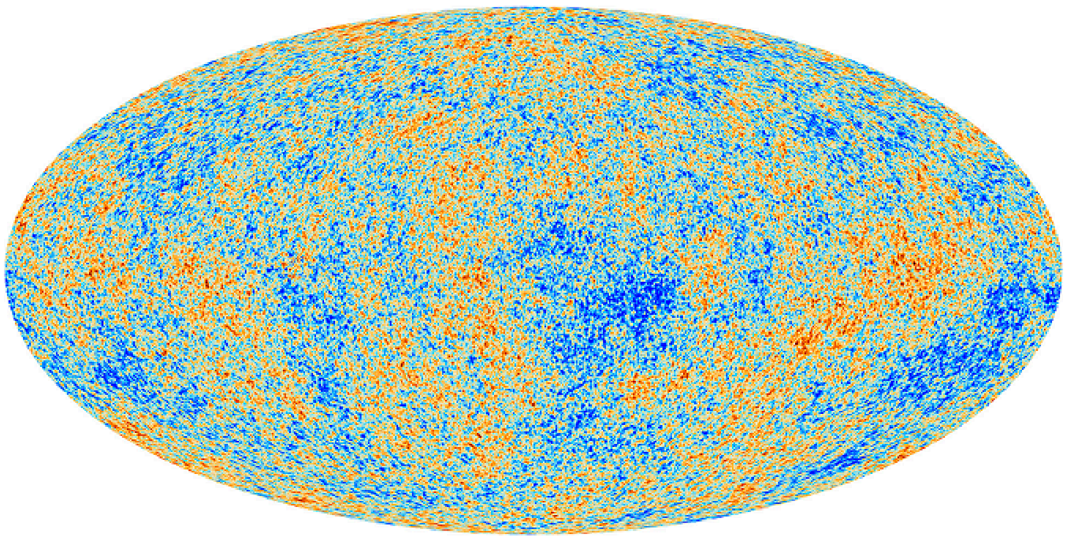
Theory of Cosmological Perturbations

Part III

— CMB anisotropy —



§ 1. Photon propagation equation

- Definitions

  · Lorentz-invariant distribution function: $f(p^\mu, x^\mu)$

  · Lorentz-invariant volume element on momentum space: $d\pi(p)$

$$\begin{aligned} d\pi(p) &= 2\theta(p^0)\delta(g_{\mu\nu}p^\mu p^\nu)\sqrt{-g}d^4p \quad (d^4p = dp^0dp^1dp^2dp^3) \\ &= 2\theta(\hat{p}^0)\delta(\eta_{ab}\hat{p}^a\hat{p}^b)d^4\hat{p} \quad (\hat{p}^a = e^a{}_\mu p^\mu \;\cdots\; \text{tetrad components}) \end{aligned}$$

(f and $d\pi$ are invariant under **local** Lorentz transformation)

  · number of photons within $d\pi(p)$ crossing 3-surface $d\Sigma_\mu$: dn

$$dn = f(p^\mu, x^\mu)d\pi(p)\, p^\alpha d\Sigma_\alpha$$

  · rate of change in dn due to collision:

$$C[f]d\pi(p)\, p^\alpha d\Sigma_\alpha$$

- Boltzmann equation

$$\left[p^\mu \frac{\partial}{\partial x^\mu} + \frac{dp^\mu}{dv}\frac{\partial}{\partial p^\mu}\right] f = C[f]\,; \qquad C[f] \;\cdots\; \text{collision term}$$

$$\frac{dp^\mu}{dv} = -\Gamma^\mu{}_{\alpha\beta}p^\alpha p^\beta\,, \quad p^\alpha = \frac{dx^\alpha}{dv} \quad (v \;\cdots\; \text{affine parameter})$$

- Boltzmann equation in a perturbed FLRW universe

  · perturbed metric (A, β_i, $H_{ij} = O(\epsilon)$)

$$\begin{aligned} ds^2 &= a^2(\eta)\left[-(1+2A)d\eta^2 + 2\beta_i dx^i d\eta + (\gamma_{ij} + H_{ij})dx^i dx^j\right] \\ &\equiv a^2(\eta)d\bar{s}^2 \quad (\Leftrightarrow \; g_{\mu\nu} = a^2 \bar{g}_{\mu\nu}) \end{aligned}$$

