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Inflation - Lecture 2

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Summer school on cosmology
ICTP, 16-18 July 2012

Inflation

lecture II

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contents

3. Non-Gaussian Curvature Perturbation

- self-interactions of inflaton/non-trivial “vacuum”

  quantum physics, subhorizon scale during inflation

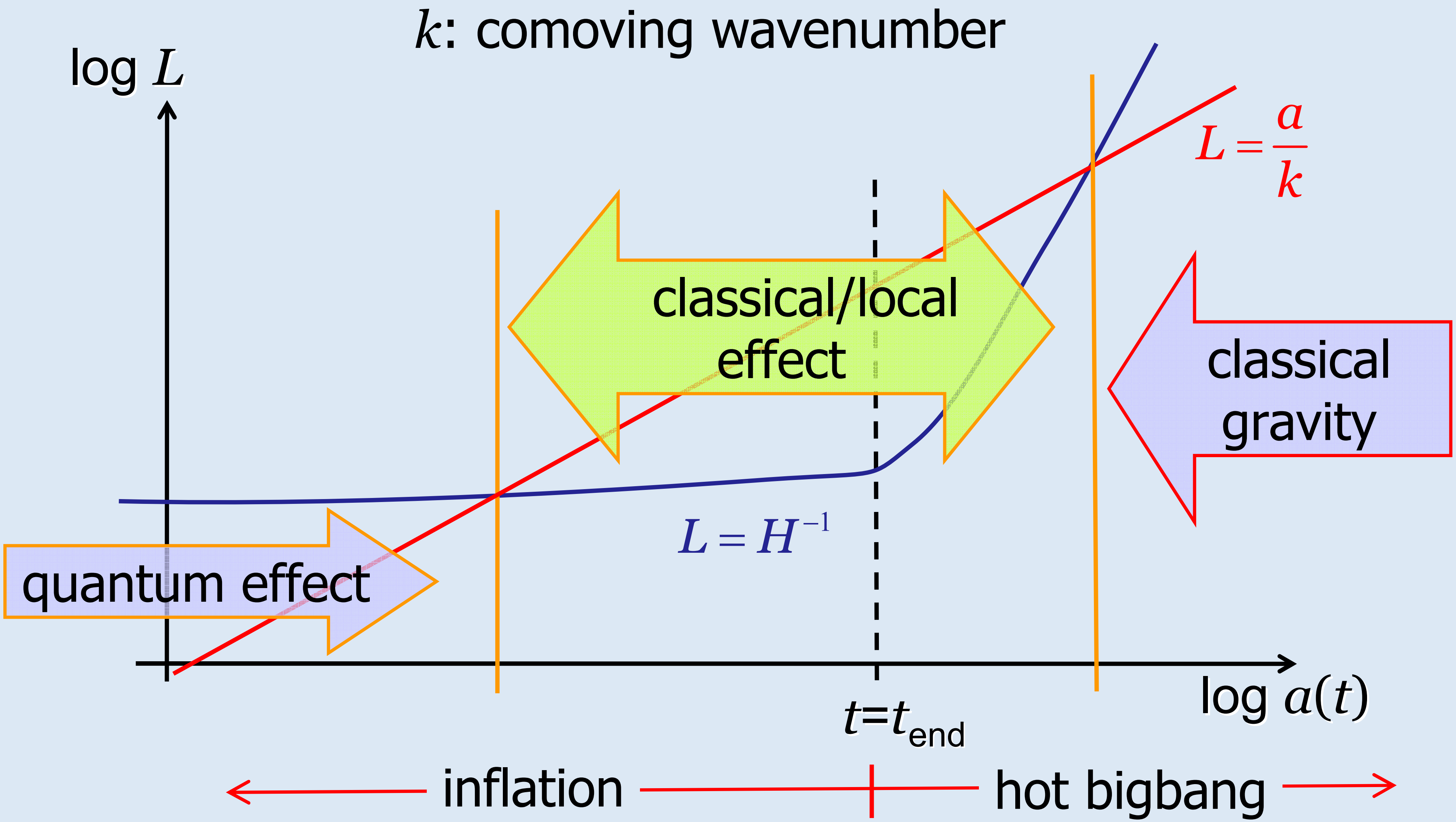
- multi-field

  classical physics, nonlinear coupling to gravity
  superhorizon scale during and after inflation

- nonlinearity in gravity

  classical general relativistic effect,
  subhorizon scale after inflation

Origin of NG and cosmic scales



Origin1 : self-interaction/non-trivial vacuum

Non-Gaussianity generated on subhorizon scales
(quantum field theoretical)

- conventional self-interaction by potential is ineffective

Maldacena ('03)

ex. chaotic inflation

$$V = \frac{1}{2} m^2 \phi^2$$ ··· free field!

(grav. interaction is Planck-suppressed) $\sim O(1/M_{Pl}^2)$

$$V = \lambda \phi^4 \;\rightarrow\; \lambda \sim 10^{-15}$$

extremely small!

- need unconventional self-interaction
  → non-canonical kinetic term can generate large NG

ex a: Non-canonical kinetic term (~DBI inflation)

Silverstein & Tong (2004),...

kinetic term: $K \sim f^{-1}(\phi)\sqrt{1-f(\phi)\dot{\phi}^2} \equiv f^{-1}\gamma^{-1}$

~ (Lorenz factor)$^{-1}$

perturbation expansion $\left(\delta\gamma = \frac{1}{2}\gamma^3\delta X; \quad X \equiv f\dot{\phi}^2\right)$

$$K = K_0 + \delta_1 K + \delta_2 K + \delta_3 K + \cdots$$

$\delta_1 K = 0$, $\quad \delta_2 K \propto \gamma^3$, $\quad \delta_3 K \propto \gamma^{3+2}$

$$\Rightarrow \quad \delta\phi \sim \delta\phi_0 + \gamma^2 \delta\phi_0^2 + \cdots$$

large NG for large γ

Bi-spectrum (3pt function) in DBI inflation

Alishahiha et al. ('04)

