QCD Phase Transitions
and
Quark Quasi-particle Picture

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YITP workshop
New Developments on Nuclear Self-consistent Mean-field Theories

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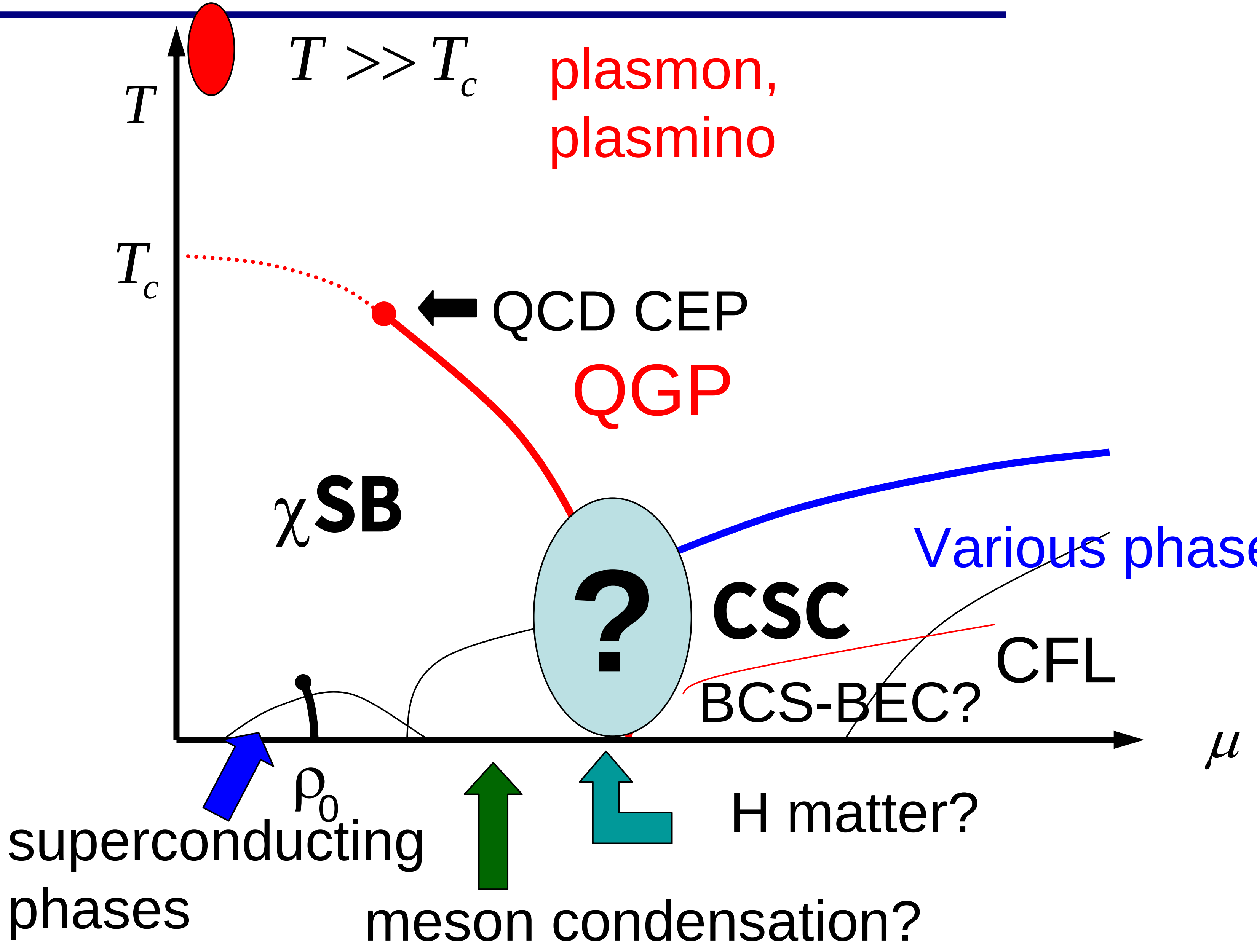
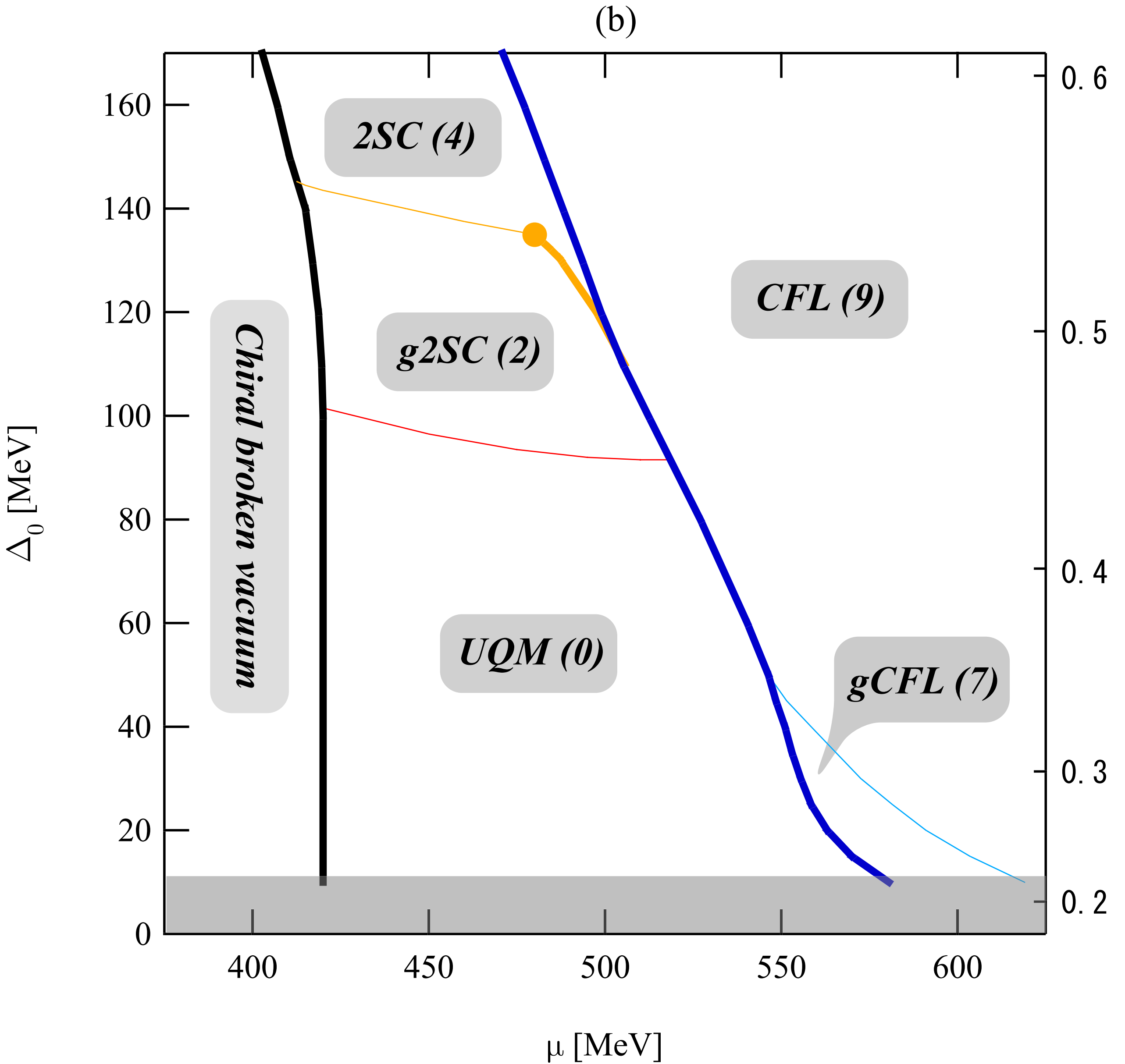
1.Introduction

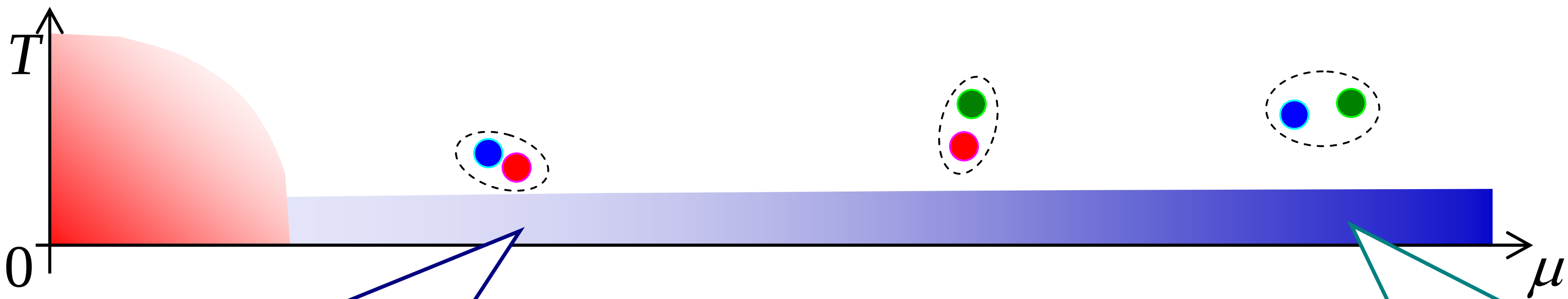
QCD phase diagram and quasi-particles



Abuki, Kitazawa and T.K.(hep-ph/04123829;
P.L.B615,102(2005))



Color Superconductivity



strong coupling!

weak coupling

$\boldsymbol{\Delta \sim 100\mathrm{MeV}}$

$\boldsymbol{\Delta / E_F \sim 0.1\text{-}0.3}$

in electric SC

$\Delta / E_F \sim 0.0001$

Mean field approx. works well.

Very Short coherence length ξ.

- There exist large fluctuations of pair field.
- Large pair fluctuations can { invalidate MFA. / cause precursory phenomena of CSC. }

cf.) Bosonization of Cooper pairs

Matsuzaki, PRD**62**,017501 (2000)

Abuki, Hatsuda, Itakura, PRD **65**, 074014 (2002)

Y.Nishida and H. Abuki (hep-ph/0504083)

4-body contact interaction with massive fermion

$$\mathcal{L}[\psi,\bar{\psi}] = \bar{\psi}\left(i\partial\!\!\!/ - m + \gamma_0\mu\right)\psi + G\left(i\psi^{\mathrm{T}}\gamma_5 C\epsilon^c\varepsilon\psi\right)\cdot\left(i\psi^{\dagger}\gamma_5 C\epsilon^c\varepsilon\psi^*\right)$$

fermion mass
fermion chemical potential

Attractive interaction in color 3, flavor 1 and $J^P=0^+$ diquark channel

Fermion pair correlation in normal phase

<Mean field + Gauss fluctuation>

S'a de Melo et al., PRL 71 (1993) 3202
Kitazawa et al.,PRD 70 (2004) 056003

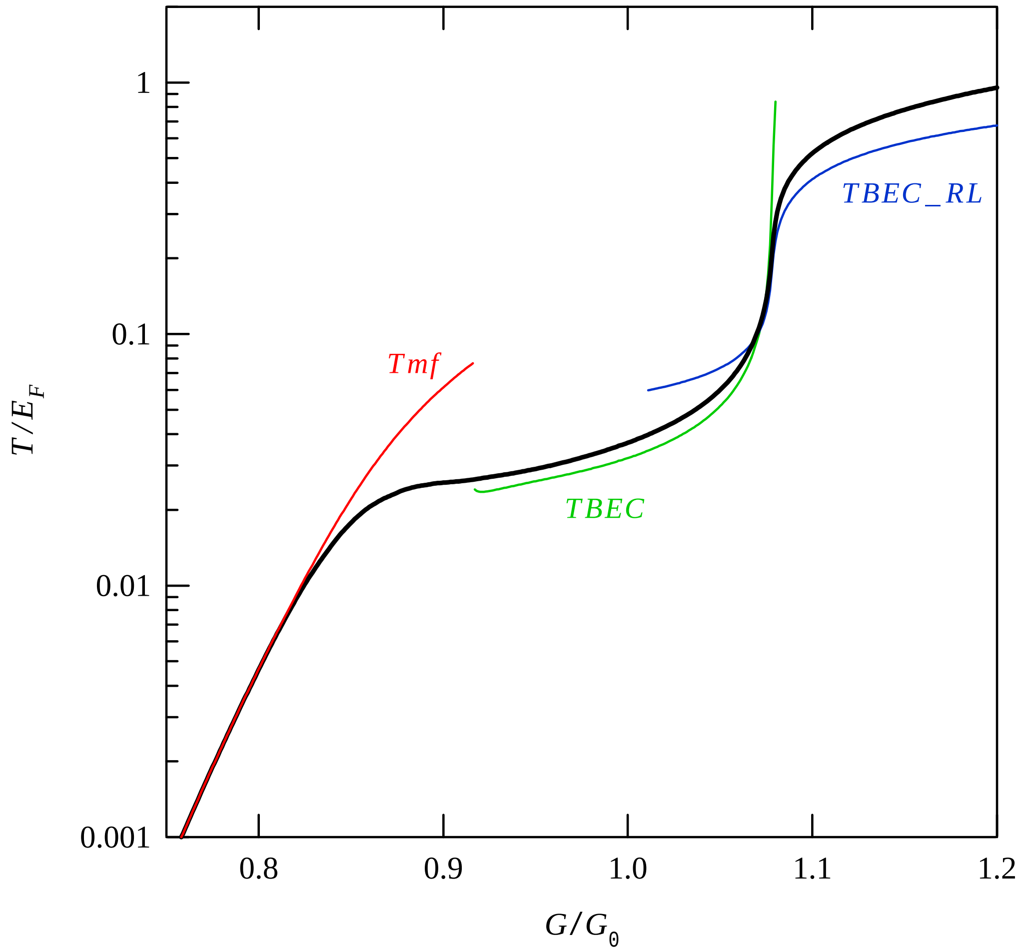
- ✓ mu-rho eq. with a fixed charge density N_{total}

  $N_{total} = N_{fermion} + N_{boson}$, where $N_{boson} = N_{bound} + N_{unstable}$

- ✓ gap eq. at critical temperature T_c
- ✓ (Thouless criterion)

$$m/\Lambda = 0.2\,,\quad k_{\mathrm{F}}/m = 0.5\ \ \left(N_{\mathrm{total}} = 2N_{\mathrm{c}}N_{\mathrm{f}}\cdot k_{\mathrm{F}}^3/6\pi^2\right),\quad G_0\cdot\Lambda^2 \simeq 2.47$$

