Growing on Different Worlds

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work in collaboration with

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Growth is about geometry

- Our beloved Kardar-Parisi-Zhang equation in 1D:

$$\partial_t h(x,t) = \nu \nabla^2 h(x,t) + \frac{\lambda}{2}|\nabla h(x,t)|^2 + \eta(x,t)$$

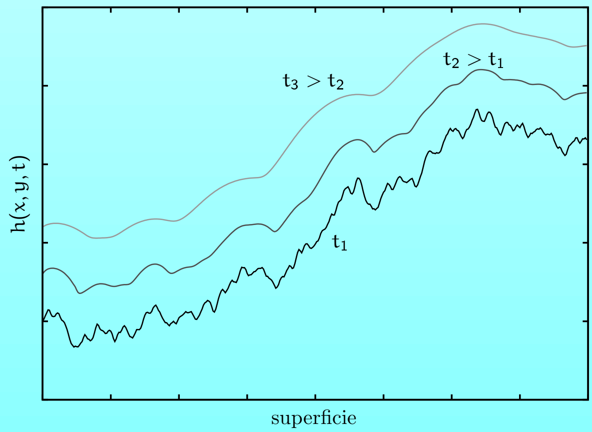
$$\langle \eta(x,t) \rangle = 0 \qquad \langle \eta(x,t)\,\eta(x',t') \rangle = D\,\delta(x-x')\delta(t-t')$$

- Assumptions:

— No overhangs.

— Small slopes.

— Euclidean space!

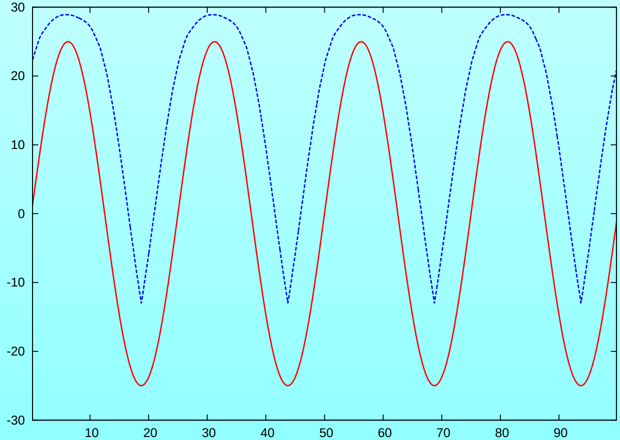


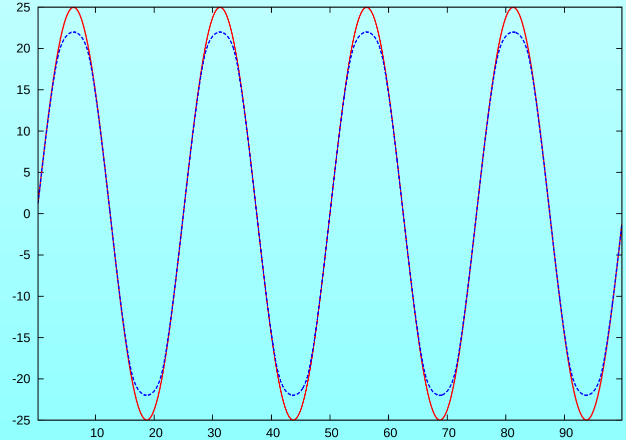
Growth is about geometry

Our proposal: the **Covariant KPZ Equation**

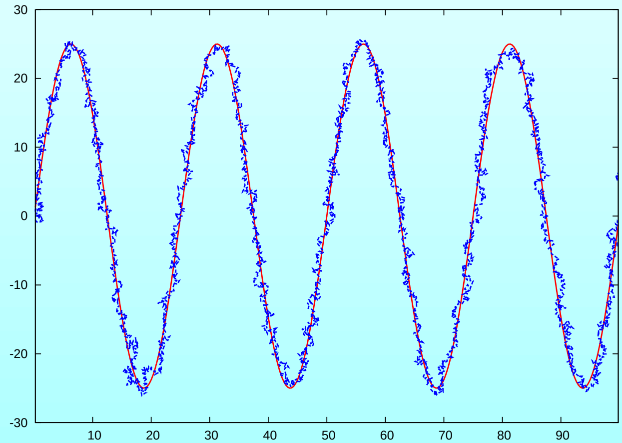
$$\partial_t \vec{r} = \left(A_0 + A_1 K(\vec{r}) + A_n \eta(\vec{r}, t)\right) \vec{n}$$