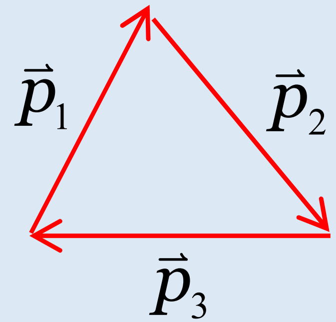
$$\left\langle \mathcal{R}_C(p_1)\mathcal{R}_C(p_2)\mathcal{R}_C(p_3)\right\rangle$$

$$\sim \delta(\sum_j p_j) f_{NL}(p_1,p_2,p_3)\left(\mathcal{R}_C(p_1)\mathcal{R}_C(p_2)+\text{cyclic}\right)$$

$$\Downarrow \qquad f_{NL} \sim \gamma^2$$

f_{NL} large for equilateral configuration

$$|\vec{p}_1| \sim |\vec{p}_2| \sim |\vec{p}_3|$$



$$\left(\vec{p}_1+\vec{p}_2+\vec{p}_3=0\right)$$

$$f_{NL} \Rightarrow f_{NL}^{\text{equil}}$$

WMAP 7yr: $-241 < f_{NL}^{\text{equil}} < 266$ (95% CL)

ex b: Non-trivial vacuum

- de Sitter spacetime = maximally symmetric $SO(4,1)$
  (same degrees of sym as Poincare (Minkowski) sym)

⇨ gravitational interaction (GI) is negligible in vacuum
(except for graviton/tensor-mode loops)

- slow-roll inflation : dS symmetry is slightly broken

  GI induces NG but suppressed by $\varepsilon \equiv -\dot{H}/H^2$

  But large NG is possible if the initial state (or state at horizon crossing) does NOT respect dS symmetry

  (ie initial state ≠ Bunch-Davies vacuum)

⇨ various types of NG :

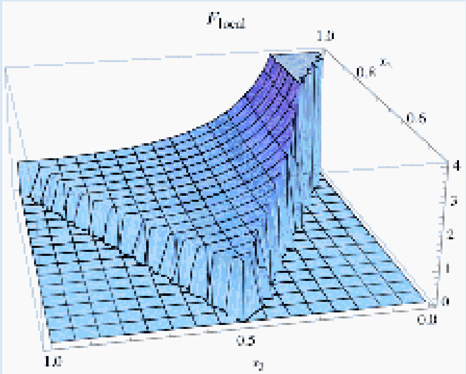
scale-dependent, oscillating, featured, folded ...

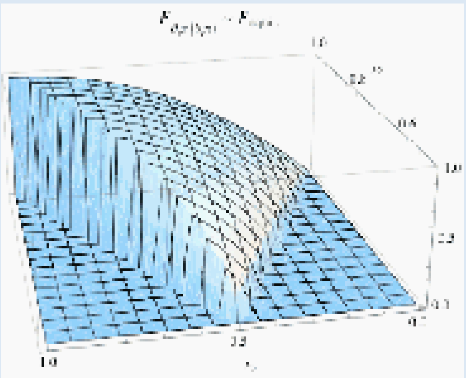
Chen et al. ('08), Flauger et al. ('10), Arroja et al. ('11),...

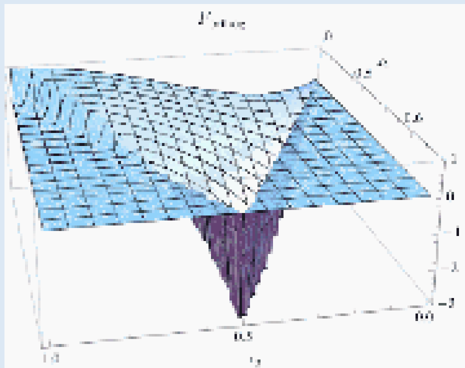
templates for primordial bispectra

(figs from Senatore, Smith & Zaldarriaga '11)

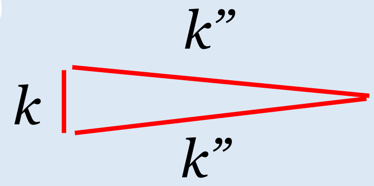
$$P_\zeta(k) = \mathcal{P}/k^3, \quad B_\zeta(k_1,k_2,k_3) = (6/5) f_{NL}(k_1,k_2,k_3)\left(P(k_1)P(k_2) + P(k_2)P(k_3) + P(k_3)P(k_1)\right)$$





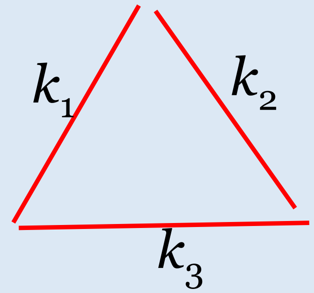


- **squeezed type** (Komatsu&Spergel 2001)
  - local in real space (f_{NL}=constant)
  - max for squeezed triangles: k<<k′,k″



$$B_\zeta(k_1,k_2,k_3) = (6/5) f_{NL}^{\text{local}} \mathcal{P}^2 \left(\frac{1}{k_1^3 k_2^3} + \frac{1}{k_2^3 k_3^3} + \frac{1}{k_3^3 k_1^3} \right)$$

- **equilateral type** (Creminelli et al 2005)
  - peaks for $k_1 \sim k_2 \sim k_3$



$$B_\zeta(k_1,k_2,k_3) = (6/5) f_{NL}^{\text{equil}} \mathcal{P}^2 \left(\frac{3(k_1+k_2-k_3)(k_2+k_3-k_1)(k_3+k_1-k_2)}{k_1^3 k_2^3 k_3^3} \right)$$

- **orthogonal type** (Senatore et al 2009)

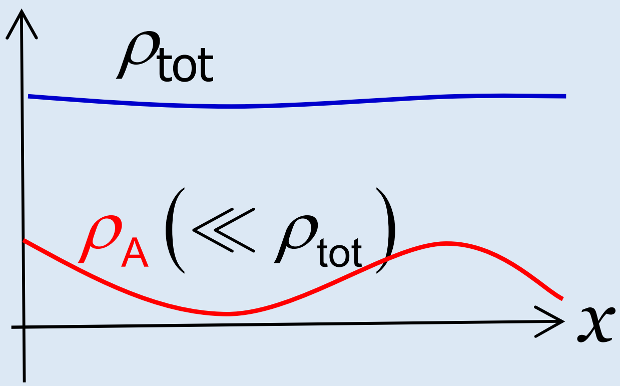
$$B_\zeta(k_1,k_2,k_3) = (6/5) f_{NL}^{\text{orthog}} \mathcal{P}^2 \left(\frac{81}{k_1 k_2 k_3 (k_1+k_2+k_3)^3} \right)$$

Origin 2 : Generation on superhorizon scales

- NG may appear if $T^{\mu\nu}$ depends nonlinearly on $\delta\phi$, even if $\delta\phi$ itself is Gaussian.

