Quark Quasi-Particle Picture at Finite Temperature and Density in Effective Models

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In collaboration with

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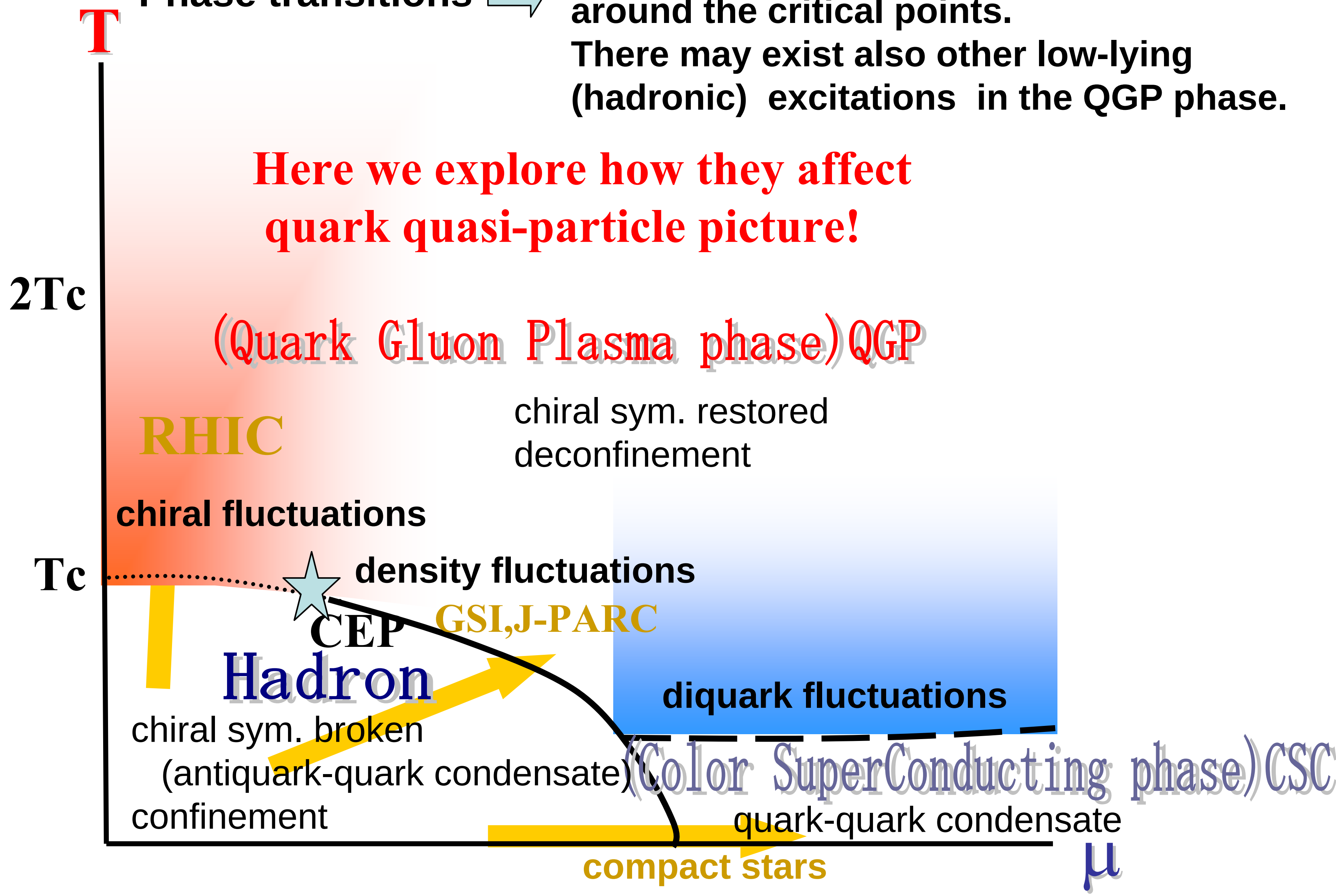
RBRC Workshop at Brookhaven National Laboratory
“Understanding QGP Through Spectral Functions and Euclidean Correlators”,
at BNL, April 23-25, 2008

Thoretical (half-conjectured) Phase Diagram of QCD

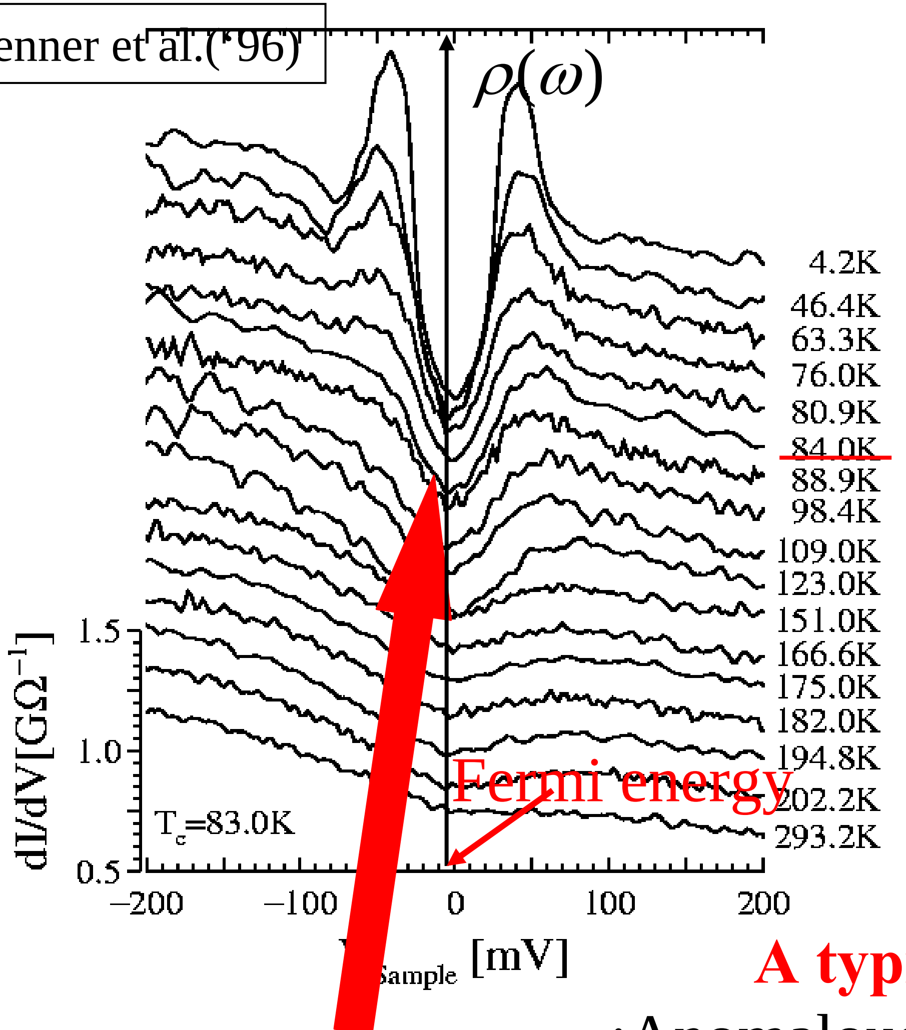
Phase transitions ⇒ large fluctuations owing to strong coupling around the critical points.
There may exist also other low-lying (hadronic) excitations in the QGP phase.



Contents

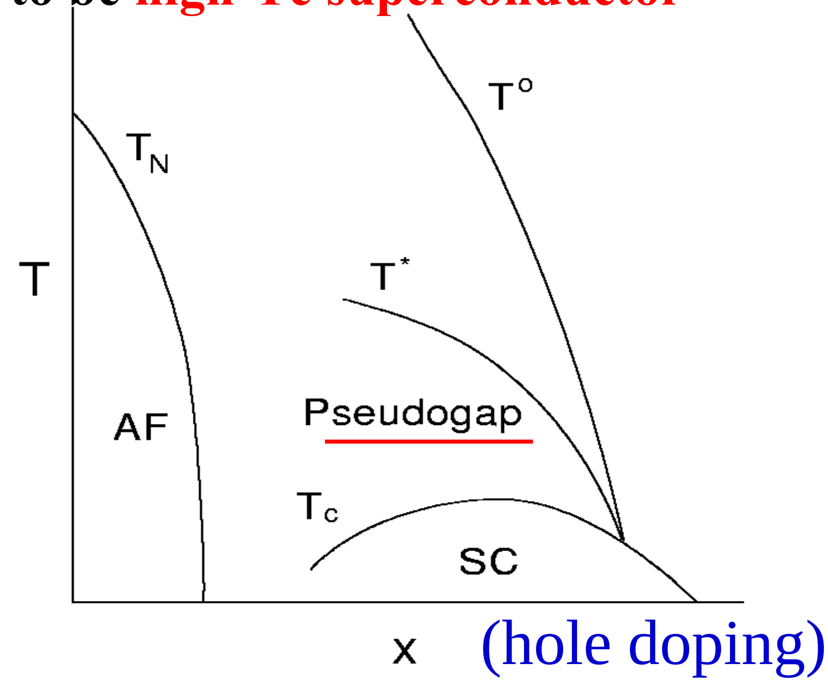
2. Diquark fluctuations and pseudogap in quark spectral function in hot and dense quark matter

Lesson from condensed matter physics on strong correlations



Pseudogap

Phase diagram of cuprates
to be high-Tc superconductor



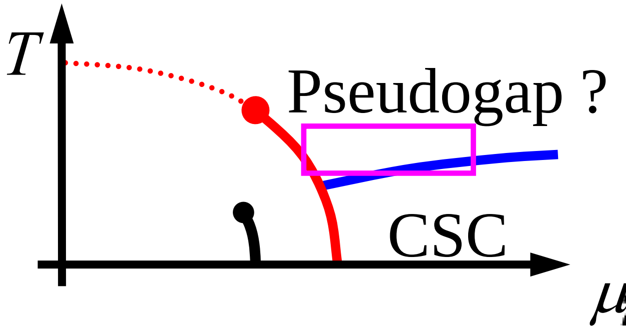
A typical non-Fermi liq. behavior!

:Anomalous depression of the **density of states** near the Fermi surface in **the normal phase**.

