Understanding pending features of the KPZ class in discrete growth models

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Interface fluctuations and KPZ universality class, Kyoto, August 2014.

Brazil - Minas Gerais

Viçosa - Minas Gerais

Typical distances

- Belo Horizonte $\longrightarrow$ 230 Km
- Rio de Janeiro $\longrightarrow$ 350 Km
- Brasília $\longrightarrow$ 950 Km
- Amazon forest $\longrightarrow \approx$ 2000 - 3000 Km
- Kyoto $\longrightarrow$ 19000 Km

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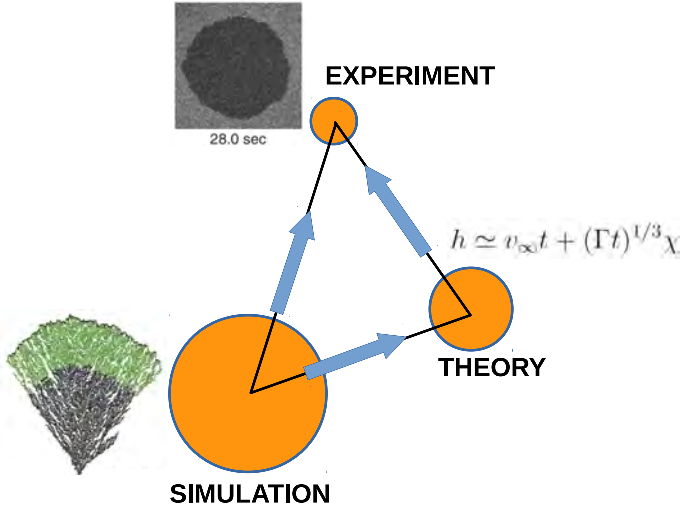
Outline

- Part I: Introduction and numerical recipes
- Part II: RSOS model in substrate dimensions $d \geq 3$
- Part III: Corrections to the scaling in ballistic growth models
- Part IV: KPZ models on enlarging flat substrates



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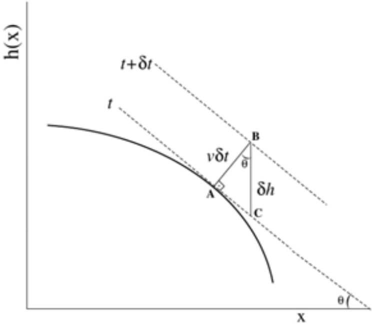
EXPERIMENT
$h \simeq v_\infty t + (\Gamma t)^{1/3} \chi$
THEORY
SIMULATION

Kardar-Parisi-Zhang (KPZ) equation

$$\frac{\partial h}{\partial t} = F + \nu \nabla^2 h + \frac{\lambda}{2}(\nabla h)^2 + \xi$$ [PRL **56**, 889 (1986)]

$$\langle \xi(x,t) \rangle = 0 \qquad \langle \xi(x,t)\xi(x',t') \rangle = D\delta(x-x')\delta(t-t')$$



- Lateral growth
- Excess velocity

$$\partial_t \langle h \rangle = F + \frac{\lambda}{2}\langle (\nabla h)^2 \rangle$$

Selected KPZ events

- Family-Viseck Ansatz [1985]

$$\langle h^2\rangle_c = t^{2\beta} f\left(\frac{L}{t^{1/z}}\right) \sim \begin{cases} t^{2\beta} & t \ll L^z \\ L^{2\alpha} & t \gg L^z \end{cases} \qquad z = \frac{\alpha}{\beta}$$

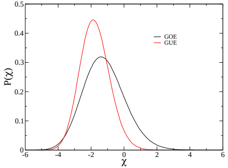
- KPZ equation [1986]

$$\partial_t h = F + \nu \nabla^2 h + \frac{\lambda}{2}(\nabla h)^2 + \xi$$

- KPZ ansatz [Krug, Meakin, Halpin-Healy, late 80's/early 90's]

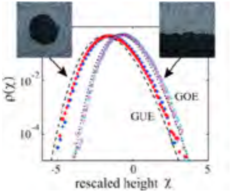
$$h = v_\infty t + s_\lambda (\Gamma t)^\beta \chi$$

- Subclasses split [Prähofer, Spohn, Johansson early 2000's]



Selected KPZ historic events

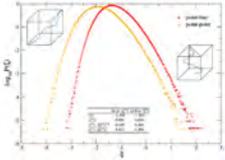
- Experimental realization [Takeuchi and Sano in early 2010's]



- KPZ equation solutions [Spohn, Sasamoto, Corwin, Calabrese, etc... in 2010's]

$$Z(x,t) = \exp\left[\frac{\lambda}{2\nu} h(x,t)\right]$$

- KPZ ansatz in $d = 2 + 1$ dimensions [Halpin-Healy, 2012]



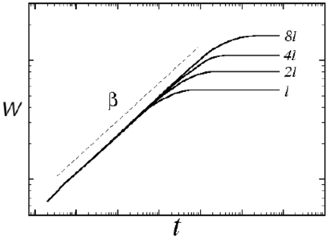
Family-Vicsek ansatz

Interface fluctuations in a scale L:

$$w^2(L,t) = \langle h^2 \rangle_c = \langle h^2 \rangle - \langle h \rangle^2$$

$$w = t^\beta f\left(\frac{L}{t^{1/z}}\right) \sim \begin{cases} t^\beta & t \ll L^z \\ L^\alpha & t \gg L^z \end{cases}$$

$$z = \frac{\alpha}{\beta}$$



Family and Vicsek, JPA **18**, L75 (1985)