This effect is small in single-field slow-roll model
(⇔ linear approximation is valid to high accuracy)
Salopek & Bond ('90)

- For multi-field models, contribution to $T^{\mu\nu}$ from each field can be highly nonlinear.



NG is always of local type:

$$f_{NL}(p_1,p_2,p_3) \rightarrow f_{NL}^{\rm local} = \text{const.}$$

WMAP 7yr: $-10 < f_{NL}^{\rm local} < 74$ (95% CL)

δN formalism for this type of NG

Origin 3：Nonlinearity in gravity

ex. post-Newtonian metric in asymptotically flat space

$$ds^2 = -\left(1 + 2\Psi - 2\Psi^2 + \cdots\right)dt^2 + \left(1 - 2\Psi + 2\Psi^2 + \cdots\right)dr^2 + \cdots$$

Newton potential

NL (post-Newton) terms (in both local and nonlocal forms)

- important when scales have re-entered Hubble horizon

  distinguishable from NL matter dynamics?

- effect on CMB bispectrum may not be negligible

  $f_{NL} \sim O(5)$?   Pitrou et al. (2010)

  (for both squeezed and equilateral types)

δN formalism

What is δN?

- δN is the perturbation in # of e-folds counted backward in time from a fixed final time t_f

  therefore it is nonlocal in time by definition

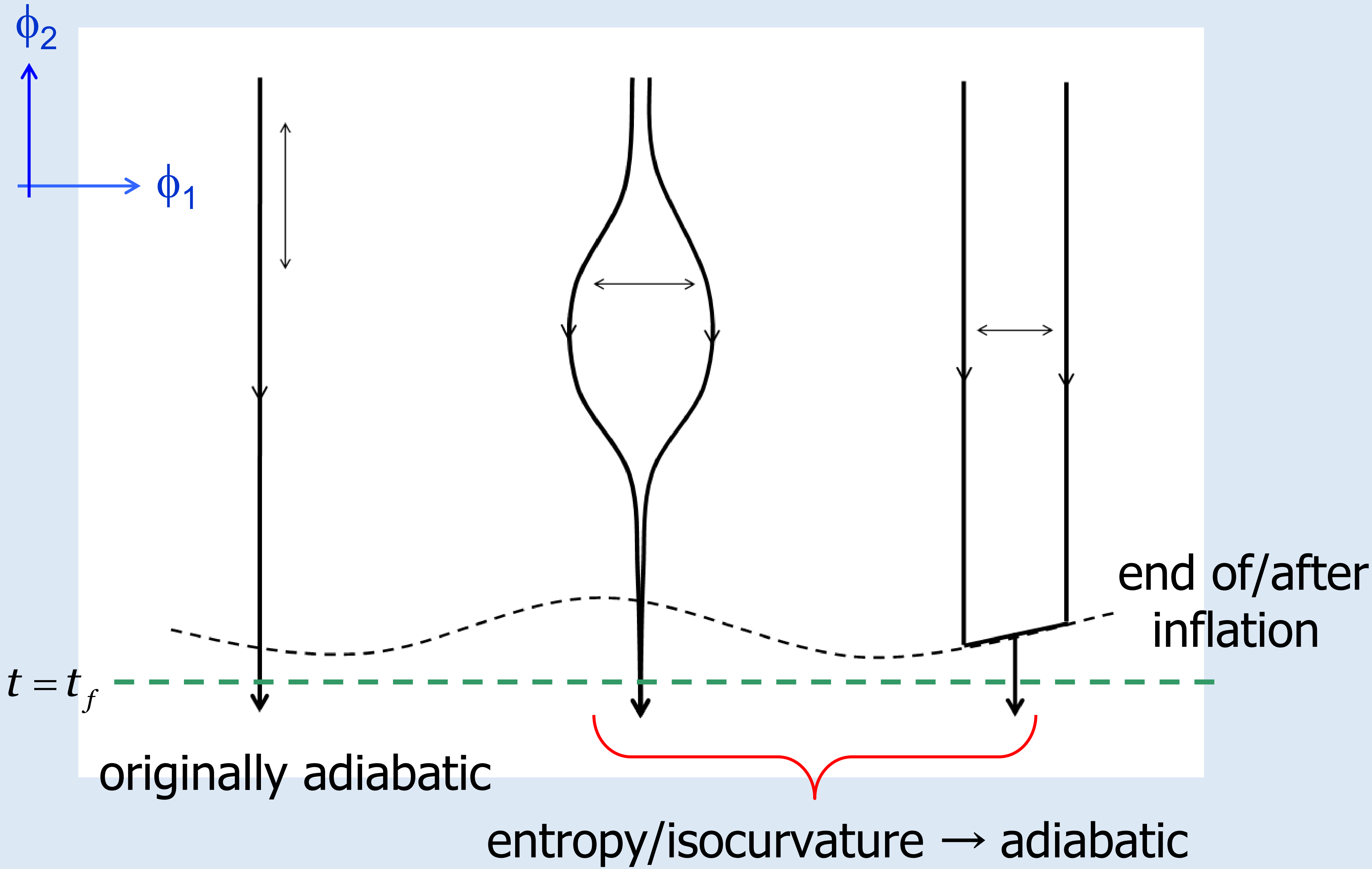
- t_f should be chosen such that the evolution of the universe has become unique by that time: "adiabatic limit"

  isocurvature perturbation that persists until today must be dealt separately

- δN is equal to conserved NL comoving curvature perturbation on superhorizon scales at $t>t_f$

- δN formalism is valid independent of gravity theory

3 types of δN



ϕ_2
ϕ_1
end of/after inflation
$t = t_f$
originally adiabatic
entropy/isocurvature → adiabatic