  · perturbed distribution function

$$\tilde{f}(p^\mu, x^\mu) = f(p^0, \eta) + \delta f(p^\mu, x^\mu)$$

- Conformal transformation (null geodesics are invariant under conformal transformation)

$$q^\mu = a^2 p^\mu = a^2 \frac{dx^\mu}{dv} \equiv \frac{dx^\mu}{d\lambda}, \quad \frac{dq^\mu}{d\lambda} = -\bar{\Gamma}^\mu{}_{\alpha\beta} q^\alpha q^\beta$$

$$\Rightarrow \quad \left[q^\mu \frac{\partial}{\partial x^\mu} + \frac{dq^\mu}{d\lambda}\frac{\partial}{\partial q^\mu}\right]\tilde{f} = a^2 C[\tilde{f}]$$

  · further transformation of the momentum variables:

$$q^\mu \quad \rightarrow \quad \hat{q} \equiv (1+A)q^0, \quad \gamma^i = \frac{q^i}{q^0}$$

$\frac{\hat{q}}{a}$ $\cdots$ photon energy seen by observer normal to η=const.

γ^i $\cdots$ directional cosine: $\gamma_{ij}\gamma^i\gamma^j = 1 + O(\epsilon)$.

$$\left[q^\mu \frac{\partial}{\partial x^\mu} + \frac{d\hat{q}}{d\lambda}\frac{\partial}{\partial \hat{q}} + \frac{d\gamma^i}{d\lambda}\frac{\partial}{\partial \gamma^i}\right]\tilde{f} = a^2 C[\tilde{f}]$$

$$\Rightarrow \quad \left[\frac{\partial}{\partial \eta} + \gamma^i \frac{\partial}{\partial x^i} + \frac{1}{q^0}\frac{d\hat{q}}{d\lambda}\frac{\partial}{\partial \hat{q}} - {}^s\Gamma^i{}_{jk}\gamma^j\gamma^k \frac{\partial}{\partial \gamma^i}\right]\tilde{f} = \frac{a^2}{q^0} C[\tilde{f}]$$

where ${}^s\Gamma^i{}_{kj}$ is the connection of γ_{ij}, and

$$\frac{d\hat{q}}{d\lambda} = \frac{d}{d\lambda}\left[(1+A)q^0\right] = q^0 \frac{d}{d\lambda}A + \frac{dq^0}{d\lambda} + O(\epsilon^2)$$

$$= q^0 \frac{d}{d\lambda}A - \bar{\Gamma}^0{}_{\mu\nu} q^\mu q^\nu \qquad \left(\frac{d}{d\lambda} = q^0\left[\frac{\partial}{\partial \eta} + \gamma^i \frac{\partial}{\partial x^i}\right] + O(\epsilon)\right)$$

$$= (q^0)^2 \left[-A_{|i}\gamma^i - \frac{1}{2}\left(H'_{ij} - 2\beta_{i|j}\right)\gamma^i\gamma^j\right]$$

Here we have used the fact $\bar{\Gamma}^0{}_{\mu\nu} = \delta\bar{\Gamma}^0{}_{\mu\nu} = O(\epsilon)$ with

$$\delta\bar{\Gamma}^0{}_{00} = A', \quad \delta\bar{\Gamma}^0{}_{0i} = A_{|i}, \quad \delta\bar{\Gamma}^0{}_{ij} = \frac{1}{2}\left(H'_{ij} - \beta_{i|j} - \beta_{j|i}\right)$$

Setting $\tilde{f} = f(\hat{q},\eta) + \delta f(\hat{q},\gamma^i,x^\mu)$,

$$\text{0th order:} \qquad \frac{\partial}{\partial\eta} f = 0 \quad \Rightarrow \quad f = f(\hat{q})\,.$$

$$\text{1st order:} \qquad \left[\frac{\partial}{\partial\eta} + \gamma^i \frac{\partial}{\partial x^i} - {}^s\Gamma^i{}_{jk}\gamma^j\gamma^k \frac{\partial}{\partial\gamma^i}\right]\delta f$$

$$= \left[A_{|i}\gamma^i + \frac{1}{2}\left(H'_{ij} - 2\beta_{i|j}\right)\gamma^i\gamma^j\right]\hat{q}\frac{d}{d\hat{q}}f + \frac{a^2}{\hat{q}}C[\tilde{f}]$$

