

**Interface fluctuations and KPZ universality class**

– unifying mathematical, theoretical, and experimental approaches

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# Step Faceting Caused by Discontinuous Surface Tension on a Crystal Surface

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Surface tension of a vicinal surface is calculated by the product-wave-function renormalization group (PWFRG) method [1], which is a classical version of the density matrix renormalization group (DMRG) method [2]. The model in this paper is a restricted solid-on-solid model with contact-type step-step attraction (p-RSOS model) [3], where “restricted” means the restricted difference of the surface height  $\Delta h_{ij} = h_i - h_j = \{0, \pm 1\}$ . The Hamiltonian of the p-RSOS model is written as follows:  $\mathcal{H} = \epsilon \sum_{\langle i,j \rangle} |h_i - h_j| + \epsilon_{\text{int}} \sum_{\langle i',j' \rangle} \delta(|h_{i'} - h_{j'}|, 2) - \Delta\mu \sum_{i=1}^N h_i$ , where  $\epsilon$  represents the energy cost to make surface height difference between the nearest neighbor (nn) sites,  $\epsilon_{\text{int}}$  represents a microscopic step-step interaction between the second nn sites,  $\delta(k, l)$  represents the Kronecker delta, and  $\Delta\mu = \mu_{\text{gas}} - \mu_{\text{crystal}}$  represents the chemical potential difference between ambient gas and the bulk crystal. Here,  $\langle i, j \rangle$  runs over all nn pairs and  $\langle i', j' \rangle$  runs over all the second nn pairs. The restricted condition is imposed implicitly. This p-RSOS model can be mapped to a 19-vertex model [3, 4].

In the case of  $\epsilon_{\text{int}} < 0$ , the step-step interaction is attractive [4]. In that case, the vicinal surface has two phase transition temperatures  $T_{f,1}$  and  $T_{f,2}$ . At high temperature  $T > T_{f,1}$ , the surface shows one-dimensional (1D) free fermion universal behavior or Gruber-Mullins-Pokrovsky-Talapov (GMPT) [5] universal behavior in the low step density limit. At temperature  $T < T_{f,1}$ , surface tension becomes discontinuous around the (111) facet edge (high step density limit). At temperature  $T < T_{f,2}$ , surface tension become discontinuous around the (001) surface (low step density limit). As the results from the discontinuous step tension, at temperature  $T < T_{f,2}$ , steps condensate to form a faceted macro step. For  $T_{f,1} < T < T_{f,2}$ , the locally merged steps, which we call “step droplets”, are created and dissociated dynamically. The surface free energy contains  $A\rho^2$  in  $f(\rho) = f(0) + \gamma\rho + A\rho^2 + B\rho^3 + O(\rho^4)$ [4], where  $\rho$  represents the step density,  $\gamma$  represents the step tension,  $B$  represents the step interaction coefficient, and  $A$  represent the “step coalescence factor”.  $A\rho^2$  represents non-universal term [4].

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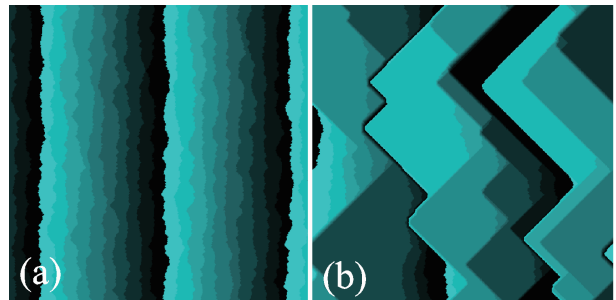


Figure 1: Top views of a vicinal surface demonstrated by the Monte Carlo calculation. (a)  $1 \times 10^3$  Monte Carlo Step per site (MCS). (b)  $3 \times 10^3$  MCS. The brighter the higher. The height levels are shown in ten gradations. Size:  $240\sqrt{2} \times 240\sqrt{2}$ . Number of steps: 24.  $k_B T / \epsilon = 0.1$  ( $T < T_{f,2}$ ).  $\epsilon_{\text{int}} / \epsilon = -0.5$ .  $\Delta\mu / \epsilon = 0.1$ .