BCS-BEC transition in QM



Y.Nishida and H. Abuki, hep-ph/0504083

Gauge Fields in a Plasma

- **In the high T limit,** $(p, \omega, m_e << \mathrm{T})$

  (hard thermal loop (HTL) approximation)

thermal mass

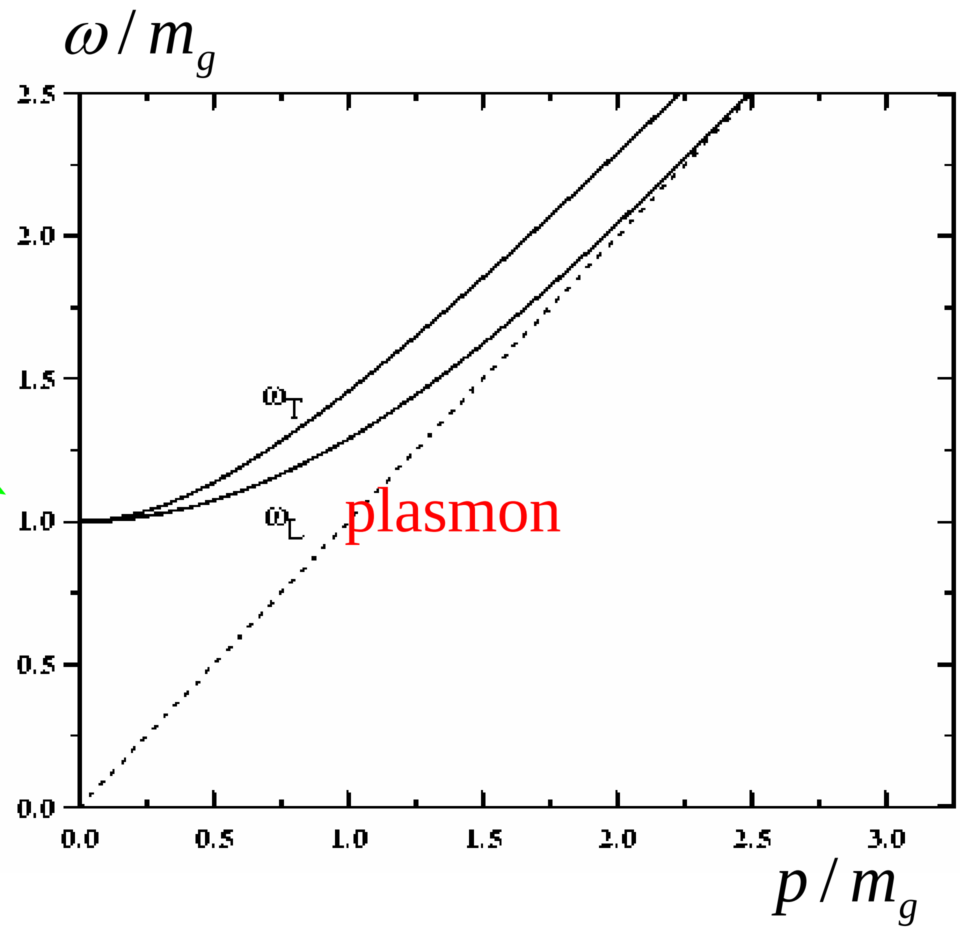
(= pole mass)

(= plasma frequency)

$$m_g^2 = \frac{1}{3} e^2 T^2$$

In QCD

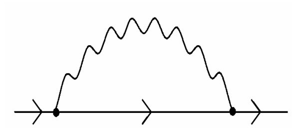
$$m_g^2 = \frac{1}{3}\left(1 + \frac{N_{\mathrm{f}}}{6}\right) g^2 T^2$$



Fermions in a Plasma

- 1-loop (g<<1) + HTL approx. $(p, \omega, m_q \ll \mathrm{T})$

$$\Sigma(\omega, p) =$$

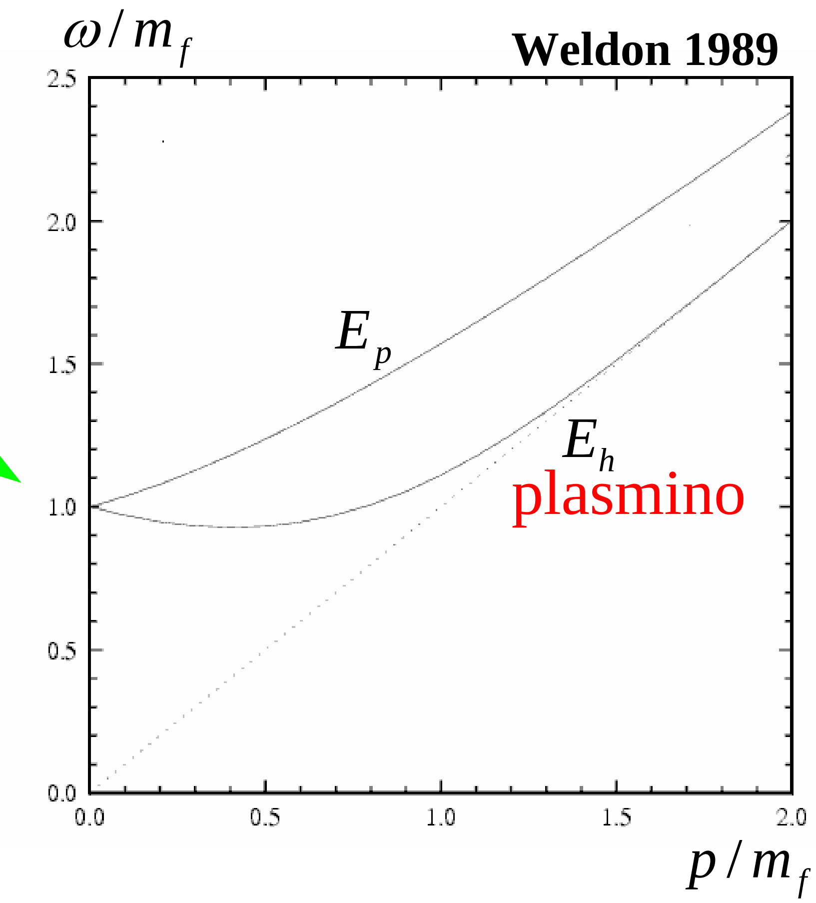


thermal masses

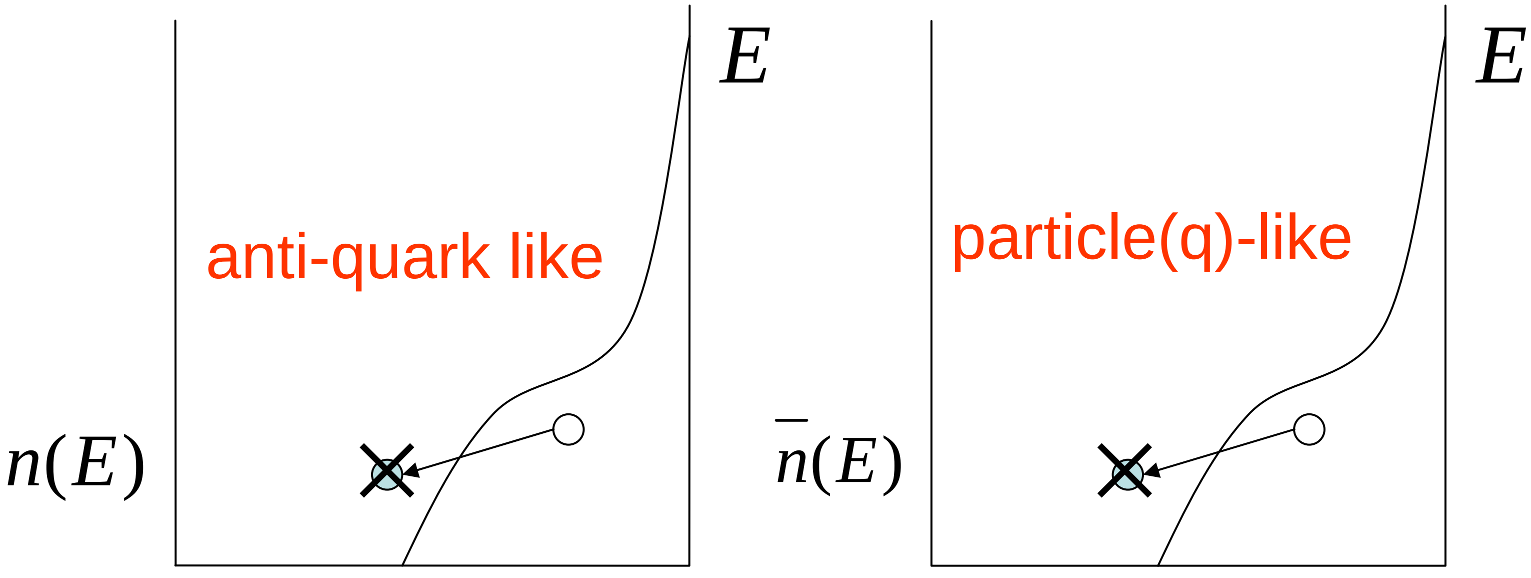
$$m_f^2 = \frac{1}{8} e^2 T^2 \quad \text{QED}$$

$$m_f^2 = \frac{1}{6} g^2 T^2 \quad \text{QCD}$$

chiral invariant



quark distribution　　　anti-q distribution



Plasmino excitation

How about when the temperature
Is lowered close to T_c ?

The wisdom of many-body theory tells us:

If a phase transition is of 2nd order or weak 1st order,

$\exists$ soft modes ; the fluctuations of the order parameter

eg. softening of 2+ phonon -→ quadrupole deformation

Gamow-Teller GR ; a soft mode of pion condensation (T.K., 1981)

Chiral Transition = a phase transition of QCD vacuum,

$\langle \bar{q}q \rangle$ being the order parameter. Lattice QCD;

There can be **hadronic excitations** (para pion and sigma)

as the soft mode of the chiral transition **in the ``QGP" phase.**

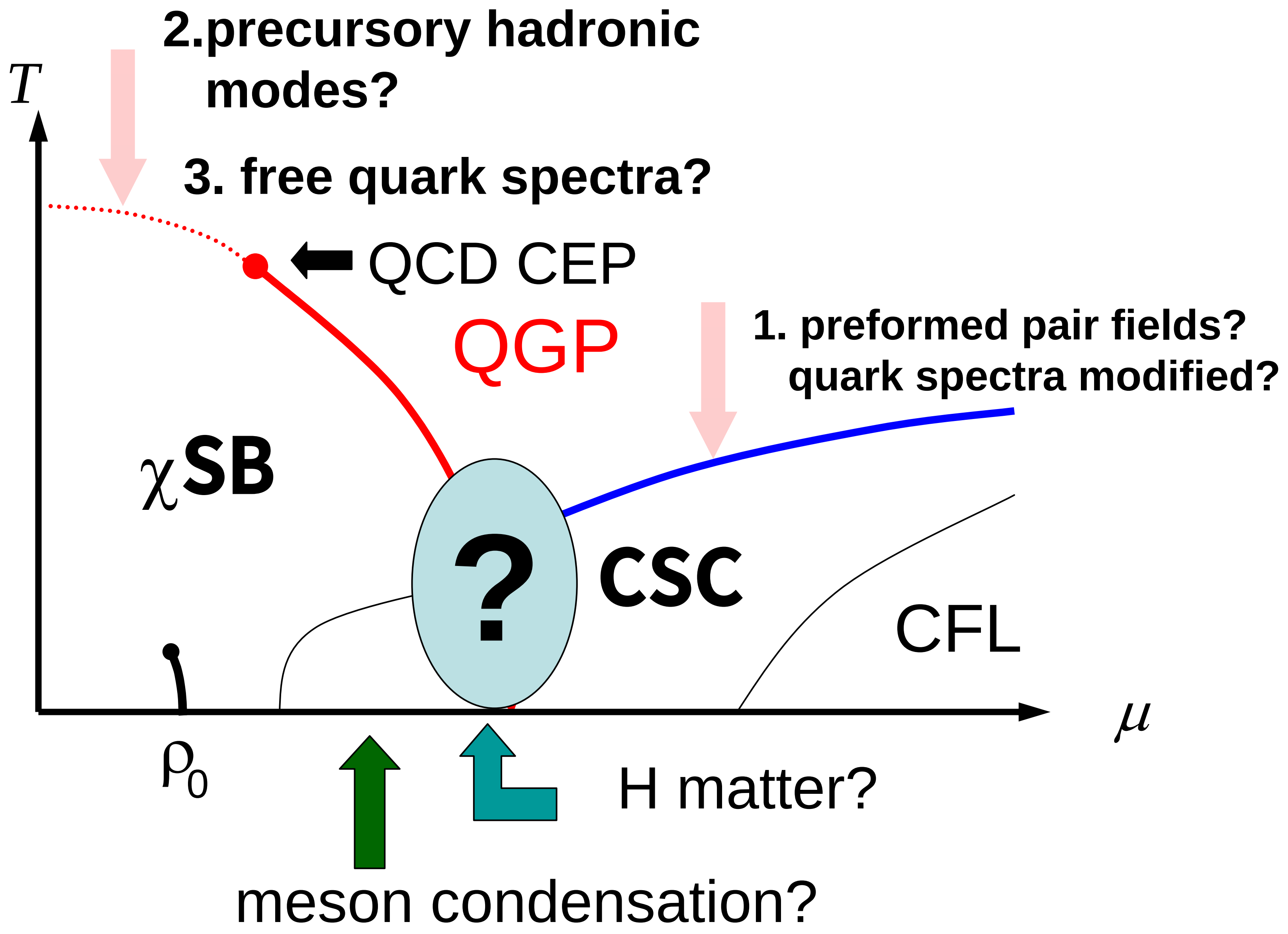
T. Hatsuda and T. K. , Phys. Rev. Lett.55('85)158; PLB71('84),1332

Prog. Theor. Phys 74 (1985), 765:

Cf. T<T_c ; the σ meson becomes the soft mode of

chiral restoration at $T \neq 0$ and/or $\rho_B \neq 0$: $m_\sigma \to 0,\ \Gamma_\sigma \to 0$

QCD phase diagram and quasi-particles

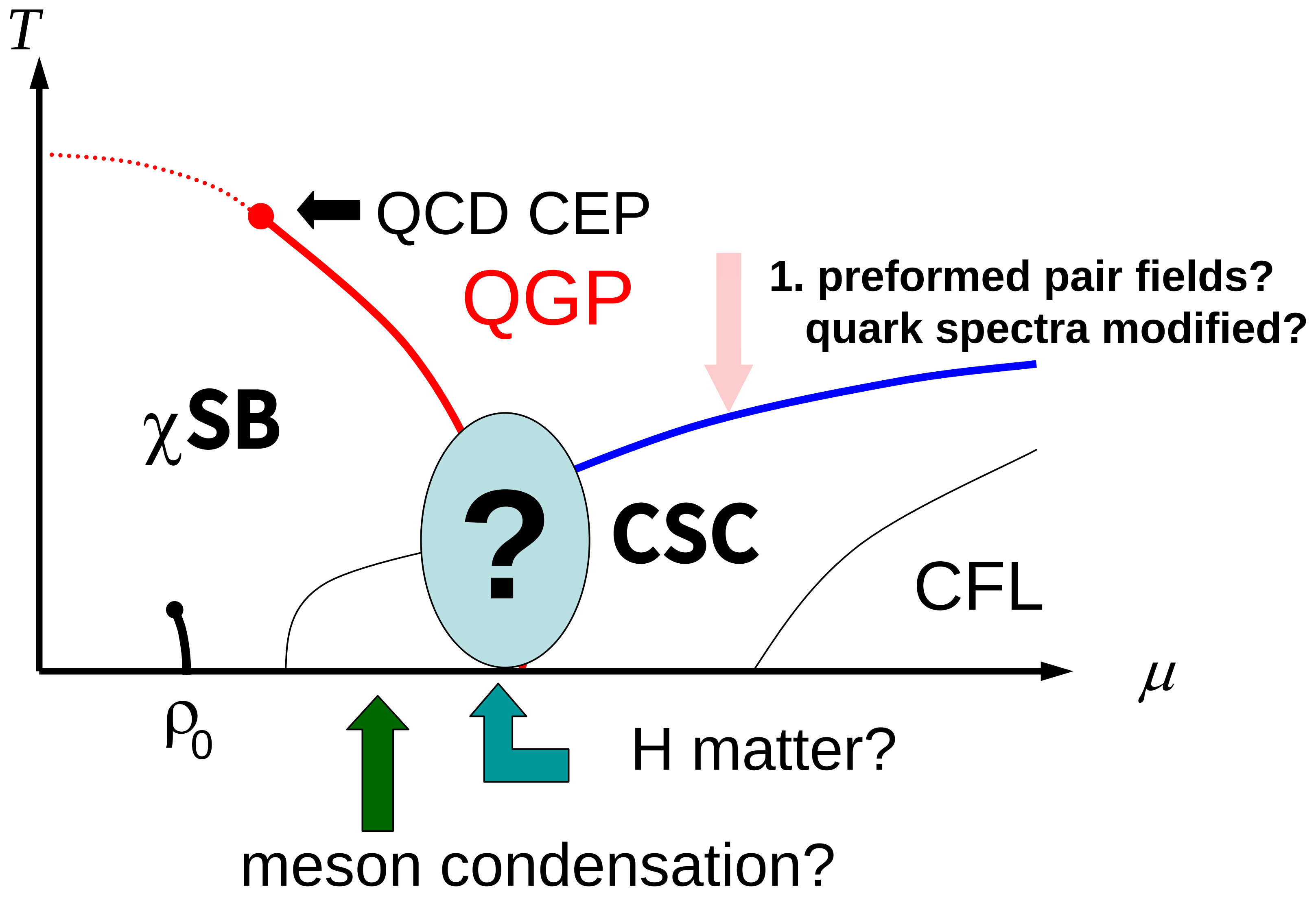


2. Precursory Phenomena of Color Superconductivity in Heated Quark Matter

Ref. M. Kitazawa, T. Koide, T. K. and Y. Nemoto,

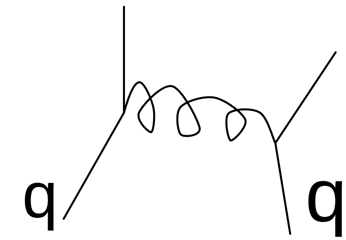
Phys. Rev. D65,091504 (2002); D70, 0965003 (2004)

QCD phase diagram



Color Superconductivity; diquark condensation

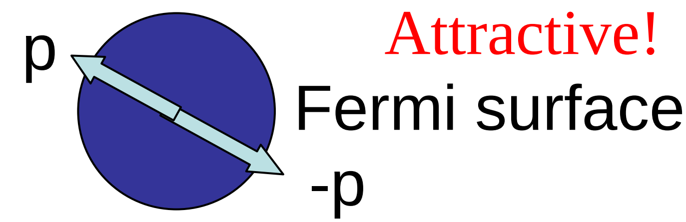
- Dense Quark Matter:
  - quark (fermion) system
  - with attractive channel in one-gluon exchange interaction.



