \cos k_0 z \sin k_0 x$$

Spin-conserved hopping by optical lattice

Spin-flipped hopping along x direction

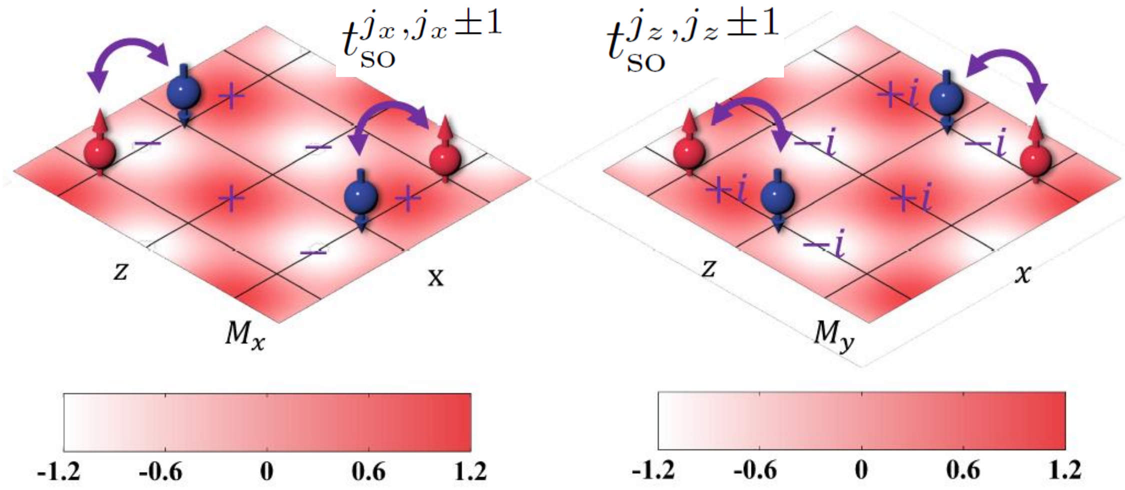
Spin-flipped hopping along y direction



A controllable crossover between 2D-1D SO coupling:

$\delta\varphi_L = \pi/2$ 2D coupling → $\delta\varphi_L = \pi$ 1D coupling

Topological physics of s-band ($\delta\varphi_L = \pi/2$)



Spin-flip hopping:

$$t_{\rm SO}^{j_x,j_x\pm1} = \pm(-1)^{j_x+j_y} t_{\rm SO}^{(0)}$$

$$t_{\rm SO}^{j_y,j_y\pm1} = \pm i(-1)^{j_x+j_y} t_{\rm SO}^{(0)}$$

The staggered factor $(-1)^{j_x+j_y}$ implies the relative (π,π) momentum transfer between spin-up and spin-down Bloch states.

The tight-binding Hamiltonian (after a gauge transformation to remove $(-1)^{j_x+j_y}$):

$$\begin{aligned} H_{\rm TI} \;=\; & -t_s \sum_{<\vec{i},\vec{j}>} (\hat{c}^\dagger_{\vec{i}\uparrow}\hat{c}_{\vec{j}\uparrow} - \hat{c}^\dagger_{\vec{i}\downarrow}\hat{c}_{\vec{j}\downarrow}) + \sum_{\vec{i}} m_z(\hat{n}_{\vec{i}\uparrow} - \hat{n}_{\vec{i}\downarrow}) + \\ & +\Big[\sum_{j_x} t_{\rm SO}^{(0)}(\hat{c}^\dagger_{j_x\uparrow}\hat{c}_{j_x+1\downarrow} - \hat{c}^\dagger_{j_x\uparrow}\hat{c}_{j_x-1\downarrow}) + {\rm H.c.}\Big] + \\ & +\Big[\sum_{j_y} i t_{\rm SO}^{(0)}(\hat{c}^\dagger_{j_y\uparrow}\hat{c}_{j_y+1\downarrow} - \hat{c}^\dagger_{j_y\uparrow}\hat{c}_{j_y-1\downarrow}) + {\rm H.c.}\Big]. \quad (2) \end{aligned}$$

XJL, K. T. Law, and T. K. Ng, PRL, 112, 086401 (2014); PRL, 113, 059901 (2014)

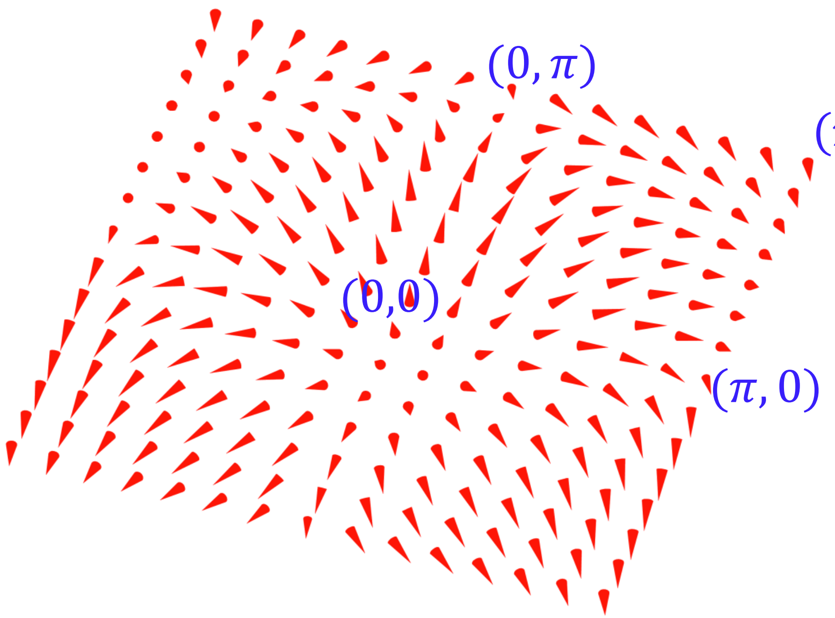
I. Non-interacting: Quantum anomalous Hall effect (s-band model)

