Momentum Fractionation on Superstrata

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Microstructures of Black Holes Workshop
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1211.0306	S. Giusto, O. Lunin, S. D. Mathur, DT
1508.01231	B. Chakrabarty, A. Virmani, DT
& work in prog.	I. Bena, E. Martinec, N. Warner, DT

Outline

1. General & technical introductions

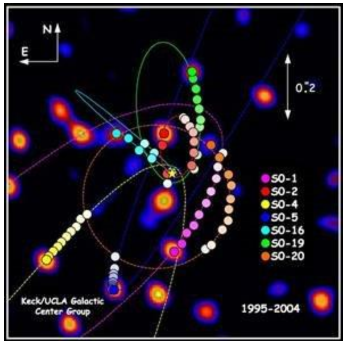
2. BPS & Non-BPS D1-D5-P black hole microstates

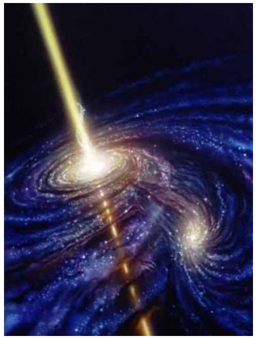
   from fractional spectral flow

3. Momentum Fractionation on Superstrata

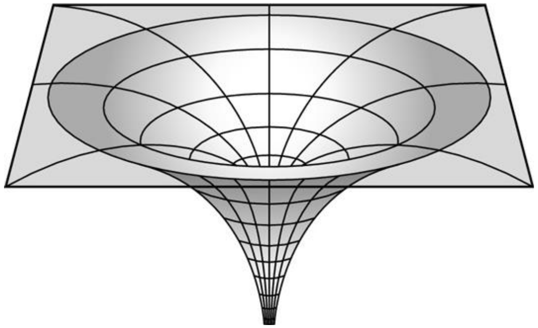
Black Holes

Physical: dark, heavy, compact bound state of matter

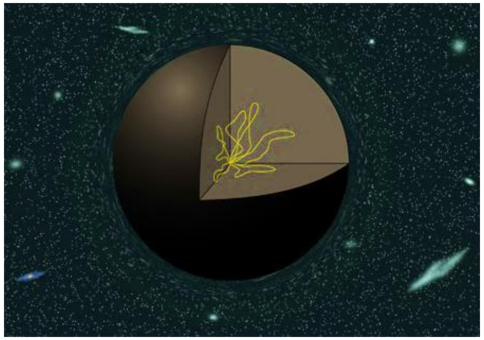




(Semi-)classical: geometry with horizon



Quantum: bound state in quantum gravity theory

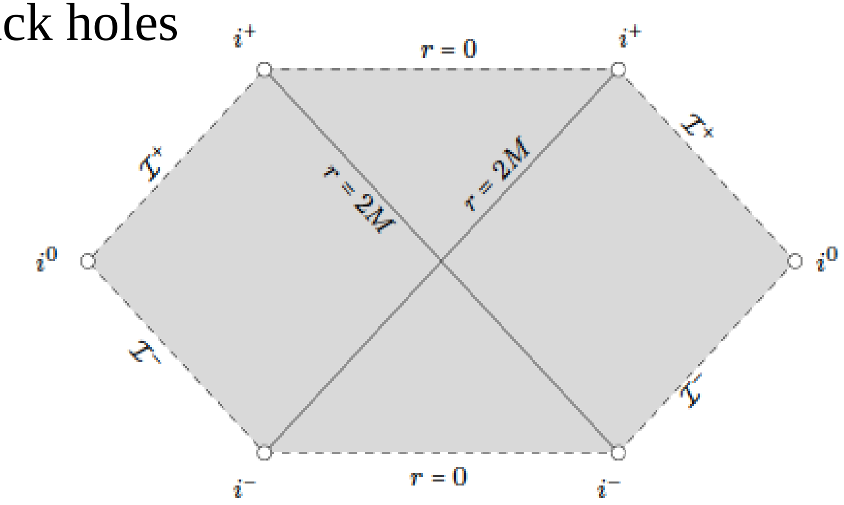


Why Black Holes?

- We desire a unified theory of fundamental laws of Nature
- Requires a theory of quantum gravity (QG)
- String theory: leading candidate

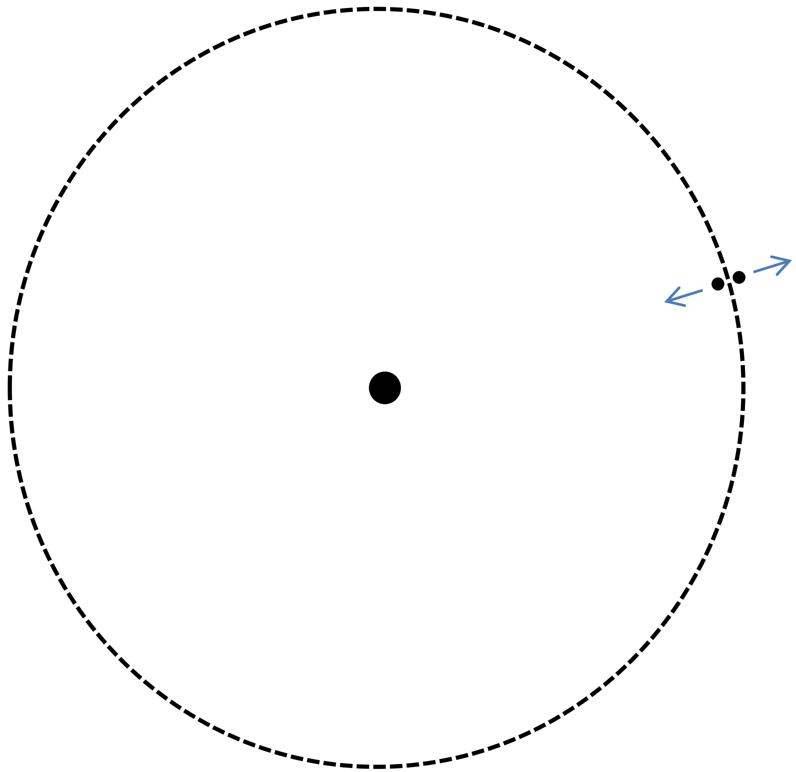
- Strong evidence for existence of Black holes
- Major test of any QG theory:

  Black hole information paradox.



