The low-lying Scalar Mesons and Related Topics

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For the SCALAR collaboration;
C.Nonaka, A. Nakamura, S. Muroya,
M. Sekiguchi, H. Wada, T.K.

σ, κ, a_0 in full QCD, Phys. Rev. D70 (2004),034504
κ in quench, Phys. Lett. B 652(2007) 250 - 254

QCD 2008
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Plan of the talk

- Introduction; motivation and objectives, partly personal
- The sigma (, a_0 and the kappa) in lattice simulation with dynamical quarks
- The kappa in finer lattice in quenched lattice simulation
- Summary and future problems

The significance of the σ meson in low energy hadron physics and QCD

1. The pole in this mass range observed in the pi-pi S-matrix.
   As a compilation of the pole positions of the σ obatined in the modern analyses: Significance of respecting chiral symmetry,unitarity and crossing symmetry to reproduce the phase shifts both in the σ (s)- and ρ, (t)-channels with a low mass σ pole;(Igi and Hikasa(1999),I. Caprini, G. Colangero and H. Leutwyler, PRL(2006)).

2. Seen in decay processes from heavy particles; $D^+ \to \pi^-\pi^+\pi^+$
   E. M. Aitala et al, Phys. Rev. Lett. (86), 770 (2001)

3. Responsible for the intermediate range attraction in the nuclear force.

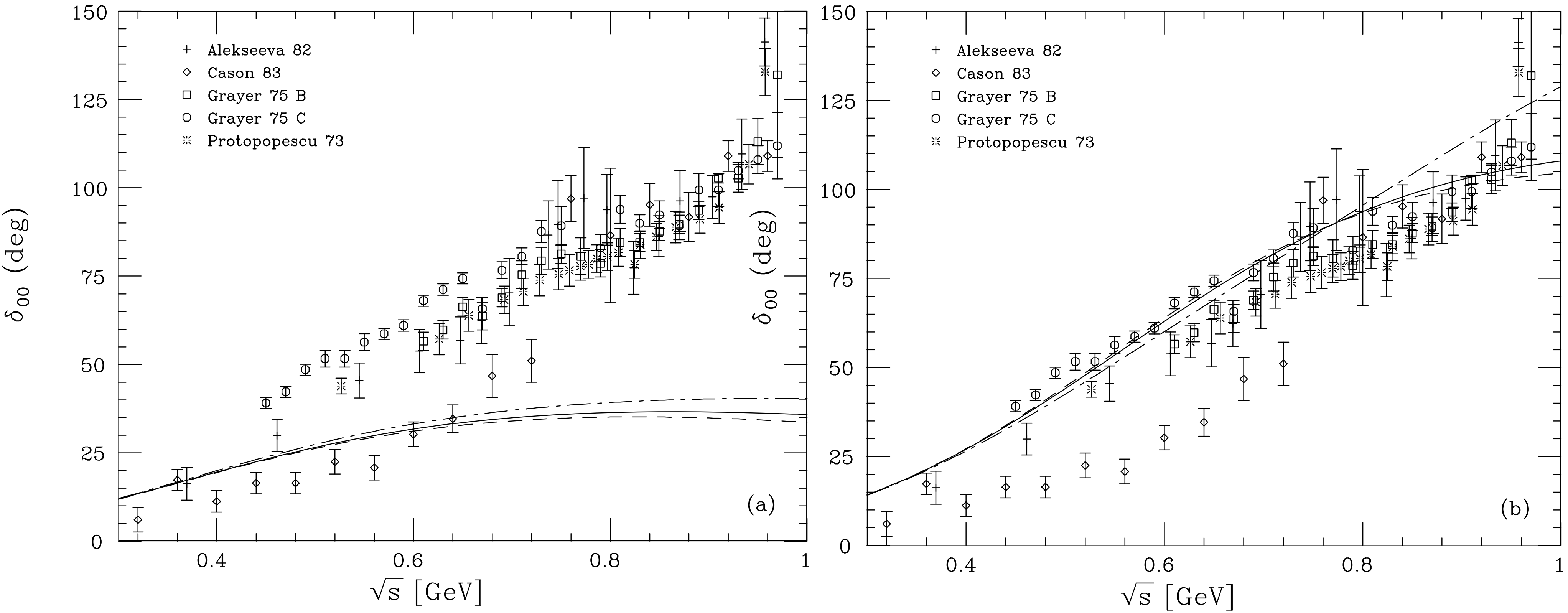
4. Accounts for Δ I=1/2 enhancement in $K^0 \to 2\pi$ compared with $K^+ \to \pi^+\pi^0$.
   E.P. Shabalin (1988); T. Morozumi, C.S. Lim and I. Sanda (1990).

5. π-N sigma term 40-60 MeV (naively » 15 MeV) ← enhanced by the collectiveness of the σ (.T.Hatsuda and T.K.(1990)) .

6. The σ : the quantum fluctuation of the chiral order parameter ↔ The Higgs particle in the WSG model

K. Igi and K. Hikasa, Phys. Rev. D59, 034005(1999)

The phase shifts in the sigma and rho channel in the N/D Method; resp. chiral symm., crossing symm and so on.



No σ but ρ in the t-channel

Both with the σ in the s- and the ρ in the t-channel

The σ in $D^+ \to \pi^- \pi^+ \pi^+$

E. M. Aitala et al, Phys. Rev. Lett. (86), 770 (2001)

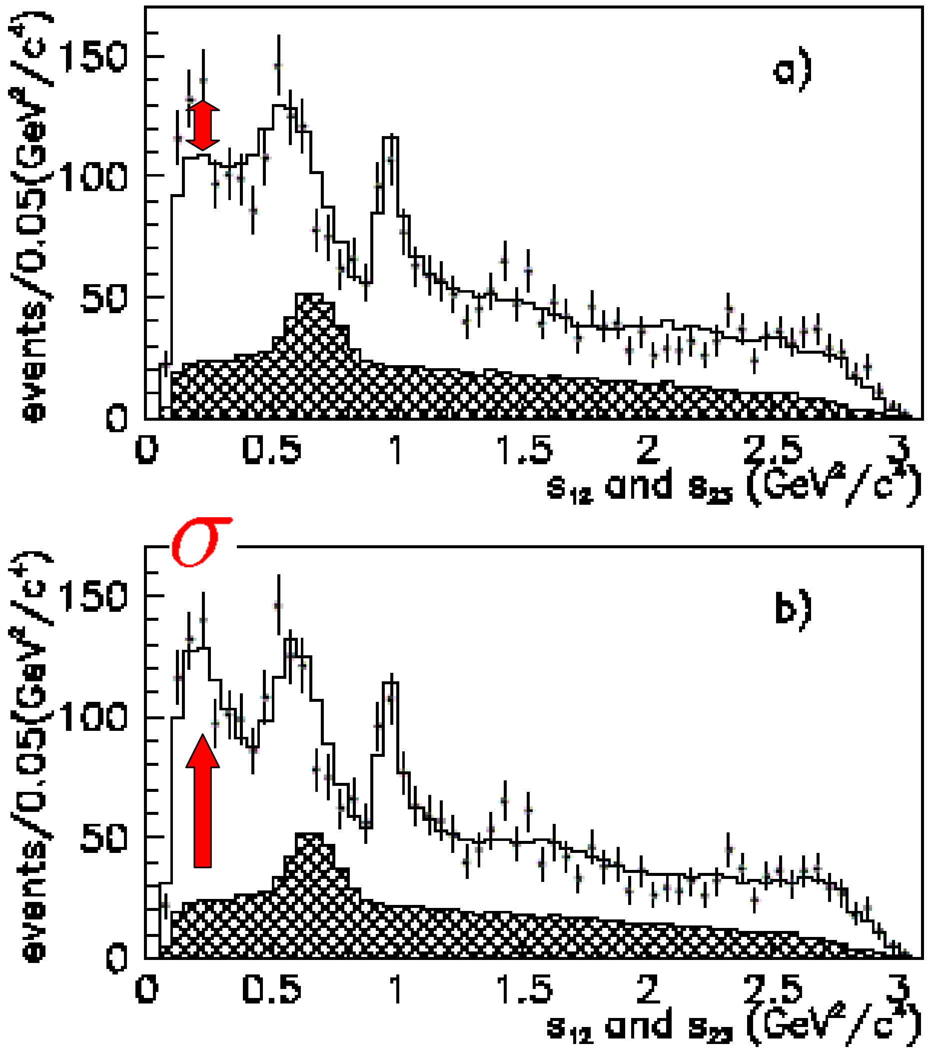


FIG. 2. s_{12} and s_{13} projections for data (error bars) and fast MC (solid line). The shaded area is the background distribution, (a) solution with the Fit 1, and (b) solution with Fit 2.