$$H_{\rm TI} = \sum_{\mathbf{q}} [c^\dagger_\uparrow(\mathbf{q}), c^\dagger_\downarrow(\mathbf{q})]\mathcal{H}(\mathbf{q})[c_\uparrow(\mathbf{q}), c_\downarrow(\mathbf{q})]^T,$$

$$\mathcal{H}(\mathbf{q}) = [m_z - 2t_0(\cos q_x a + \cos q_y a)]\sigma_z + 2t_{\rm so}\sin q_x a\sigma_y + 2t_{\rm so}\sin q_y a\sigma_x$$

This is the minimal single-band SO coupled QAH model.

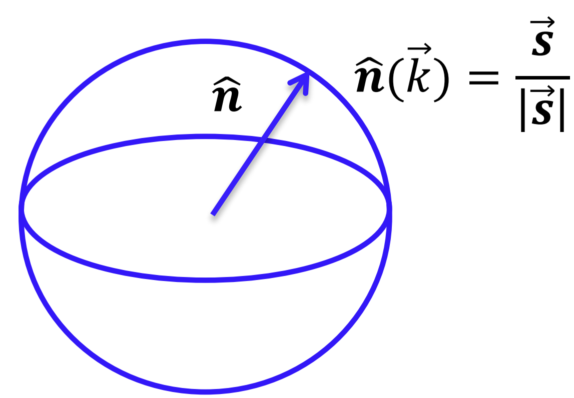
- 2D spin texture (magnetic skyrmion) in **k**-space:



Mapping: $\vec{k} \mapsto \hat{\boldsymbol{n}}$

- Chern number (Qi, Wu, Zhang, PRB 2006):

$$\mathrm{Ch}_1 = \begin{cases} \mathrm{sgn}(m_z), & \text{for } 0 < |m_z| < 4t_0, \\ 0, & \text{for } |m_z| \geq 4t_0, m_z = 0. \end{cases}$$

