Massive states localized on a de Sitter brane

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D. Langlois and MS, hep-th/0302069

§1. Disappearance of massive states in RS braneworld

Dubovsky, Rubakov and Tinyakov, PRD **62**, 105011 (2000)

For a bulk scalar with M^2 in a RS2 (single) braneworld,

$$[\Box_5 - M^2]\phi = 0\,; \quad \phi = u_n(z)\psi_n(x^\mu)$$

$$\Rightarrow \quad \left[-b^{-3}(z)\partial_z b^3(z)\partial_z + M^2 b^2(z)\right] u_n = m_n^2\, u_n\,, \quad \left[\Box_4 - m_n^2\right]\psi_n = 0.$$

$b(z)$: (conformal) warp factor

· m_n^2 $\cdots$ separation constant (=effective 4d mass^2).

· Bound state at $m^2 = 0$ (zero mode) only for $M^2 = 0$.

· All massive states belong to (continuous) KK spectrum.

· However, for 'outgoing-wave' boundary condition,
there exists a 'quasi-normal mode',

$$M_{qnm} \approx \pm\frac{M}{\sqrt{2}} - i\frac{\sqrt{2}\,\pi}{32}M^3\ell^2 \quad (M^2\ell^2 \ll 1)$$

- Imaginary part describes decay into bulk.

§2. Bound states on dS branes and effective potential

Himemoto, Tanaka and MS, PRD **65**, 104020 (2002)

For a dS brane with Hubble H, a massive bound state exists if $M^2 \lesssim H^2$,

$$M_b^2 \approx \frac{M^2}{2} \quad \text{for} \quad H^2\ell^2 \ll 1.$$

$\Leftrightarrow$ consistent with effective 4d field picture:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_5}{3}\left(\frac{1}{4}\dot{\phi}^2 + \frac{1}{2}V(\phi)\right) + \frac{E^t{}_t}{3}, \qquad E^t{}_t = \frac{8\pi G_5}{4}\dot{\phi}^2 + \frac{3C}{a^4}$$

$$\Rightarrow \quad \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_5}{3}\left(\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}V(\phi)\right) + \frac{C}{a^4}$$

or

$$\varphi = \sqrt{\ell}\phi, \quad V_{\text{eff}}(\varphi) = \frac{\ell}{2}V(\varphi/\sqrt{\ell}), \quad G_4 = \frac{G_5}{\ell}$$

$$\Rightarrow \quad \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_4}{3}\left(\frac{1}{2}\dot{\varphi}^2 + V_{\text{eff}}(\varphi)\right) + \frac{C}{a^4}$$

How far does this picture work?

§3. Bulk scalar coupled to brane tension

$$S = \int d^5x\sqrt{-g}\left(\frac{1}{16\pi G_5}R - \frac{1}{2}(\nabla\phi)^2 - V_5(\phi)\right) - \int d^4x\sqrt{-q}\,\sigma(\phi)$$

$$8\pi G_5 V_5(0) = \Lambda_5 = -6\,\ell^2$$

⋆ Friedmann equation on the brane: Maeda & Wands (2000)

$$H^2 = \frac{8\pi G_5}{3}\left(\frac{1}{4}\dot\phi^2 + \frac{1}{2}V_5 + \frac{8\pi G_5}{12}\sigma^2 - \frac{1}{16}\left(\frac{\partial\sigma}{\partial\phi}\right)^2\right) + \frac{E^t{}_t}{3}$$

If $\ {}^\exists V_{\text{eff}}$ s.t. $\ddot\Phi + 3H\dot\Phi + V'_{\text{eff}} = -J\,, \quad \Phi \equiv \sqrt{\ell}\,\phi\,, \quad G_4 \equiv G_5/\ell$

$$\Rightarrow \quad \begin{cases} H^2 = \dfrac{8\pi G_4}{3}\left[\dfrac{1}{2}\dot\Phi^2 + V_{\text{eff}}(\Phi) + \rho_E\right], \\ \dot\rho_E + 4H\rho_E = J\dot\Phi\,; \quad V_{\text{eff}} = \ell\left(\dfrac{1}{2}V_5 + \dfrac{8\pi G_5}{12}\sigma^2 - \dfrac{1}{16}\left(\dfrac{\partial\sigma}{\partial\phi}\right)^2\right), \end{cases}$$

If valid, perhaps related to AdS/CFT correspondence.