$[3]_c \times [3]_c = [\bar{3}]_c + [6]_c$

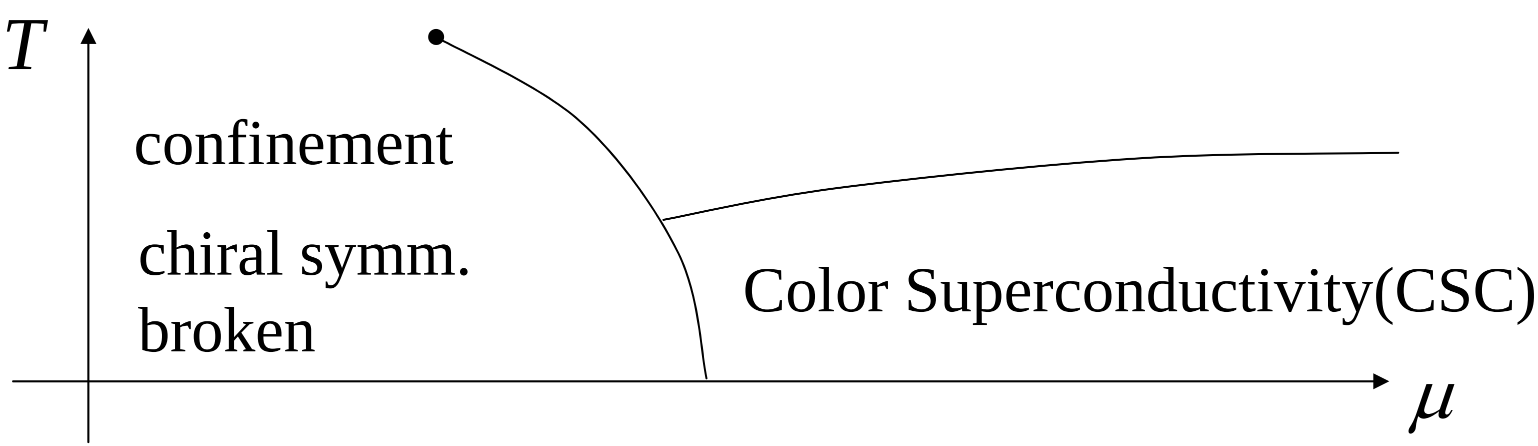
Attractive!

⇨ Cooper instability at sufficiently low T

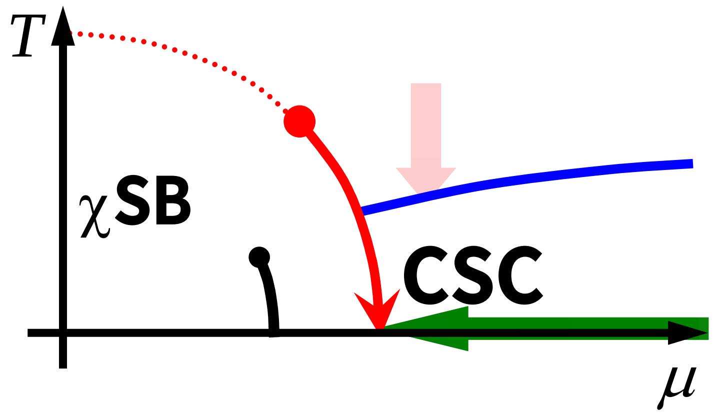


⇨ $SU(3)_c$ gauge symmetry is broken!

- Δ~100MeV at moderate density μ_q~ 400MeV



Nature of CSC in Intermediate Density (μ_q~500MeV)



- gap Δ ~ 100MeV at T=0
- order of T_c ~ 50MeV

$$\Delta / E_F \sim 0.1$$

in electric SC
$\Delta / E_F \sim 0.0001$

owing to Strong coupling nature of QCD

implies,

Short correlation length of Cooper pairs ⇨ Large fluctuation of the pair field is expected even at T>0 !

➡ may be relevant to newly born neutron stars
or intermediate states in heavy-ion collisions
(GSI, J-PARC)

Collective Mode in CSC

Response Function of Pair Field

Linear Response

- external field: $H_{ex} = \int d\mathbf{x} \left(\Delta_{ex}^{\dagger} \bar{\psi}^{C} i\gamma_5 \tau_2 \lambda_2 \psi + \text{h.c.} \right)$

  ⇒ expectation value of induced pair field:

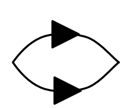
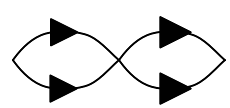
$$\left\langle \bar{\psi}(x) i\gamma_5 \tau_2 \lambda_2 \psi^{C}(x) \right\rangle_{ex} = i \int_{t_0}^{t} ds \left\langle \left[H_{ex}(s), O(\mathbf{x}, t) \right] \right\rangle$$

$$\begin{cases} \Delta_{ind}(x) = -2G_C \left\langle \bar{\psi}(x) i\gamma_5 \tau_2 \lambda_2 \psi^{C}(x) \right\rangle_{ex} = \int dt' \int d\mathbf{x} D^{R}(x, x') \Delta_{ex}(x') \\ D^{R}(\mathbf{x}, t) = -2G_C \underbrace{\left\langle \left[\bar{\psi}(x) i\gamma_5 \tau_2 \lambda_2 \psi^{C}(x), \bar{\psi}(0) i\gamma_5 \tau_2 \lambda_2 \psi^{C}(0) \right] \right\rangle \theta(t)} \end{cases}$$

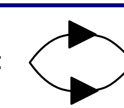
⤷ Retarded Green function

- Fourier transformation ⇒ $\Delta^{\dagger}(\mathbf{k}, \omega_n)_{\mathrm{ind}} = \mathcal{D}(\mathbf{k}, \omega_n) \Delta^{\dagger}(\mathbf{k}, \omega_n)_{\mathrm{ext}}$

  with Matsubara formalism

- RPA approx.: $\mathcal{D}(\mathbf{k}, \omega_n) =$  $+$  $+ \cdots$

$$= -\frac{G_C Q(\mathbf{k}, \omega_n)}{1 + G_C Q(\mathbf{k}, \omega_n)}$$

with $Q(\mathbf{k}, \omega_n) =$ 

After analytic continuation to real time,

$$\begin{aligned} D^R(\mathbf{k},\omega) &= -G_c Q(\mathbf{k},\omega)/(1+G_c Q(\mathbf{k},\omega)), \\ &\equiv -G_c Q(\mathbf{k},\omega)\cdot \Xi(\mathbf{k},\omega) \\ \Xi^{-1}(\mathbf{k},\omega) &\equiv 1+G_c Q(\mathbf{k},\omega). \end{aligned}$$

The spectral function;

$$\rho(\mathbf{k},\omega) = -\frac{1}{\pi}\mathrm{Im}D^R(\mathbf{k},\omega)$$

An important obeservation: at $T = T_c$;

$$\Xi^{-1}(\mathbf{k}=\mathbf{0},\omega=0)=0$$

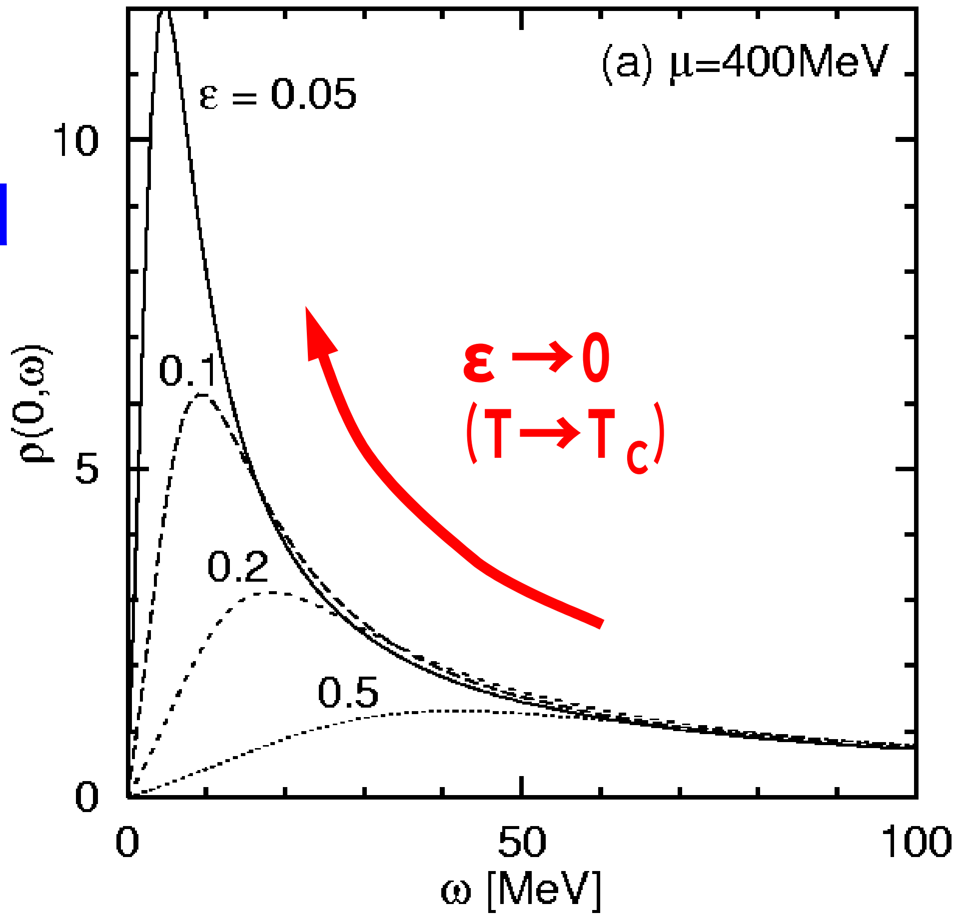
Equivalent with the gap equation (Thouless criterion)

Precursory Mode in CSC

(Kitazawa, Koide, Nemoto and T.K., PRD 65, 091504(2002))

Spectral function of the pair field at T>0

$$\rho(\mathbf{k}=\mathbf{0},\omega)$$



at k=0

$$\varepsilon = \frac{T - T_C}{T_C}$$

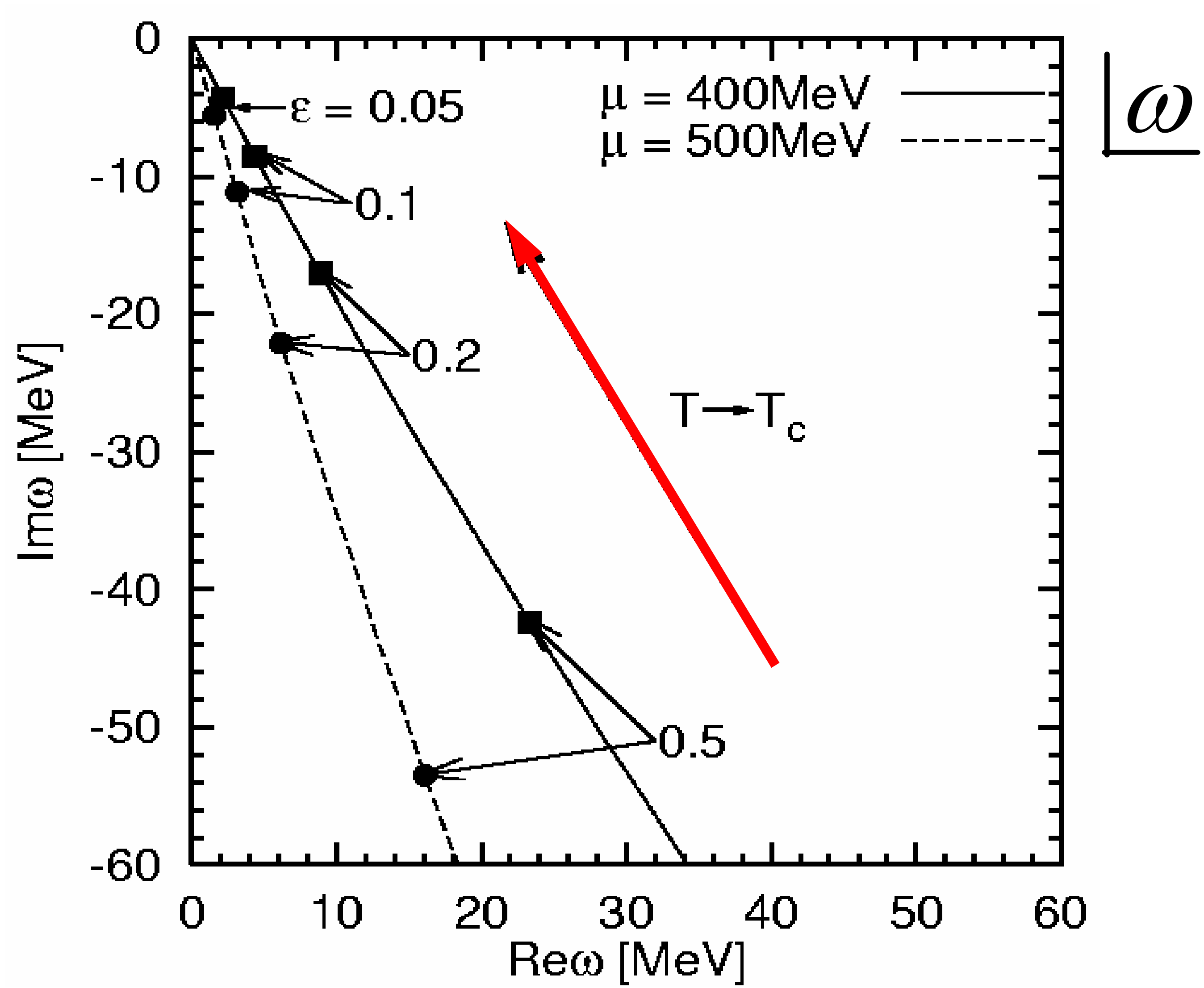
- As T is lowered toward T_C,

  The peak of ρ becomes sharp. (Soft mode) ⇒ Pole behavior

- The peak survives up to **ε~ 0．2** ⇔ electric SC： **ε ~ 0.005**

The pair fluctuation as the soft mode;

--- movement of the pole of the precursory mode---



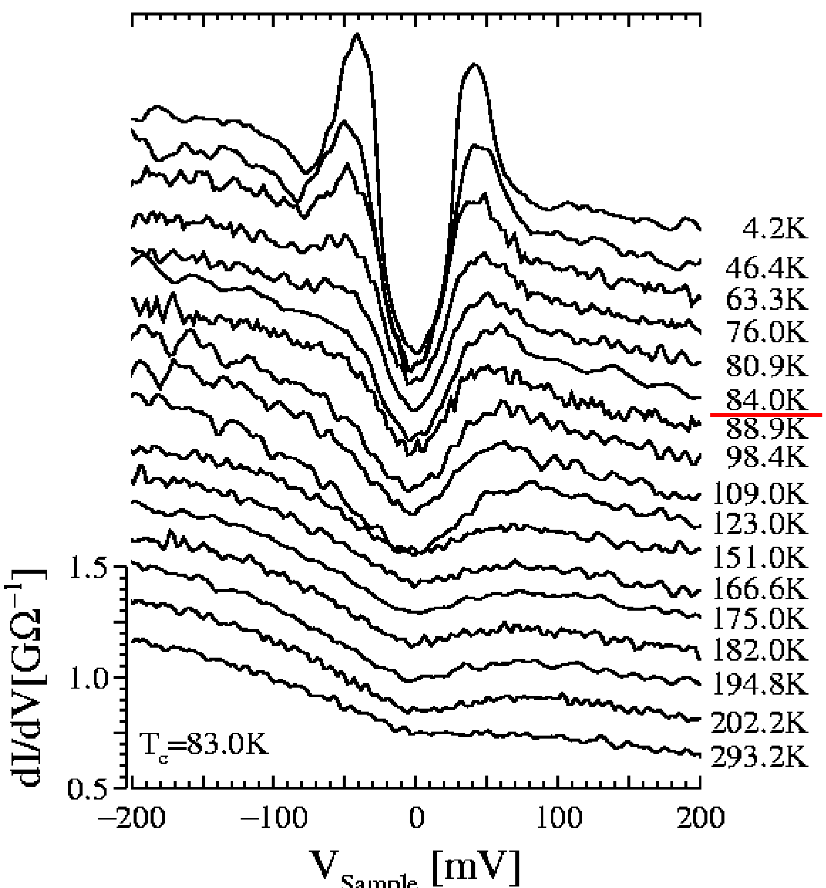
How does the soft mode affect the quark spectra?