KPZ exponents ($d = 1 + 1$) : $\alpha = 1/2$, $\beta = 1/3$, $z = 3/2$

$$\alpha + z = 2$$

[KPZ, PRL **56**, 889 (1986)]

Non-universal correction in the KPZ ansatz

$$h = v_\infty t + (\Gamma t)^\beta \chi_{TW} + \eta + \cdots \Longrightarrow q = \frac{h - v_\infty t}{(\Gamma t)^\beta} = \chi_{TW} + ct^{-\beta} + \cdots$$

- Experiments in turbulent crystal liquids

  Takeuchi, Sano PRL **104**, 230601 (2010); JSP **147** 853 (2012)

- Explicit solutions of 1+1 KPZ Eq.

  Sasamoto and Spohn PRL 104 (23), 230602 (2010)

- Solvable models in $d = 1 + 1$ (PNG, PASEP, etc..)

  Ferrari, Frings JSP **144** (6), 1123 (2011).

- Simulations in $d = 1 + 1$ (Ballistic deposition, Eden, RSOS, etc...)

  Alves, Oliveira, Ferreira JSTAT P05007 (2013);
  EPL **96** 48003 (2011); PRE **85** 010601(R) (2012);

- Simulations in $d = 2 + 1$ dimensions

  Alves, Oliveira, Ferreira PRE **87** 040102(R) (2013)

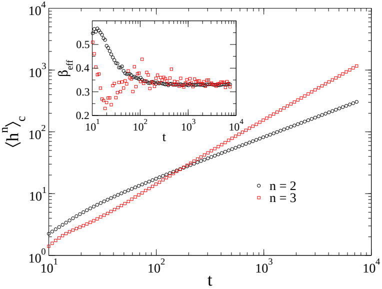
Determination of scaling exponents

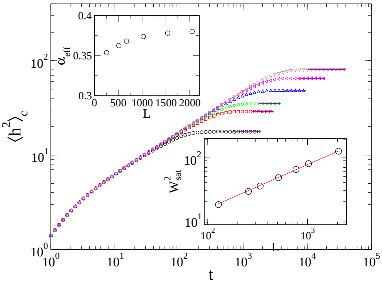
$$L \to \infty \quad \langle h^n \rangle_c \simeq t^{n\beta}$$

$$t \to \infty \quad \langle h^n \rangle_c \simeq L^{n\alpha}$$

$$\beta_{eff} = \frac{1}{n}\frac{d\ln\langle h^n \rangle_c}{d\ln t} \quad n \geq 2$$

$$\alpha_{eff} = \frac{1}{n}\frac{d\ln\langle h^n \rangle_c}{d\ln L} \quad n \geq 2$$

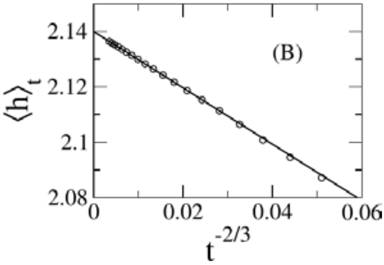




Determination of non-universal parameters

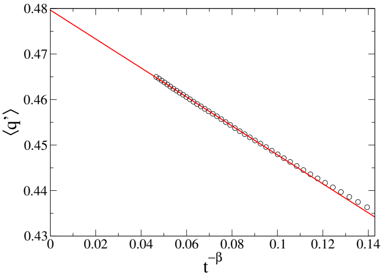
Asymptotic velocity

$$\partial_t \langle h \rangle \simeq v_\infty + \Gamma^\beta \langle \chi_{TW} \rangle t^{\beta-1}$$



Mean shift

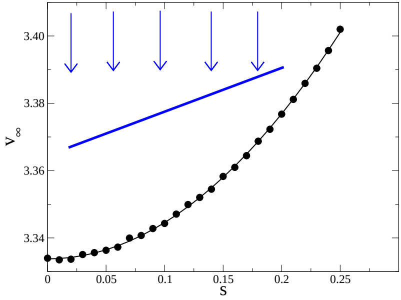
$$q' = \frac{h - v_\infty t}{t^\beta} \simeq \Gamma^\beta \chi_{TW} + \eta t^{-\beta}$$



Amplitude of fluctuations

Coefficient λ

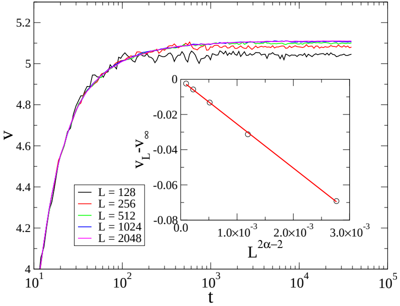
$$v_\infty(s) = v_\infty(0) + \frac{\lambda}{2}s^2$$



Krug-Spohn, PRL **64** 2332 (1989)

Parameter Γ

$$v(L) - v_\infty \simeq -\frac{A\lambda}{2}L^{2\alpha-2} \qquad \Gamma = \lambda A^{1/\alpha}$$