The mechanism of the pseudogap in High-TcSC is **still controversial**, but see,

Y. Yanase et al, Phys. Rep. 387 (2003),1, where the essential role of pair fluc. is shown.

Possible pseudogap formation in heated quark matter

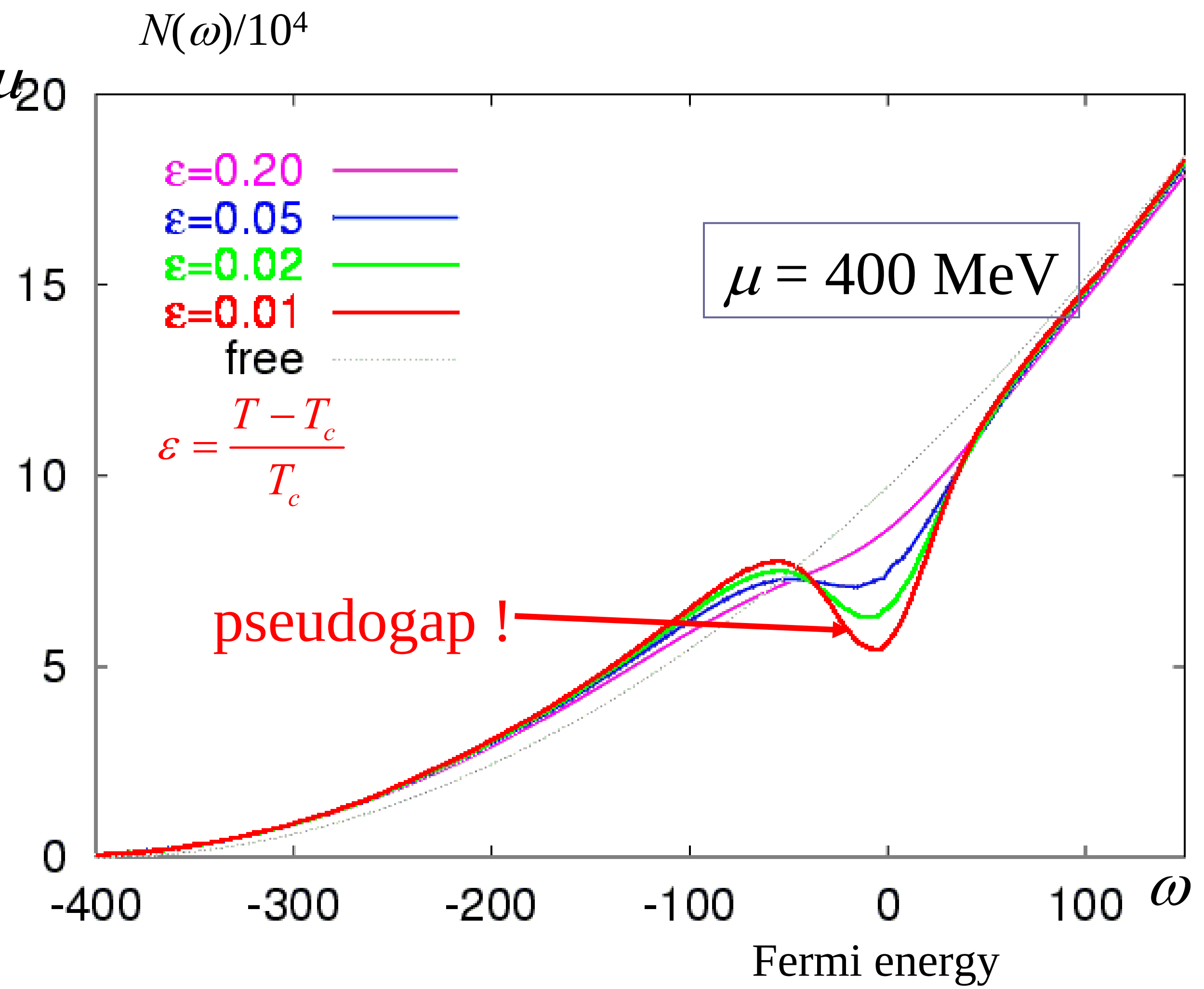


M. Kitazawa, T. Koide, T. K. and Y. Nemoto
Phys. Rev. D70, 956003(2004);
Prog. Theor. Phys. 114, 205(2005),

Pseudogap is formed above Tc of CSC in heated quark matter!

How?

$$N(\omega)=\int\frac{d^3\mathbf{k}}{(2\pi)^3}\rho(\mathbf{k},\omega)$$

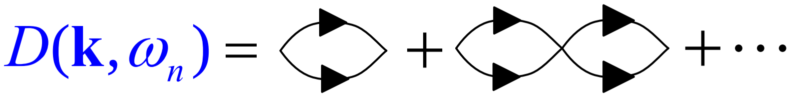


Mechanism of the pseudogap formation

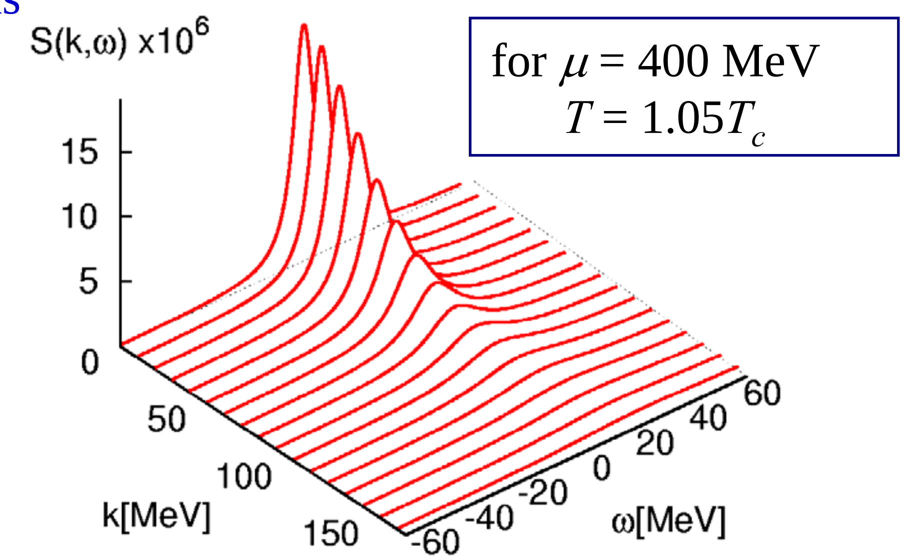
1. Development of precursory diquark fluctuations above Tc

Dynamical Structure Factorof diquark fluctuations

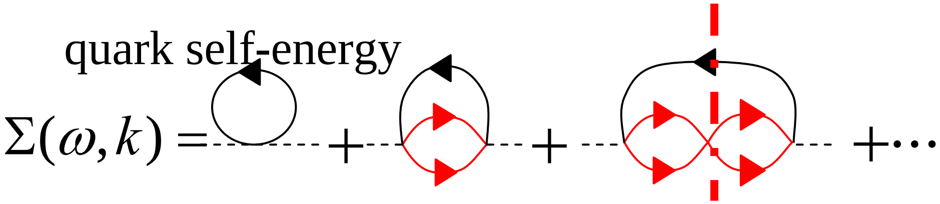
$$S(\mathbf{k}\omega) = -\frac{1}{1-e^{-\beta\omega}}\frac{1}{\pi}\mathrm{Im}\,D(\mathbf{k}\omega)$$



sharp peak at the origin
(= diffusive over-damping mode)



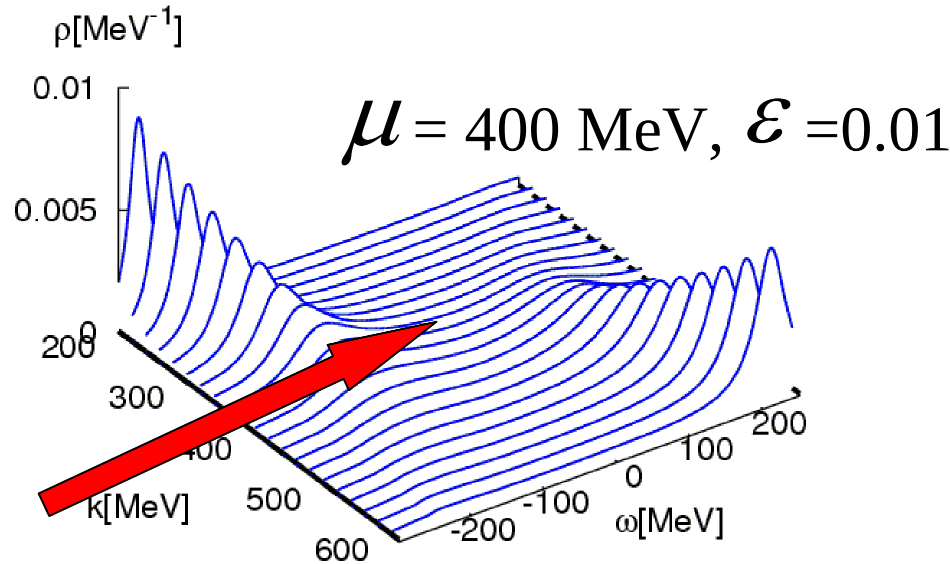
2.Coupling of quarks with the diquark fluctuations



Cherenkov-like emission of
diquark mode around Fermi energy

⇩

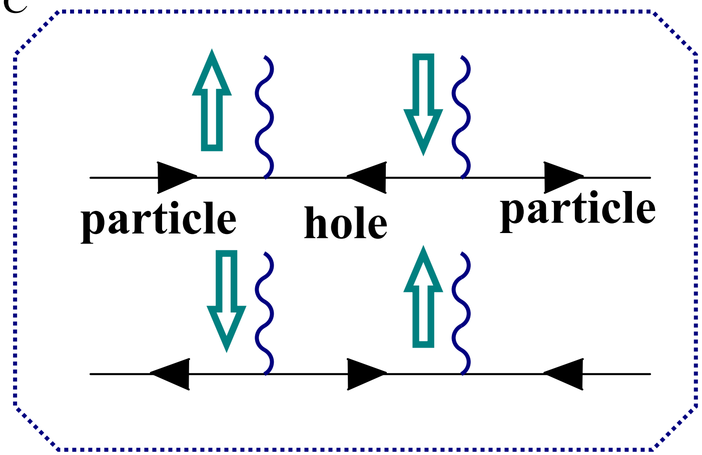
Depression of the quark spectral
Function around the Fermi energy



Level repulsion or Pseudo gap due to resonant scattering

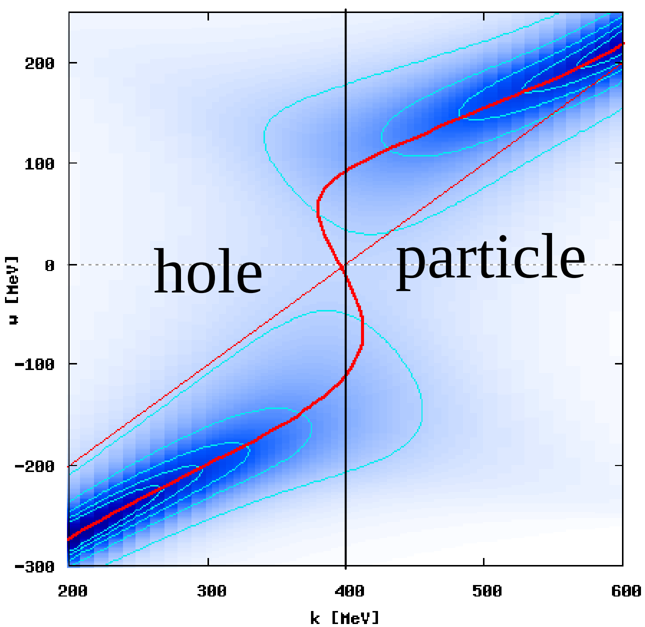
G_C=4.67GeV^{-2} M. Kitazawa, T.K. and Y. Nemoto, Phys. Lett.B 631(2005),157

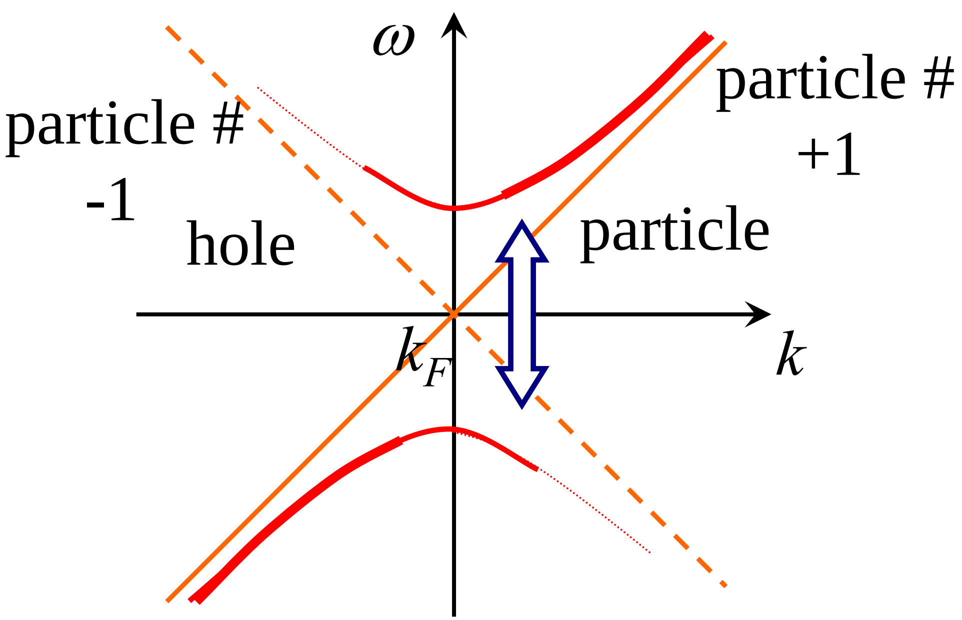
Janko, Maly, Levin, PRB**56**,R11407 (1995)



Mixing between particles and holes owing to the Landau damp. by the collective diquark mode.