---- formation of pseudogap ----

Ref. M. Kitazawa, T. Koide, T. K. and Y. Nemoto
Phys. Rev. D70, 956003(2004);
hep-ph/-5-2035; Prog. Theor. Phys. in press,
M.Kitazawa, T.K. and Y. Nemoto, hep-ph/0505070

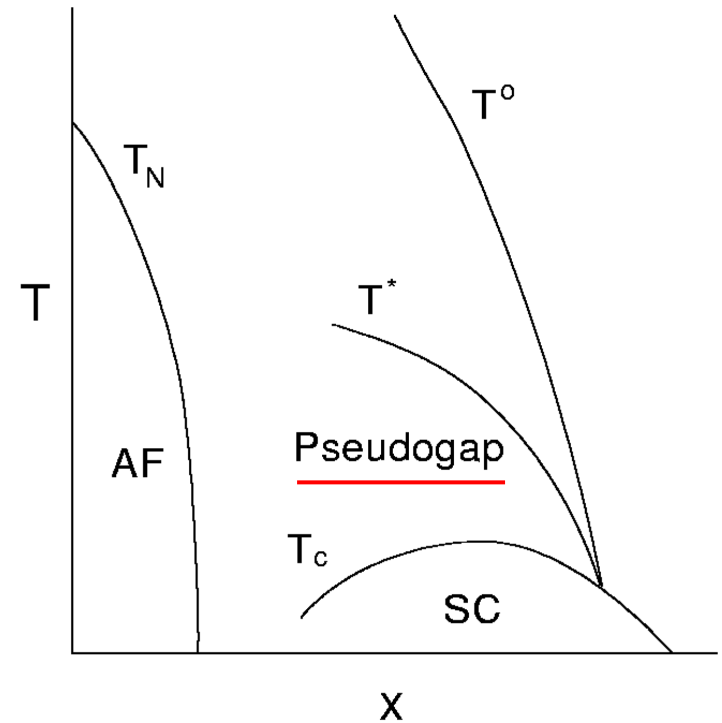
Pseudogap

:Anomalous depression of the density of state near the Fermi surface in the normal phase.



Renner et al.('96)

Conceptual phase diagram of HTSC cuprates

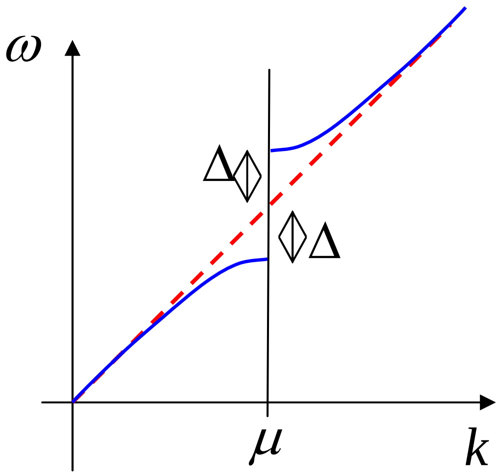


The origin of the pseudogap in HTSC is **still controversial**.

Density of State in BCS theory

- Quasi-particle energy:

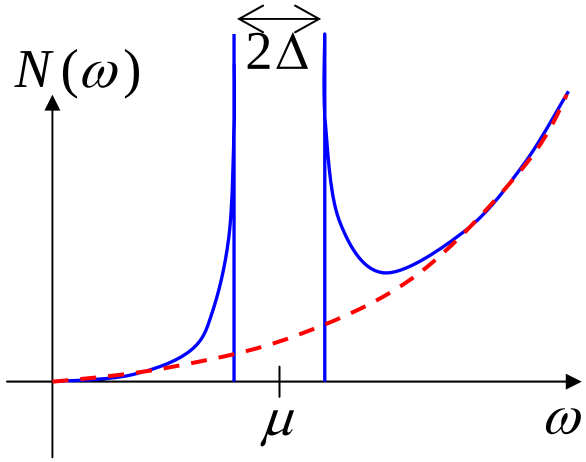
$$\omega = \operatorname{sgn}(k-\mu)\sqrt{(k-\mu)^2 + \Delta^2}$$



- Density of State:

$$N(\omega) \propto k^2 \frac{dk}{d\omega}$$

$$\frac{d\varepsilon}{dk} = \frac{k-\mu}{\sqrt{(k-\mu)^2 + \Delta^2}}$$



⇨ The gap on the Fermi surface becomes smaller as T is increased, and it closes at T_c.

Density of State $N(\omega)$

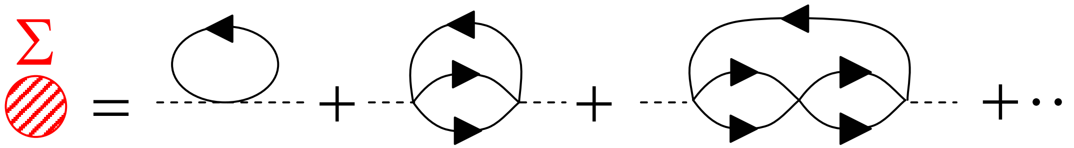
$$N = \int d^3x \left\langle \bar{\psi}\gamma^0\psi \right\rangle$$

$$N(\omega) = \int \frac{d^3\mathbf{k}}{(2\pi)^3}\rho^0(\mathbf{k},\omega) \Leftarrow \rho^0(\mathbf{k},\omega) = \frac{1}{4}\mathrm{Tr}\left[\gamma^0 \,\mathrm{Im}\, G^R(\mathbf{k},\omega)\right]$$

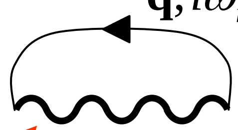
T-matrix Approximation

$$G(\mathbf{k},\omega_n) = \frac{1}{G^0(\mathbf{k},i\omega_n) - \Sigma(\mathbf{k},i\omega_n)}$$

$\Sigma(\mathbf{k},\omega_n) =$



$\equiv$



$$= T\sum_m \int \frac{d^3q}{(2\pi)^3}\Xi(\mathbf{k}+\mathbf{q},\omega_n+\omega_m)G^0(\mathbf{q},\omega_m)$$

$$G^0(\mathbf{k},i\omega_n) = \left[(i\omega_n+\mu)\gamma^0 - \mathbf{k}\cdot\vec{\gamma}\right]^{-1} = \longrightarrow$$:free progagator

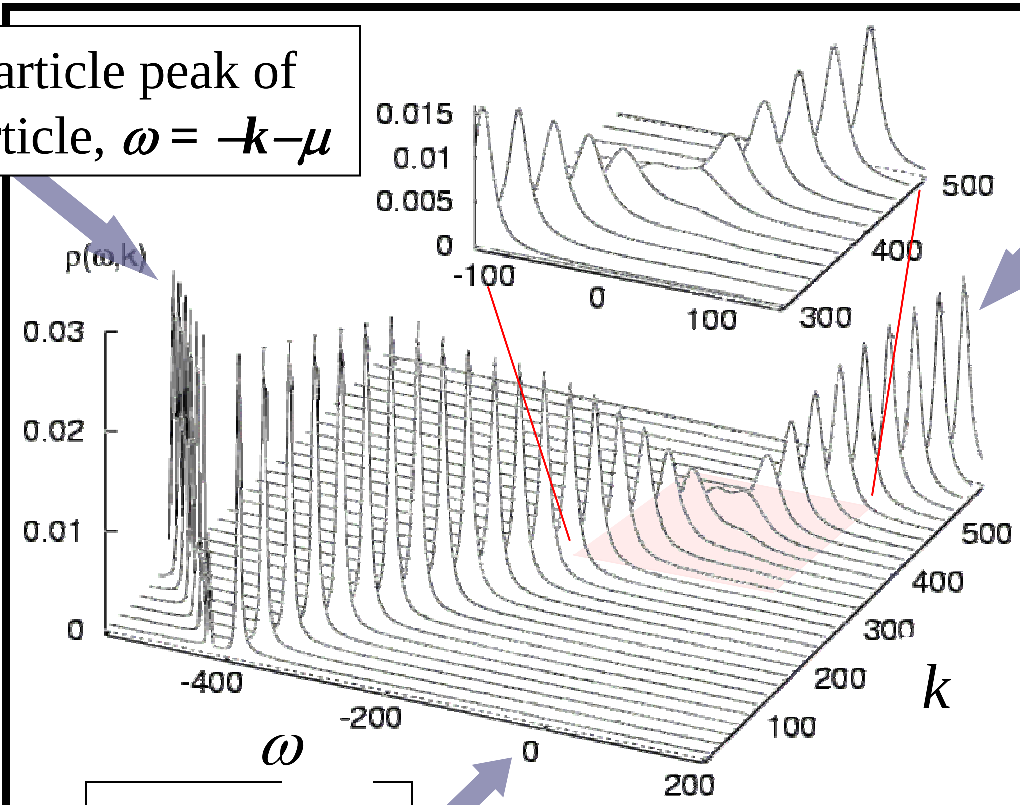
Soft mode

1-Particle Spectral Function

μ= 400 MeV
ε=0.01

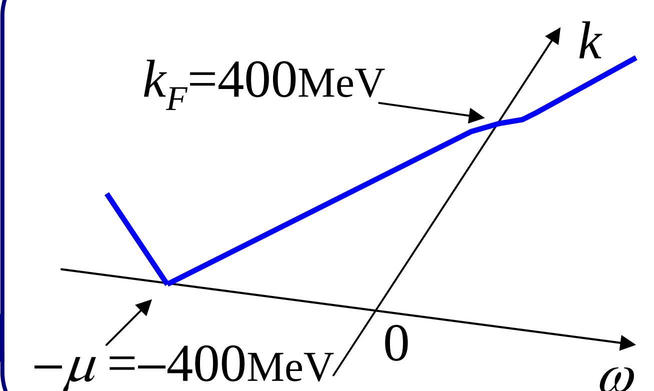
quasi-particle peak of anti-particle, $\omega = -k - \mu$

quasi-particle peak, $\omega = \omega_{-}(k) \sim k - \mu$

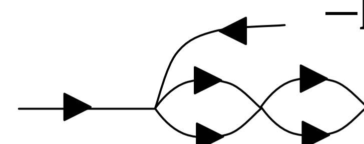


Fermi energy

position of peaks



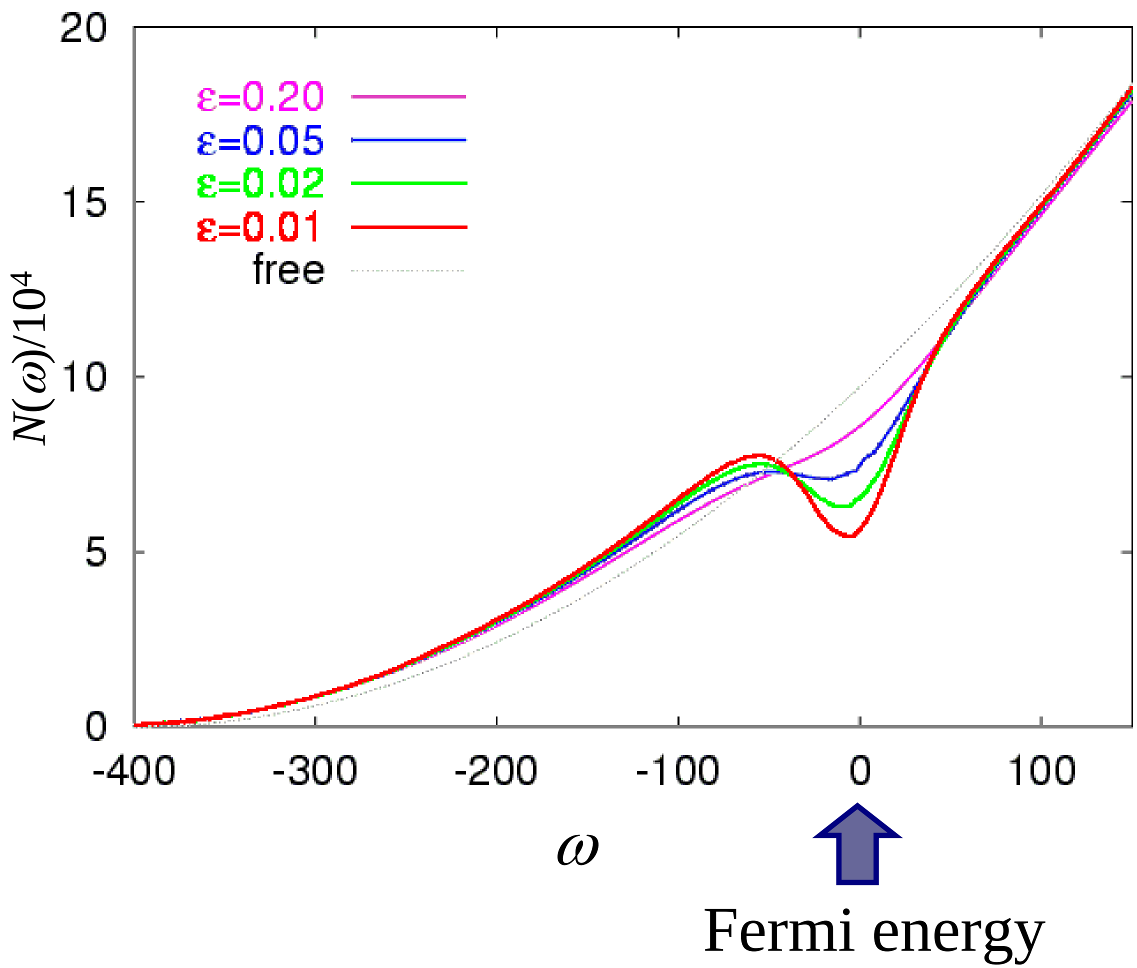
- quasi-particle peaks at $\omega = \omega_{-}(k) \sim k - \mu$ and $\omega = -k - \mu$.
- Quasi-particle peak has a depression around the Fermi energy due to **resonant scattering**.