<u>0th order:</u>

Assuming thermal equilibrium at sufficiently early stage $(\eta = \eta_i)$,

$$f(\hat{q}) = \frac{1}{e^{\hat{q}/(a_i T_i)} - 1} \quad (T_i \ \cdots \ \text{temperature at } \eta = \eta_i)$$

Define 'temperature' T by $T = a_i T_i / a$ irrespective of thermal equilibrium or not. Then

$$f(\hat{q}) = f(a\,q) = \frac{1}{e^{q/T} - 1}\,; \quad q \equiv \frac{\hat{q}}{a} \ \cdots \ \text{physical energy}$$

Planck distribution even if photons become out of thermal equilibrium.

<u>1st order:</u>

· Evaluation of collision term

For $T \ll m_e = 0.5\,\mathrm{MeV} \sim 5\times 10^9\mathrm{K}$, electrons are non-relativistic.

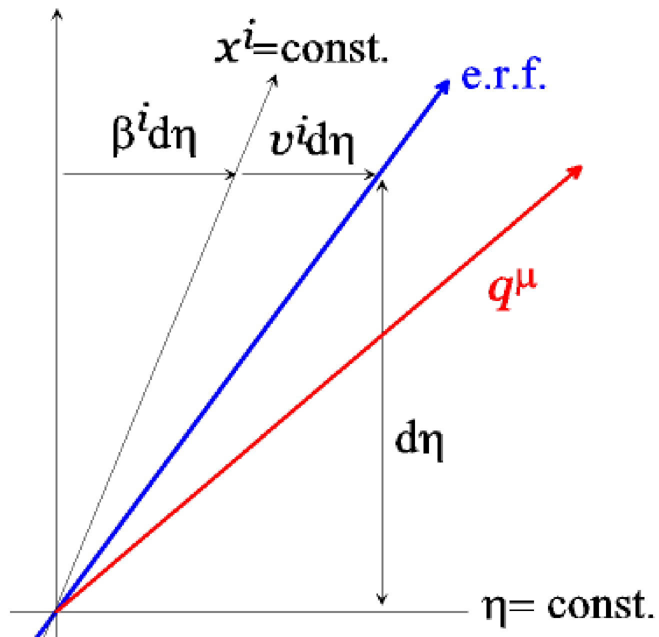
$\Rightarrow$ Thomson scattering of photons by electrons ($\sigma_T = 8\pi r_e^2/3 \approx 6.7\times 10^{-25}\mathrm{cm}^2$):

$$\frac{1}{q}C[\tilde{f}] = \frac{3}{8\pi} n_e \sigma_T \int d\bar{\Omega}_* \frac{1+\cos^2\bar{\theta}}{2}\left[\tilde{f}(q_*, \gamma^i_*) - \tilde{f}(q, \gamma^i)\right]$$

(q_*, γ^i_*) ··· photon momentum after scattering

$d\bar{\Omega}_*$ ··· solid angle sustained by γ^i_* in electron rest frame (e.r.f.)

$\bar{\theta}$ ··· scattering angle in e.r.f.



Let $v^i_{(e)}$ be the electron 3-velocity. Then

$$\bar{q}_* = q_*\left[1 - \left(\beta_i + v_{(e)i}\right)\gamma^i_*\right]$$

Because the photon energy is unchanged under Thomson scattering,

$$\bar{q}_* = \bar{q} = q\left[1 - \left(\beta_i + v_{(e)i}\right)\gamma^i\right]$$

Hence

$$\tilde{f}(q_*, \gamma_*^i) = f(q_*) + \delta f(q_*, \gamma_*^i) = f(q) + (q_* - q)\frac{\partial}{\partial q} f + \delta f(q, \gamma_*^i)$$
$$= f(q) + \left(q\frac{\partial}{\partial q} f(q)\right)\left(\beta_i + v_{(e)i}\right)\left(\gamma_*^i - \gamma^i\right) + \delta f(q, \gamma_*^i)$$

and

$$d\bar{\Omega}_* = \left(\frac{q_*}{\bar{q}_*}\right)^2 d\Omega_* = \left(1 + 2(\beta_i + v_{(e)i})\gamma_*^i\right) d\Omega_* = d\Omega_* + O(\epsilon)$$

