Theory of Cosmological Perturbations

preliminary

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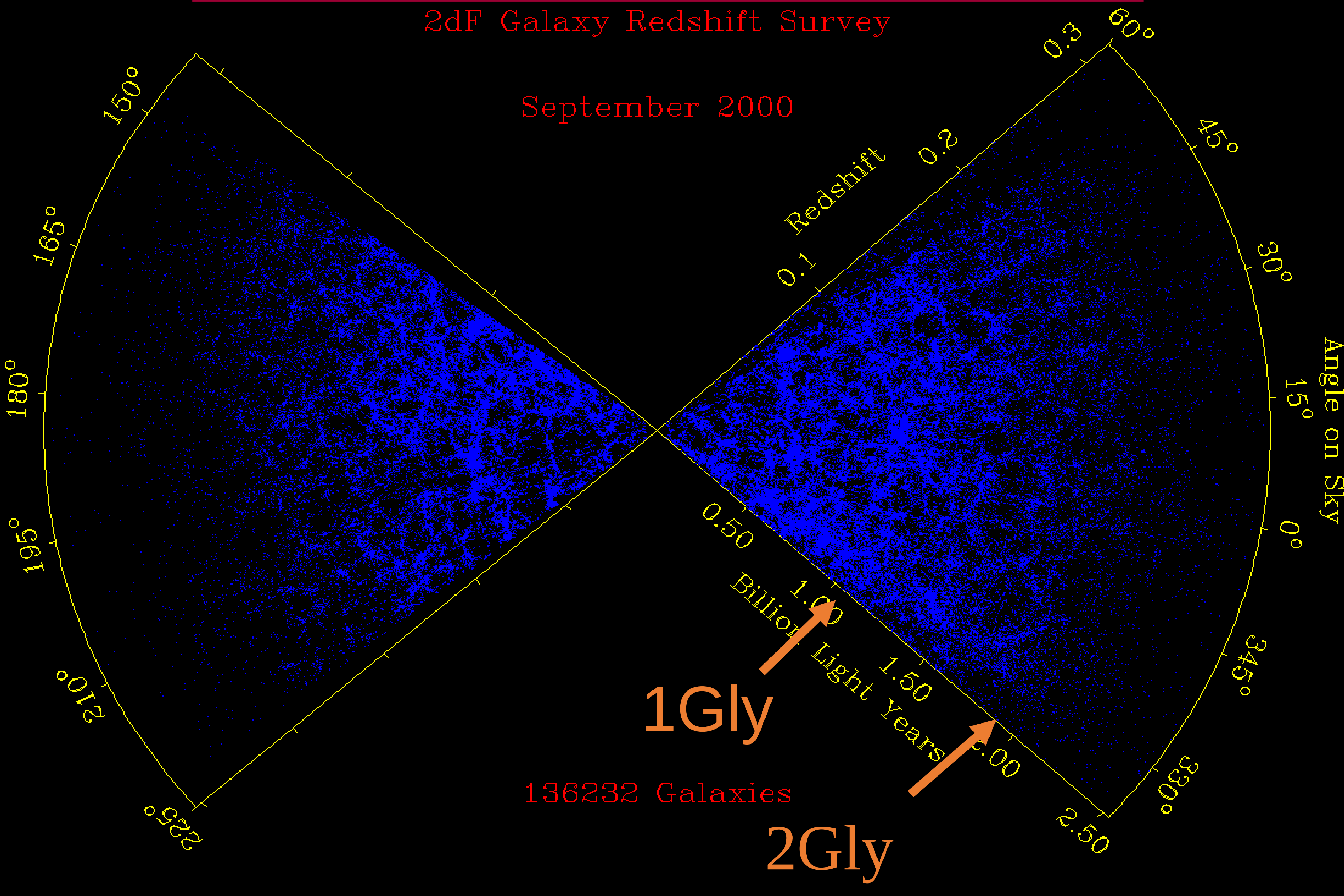
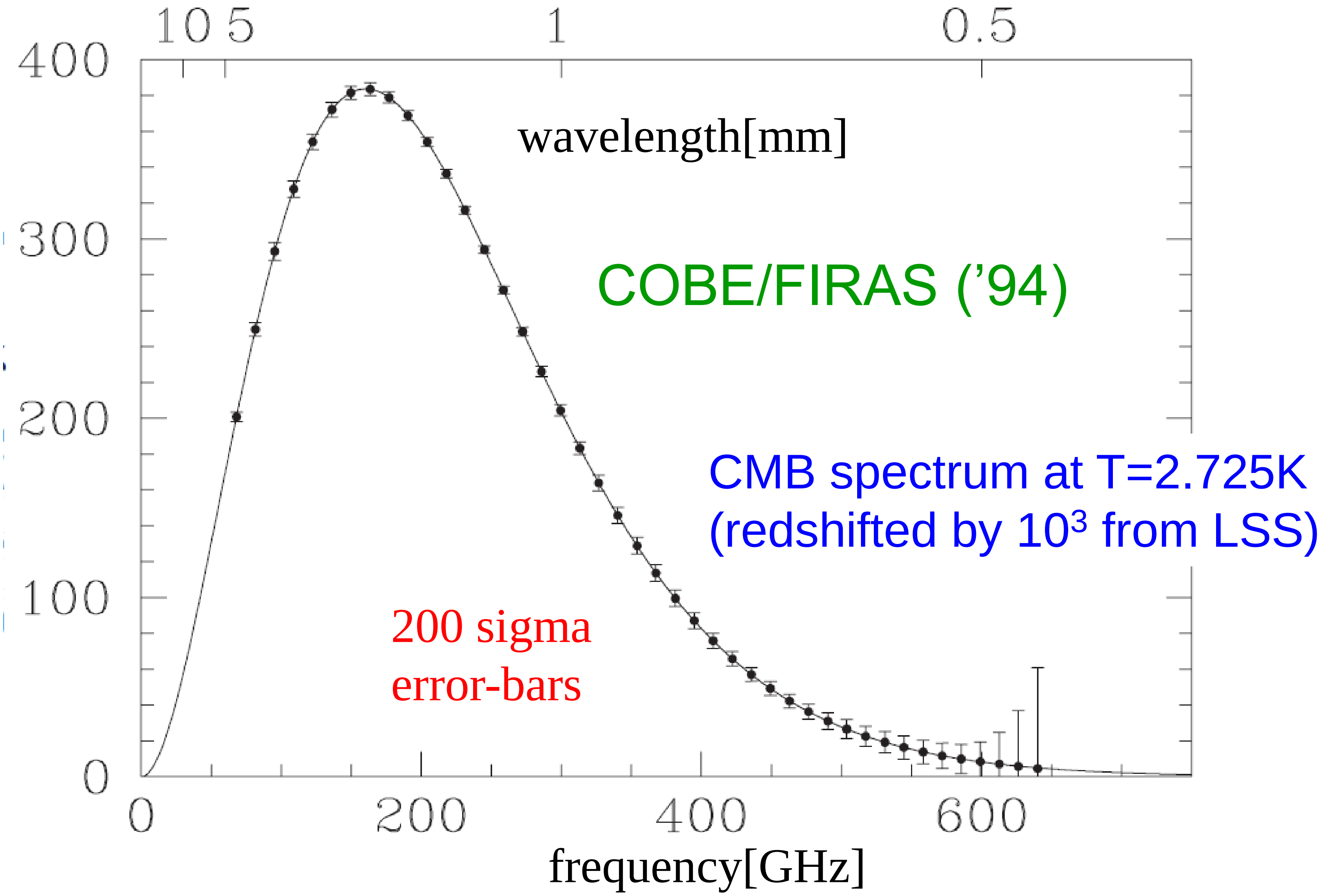
Progress in Cosmology