$-\mathbf{k}, k-\mu$: on-shell

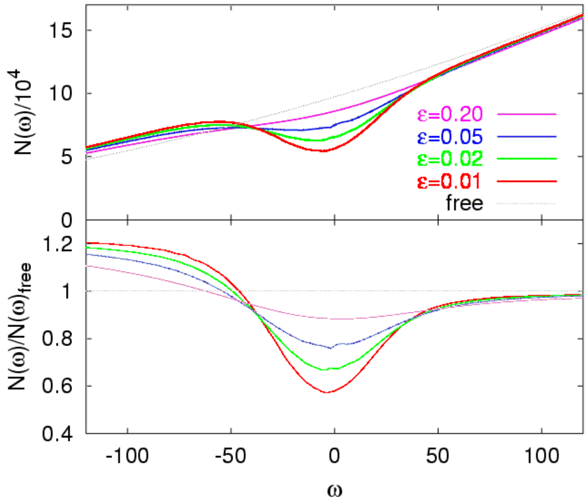
$\mathbf{0}, 0$: the soft mode

Density of state of quarks in heated quark matter



μ = 400 MeV

- **<u>Pseudogap structure manifests itself in $N(\omega)$.</u>**
- The pseudogap survives up to <u>$\varepsilon \sim 0.05$</u> <u>(5% above T_C).</u>

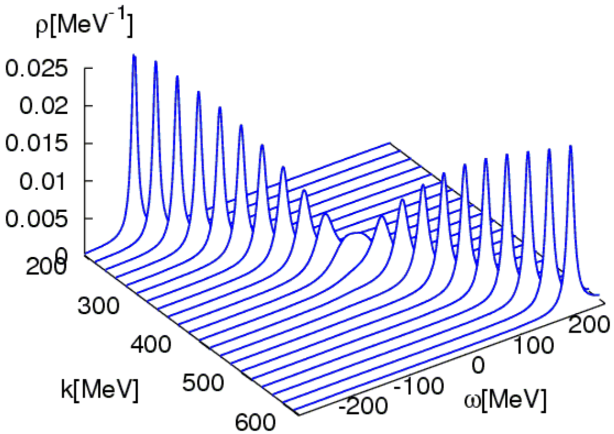


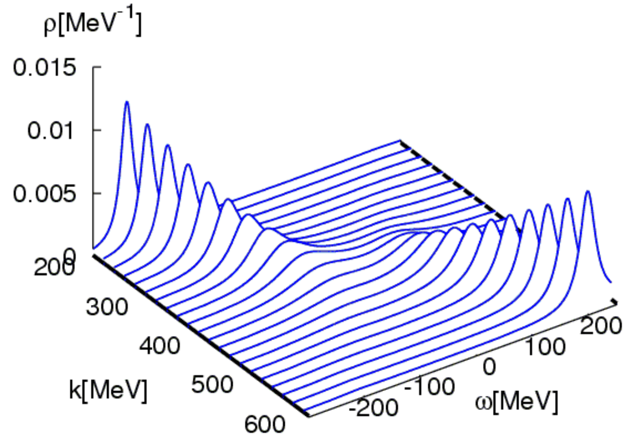
Diquark Coupling Dependence

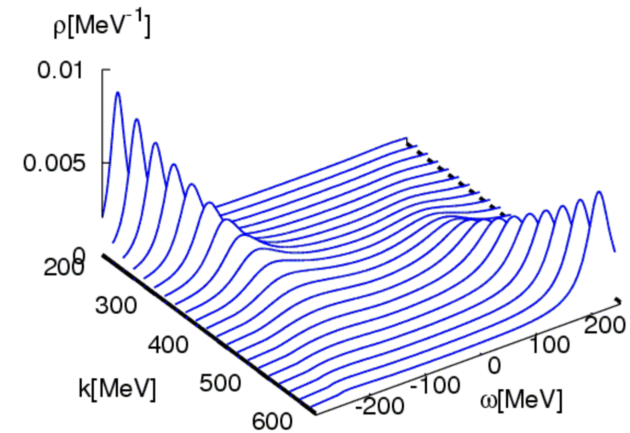
μ= 400 MeV
ε=0.01

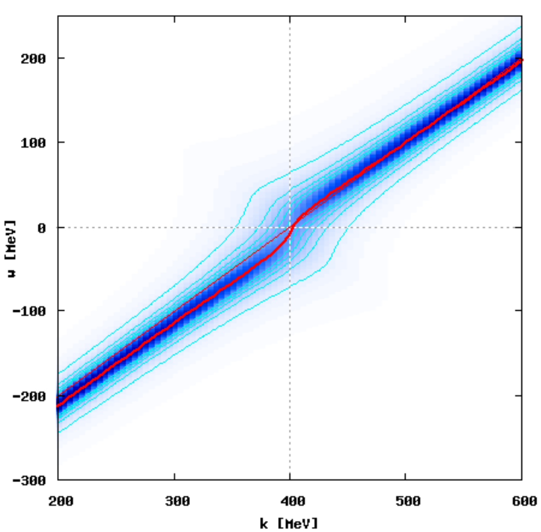
stronger diquark coupling G_C

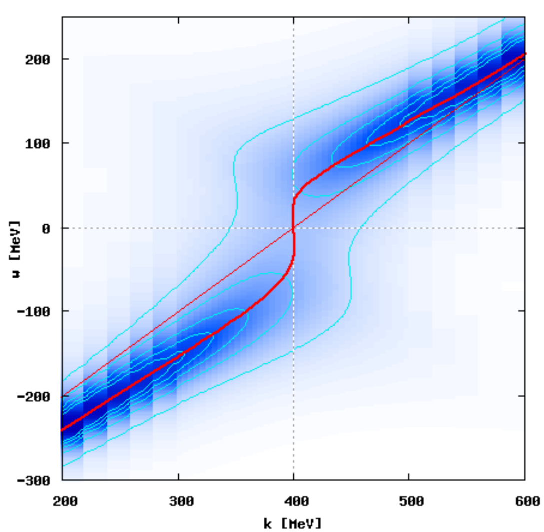
G_C	×1.3	×1.5

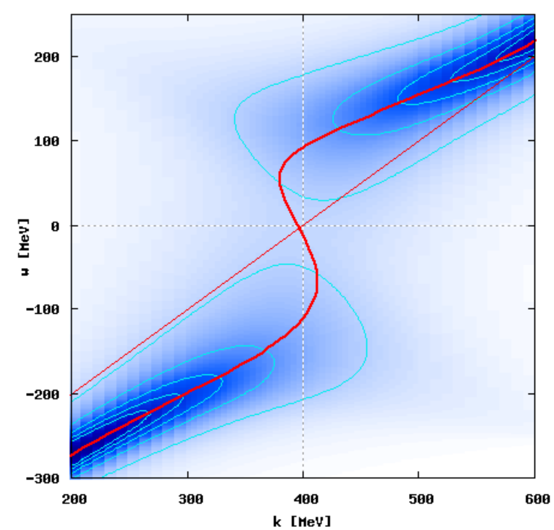






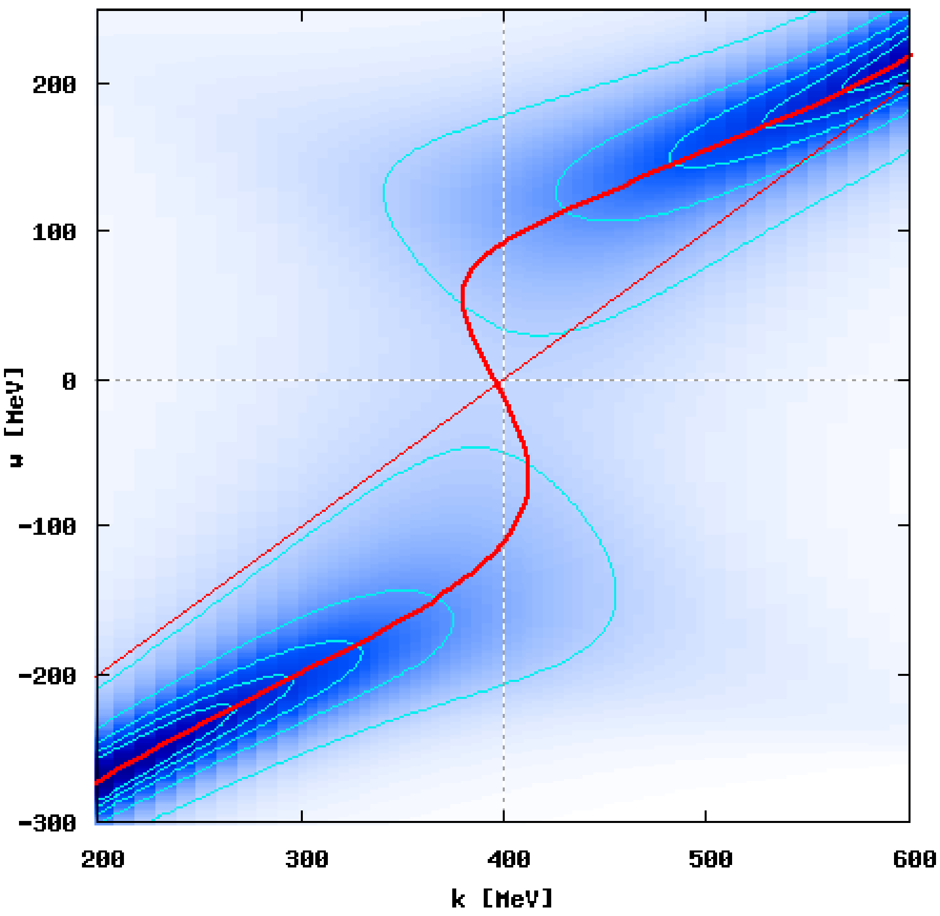




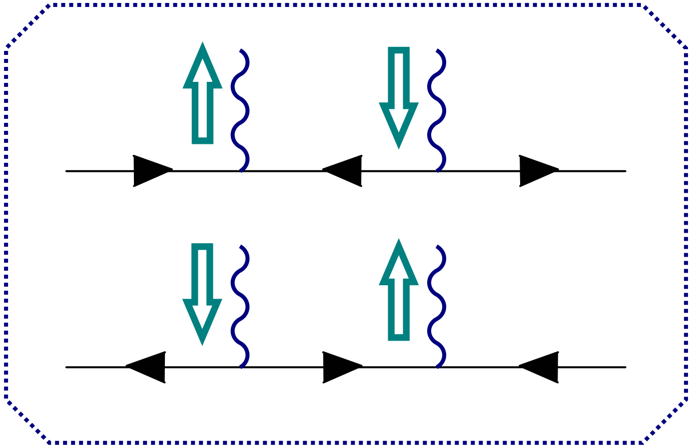


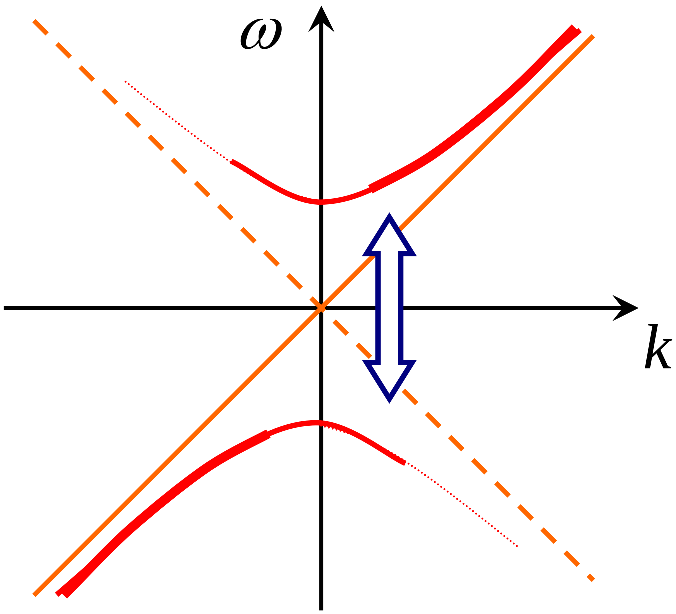
Resonance Scattering of Quarks

G_C=4.67GeV^{-2}



Mixing between quarks and holes





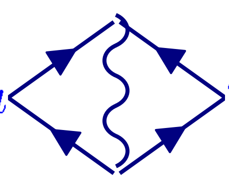
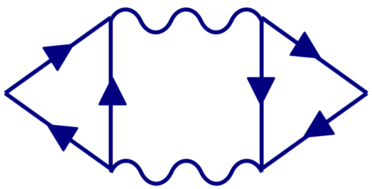
(M.Kitazawa, Y. Nemoto, T.K. hep-ph/0505070)

Summary of this section

- There may exist a wide T region where the precursory soft mode of CSC has a large strength.
- The soft mode induces the pseudogap, Typical Non-Fermi liquid behavior ⇐ resonant scattering

Future problems:

effects of the soft mode on . **H-I coll. & proto neutron stars**

⇒ eg.1) γ_μ  γ_ν  ⇒ anomalous lepton pair production

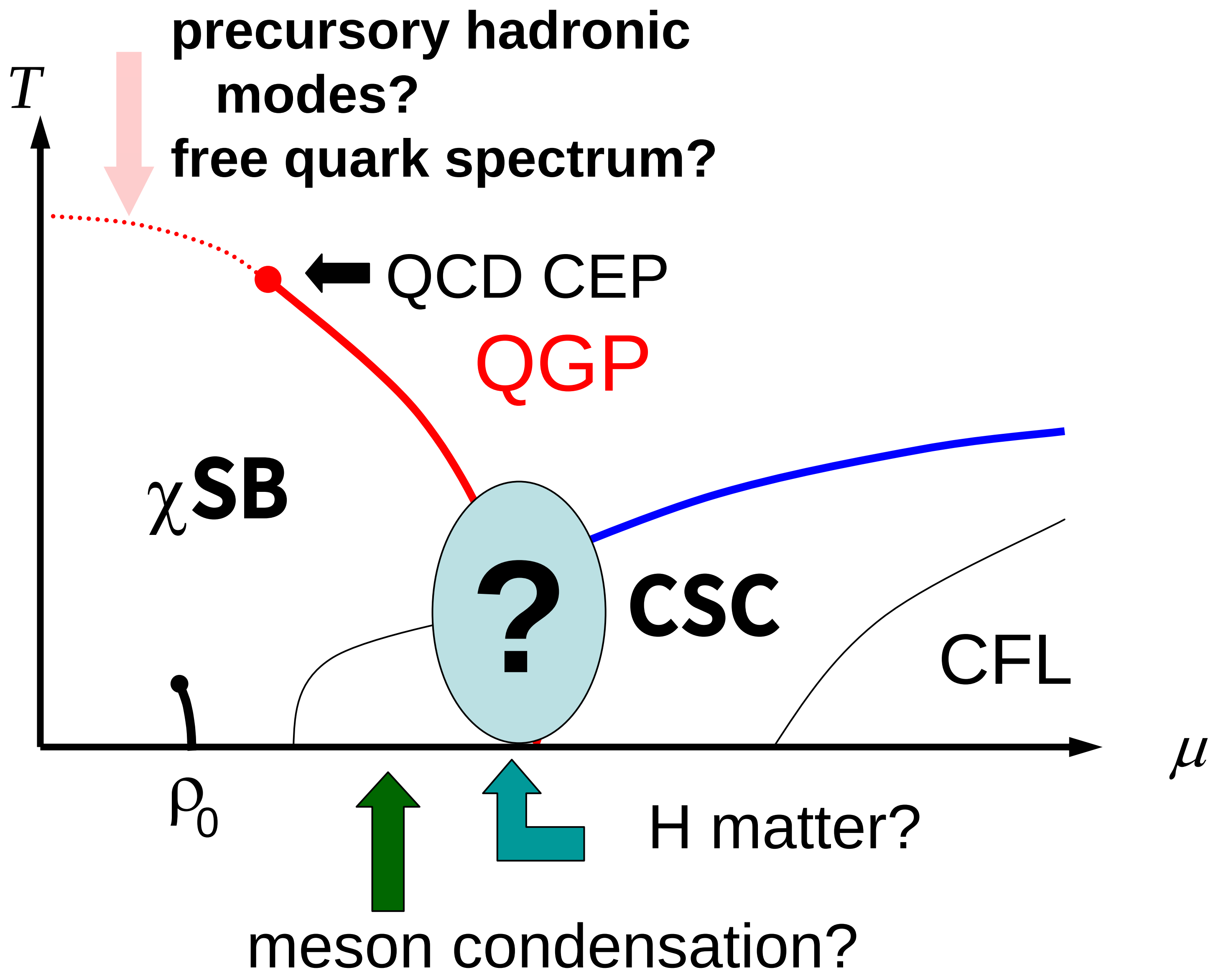
collective mode:  (ω,k)

eg.2) C_V ; cooling of newly born stars

(M.Kitazawa and T.K in progress.)