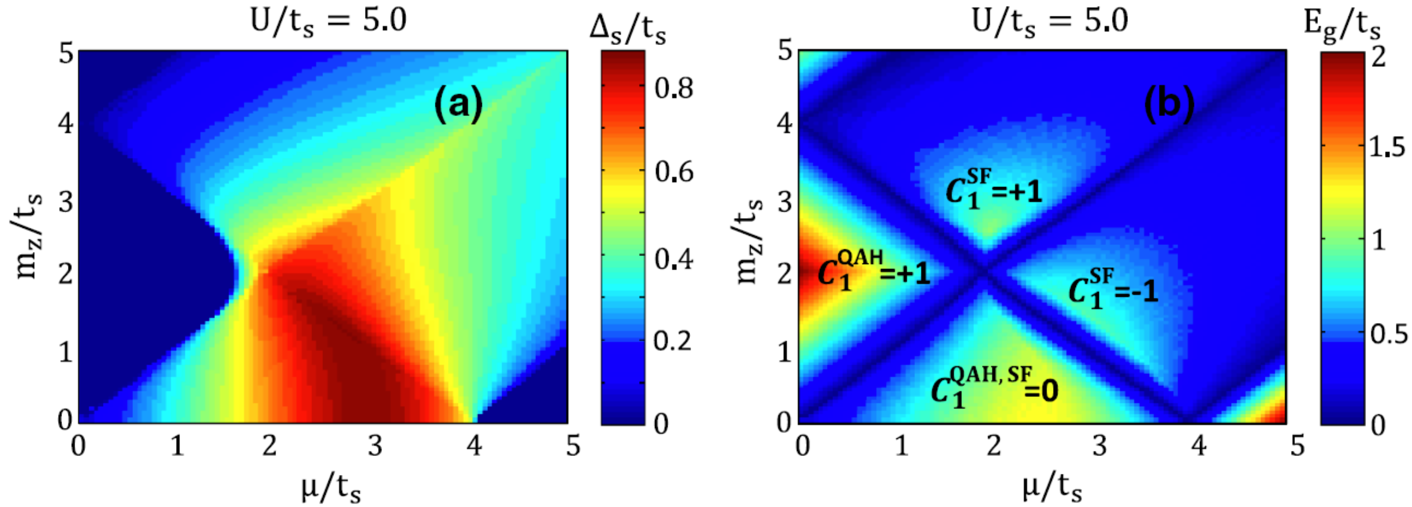


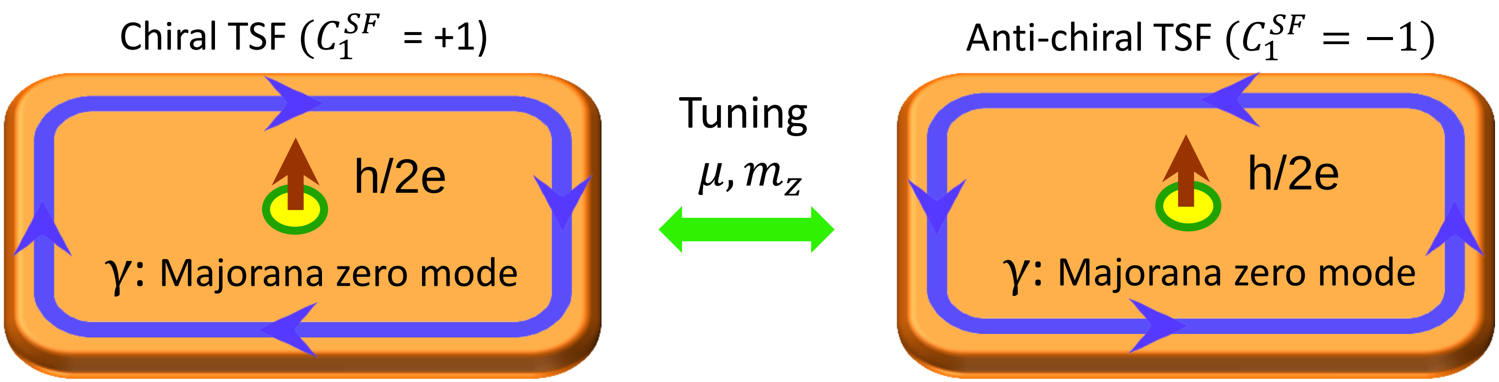
XJL, K. T. Law, and T. K. Ng, PRL, 112, 086401 (2014); PRL, 113, 059901 (2014)

II. Interacting regime: Chiral topological superfluids

Attractive Hubbard model: $$H = \sum_{\vec{k}} C^{\dagger}(\vec{k})\,\mathcal{H}_s(\vec{k})\,C(\vec{k}) - \sum_i U n_{i\uparrow} n_{i\downarrow}$$

Phase diagram:





- One Majorana zero bound state $\gamma(E=0)$ exists in each vortex core. Majorana bound modes obey non-Abelian statistics (Reed & Green, PRB, ’00; Ivanov, PRL, ’01; Alicea et al., Nat. Phys., ’11)

XJL, K. T. Law, and T. K. Ng, PRL, 2014.

Berezinsky-Kosterlitz-Thouless transition:

Phase fluctuation:

$$\Delta_s = \Delta_0 e^{i\theta(\mathbf{r})}$$

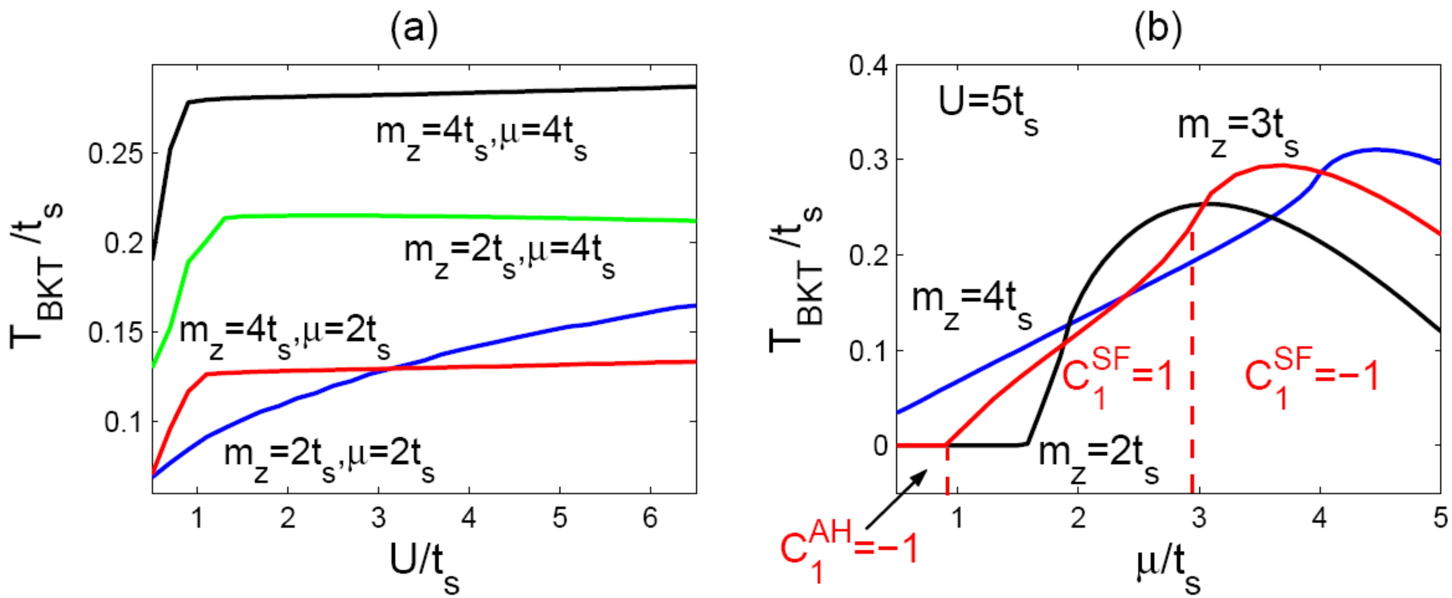
SF stiffness

Effective action: $S_{eff} = S_0(\Delta_0) + S_{fluc}(\Delta_0, \nabla\theta)$

To second-order expansion: $S_{fluc}(\Delta_0, \nabla\theta) = \mathbf{Tr}\sum_n \frac{1}{n}[\mathcal{G}_0(\Delta_0)\Sigma(\nabla\theta)]^n \approx \frac{1}{2}\int d^2r\rho_s(\nabla\theta)^2$

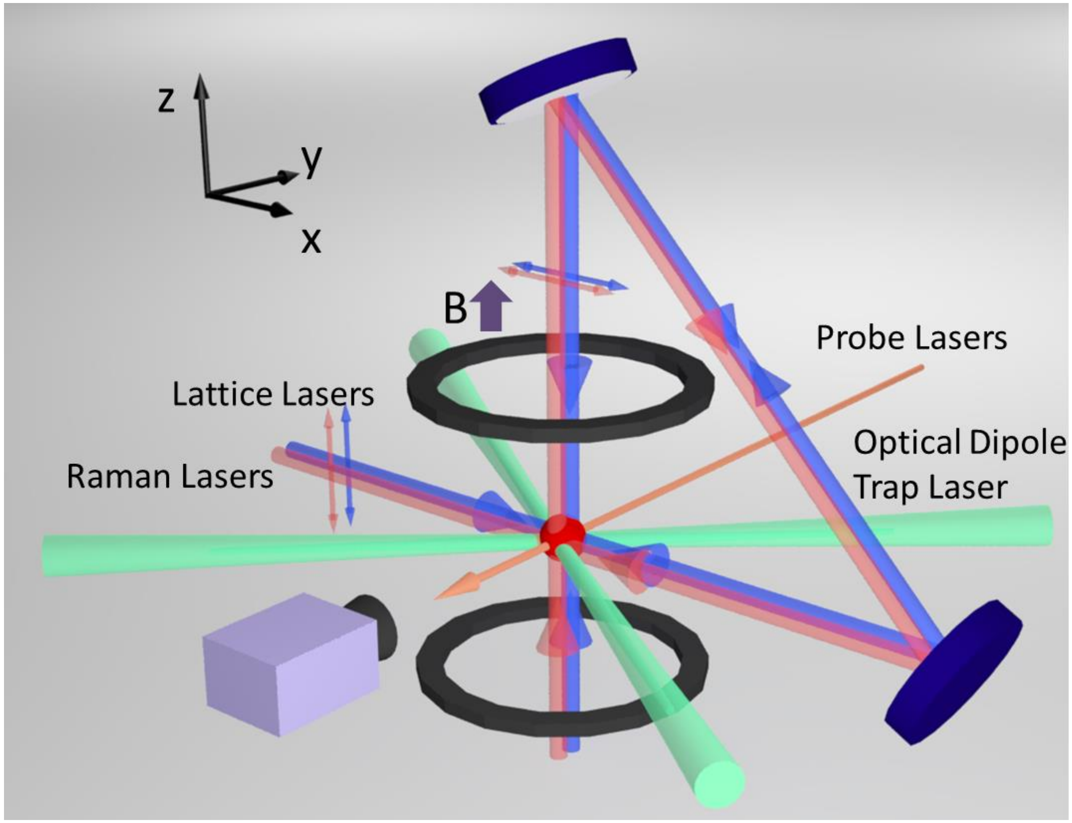
BKT temperature:

$$T_{\rm BKT} = \frac{\pi}{2}\rho_s(\Delta_s, T_{\rm BKT})$$



XJL, K. T. Law, and T. K. Ng, PRL, 112, 086401 (2014).

Experimental results



^{87}Rb condensate with
1.5×10^5 atoms
in optical dipole trap

Lattice and Raman coupling lasers are from the same fiber to make sure the spatial modes are exactly the same.