- 1916~ GR cosmological models
- 1929 Hubble's law
- 1946~ Big-bang model
- 1965 Discovery of CMB
- 1980~ Revelation of Large Scale Structure
  Birth of Particle Cosmology
- 1992 Detection of CMB Anisotropy
- 2003 Existence of Dark Energy
- 2013 Strong Evidence for Inflation
- 21st Century Era of Precision Cosmology

Large Scale Structure



CMB spectrum by COBE

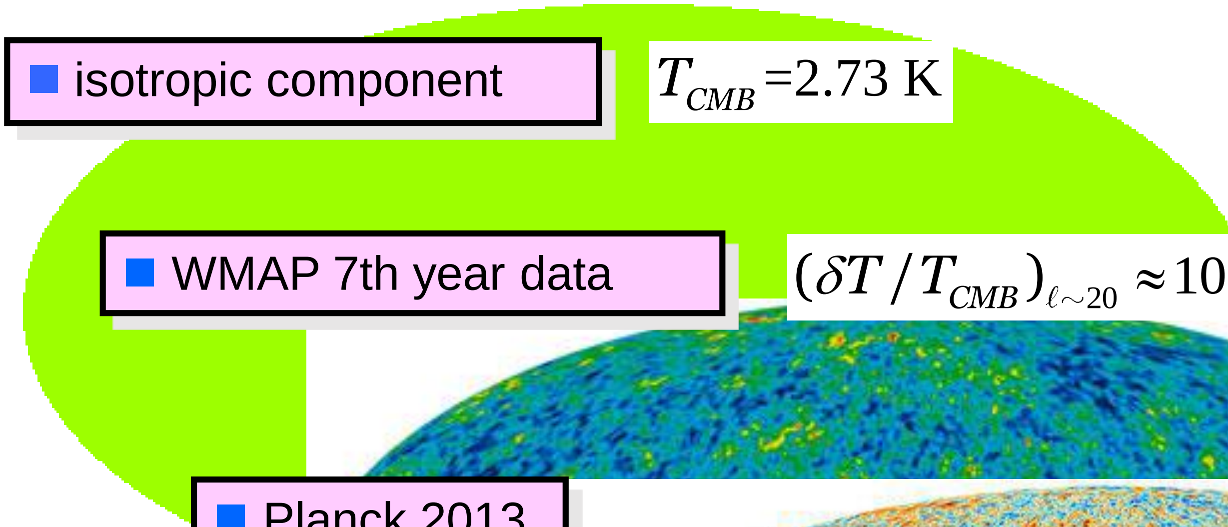


CMB Full Sky Map

COBE-DMR (199
WMAP (2003~)
Planck (2013~)