3. Precursory Hadronic Mode
and
Single Quark Spectrum
above
Chiral Phase Transition

QCD phase diagram and quasi-particles



Chiral Transformation

$$\frac{1-\gamma_5}{2}q_i \equiv q_{iL} \to L_{ij}q_{jL},$$ (left handed)

$$\frac{1+\gamma_5}{2}q_i \equiv q_{iR} \to R_{ij}q_{jR},$$ (right handed)

$$L = \exp(i\boldsymbol{\theta}_L \cdot \boldsymbol{\lambda}/2) \equiv U(\boldsymbol{\theta}_L),$$

$$R = \exp(i\boldsymbol{\theta}_R \cdot \boldsymbol{\lambda}/2) = U(\boldsymbol{\theta}_R),$$

$$\boldsymbol{\theta}_{L,R} \cdot \boldsymbol{\lambda} = \sum_{a=0}^{8} \theta^a_{L,R}\lambda^a.$$

Chirality: $\gamma_5 q_L = -q_L, \quad \gamma_5 q_R = q_R.$

For $N_f = 3$, the chiral transformation forms

:直積群 $U_L(3) \otimes U_R(3) \simeq (U_L(1) \otimes U_R(1)) \otimes SU_L(3) \otimes SU_R(3)$

Chiral Invariance of Classical QCD Lagrangian
in the chiral limit (**m=0**)

$$\bar{q}\gamma^\mu q = \bar{q}_L\gamma^\mu q_L + \bar{q}_R\gamma^\mu q_R$$

$$\rightarrow \quad \bar{q}_L L^\dagger \gamma^\mu L q_L + \bar{q}_R R^\dagger \gamma^\mu R q_R$$

$$= \quad \bar{q}_L\gamma^\mu q_L + \bar{q}_R\gamma^\mu q_R$$

$$= \bar{q}\gamma^\mu q$$ invariant!

In the chiral limit (m=0),

$\bar{q}\,\gamma^\mu D_\mu q$; Chiral invariant

$$D_\mu = \partial_\mu - i g t^a A^a_\mu$$

$$\mathcal{L}^{cl}_0 = \bar{q}(i\gamma^\mu D_\mu - \cancel{m})q - \frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a$$;Chiral invariant!

The notion of Spontaneous Symmetry Breaking

Q^a the generators of a continuous transformation

$\partial^\mu j^a_\mu = 0$; $j^a_\mu(x)$ Noether current $Q^a = \int d\mathbf{x}\, j^a_0(x)$

eg. Chiral transformation for $SU_L(2) \otimes SU_R(2)$

$Q^a_5 = \int d\mathbf{x}\, \bar{q}\gamma^0\gamma_5\tau^a/2q$ Notice; $[iQ^a_5, \bar{q}(x)i\gamma_5\tau^b q(x)] = -\delta^{ab}\bar{q}(x)q(x)$

The two modes of symmetry realization in the vacuum $|0\rangle$:

a. Wigner mode $Q^a|0\rangle = 0 \quad \forall a$

b. Nambu-Goldstone mode $Q^a|0\rangle \neq 0 \quad \exists a$

The symmetry is spontaneously broken.

Now, $\langle 0|\bar{q}q|0\rangle = \langle 0|[Q^a_5, \bar{q}\gamma_5\tau^a q]|0\rangle$

$\langle 0|\bar{q}q|0\rangle \neq 0 \Rightarrow Q^a_5|0\rangle \neq 0$

Chiral symmetry is spontaneously broken!

Chiral invariant forms : $N_f = 2$

transformation:

Def.

:

:

,

Any function of ; Invariant!

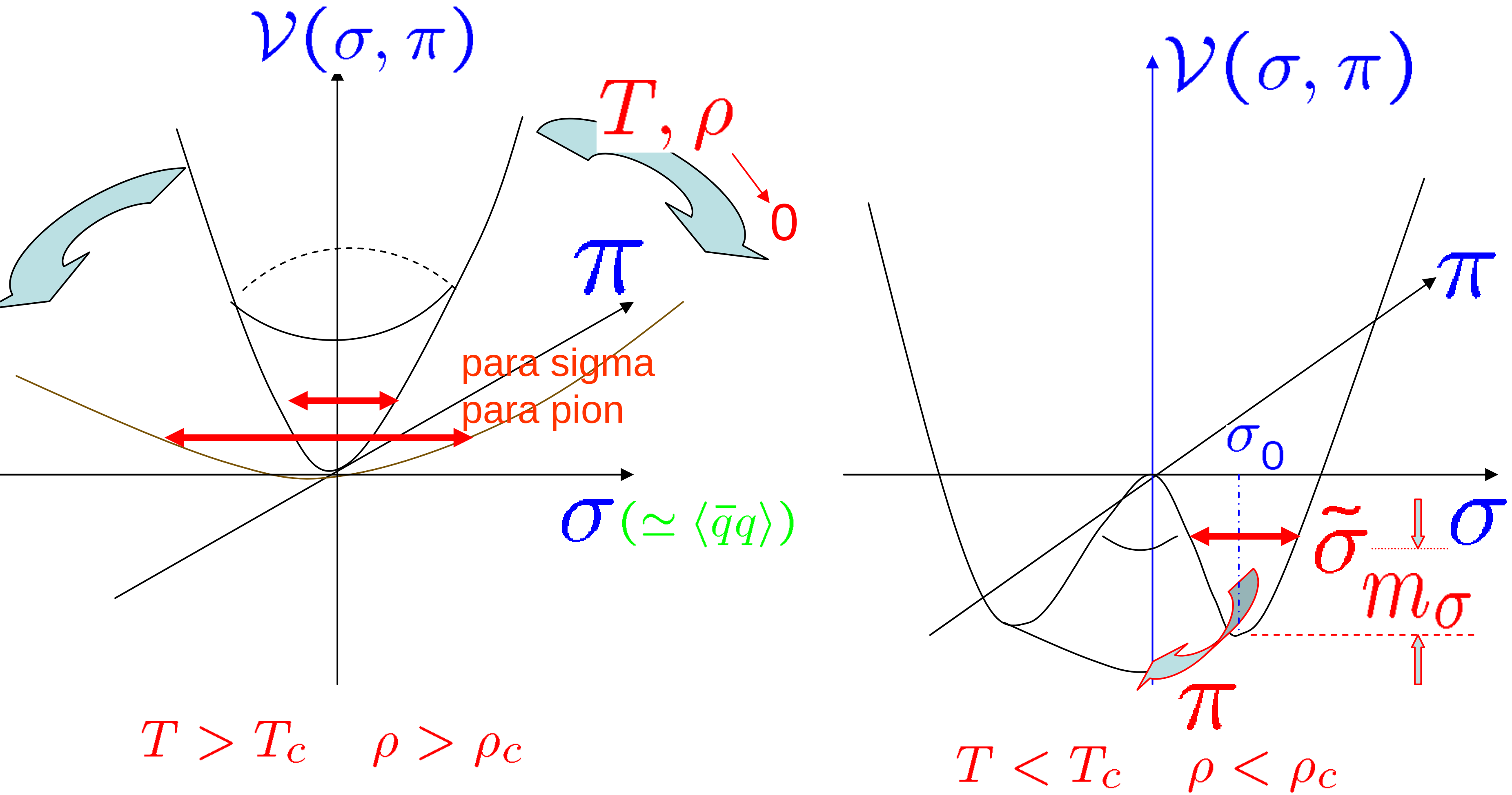
eg.1

(Linear sigma model)

eg.2

(Nambu-Jona-Lasinio model)

Chiral Transition and the collective modes



$$\sigma = \sigma_0 + \tilde{\sigma}$$

c.f. Higgs particle in WSH model

ϕ ; Higgs field ➡ $\phi = \langle \phi \rangle + \tilde{\phi}$

Higgs particle

Hadronic Modes in the ‘QGP’ Phase

The `para-sigma’ and `para-pion’

T. Hatsuda and T. K.,(1985)

The driving force leading to the phase transition should be strong enough to form the collective modes even at $T > T_c$

T. Hatsuda and T. K. , Phys. Rev. Lett.55(’85)158; PLB71(’84),1332 ; Prog. Theor. Phys 74 (1985), 765.

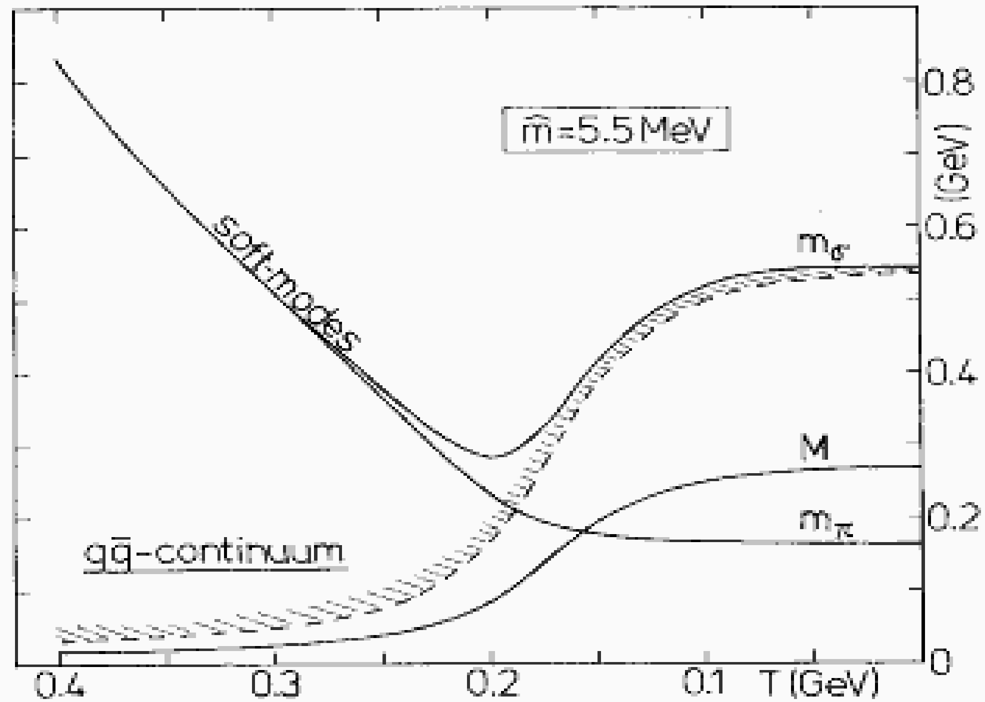


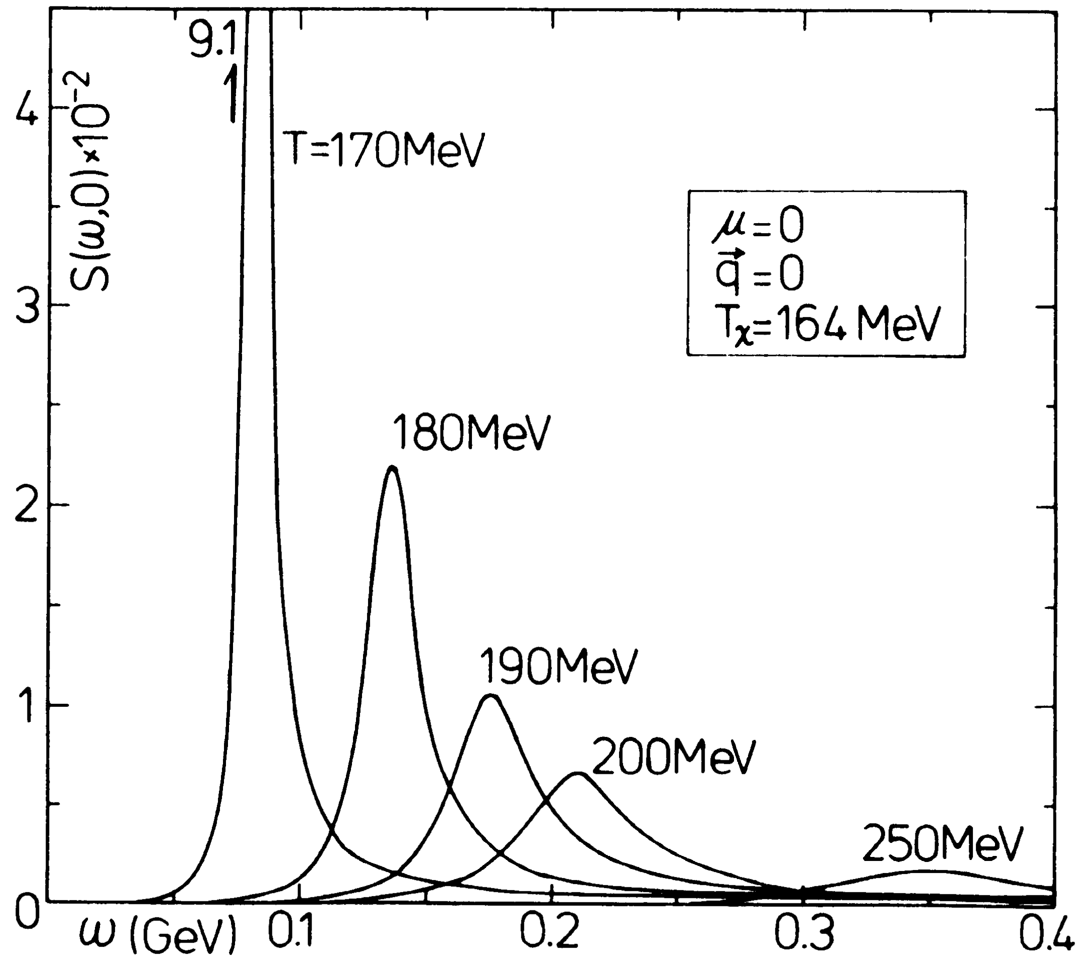
FIG. 3. Dynamical quark mass $M = M_D(T, \mu) + \hat{m}$, and the masses of σ mode (m_σ) and π mode (m_π). The dashed line denotes the $2M$ threshold from which the $q\bar{q}$ continuum starts.

Large ← T

The spectral function of the degenerate ``para-pion" and the ``para-sigma" at T>Tc for the chiral transition: Tc=164 MeV

T. Hatsuda and T.K. (1985)

The idea is rediscovered and revived :
E. Shuryak and I. Zahed, hep-ph/307267,
G.E. Brown,C.H. Lee, M. Rho and E. Shuryak, hep-ph/0312175;
M.Gyulassy and L.McLerran, Nucl-th/0405013



How does the soft mode affect a single quark spectrum near Tc?

Y. Nemoto, M. Kitazawa ,T. K. (in preapration)

Method

- **low-energy effective theory of QCD**

  4-Fermi type interaction (Nambu-Jona-Lasinio with 2-flavor)

$$L = \bar{q} i\gamma \cdot q + G_S[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2]$$

τ：SU(2) Pauli matrices

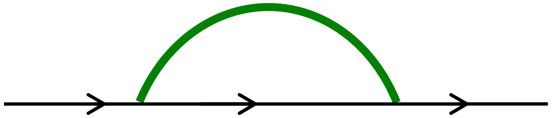
$G_S = 5.5 \cdot 10^{-6}\mathrm{GeV}^{-2}, \Lambda = 631\mathrm{MeV}$ $m_u = m_d = 0$ chiral limit

The parameters are determined so as to reproduce m_π and f_π in the chiral lim.