The Information Paradox

BH Horizon:
normal lab physics
(small curvature)



Hawking radiation:
pair creation

→ entangled pair

- Endpoint of process: violation of unitarity or exotic remnants. Hawking '75
- Conclusions robust including small local corrections Mathur '09

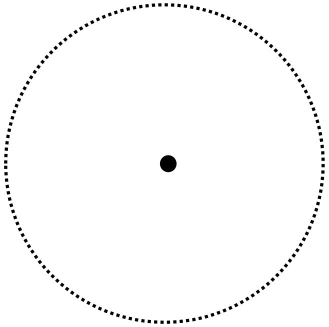
Black hole hair

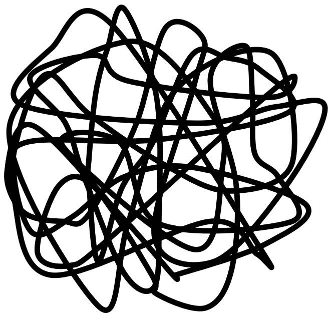
- Bekenstein-Hawking entropy $S \;\rightarrow\; e^S$ microstates
- Can physics of individual microstates modify Hawking's calculation?
- Many searches for Black hole 'hair': deformations at the horizon.
- In classical gravity, many 'no-hair' theorems resulted

Israel '67, Carter '71, Price '72…

In String theory, we do find hair. This suggests that

- Quantum effects important at would-be-horizon (fuzz)
- Bound states have non-trivial size (ball)





"Fuzzball"

D1-D5-P: large BPS black hole

- D1-D5-P black hole is a macroscopic BPS black hole in 5D

- Entropy reproduced from microscopic degrees of freedom

  Strominger, Vafa ’96

- Large classes of three-charge microstate geometries constructed

  Mathur, Bena, Warner, Giusto,
  de Boer, Russo, Shigemori,…

- Recent significant progress: construction of superstratum

  Bena, de Boer, Shigemori, Warner ’11
  Bena, Giusto, Russo, Shigemori, Warner ’15

Large BPS, non-BPS, non-extremal

Despite much progress, important questions remain:

1. Can the physics of fractionated momentum be described in sugra?
2. Can one construct (many) solutions which have angular momenta within the black hole regime, and which have large three-charge throats?

E.g. Denef; Bena, Wang, Warner "Scaling Solutions"

Both these questions are key to understanding (more) typical states of large BPS black holes.

3. Can the program be fully extended to non-extremal black holes?

This is crucial to studying Hawking radiation, but only a handful of solutions known.

Jejjala, Madden, Ross, Titchener '05, …

Bena, Bossard, Katmadas, Turton '15

- This talk: Momentum fractionation in large BPS & non-extremal solutions.

The D1-D5 system on T^4

D1-D5 system on T^4: setup

Work in type IIB string theory on

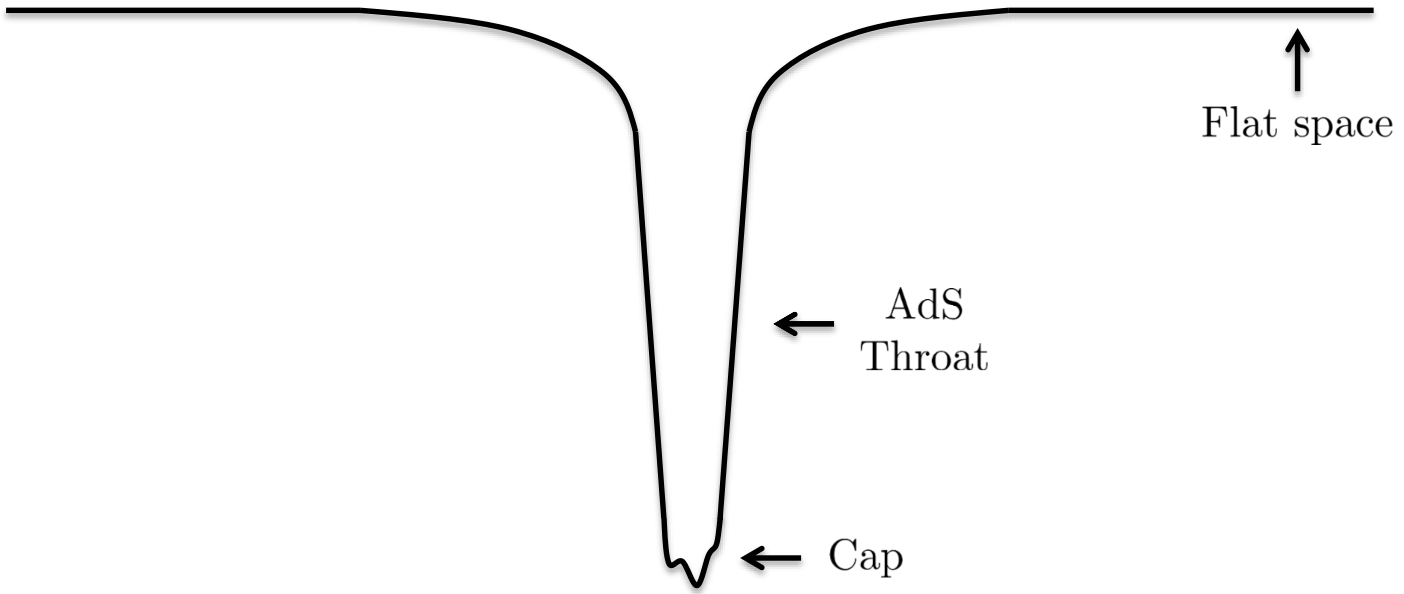
$$\begin{array}{ccccc} \mathbb{R}^{1,4} & \times & S^1 & \times & T^4 \\ t, x^\mu & & y & & z^i \end{array}$$

- Radius of S^1 : R_y
- Wrap n_1 D1 branes on S^1
- Wrap n_5 D5 branes on $S^1 \times T^4$

For states which have geometrical descriptions, the geometry will have D1 and D5 charges given by

$$Q_1 = \frac{g_s \alpha'^3}{V} n_1 \,, \qquad Q_5 = g\alpha' n_5$$

To get an AdS throat, take $(Q_1\, Q_5)^{1/4} \ll R_y$. Structure of geometry is then:



The throat is locally $AdS_3 \times S^3 \times T^4$.

D1-D5 CFT & AdS/CFT

- Worldvolume gauge theory on D1-D5 bound state flows in IR to a (4,4) SCFT.
- Orbifold point in moduli space: Free SCFT on $(T^4)^N/S_N$, $N = n_1 n_5$.

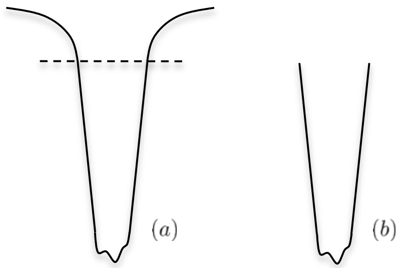
Symmetry generators:

Vafa '95, Douglas '95

- Virasoro$_L \times$ Virasoro$_R$ $\qquad L_{-n} \longleftrightarrow AdS_3$
- R-symmetries $SU(2)_L \times SU(2)_R$ – indices $\alpha, \dot{\alpha}$ $\qquad J^a_{-n} \longleftrightarrow S^3$
- U(1) currents of T^4 translations $\qquad J^i_{-n} \longleftrightarrow T^4$

  T^4 : broken SO(4) ~ $SU(2)_1 \times SU(2)_2$ – indices A, $\dot{A}$

& Susy generators



Maldacena '97

Fields & Twist operators

- Orbifold CFT on $(T^4)^N/S_N$: N copies of T^4 sigma model, fields:

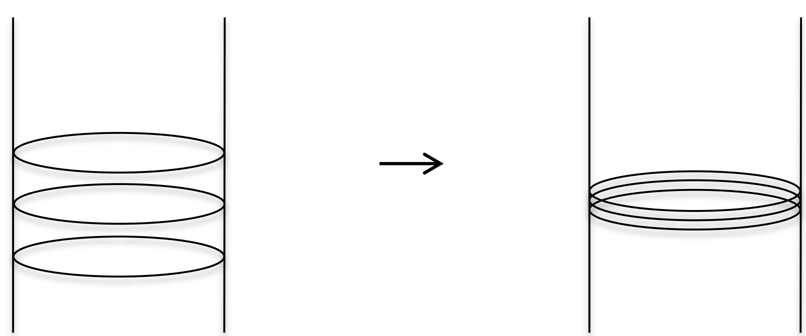
$$X_{A\dot{A}} \qquad \psi^{\alpha A} \qquad \bar{\psi}^{\dot{\alpha} A}$$

- Twist operators: permute fields, 'link together' different copies. On cylinder with coordinate $w = \tau + i\sigma$, action is

$$\sigma_k : \qquad \begin{array}{l} X^{(1)} \to X^{(2)} \to \cdots \to X^{(k)} \to X^{(1)} \\ \psi^{(1)} \to \psi^{(2)} \to \cdots \to \psi^{(k)} \to -\psi^{(1)}. \end{array}$$

- The operator σ_k links together k copies of the sigma model to effectively make a single CFT on a circle k times longer.