R-L, Santalla & Cuerno, JSTAT 2011





Growth is about geometry



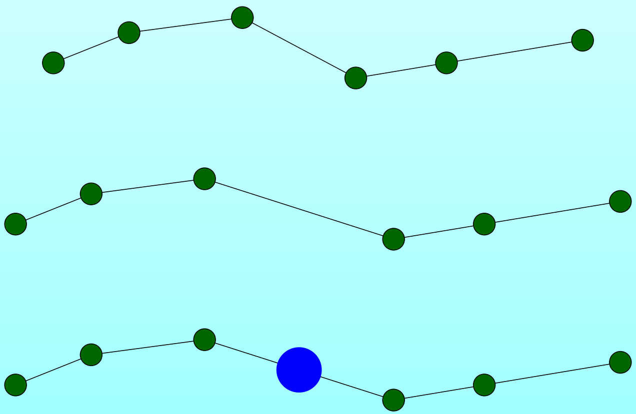
- Our noise is **multiplicative** and correlated with the interface.

纷纷纭纭，斗乱 而不可乱也

“Simulated disorder implies perfect discipline”, Sun Zi

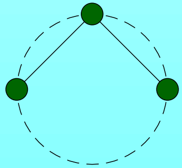
Discrete Geometry can be good Geometry

- Our discretization attempts to be **geometrically natural**
- The interface is simulated as an **quark-like** string of points.



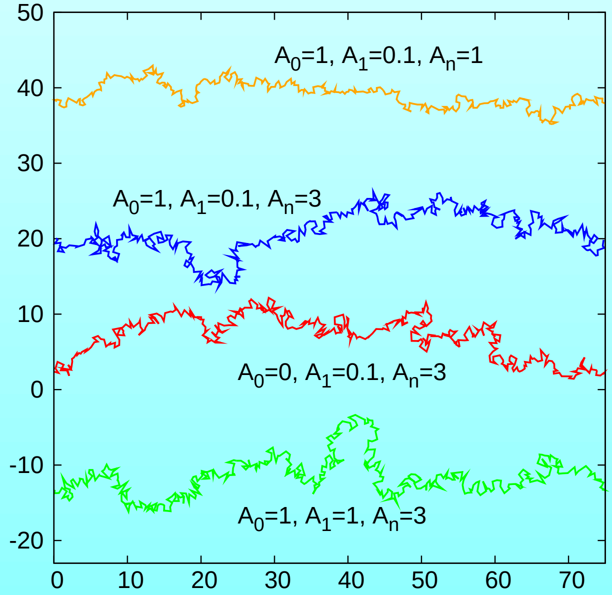
- Curvature is estimated from the circumscribed circle