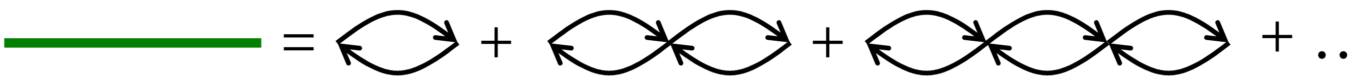
- **Chiral phase transition takes place at Tc=193.5 MeV(2nd order).**

- **Self-energy of a quark (above Tc)**

$$\Sigma(\omega_n, \vec{p}) = \mathrm{T}\sum_m \int \frac{\mathrm{d}^3 q}{(2\pi)^3} D(\omega_n - \omega_m, \vec{p}-\vec{q}) G_0(\omega_m, \vec{q})$$



$D(\omega_n, \vec{p}) =$



scalar and pseudoscalar parts

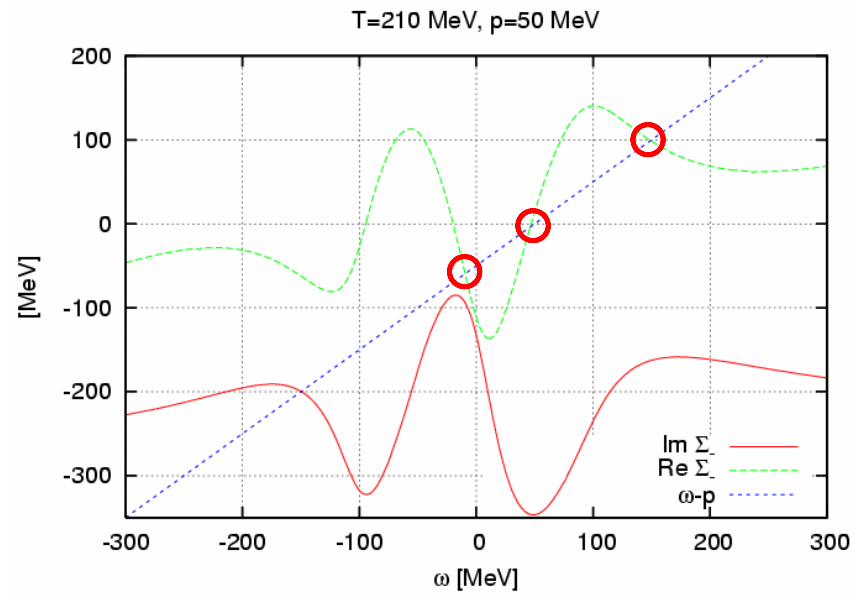
$\Sigma^R(\omega, p) = \Sigma(\omega_n, p)|_{i\omega_n = \omega + i\varepsilon}$: imaginary time → real time

Self-Energy and Spectral Func.

self-energy

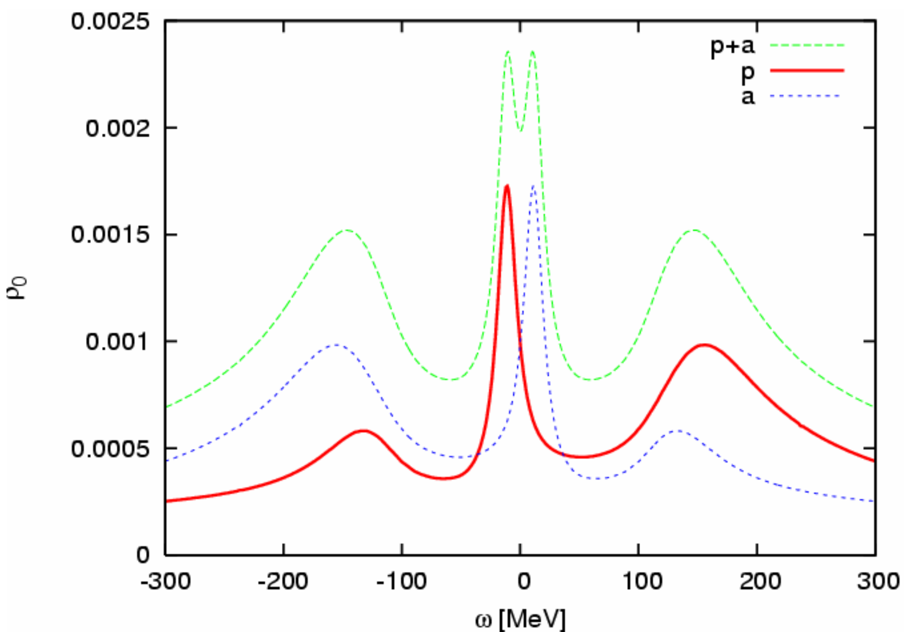
dispersion relation for a quark

$$\omega - |\vec{p}| - \mathrm{Re}\,\Sigma\_(\omega, p) = 0$$



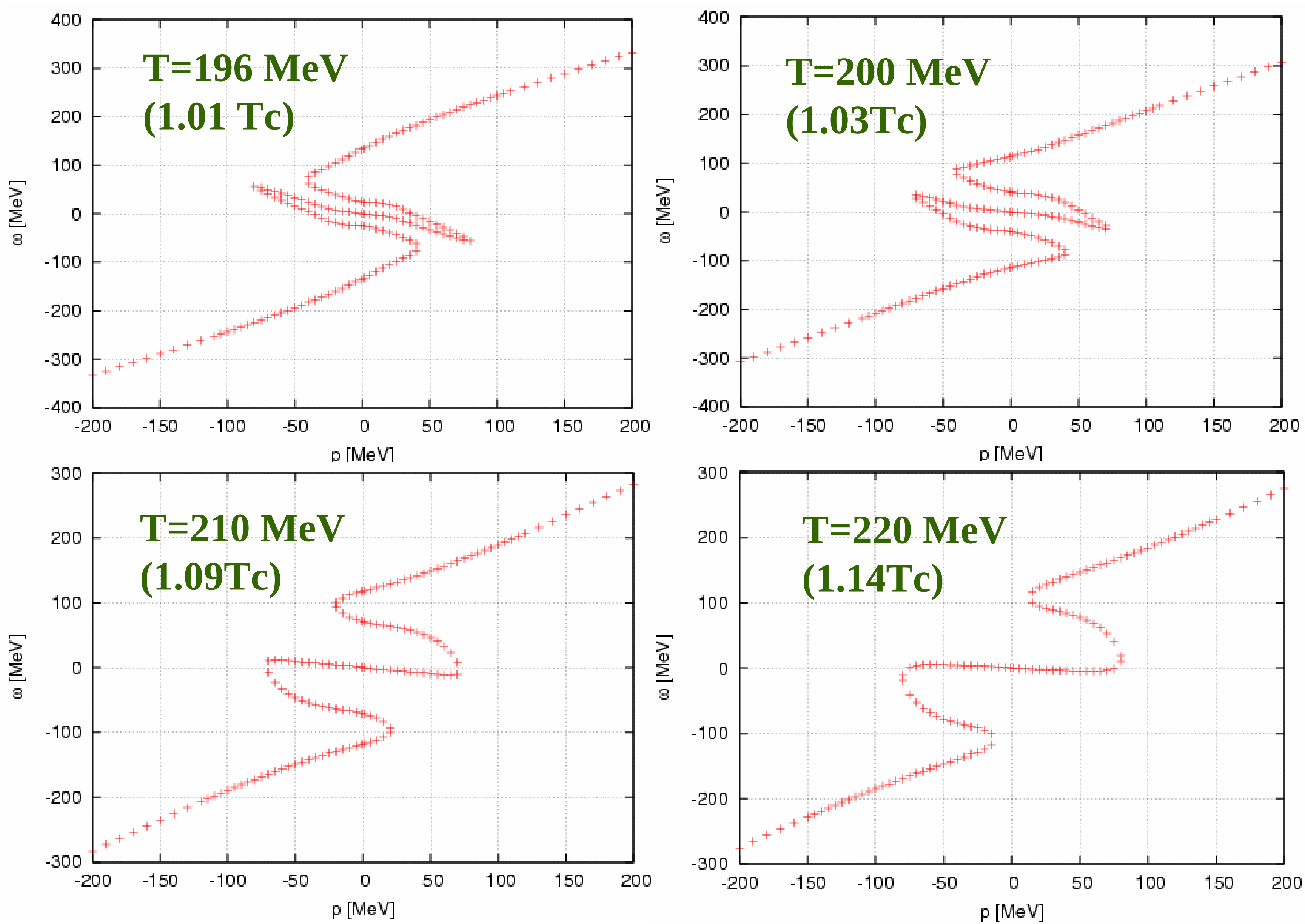
spectral function

three peaks: one is at ω>0 and the others are at ω<0.

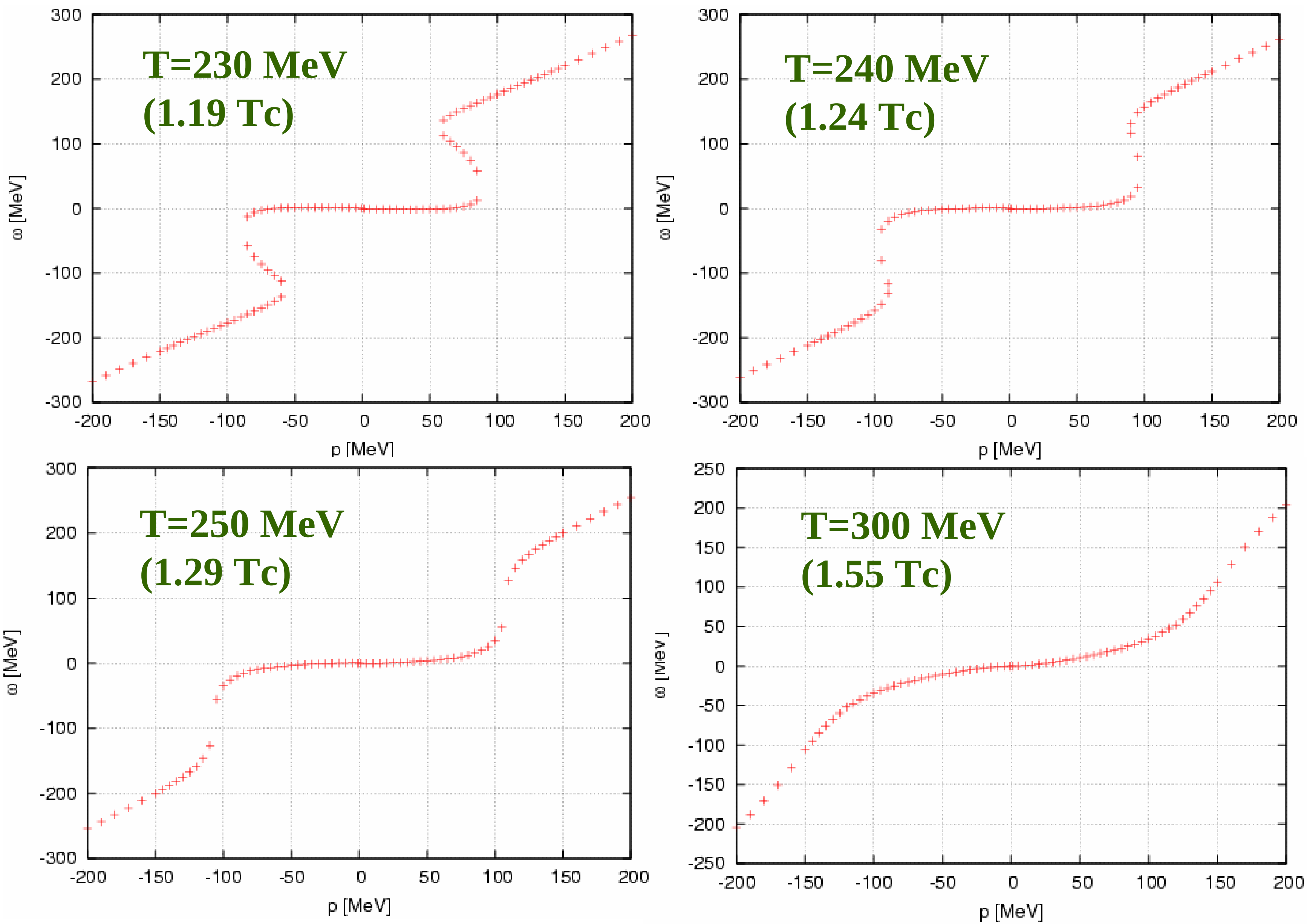


Antiquark sector is similar.