§4. Exact solutions

Cai et al. ('98), Chamblin & Reall ('99), Langlois & R-Martinez ('01), ⋯

$$V(\phi) = V_0 \exp\left(-\frac{2}{\sqrt{3}}\lambda\kappa\phi\right), \quad \sigma(\phi) = \sigma_0 \exp\left(-\frac{\lambda}{\sqrt{3}}\kappa\phi\right).$$

This gives a bulk solution:

$$ds^2 = -h(R)dT^2 + \frac{R^{2\lambda^2}}{h(R)}dR^2 + R^2 d\vec{x}^2\,;$$

$$h(R) = -\frac{\kappa^2 V_0/6}{1-(\lambda^2/4)}R^2 - \mathcal{C}R^{\lambda^2-2},$$

$$\phi = \frac{\sqrt{3}\,\lambda}{\kappa}\ln(R)\,; \quad \kappa^2 = 8\pi G_5$$

and Friedmann equation on the brane:

$$H^2 = \left[\frac{\kappa^4}{36}\sigma_0^2 + \frac{\kappa^2 V_0/6}{1-(\lambda^2/4)}\right] R^{-2\lambda^2} + \mathcal{C}R^{-4-\lambda^2}.$$

This conforms to the effective potential picture with

$$V_{\rm eff} = \frac{1}{2}V + \frac{\kappa_5^2}{12}\sigma^2 - \frac{1}{16}\sigma'^2 = \left[\frac{V_0}{2} + \frac{\kappa^2}{12}\left(1 - \frac{\lambda^2}{4}\right)\sigma_0^2\right]\exp\left(-\frac{2}{\sqrt{3}}\lambda\kappa\phi\right)$$
$$\equiv V_{\rm eff,0}\,\exp\left(-\frac{2}{\sqrt{3}}\lambda\kappa\phi\right).$$

and

$$J = -\left(1 - \frac{\lambda^2}{2}\right)H\dot{\phi}\,.$$

The dark energy density ρ_E is

$$\kappa^2\rho_E = 3\left(1 - \frac{\lambda^2}{4}\right)\mathcal{C}R^{-4-\lambda^2} - \frac{\kappa^2}{4}\dot{\phi}^2$$
$$= 3\left(1 - \frac{\lambda^2}{2}\right)\mathcal{C}R^{-4-\lambda^2} - \frac{\lambda^2/4}{1-\lambda^2/4}V_{\rm eff,0}\,R^{-2\lambda^2}.$$

Energy flows onto the brane if $\lambda^2 < 2$.

§5. Mode analysis in the case of quadratic potential and coupling

$$V(\phi) = \frac{1}{2}M^2\phi^2, \quad \sigma(\phi) = \sigma_0 + \frac{\alpha}{\ell}\phi^2.$$

- Assumptions:

  AdS$_5$ background with dS brane at $r = r_0$:

$$ds^2 = dr^2 + H^2\ell^2 \sinh^2(r/\ell)(-dt^2 + \cosh^2 Ht d\Omega_{(3)});$$

$$H\ell = \frac{1}{\sinh(r_0/\ell)}$$

  No back reaction of ϕ to the geometry.

- mode decomposition; $\phi = u_\mu(z)\psi_\mu(t)$ $(z = \cosh r/\ell)$:

$$u(z) = \frac{P^{\mu}_{\nu-1/2}(z)}{(z^2-1)^{3/4}}; \quad \nu = \sqrt{M^2\ell^2+4}$$

$$\text{b.c.}: \quad \frac{d}{dz}u + \frac{\alpha}{(z^2-1)^{1/2}}u = 0 \quad \text{at } z = z_0\,.$$