From these,

$$\frac{1}{q} C[\tilde{f}] = \frac{3}{8\pi} n_e \sigma_T \int d\Omega_* \frac{1 + (\gamma_*^k \gamma_k)^2}{2}$$
$$\times \left[q\frac{\partial f}{\partial q}(\beta_i + v_{(e)i})(\gamma_*^i - \gamma^i) + \delta f(q, \gamma_*^i) - \delta f(q, \gamma^i)\right]$$
$$= n_e \sigma_T \left[\langle \delta f \rangle_\Omega - \delta f - \left(q\frac{\partial f}{\partial q}\right)(\beta_i + v_{(e)i})\gamma^i + \frac{3}{4} Q_{ij} \gamma^i \gamma^j\right]$$

$$\langle \delta f \rangle_\Omega = \frac{1}{4\pi} \int \delta f d\Omega$$
$$Q_{ij} = \frac{1}{4\pi} \int (\gamma_i \gamma_j - \frac{1}{3}\gamma_{ij}) \delta f d\Omega$$

Since $f = f(\hat{q}) = f(aq)$

$$q\frac{\partial f}{\partial q} = \hat{q}\frac{df}{d\hat{q}}$$

$$\begin{aligned}\Rightarrow \quad & \left[\frac{\partial}{\partial \eta} + \gamma^i \frac{\partial}{\partial x^i} - {}^s\Gamma^i{}_{jk}\gamma^j\gamma^k \frac{\partial}{\partial \gamma^i}\right]\delta f \\ & = \left[A_{|i}\gamma^i + \frac{1}{2}(H'_{ij} - \beta_{i|j} - \beta_{j|i})\gamma^i\gamma^j\right]\hat{q}\frac{df}{d\hat{q}} \\ & \quad + a n_e \sigma_T \left[\langle \delta f \rangle_\Omega - \delta f - \left(\hat{q}\frac{df}{d\hat{q}}\right)(\beta_i + v_{(e)i})\gamma^i + \frac{3}{4}Q_{ij}\gamma^i\gamma^j\right]\end{aligned}$$

Define

$$\Theta \equiv \frac{\delta f}{\left(-\hat{q}\dfrac{df}{d\hat{q}}\right)},$$

and rescale the affine parameter λ as

$$\frac{1}{\hat{q}}\frac{d}{d\lambda} \rightarrow \frac{d}{d\lambda} \equiv \left[\frac{\partial}{\partial \eta} + \gamma^i \frac{\partial}{\partial x^i} - {}^s\Gamma^i{}_{jk}\gamma^j\gamma^k \frac{\partial}{\partial \gamma^i}\right]$$

Then

$$\frac{d}{d\lambda}\Theta = -\left[A_{|i}\gamma^i + \frac{1}{2}(H'_{ij} - \beta_{i|j} - \beta_{j|i})\gamma^i\gamma^j\right]$$
$$+an_e\sigma_T\left[\langle\Theta\rangle_\Omega - \Theta - (\beta_i + v_{(e)i})\gamma^i + \frac{3}{4}\Pi_{ij}\gamma^i\gamma^j\right]$$
$$\Pi_{ij} \equiv \langle(\gamma_i\gamma_j - \frac{1}{3}\gamma_{ij})\Theta\rangle_\Omega$$

Equation for Θ is independent of the photon energy q.

If we consider a temperature fluctuation in $f = (e^{q/T} - 1)^{-1}$ by setting $T \to T + \delta T$

$$\delta_T f = f(T + \delta T) - f(T) = \left(-\hat{q}\frac{df}{d\hat{q}}\right)\frac{\delta T}{T}$$

$$\Rightarrow \quad \frac{\delta T}{T} = \frac{\delta_T f}{\left(-\hat{q}\dfrac{df}{d\hat{q}}\right)} \qquad \left(\Leftrightarrow \; \Theta = \frac{\delta f}{\left(-\hat{q}\dfrac{df}{d\hat{q}}\right)}\right).$$

We may regard Θ as the temperature fluctuation.