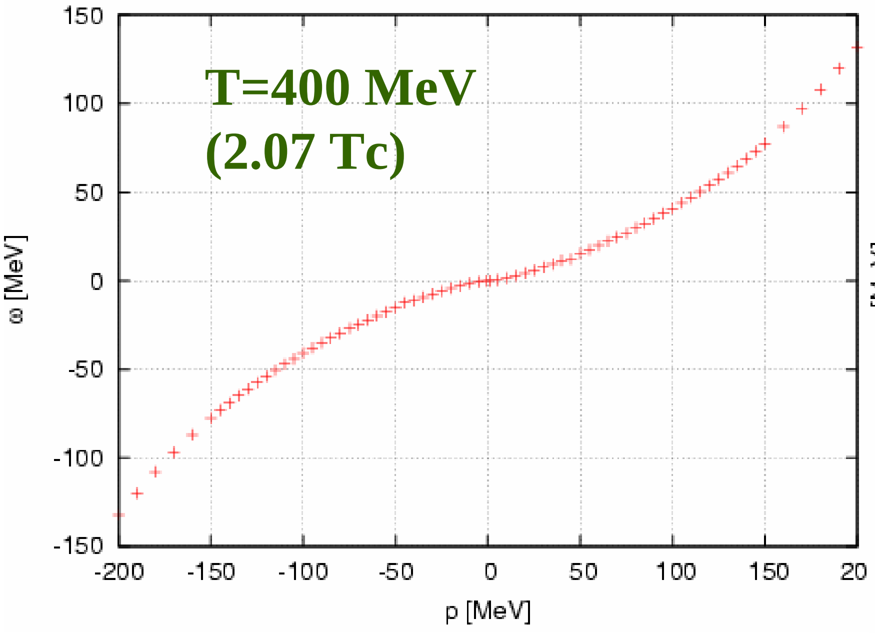
Dispersion Relations of Quarks

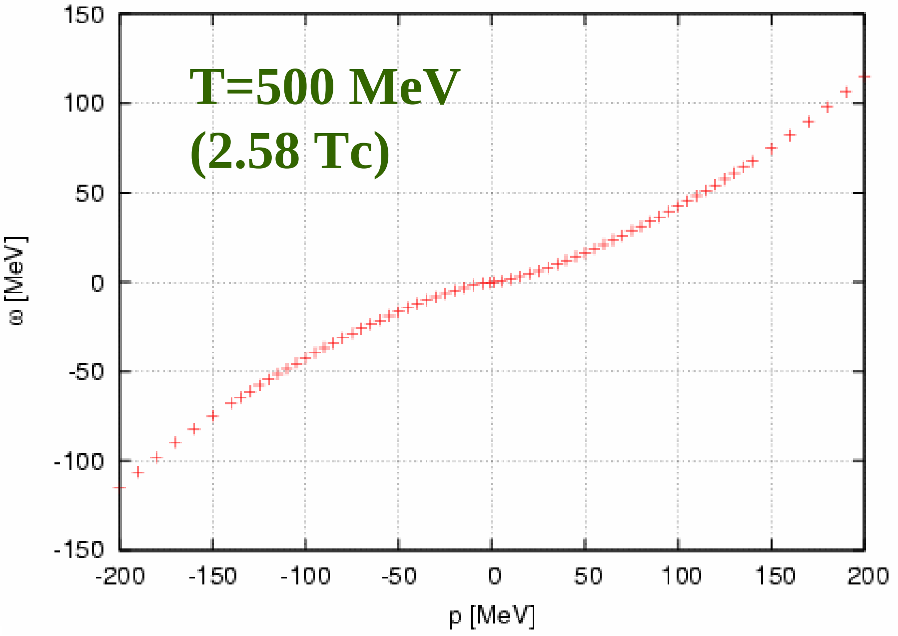


Dispersion Relations

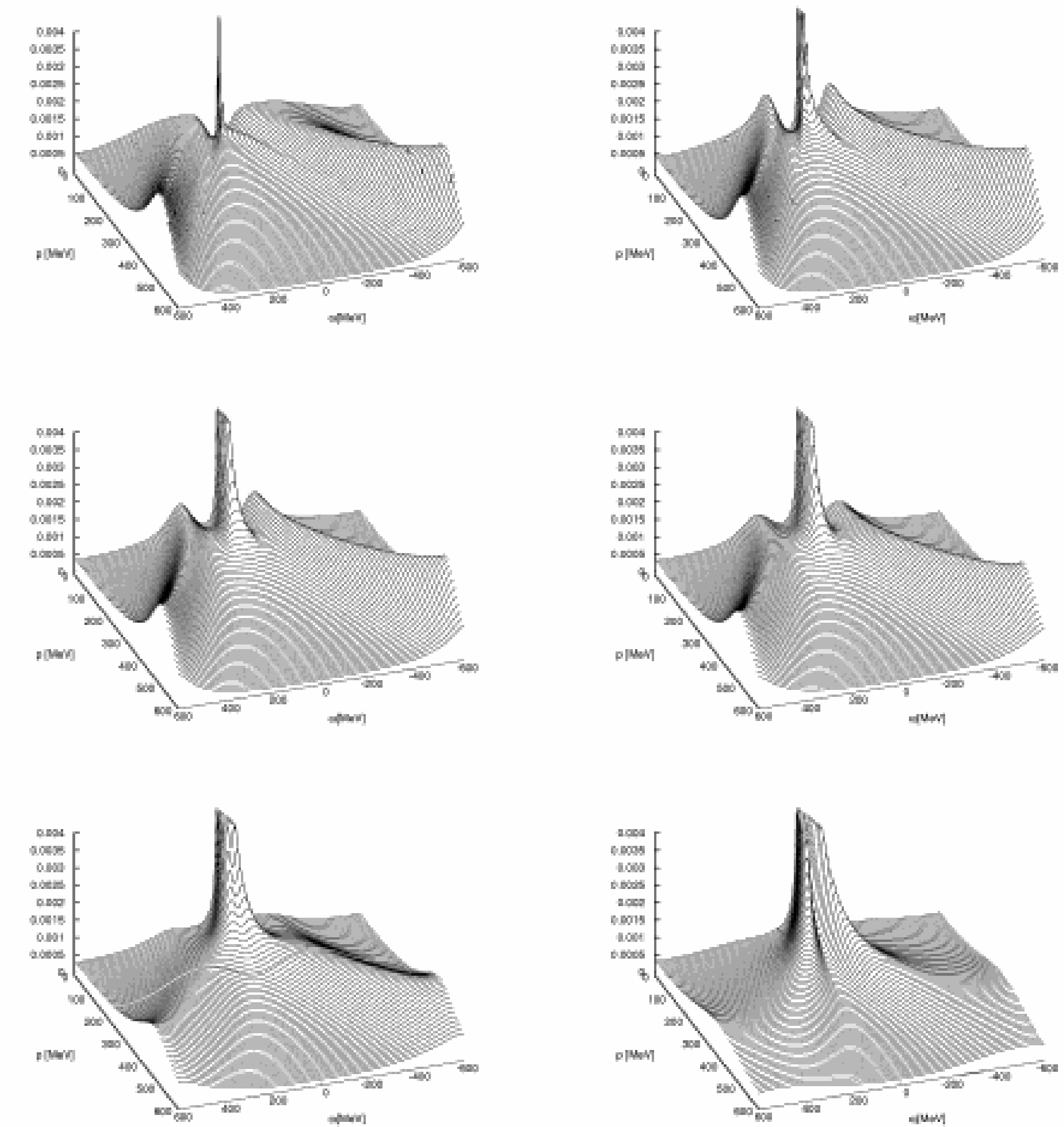


Dispersion Relations

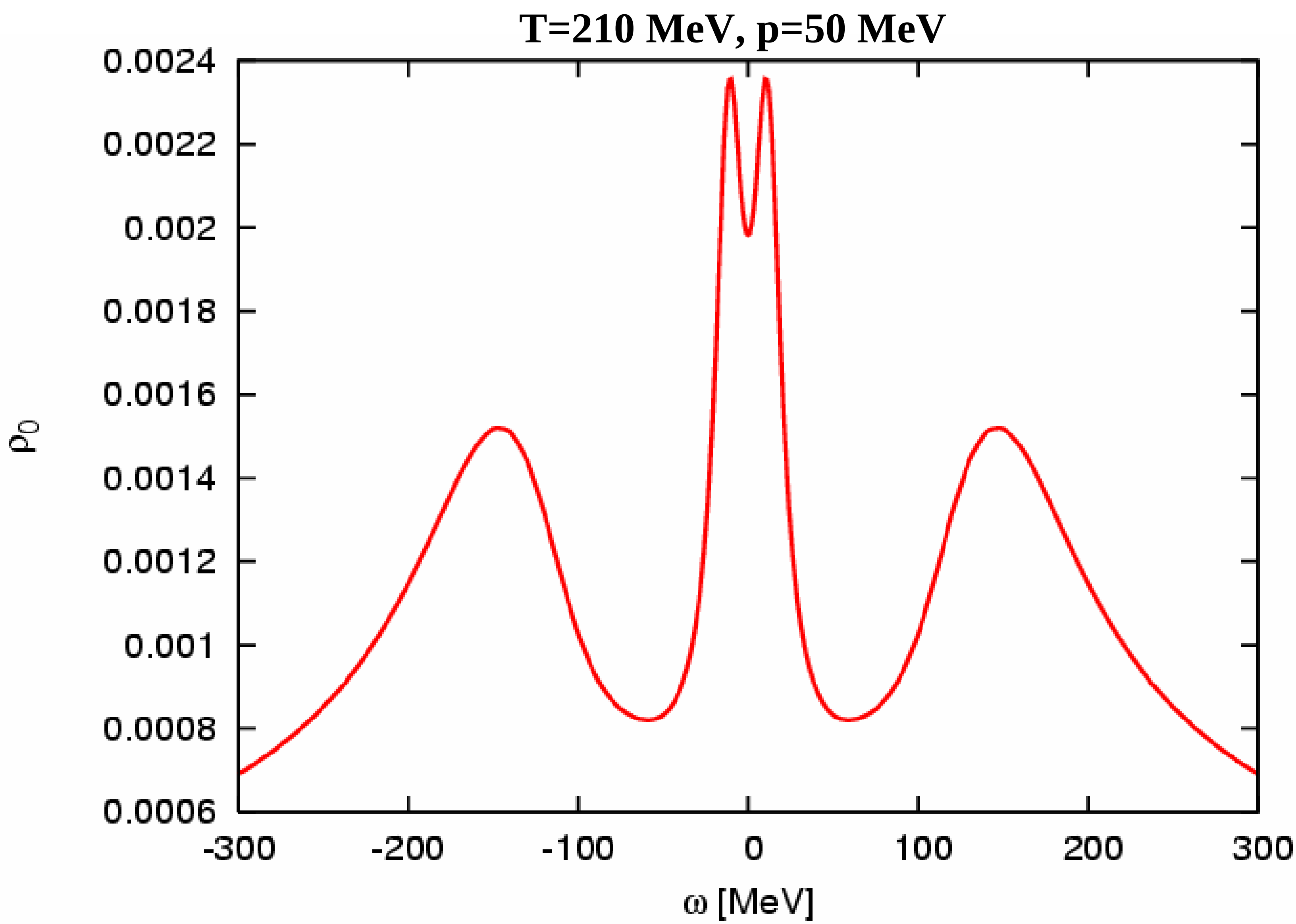




Spectral function of the quarks



Spectral Function of Quarks



Summary of this section

- **Near (above) Tc, the quark spectrum at long-frequency and long wave-length is modified drastically by the soft mode for the chiral condensate, $\langle \bar{q} q \rangle$.**

- **The many-peak structure of the spectral function can be understood in terms of two resonant scatterings at small ω and p of a quark and an antiquark.**

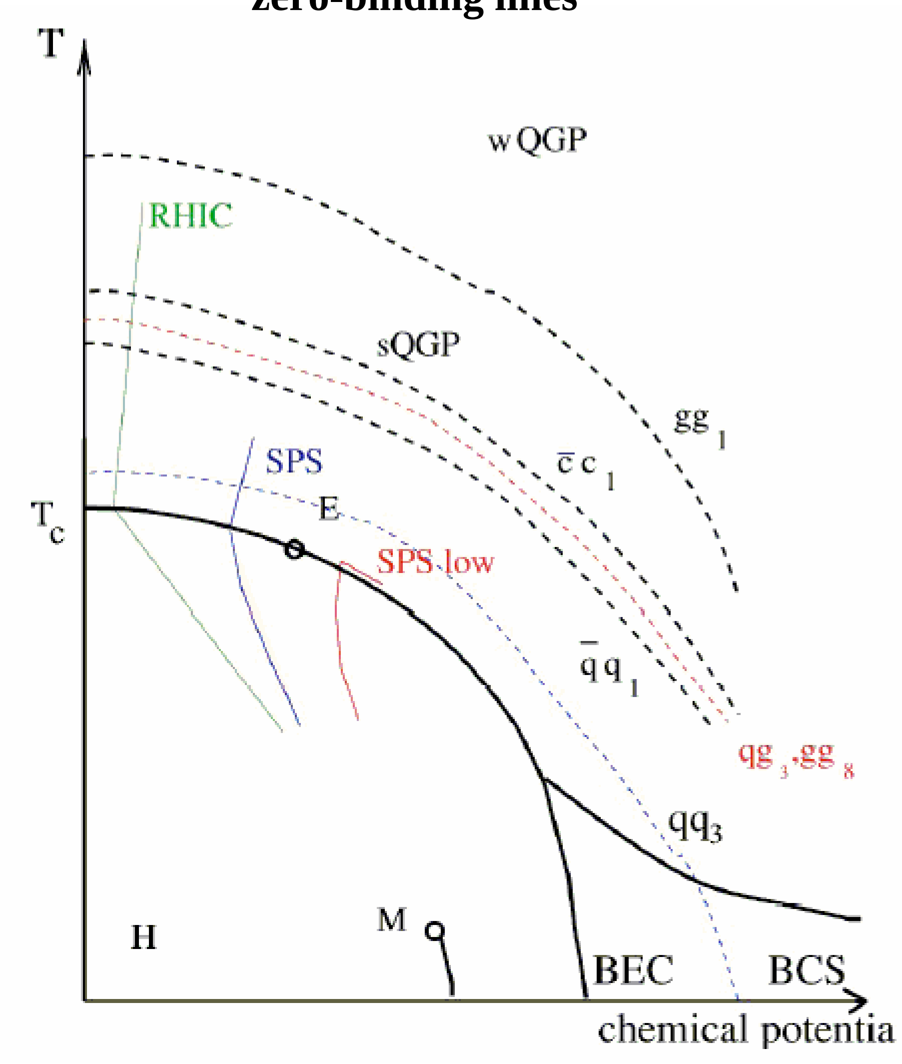
- **CSC : The Fermi surface is significant.**
  Chiral: Antiquarks are significant. (antiquark holes)

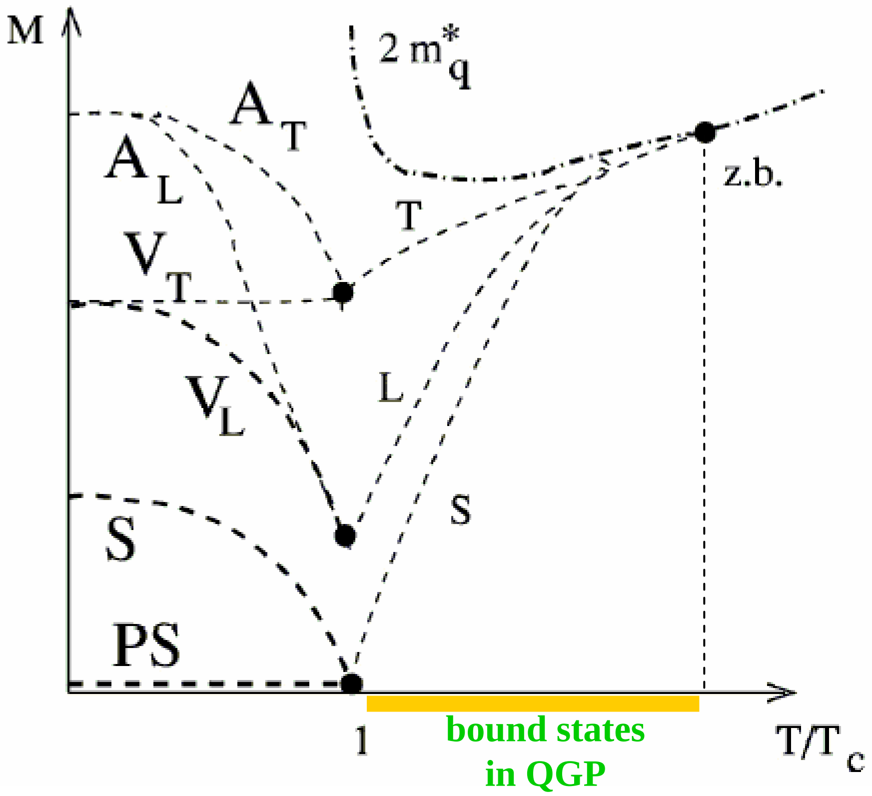
Future

- finite quark mass effects. (2nd order → crossover)
- finite density (tricritical point, critical end-point)
- phenomenological applications

From E.V.Shyryak and I.Zahed, PRC70,021901('04)

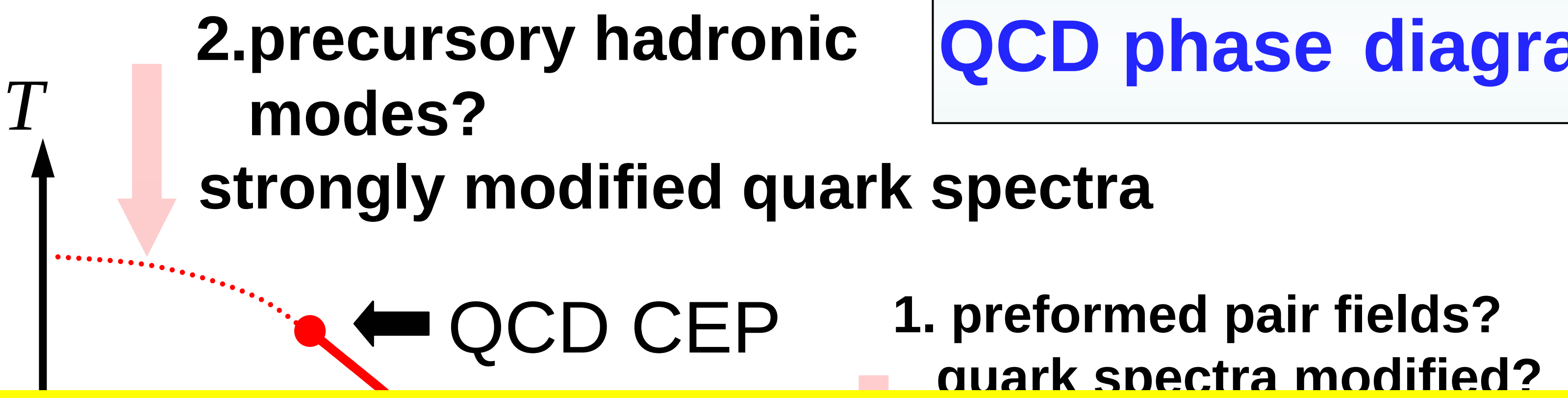
zero-binding lines





Summary of the Talk

QCD phase diagram



`QGP' itself seems surprisingly rich in physics!

Condensed matter physics of strongly coupled Quark-Gluon systems will constitute a new field of fundamental physics.