Krug-Meakin, JPA **23**, L987 (1990)

Part II

RSOS model in high dimensional substrates

with Sidiney G. Alves and Tiago J. Oliveira (Univ. Fed. Viçosa)

ArXiv:1405.0974 to appear in PRE Rapid Communication

KPZ class in higher dimensions

Simulations

d	α	β	z
2	0.395(5)	0.245(5)	1.58(10)
3	0.29(1)	0.184(5)	1.60(10)
4	0.245(5)	0.15(1)	1.91(10)
5	0.22(1)	0.115(5)	1.95(15)

Ódor *et al.* PRE **81**, 031112 (2011)

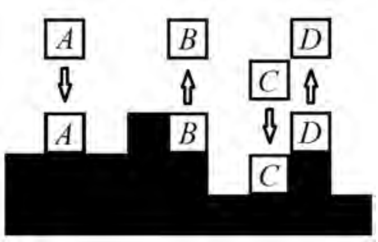
Central theoretical issue:

Upper critical dimensions d_u ($\alpha = \beta = 0$ and $z = 2$):

- Mode-coupling theory and field theoretical approaches $2.8 < d_u \leq 4$
- Renormalization group and simulations $d_u > 4$
- Some works suggest $d_u = \infty$

Concise review: Pagnani and Parisi PRE **87** 010102(R) (2013)

The restricted solid-on-solid (RSOS) model



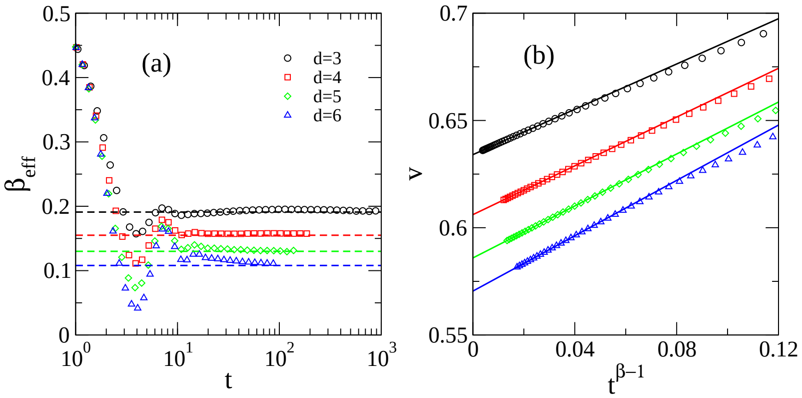
Depositions producing NN height differences $|\Delta h| > m$ are rejected.

Kim and Kosterlitz PRL **72** 2289 (1989).

Some recentest advances:

- Very precise simulations "prove" that $d_u > 4$ [Pagnani and Parisi, PRE **87** 010102(R) (2013)]
- Restriction parameter $m > 1$ improves scaling in $d = 4 + 1$ [Kim and Kim PRE 88, 034102 (2013)]
- Scaling exponents using $m > 1$ support $d_u > 11$ [Kim and Kim JSTAT (2014) P07005]

Universal and non-universal quantities in $d = 3 - 6$



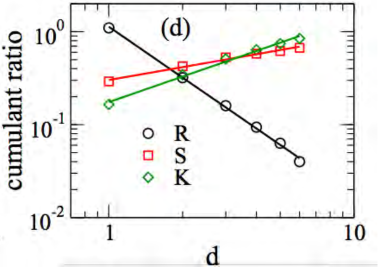
Alves, Oliveira, Ferreira ArXiv:1405.0974

Dimensionless cumulant ratios

$$R = \frac{\langle h^2 \rangle_c}{\langle h \rangle^2} \to \frac{\langle \chi^2 \rangle_c}{\langle \chi \rangle^2} \qquad S = \frac{\langle h^3 \rangle_c}{\langle h^2 \rangle_c^{3/2}} \to \frac{\langle \chi^3 \rangle_c}{\langle \chi^2 \rangle_c^{3/2}} \qquad K = \frac{\langle h^4 \rangle_c}{\langle h^2 \rangle_c} \to \frac{\langle \chi^4 \rangle_c}{\langle \chi^2 \rangle_c^2}$$

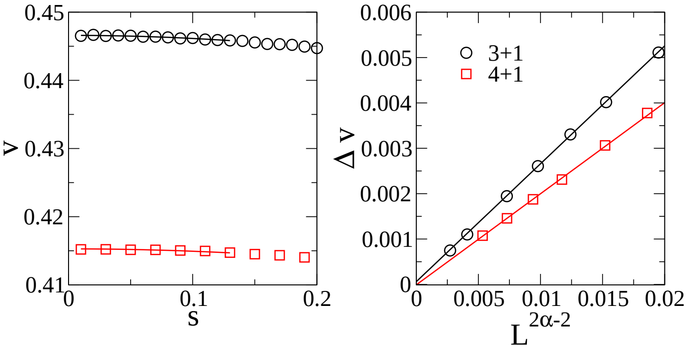
Model	β	v_∞	R	S	K
$d=3$					
RSOS ($m=2$)	0.189	0.44650	0.156	0.53	0.50
RSOS ($m=4$)	0.191	0.6340	0.163	0.53	0.52
$d=4$					
RSOS ($m=2$)	0.150	0.41518	0.093	0.57	0.63
RSOS ($m=4$)	0.155	0.6059	0.096	0.59	0.65
$d=5$					
RSOS ($m=2$)	0.13	0.39356	0.064	0.61	0.73
RSOS ($m=4$)	0.13	0.5858	0.063	0.63	0.76
$d=6$					
RSOS ($m=4$)	0.11	0.57055	0.042	0.66	0.83
RSOS ($m=8$)	0.10	0.7380	0.037	0.68	0.86

Different parameters m (different λ) yield the same cumulant ratios, which should be model-dependent for $d > d_u$.