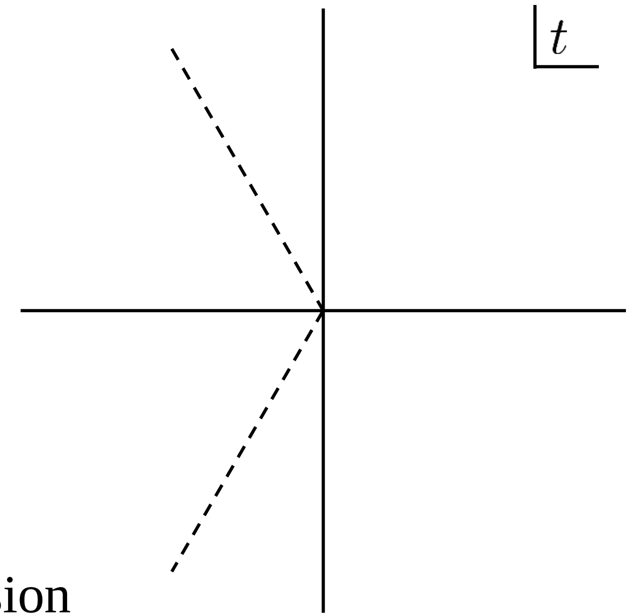
σ_3 :



'Component String'

Twist operators

- Let us define the twist operator σ_k in more detail.
- We map cylinder to plane $z = e^w$; let us construct twist operator at $z = 0$
- We pass to a local covering space via a map of the local form $z \simeq t^k$
- A generic point in the z plane has k images in the t plane
- The k copies of the fields in the z plane map to one single-valued field in the t plane
- We insert the identity operator at the origin of the t plane; this gives the lowest-dimension operator in this twist sector.



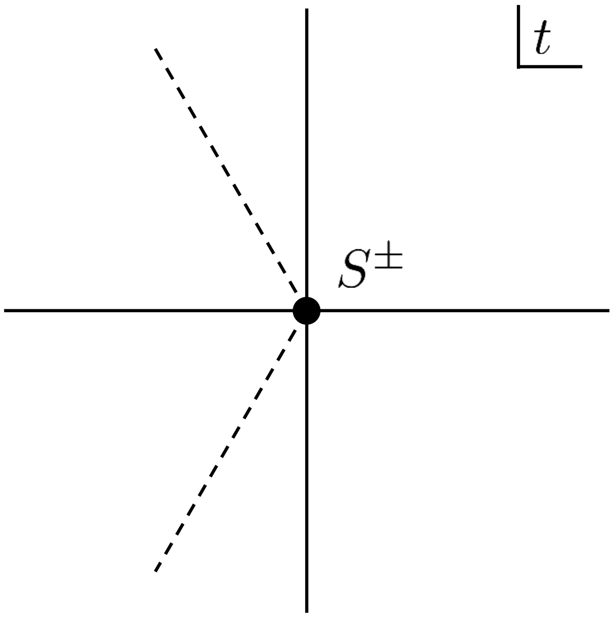
Excited twist operators

- One can also define excited (spin-)twist operators, by dressing the 'bare' twist σ_k

- We are particularly interested in the twist operators obtained by inserting a spin field $S^{\pm}$ at the origin of the t plane

- In the z plane, we denote this operator as

$$\sigma_k^{\pm} = S_k^{\pm} \sigma_k$$



- At the origin of the t plane, this creates a Ramond vacuum.
- In the original cylinder/plane, we denote this state by

$$|0_R^{\pm}\rangle_k$$

Excited twist operators

- Note that the dressed twist operator $\sigma_k^{\pm}$ imposes the following monodromies on the fields on the original cylinder:

$$\sigma_k^{\pm}: \qquad \begin{array}{l} X^{(1)} \to X^{(2)} \to \cdots \to X^{(k)} \to X^{(1)} \\ \psi^{(1)} \to \psi^{(2)} \to \cdots \to \psi^{(k)} \to \psi^{(1)}. \end{array}$$

- Adding in the antiholomorphic fields, we also have a right-moving spin field $\bar{S}_k^{\dot{\alpha}}$ which gives

$$\sigma_k^{\alpha\dot{\alpha}} = S_k^{\alpha} \bar{S}_k^{\dot{\alpha}} \sigma_k$$

T^4-invariant, bosonic R-R ground states

- Acting with $\sigma_k^{\alpha\dot{\alpha}}$ on the NS-NS vacuum yields four R-R ground states in each twist sector,

$$|\alpha\dot{\alpha}\rangle_k = \sigma_k^{\alpha\dot{\alpha}}|0_{\rm NS}\rangle$$

- Acting with fermion zero modes on L and R sectors, we obtain four ground states,

$$|AB\rangle_k = \epsilon_{\alpha\beta}\epsilon_{\dot{\alpha}\dot{\beta}}\psi_0^{\alpha A}\psi_0^{\beta B}|\beta\dot{\beta}\rangle_k$$

- The singlet in the above gives the fifth T^4–invariant ground state in each twist sector,

$$|00\rangle_k = \epsilon_{AB}|AB\rangle_k \, .$$

Spectral Flow

- Spectral flow (SF) is a continuous transformation which modifies the boundary conditions of fermions. It acts on both operators and states.
- Independent SF transformations in L and R-moving sectors.
- For L-moving states, under SF with parameter α ,

  the weights and $\mathrm{SU}(2)_R$ charges transform as

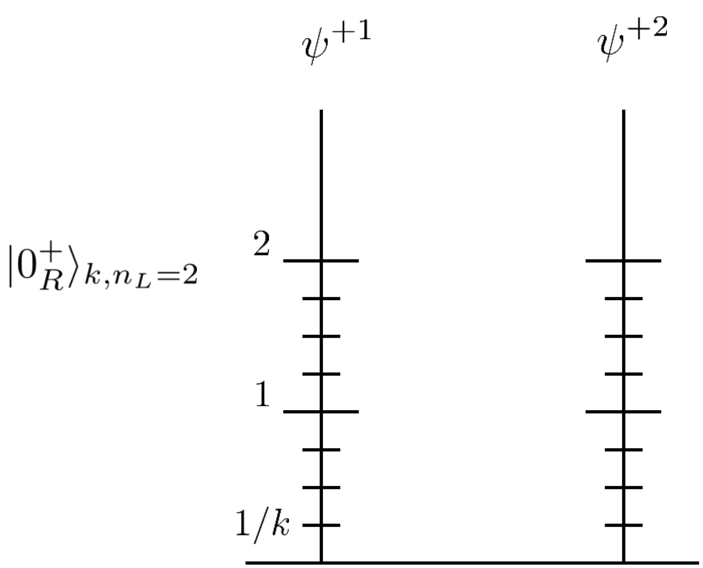
$$h' = h + 2\alpha j^3 + \alpha^2 \frac{c}{6} \,, \qquad j^{3\prime} = j^3 + \alpha \frac{c}{6} \,.$$

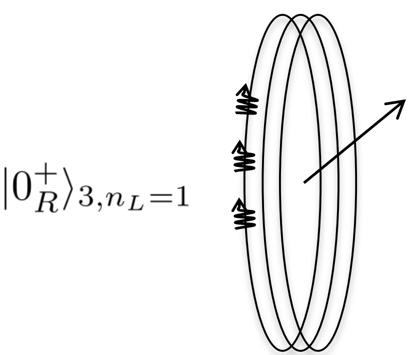
- Starting with the L-moving NS vacuum, spectral flow by $\alpha = \frac{1}{2}$ results in the Ramond ground state $|0_R^+\rangle$.