Without sigma pole

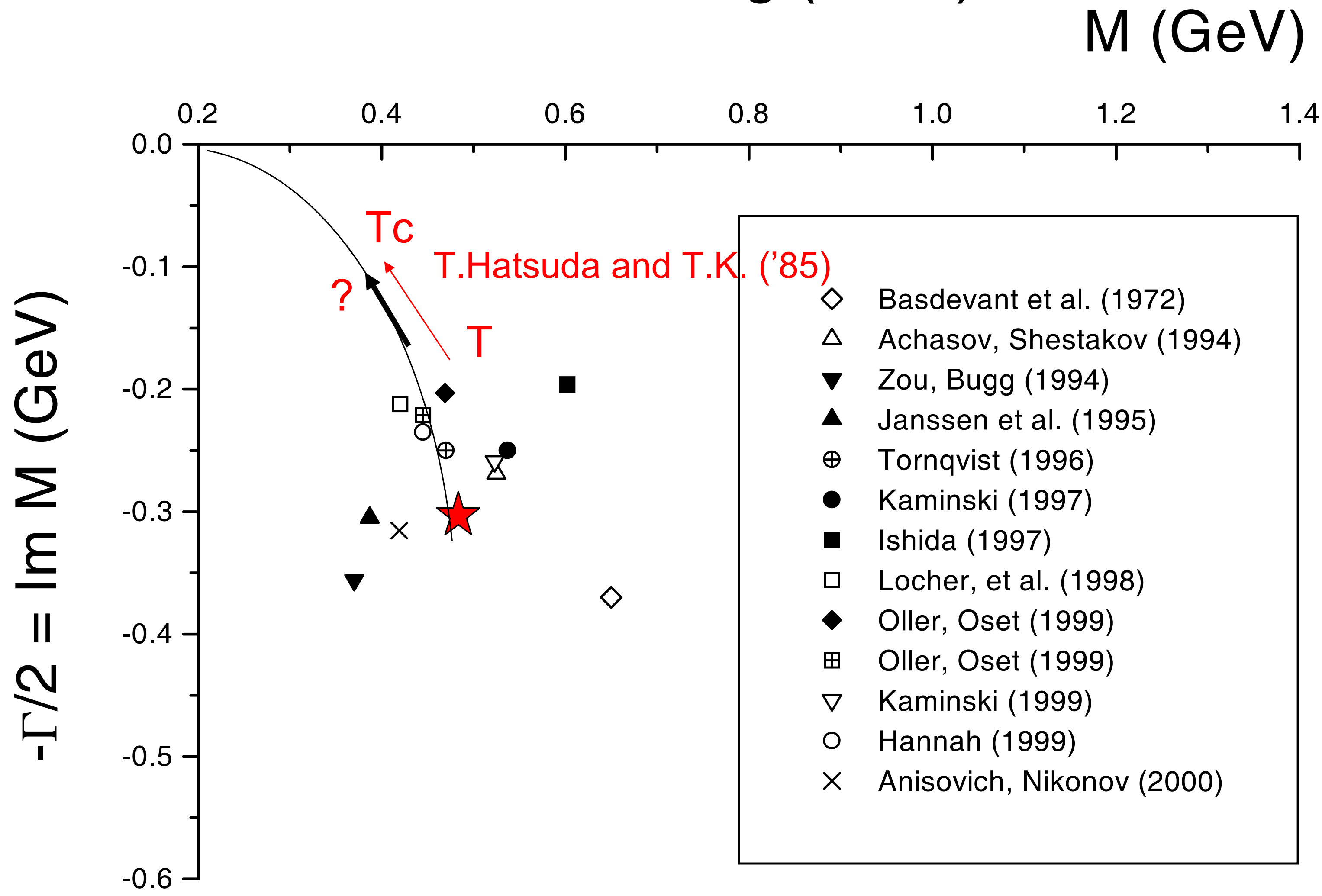
With a sigma pole:

$$m_\sigma = 478 \pm {}^{24}_{23} \pm 17 \text{ MeV}$$

$$\Gamma_\sigma = 324 \pm {}^{42}_{40} \pm 2117 \text{ MeV}$$

The poles of the S matrix of the pi-pi scattering in the complex mass plane for the sigma meson channel:

complied in Z. Xiao and H.Z. Zheng (2001)



See also, I. Caprini, G. Colangero and H. Leutwyler, PRL(2006);
H. Leutwyler, hep-ph/0608218 ; M_sigma=441 – i 272 MeV

The significance of the σ -degrees of freedom in chiral transition at finite T in lattice QCD

(F.Karsch, Lect. Notes 583 (2002), 209)

Chiral susceptibility

$$\chi_m = \frac{\partial}{\partial m}\langle\bar{\psi}\psi\rangle = \langle(\bar{\psi}\psi)^2\rangle$$

$J^{PC} = 0^{++}$

Polyakov loop and its fluc's.

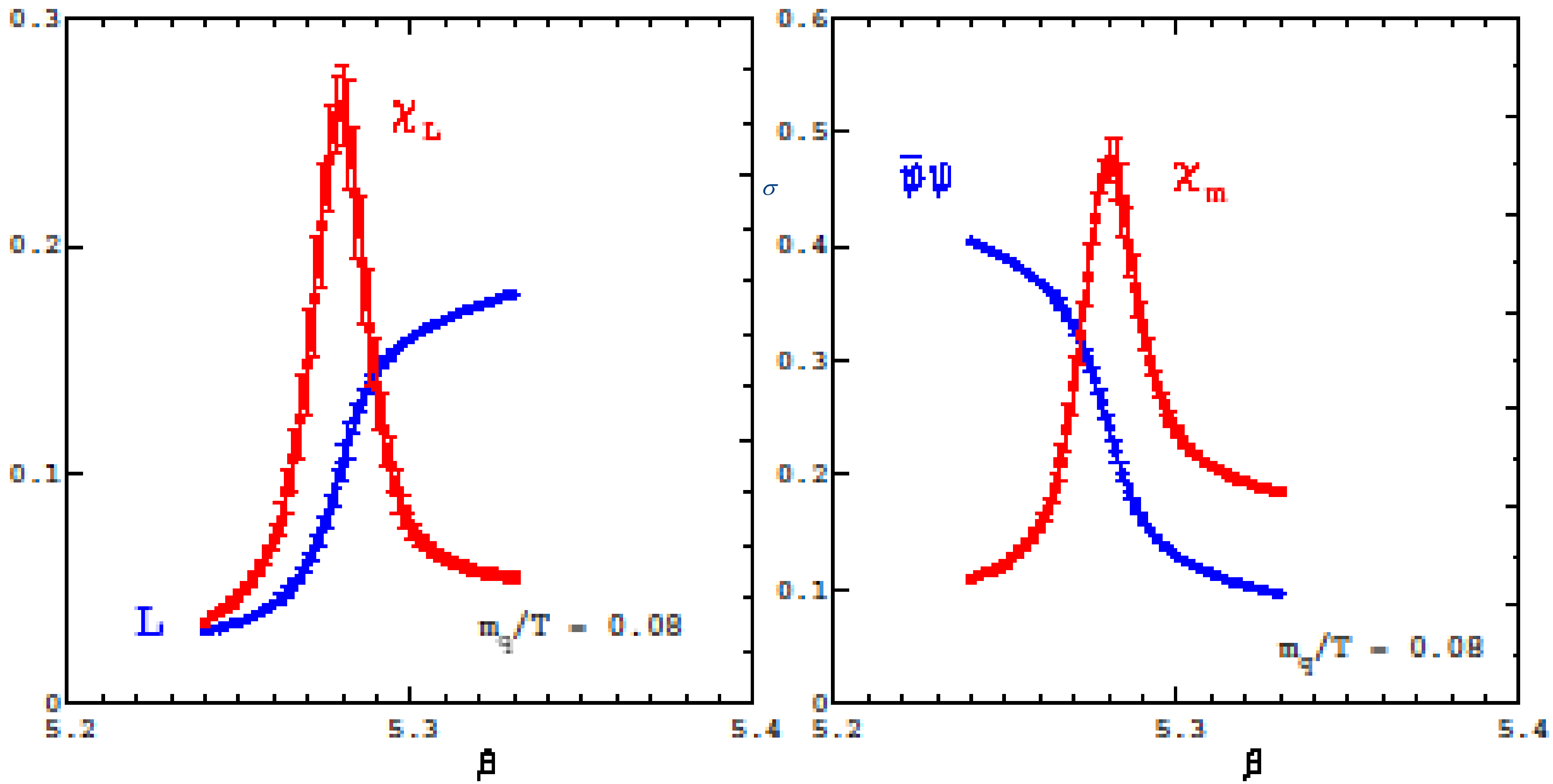


Fig. 2. Deconfinement and chiral symmetry restoration in 2-flavour QCD: Shown

The softening of the σ-like excitation around the critical temperature of chiral transition