Note: data extrapolated for $t \to \infty$

KPZ "machinery" in $d = 3 + 1$ *and* $d = 4 + 1$

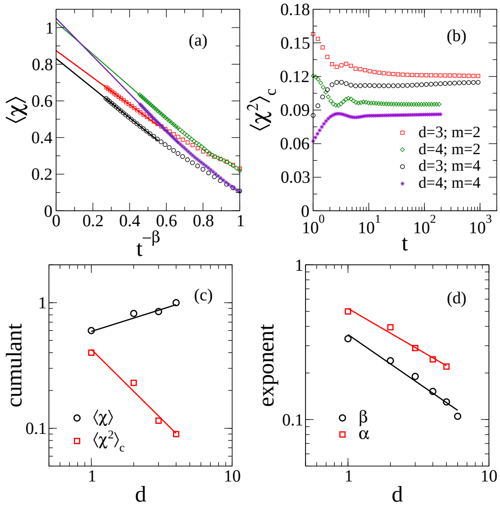


Universal and non-universal quantities

TABLE II. Estimates of nonuniversal parameters (A, λ, Γ) for the RSOS model in $d = 1$–4 dimensions. Height restriction parameters are shown in brackets. The estimates of the first and second cumulants of χ are shown in the last columns. Results for $d = 1$ were extracted from Ref. [8], where a factor of a different convention $\Gamma = |\lambda|A/2$ was used. Our results in $d = 1$ and 2 with $m = 1$ are in agreement with former reports [11,16].

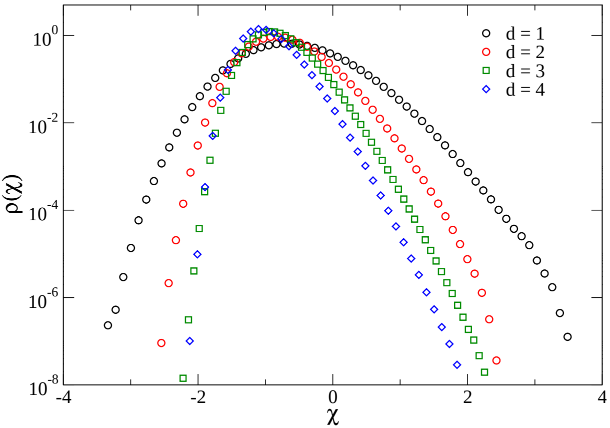
d [m]	A	λ	Γ	$\langle\chi\rangle$	$\langle\chi^2\rangle_c$
1 [1]	0.81	−0.77	0.51	−0.60	0.40
2 [1]	1.22(4)	−0.41(1)	0.68(6)	−0.83(2)	0.23(1)
2 [2]	4.5(1)	−0.121(3)	5.5(2)	−0.82(2)	0.23(1)
3 [2]	5.8(2)	−0.090(2)	38(3)	−0.86(2)	0.12(1)
3 [4]	19(2)	−0.024(2)	600(50)	−0.82(3)	0.11(1)
4 [2]	8(1)	−0.05(1)	240(50)	−1.00(4)	0.09(1)
4 [4]	25(2)	−0.015(2)	7600(900)	−0.98(5)	0.09(1)

Universal quantities



$$q = \frac{h - v_\infty t}{(\Gamma t)^\beta} = \chi + ct^{-\beta} + \cdots$$

Distributions



$$q = \frac{h - v_\infty t - \langle \eta \rangle}{(\Gamma t)^\beta}$$

Conclusions of Part II

- The theoretical machinery developed for the KPZ equation in $d = 1 + 1$ holds up to $d = 6$.
- Interface height distributions are universal for all investigated dimensions.
- Fluctuations are not negligible $\Longrightarrow d_u > 6$.
- Extrapolation for $d \geq 7$ supports $d_u = \infty$.

Part III

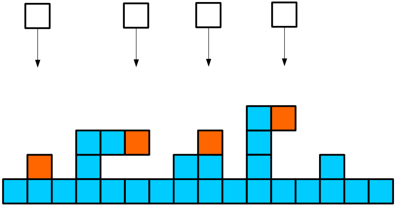
Scaling Corrections in ballistic growth models in $d = 2 + 1$

with Sidiney G. Alves and Tiago J. Oliveira (Univ. Fed. Viçosa)

ArXiv:XXXX.YYYY

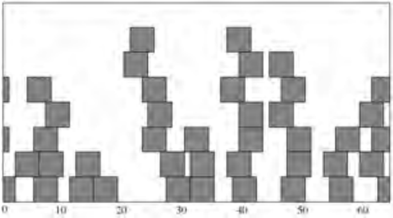
Models

Ballistic deposition



Vold, J. Colloid Sci. **14**, 168 (1959).