Excited states with filled Fermi seas

- Starting with the L sector twisted Ramond vacuum $|0_R^+\rangle_k$, spectral flow by n_L units maps this R ground state to an excited Ramond sector state.

- This state has Fermi seas filled to an integer level n_L in units of $1/k$.



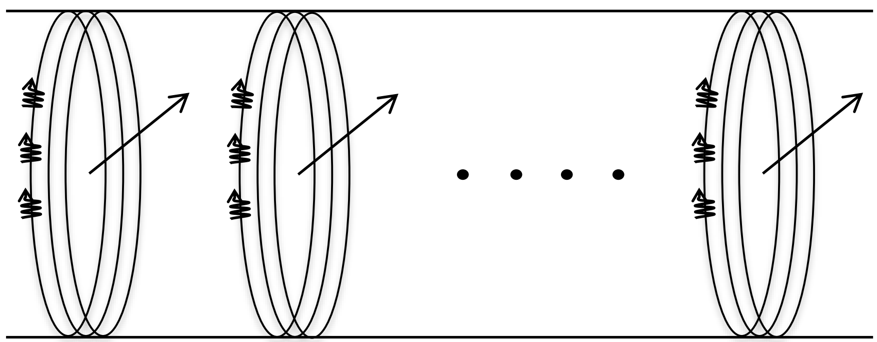


Excited states with filled Fermi seas

- Full CFT: n_C = N/k component strings

- States with all component strings equal:

$|0_3^+\rangle_{n_L=1}$



- Gravitational duals have been constructed.

Giusto, Mathur, Saxena ’04

Lunin ’04

BPS microstates at the cap

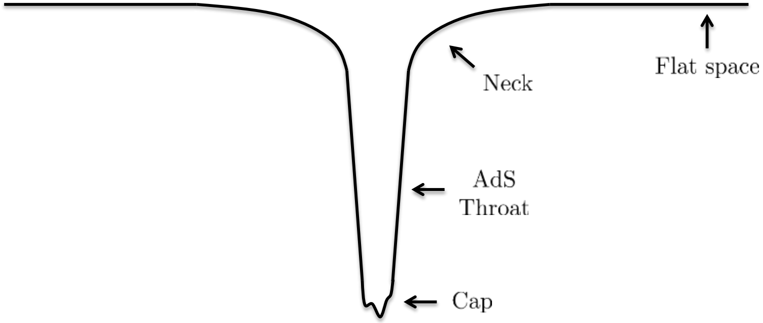
D1-D5-P Superdescendents

- One can construct gravitational solutions dual to superdescendents of the D1-D5 CFT:

$$L_{-n} \longleftrightarrow AdS_3$$

$$J^a_{-n} \longleftrightarrow S^3$$

$$J^i_{-n} \longleftrightarrow T^4$$



Mathur, DT 1112.6413, JHEP

Mathur, DT 1202.6421, NPB

Lunin, Mathur, DT 1208.1770, NPB

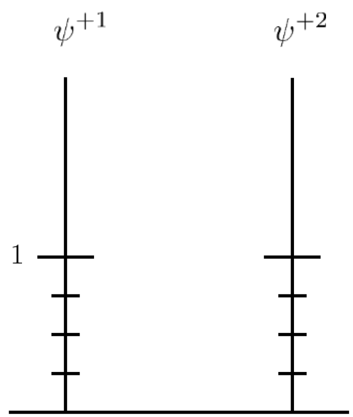
Giusto, Russo 1311.5536, JHEP

- We desire to go further and construct solutions dual to more typical states.

Microstates from Fractional spectral flow

- Look for states which cannot be written in terms of symmetry algebra generators acting on a ground state.

- Fermi seas filled to integer level can be written in this way:



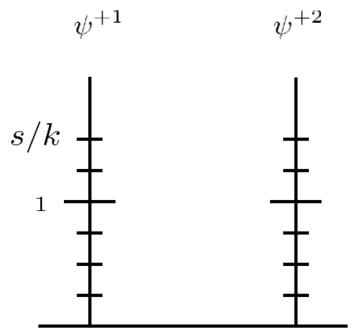
$$J^+_{-2}(J^+_{-1})^{k-1}|0^+_k\rangle_R$$

- R-symmetry current (n integer, m in units of 1/k)

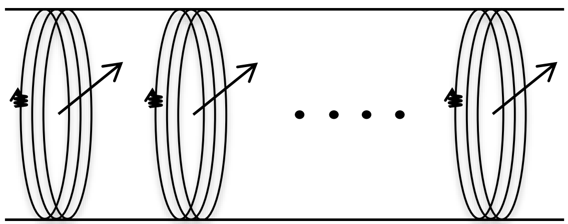
$$J^+_{-n} \;=\; \sum_{\substack{\text{copies} \\ m}} \psi^{+1}_{-m}\psi^{+2}_{-(n-m)}$$

Microstates from Fractional spectral flow

- Consider Fermi seas filled to fractional level s/k :

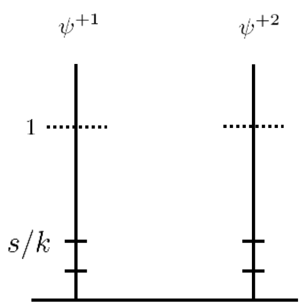


$s = 1$
$k = 3$



Fractional spectral flow
= spectral flow on cover

- For $\quad s = nk + s' \,, \qquad 0 < s' < k-1$
these states cannot be obtained by acting
with symmetry algebra generators on R ground states.



$$J^+_{-n} \;=\; \sum_{\substack{\text{copies}\\ m}} \psi^{+1}_{-m} \psi^{+2}_{-(n-m)}$$

Microstates from Fractional spectral flow

- Gravitational descriptions of the states: previously known
- Generalizations of the gravitational solutions describing states with integer spectral flow parameter n_L to fractional values s/k .
- Interesting orbifold structure in cap

Jejjala, Madden, Ross, Titchener '05

- Corresponds to most general Bena-Warner solution with 2 centers

Bena, Warner '05
Berglund, Gimon, Levi '05

Microstates from Fractional spectral flow

- Most general Bena-Warner solution with 2 centers:

$$
\begin{aligned}
V &= -\frac{s}{|\mathbf{x}|} + \frac{s+1}{|\mathbf{x}-\mathbf{c}|}\,, \\
K_I &= d_I \left(\frac{1}{|\mathbf{x}|} - \frac{1}{|\mathbf{x}-\mathbf{c}|} \right), \ldots \\
d_1 &= \frac{g\,\alpha'}{2\,R}\,k_1\,, \quad d_2 = \frac{g\,\alpha'^3}{2\,V\,R}\,k_2\,, \quad d_3 = \frac{R}{2}\,k_3, \ldots
\end{aligned}
$$