# Interfacial properties of a discrete model for cell cultures

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We propose and study by means of Monte Carlo numerical simulations a minimal discrete model for avascular tumor growth, which can also be applied for the description of cell cultures in vitro. The interface of the tumor is self-affine and its width can be characterized by the following exponents: (i) the growth exponent  $\beta = 0.32(2)$  that governs the early time regime, (ii) the roughness exponent  $\alpha = 0.49(2)$  related to the fluctuations in the stationary regime, and (iii) the dynamic exponent  $z = 1.49(2)$ , which measures the propagation of correlations in the direction parallel to the interface, e.g.  $x_{\parallel} \propto t^{1/z}$ , where  $x_{\parallel}$  is the parallel correlation length. So, the interface belongs to the KPZ universality class in agreement with recent experiments of cell cultures in vitro. Furthermore, density profiles of the growing cells are rationalized in terms of traveling waves that are solutions of the Fisher-Kolmogorov equation. In this way, we achieved excellent agreement between the simulation results of the discrete model and the continuous description of the growth front of the culture/tumor.

## Interface localization-delocalization transitions in confined ferromagnets

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We present a numerical study of the localization-delocalization transition of the interface between magnetic domains of different orientation, in an Ising magnet confined between two walls separated by a distance  $L$ , where short-range inhomogeneous surface magnetic fields act. So, samples are assumed to have a size  $L \times M$ ,  $L$  being the width and  $M$  the length, respectively. The localization-delocalization transition of the interface becomes a true phase transition when the thermodynamic limit ( $L \rightarrow \infty$ ,  $M \rightarrow \infty$ ) is properly taken, as it follows from scaling arguments. By considering surface fields varying spatially with a given wavelength or period ( $\lambda$ ),  $H_1(x, \lambda)$  with  $1 \leq x \leq M$ , we found that the wetting temperature is given by the exact result of Abraham [Phys. Rev. Lett. **44**, 1165, (1980)] provided that an effective field given by the spacial average value ( $H_{eff} \equiv \frac{1}{\lambda} \int_0^\lambda H_1(x, \lambda) dx > 0$ ) is considered. The above results hold in the low wavelength regime, while for  $\lambda \rightarrow \infty$  and a bivaluated surface field (i.e.,  $H_{max}$  for  $x \leq M/2$ , and  $\delta H_{max}$  for  $x > M/2$ , with  $0 < \delta < 1$ ), one observes two almost independent wetting transitions, both being compatible with Abraham's exact results corresponding to  $H_{max}$  and  $\delta H_{max}$ , respectively. On the other hand, for  $H_1(x, \lambda) \neq 0$  but  $H_{eff} = 0$  bulk standard critical behavior results is observed.

# A Study of Correlations in the Dyson Model

Sergio Andraus

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Dyson's Brownian motion model (henceforth referred to as the Dyson model) is a family of stochastic processes realized by multiple particles in one dimension which undergo independent Brownian motions while repelling each other through a logarithmic potential with coupling constant  $\beta/2$ , where  $\beta > 0$ . For particular values of  $\beta$ , the Dyson model describes the time evolution of the eigenvalues of real symmetric ( $\beta=1$ ), complex Hermitian ( $\beta=2$ ) and quaternionic self-dual ( $\beta=4$ ) matrices whose entries are independent Brownian motions up to symmetry constraints [1]. The case where  $\beta=2$  is also known as the scaling limit of the vicious walker model [2], that is, as the Brownian motion of several particles conditioned never to collide [3]. This non-colliding Brownian motion has been widely studied due to its applications in polymer physics, wetting and melting transitions and growth models. Indeed, the Dyson model with  $\beta=2$  possesses several mathematical properties which allow for the calculation of several quantities of interest, such as particle densities and single- and multi-time correlation functions.

The cases where  $\beta$  is not equal to 2, however, are far less understood. In fact, the single-time correlation functions have only been calculated in the matrix cases ( $\beta=1,4$ ) and when  $\beta$  is an even integer [4]. The study of correlations in the Dyson model for other values of  $\beta$  remains an open problem.

In the present work, we focus on the exact calculation of the correlation function of the Dyson model for two particles as well as their collision probability in time for  $\beta > 0$ . For this purpose, we use the fact that the two-particle Dyson model can be transformed into a Bessel process of  $d=\beta+1$  dimensions [5]. In addition, we use numerical simulations to investigate the correlation functions and collision probabilities in the case where the number of particles is arbitrary.