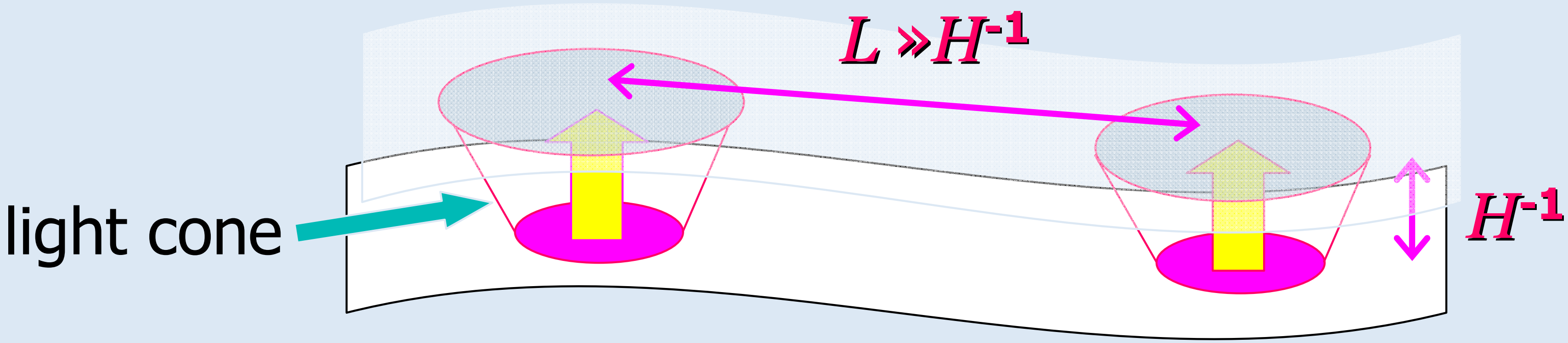
Separate Universe approach (~ spatial gradient expansion)

- On superhorizon scales, spatial gradient expansion is valid:

$$\left|\frac{\partial}{\partial x^i}Q\right| \ll \left|\frac{\partial}{\partial t}Q\right| \sim HQ;\;\; H \sim \sqrt{G\rho}$$

Belinski et al. '70, Tomita '72, Salopek & Bond '90, …

This is a consequence of causality:



- At lowest order, no signal propagates in spatial directions.

Field equations reduce to ODE's

metric on superhorizon scales

- gradient expansion:

$$\partial_i \rightarrow \varepsilon \partial_i \,, \quad \varepsilon = \text{expansion parameter}$$

- metric:

$$ds^2 = -\mathcal{N}^2 dt^2 + e^{2\alpha} \tilde{\gamma}_{ij} \left(dx^i + \beta^i dt\right)\left(dx^j + \beta^j dt\right)$$

$$\det \tilde{\gamma}_{ij} = 1, \quad \beta^{\mathrm{i}} = O(\varepsilon)$$

the only non-trivial assumption

contains GW (~ tensor) modes

$$e^{\alpha(t,x^i)} = a(t) e^{\mathcal{R}(t,x^i)}; \; \mathcal{R} \sim$$ NL curvature perturbation

fiducial `background'

Local Friedmann equation & δN formula

Lyth, Malik & MS ('05)

$$\tilde{H}^2(t,x^i) = \frac{8\pi G}{3}\rho(t,x^i) + O(\varepsilon^2)$$

$$\tilde{H} \equiv \frac{\partial}{\partial\tau}\alpha = \frac{\partial}{\mathcal{N}\partial t}[\ln a + \mathcal{R}]$$

··· geometrical def of "Hubble"

x^i : comoving (Lagrangean) coordinates.

$d\tau = \mathcal{N}\, dt$: proper time along fluid flow

exactly the same as the homogeneous background

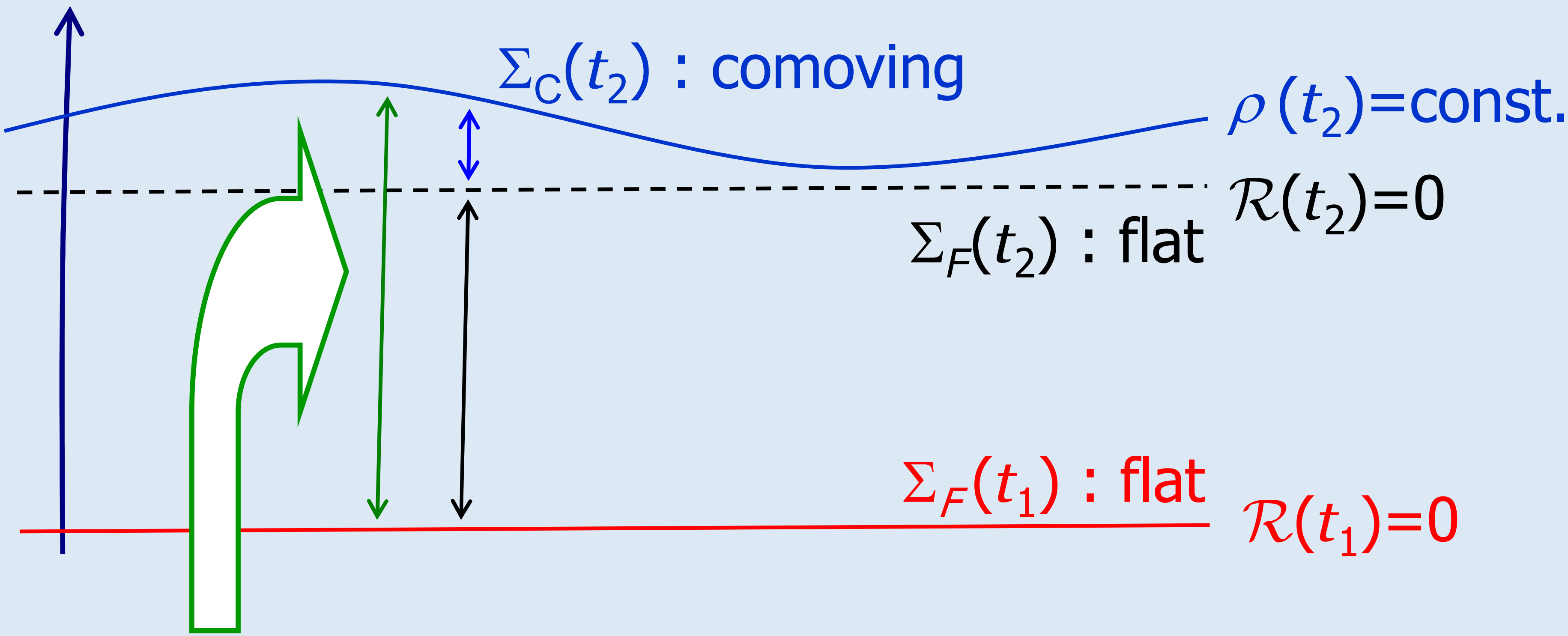
$$N(t_2,t_1) \equiv \int_{t_1}^{t_2} \tilde{H}\, d\tau = N_0(t_2,t_1) + \mathcal{R}(t_2,x^i) - \mathcal{R}(t_1,x^i)$$