$$K_i = \frac{2 \sin \alpha_i}{d_{i-1,i+1}}$$



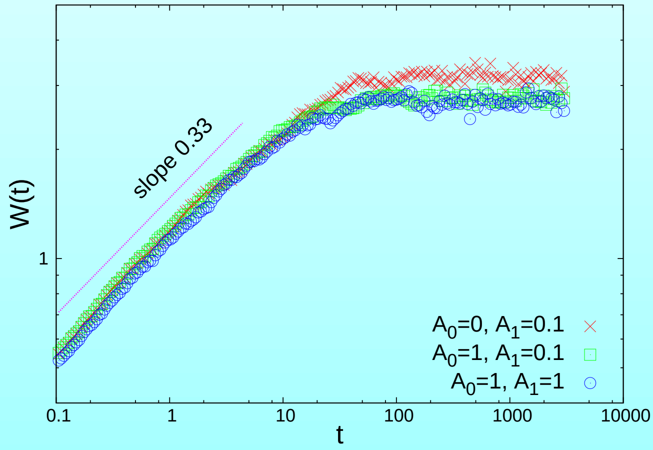
Results in band geometry

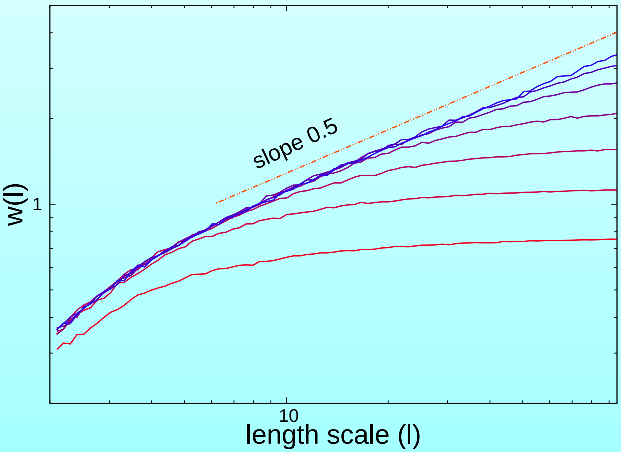
$$\partial_t \vec{r} = \left(A_0 + A_1 K(\vec{r}) + A_n \eta(\vec{r}, t)\right) \vec{n}(\vec{r}) \quad + \text{Removal of self-intersections.}$$



Scaling in band geometry

- Family-Vicsek scaling: $W(t) \sim t^{\beta}$, $\beta = 1/3$; $\quad w(\ell) \sim \ell^{\alpha}$, $\alpha = 1/2$.



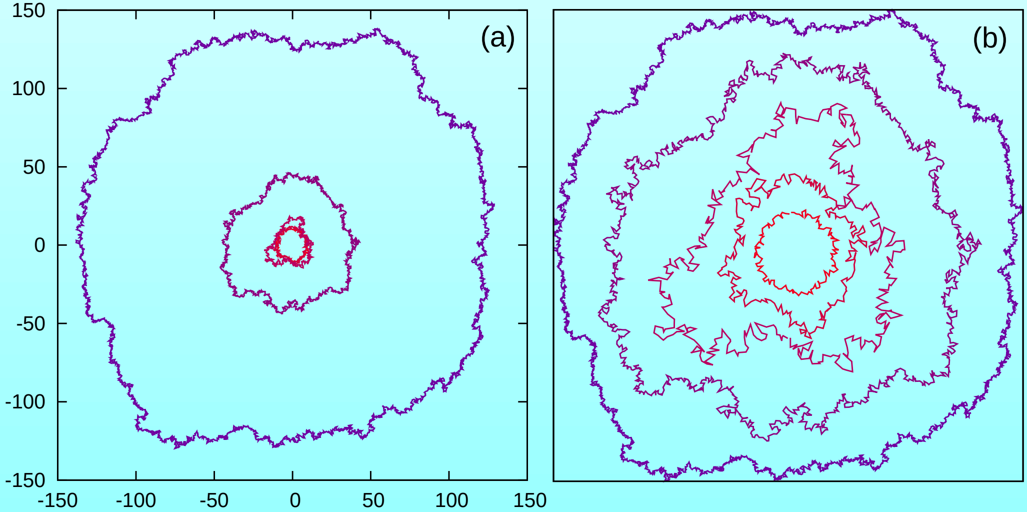


- For low noise, $\beta = 1/4$ and $\alpha = 1/2 \rightarrow$ **Edwards-Wilkinson**.
- For small systems, $\beta = 1/3$ and $\alpha = 2/3 \rightarrow$ **Self-Avoiding Walk**.

Now... circular!