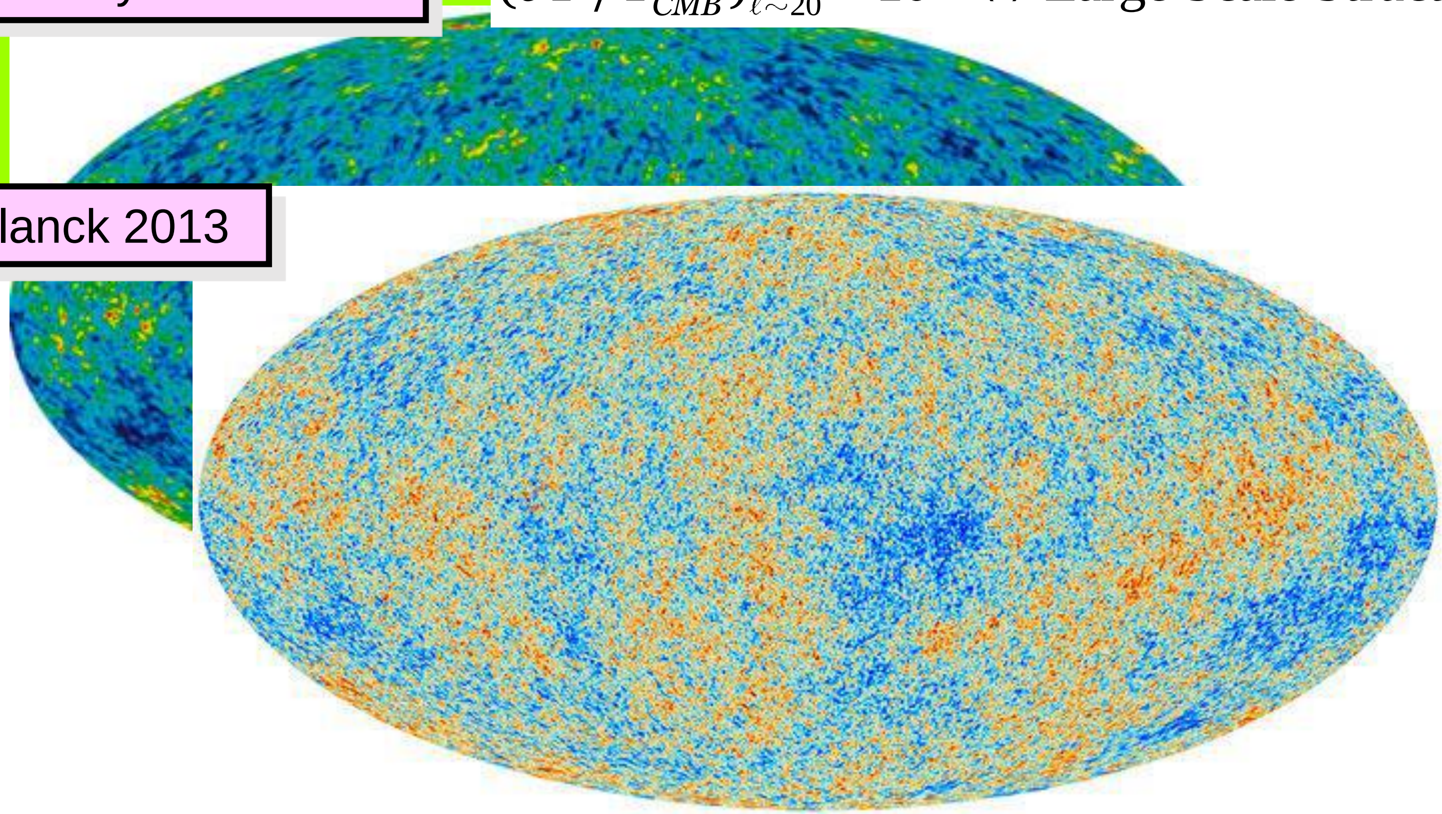
- isotropic component

$T_{CMB} = 2.73$ K



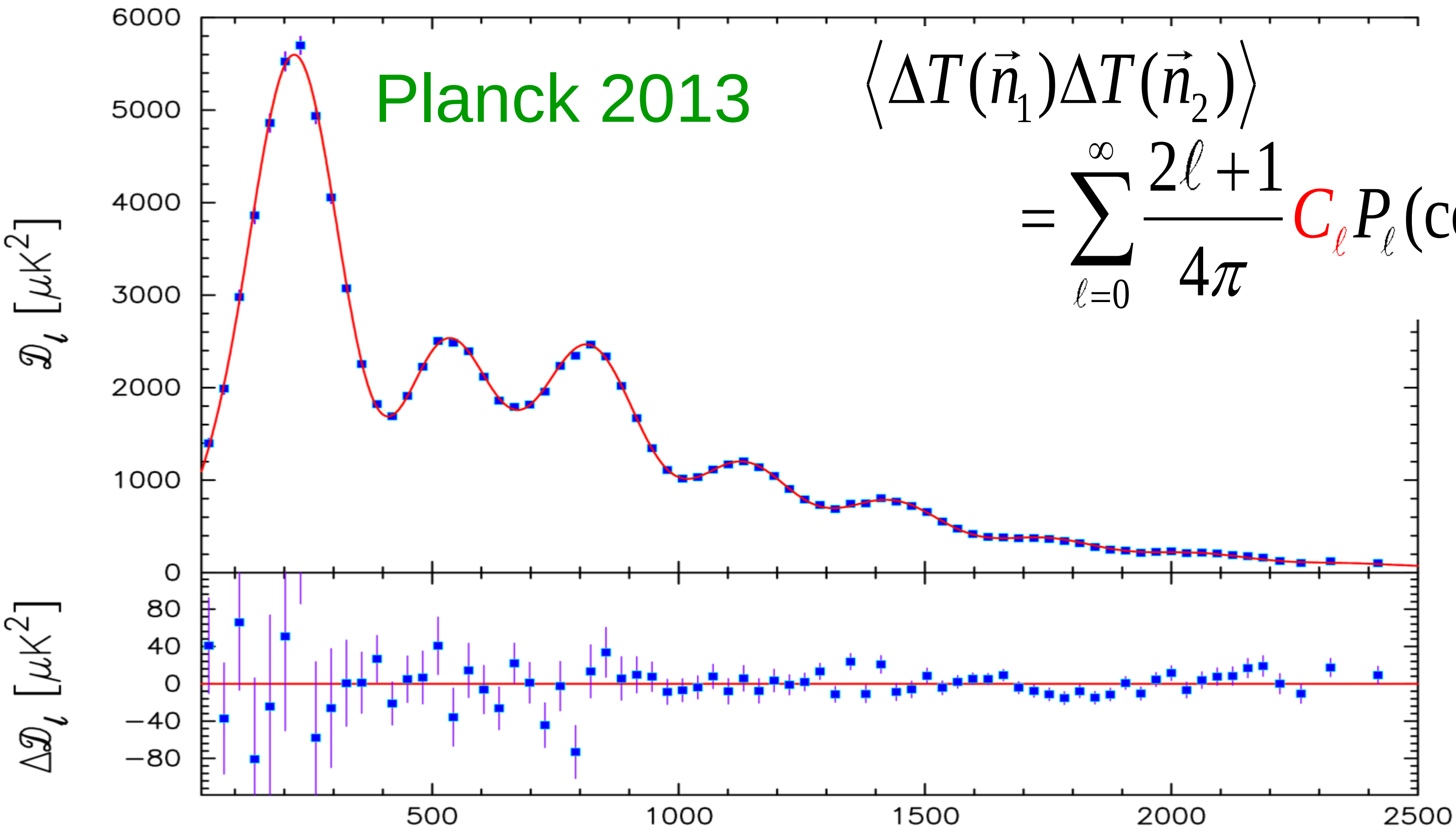
- WMAP 7th year data

$(\delta T / T_{CMB})_{\ell \sim 20} \approx 10^{-5} \Leftrightarrow$ Large Scale Structure