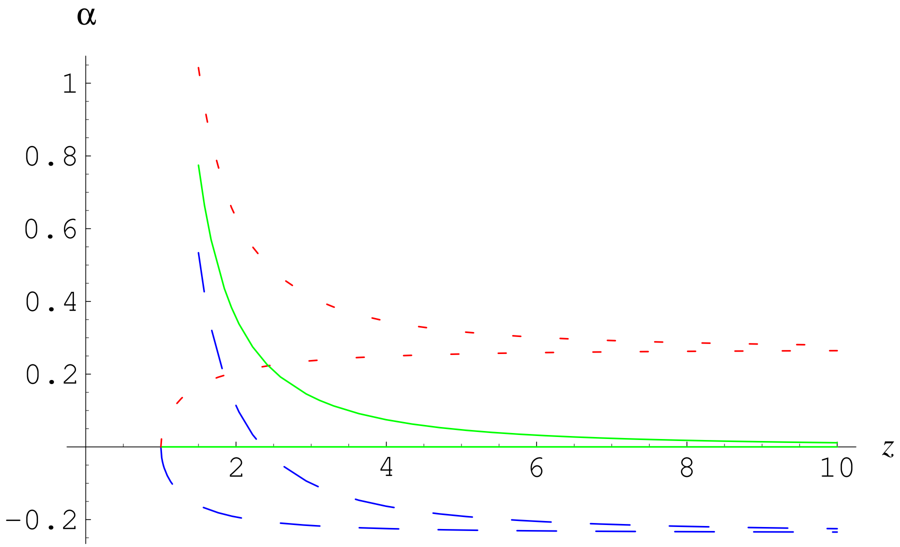
bound state $\Leftrightarrow$ $\mu < 0$ with $m^2_{(4)} = (9/4 - \mu^2)H^2$.

- critical values of α:

$$\alpha_{\rm zm}(z, M) \equiv \alpha(\mu = -3/2; z, M) = -M^2\ell^2 \, \frac{P^{-5/2}_{\nu-1/2}(z)}{P^{-3/2}_{\nu-1/2}(z)},$$

$$\alpha_{\rm bs}(z, M) \equiv \alpha(\mu = 0; z, M) = \frac{(\nu - 1/2)P_{\nu-3/2}(z) - (\nu - 2)zP_{\nu-1/2}(z)}{(z^2 - 1)^{1/2}P_{\nu-1/2}(z)}.$$

where $\quad z = z_0 = \dfrac{\sqrt{1 + H^2\ell^2}}{H\ell}$



Range of α ($\alpha_{\rm bs} > \alpha > \alpha_{\rm zm}$) as a function of z:

$M^2\ell^2 = -1$ (short-dashed),
$M^2\ell^2 = 0$ (continuous),
$M^2\ell^2 = 1$ (long-dashed).

⋆ Negative α favors the existence of bound state (tachionic state).

- critical values of α expected from the effective potential

$$\mathcal{M}_{\rm eff}^2 = \frac{M^2}{2} + 2\frac{\alpha}{\ell^2}\sqrt{1+H^2\ell^2} - \frac{\alpha^2}{2\ell^2} = \frac{M^2}{2} + \frac{2\alpha}{\ell\ell_0} - \frac{\alpha^2}{2\ell^2}\,;$$

$$H^2 = \frac{1}{\ell_0^2} - \frac{1}{\ell^2}\,, \quad \ell_0 = \frac{4\pi G_5 \sigma_0}{6}$$

$$\mathcal{M}_{\rm eff}^2 = 0$$

$$\to \quad \hat{\alpha}_{\rm zm}(z, M) = \frac{2z}{\sqrt{z^2-1}} - \sqrt{\frac{4z^2}{z^2-1} + M^2\ell^2}.$$

$$\mathcal{M}_{\rm eff}^2 = \frac{9/4}{\ell^2(z^2-1)}$$

$$\to \quad \hat{\alpha}_{\rm bs}(z, M) = \frac{2z}{\sqrt{z^2-1}} - \sqrt{\frac{4z^2}{z^2-1} + M^2\ell^2 - \frac{9/2}{(z^2-1)}}.$$