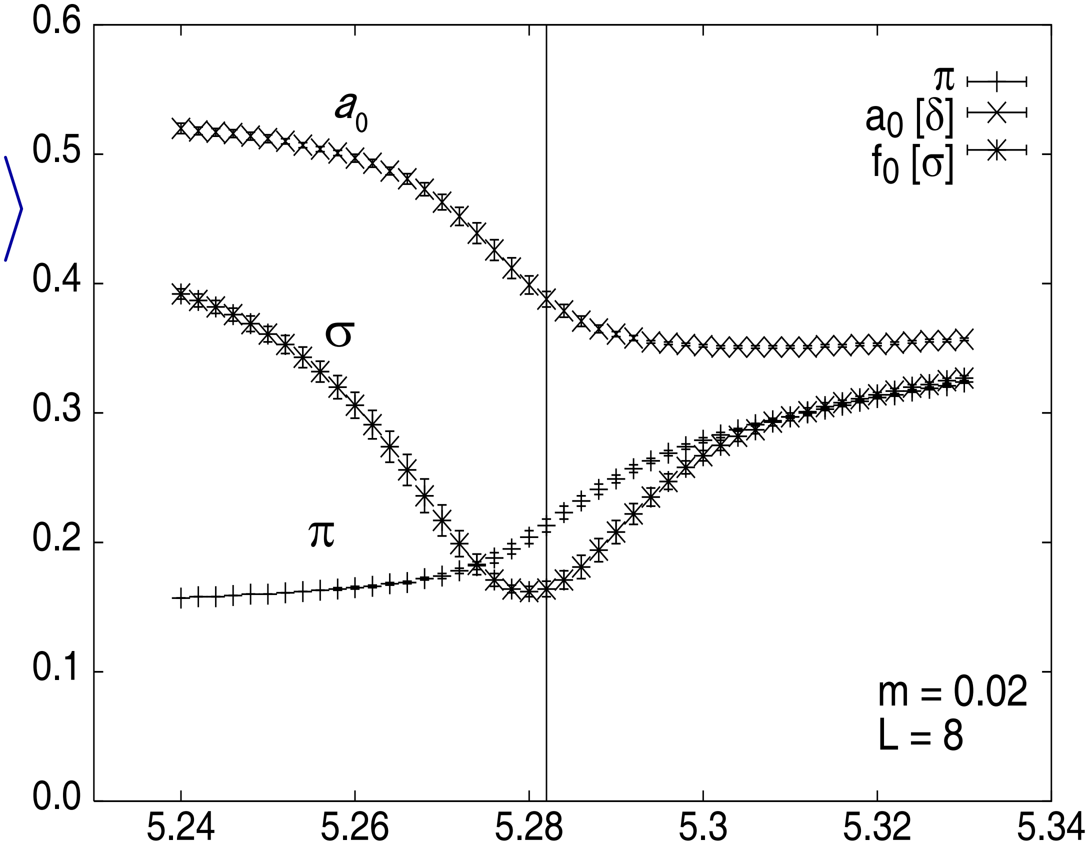
(F.Karsch, Lect. Notes 583 (2002), 209)

The generalized mass;

$$m_\sigma^2 = \chi_m^{-1}$$

$$\chi_m = \frac{\partial}{\partial m}\left\langle \bar{\psi}\psi \right\rangle = \left\langle (\bar{\psi}\psi)^2 \right\rangle$$

$$J^{PC} = 0^{++}$$



Remark: m_σ is not the dynamical mass as a pole of the time-correlator.

Issues with the low-mass σ meson in QCD

- In the constituent quark model; $J^{PC} = 0^{++} \rightarrow {}^3P_0$

  ➡ the mass in the 1.2 --- 1.6 GeV region.

Some mechanism needed to down the mass with ~ 800 MeV;

- (i) Color magnetic interaction between the di-quarks. (Jaffe; 1977)

  with the bag-model wave functions. ➡ **All the low-lying scalars are tetra quarks!**

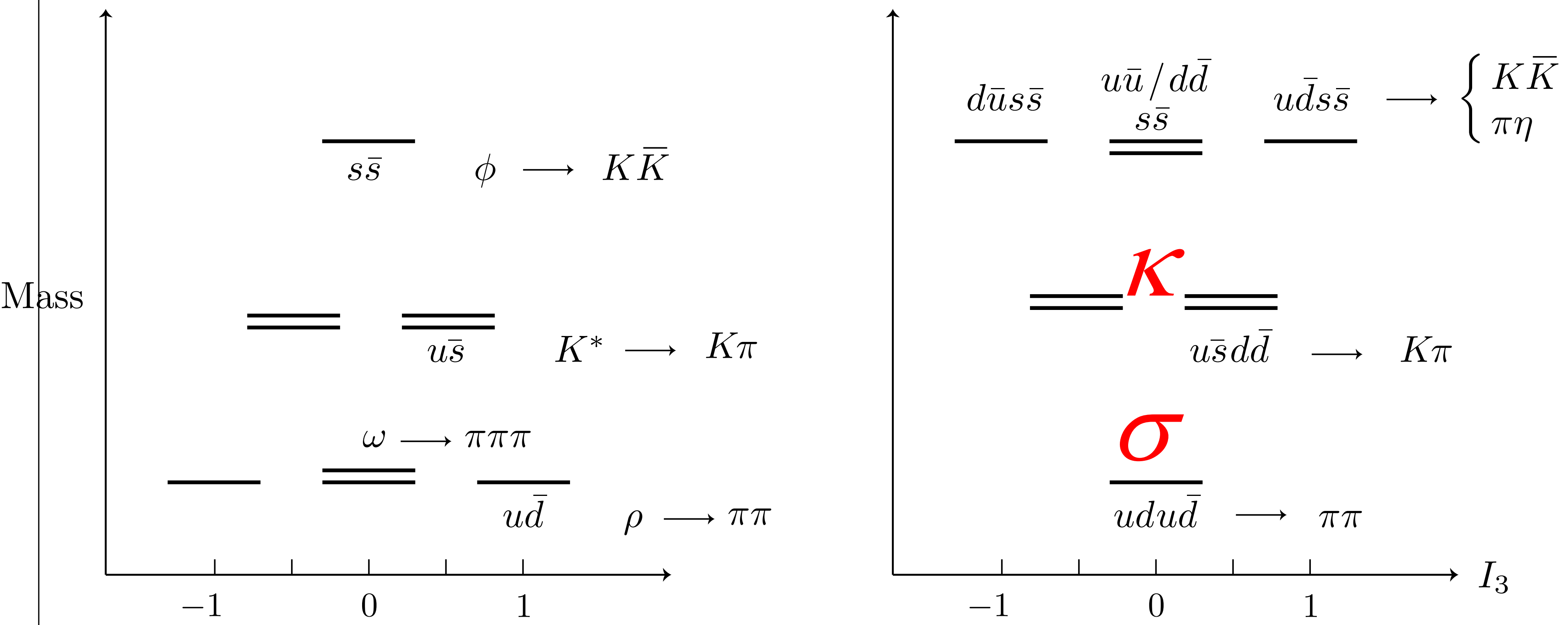
- (ii) The collectiveness of the scalar mode as the ps mode; a superposition of $q\bar{q}$ states. ⬅ Chiral symmetry (NJL)

- (iii) The π-π molecule as suggested in π- π scatt.

. (vi) a mixed state with scalar glue ball states

Scalar Mesons in the tetra-quark picture

$\bar{q}^2$-q^2 scheme (Jaffe(1977), Alford and Jaffe (2000))



1(a) 1(b)

Dynamical Chiral Symmetry Breaking and the sigma meson

Y. Nambu, **117** (1960), 648; Gauge invariance in Superconductivity → Appearance of a collective mode in the broken phase coupling to the longitudinal part of the current.

Gauge invariance

(Bogoliubov-Anderson)

Y. Nambu, PRL **4** (1960), 380;

Axial gauge (chiral) symm.

Y. Nambu and G. Jona-Lasinio, **122** (1960), 345;
Dynamical model of elementary particles based on an analogy with superconductivity.

The pion ; a (massless) collective mode associated with the dynamical breaking of chiral symmetry.

A scalar meson with the mass $2m_f$ appears as another collective mode than the pion. The sigma is a Higgs in QCD.

$$m_f \approx 300\text{MeV} \rightarrow m_\sigma = 2m_f \approx 600\text{MeV}$$

The feature essentially does not change with U_A(1) anomaly term incorporated;
T. Hatsuda and T.K.. Phys.Lett.B206 (1988), Z. Phy. C51 (1991)

Objective of Scalar Collaboration

- Confidence level of the sigma meson (and other scalar mesons, κ) has been increasing, and its physical significance in hadron physics and QCD is apparent.
- Using Lattice QCD, we have been (and will be) addressing the following Question about the scalar mesons:

  Are you a pole in QCD ?

  i.e., the σ and other low-lying scalar mesons are resonances in QCD or something else ?

, since as early as 2001-2002.