- Planck 2013

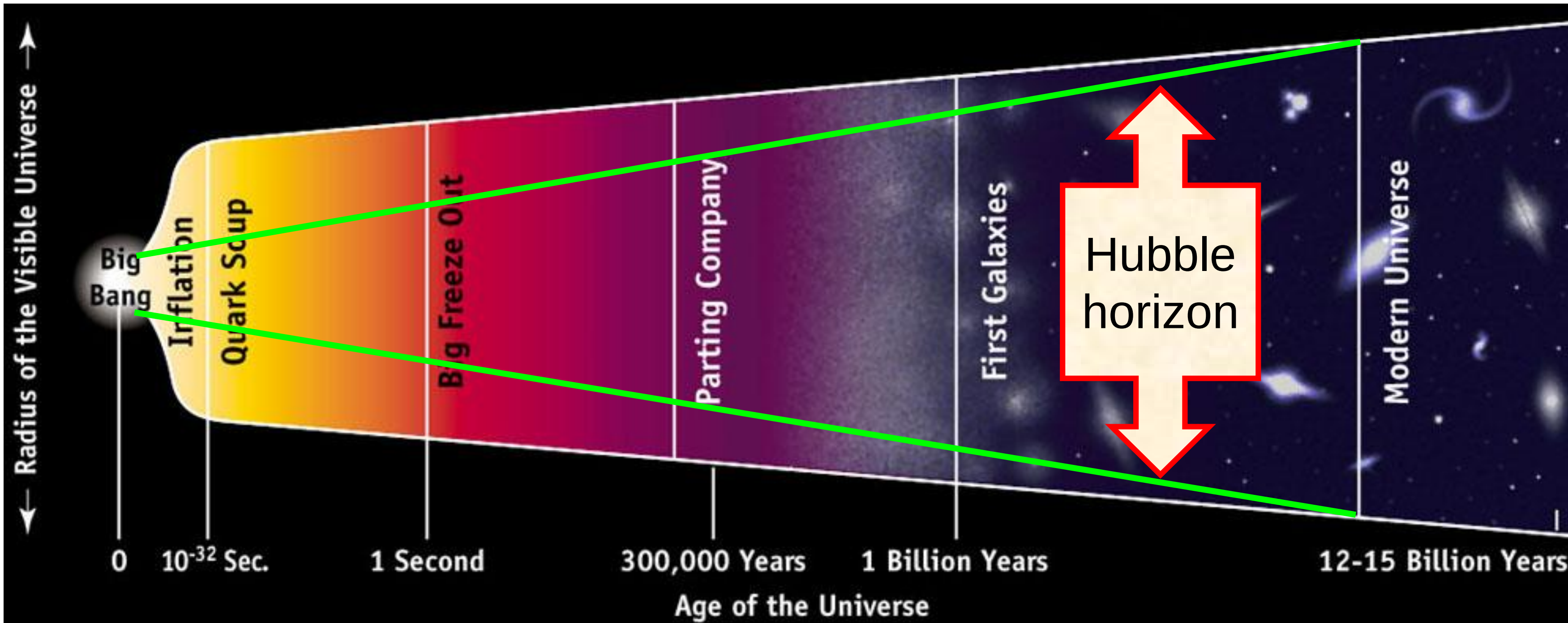
CMB angular power spectrum



$$\langle \Delta T(\vec{n}_1)\Delta T(\vec{n}_2)\rangle = \sum_{\ell=0}^{\infty} \frac{2\ell+1}{4\pi} C_\ell P_\ell(\cos\theta).$$

highly Gaussian fluctuations
almost scale-invariant spectrum
strong evidence for inflation

Cosmic Thermal History & Hubble Horizon



baryogenesis?