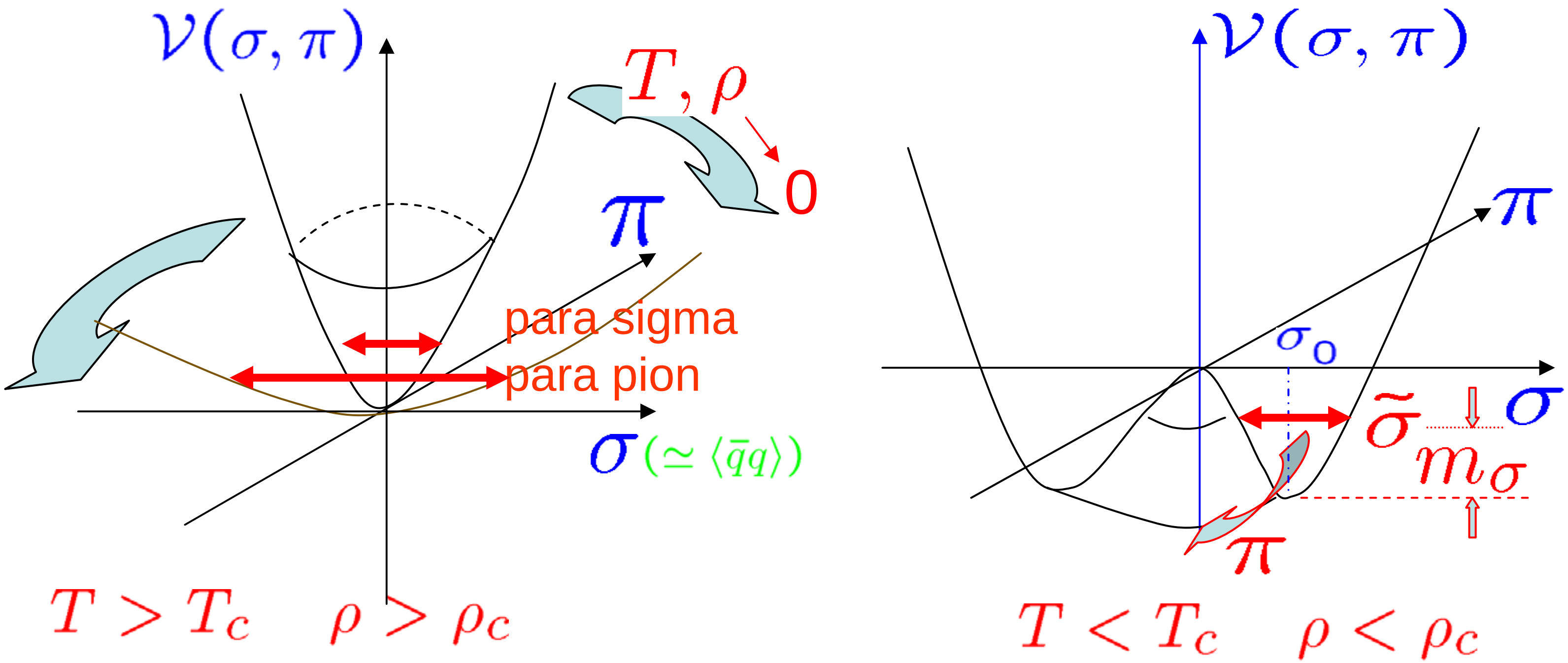
level repulsion of energy level





3. Soft modes of chiral transition and anomalous quasi-quark spectrum

Chiral Transition and the collective modes



The low mass sigma in vacuum is now established:

pi-pi scattering; Colangero, Gasser, Leutwyler('06) and many others

Full lattice QCD ; SCALAR collaboration ('03)

q-qbar, tetra quark, glue balls, or their mixed st's?

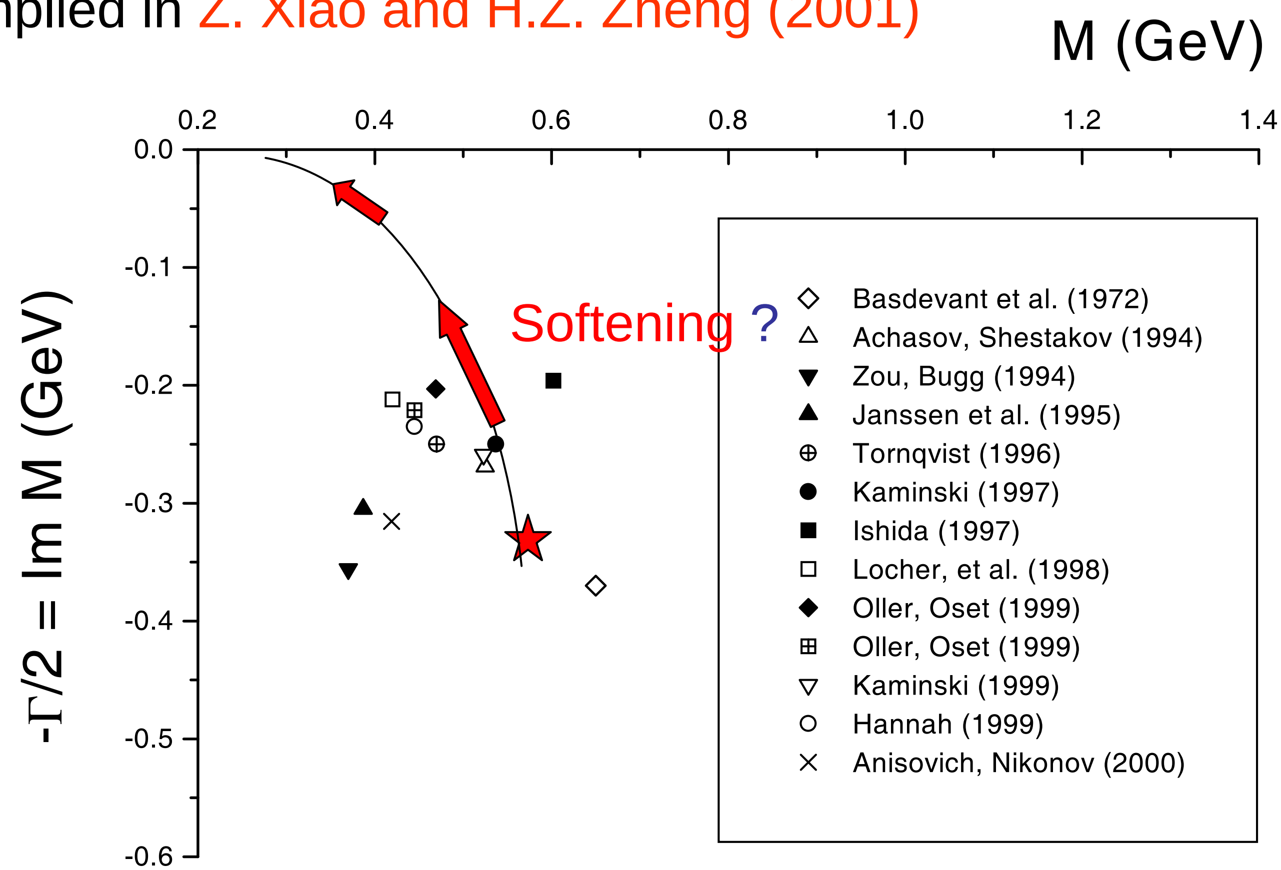
c.f. The sigma as the Higgs particle in QCD $\sigma = \sigma_0 + \tilde{\sigma}$

ϕ ; Higgs field ⇒ $\phi = \langle \phi \rangle + \tilde{\phi}$

Higgs particle

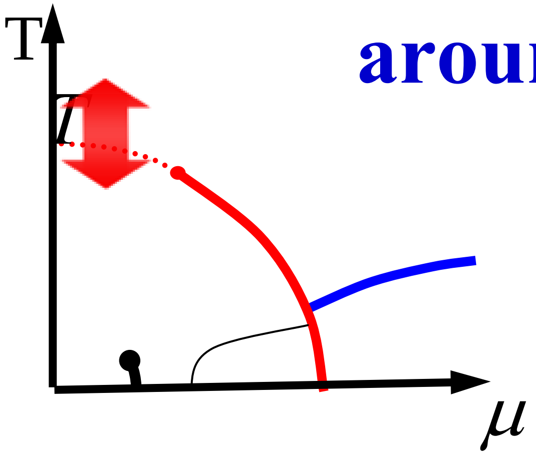
Digression:The poles of the S matrix in the complex mass plane f the sigma meson channel:

complied in Z. Xiao and H.Z. Zheng (2001)



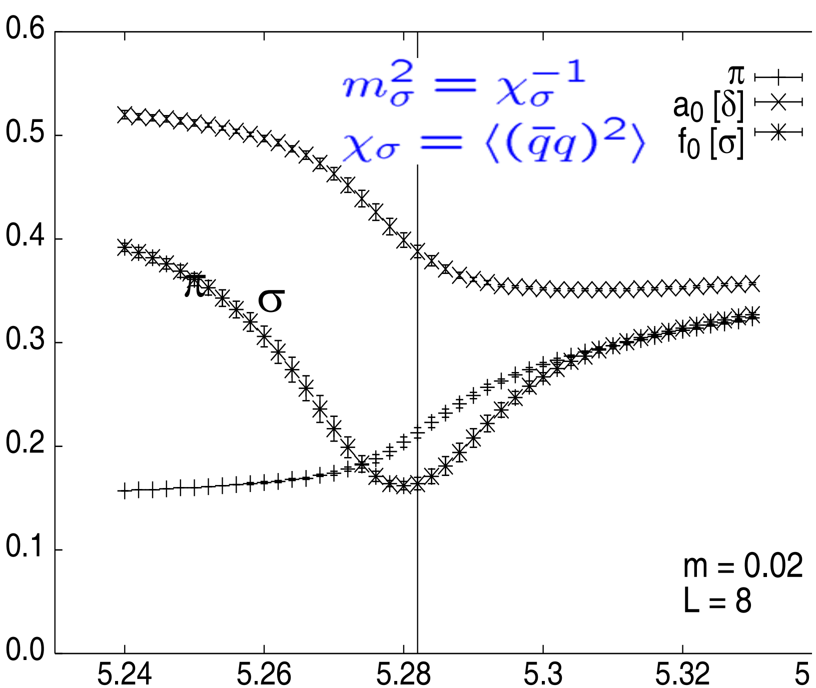
See also, I. Caprini, G. Colangero and H. Leutwyler, PRL(2006);
H. Leutwyler, hep-ph/0608218 ; M_sigma=441 – i 272 MeV