The Scalar mesons on the Lattice

---- A full QCD calculation -----

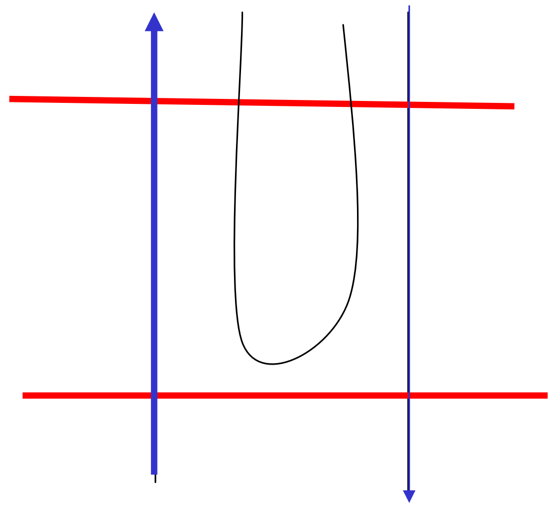
The Scalar Collaboration:

S. Muroya,A. Nakamura,C. Nonaka,M. Sekiguchi,
H. Wada,T. K.

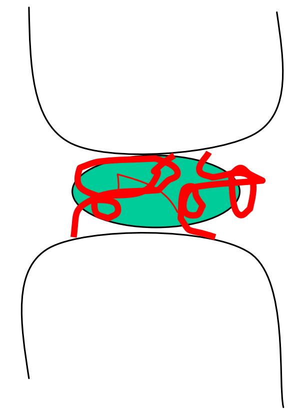
(Phys. Rev. D70, 034504(2004);
arXiv: 0310312(hep-ph))

A first exploratory work on the sigma in lattice QCD with dynamical quarks.

The **full** QCD simulation is necessary to properly describe the sigma with the possible contents, i.e., the glueball, tetra quarks and so on.



MM or **tetra quark**
as well as qq-bar



Glue ball

Previous Lattice QCD simulations of the scalars

- W. Lee and D. Weingarten
  - Phys. Rev. **D61** (1999) 014015
  - Quench
  - Mixing of Glue-ball
- UKQCD C.McNeile and C.Michael
  - Phys. Rev. **D63** (2001) 114503
  - Full QCD $m_\sigma < m_\pi$?
- Alford and Jaffe, Nucl.Phys. B578 (2000)367.
  - Quench
  - $\sigma = qq\overline{qq}$ $E(q\overline{q}q\overline{q}) < E(q\overline{q} + q\overline{q})$

Details of our Calculation
– Simulation parameters –

Lattice size : $8^3 \times 16$ ⟵ Very small !

$\beta = 4.8$ ⟵ Very strong coupling !

$\kappa = 0.1846,\ 0.1874,\ 0.1891$

> well established by CP-PACS,
> a = 0.197(2) fm , $\kappa_c = 0.19286(14)$
> (CP - PACS, Phys. Rev. D60(1999)114508)

Wilson Fermions & Plaquette gauge action

Point source

Number of the Z2 noise = 1000 ⟵ Very large !

Operator for σ Meson

I = 0, $J^{PC}=0^{++}$

c = 1,2,3 ⋯ color

α = 1,2,3,4 ⋯ Dirac spin

$$\sigma(x) \equiv \sum_{c=1}^{3} \overline{\psi}_c(x)\psi^c(x)$$

$$= \sum_{c=1}^{3}\sum_{\alpha=1}^{4} \frac{\overline{u}_\alpha^c(x)u_\alpha^c(x) + \overline{d}_\alpha^c(x)d_\alpha^c(x)}{\sqrt{2}}$$

$$\psi \equiv \begin{pmatrix} u \\ d \end{pmatrix}$$

Full QCD with disconnected diagram

⇓

glueball, tetraquark, meson-meson states are all coupled.

Cf. The ss-bar component of the sigma is small in the three-flavor NJL model with the determinatal anomaly term.
T. Hatsuda and T.K. ('88)

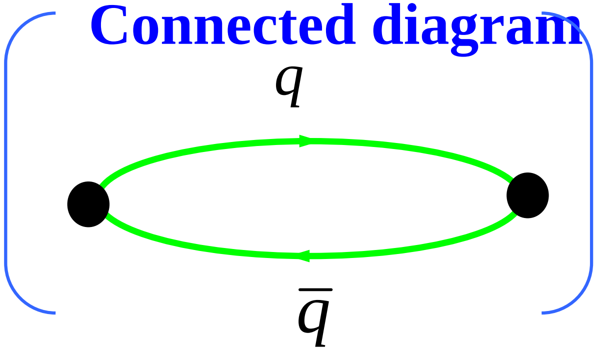
Propagator for σ meson (2)

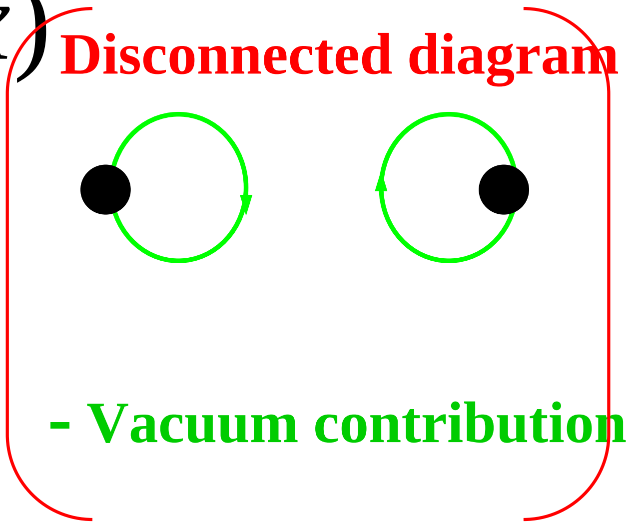
$$G(x,y) = -\langle Tr W^{-1}(x,y) W^{-1}(y,x) \rangle$$

$$+ 2 \langle (\sigma(x) - \langle \sigma \rangle)(\sigma(y) - \langle \sigma \rangle) \rangle$$