- Four integers $s,\ k_1, k_2, k_3$ specify the geometry

- Alternatively, can take four parameters to be n_1, n_5, k, s . ($k = k_3$)

- Metric:

$$
\begin{aligned}
ds^2 \quad = \quad & -\frac{1}{h}\,(dt^2-dy^2)+\frac{Q_p}{h\,f}\,(dt-dy)^2+h\,f\left(\frac{dr^2}{r^2+a^2\,(\gamma_1+\gamma_2)^2\,\eta}+d\theta^2\right)\\
&+h\left(r^2+a^2\,\gamma_1\,(\gamma_1+\gamma_2)\,\eta-\frac{Q_1Q_5\,a^2\,(\gamma_1^2-\gamma_2^2)\,\eta\,\cos^2\theta}{h^2\,f^2}\right)\cos^2\theta\,d\psi^2\\
&+h\left(r^2+a^2\,\gamma_2\,(\gamma_1+\gamma_2)\,\eta+\frac{Q_1Q_5\,a^2\,(\gamma_1^2-\gamma_2^2)\,\eta\,\sin^2\theta}{h^2\,f^2}\right)\sin^2\theta\,d\phi^2\\
&+\frac{Q_p\,a^2\,(\gamma_1+\gamma_2)^2\,\eta^2}{h\,f}\,(\cos^2\theta\,d\psi+\sin^2\theta\,d\phi)^2\\
&-\frac{2\,\sqrt{Q_1Q_5}\,a}{h\,f}\,(\gamma_1\,\cos^2\theta\,d\psi+\gamma_2\,\sin^2\theta\,d\phi)\,(dt-dy)\\
&-\frac{2\,\sqrt{Q_1Q_5}\,a\,(\gamma_1+\gamma_2)\,\eta}{h\,f}\,(\cos^2\theta\,d\psi+\sin^2\theta\,d\phi)\,dy+\sqrt{\frac{H_1}{H_5}}\,\sum_{a=1}^{4}dz_a^2\,,
\end{aligned}
$$

$$
\begin{aligned}
& a=\frac{\sqrt{Q_1\,Q_5}}{R}\,,\quad Q_p=-a^2\,\gamma_1\,\gamma_2\quad \eta=\frac{Q_1\,Q_5}{Q_1\,Q_5+Q_1\,Q_p+Q_5\,Q_p}\,,\\
& f=r^2+a^2\,(\gamma_1+\gamma_2)\,\eta\,(\gamma_1\,\sin^2\theta+\gamma_2\,\cos^2\theta)\,,\\
& H_1=1+\frac{Q_1}{f}\,,\quad H_5=1+\frac{Q_5}{f}\,,\quad h=\sqrt{H_1\,H_5}\,.
\end{aligned}
$$

$$
\gamma_1=-\frac{s}{k}\,,\quad \gamma_2=\frac{s+1}{k}\,.
$$

Orbifold structure in cap

- Coordinates in which the cap is locally $AdS_3 \times S^3 \times T^4$:

$$\tilde{\psi} = \psi - \frac{y}{kR_y} + \frac{s}{kR_y}(t-y), \quad \tilde{\phi} = \phi - \frac{t}{kR_y} - \frac{s}{kR_y}(t-y)$$

(~ "Fractional spectral flow"). Cap metric: $\tilde{y} = \frac{y}{R_y}, \quad \tilde{t} = \frac{t}{R_y}, \quad \tilde{r} = \frac{r}{a}$

$$ds_6^2 = \sqrt{Q_1 Q_5}\left[-(\tilde{r}^2+\gamma^2)dt^2 + \frac{d\tilde{r}^2}{\tilde{r}^2+\gamma^2} + \tilde{r}^2 d\tilde{y}^2 + d\theta^2 + \cos^2\theta d\tilde{\psi}^2 + \sin^2\theta d\tilde{\phi}^2\right].$$

- Periodicities of the angles at infinity induce the periodicities:

$$\begin{aligned} A: &\quad (\tilde{y},\tilde{\psi},\tilde{\phi}) \to (\tilde{y},\tilde{\psi},\tilde{\phi}) + 2\pi\left(\frac{1}{k}, -\frac{s+1}{k}, \frac{s}{k}\right), \\ B: &\quad (\tilde{y},\tilde{\psi},\tilde{\phi}) \to (\tilde{y},\tilde{\psi},\tilde{\phi}) + 2\pi(0,1,0), \\ C: &\quad (\tilde{y},\tilde{\psi},\tilde{\phi}) \to (\tilde{y},\tilde{\psi},\tilde{\phi}) + 2\pi(0,0,1). \end{aligned}$$

- $(\tilde{y},\tilde{\psi})$ shrink at $(r = 0 , \theta = \pi/2)$; $(\tilde{y},\tilde{\phi})$ shrink at $(r = 0 , \theta = 0)$.
  $\rightarrow$ Potential fixed points.

Orbifold structure in cap

We have the following four possibilities: (gcd = greatest common divisor)

1. $\gcd(k,s) = \gcd(k,s+1) = 1$: Geometry completely regular

2. $\gcd(k,s) = l_1 > 1\,, \quad \gcd(k,s+1) = 1$: $\mathbb{Z}_{l_1}$ orbifold at $(r=0, \theta = \frac{\pi}{2})\,.$

3. $\gcd(k,s) = 1\,, \quad \gcd(k,s+1) = l_2 > 1$: $\mathbb{Z}_{l_2}$ orbifold at $(r=0, \theta = 0)\,.$

4. $\gcd(k,s) = l_1 > 1\,, \quad \gcd(k,s+1) = l_2 > 1$: $\mathbb{Z}_{l_1}$ orbifold at $(r=0, \theta = \frac{\pi}{2})\,,$
$\mathbb{Z}_{l_2}$ orbifold at $(r=0, \theta = 0)\,.$

Microstates from Fractional spectral flow

- In the supergravity, quantization of fluxes and charges requires

$$\frac{s(s+1)}{k} \in \mathbb{Z}$$

- In the CFT, the momentum on each component string must be an integer, which is exactly the above condition.

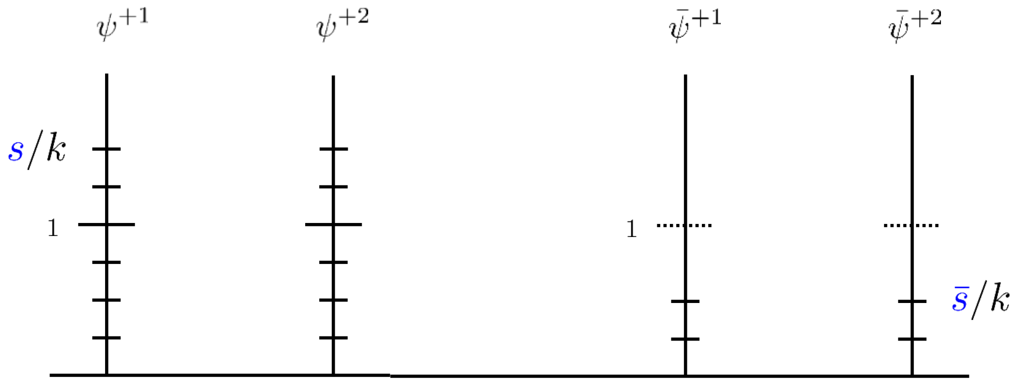
- So the CFT knows about supergravity regularity, and vice versa.