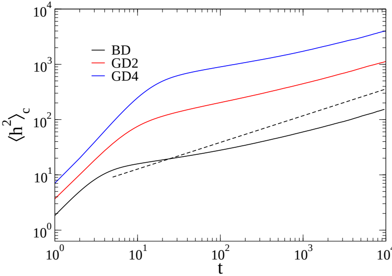
Grain deposition

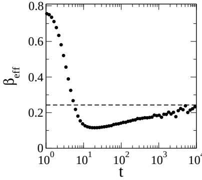


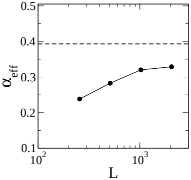
Oliveira and Reis, J. Appl. Phys. 101, 063507 (2007).

Motivation

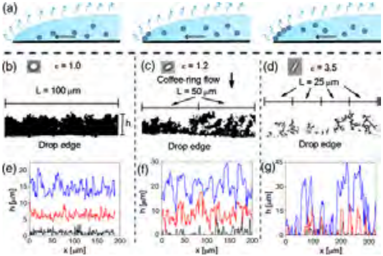
“Bad” scaling properties of DB models in high dimensions



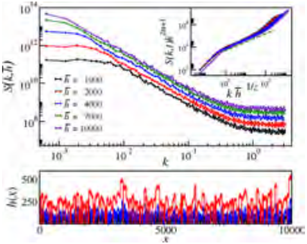




Scaling in experiments



Yunker et al., PRL **110**, 035501 (2013)



Nicoli et al., PRL **111**, 209601 (2013)

Intrinsic width

Heuristic formula:

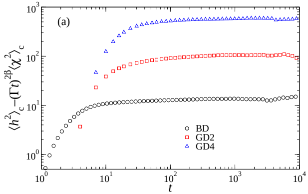
$$\langle h^2\rangle_c \simeq \underbrace{L^{2\alpha} f\left(\frac{t}{L^z}\right)}_{\text{long wavelength}} + \underbrace{w_i^2}_{\text{short wavelength}}$$

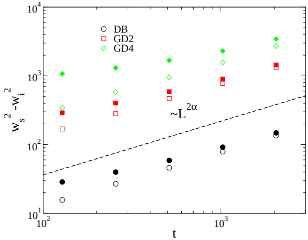
Kertész and Wolf, JPA **21** 747 (1988).

KPZ ansatz:

$$w_i^2 = \lim_{t\to\infty}\left[\langle h^2\rangle_c - (\Gamma t)^{2\beta}\langle\chi^2\rangle_c\right].$$

$w_i = 3.6(2)$ (BD) , 10.0(5) (GD2) and 26(2) (GD4).





$$\langle h^2\rangle_c = (\Gamma t)^{2\beta}\langle\chi^2\rangle_c + 2(\Gamma t)^{\beta}\text{cov}(\chi,\eta) + \langle\eta^2\rangle_c + \cdots$$

$$w_i^2 = \text{const.} \Longrightarrow \text{cov}(\chi,\eta) \approx 0$$

Height increment fluctuations

Let $\delta h = h(\mathbf{x}, t+\delta t) - h(\mathbf{x}, t)$ is the height increment in a time step.

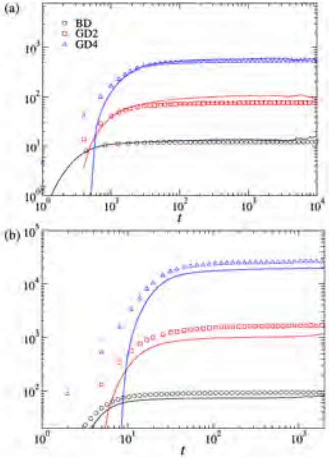
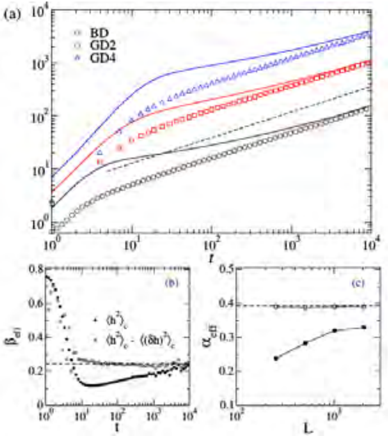
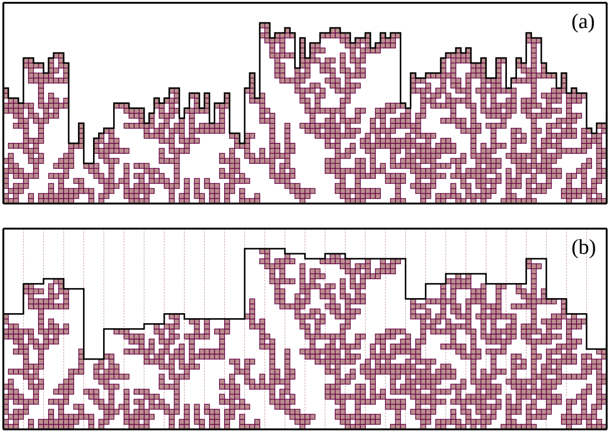


FIG. 4: (a) Second cumulant of δh (symbols) and $\langle h^2\rangle_c - (\Gamma t)^{2\beta}\langle\chi^2\rangle_c$ (lines) against time. (b) Third cumulant of δh (symbols) and $\langle h^3\rangle_c - (\Gamma t)^{3\beta}\langle\chi^3\rangle_c$ (lines) against time.