§ 2. Gauge-invariant formulation

Gauge transformation induced by $\bar{x}^\mu = x^\mu + \xi^\mu, \quad \xi^\mu = (T, L^i)$:

$$p^\mu \;\to\; \bar{p}^\mu = \frac{\partial \bar{x}^\mu}{\partial x^\nu} p^\nu = p^\mu + \xi^\mu{}_{,\nu} p^\nu$$

This gives

$$\begin{aligned}
\hat{q} \;\to\; \bar{\hat{q}} &= (1 + \bar{A}(\bar{x}))\bar{q} = a^2(\bar{\eta})(1 + \bar{A}(\bar{x}))\bar{p}^0 \\
&= a^2(\eta)\left(1 + 2\frac{a'}{a}T\right)\left(1 + A(x) - \frac{a'}{a}T - T'\right)\left(p^0 + T_{,\nu}p^\nu\right) \\
&= \hat{q}\left(1 + \mathcal{H}T + T_{|i}\gamma^i\right) \;; \quad \mathcal{H} \equiv \frac{a'}{a}
\end{aligned}$$

Hence, from $\bar{f} + \overline{\delta f} = f + \delta f$,

$$\overline{\delta f} = \delta f - \hat{q}\frac{df}{d\hat{q}}\left[\mathcal{H}T + T_{|i}\gamma^i\right]$$

$$\Leftrightarrow \quad \bar{\Theta} = \Theta + \mathcal{H}\,T + T_{|i}\gamma^i\,.$$

For scalar-type metric perturbations, in addition to A,

$$\frac{1}{2}H^S_{ij} = \mathcal{R}\gamma_{ij} + H_{T|ij}\,, \quad \beta_i = B_{|i}\,, \quad v_{(\mathrm{x})i} = -U_{(\mathrm{x})|i}\,; \quad \text{`x'=electron, photon, etc..}$$

These transform under a gauge transformation $\xi^\mu = (T, L^{|i})$,

$$\begin{aligned}\bar{A} &= A - T' - \mathcal{H}T\,, \quad \bar{B} = B + T + L' \\ \bar{\mathcal{R}} &= \mathcal{R} - \mathcal{H}T\,, \quad \bar{H}_T = H_T + L\,, \quad \bar{U}_{\mathrm{x}} = U_{(\mathrm{x})} + L'\,.\end{aligned}$$

A convenient choice of gauge-invariant variables

$$\begin{aligned}&\Psi = A - \sigma' - \mathcal{H}\sigma\,; \quad \sigma = H_T' - B\,, \\ &\Phi = \mathcal{R} - \mathcal{H}\sigma\,, \\ &V_{(\mathrm{x})} = U_{(\mathrm{x})} - H_T'\,, \\ &\Theta_s = \Theta + \mathcal{H}\sigma + \sigma_{|i}\gamma^i\end{aligned}$$

These correspond to perturbations variables defined in Newton (logitudinal) slicing for which $\sigma = 0$. In terms of these gauge-invariant variables,

$$\frac{d}{d\lambda}\Theta_s = -(\Phi' + \Psi_{|i}\gamma^i + \frac{1}{2}H^{T\prime}_{ij}\gamma^i\gamma^j) + an_e\sigma_T\left[\langle\Theta_s\rangle_\Omega - \Theta_s + V_{(e)|i}\gamma^i + \frac{3}{4}\Pi_{ij}\gamma^i\gamma^j\right]$$

H^T_{ij} $\cdots$ tensor (GW) perturbation, $\quad \Pi^{ij} = \left\langle\left(\gamma^i\gamma^j - \frac{1}{3}\gamma^{ij}\right)\Theta\right\rangle_\Omega$ is gauge-invariant.