Where

$$\sigma(x) \equiv Tr W^{-1}(x,x) = \overline{\psi}(x)\psi(x)$$



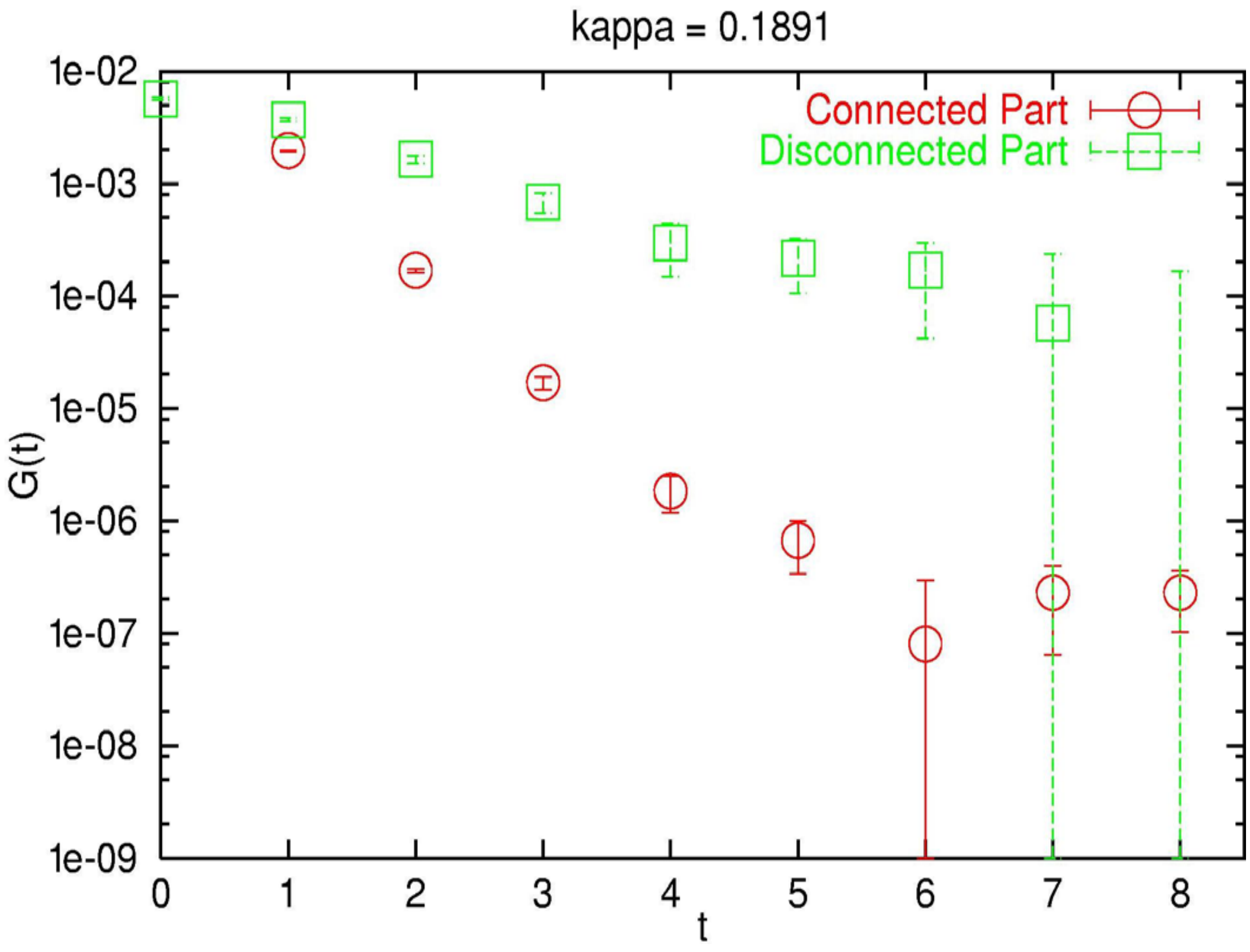


m_π / m_ρ

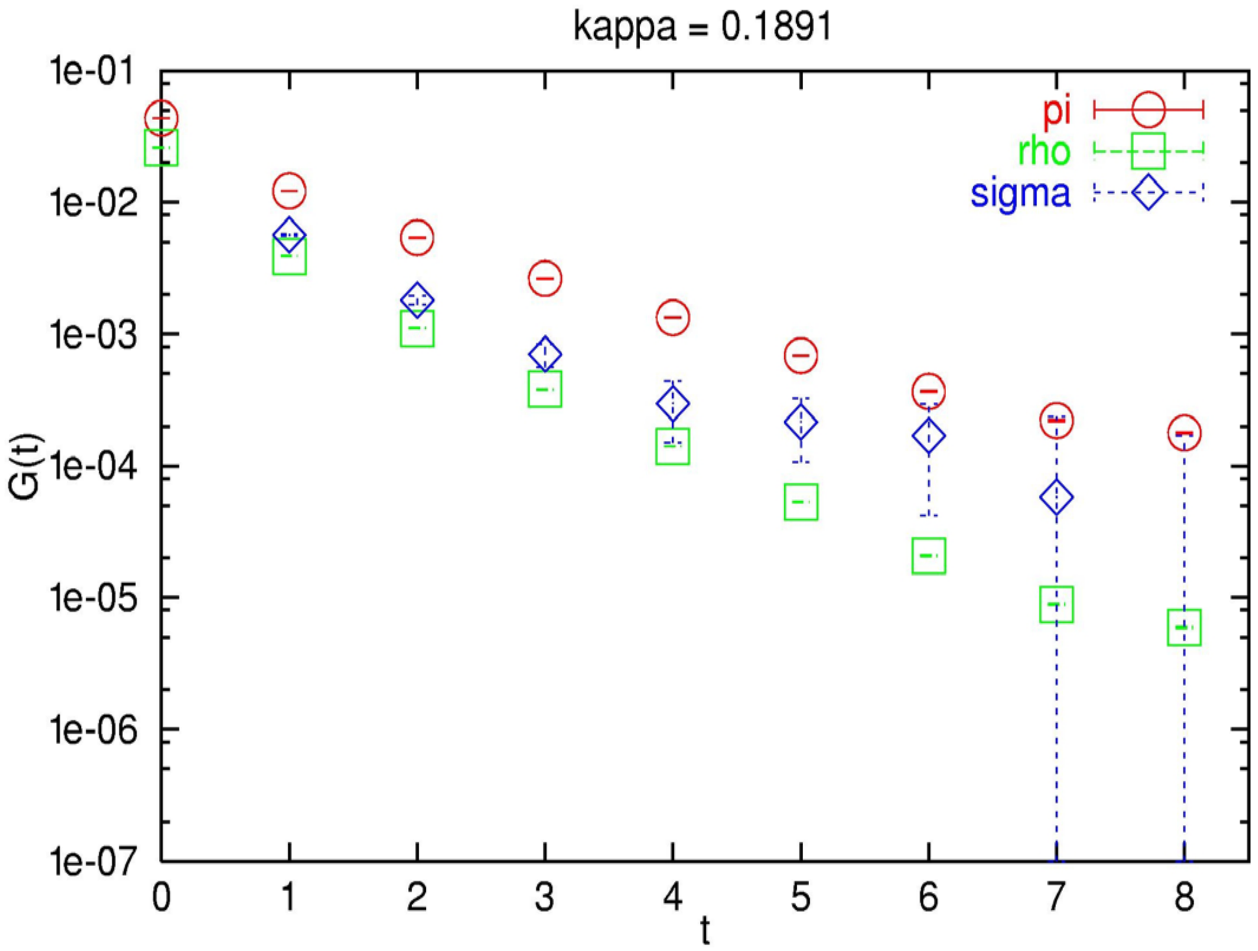
κ	m_π/m_ρ (Our Results)	m_π/m_ρ (CP-PACS)
0.1846	0.8245 ± 0.0012	0.8291 ± 0.0012
0.1874	0.7573 ± 0.0015	0.7715 ± 0.0017
0.1891	0.6928 ± 0.0023	0.7026 ± 0.0032

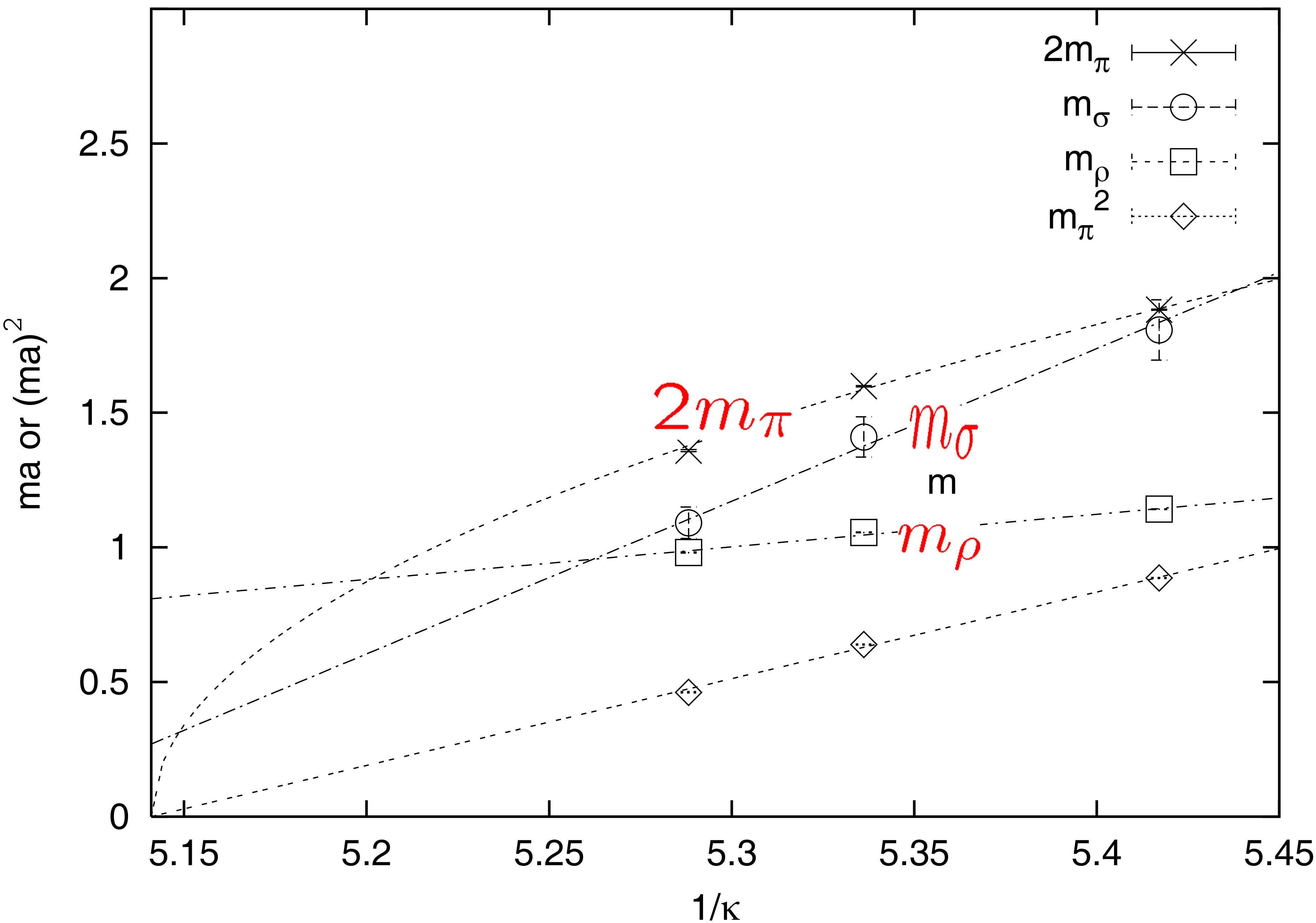
nearly equal

σ meson propagators
Connected Part & Disconnected Parts (κ = 0.1891)



Propagators for π, ρ, σ mesons (κ =0.1891)





2m_π
m_σ
m_ρ
m_π^2
2.5
2
1.5
1
0.5
0
ma or (ma)^2
5.15
5.2
5.25
5.3
5.35
5.4
5.45
1/κ
2m_π
m_σ
m
m_ρ

m_σ / m_ρ

$u\bar{d} \sim a_0(\delta)$

κ	m_σ/m_ρ	m_{con}/m_ρ
0.1846	1.583 ± 0.098	2.400 ± 0.018
0.1874	1.336 ± 0.071	2.436 ± 0.025
0.1891	1.112 ± 0.060	2.481 ± 0.031

Phys. Rev. D70, 034504(2004); arXiv: 0310312(hep-ph))

We conclude

the sigma shows a pole behavior

and $m_{\pi} < m_{\sigma} \leq m_{\rho}$

Here the disconnected diagram plays essential role.