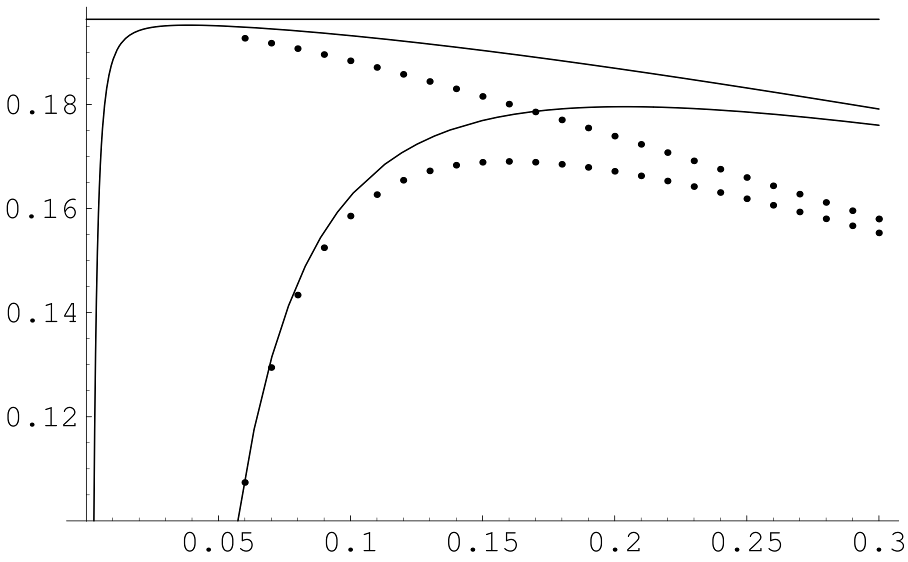
- These values agree well with those by mode analysis for $H^2\ell^2 \ll 1$.
- The agreement is quite good even for $H^2\ell^2 \lesssim 1$.

§6. Quasi-normal modes for quadratic potential and coupling

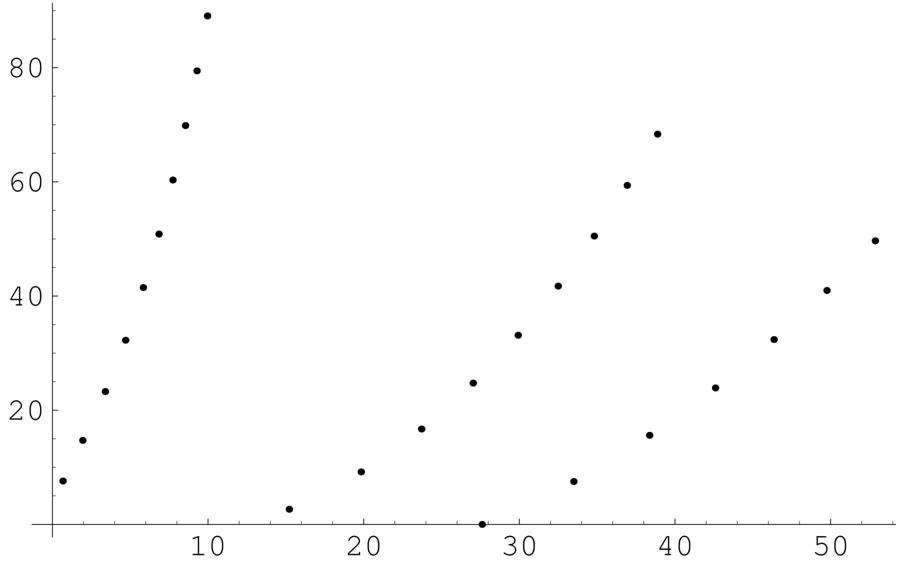
For $H^2\ell^2 \ll M^2\ell^2 \ll 1$ and $H^2\ell^2 \ll \alpha \ll 1$,

$$\mathrm{Re}\left[m^2_{(4)}\right] = \mathrm{Re}\left[H^2\left(9/4-\mu^2\right)\right] \approx \frac{M^2\ell^2+4\alpha}{2\ell^2}$$

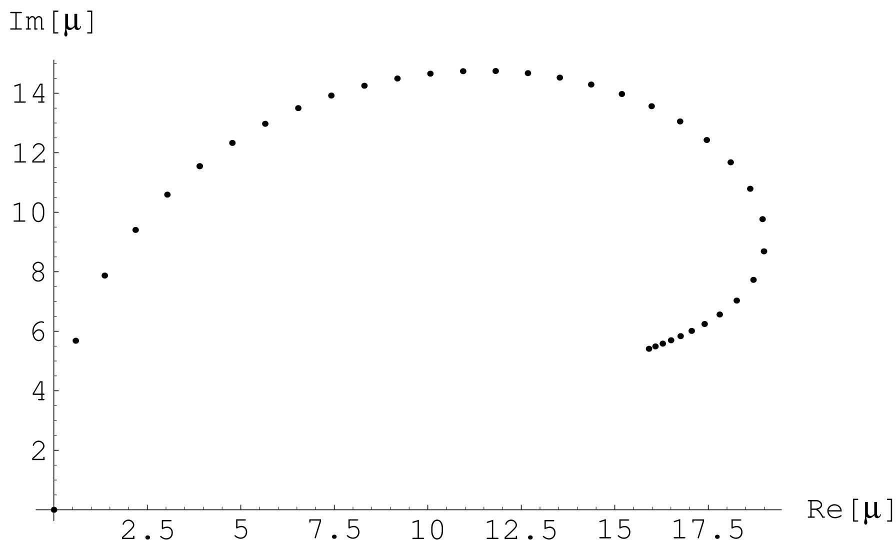
$$\mathrm{Im}\left[m^2_{(4)}\right] = \mathrm{Im}\left[H^2\left(9/4-\mu^2\right)\right] \approx \pm\frac{\pi}{16\ell^2}\left(M^2\ell^2+4\alpha\right)^2 .$$