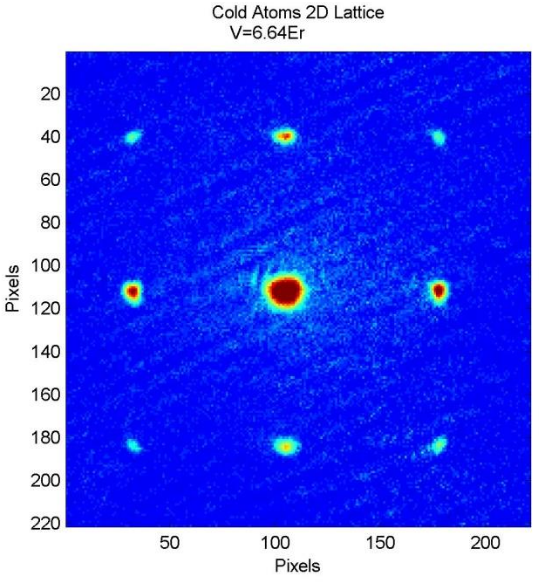
2D Lattice and Raman Laser Wavelength: 767nm
Frequency difference $\delta\omega=2\pi\times35$MHz,
Bias field: ~50 Gauss, 2-level system

Detection: TOF + Stern Gelach,
Spin and momentum-resolved absorption image

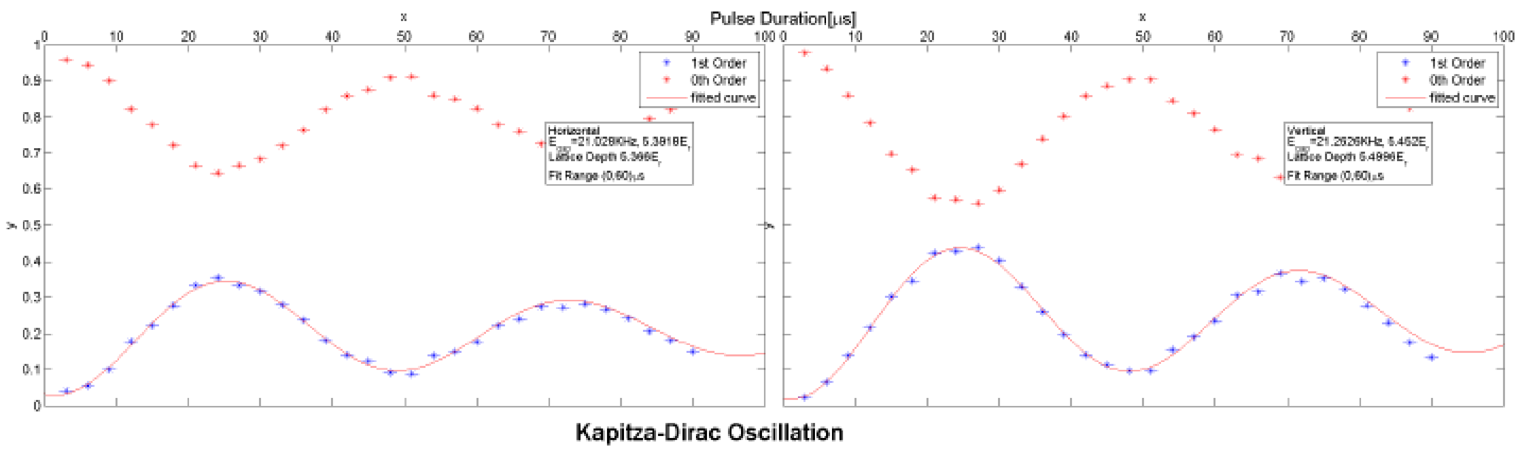
PKU+USTC: Z. Wu et al., Science, 354, 83-88 (2016).