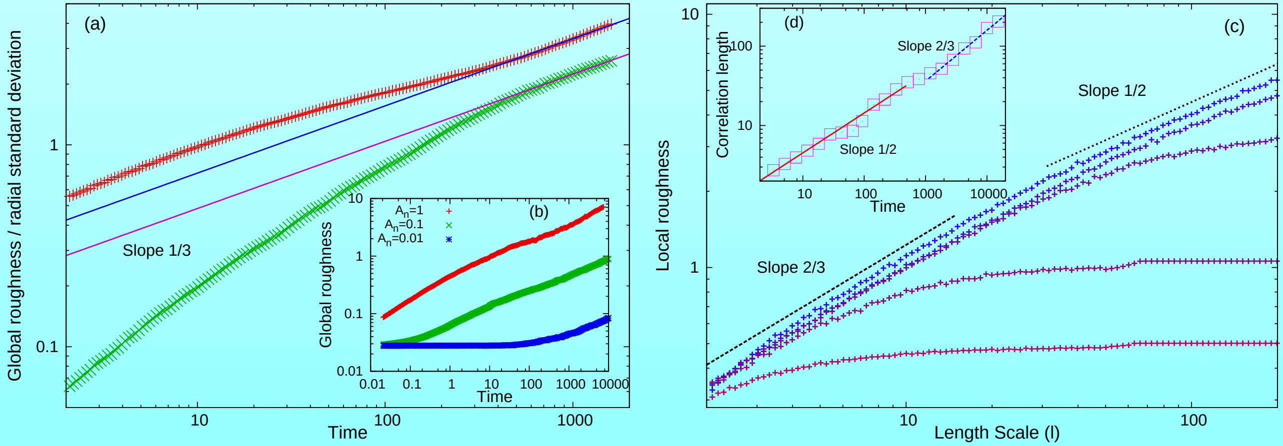
Santalla, R-L & Cuerno, PRE 2014

$$\partial_t \vec{r} = (A_0 + A_1 K(\vec{r}) + A_n \eta(\vec{r}, t))\,\vec{n}(\vec{r}) \quad + \text{Removal of self-intersections.}$$



Scaling in circular geometry

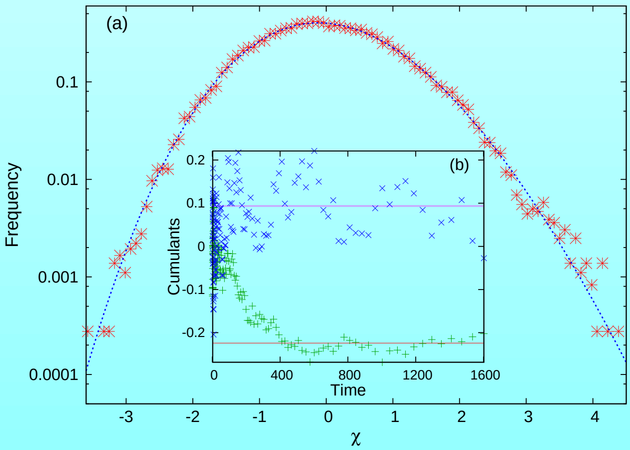
- Short-time regime: no growth, $\beta = 1/3$, $\alpha = 2/3 \rightarrow$ **Self-Avoiding Walk**.
- Transition time: $\beta = 1/4$, $\alpha = 1/2 \rightarrow$ **Edwards-Wilkinson**.
- Long-time regime: $\beta = 1/3$, $\alpha = 1/2$, $\rightarrow$ **KPZ**.



Scaling in circular geometry

- Noise makes you grow faster $\rightarrow$ renormalization of growth rate.
- Radial fluctuations:

$$R(t) \sim Vt + \Gamma t^{1/3} \chi$$



From KPZ to KPZ

• Knizhnik-Polyakov-Zamolodchikov (KPZ_2) studied effect of fluctuating geometry on a critical 2D system.

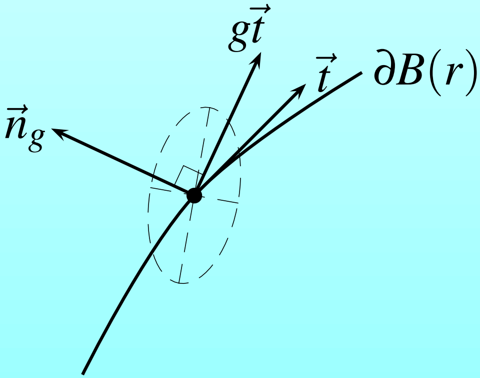
• Fluctuating geometry **is relevant**, and changes the critical exponents of a primary conformal field:

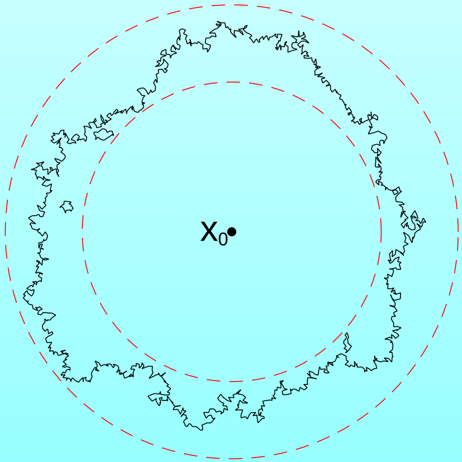
$$\Delta = \frac{\sqrt{1-c+24\Delta^{(0)}} - \sqrt{1-c}}{\sqrt{25-c} - \sqrt{1-c}}$$

• So, $KPZ_1 \rightarrow KPZ_2$... How does a fluctuating geometry affect growth?