• Qualitative derivation of CMB anisotropy

Before recombination

$$\frac{a n_e \sigma_T}{\mathcal{H}} = \frac{n_e \sigma_T}{H} \approx 6 \times 10^4 \frac{n_e}{n_b} \Omega_b h^2 \left(\frac{T}{1\,\mathrm{eV}}\right) \left[1 + 10\Omega_b h^2 \left(\frac{T}{1\,\mathrm{eV}}\right)\right]^{-1/2} \gg 1$$

$$\Rightarrow \quad \Theta_s = \frac{1}{4}\Delta_{s,r} + V_{b|i}\gamma^i + O\left(\frac{H}{n_e \sigma_T}\right)$$

where $\Delta_{s,r} = \delta\rho/\rho = 4\delta T/T$ on Newton slices. $V_b (= V_{(e)})$ is the baryon velocity potential. Thus, Θ_s at the last scattering surface is approximately

$$\Theta_s = \frac{1}{4}\Delta_{s,r} + V_{b|i}\gamma^i$$

$\Delta_{s,r}$ is related to the radiation density perturbation on the matter comoving slices, Δ_r, as

$$\Delta_{s,r} = \Delta_r - 4\mathcal{H}\,V$$

where V is the matter shear velocity potential.

Neglecting the difference between V and V_b,

$$\Theta_s = \frac{1}{4}\Delta_r - \mathcal{H}\,V + V_{|i}\gamma^i \,.$$

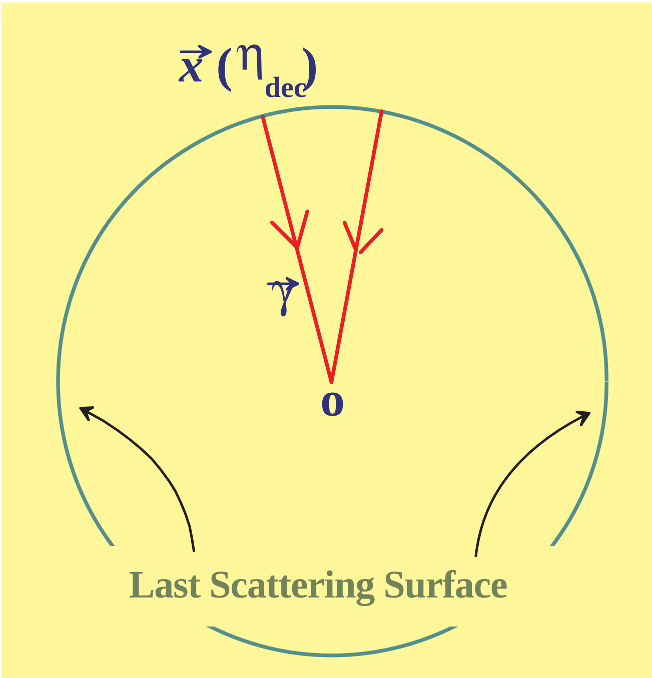
After recombination

For dust-dominated universe, $\Phi = -\Psi$. Then, ignoring H^T_{ij} (tensor) for the moment,

$$\frac{d}{d\lambda}\Theta_s = -\Phi' - \Psi_{|i}\gamma^i = 2\Psi' - \frac{d}{d\lambda}\Psi \quad \Rightarrow \quad \frac{d}{d\lambda}(\Theta_s + \Psi) = 2\Psi'$$

Integrating this along a photon path to the present ($\lambda = \lambda_0$) gives

$$(\Theta_s + \Psi)(\lambda_0) = (\Theta_s + \Psi)(\lambda_d) + \int_{\lambda_d}^{\lambda_0} 2\Phi' d\lambda$$



$$x^\mu(\lambda_d) = (\eta_d, x^i(\eta_d))$$

$$x^\mu(\lambda_0) = (\eta_0, 0)$$

$$\Rightarrow x^i(\eta_d) = -\gamma^i(\eta_0 - \eta_d)$$

Temperature anisotropy measured on the matter comoving frame:

$$\Theta_c = \Theta_s + \mathcal{H}\,V + V_{|i}\gamma^i$$

Therefore, recovering tensor contribution, we obtain

$$\Theta_c(\lambda_0) = \left(-\Psi - \mathcal{H}\,V - V_{|i}\gamma^i\right)_0 + \left(\Psi - \mathcal{H}\,V + V_{|i}\gamma^i + \frac{1}{4}\Delta_r\right)_d + \int_{\lambda_d}^{\lambda_0}\left(2\Psi' - \frac{1}{2}H_{ij}^{T}{}'\gamma^i\gamma^j\right)d\lambda$$

$(-\Psi - \mathcal{H}\,V)_0$ $\cdots$ Monopole: unobservable.