Creation of the square Lattice

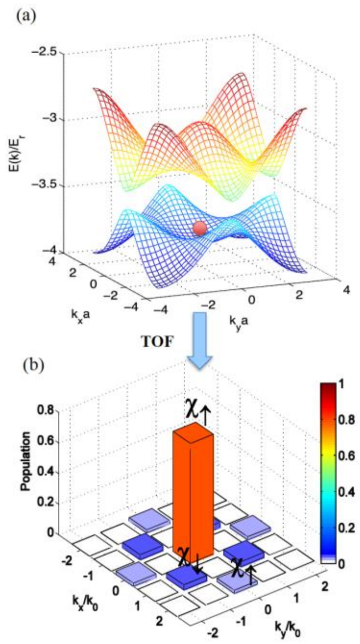
Observe the 2D optical lattice



Kapiza-Dirac diffraction



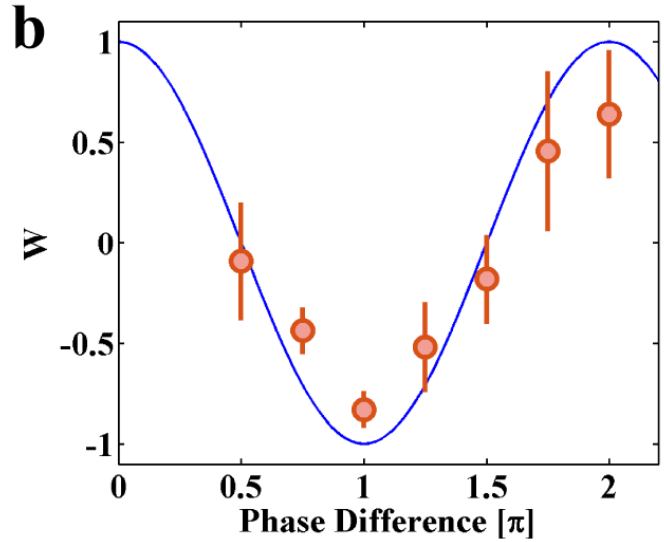
2D-1D SOC crossover

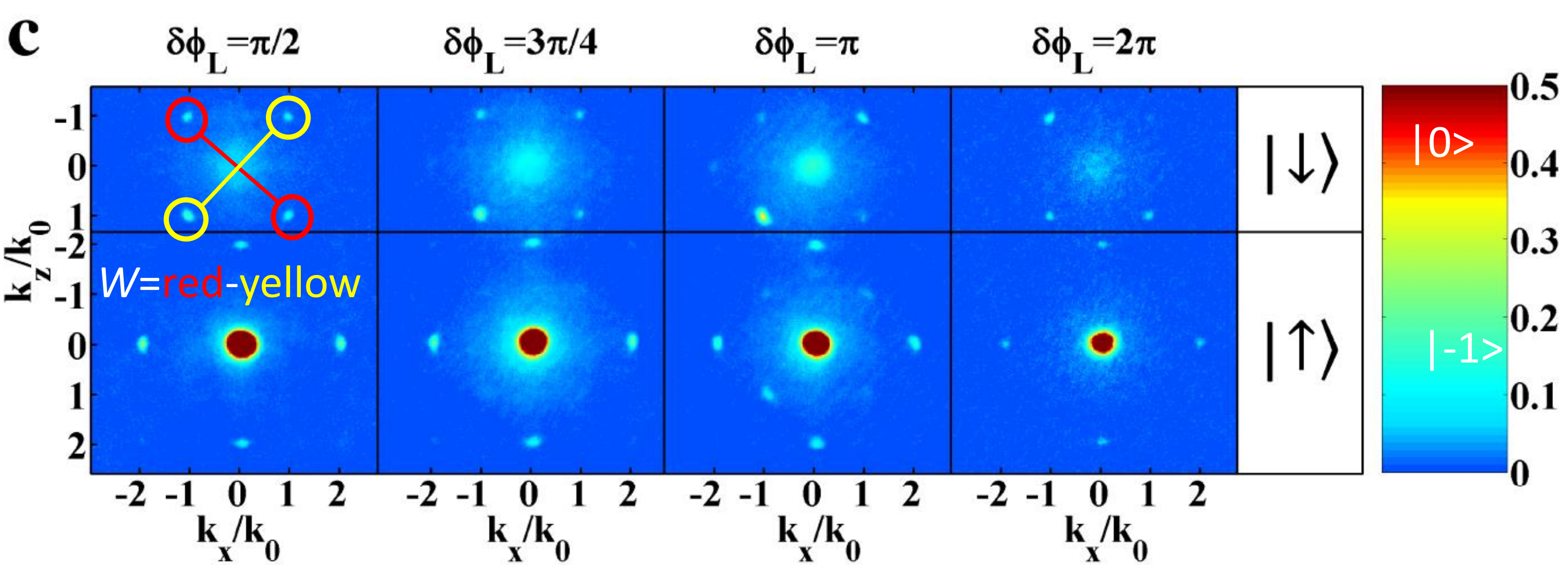


- Adiabatically ramp up the lattice and Raman coupling
- Probe: TOF + Stern Gerlach

The crossover between 2D and 1D SO coupling is observed.

W=red-yellow





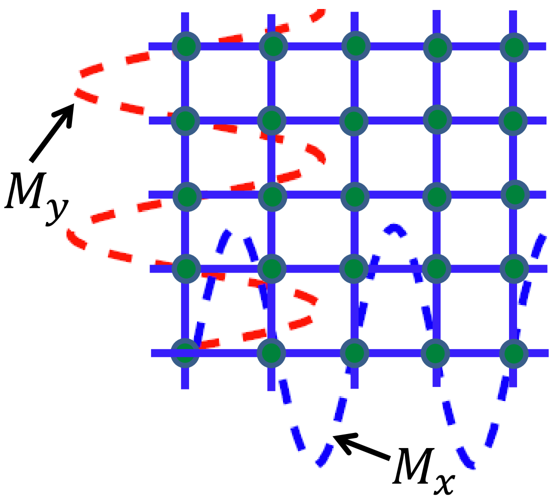
Lattice depth: $V_0 = 4.16E_\mathrm{r}$ Raman coupling: $M_0 = 1.32E_\mathrm{r}$

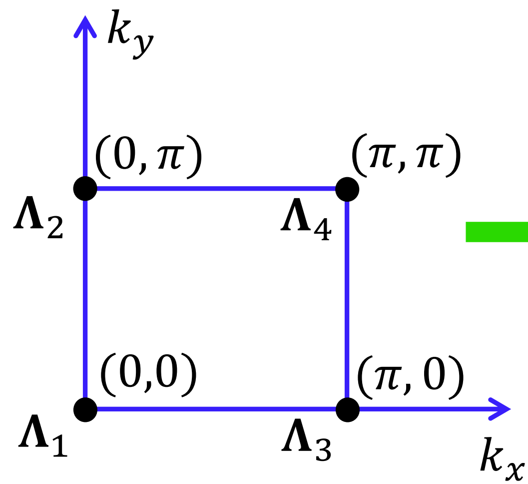
How to measure the band topology?