The flavored scalar mesons are not light as observed.

m_{κ} ~ 1.8 GeV> 0.8 GeV m_{a0} ~ 1.9 GeV > 0.98 GeV

Caviats; the lattice is still coarde (~0.2fm).

The kappa meson

- 0^+ scalar meson with the strangeness
- Recent experimental candidates:
  - Fermilab E791: hep-ex/0204018, (PRL89(02)12801).
  - BES:hep-ex/0304001.
- Both observed a candidate near 800MeV
- Even 660 MeV!;

Eur. Phys. J. C48('06), 553

(hep-ph/0607133)

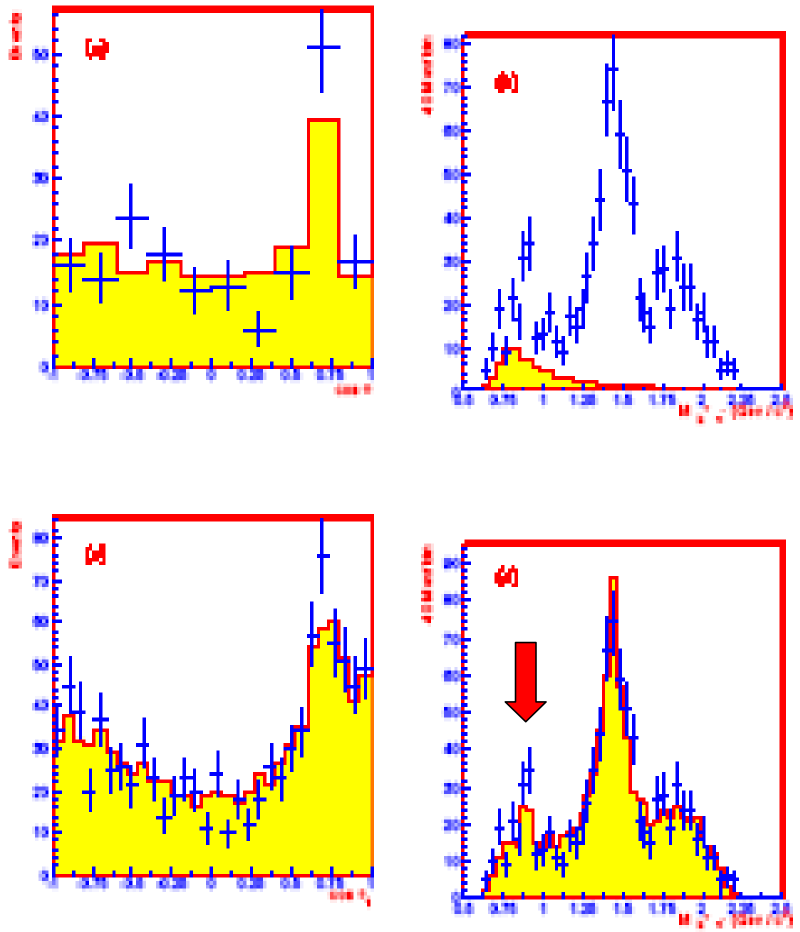


FIG. 4. (a) The final fit to the angular distribution of K^+ in $K^+\pi^-$ rest frame in κ mass region. The crosses stand for the data and histogram is our final fit. ($M_{K^+\pi^-} < 1.1$ GeV) (b) The κ-particle contribution. The crosses are the data and the shaded histogram is the contribution from κ-particle. (c) The final global fit to the angular distribution. θ_1 is the polar angle of K^+ particle in the κ rest reference. (d)The final global fit to the invariant mass spectrum.

Quench simulation of the kappa

H. Wada et al (SCALAR collaboration),
Phys. Lett. B(2007); aeXiv:hep-lat/0702023

c.f. m_κ ~ 1.8 GeV> 0.8 GeV
with a~0.2 fm

- Wilson fermion,
- Plaquette gauge action
- 20^3 x24,
- a=0.1038(33) fm $\beta = 5.9$
- Hopping parameters;

  h_u,d=0.1589, 0.1583, 0.1574

  h_s=0.1557, 0.1566

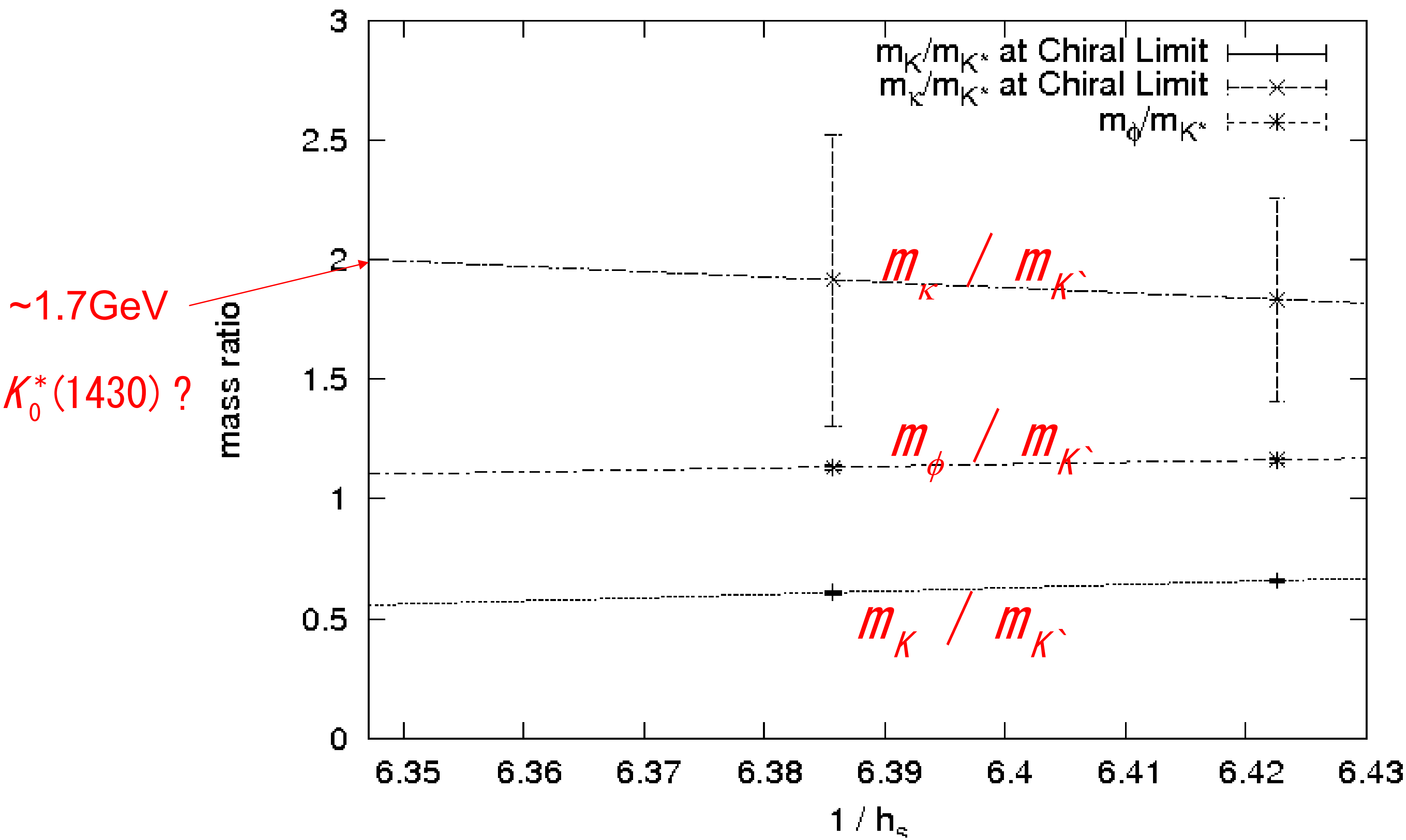


Fig. 5. The ratios *mK=mK¤* and *m·=mK¤* at chiral limit, and *mÁ=mK¤* for *s* quark hopping parameters *hs* = 0.1566 and 0.1557.

Summary

- The sigma meson and other low-lying scalar mesons are still a source of debates.

  The understanding of the nature or the even (non-)existence is important for a deep understanding of the QCD vacuum as well as the QCD/hadron dynamics.
- A full QCD lattice simulation suggests the existence of a low-lying sigma as a pole in QCD; its physics content, i.e., a tetra quark, a hybrid with the glue ball or the qq-bar collective state, is obscure: the disconnected diagram gives the dominant contribution. $m_\pi < m_\sigma < m_\rho$ in the chiral limit.
- A quenched lattice calculation suggests that the kappa can not be a normal qq-bar state.
- Exploring the possible change of the spectral function in the scalar channel in the hot and/or dense medium would be interesting.

What's next ?

What's next ?

MANY!

- Better Simulation
  - Larger lattice; continuum limit → Glue ball is strongly dep. on a, lattice spacing. (UKQCD)
  - Error reduction, eg. smearing
    - Large errors come from

$$\langle (\sigma(x) - \langle \sigma \rangle)(\sigma(y) - \langle \sigma \rangle) \rangle$$

--variational method with multiple interpolating op's.
an explicit inclusion of tetraquark operator.