Giusto, Lunin, Mathur, DT 1211.0306, JHEP

Non-BPS microstates at the cap

Non-BPS states from fractional spectral flow

- Consider Fermi seas filled to fractional level s/k in left-moving sector, and $\bar{s}/k$ in right-moving sector



- By our earlier arguments, they cannot be written in terms of chiral algebra generators acting on RR ground states
- Expect these states to be dual to geometries with nontrivial cap structure.

The JMaRT solutions

- The JMaRT metric is that of the general non-BPS Cvetic-Youm D1-D5-P solution, which includes both black hole solutions and smooth solitons:

$$
\begin{aligned}
\mathrm{d}s^2 \;=\;& -\frac{f}{\sqrt{\tilde{H}_1\tilde{H}_5}}(\mathrm{d}t^2-\mathrm{d}y^2)+\frac{M}{\sqrt{\tilde{H}_1\tilde{H}_5}}(s_p\mathrm{d}y-c_p\mathrm{d}t)^2\\
&+\sqrt{\tilde{H}_1\tilde{H}_5}\left(\frac{r^2\mathrm{d}r^2}{(r^2+a_1^2)(r^2+a_2^2)-Mr^2}+\mathrm{d}\theta^2\right)\\
&+\left(\sqrt{\tilde{H}_1\tilde{H}_5}-(a_2^2-a_1^2)\frac{(\tilde{H}_1+\tilde{H}_5-f)\cos^2\theta}{\sqrt{\tilde{H}_1\tilde{H}_5}}\right)\cos^2\theta\mathrm{d}\psi^2\\
&+\left(\sqrt{\tilde{H}_1\tilde{H}_5}+(a_2^2-a_1^2)\frac{(\tilde{H}_1+\tilde{H}_5-f)\sin^2\theta}{\sqrt{\tilde{H}_1\tilde{H}_5}}\right)\sin^2\theta\mathrm{d}\phi^2\\
&+\frac{M}{\sqrt{\tilde{H}_1\tilde{H}_5}}(a_1\cos^2\theta\mathrm{d}\psi+a_2\sin^2\theta\mathrm{d}\phi)^2\\
&+\frac{2M\cos^2\theta}{\sqrt{\tilde{H}_1\tilde{H}_5}}[(a_1c_1c_5c_p-a_2s_1s_5s_p)\mathrm{d}t+(a_2s_1s_5c_p-a_1c_1c_5s_p)\mathrm{d}y]\mathrm{d}\psi\\
&+\frac{2M\sin^2\theta}{\sqrt{\tilde{H}_1\tilde{H}_5}}[(a_2c_1c_5c_p-a_1s_1s_5s_p)\mathrm{d}t+(a_1s_1s_5c_p-a_2c_1c_5s_p)\mathrm{d}y]\mathrm{d}\phi+\sqrt{\frac{\tilde{H}_1}{\tilde{H}_5}}\sum_{i=1}^{4}\mathrm{d}z_i^2
\end{aligned}
$$

where

$$
\begin{aligned}
\tilde{H}_i &= f+M\sinh^2\delta_i, \quad f = r^2+a_1^2\sin^2\theta+a_2^2\cos^2\theta,\\
c_i &= \cosh\delta_i\,, \qquad s_i = \sinh\delta_i
\end{aligned}
$$

Cvetic, Youm ’96

Jejjala, Madden, Ross, Titchener ’05

Smooth JMaRT solitons

- Smooth soliton solutions are obtained by requiring smoothness and absence of horizons. This results in a set of conditions on the parameters.

- For generality let us focus on the case of three non-zero charges
  - (two non-zero charges: similar and a little simpler).

- One of the conditions is that some combination of the periodic coordinates y , ψ , ϕ shrinks smoothly in the cap, forming a 5D bolt
  - (5D bolt: fixed point which is a three-surface, in this case S^3).

Jejjala, Madden, Ross, Titchener '05

- This fixes the radius of the y coordinate at infinity, R_y in terms of other parameters; from a string theory point of view, we consider R_y to be a modulus and then the parameters of the solution are fixed in terms of R_y .

Ergoregion emission and CFT

- The JMaRT solutions have an ergoregion instability
- This can be derived by solving the free massless scalar wave equation, and finding modes which are regular in the cap, outgoing at infinity, and grow with time

Cardoso, Dias, Hobdevo, Myers '05

- Using AdS/CFT this has been interpreted as Hawking radiation from these states, which is enhanced to a classical effect due to the special nature of the states.
  - (Analogous to laser radiation vs thermal radiation)

Chowdhury, Mathur '07

- Previously, AdS/CFT studied only in special cases, all involving integer spectral flow.

Chowdhury, Mathur '07, '08

Avery, Chowdhury, Mathur '09

Avery, Chowdhury '09

JMaRT parameters

Parameters:

- n_1 , n_5 : number of D1 and D5 branes
- R_y : Radius of the y circle at infinity
- m, n : parameterize the two angular momenta
- k : orbifold parameter

When $k > 1$ the geometry may have orbifold singularities

or may be completely smooth,

depending on common divisors between m, n, k .

CFT duals of orbifolded JMaRT solitons

- Conjecture: the general family of JMaRT solitons, including orbifolds, is dual to states with Fermi seas filled to fractional level s/k in left-moving sector and $\bar{s}/k$ in right-moving sector, obtained from fractional spectral flow.

  Giusto, Lunin, Mathur, DT 1211.0306, JHEP

- Identification of parameters:

$$m = s + \bar{s} + 1\,, \qquad n = s - \bar{s}$$

- Classified orbifold structures classified in cap
  - Similar to BPS case, with additional possibility of an orbifold singularity all over S^3

- Computed emission spectrum and emission rate from massless scalar wave equation & matched to CFT

  Chakrabarty, DT, Virmani, 1508.01231, JHEP

Momentum Fractionation on Superstrata

The Superstratum

- Superstratum: three-charge object described by supergravity solutions that depend on arbitrary functions of two variables

- Asymptotically AdS supergravity solutions constructed, holographic dual states proposed and tested

- So far, dual states involve integer-moded generators acting on R-R ground states

Bena, Giusto, Russo, Shigemori, Warner ’15

Giusto, Moscato, Russo ’15

Momentum Fractionation

Given our understanding of BPS supergravity solutions dual to states involving fractionated momenta, let us investigate whether we can probe momentum fractionation on superstrata.

- We will work in asymptotically flat space
- In doing so, we will also construct the asymptotically-flat generalizations of some asymptotically AdS superstratum solutions.

Spectral interchange

- Given a D1-D5-P microstate geometry, one can interchange the v and ψ coordinates and generate a new solution

Niehoff, Warner ’13

- We can use this technique to add momentum to a multiwound supertube:

1. Take a multiwound supertube, with winding k
2. Perform spectral interchange
3. Add ψ–dependent charge densities on the transformed supertube
4. Perform spectral interchange back to generate a v–dependent solution with non-zero momentum charge

BPS D1-D5-P solutions in 6D

- The metric takes the form

$$ds_6^2 = -\frac{2}{\sqrt{\mathcal{P}}}\,(dv+\beta)\left[du+\omega-\frac{Z_3}{2}(dv+\beta)\right]+\sqrt{\mathcal{P}}\,ds_4^2$$

We have an almost-linear system (c.f. Giusto's talk):

1. Base metric ds_4^2 , one-form β
2. Scalars $Z_1,\, Z_2,\, Z_4$, two-forms $\Theta_1,\, \Theta_2,\, \Theta_4$
3. Scalar Z_3 , one-form ω

BPS D1-D5-P solutions in 6D

- For solutions which are v and ψ-independent, one can write the solution in terms of 8 functions,

$$V, \; K_1, \; K_2, \; K_3, \; L_1, \; L_2, \; L_3, \; M$$

These functions must be harmonic on the 3D base.