Inversion symmetric quantum anomalous Hall insulators: $PH(x,z)\,P^{-1} = H(x,z), \quad P = \sigma_z \otimes R_{2D}$.

XJL, K. T. Law, and T. K. Ng, and P. A. Lee, PRL, 111, 120402 (2013).

XJL, Liu, Law, W. V. Liu, and Ng, New J. Phys. 18, 035004 (2016).





σ_z: "parity operator"

Four parity eigenstates:

$$\sigma_z |u_\pm(\Lambda_i)\rangle = \xi^{(\pm)} |u_\pm(\Lambda_i)\rangle$$

eigenvalues: $\xi^{(\pm)} = \pm 1$

At inversion symmetric momenta: $\sigma_z \mathcal{H}(\Lambda_i)\sigma_z^{-1} = \mathcal{H}(\Lambda_i)$

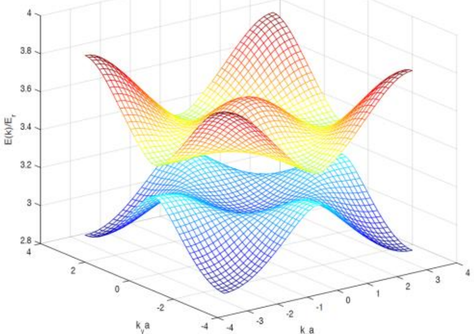
The topology is determined by:

$$\prod_i \xi^{(-)}(\Lambda_i) = \begin{cases} +1 & \text{trivial} \\ -1 & \text{topological} \end{cases}$$

Therefore, the topological phase can be detected by only measuring the spin polarization of Bloch states at four symmetric momenta.

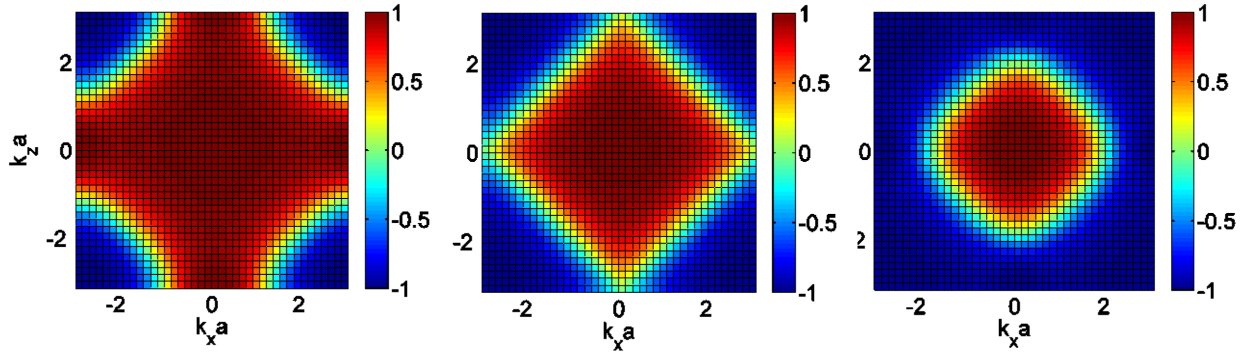
Spin texture with hot atoms

upper band



lower band

Spin polarization σ_z in the lowest band



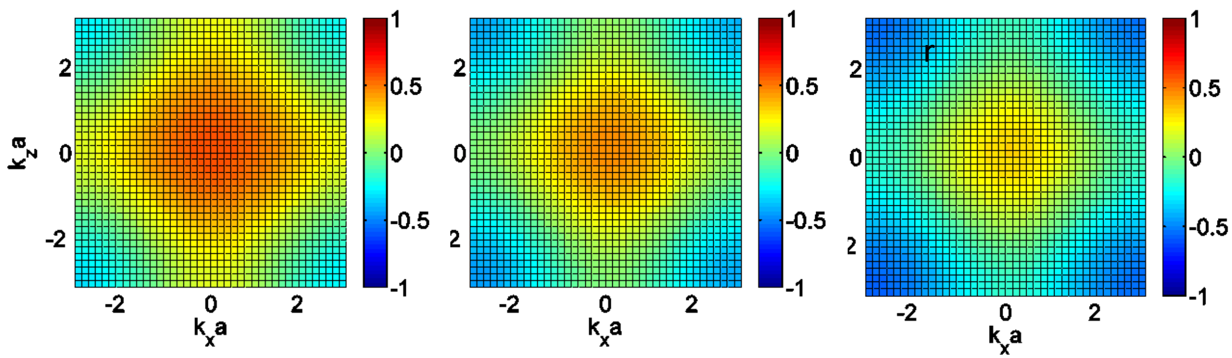
m_z=-0.1E_r m_z=0E_r m_z=0.1E

r

To fill the energy band with thermal atoms (for Bosons) with low temperature to see the feature of spin texture

both lower band and upper band are populated, the visibility of spin-polarization is decreased when T increases.