Binning method for ballistic growth

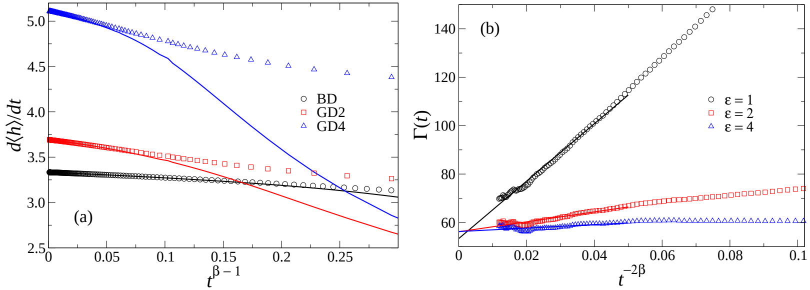


Method: *The surface is binned using boxes of size ε where only the highest site inside a bin is used to reconstruct teh surface.*

Non-universal parameters

According to the KPZ ansatz, Γ can also be obtained using

$$\Gamma(t) \simeq \left[\frac{\langle h^2\rangle_c}{t^{2\beta}\langle\chi^2\rangle_c}\right]^{1/2\beta} = \Gamma(\infty) + ct^{-2\beta} + \cdots., \qquad \langle\chi^2\rangle_c = 0.235^{a)}$$



[a)] Halpin-Healy, PRL (2012)

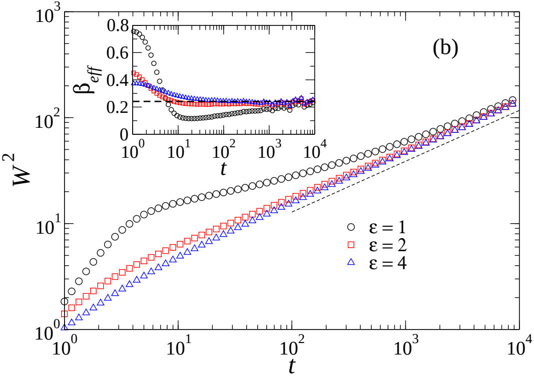
Conclusion: Binning method does not change the non-universal parameters.

Non-universal parameter for ballistic models in $d = 2 + 1$

Model	v_∞	λ	Γ
BD	3.33396(3)	2.15(10)	57(7)
GD2	3.6925(1)	0.35(3)	$3.5(3)\times 10^3$
GD4	5.1124(1)	0.76(3)	$4.3(7)\times 10^4$

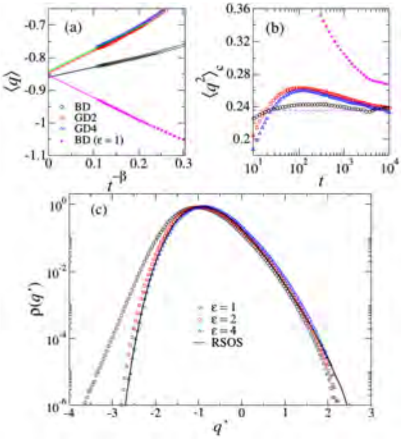
Table: Non-universal parameters for ballistic models using Krug-Meakin analysis [JPA **23**, L987 (1990)] for binning windows of size $\epsilon = 4$.

Scaling exponents with binning



Intrinsic width for BD:
$w_i = 3.6(2)$ ($\varepsilon = 1$); $w_i = 1.5(2)$ ($\varepsilon = 2$); and 0.8(2) ($\varepsilon = 4$)

Binning method to measure χ



model	β	α	$\langle\chi\rangle$	$\langle\chi^2\rangle_c$	S	K
BD	0.239(15)	0.389(3)	0.86(2)	0.235(15)	0.41(2)	0.31(3)
GD2	0.225(15)	0.375(5)	0.85(2)	0.24(2)	0.43(3)	0.32(3)
GD4	0.237(18)	0.375(15)	0.84(3)	0.24(2)	0.44(3)	0.35(5)

TABLE II: Universal quantities determined for ballistic models either discounting the intrinsic width (β and α) or using surfaces constructed with $\varepsilon = 2l$ (the other quantities). Uncertainties in cumulants and cumulant ratios were obtained propagating the uncertainties in the non-universal parameters v_∞ and Γ given in Table I.

In agreement with the recent characterization of the KPZ ansatz in d=2+1 [Halpin-Healy, PRL 2012, PRE 2013; OAF PRE 2013].

$$q' = \frac{h - v_\infty t - \langle\eta\rangle}{(\Gamma t)^\beta}$$

Conclusions of Part III

- Strong corrections to the scaling of ballistic growth models are strongly related to the fluctuations of height increments, $\langle (\delta h)^2 \rangle$
- Discounting $\langle (\delta h)^2 \rangle$ from $\langle h^2 \rangle_c$, an excellent agreement with the KPZ exponents are found.
- Binning method is able to reduce corrections to the scaling allowing to determined the universal properties of the height fluctuations.
- The method can be applied to unveil universality in experimental systems with large steps in surface.