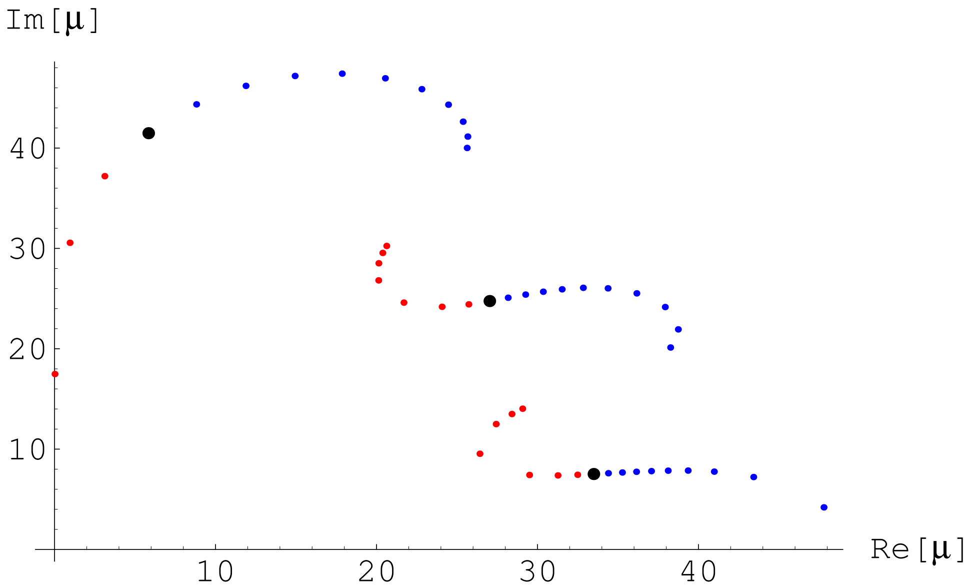
$-\mathrm{Im}[m^2_{(4)}]/(M^4\ell^2)$ as a function of $M\ell$ (lower: $z=50$, upper: $z=1000$). Both for $\alpha=0$. The real curves are analytical estimates.



Evolution in the complex μ-plane when M varies ($\alpha = 0$, $z = 10$), up to $M\ell = 10$ for each branch. The increment is $\Delta(M\ell) = 1$.



Evolution when α varies ($M\ell = 0$, $z = 10$), from 0 to 7 in the clockwise direction. The increment is $\Delta\alpha = 0.2$.



Variation when α changes (for $M\ell = 5$, $z = 10$). The increment $\Delta\alpha = 1$.
The big dots (●) correspond to the modes for $\alpha = 0$.
The points in red correspond to negative α and in blue to positive α.

- Thus, when $M\ell \gtrsim 1$, there appear additional quasi-normal modes and the number of qnm's increase as $M\ell$ increases.
- The behavior of the quasi-normal modes in the complex μ-plane as a function of α is quite complicated when $M\ell \gg 1$.

§7. Summary

- Effective potential approach gives a good description of the dynamics on the brane when $H^2\ell^2 \ll 1$.
- Decay of the scalar field out to the bulk may be evaluated by the imaginary part of quasi-normal modes.
- Distribution of quasi-normal modes becomes quite complicated when $M\ell \gg 1$.
  —- Need more studies to make quantitative predictions.
- Application to inflation?
- Dynamical self-tuning of cosmological constant?
- .....