m_z=-0.1E_r m_z=0E_r m_z=0.1E



T=100nK

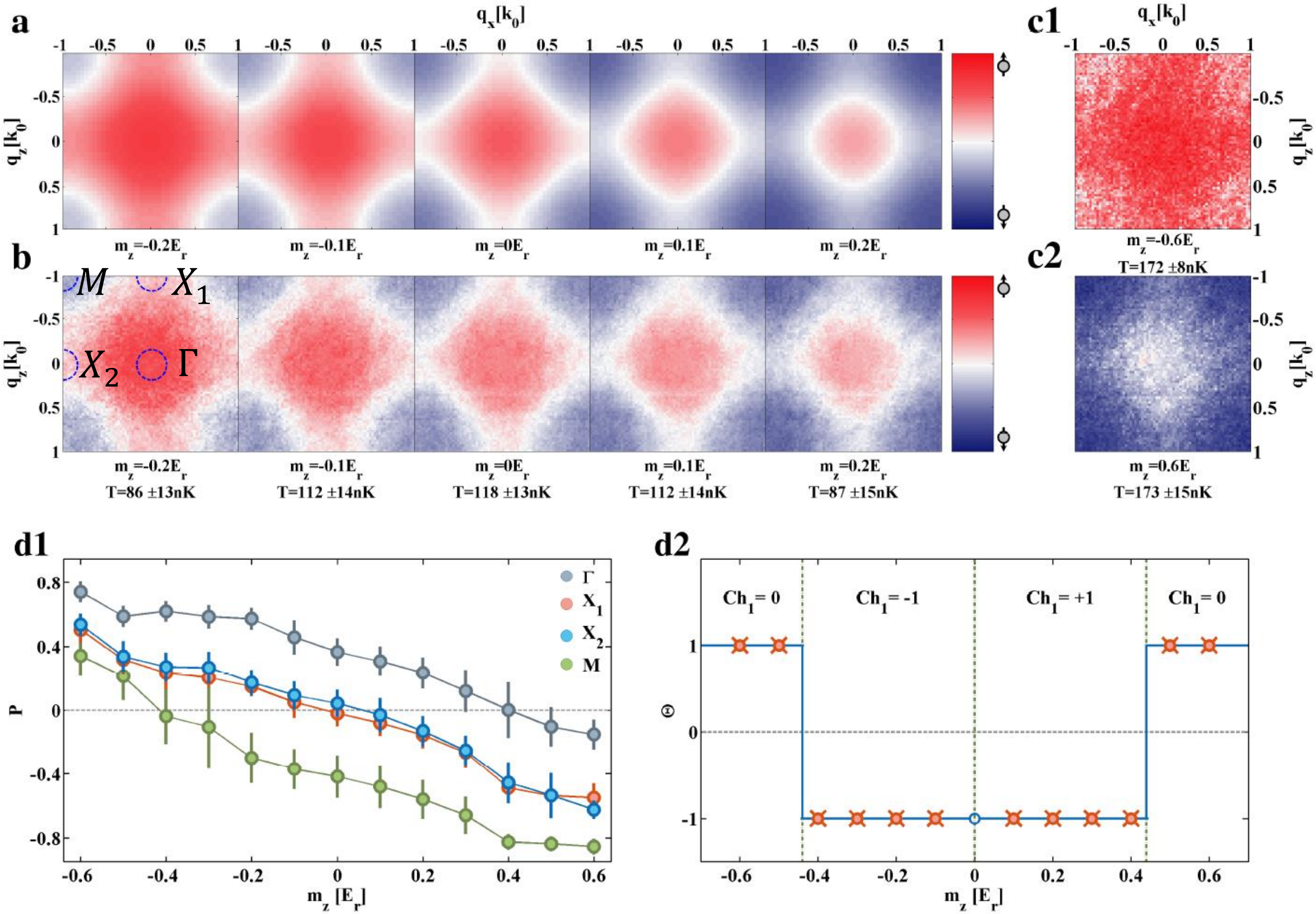
Spin texture and band topology

Spin texture measurement in FBZ

Theory

Experiment

Polarization at four symmetric momenta of the FBZ.



$$Ch_1 = \frac{\nu}{2}\sum_i sgn[\xi^{(-)}(\mathbf{\Lambda}_\mathrm{i})]$$

Z. Wu et al., Science, 354, 83-88 (2016).

Summary

- Proposed a minimal optical Raman lattice scheme to realize 2D SOC and topological bands.
- Successfully realize in experiment 2D SO coupling with 87Rb quantum degenerate atom gas. The SO coupling effects and topological bands are measured.

References:
XJL, Z.-X. Liu, M. Cheng, PRL, 110, 076401 (2013).
XJL, K. T. Law, and T. K. Ng, and P. A. Lee, PRL, 111, 120402 (2013).
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Next issues in theory and experiment:

- Realization of 2D SOC with fermions. Topological superfluids. Majorana zero modes.
- Generalized to higher dimensional systems
- Many-body and few body physics, quenching dynamics, high orbital bands, other lattice configurations.

Acknowledgement

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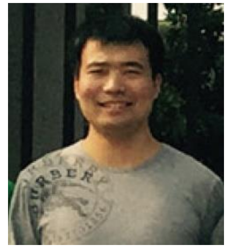

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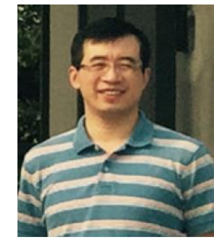

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