Fluctuations of chiral order parameter around Tc in Lattice QCD



Cf. Lattice Calculation of the *generalized masses*

F. Karsch, Lect. Note Phys. **583** (2002), 20



the softening of the σ with increasing T

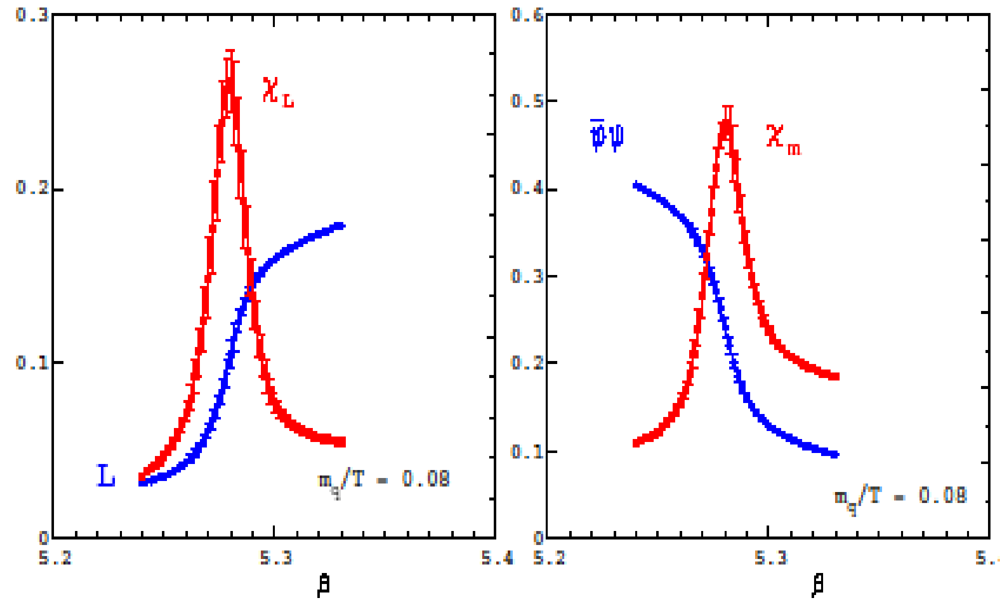
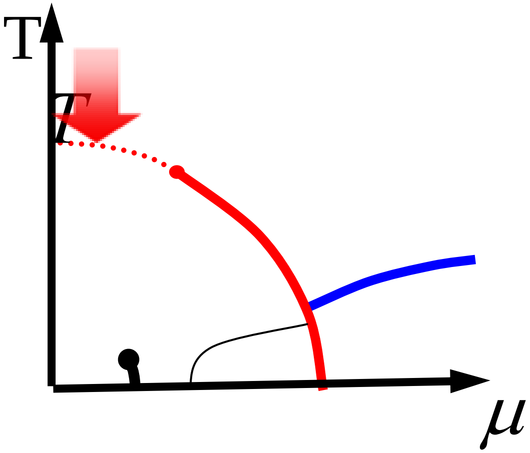


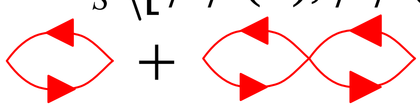
Fig. 2. Deconfinement and chiral symmetry restoration in 2-flavour QCD: Shown is $\langle L \rangle$ (left), which is the order parameter for deconfinement in the pure gauge limit ($m_q \to \infty$), and $\langle \bar\psi\psi \rangle$ (right), which is the order parameter for chiral symmetry breaking in the chiral limit ($m_q \to 0$). Also shown are the corresponding susceptibilities as a function of the coupling $\beta = 6/g^2$.

The spectral function of the degenerate ``para-pion" and the ``para-sigma" at T>Tc for the chiral transition: Tc=164 MeV

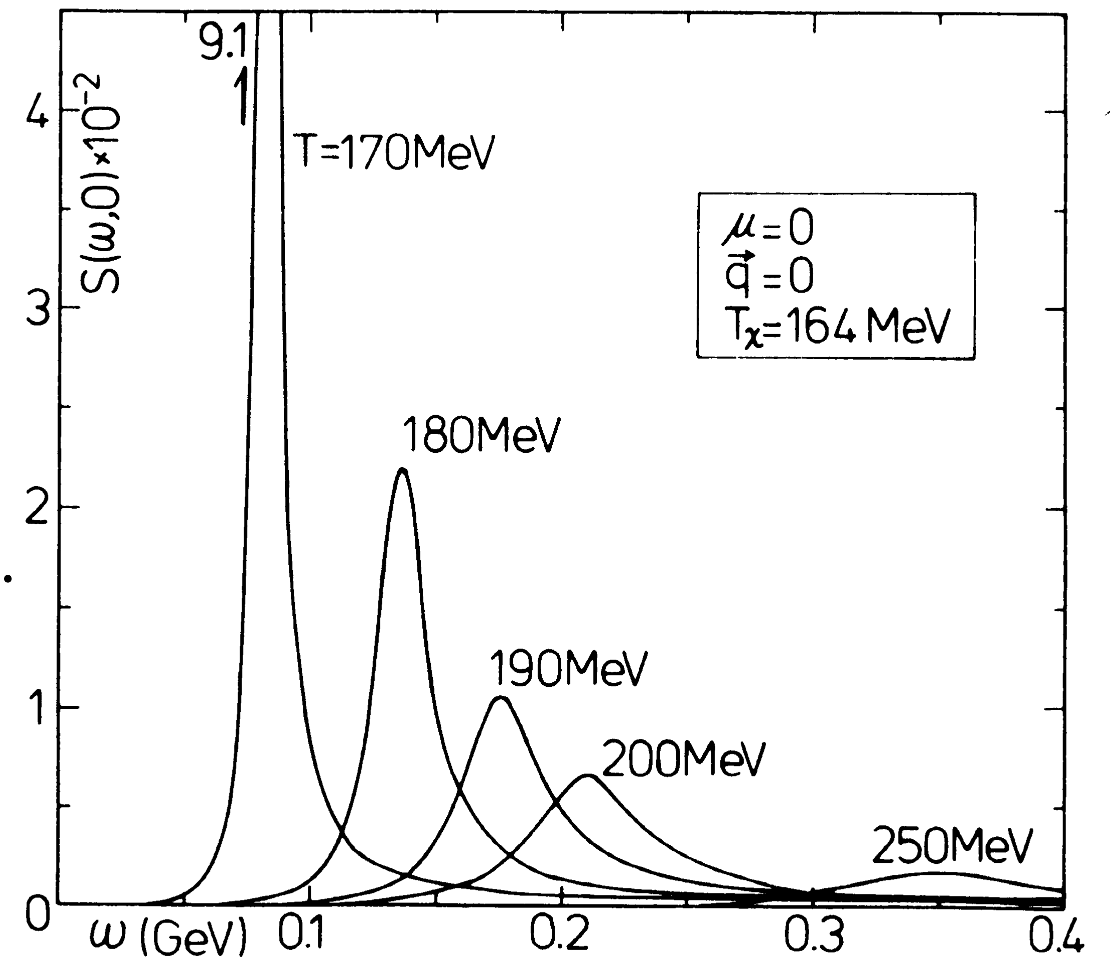
T. Hatsuda and T.K. (1985)



$$D_\sigma(x) = -2G_S \left\langle \left[\bar{\psi}\psi(x), \bar{\psi}\psi(0) \right] \right\rangle$$

=  + ⋯

$$\rho_\sigma(p,\omega) = -\frac{1}{\pi}\,\mathrm{Im}\,D_\sigma(p,\omega)$$



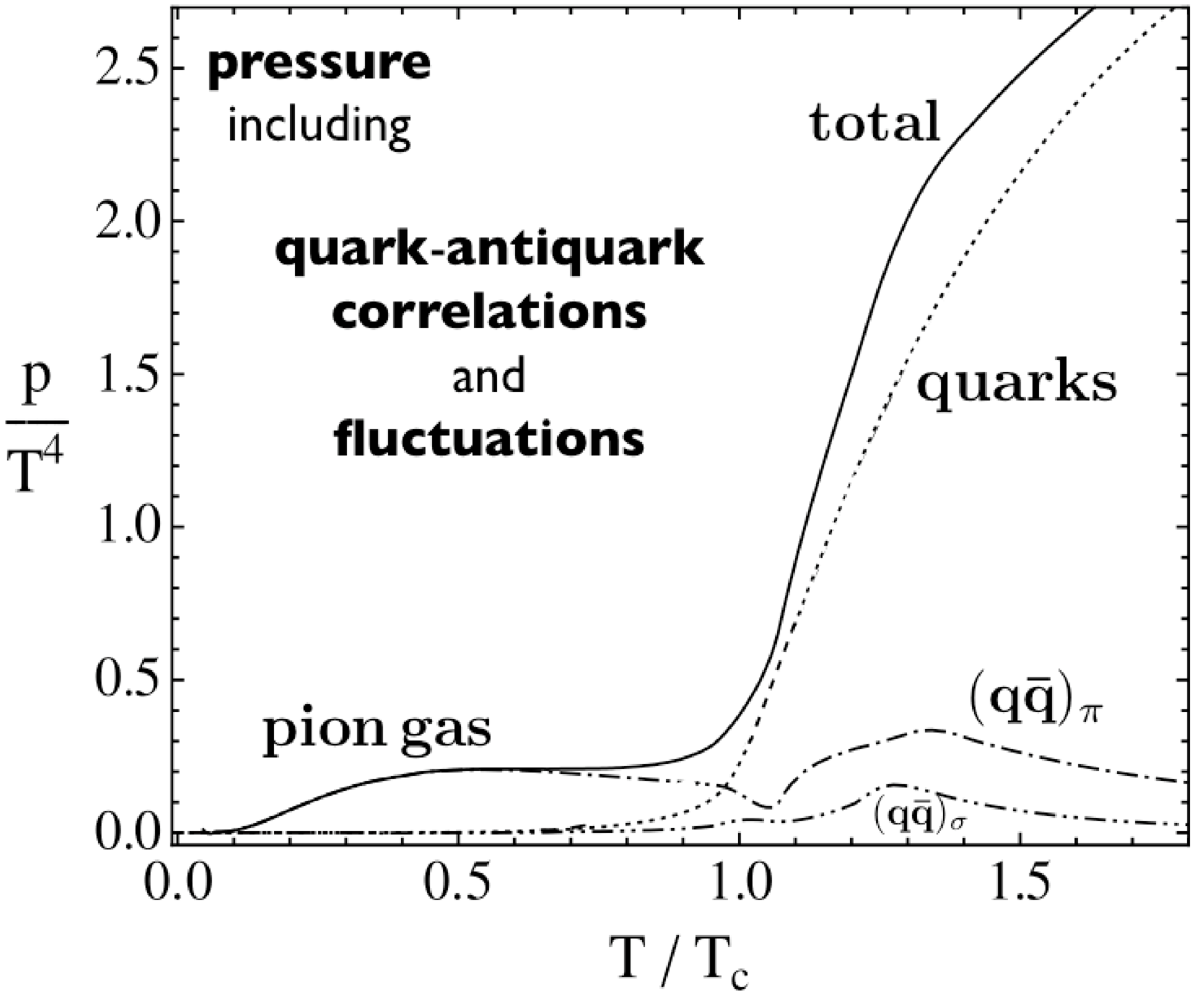
(NJL model cal.)