Bend it like Riemann!

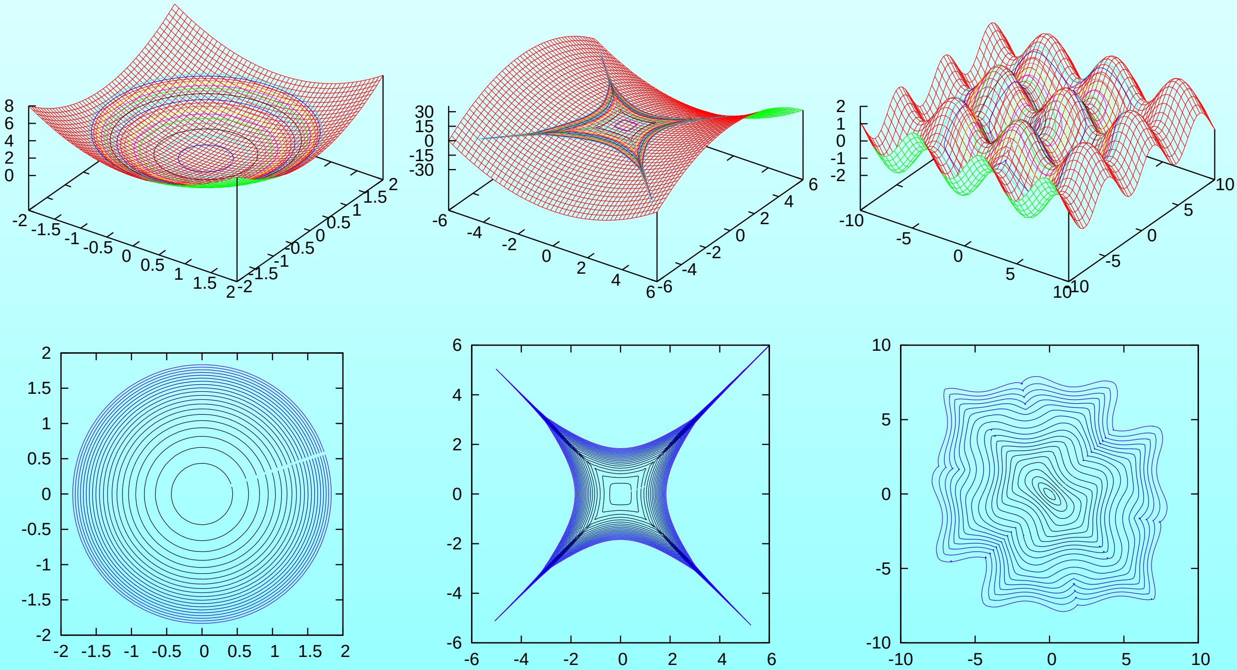
- What about a curved background space? How will KPZ look like?
- Establish an arbitrary Riemannian metric $g_{\mu\nu}(\vec{r})$.
- No noise, no curvature: the **ball equation**: $\partial_t \vec{r} = \vec{n}_g(\vec{r})$





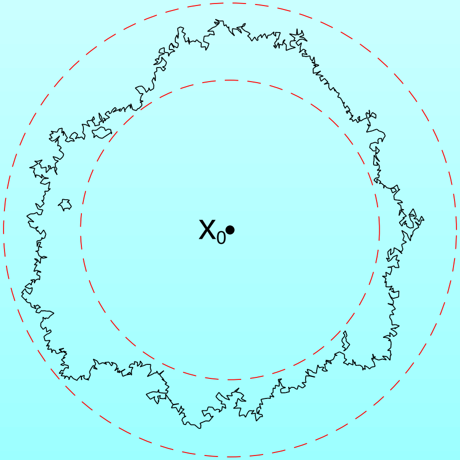
Santalla, R-L, LaGatta, Cuerno, ArXiv 2014