  - volume dependence; resonance or scattering
  - chiral fermions, Nc-dependence, etc

- Observables which are sensitive to the inner structure? Possible role of axial anomaly?....
- the sigma at finite T?

The κ

- full QCD simulation with multiple interpolating op's including a tetraquark
- operator with variational method, and analysis of the volume dependence.

Actually, there have been a quite remarkable development in the studies of the low-lying scalar mesons using (un)quenched lattice QCD. However, no full-QCD simulation with disconnected diagrams included and these conditions satisfied so far, despite of the vigorous activities on this problem.

See the following nice review articles for more detailed accounts:
Craig McNeil, arXiv:0710.0985, arXiv:0710.24708hep-lat]
Sasa Prelovsek, arXiv:0804.2549[hep-lat]

BACK UPS

N_c dependence of the physical contentof the scalar mesons

Nc−dependence of the nature of the sigma:

T. Shaefer (‘03); Instanton liq. model

Nc=3; sigma ~ $q\bar{q} + (q\bar{q})^2$

Nc>3 m_sigma increases, ~ $q\bar{q}$

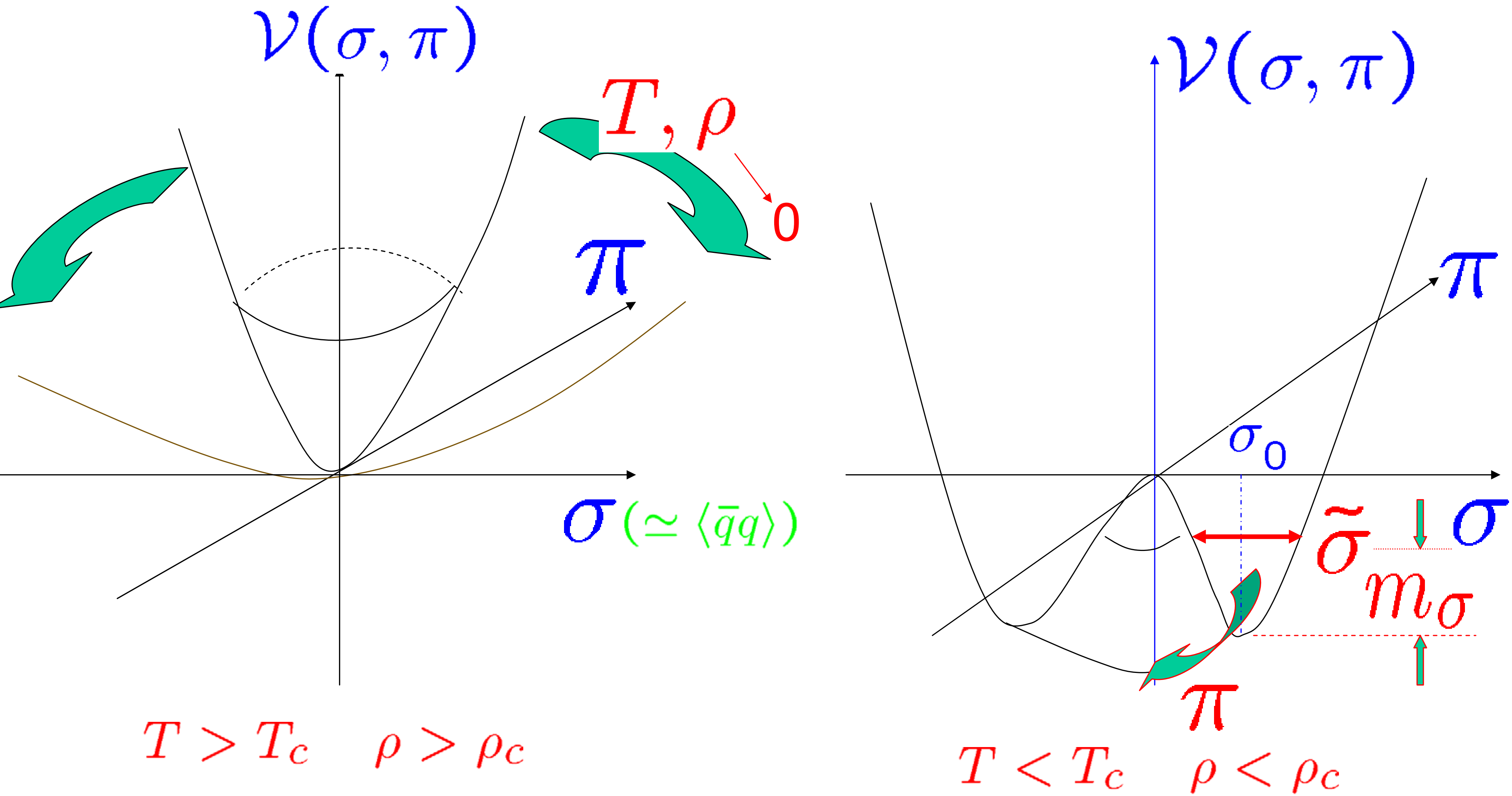
J.R. Pelaez(03, 06); unitarized chiral perturbation

large Nc, $qqqq$

Harada et al;

Nc<6, di-quark-anti-diquark is necessary

Chiral Transition and the collective modes



$$\sigma = \sigma_0 + \tilde{\sigma}$$

c.f. Higgs particle in WSG model

ϕ ; Higgs field ➡ $\phi = \langle \phi \rangle + \tilde{\phi}$

Higgs particle

Statistics

[κ = 0.1846]

1110 configurations from 2070th trajectory

[κ = 0.1874]

860 configurations from 2000th trajectory

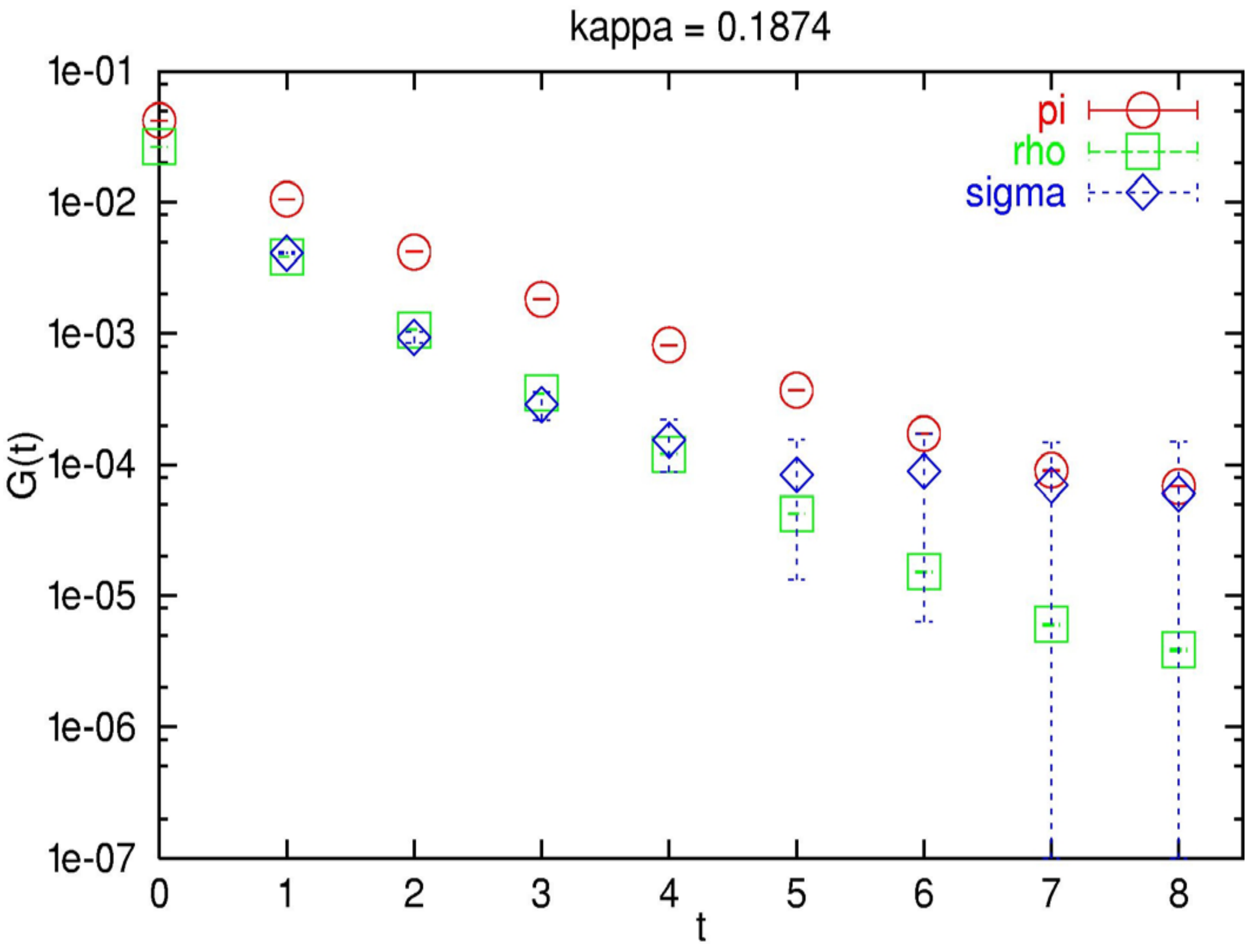
[κ = 0.1891]