$(-V_{|i}\gamma^i)_0$ $\cdots$ Dipole: our peculiar motion relative to CMB rest frame.

$(\Psi - \mathcal{H}\,V)_d$ $\cdots$ Sachs-Wolfe effect. $\left(V_{|i}\gamma^i\right)_d$ $\cdots$ Doppler effect.

$\left(\frac{1}{4}\Delta_r\right)_d$ $\cdots$ Intrinsic temperature fluctuation at LSS.

$2\int_{\lambda_d}^{\lambda_0}\Psi' d\lambda$ $\cdots$ Integrated SW effect. $\int_{\lambda_d}^{\lambda_0}\left(-\frac{1}{2}H_{ij}^{T}{}'\gamma^i\gamma^j\right)d\lambda$ $\cdots$ Tensor effect.

Large angular scale anisotropy

Large angle ⇔ small wavenumber k:

$$\theta \gtrsim 10^{-2} \ (\gtrsim 1^\circ) \quad \Leftrightarrow \quad \frac{k}{aH} \lesssim 1 \ \cdots \ \text{superhorizon scale at LSS}$$

For adiabatic perturbations, at LSS (assuming dust-dominance),

$$\mathcal{H}V = \frac{2}{3}\Psi\,, \quad \Delta_r \approx \frac{3}{4}\Delta_{tot} \approx -\frac{8}{9}\Psi\frac{k^2}{a^2H^2} \ll \Psi \quad \text{for} \ \left(\frac{k^2}{a^2H^2}\right)_d \ll 1$$

$$\Rightarrow \quad \Theta_{ad} = \frac{1}{3}\Psi_d + \text{ISW}$$

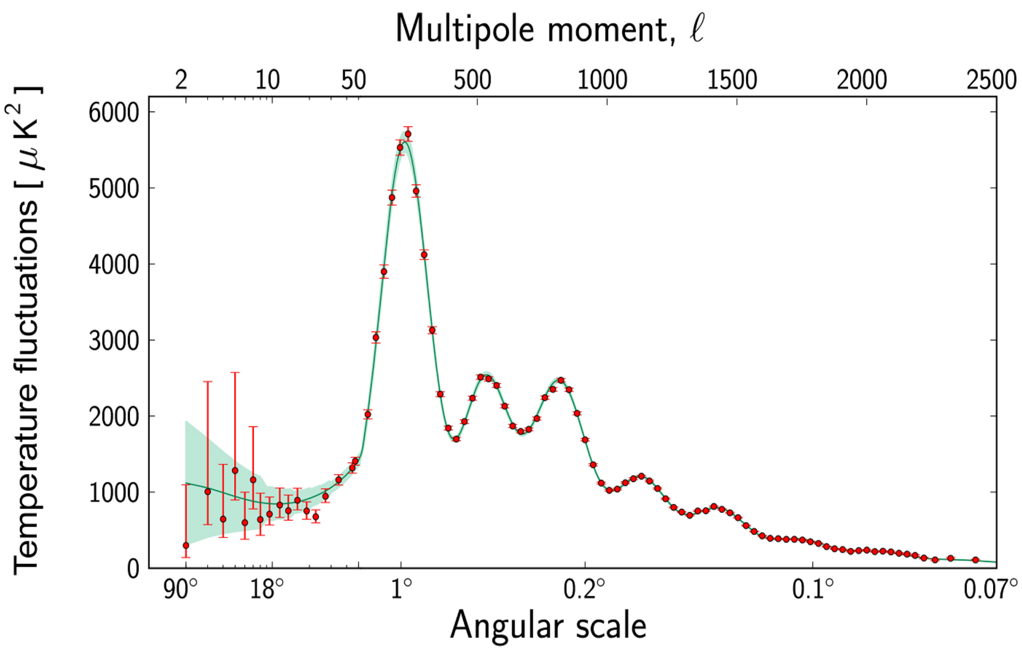
For isocurvature perturbations,

$$S \equiv \left(\frac{\delta\rho}{\rho}\right)_{\text{dust}} - \frac{3}{4}\left(\frac{\delta\rho}{\rho}\right)_{\text{rad}} \ \text{at} \ \eta \to 0\,, \quad \frac{1}{4}\Delta_r \approx -\frac{1}{3}S = \frac{5}{3}\Psi \ \text{at} \ \eta = \eta_d.$$

$$\Rightarrow \quad \Theta_{iso} = 2\Psi_d + \text{ISW}$$

SW part of Θ_{iso} is enhanced by factor 6 relative to Θ_{ad}
⇒ Stringent constraint on isocurvature perturbations

§ 3. Cosmological parameter dependence



Observed temperature anisotropy spectrum by Planck.