We can express spectral inversion as a transformation on these functions.

Charge densities and regularity

- In the spectrally inverted frame, the ψ-dependent solution takes the form

$$
\begin{aligned}
V &= \frac{k}{\mu_+}\,, \qquad K^1 = K^2 = \frac{1}{R}\,, \qquad K^3 = \frac{R}{\mu_-}\,, \\
L_1 &= \frac{\bar{Q}_1}{k}\,\lambda_1(\psi,\vec{r})\,, \qquad L_2 = \frac{\bar{Q}_2}{k}\,\lambda_2(\psi,\vec{r})\,, \qquad L_3 = \left(k + \frac{\bar{Q}_1+\bar{Q}_2}{kR^2}\right)\,, \\
M &= -\frac{1}{2}R + \frac{1}{2}\frac{\bar{Q}_1\bar{Q}_2}{k^2R}\,j(\psi,\vec{r})
\end{aligned}
$$

where $\lambda_A,\, j$ are harmonic on the 4D base, and so may be written in terms of a Green's function:

$$
\begin{aligned}
\lambda_A(\psi,\vec{r};\,\vec{r}\,') &= 4\pi \int_0^{4\pi} \widehat{G}(\psi,\vec{r};\,\psi',\vec{r}\,')\,\rho_A(\psi')\;d\psi'\,, \\
j(\psi,\vec{r};\,\vec{r}\,') &= 4\pi \int_0^{4\pi} \widehat{G}(\psi,\vec{r};\,\psi',\vec{r}\,')\,\hat{\rho}(\psi')\;d\psi'\,.
\end{aligned}
$$

Charge densities and regularity

- When adding ψ–dependent charge densities, one must obey the following conditions for regularity of the solution at the supertube:

$$kR\left(\hat{\rho}-1\right)+\frac{1}{kR}\left[\bar{Q}_1\left(\rho_1-1\right)+\bar{Q}_2\left(\rho_2-1\right)\right] \;=\; 0\,,$$

$$\hat{\rho} \;=\; \rho_1\rho_2\,.$$

- Satisfying these requires an infinite set of Fourier modes
  (Cut off by $N_1 N_5$ at quantum level)

- More convenient to introduce the field Z_4 and then perform a 'coiffuring' procedure – namely to arrange Fourier modes to obtain regularity.
- This can be done with a finite set of supergravity modes.

Charge densities and regularity

- We can solve for the Green's function on the background in closed form.

- When we spectrally invert back, the ψ–dependence becomes v–dependence.

- We add momentum along the v fiber sourced at the center of space, where the v fiber becomes the combination of angles

$$\zeta = \frac{v}{2R} - k\varphi_2 \,.$$

Turning on Z_4 and Θ_4

- After turning on Z_4, Θ_4 one obtains a solution to the first layer of BPS equations with Z_1, Z_2, Z_4 given by

$$Z_A = 1 + \frac{Q_A}{\Sigma}\left(1 + \Delta^{kn}\left(b_A\, e^{-in\zeta} + \bar{b}_A\, e^{in\zeta}\right)\right), \qquad A = 1, 2,$$

$$Z_4 = \frac{\Delta^{kp}}{\Sigma}\left(b_4\, e^{-ip\zeta} + \bar{b}_4\, e^{ip\zeta}\right)$$

single oscillating mode per field

where

$$\zeta = \frac{v}{2R} - k\varphi_2, \qquad \Delta = \frac{a\,\cos\theta}{(r^2 + a^2)^{\frac{1}{2}}},$$

- We can arrange smoothness by relating the mode numbers n and p, and the coefficients b_A, b_4.

Coiffuring: Style 1

- We arrange smoothness in two ways, which we denote Styles 1 & 2.

- In Style 1, we give (Z_4, Θ_4) the same mode-dependence as (Z_A, Θ_A) .
- This means that we take $p = n$.
- Then smoothness fixes b_1 and b_2 in terms of b_4 via

$$b_1 = \frac{i\, b_4}{Q_1}\sqrt{\frac{Q_1 + a^2}{Q_2 + a^2}}\,, \qquad b_2 = -\frac{i\, b_4}{Q_2}\sqrt{\frac{Q_2 + a^2}{Q_1 + a^2}}\,.$$

Coiffuring: Style 1

- The momentum charge is equipartitioned between terms coming directly from b_4 and terms coming from b_1 and b_2 :

$$Q_P = \frac{2|b_4|^2}{k^2 R_y^2} - \frac{Q_1 Q_2}{k^2 R_y^2} \left(b_1 \bar{b}_2 + b_2 \bar{b}_1\right) = \frac{4|b_4|^2}{k^2 R_y^2}$$

- The angular momenta are

$$\begin{aligned} J_L &= \frac{1}{2}\frac{Q_1 Q_2}{k R_y} + \frac{1}{2} k R_y Q_P = J_R + k R_y Q_P \,, \\ J_R &= \frac{1}{2}\frac{Q_1 Q_2}{k R_y} - \frac{1}{2} k R_y Q_P \,. \end{aligned}$$

Coiffuring: Style 2

- In Style 2, we set $b_2 = 0$ and take the (Z_1, Θ_1) oscillations to have twice the mode-number of the (Z_4, Θ_4) oscillations, *i.e.* $n = 2p$.

- Then smoothness fixes b_1 in terms of b_4 via

$$Q_1(Q_2 + a^2)b_1 \;=\; b_4^2 \,.$$

Coiffuring: Style 2

- Since $b_2 = 0$, the momentum charge comes only from b_4 terms:

$$Q_P = \frac{2|b_4|^2}{k^2 R_y^2}$$

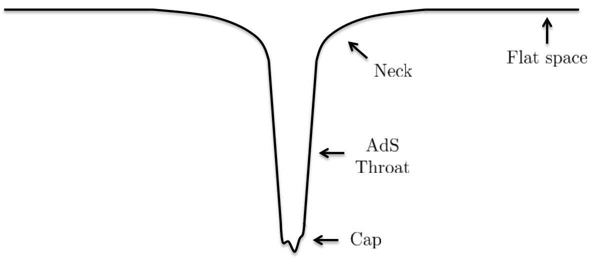
- The angular momenta take the same form as in Style 1 in terms of Q_P,

$$\begin{aligned} J_L &= \frac{1}{2}\frac{Q_1 Q_2}{k R_y} + \frac{1}{2} k R_y Q_P = J_R + k R_y Q_P \,, \\ J_R &= \frac{1}{2}\frac{Q_1 Q_2}{k R_y} - \frac{1}{2} k R_y Q_P \end{aligned}$$

although they differ from Style 1 when expressed in terms of b_4.

Coiffuring: Style 2

In the special case $k = 1$, our Style 2 solution is the extension to asymptotically flat space of one of the Habemus asymptotically AdS solutions.



- We can exploit this to gain some intuition on possible dual CFT states of our general class of solutions (when restricted to the AdS region).