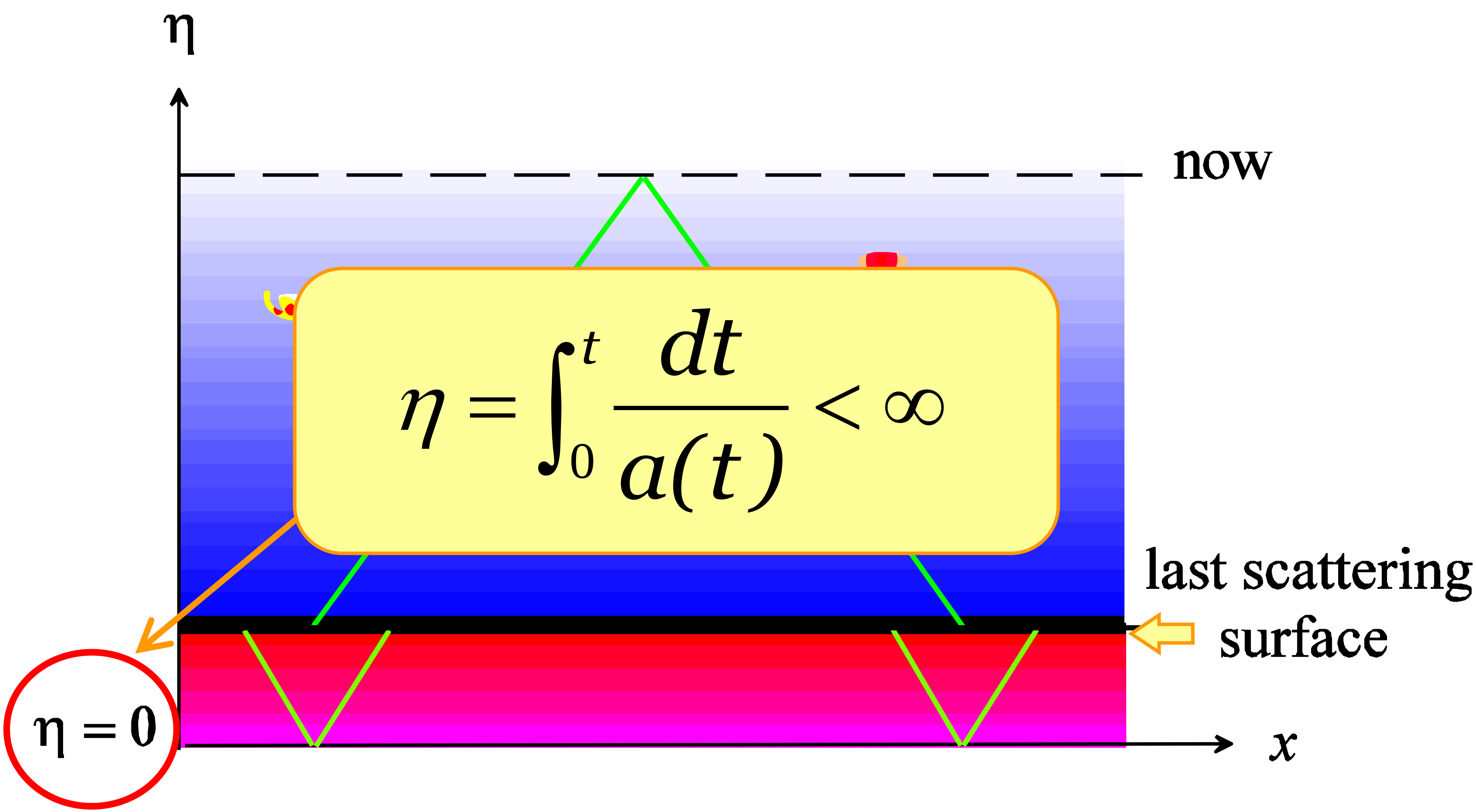
ν-decoupling:
nucleosynthesis

baryon-photon
decoupling:
last scattering surface

now

Horizon Problem

$$ds^2 = -dt^2 + a^2(t)d\vec{x}^2 = a^2(\eta)\left(-d\eta^2 + d\vec{x}^2\right)$$



Evolution of Inhomogeneities

- evolution on scales greater than Hubble horizon

$$M(r) = \frac{4\pi}{3}\rho r^3 \Rightarrow r_{\mathrm{g}} = \frac{2GM}{c^2} = \frac{8\pi G}{3c^2}\rho r^3$$

$$\frac{8\pi G}{3}\rho \approx H^2 = \frac{c^2}{r_{\mathrm{H}}^2} \quad \therefore\ r_{\mathrm{g}} = \left(\frac{r}{r_{\mathrm{H}}}\right)^2 r > r \ \textbf{for}\ r > r_H$$

"gravitational" radius > r >Hubble radius

⇩

- general relativistic treatment is mandatory

general relativistic cosmological perturbation theory

selected references

basics on cosmological perturbations

- H. Kodama and M. Sasaki
  Cosmological Perturbation Theory
  Prog. Theor. Phys. Suppl. 78 (1984) 1-166.

very good textbook on early universe cosmology including inflation

- V. Mukhanov,
  Physical foundations of cosmology
  Cambridge, UK: Univ. Pr. (2005)

classic textbook on quantum field in curved spacetime

- N.D. Birrell and P.C.W. Davies
  Quantum Fields in Curved Space
  Cambridge Mongr. Math. Phys. (1982)