Para-pion and para-sigma modes are still seen in PNJL model

P=Polyakov-loop coupled

K. Fukushima (2004)
C. Ratti, M. Thaler,
W. Weise (2006)

PNJL model



S. Rößner, T. Hell, C. Ratti, W. Weise.arXiv:0712.3152 [hep-ph]

W. Weise, talk at NFQCD2008 at YITP, Kyoto.

Quark Spectral Function

$$G(p_0,p)=\frac{1}{G^0(p_0,p)-\Sigma(p_0,p)}$$

- **Quark Self-energy at finite T**

$$\Sigma(p_0,p)=\Sigma^0(p_0,p)\gamma_0-\Sigma^V(p_0,p)\vec{\gamma}\cdot\vec{p}+\underbrace{\Sigma^S(p_0,p)}_{=0}$$

- **Quark Spectral function**

$$\rho(p_0,p)=\rho^0(p_0,p)\gamma_0-\rho^V(p_0,p)\boldsymbol{\gamma}\cdot\mathbf{p}+\underbrace{\rho^S(p_0,p)}_{=0}\quad\text{(chiral limit)}$$

$$\rho(p^0,p)=\underbrace{\rho_+(p^0,p)}_{\text{particle}}\Lambda_+\gamma^0+\underbrace{\rho_-(p^0,p)}_{\text{antiparticle}}\Lambda_-\gamma^0$$

particle **antiparticle**

$$\rho(p_0,p)=-\frac{1}{\pi}\mathrm{Im}G(p_0,p)$$

$$=\rho_+(p_0,p)\Lambda_+\gamma^0+\rho_-(p_0,p)\Lambda_-\gamma^0$$

at zero density

$$\rho_+(p_0,p)=\rho_-(-p_0,p)$$

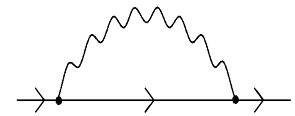
c.f: for a free quark

$$\rho_0(p_0,p)\sim\delta(p_0-p)+\delta(p_0+p)$$

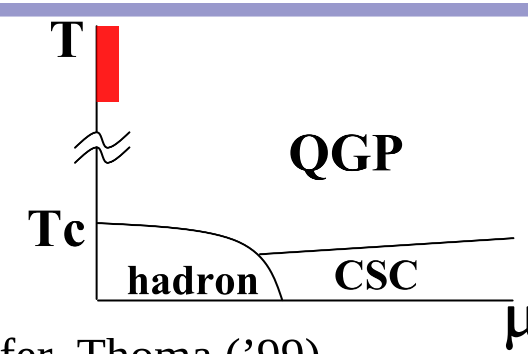
Digression: Quarks at very high T (T>>Tc)
---- physical origin of plasmino and thermal mass ---

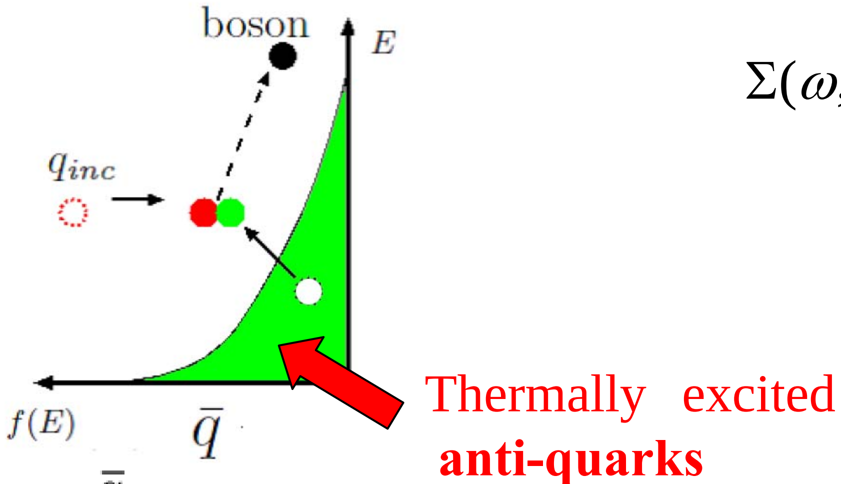
- 1-loop (g<<1) + HTL approx. $(p, \omega, m_q << T)$

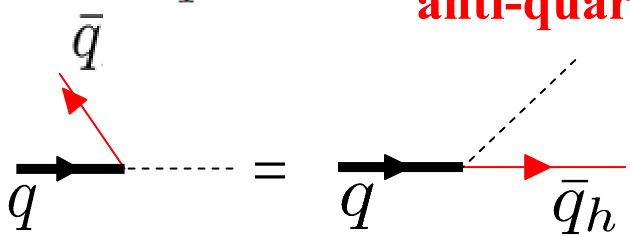
$$\Sigma(\omega, p) =$$

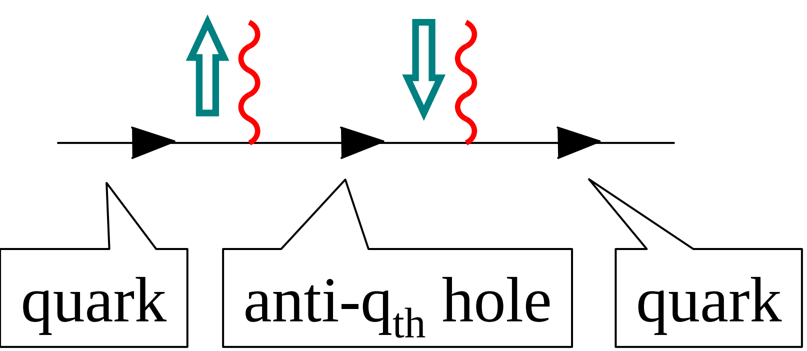


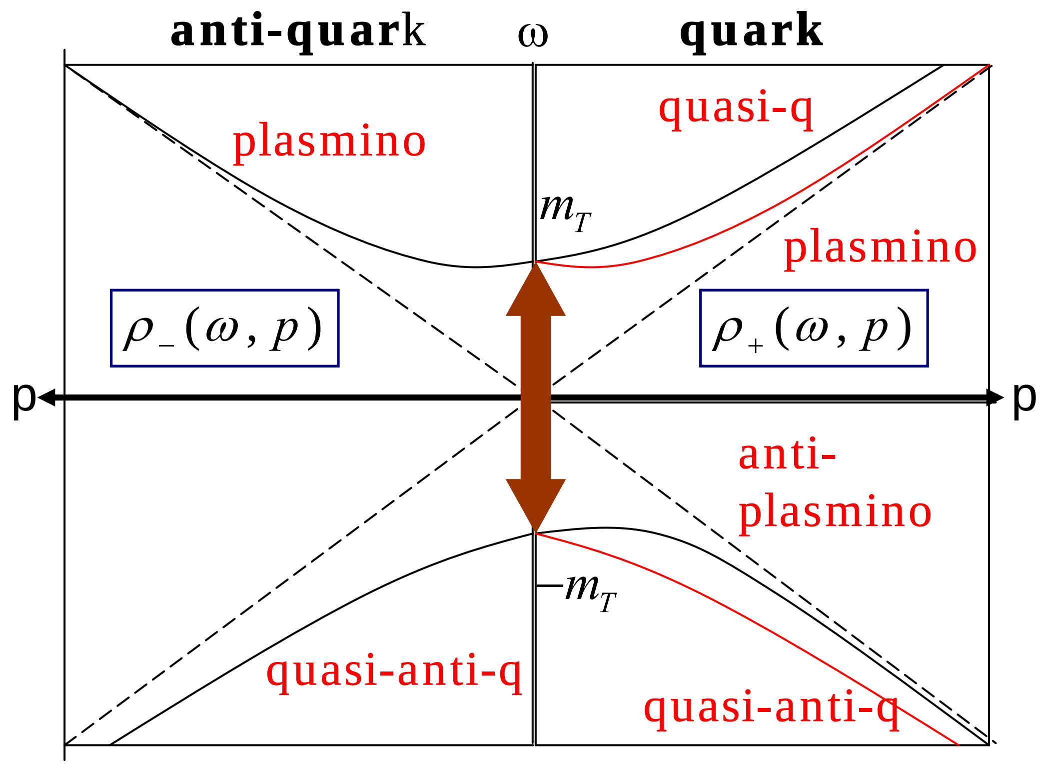
Klimov, Weldon('82), Pisarski('89), A.Schaefer, Thoma ('99)







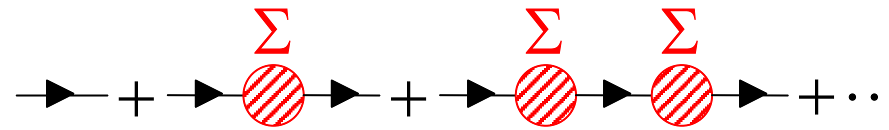




Quarks coupled to chiral soft modes near Tc

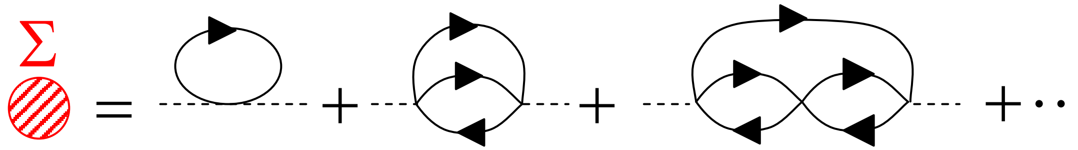
We incorporate the fluctuation mode into a single particle Green function of a quark through a self-energy.

$$G(\omega_n, p) = \frac{1}{G_0(\omega_n, p) - \Sigma(\omega_n, p)} =$$

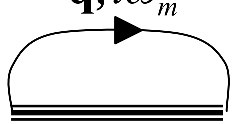


Non self-consistent T-approximation (1-loop of the fluctuation mode)

$$\Sigma(\omega_n, p) =$$



$$\equiv$$



$$= T\sum_m \int \frac{d^3q}{(2\pi)^3} D(\omega_n + \omega_m, p+q) G_0(\omega_m, q)$$

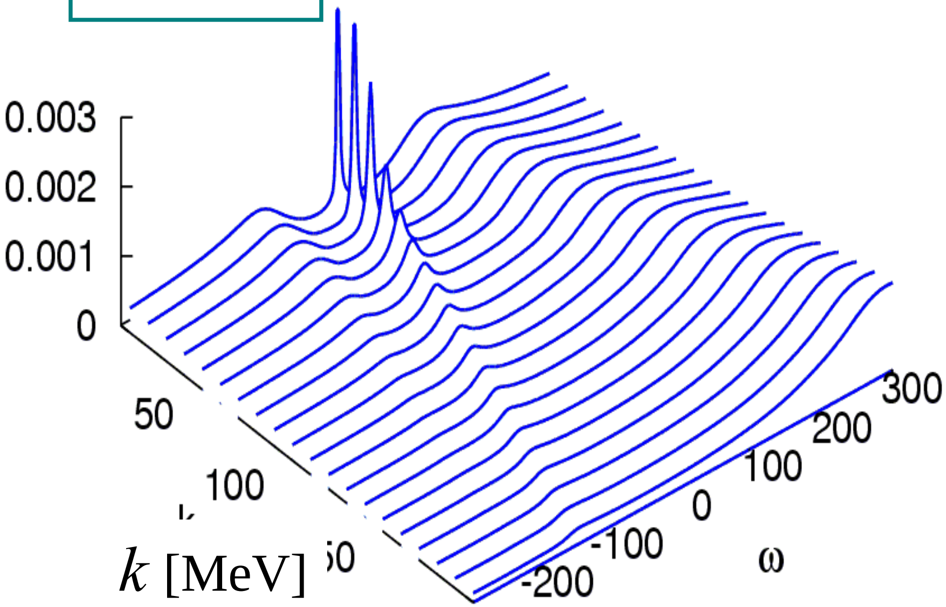
N.B. This is a complicated multiple integral owing to the compositeness of the para-sigma and para-pion modes.