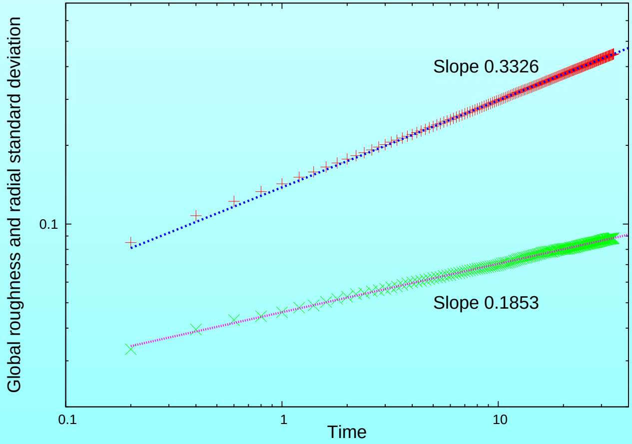
Deterministic growth on curved surfaces



Drunk Euclid still rules

- Random static $g_{\mu\nu}$, smooth, short-range correlators.
- $W(t) \sim t^{1/3}$ and $\sigma_R \sim t^{1/6}$...

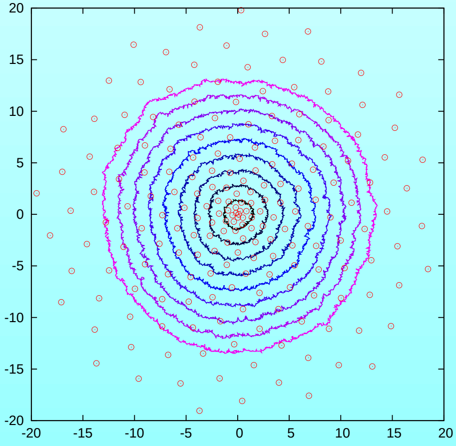


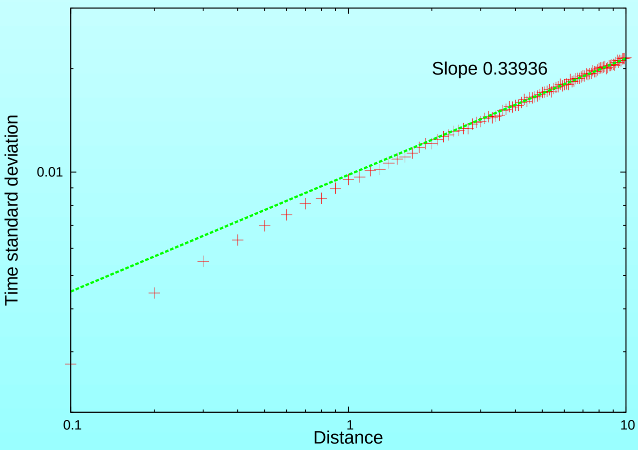


- Exponent z also appears in the **random geodesics**: $z = 3/2$.
- So, $z = 3/2$, $\alpha = 1/2$... BUT fluctuations are not TW!!!

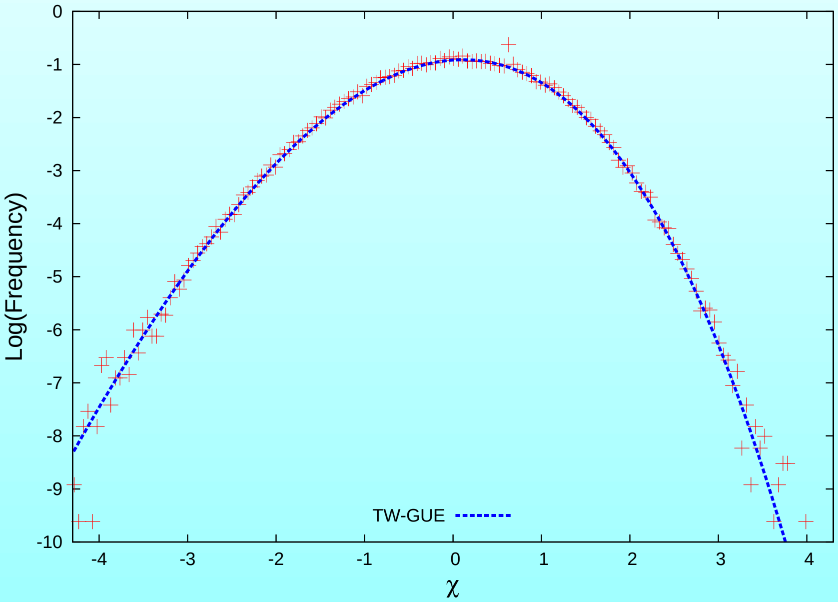
Arrival times

- Yet... KPZ is hidden!!
- Inspired by First Passage Percolation (FPP)...
- Look at **arrival times!**