$$N_0(t_2,t_1) \equiv \ln\left[a(t_2)/a(t_1)\right]$$

Nonlinear δN - formula

Choose flat slice at $t = t_1$ [$\Sigma_F(t_1)$] and
comoving (=uniform density) at $t = t_2$ [$\Sigma_C(t_2)$] :

('flat' slice: $\Sigma(t)$ on which $\mathcal{R} = 0 \leftrightarrow e^{\alpha} = a(t)$)



$$N(t_2,t_1) \equiv N_0(t_2,t_1) + \delta N_F(t_2,t_1;x^i)$$

$$\delta N_F(t_2,t_1;x^i) = \mathcal{R}_C(t_2,x^i)$$

How do we relate δN to matter evolution?

need eqn relating 'expansion' with matter 'evolution'

energy conservation!

$$\frac{d}{d\tau}\rho + 3\tilde{H}\left(\rho + p\right) = 0 \quad \Rightarrow \quad \tilde{H} \equiv -\frac{1}{3(\rho+p)}\frac{\partial}{\partial\tau}\rho$$

$$\Rightarrow \quad N(t_2,t_1) = -\int_{t_1}^{t_2} dt \frac{1}{3(\rho+p)}\frac{\partial\rho}{\partial t}$$

$$\delta N_F\left(t_2,t_1;x^i\right) = -\frac{1}{3}\int_{\Sigma_F(t_1)}^{\Sigma_C(t_2)} \left.\frac{\partial_t\rho}{\rho+P}\right|_{x^i} dt + \frac{1}{3}\int_{\Sigma_F(t_1)}^{\Sigma_C(t_2)} \left.\frac{\partial_t\rho}{\rho+P}\right|_{0} dt$$

$x^i=0$: fiducial background trajectory

$\rho\,(x^i,t_2) = \rho\,(0,t_2)$ = uniform on $\Sigma_C(t_2)$

matter fluctuates only on the initial flat slice

- Nonlinear δN for multi-component inflation :

$$\delta N = N\left(\phi^A + \delta\phi^A\right) - N\left(\phi^A\right)$$

$$= \sum_n \frac{1}{n!} \frac{\partial^n N}{\partial\phi^{A_1} \partial\phi^{A_2} \cdots \partial\phi^{A_n}} \delta\phi^{A_1} \delta\phi^{A_2} \cdots \delta\phi^{A_n}$$

where $\delta\phi = \delta\phi_F$ is fluctuation on initial flat slice at or after horizon-crossing.

$\delta\phi_F$ may contain non-Gaussianity from subhorizon (quantum) interactions

eg, in DBI inflation

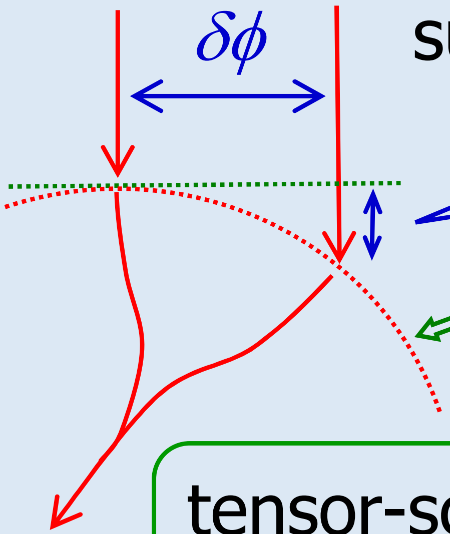
NG generation on superhorizon scales

two efficient mechanisms to convert
isocurvature to curvature perturbations:

- curvaton-type   Lyth & Wands ('01), Moroi & Takahashi ('01),...

$\rho_{curv} << \rho_{tot}$ ⇔ highly nonlinear dep on $\delta\phi_{curv}$

- multi-brid inflation MS ('08), Naruko & MS ('08),...



sudden change/transition in the trajectory

$$\delta N = \partial_a N \delta\phi^a + \frac{1}{2}\partial^2_{ab} N \delta\phi^a \delta\phi^b + \cdots$$

curvature of this surface determines sign of f_{NL}
Huang ('09)

tensor-scalar ratio r may be large in multi-brid models,
while it is always small in curvaton-type if NG is large.