Part IV

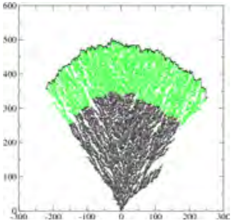
Interface fluctuations in the deposition on enlarging flat substrates

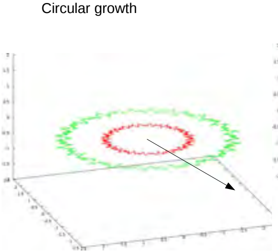
with Ismael. S. S. Carrasco, Tiago J. Oliveira (UFV), Kazumasa A. Takeuchi (Univ. of Tokyo)

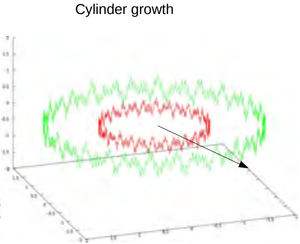
ArXiv:XXXX.YYYY

Motivation

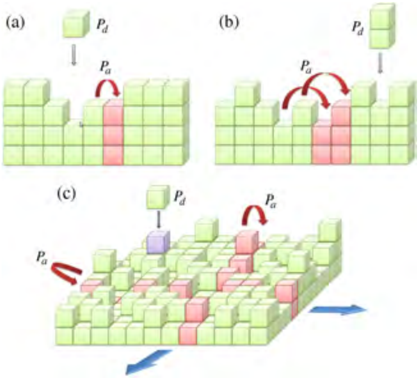
- Computational limitations of anisotropic systems
- Equivalence between interface fluctuations in radial growth and increasing substrates







Deposition on enlarging substrates



- Square lattice with $N = L^{d_s}$
- Periodic boundary cond (cylinder).
- Constant enlarging rate ω.
- Initial substrate size $L_0 \sim \omega$
- Implementation

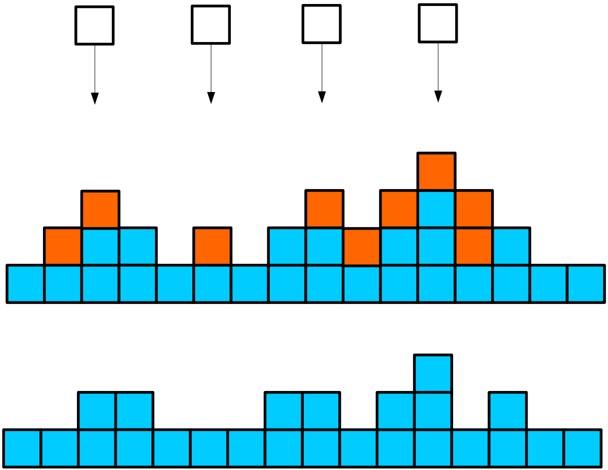
$$P_{dep} = \frac{N}{N + \omega d_s}$$

$$P_{dup} = \frac{\omega d_s}{N + \omega d_s}$$

$$\Delta t = \frac{1}{N + \omega d_s}$$

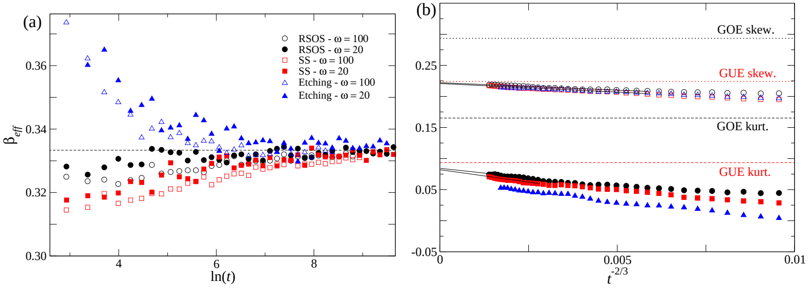
Masoud et al. [Jstat L02001 (2012)] did similar simulations using a different rule and verified that scaling exponents are the same as the fixed-size case.

Etching model ($\lambda > 0$)



Mello et al. PRE **63** 041113 (2001)

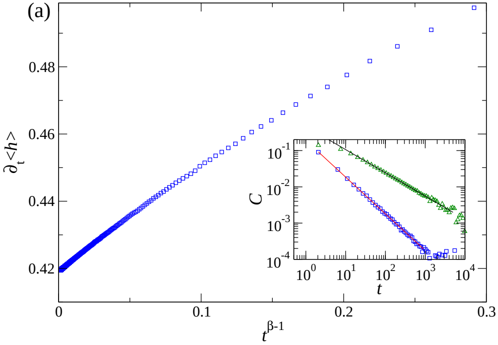
Universal quantities in d=1+1



Logarithmic corrections

- If deposition is turned off, duplication leads to a decay $\langle|\nabla h|^2\rangle \sim t^{-1}$.
- Assuming a correction t^{-1} in the gradient, a logarithmic is found.

$$h \simeq v_\infty t + s_\lambda(\Gamma t)^\beta \chi + \eta + s_\lambda \zeta \ln t + \dots$$