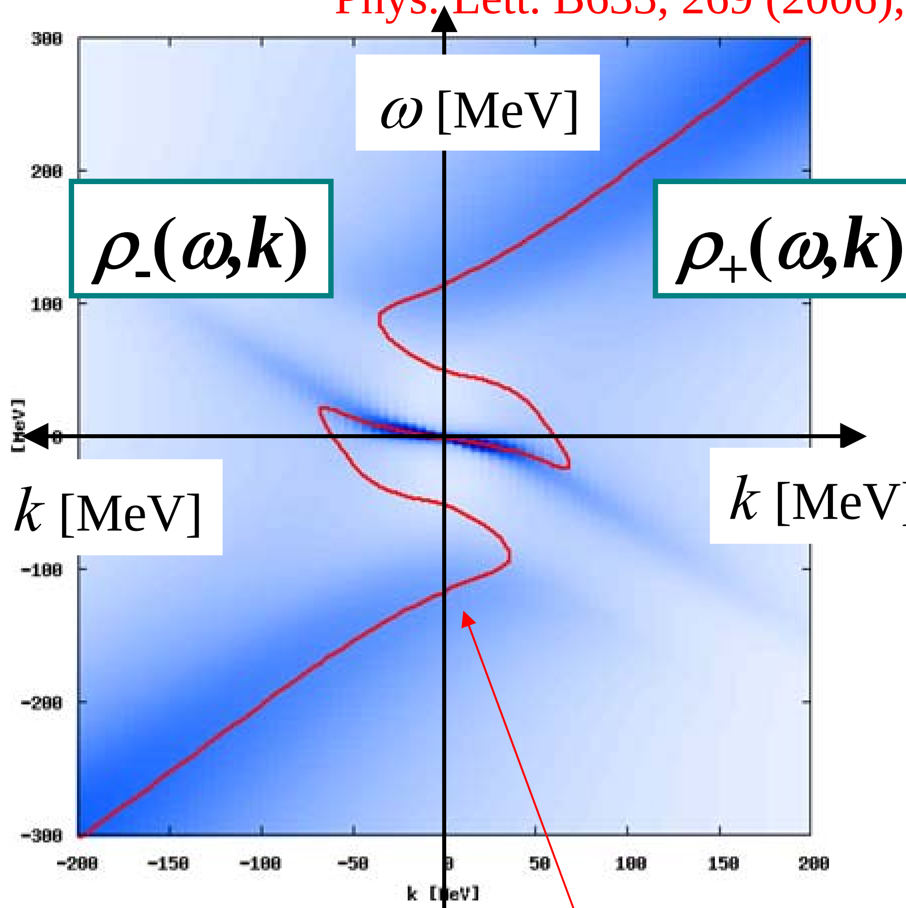
Spectral Function of Quark

$$\varepsilon = \frac{T - T_c}{T_c}$$

$$\rho(p^0, p) = \rho_+(p^0, p)\Lambda_+\gamma^0 + \rho_-(p^0, p)\Lambda_-\gamma^0$$

Kitazawa, Nemoto and T.K., Phys. Lett. B633, 269 (2006),





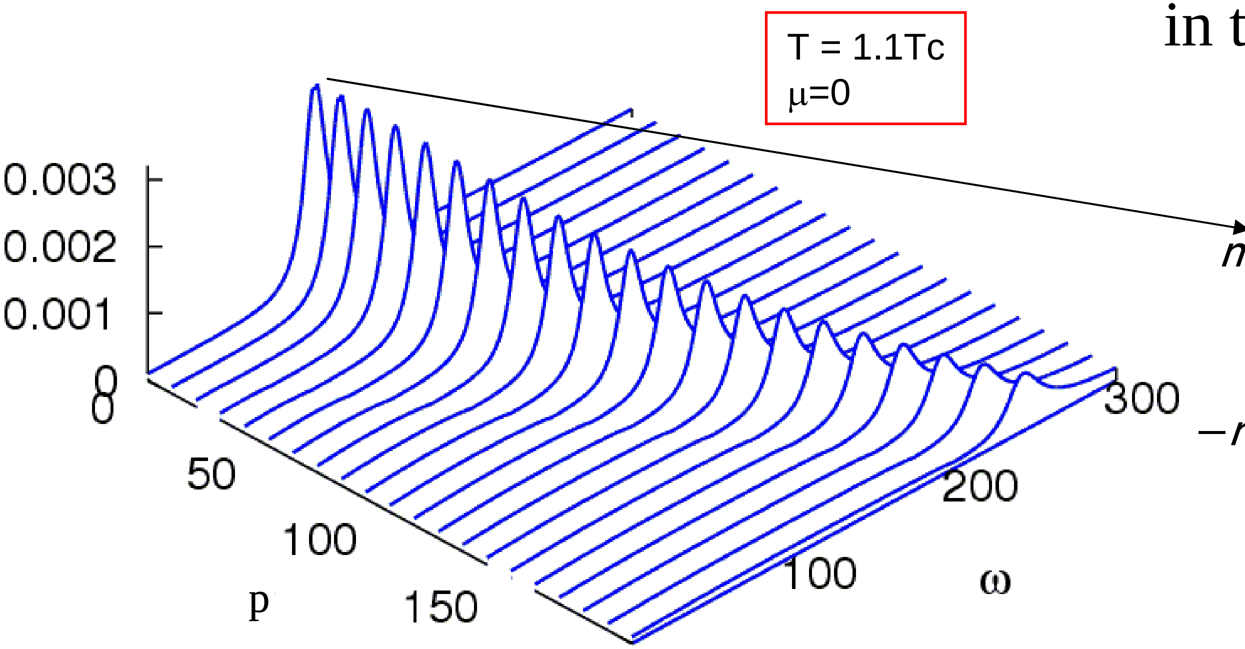
- Three-peak structure emerges.
- The peak around the origin is the sharpest.

Quasi-dispersion relation for eye-guide;

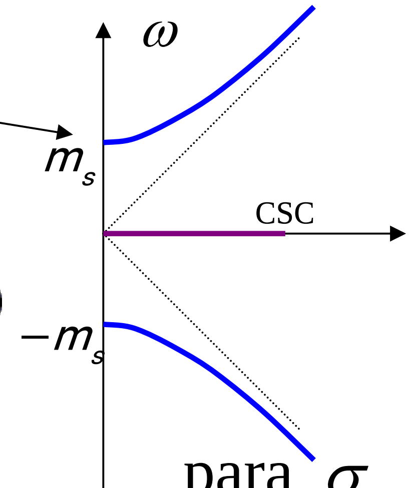
$$\Sigma^{\pm}(p_0, p) = \Sigma^0(p_0, p) \mp \Sigma^V(p_0, p) \qquad \mathrm{Re}\left[S_+(\omega, \mathbf{p})\right]^{-1} = \omega - |\mathbf{p}| - \mathrm{Re}\,\Sigma_+(\omega, \mathbf{p}) = 0$$

Fluctuations of the chiral codensate

- Spectrum of the fluctuations



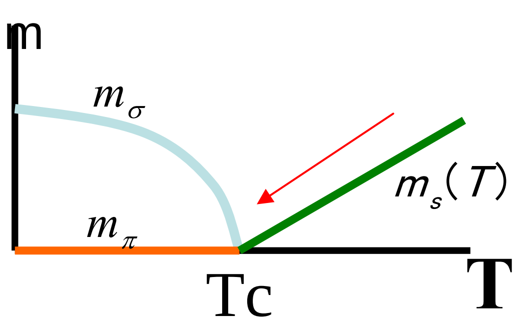
- sharp peak in time-like region



para σ, π-mode

$$\omega_s \approx \pm\sqrt{p^2 + m_s^2(T)}$$

propagating mode

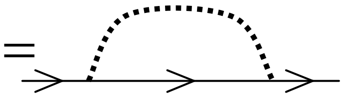


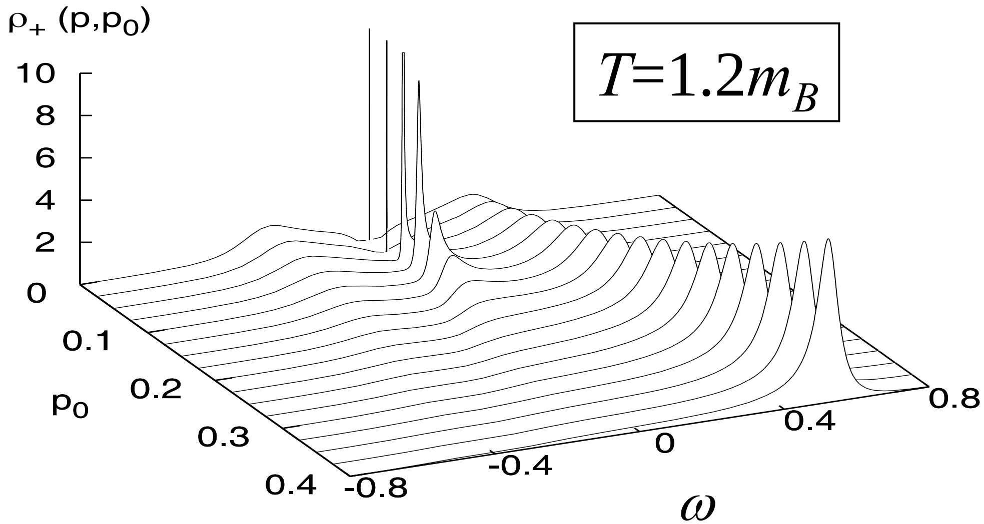
c.f.: diffusion-like mode
in diquark fluctuations
$\omega \sim p^2$

Quark Spectrum in Yukawa models

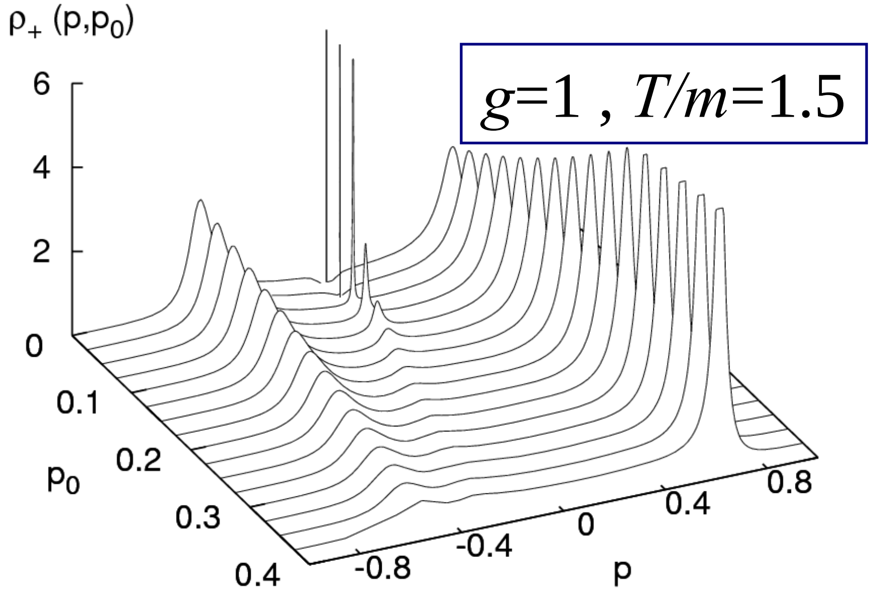
Kitazawa, Nemoto and T.K.,
Prog. Theor. Phys.117, 103(2007),

Near the critical point , the soft-modes may be represented by an elementary boson. ⇒ quark + massive boson. Yukawa model!