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# Roughening Dynamics of Radial imbibition in a Hele-Shaw Cell

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We studied the radial imbibition of water in a porous medium in a Hele-Shaw cell. Washburn's law for radial imbibition is confirmed in our experiment. We check the roughening dynamics of forced fluid imbibition and spontaneous imbibition. For forced fluid imbibition, Radial imbibition follows scaling dynamics and shows anomalous roughening dynamics when the front invades the porous medium. The roughening dynamics depend on the flow rate of the injected fluid. The growth exponents increase linearly with an increase in the flow rate while the roughness exponents decrease with an increase in the flow rate. For spontaneous imbibition, we found a growth exponent 0.6 that is almost independent of the pressure applied at the liquid inlet. The roughness exponent decreased when the pressure increased. The roughening dynamics of spontaneous imbibition obeys Family-Vicsek scaling relation. We conclude that roughening dynamics of radial imbibition is markedly different from one dimensional imbibition with a fixed-size planar interface window.

# Dynamic phase transition by coupling Ising spins to 1D KPZ-type surface growth

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It is examined whether  $1 + 1$  dimensional directed polymers in a random medium can have long-range order of sign if their amplitudes change sign with probability  $p$ . If such order exists even at nonzero  $p$ , it would then be an example of one-dimensional (1D) dynamic phase transition induced by nonequilibrium process. The question then arises whether degrees of freedom coupled to interfaces can undergo genuine phase transitions in 1D growing stationary states. Reinterpreting the sign order as the ferromagnetic order of Ising spins coupled to the Kardar-Parisi-Zhang (KPZ) type surface growth, we numerically show that such a transition is actually a crossover phenomenon observed in finite systems, and provide a finite-size scaling description of our observations. This issue was addressed a decade ago and answered in the negative sense [1], but we also find a case where those surface degrees of freedom behaved as if in a quasi-critical phase, slaved to the KPZ interface fluctuations. Since the same issue resurfaced recently in the context of magneto-resistance in disordered variable range hopping conductors [2, 3], representing completely different physics but mathematically linked to ours, some new results are also presented on the magneto-resistance application.

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## Information thermodynamics for coupled Brownian particles

Sosuke Ito

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We study nonequilibrium thermodynamics of complex information flows induced by interactions between multiple Brownian particles. We have a novel generalization of the second law, which includes the information flow, i.e., transfer entropy [1,2].

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# Domain coarsening on ballistic deposition model with surface diffusion

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We study domain coarsening of ballistic deposition model with a surface diffusion on two-dimensional substrate. An underlying domain catches a deposited particle and the domain grows. The domains overhang neighboring ones and the resulting number density of domains  $\rho$  decreases with the average height  $h$  of the interface. The domain density  $\rho$  is expected to decrease as  $h^{-\gamma}$ . The characteristic domain size is described as  $\rho^{-1/d}$  with the interfacial dimension  $d$ . This size is related to the correlation length of the system, which is written as  $h^{1/z}$  with the dynamic exponent  $z$ . The new scaling law of domain coarsening is obtained as  $\rho \sim h^{-d/z}$ . As it is said that the scaling class of ballistic deposition model is the KPZ class, the exponent  $\gamma$  is expected to be about 2 on the two-dimensional substrate. We performed the kinetic Monte Carlo simulation of the  $(2 + 1)$ -dimensional ballistic deposition model. The magnitude of the obtained power is in good agreement with that from the scaling analysis. This relation is also valid for the case with surface diffusion, which flattens the interface.

## Restricted solid-on-solid model in higher dimensions and no upper critical dimension in KPZ Eq.

Jin Min Kim

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A restricted solid-on-solid growth model is studied in higher dimensions with a proper restriction parameter. The interface width  $W(t)$  is measured as a function of time  $t$  and the growth exponent  $\eta$  is estimated through the relation  $W(t) \sim t^\eta$  in higher dimensions. The obtained  $\eta$  is a little bit less than but close to  $1/(d + 1)$ . Our numerical simulation results indicate that the upper critical dimension of the Kardar Parisi Zhang equation is larger than  $d = 11 + 1$ .

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# Fragility and robustness of the Kardar-Parisi-Zhang universality class

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The paradigmatic Kardar-Parisi-Zhang (KPZ) equation for a rough interface is recently proving itself as a remarkable instance of universality. The exact asymptotic height distribution function has been obtained for  $d=1$ : it is given by the largest-eigenvalue distribution of large random matrices in the Gaussian unitary (GUE) (orthogonal, GOE) ensemble, the Tracy-Widom (TW) distribution, for globally curved (flat) interfaces. Such distribution functions and limiting processes are shared by discrete models, continuum equations, and by experimental systems. This behavior generalizes to  $d=2$ , where generalizations of the TW distributions occur.