730 configurations

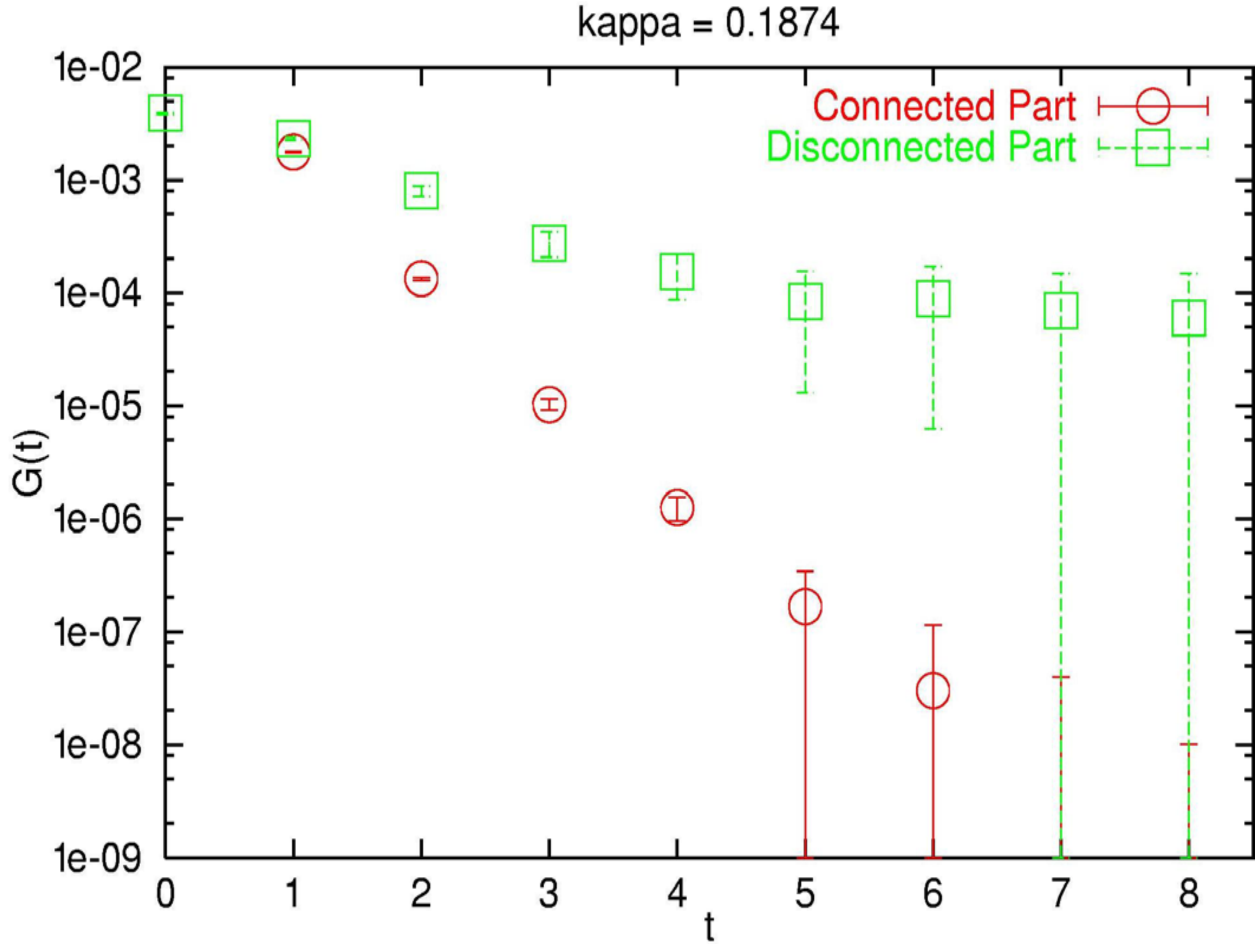
from 2010th trajectory (cold start)

2000th trajectory (hot start)

Propagators for π, ρ, σ mesons (κ =0.1874)



σ meson propagators
Connected Part & Disconnected Parts （ κ = 0.1874 ）



Extrapolation

0.8093

⇓

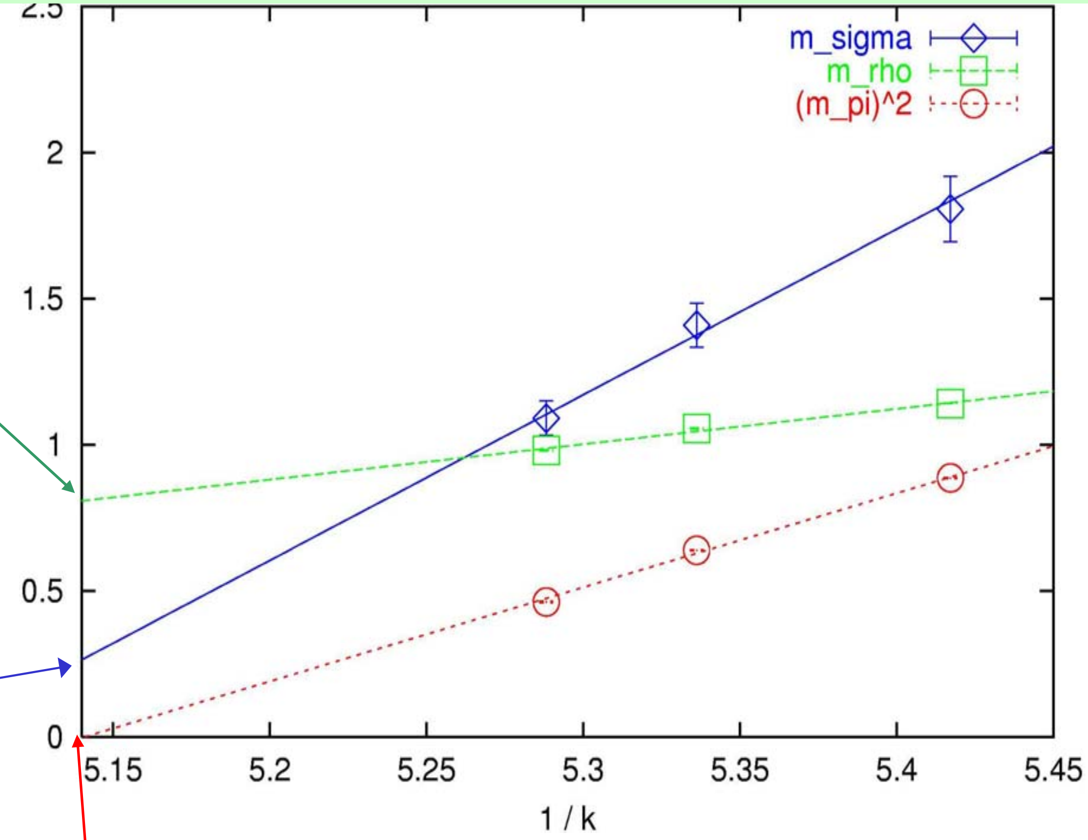
a = 0.207±0.009 fm

(CP-PACS
a = 0.197(2) fm)

0.270

$$\frac{m_\sigma}{m_\rho} = \frac{0.270}{0.8093} \approx 0.334$$

m_σ=257MeV



5.1410±0.0747 ⇒ κ c = 0.1945±0.0029

(CP-PACS κ c = 0.19286(14))

T.K. and T. Hatsuda, Phys. Lett. B240 (1990) 209

Light quark contents of baryons

B	$\langle \bar{u}u \rangle_B$	$\langle \bar{d}d \rangle_B$	$\langle \bar{s}s \rangle_B$
P (938)	4.97 (2)	4.00 (1)	0.53 (0)
Λ^0 (1115)	3.63(1)	3.63(1)	1.74(1)
Δ^{++} (1232)	3.66(2)	0.76 (0)	0.26 (0)
Ω^- (1672)	0.72 (0)	0.72 (0)	3.71 (3)

The numbers in (,) are those in the naive quark model.

The quark content (or the scalar charge of the quarks) isenhanced by the collective σ mode in the scalar channel!

$$\Sigma_{\pi N} = \hat{m}\langle \bar{u}u + \bar{d}d \rangle_N$$
$$= 5.5\text{MeV} \times (4.97 + 4)$$
$$\simeq 50\,\text{MeV}$$
$$>> 5.5 \times (2+1) \simeq 17\,\text{MeV}$$

C.f. $y \equiv \frac{2\langle \bar{s}s \rangle_N}{\langle \bar{u}u + \bar{d}d \rangle_N} = 0.12$

The empirical value of π-N Sigma term is reproduced due to the enhancement of the scalar charge due to the σ-mesonic collective mode!

See also T.K., Supplement of Prog. Theor. Phys. 120