$\Sigma(i\omega_n, \mathbf{p}) =$  (at one-loop)



Massie scalar/pseudoscalar boson

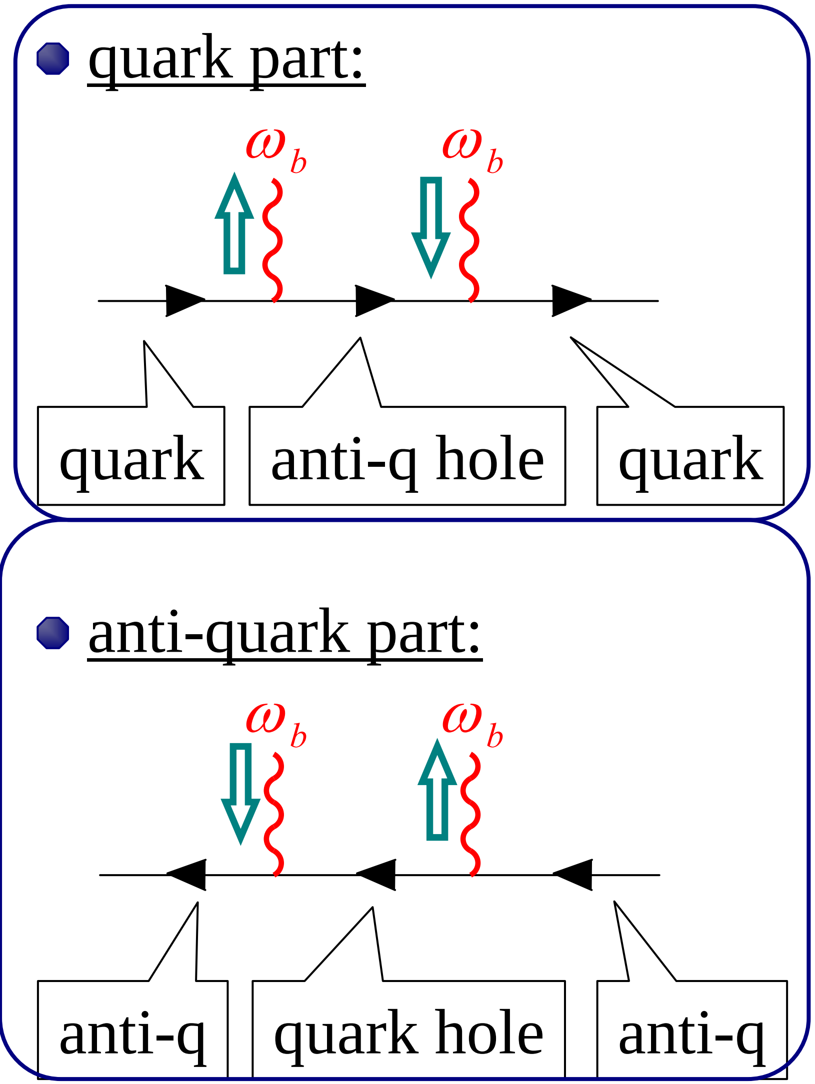


Massie vector/axial vector boson

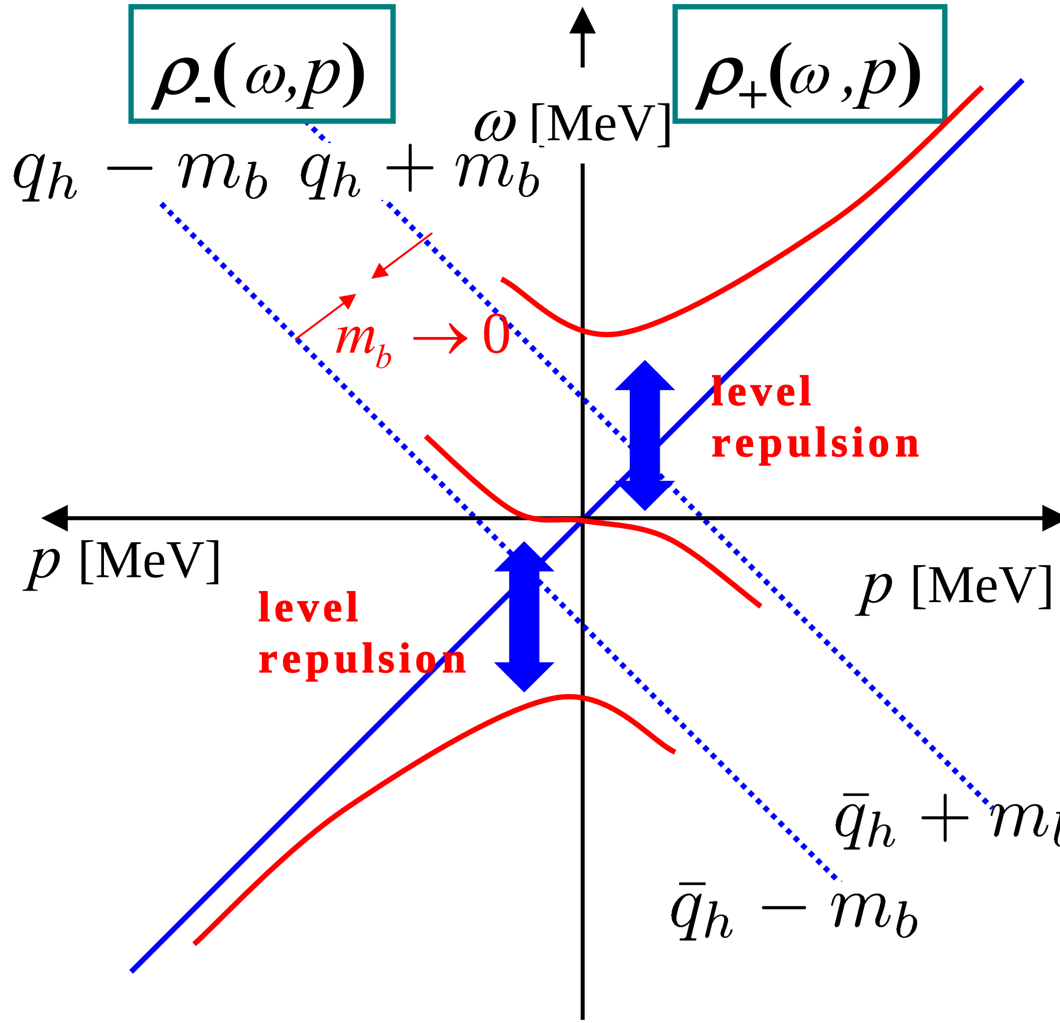
The 3-peak structure emerges irrespective of the type of the boson at $T \sim m_B$.

Remark: Bosonic excitations in QGP may include $\sigma, \pi, \rho, J/\psi, \ldots$ / glue balls… How about the case of Vector manifestaion.(B-R,H-Y-S)

Mechanism of the 3-peak fromation



The level crossing is shifted by the mass of the fluctuation modes.

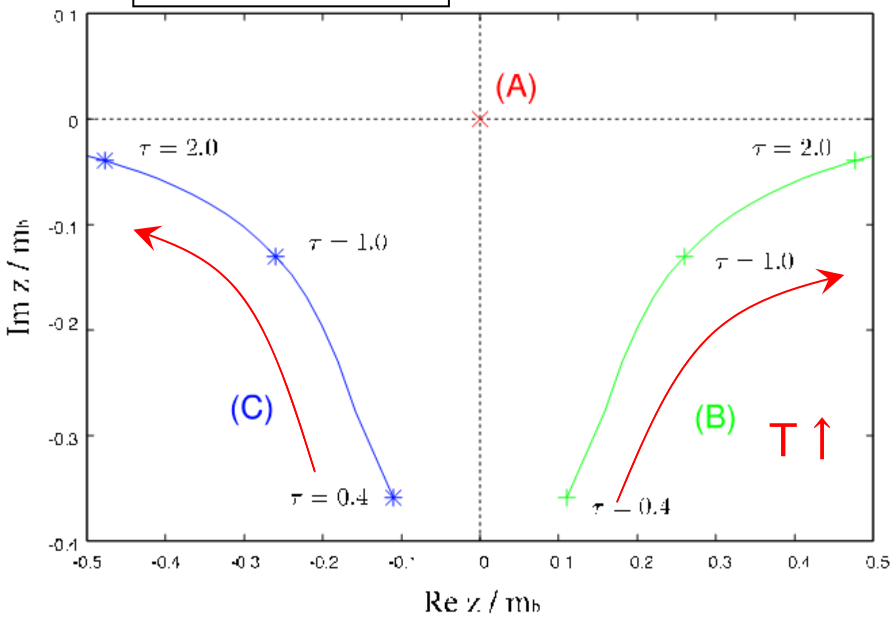


$m_b \to 0$: **the HTL result only with the normal quark and plasmino.**

Discussions

The complex quasi-quark pole --- ‘Gauge independence’

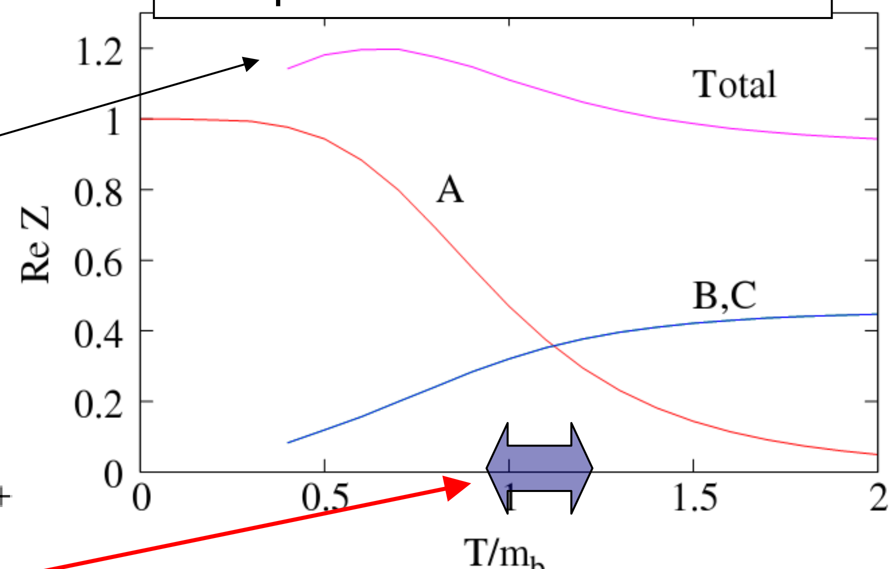
Pole position



Mitsutani, Kitazawa, Nemoto, T.K.
Phys. Rev. D77, 045034 (2008)

There are three poles corresponding to the three peaks in the spectral function; the pole distribution is symmetric with respect to the imaginary axis because $m_f = \mu = 0$