In this contribution we consider the dependence on substrate dimensionality of the asymptotic scaling behavior of a family of nonlocal equations that feature the basic symmetries of the KPZ equation [1]. Even for cases in which, as expected from universality arguments, these models display KPZ critical exponent values, their behavior deviates from KPZ scaling for increasing system dimensions [2]. Such a fragility of KPZ universality contradicts naive expectations, and questions straightforward application of universality principles for the continuum description of experimental systems. Still, we find that the ensuing TW(-like) distributions coincide with those of the KPZ class in one (two) dimensions, demonstrating the robustness of the former under changes of the critical exponent values.

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## Extensions of duality relations between stochastic differential equations and birth-death processes

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Extensions of duality relations between stochastic differential equations and birth-death processes are shown. The conventional duality relations are widely used in studies of non-equilibrium physics; for example, the brownian momentum process (BMP), which is a toy model of heat conduction, can be analyzed by its dual particle hopping model. However, there is no guarantee that a stochastic differential equation has a corresponding dual birth-death process. Employing an algebraic approach, i.e., the Doi-Peliti formalism, I will show that the duality relation is easily derived. In addition, it is possible to extend the conventional duality relations. The extension enables us to derive birth-death processes from various types of stochastic differential equations.



# Random geometry and the Kardar-Parisi-Zhang universality class

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We study numerically a continuum analogue [2] of first-passage percolation (FPP) [3]. Specifically, we consider short-range-correlated random metrics on  $\mathbb{R}^2$  which are flat on average, and focus on the statistical properties of balls and geodesics. Connection to FPP is achieved by assuming an underlying Euclidean metric and by considering distances as times required to travel between different points. The spread of walkers along geodesics defines balls [2], see Fig. 1.

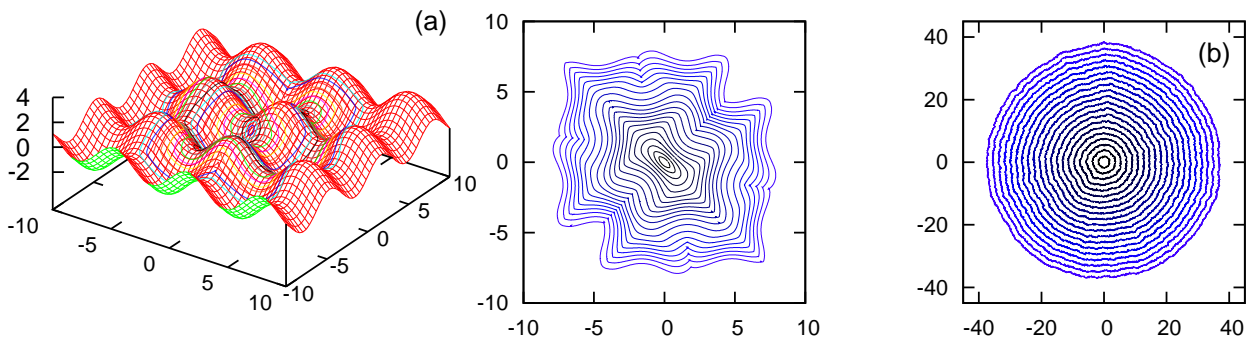


Figure 1: Numerical examples of: (a) balls in a non-random metric and (b) balls in a random metric.

Random metrics are generated point-wise, assuming a random distribution for the eigenvalues of the metric tensor and the directions of the eigenvectors. We have devised a discretization scheme which relies upon Huygens' principle, which is stable and geometrically natural.

Scaling for the roughness of balls and the lateral spread of minimizing geodesics feature Kardar-Parisi-Zhang (KPZ) exponents, as expected [3]. In many known instances of circular geometry in the context of the KPZ universality class, radial fluctuations follow the Tracy-Widom (TW) distribution for the Gaussian Unitary Ensemble of random matrices [4, 5]. However, the radial fluctuations *do not* follow TW statistics. Nonetheless, by exploiting the analogy between our random metric model and FPP, we find that TW statistics hold for a different observable, namely, the fluctuations of arrival times.

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# Numerical simulation of polymer thin film growth by vapor deposition

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To understand anomalous kinetic roughening with multifractality in vapor deposition polymerization (VDP) growth, we consider a simple model for the polymer thin film growth by vapor deposition, along with monomer diffusion, polymer nucleation, limited active end bonding, and shadowing effects. Through the extensive numerical simulations, we observe the power-law distribution of height differences, which is clear evidence of anomalous scaling and multiscaling behavior. It is argued that such anomalies in the VDP growth are attributed to the instability induced by the non-local shadowing effects in polymerization. As varying the ratio of diffusion coefficient to deposition rate, our results will be compared with recent experimental data.