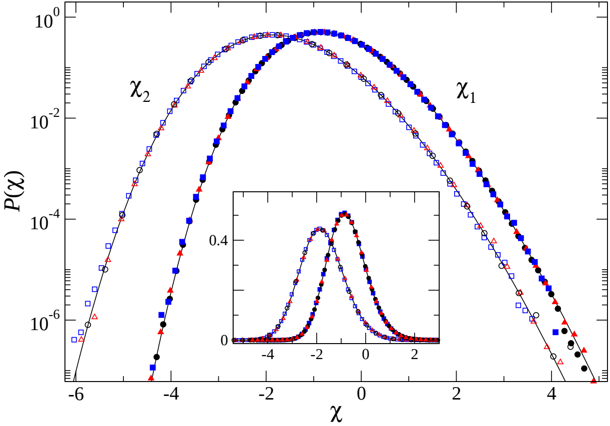


$$C = \partial_t\langle h\rangle - v_\infty - s_\lambda \Gamma^\beta t^{\beta-1}\langle\chi\rangle$$

$$\langle\chi\rangle = \langle GUE\rangle \to C \sim t^{-1}$$

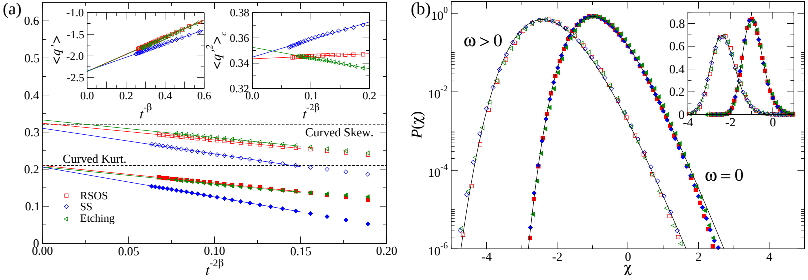
$$\langle\chi\rangle = \langle GOE\rangle \to C \sim t^{\beta-1}$$

Distributions in $d = 1 + 1$



$$\chi \equiv \frac{h - v_\infty t - \langle \eta \rangle - s_\lambda \zeta \ln t}{s_\lambda (\Gamma t)^\beta}$$

Universal distributions in $d = 2 + 1$



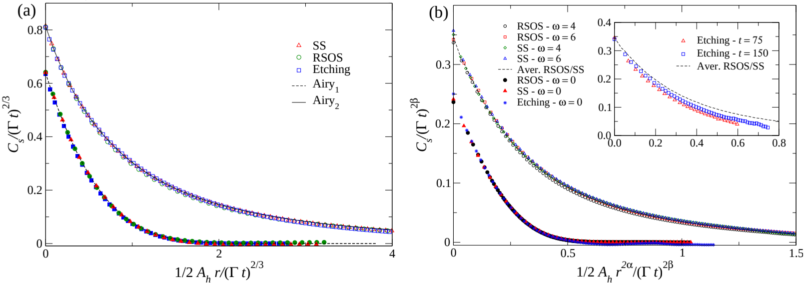
	$\langle\chi\rangle_c$	$\langle\chi^2\rangle_c$	S	K
RSOS	-2.34(3)	0.341(5)	0.328(4)	0.210(4)
SS	-2.37(5)	0.336(6)	0.329(7)	0.206(3)
Etching	-2.36(3)	0.346(8)	0.336(6)	0.21(1)

In agreement with the curved KPZ subclass in d=2+1 [Halpin-Healy, PRL 2012 and PRE 2013; OAF PRE 2013]

Spatial covariance

$$C_s(r,t) = \left\langle \tilde{h}(x,t)\tilde{h}(x+r,t) \right\rangle \simeq (\Gamma t)^{2\beta}\Psi[A_h r/(\Gamma t)^{2\beta}],$$

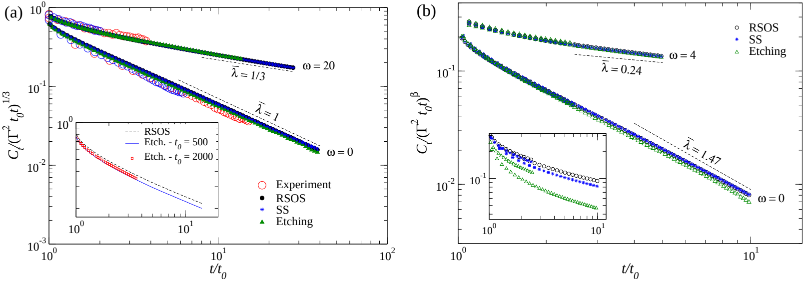
$d = 1+1$



In $d = 1+1$ dimensions we have the Airy$_1$ (flat) and Airy$_2$ (curved) processes [see review by Kriecherbauer and Krug JPA **43**, 403001 (2010)] .

Temporal covariance

$$C_t(t,t_0) = \left\langle \tilde{h}(x,t_0)\tilde{h}(x,t) \right\rangle \simeq (\Gamma^2 t_0 t)^{\beta} \Phi(t/t_0),$$



See [Takeuchi and Sano, JSP **147**, 853 (2012)] for temporal correlation function.

Conclusions of Part IV

- Enlarging substrates belong to KPZ subclass for curved systems in both $d = 1 + 1$ and 2+1.
- Spatial and temporal covariances for the curved subclass in $d = 2 + 1$ are presented.
- Question: Does curvature indeed matter?