T-dependence of the residues



The sum of the three residues approximately satisfy the sum rule

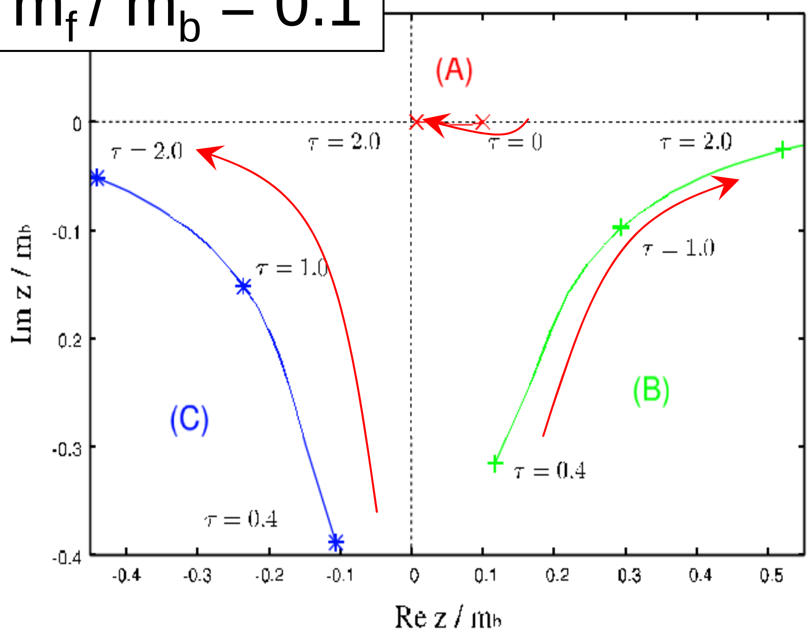
$$Z = \left[1 - \frac{\partial \Sigma_+^R(z)}{\partial z}\right]\Bigg|_{z=z_+}$$

The three residues comparable at T ~ m_b which support 3-peak structure

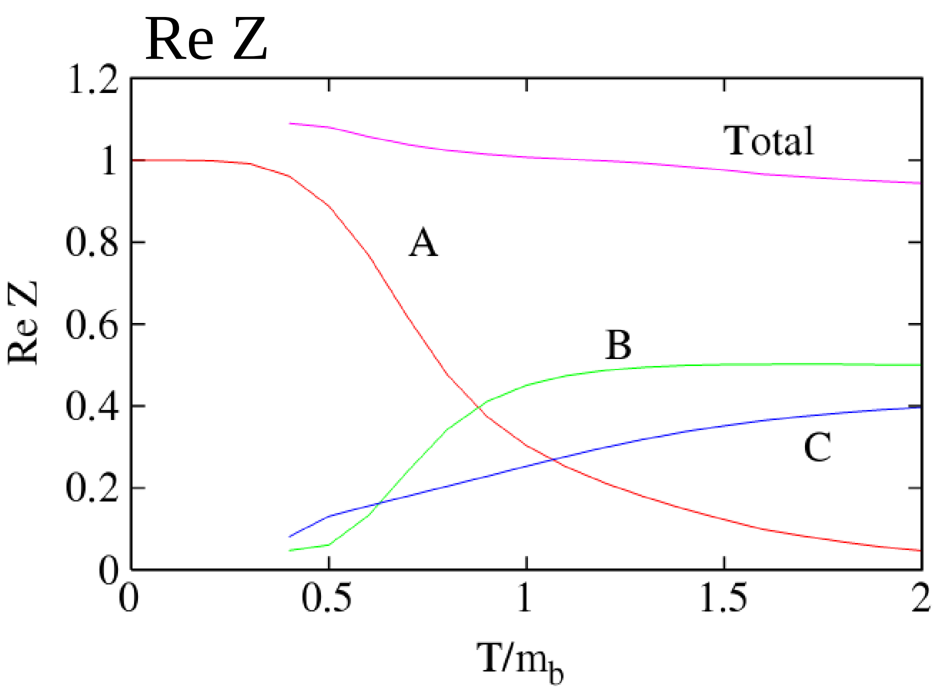
Finite quark mass effects

Mitsutani, Kitazawa, Nemoto, T.K.
Phys. Rev. D77, 045034 (2008)

$m_f / m_b = 0.1$



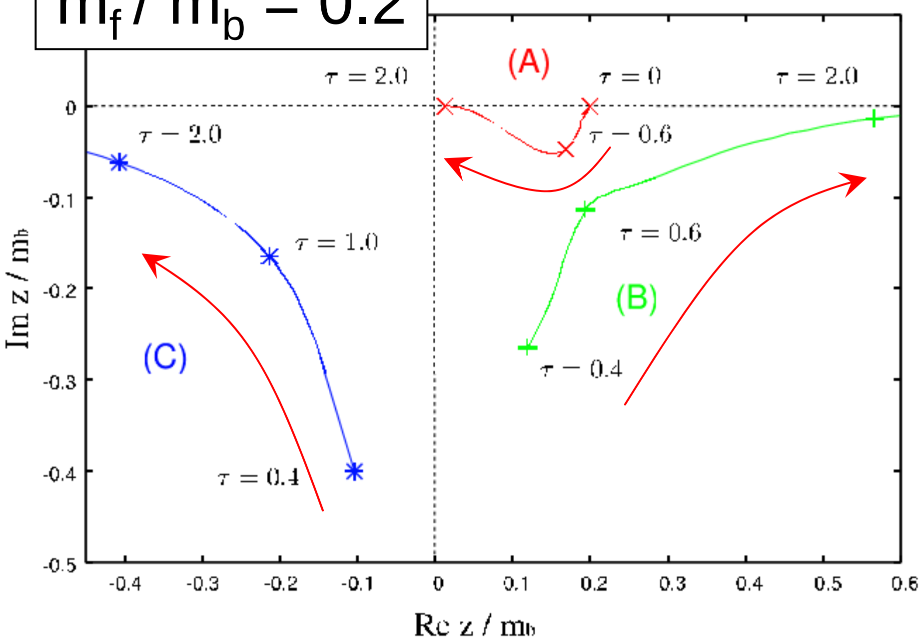
- There still exist three poles.
- The pole at T=0 (red) moves toward the origin as T is raised.
- The pole in the $\omega < 0$-region has a larger imaginary part than that in the positive-ω region for the same T.



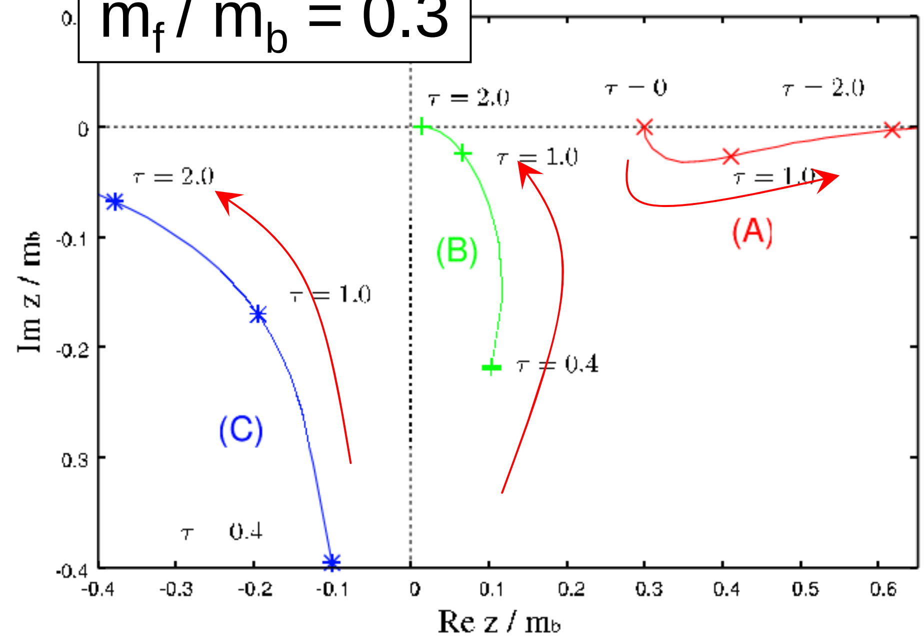
- The residue at the pole in the negative ω region is suppressed at $T \sim m_b$, corresponding to the suppression of the peak in the negative-energy region.
- The sum of the residues approximately satisfy the sum rule also in this case.

Structure change of the pole behavior

$m_f / m_b = 0.2$



$m_f / m_b = 0.3$



The pole at T=0 moves toward the origin as T is raised.
This behavior is qualitatively the same as in the case of lower masses.

The pole at T=0 moves toward the large-ω region as T is raised.
This behavior is qualitatively different from that in the smaller mass cases..

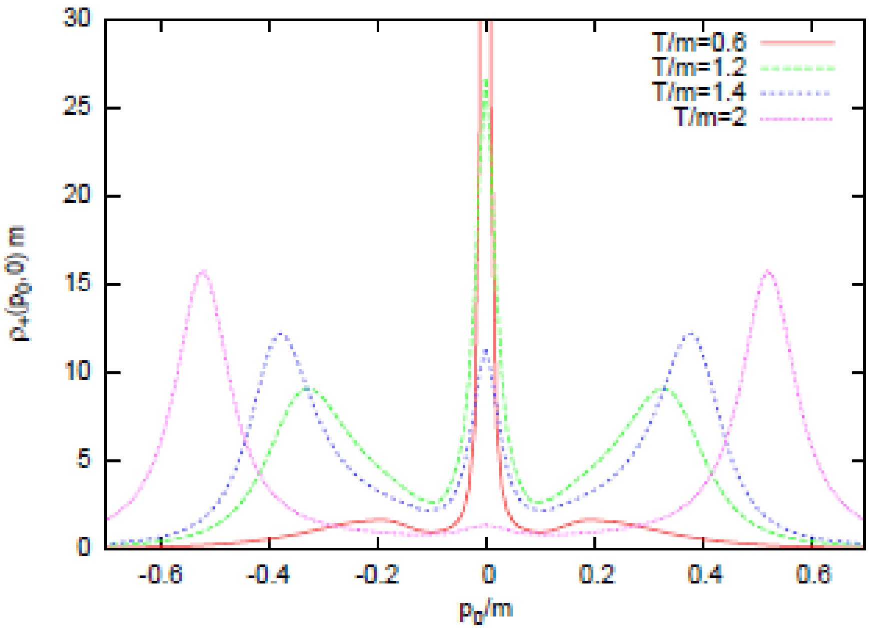
The physics contents of the three poles change at a critical mass m_f^{crit}. We find $m_f^{crit} / m_b \approx 0.21$

Beyond one-loop

Schwinger-Dyson approach for
lin. sigma model; Harada-Nemoto('08)

The three peak structure in the quark spectral function **is still there** for small momenta, although the central peak gets to have **a width** owing to **multiple scattering**.

Harada, Nemoto,0803.3257(hep-ph)



Possible confirmation in Lattice QCD

Unquenched lattice simulation with hopefully chiral fermion action on a large lattice is
necessary for accommodate the possible chiral fluctuations with energy comparable to $T_c \sim 200$ MeV.

4. Summary and concluding remarks

☆ If a QCD phase transition is of a second order or close to that, there should exist specific soft modes, which may be easily thermally excited.

☆ In the fermion-boson system with $m_F << m_B$, the fermion spectral function has a 3-peak structure at 1-loop approximation at $T \sim m_B$.

⇒ If the chiral transition is close to a second order, quarks may have a 3-peak structure **in the QGP** phase near T_c.

☆ The physical origin of the 3-peak structure is the Landau damping of quarks and anti-quarks owing to the thermally excited massive boson, which induces a mixing between quarks and anti-quark hole,

☆ The boson may be vector-type or glueballs.

Future problems:

Full self-consistent calculation
Confirmatin in the lattice QCD
experimental observables ; eg. Lepton-pair production (PHENIX?)
transport coefficients
Soft mode (density-fluctuations) at the CEP and quark spectrum