Arrival times

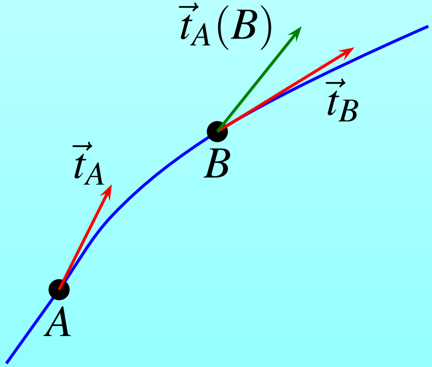


- No pre-asymptotic!
- One might suspect a **quenched KPZ**...

Curavature

- Parallel Transport: $\vec{v}(A) \to \vec{v}(B) = \Gamma(A \to B)\,\vec{v}(A)$.
- $\Gamma(A \to B)$ is given by the Christoffel symbols.
- **Geodesic curvature**: Angular deviation per unit length

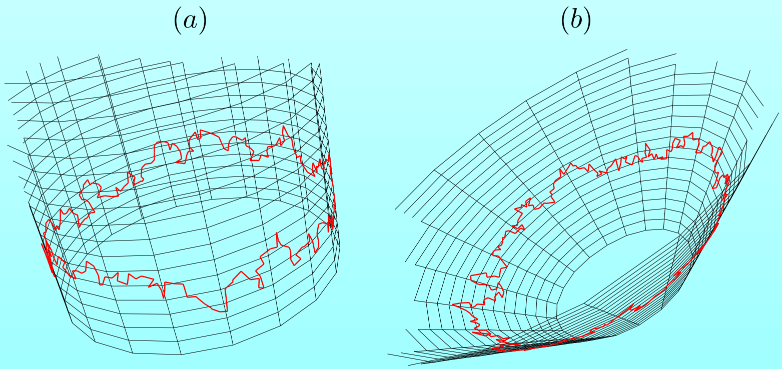
$$k_g(A) = \lim_{B \to A} \frac{\alpha(\vec{t}_A(B), \vec{t}_B)}{d(A, B)}$$



- Measures how much your trajectory deviates from a geodesic.

Topology & KPZ

- KPZ on a **cylinder** $\rightarrow$ TW-GOE fluctuations.
- KPZ on a **plane** $\rightarrow$ TW-GUE fluctuations.
- *Is there something in between?*



- Possibility: KPZ on cones.

Santalla,R-L,Celi,Cuerno, under progress.

Topology & KPZ

- Will cones show intermediate fluctuations between TW-GOE & TW-GUE?

Against:

— A small amount of "U" changes the universality.

In favor:

— There is a natural continuous family of of TW distributions.

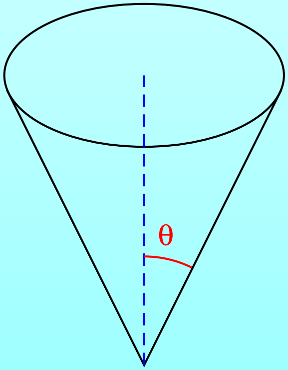
— We have a (topological) constant of motion.