Dual CFT description

- Let us make some tentative remarks on the dual CFT states, based on work in progress.
- We focus first on our Style 2 solutions, due to their relation to particular Habemus solutions for $k = 1$.
- The dual CFT states proposed in Habemus for this solution involve two types of strand:

$$|{+}{+}\rangle_1 \,, \qquad (J^+_{-1})^p |00\rangle_p$$

where here J^+_{-1} is the current of the individual strand, rather than the current of the full CFT.

$$J^+_{-n} = \sum_{\substack{\text{copies} \\ m}} \psi^{+1}_{-m} \psi^{+2}_{-(n-m)}$$

Dual CFT description

- On these excited strands, the operator J^{+}_{-1} is applied the maximal amount of times, and the state on each strand has

$$(J^{+}_{-1})^{p}|00\rangle_{p} \;:\qquad n_{p} \equiv h - \bar{h} = p\,, \qquad j^{3} = p\,.$$

- This agrees with the phase dependence $\zeta \;=\; \frac{v}{2R} - \varphi_{2}\,.$

- Such a state can also be generated by spectral flow from $|00\rangle_{p}$ with $\alpha = 1$.

Dual CFT description

- To generalize to $k > 1$, note that now we have the phase dependence

$$\zeta = \frac{v}{2R} - k\varphi_2 \,.$$

- This suggests that our states involve the action of powers of the fractionated generator $J^+_{-\frac{1}{k}}$ on each strand.

- In order to have integer momentum on the CFT strand, we must act a multiple of k times:

$$\left(J^+_{-\frac{1}{k}}\right)^{k\hat{p}} \,, \qquad \hat{p} \in \mathbb{Z} \,.$$

Momentarily we will relate $\hat{p}$ to the p in the supergravity solutions.

Dual CFT description

- In order to have integer momentum in the CFT, we must act a multiple of k times:

$$\left(J^{+}_{-\frac{1}{k}}\right)^{k\hat{p}}$$

- The length of $|00\rangle$ strands for which this is the maximum number of times that $J^{+}_{-\frac{1}{k}}$ can be applied is $k^2\hat{p}$. This suggests the state

$$\left(J^{+}_{-\frac{1}{k}}\right)^{k\hat{p}} |00\rangle_{k^2\hat{p}} : \qquad n_p \equiv h - \bar{h} = \hat{p}\,, \qquad j^3 = k\hat{p}\,.$$

- This state is a fractional spectral flow of $|00\rangle_{k^2\hat{p}}$ by $\alpha = \dfrac{1}{k}.$

Dual CFT description: Style 2

- So for Style 2, it is natural to conjecture that the dual states are coherent states involving two types of strand, where $\hat{p} = p$:

$$|{+}{+}\rangle_k \,, \qquad\qquad \left(J^+_{-\frac{1}{k}}\right)^{kp} |00\rangle_{k^2 p} \,.$$

- It would be very interesting to scrutinize this with the technology of holographic renormalization, as has been done for $k = 1$.

Kanitscheider, Skenderis, Taylor ’06, ’07

Taylor ’07

Giusto, Moscato, Russo ’15

Dual CFT description: Style 1?

- What about Style 1?
- Both Style 1 and Style 2 obey similar relations

$$
\begin{aligned}
J_L &= \frac{1}{2}\frac{Q_1 Q_2}{kR_y} + \frac{1}{2}kR_y Q_P \;=\; J_R + kR_y Q_P \,, \\
J_R &= \frac{1}{2}\frac{Q_1 Q_2}{kR_y} - \frac{1}{2}kR_y Q_P \,.
\end{aligned}
$$

- The main difference is that in Style 1 the momentum charge is "equipartitioned":

$$
Q_P \;=\; \frac{2|b_4|^2}{k^2 R_y^2} - \frac{Q_1 Q_2}{k^2 R_y^2}\left(b_1 \bar{b}_2 + b_2 \bar{b}_1\right) \;=\; \frac{4|b_4|^2}{k^2 R_y^2}
$$

Dual CFT description: Style 1?

Some possible ingredients:

- One can introduce more types of momentum-carrying strands.
- If we follow the logic that led us to $\left(J^{+}_{-\frac{1}{k}}\right)^{k\hat{p}} |00\rangle_{k^2\hat{p}}$,

  but start with $|{+}{+}\rangle$ & $|{-}{-}\rangle$ ground states, we find two new candidates:

$$\left(J^{+}_{-\frac{1}{k}}\right)^{k\hat{p}} |{+}{+}\rangle_{k^2\hat{p}+k} \,, \qquad\qquad \left(J^{+}_{-\frac{1}{k}}\right)^{k\hat{p}} |{-}{-}\rangle_{k^2\hat{p}-k}$$

- One immediate constraint: to match the conserved charges, the average numbers of these strands in the coherent state must be equal.
- It appears to be necessary to combine many strands of different lengths.

Summary

- Fractionated degrees of freedom can be probed in supergravity
- This allows us to access more typical states in the dual CFT
- We have studied both BPS and non-BPS examples
- In BPS case, fractionation on superstrata is a step towards understanding typical states of the black hole.

Outlook

Other recent work:

- Multi-center generalizations of JMaRT solutions

  Bena, Bossard, Katmadas, DT 1511.03669

- Thermalization in D1-D5 CFT away from orbifold point

  Carson, Hampton, Mathur, DT 1405.0259, JHEP
  Carson, Mathur, DT 1406.6977, NPB
  Carson, Hampton, Mathur, DT 1410.4543, JHEP

Future:

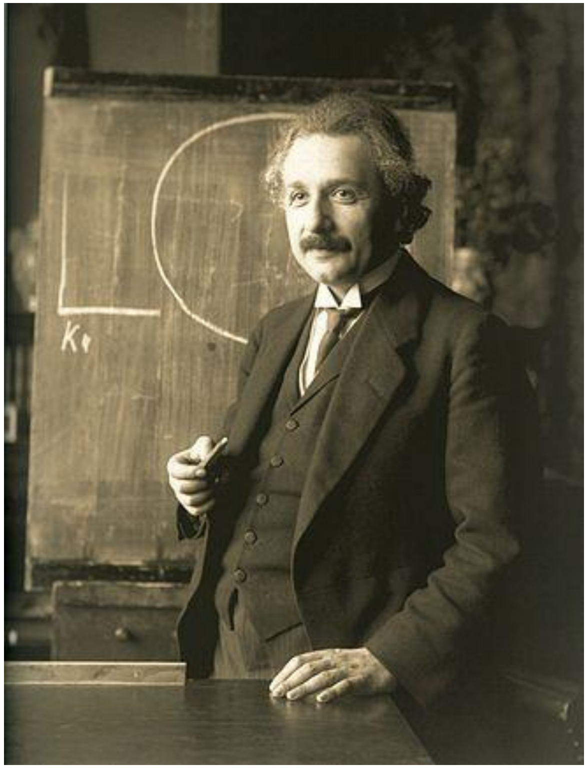
- More general asymptotically flat superstrata
- CFT duals of multi-center BPS D1-D5-P solutions?
- CFT duals of scaling solutions?
- How far can holographic technology be pushed explicitly?
- More general multi-center non-BPS solutions

On 25 November, 1915, exactly 100 years ago today,

Einstein presented the final form of

the equations of General Relativity to

the Prussian Academy of Sciences.



Let us celebrate this anniversary!

Thanks!