Curvaton model

Lyth & Wands ('01)
Moroi & Takahashi ('01)

Inflation driven by inflaton = ϕ

Final curvature perturbation dominated by curvaton = χ

$$V_{tot} = V(\phi) + \frac{1}{2} m_\chi^2 \chi^2$$

$$m_\chi^2 \ll H^2 \approx \frac{8\pi G V(\phi)}{3}$$

during inflation: $V(\phi) \gg \frac{1}{2} m_\chi^2 \chi^2$

curvature perturbation is still dominated by ϕ

$$\delta\phi \sim \frac{H}{2\pi}, \quad \delta\chi \sim \frac{H}{2\pi} \quad \Rightarrow \quad \left| V'(\phi)\delta\phi \right| \gg m_\chi^2 \left| (\chi + \delta\chi)^2 - \chi^2 \right|$$

after inflation, ϕ thermalizes. χ undergoes damped oscillation

$$\Rightarrow \begin{cases} \rho_\phi = \rho_\gamma \propto a^{-4} \\ \rho_\chi \propto a^{-3} \end{cases} \Rightarrow \quad \mathcal{R}_C \sim \frac{4\rho_\gamma \mathcal{R}_\phi + 3\rho_\chi \mathcal{R}_\chi}{4\rho_\gamma + 3\rho_\chi}$$

$f_{NL} \sim 1/q$

Assume $\delta\chi$ dominates the final curvature perturbation:

$$\mathcal{R}_C \approx \frac{q}{4-q}\left(2\frac{\delta\chi}{\chi} + \left(\frac{\delta\chi}{\chi}\right)^2 + \cdots\right) \approx \left(\frac{q}{2}\frac{\delta\chi}{\chi}\right) + \frac{1}{q}\left(\frac{q}{2}\frac{\delta\chi}{\chi}\right)^2$$

$\uparrow$ $q \ll 1$

$$q \equiv \left.\frac{\rho_\chi}{\rho_\chi + \rho_\gamma}\right|_{t=t_{decay}}$$ … density fraction when χ decays

large NG if $q << 1$

Enqvist & Nurmi ('05)

tensor-scalar ratio will be strongly suppressed:

$$r = \frac{P_T(k)}{P_{\mathcal{R}\chi}(k)} = \frac{P_T(k)}{P_{\mathcal{R}\phi}(k)}\frac{P_{\mathcal{R}\phi}(k)}{P_{\mathcal{R}\chi}(k)} \ll \frac{P_T(k)}{P_{\mathcal{R}\phi}(k)} \ll 1$$

Multi-brid inflation

"multi"-field hy"brid" inflation

MS (2008)

$$L_\phi = -\frac{1}{2}\sum_A g^{\mu\nu}\partial_\mu\phi_A\partial_\nu\phi_A - V(\phi)$$

- slow-roll eom $(8\pi G = M_{\mathrm{Planck}}^{-2} = 1)$

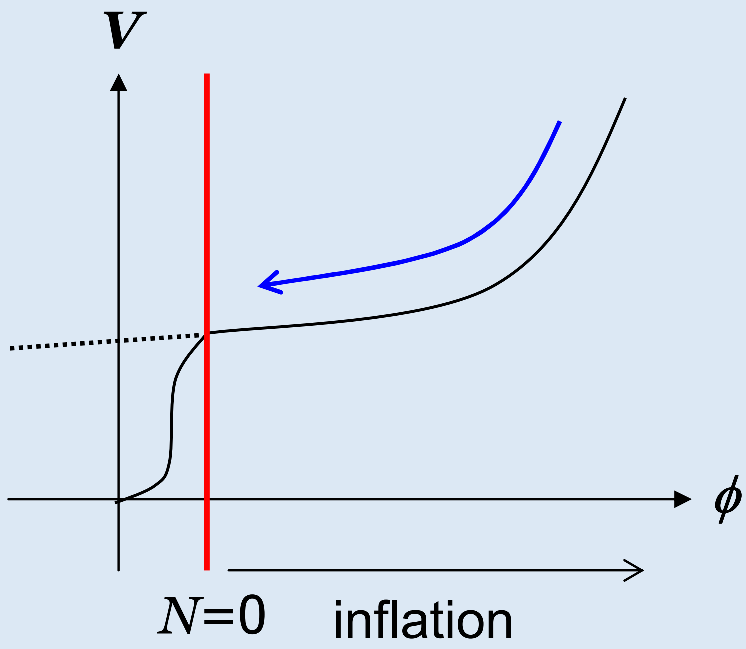
$$\frac{d\phi_A}{dt} = -\frac{1}{3H}\frac{\partial V}{\partial\phi_A}, \quad 3H^2 = V$$

N as a time variable: $dN = -Hdt$

$$\Rightarrow \quad \frac{d\phi_A}{dN} = \frac{1}{3V}\frac{\partial V}{\partial\phi_A}$$

$$\Rightarrow \quad \phi_A = \phi_A(N, \phi_A^0)$$

… slow-roll ends at $F(\phi_A)$=0.



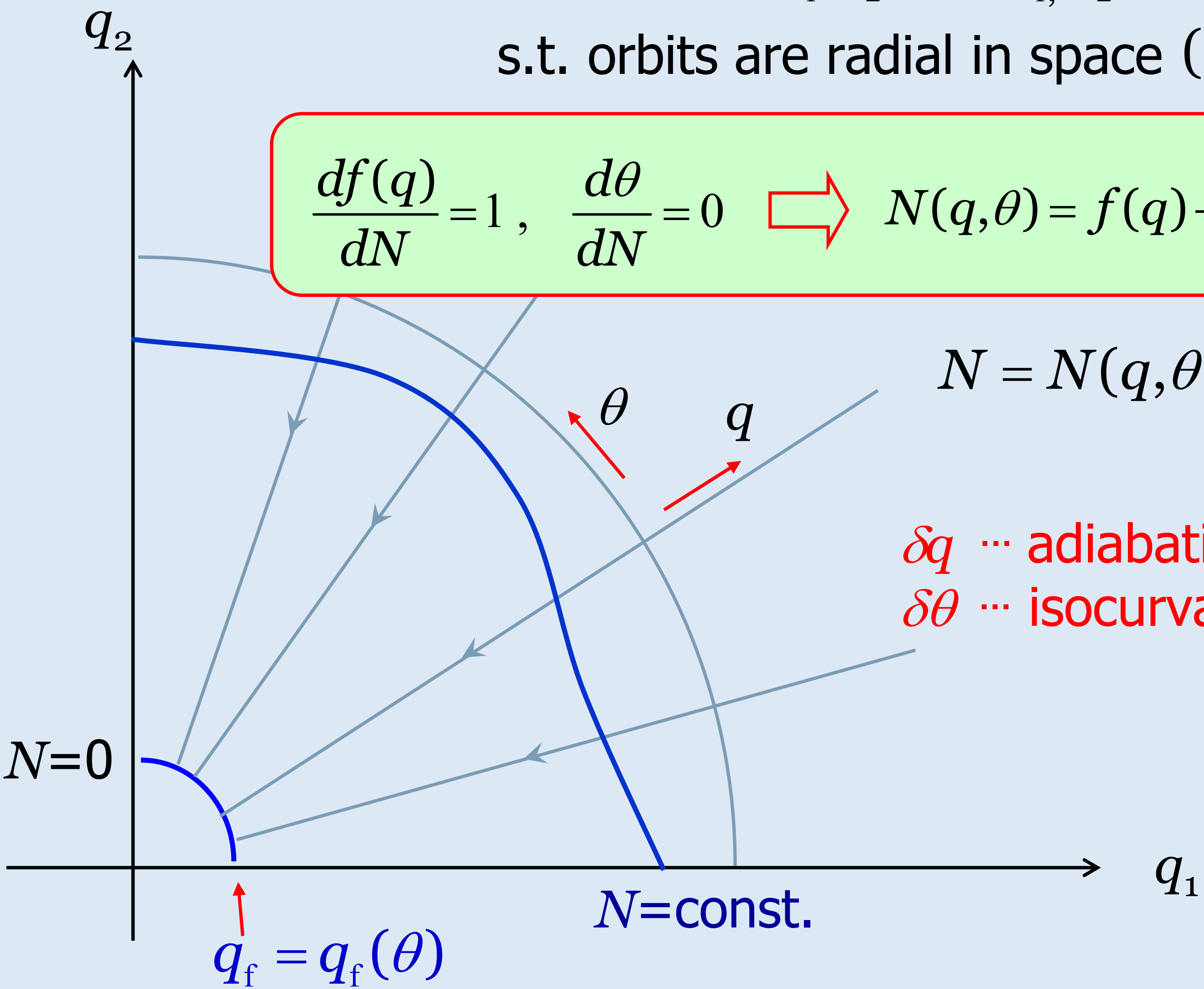
2-dim case:

coord trans $(\phi_1,\phi_2) \to (q_1, q_2)$

s.t. orbits are radial in space (q_1, q_2)

$$\frac{df(q)}{dN}=1\,, \quad \frac{d\theta}{dN}=0 \quad \Rightarrow \quad N(q,\theta)=f(q)-f(q_f(\theta))$$

$$N = N(q,\theta) = N(\phi_1,\phi_2)$$

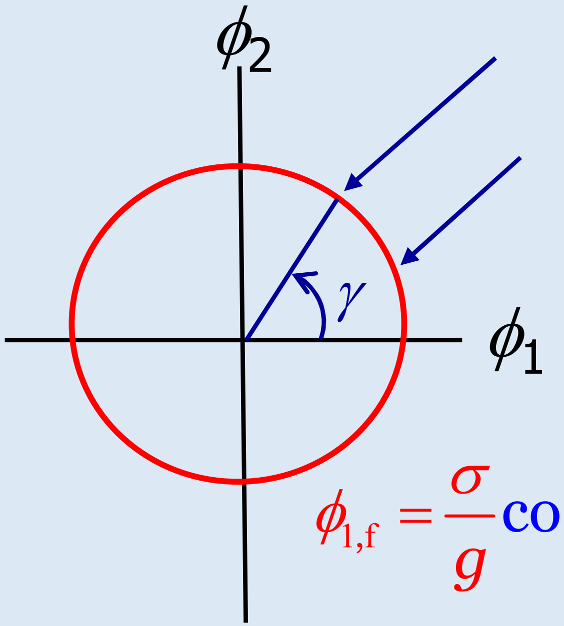


δq ··· adiabatic pertn

$\delta\theta$ ··· isocurvature pertn

analytical multi-brid model

➢ Exponential potential: $V = V_0 \exp\left[m_1\phi_1 + m_2\phi_2\right]$



Inflation ends at $g^2\left(\phi_1^2 + \phi_2^2\right) = \sigma^2$

realized by a waterfall field χ :

$$V_0 = \frac{1}{2} g^2 \left(\phi_1^2 + \phi_2^2\right)\chi^2 + \frac{\lambda}{4}\left(\chi^2 - \frac{\sigma^2}{\lambda}\right)^2$$

$$\phi_{1,f} = \frac{\sigma}{g}\cos\gamma, \quad \phi_{2,f} = \frac{\sigma}{g}\sin\gamma$$

trajectory specified by "γ"

- δN to 2nd order in δφ :

$$\delta N = \frac{\delta\phi_1 \cos\gamma + \delta\phi_2 \sin\gamma}{m_1\cos\gamma + m_2\sin\gamma} + \frac{g}{2\sigma}\frac{(m_2\delta\phi_1 - m_1\delta\phi_2)^2}{(m_1\cos\gamma + m_2\sin\gamma)^3}$$

$$\Rightarrow \quad \delta N = \delta_L N + \frac{3}{5} f_{NL}^{\text{local}} \left(\delta_L N + S \right)^2$$

linear entropy perturbation contributes at 2nd order

$$\delta_L N \equiv \frac{\delta\phi_1 \cos\gamma + \delta\phi_2 \sin\gamma}{m_1 \cos\gamma + m_2 \sin\gamma}, \quad S \equiv \frac{\delta\phi_1 \sin\gamma - \delta\phi_2 \cos\gamma}{m_2 \cos\gamma - m_1 \sin\gamma}$$

"true" entropy perturbation

- curvature perturbation spectrum

$$P_S(k) = \frac{1}{(m_1 \cos\gamma + m_2 \sin\gamma)^2} \left(\frac{H}{2\pi} \right)^2 \Bigg|_{k=Ha}$$

spectral index: $n_s = 1 - (m_1^2 + m_2^2)$

tensor/scalar: $r \equiv \dfrac{P_T(k)}{P_S(k)} = 8(m_1 \cos\gamma + m_2 \sin\gamma)^2$

non-Gaussianity: $f_{NL}^{\text{local}} = \dfrac{5g}{6\sigma} \dfrac{(m_2 \cos\gamma - m_1 \sin\gamma)^2}{m_1 \cos\gamma + m_2 \sin\gamma}$

just for fun ...