Gauss-Bonnet Formula

- The Gauss-Bonnet theorem states:

$$\int_{\partial M} k_g \, ds + \int_M K \, dA = 2\pi \chi(D)$$

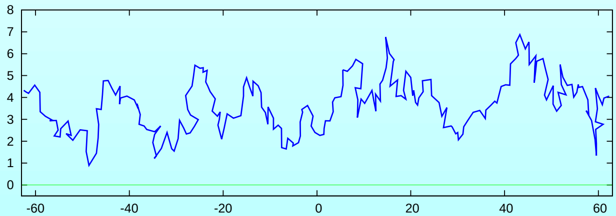
With K the Gaussian curvature of the surface. In our case:

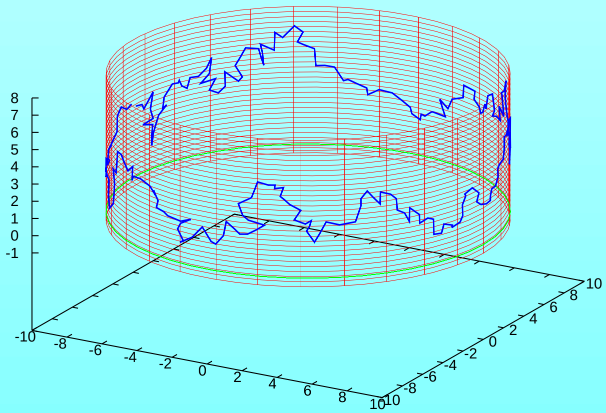


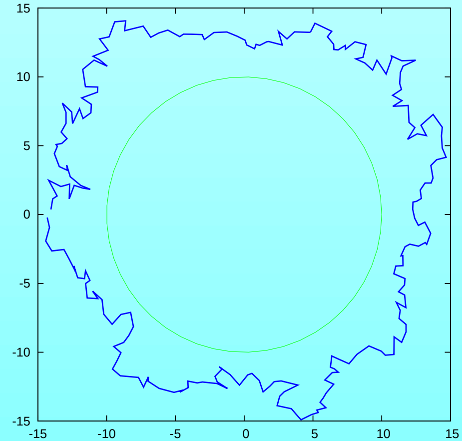
$$\int_{\partial M} k_g \, ds = 2\pi \sin(\theta)$$

Analogy with Gauss law → *the tip is our sun*!

Our map of the cones

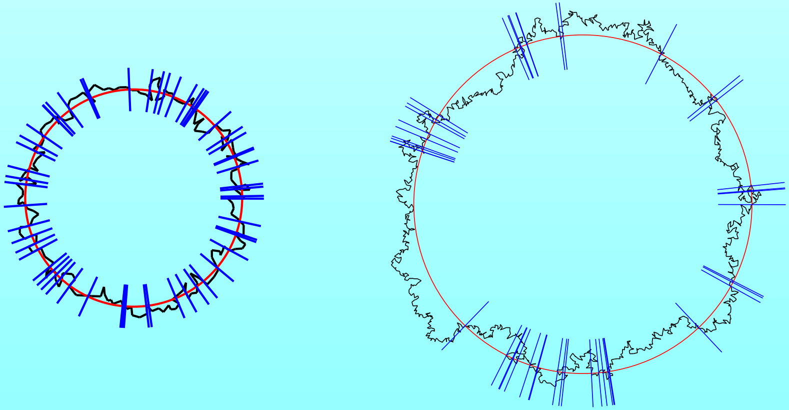






Measuring with bent rulers

- Another problem: finding covariant measures.
- In cones: radial distances are preserved, azimuthal distances are normalized.
- Roughness is straightforward, but morphology can be tricky.
- Nice possibility: expected distance to crossing with fitting circle.



Preliminary results

• For large enough noise, initial stage is SAW.

— $W(t) \sim t^{1/3}$.

— $\xi(t) \sim t^{1/2}$

• For intermediate times, a (short) EW behavior.

• For long times, KPZ behavior.

• (Likely) GUE behavior for all $\theta \neq 0$, but with a long crossover time.

• Thus, GOE would be a **black hole indicator**, the Hawking radiation of growth processes.

Take-home message

Growth is about geometry

Young droplets are self-avoiding walks

Euclid rules even when drunk

From KPZ to KPZ?

How to grow in different worlds

The tip is our Sun

How to measure with bent rulers

Tracy-Widom feels the black holes

Thank you for your attention!

- Please, visit our web: `http://moria.uc3m.es/kpz`

- Please, visit our papers:

RL, Santalla, Cuerno, JSTAT 2011, `1105.1727`.

Santalla, RL, Cuerno, PRE 2014, `1312.7696`.

Santalla, RL, LaGatta, Cuerno, submitted 2014, `1407.0209`.