## Path condensation in nonequilibrium Langevin systems

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We study the dynamics of a Brownian particle driven by the noisy Burgers (KPZ) equation. We conjecture that the probability density of trajectories shows a condensation phenomenon in which two independent trajectories have a non-zero overlap. We present theoretical and numerical results supporting this conjecture.

# Current fluctuations in Totally Asymmetric Inclusion Process

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Symmetric inclusion process (SIP) is a stochastic lattice gas introduced in [1,2] and further studied in [3,4,5]. It is a Bosonic counter model to the well-known Fermionic symmetric exclusion process (SEP). While there is an effective repulsion between particles in SEP, such that there can not be more than one particle per sites in SEP, in SIP the particles attract each other, and in fact prefer to jump to the sites that are more occupied. In this talk, we study totally asymmetric inclusion process (TASIP) in 1D. We consider different initial conditions, for instance a step-like initial condition in which there are fixed number of particles per sites only to the left of the origin at time zero. We look at the statistics of the integrated current, which is the total number of particles passed through the origin at the time  $t$  and also the position of the right-most particle. As in TASEP, the position of the right-most particle can be thought of as a growing interface. We investigate the effect of initial conditions and the system size, and show evidence from numerical experiments that the integrated current fluctuations in TASIP are super-diffusive. This puts TASIP in a different universality class than the TASEP which is sub-diffusive. We also comment on analytical methods to study this problem rigorously and possible comparisons with the numerical results.

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# Strong Anisotropy in Surface Kinetic Roughening: Theory and Experiments

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Frequently, two-dimensional surfaces driven out of equilibrium by growth or erosion processes, as in thin-film production, solid fracture, etc. self-organize so as to display anisotropic scale-invariant behavior (kinetic roughening), with different scaling behaviors along the different substrate directions (strong anisotropy, SA).

We formulate a SA scaling Ansatz which, albeit equivalent to other proposals in the literature, is more naturally adapted to the type of observables frequently measured in experiments, like height structure factors.<sup>2</sup> We test our Ansatz on paradigmatic stochastic equations displaying SA, including the Hwa-Kardar (HK) system. We compare the latter with a linear non-local equation which has the exact same scaling exponents, thanks to suitable scaling relations. We thus elucidate slow convergence properties of certain observables, which can otherwise hinder a clear-cut assessment of SA. Our Ansatz has been successfully tested against experimental data from silicon targets eroded by low-energy ion-beam sputtering.<sup>3</sup>

From a general perspective, we investigate conditions for the occurrence of strong anisotropy in stochastic height equations. We consider anisotropic generalizations of several well-known models, such as the KPZ and the conserved KPZ equations, as representative cases in which the dynamics is non-conserved and conserved, respectively.<sup>4</sup> Dynamic renormalization group analysis and direct numerical simulations lead us to identify special (non-generic) conditions from the shape of the terms appearing in the dynamical equation, in order for asymptotic SA to actually take place.

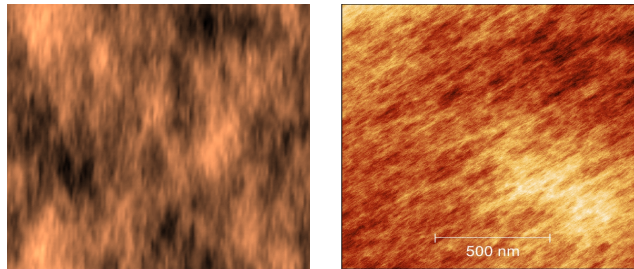


Figure 1: Examples of strongly anisotropic surfaces. Left panel: morphology generated by numerical simulations of a linear strongly anisotropic model, the  $\mathcal{L}$ - $\mathcal{K}$  equation.<sup>2</sup> Right panel: STM image for a silicon sample eroded by low-energy ion-beam sputtering.<sup>3</sup>

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# Scaling Limit for Brownian Motions with One-sided Collisions

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We consider Brownian motions with one-sided collisions, meaning that each particle is reflected at its right neighbour. For a finite number of particles a Schütz-type formula is derived for the transition probability. We investigate an infinite system with periodic initial configuration, *i.e.*, particles are located at the integer lattice at time zero. The joint distribution of the positions of a finite subset of particles is expressed as a Fredholm determinant with a kernel defining a signed determinantal point process. In the appropriate large time scaling limit, the fluctuations in the particle positions are described by the  $\text{Airy}_1$  process. Corresponding results for an infinite system with Poisson initial configuration are in progress.