$$1 = M_{Pl} = (8\pi G)^{-1/2} = 2.43 \times 10^{18}\,\mathrm{GeV}$$

model parameters: $m_1^2 \sim 0.005,\ m_2^2 \sim 0.035$

assume $m_1 \cos\gamma \gtrsim m_2 \sin\gamma \quad (\Leftrightarrow \gamma \ll 1)$

outputs:

$$n_s = 1 - (m_1^2 + m_2^2) \sim 0.96$$
$$r \approx 8 m_1^2 \sim 0.04$$

} indep. of waterfall field

$$3H^2 = \sigma^4 / 4\lambda \sim 1.5 \times 10^{-9} \quad \left(\Leftrightarrow P_S(k) \sim 2.5 \times 10^{-9}\right)$$

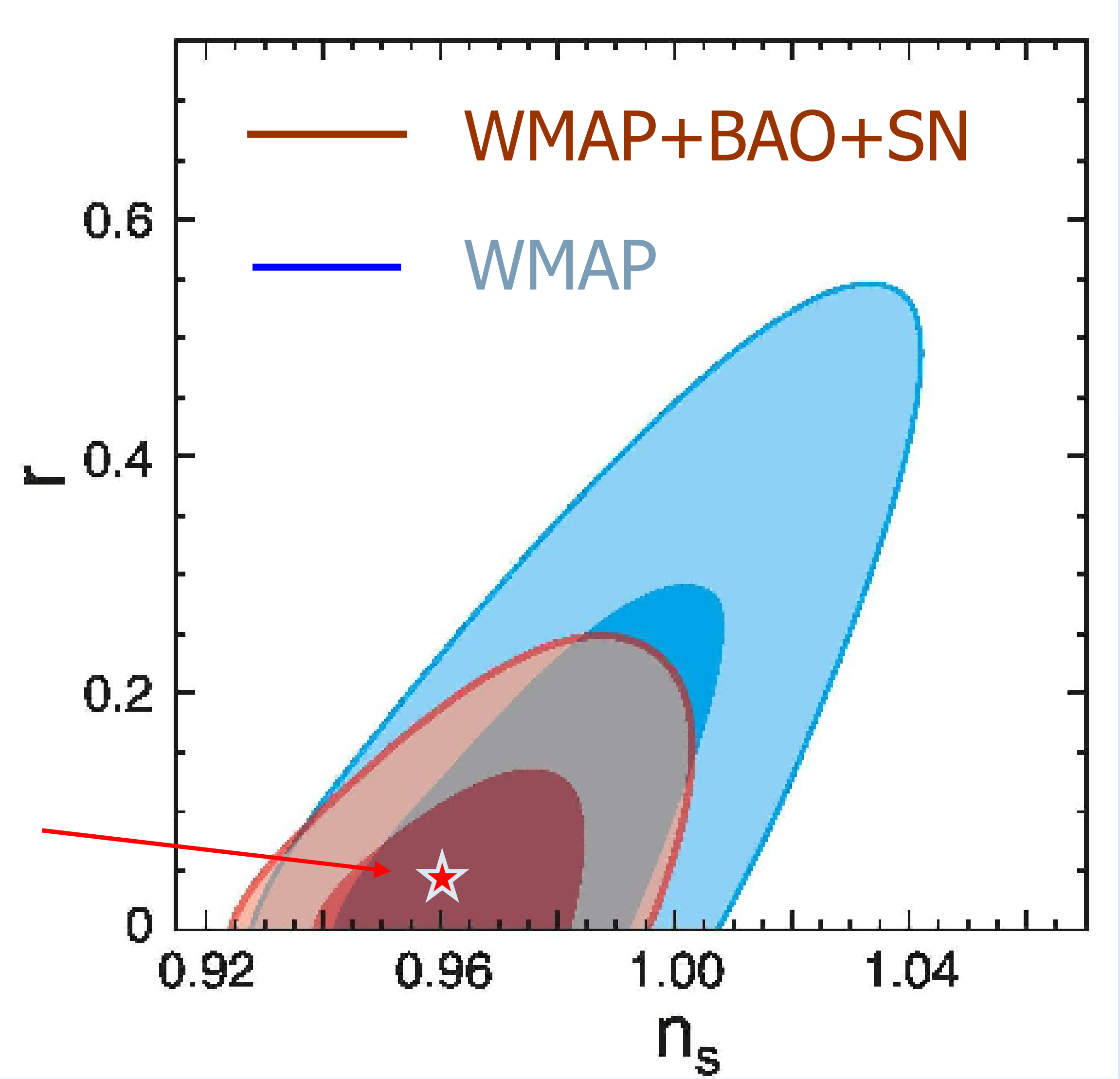
$$\Rightarrow \quad \sigma^2 \sim \lambda^{1/2} \times 10^{-4}$$

$$\Rightarrow \quad f_{NL}^{\mathrm{local}} \approx \frac{5 g m_2^2}{6 \sigma m_1} \sim 40 \frac{g}{\lambda^{1/4}}$$

WMAP 5yr constraint on r & n_s

Komatsu et al. '08

example



and f_{NL}^{local} can be ~ 50 as well.

5. Summary

- inflation explains observed structure of the universe

  flatness: $\Omega_0=1$ to good accuracy

  curvature perturbation spectrum

  almost scale-invariant
  almost Gaussian

- inflation also predicts scale-invariant tensor spectrum

  will be detected soon if tensor-scalar ratio $r>0.1$

any new/additional features?

non-Gaussianities

- 3 origins of NG in curvature perturbation

1. subhorizon ··· quantum origin
2. superhorizon ··· classical (local) origin
3. NL gravity ··· late time classical dynamics

NG from inflation

need to be quantified

- DBI-type model: origin 1.

  f_{NL}^{equil} may be large

- non BD vacuum: origin 1.

  any type of f_{NL} may be large

  could be spatially localized: "NG bubbles in the sky"

  Sugimura, Yamauchi & MS in prep.

- multi-field model: origin 2.

  f_{NL}^{local} may be large: In curvaton-type models $r \ll 1$. Multi-brid model may give $r \sim 0.1$.

Identifying properties of non-Gaussianity is extremely important for understanding physics of the early universe

not only bispectrum(3-pt function) but also trispectrum or higher order n-pt functions may become important.

Confirmation of primordial NG?

PLANCK (February 2013?) ...