⋆ CMB anisotropy depends on

· Nature of primordial perturbation spectrum (spectral index, ad or iso, etc...)

· Physics at decoupling time (baryon density, CDM density, etc...)

· Evolution of the universe after decoupling (cosmological constant, dark energy, etc....)

⋆ Physics at decoupling is determined by the values of ρ_b and ρ_{CDM}.

· Peaks are caused by accoustic oscillations of the baryon-photon fluid:

$$\Psi - \mathcal{H}V \propto F(\rho_b, \rho_m)\cos(c_s k\eta_d) \quad (c_s \ \cdots \ \text{sound velocity}) \tag{1}$$

· Baryon and CDM density in terms of cosmological parameters:

$$(\rho_\mathrm{x})_d = (1+z_d)^3(\rho_\mathrm{x})_0 = (1+z_d)^3(\rho_{cr})_0\Omega_\mathrm{x} \propto \Omega_b h^2$$
$$\mathrm{x} = \mathrm{b(baryon)\ or\ CDM}$$

Thus, physics at decoupling is the same for the same values of $(\Omega_b h^2, \Omega_{CDM}h^2)$.
$\Rightarrow$ degeneracy in CMB anisotropy.

⋆ Evolution after decoupling is affected by ρ_Λ (cosmological constant)

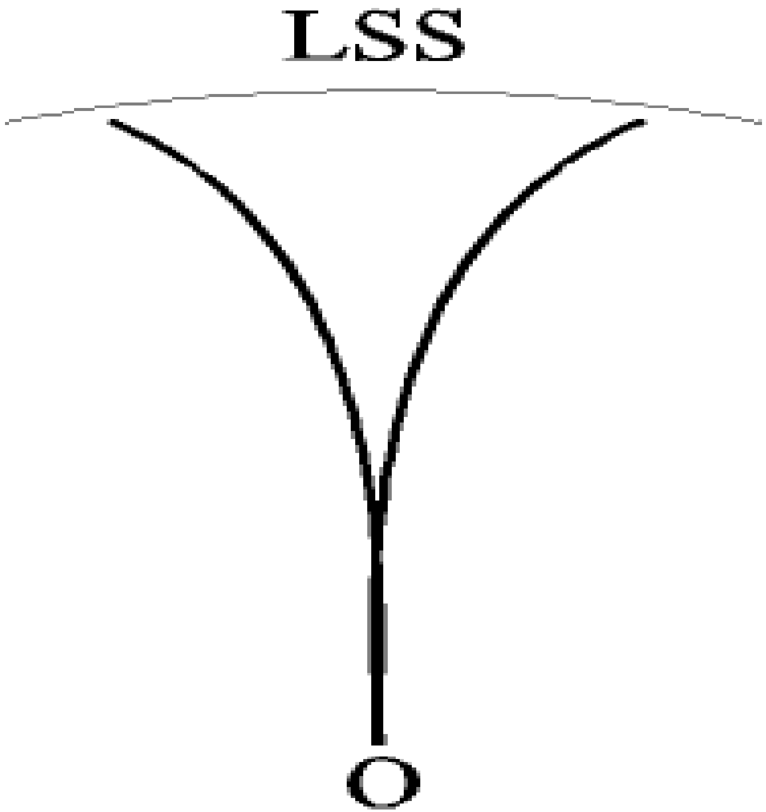
· ρ_Λ is important at $z \lesssim O(1)$: Late time ISW effect.
$\Rightarrow$ Larger $\Omega_\Lambda h^2$ causes larger power on large angular scales (small ℓ).

· ρ_Λ causes increase in the conformal distance to LSS: $\eta_0 - \eta_d$ increases.
$\Rightarrow$ The same physical length on LSS is sustained by a smaller angle.

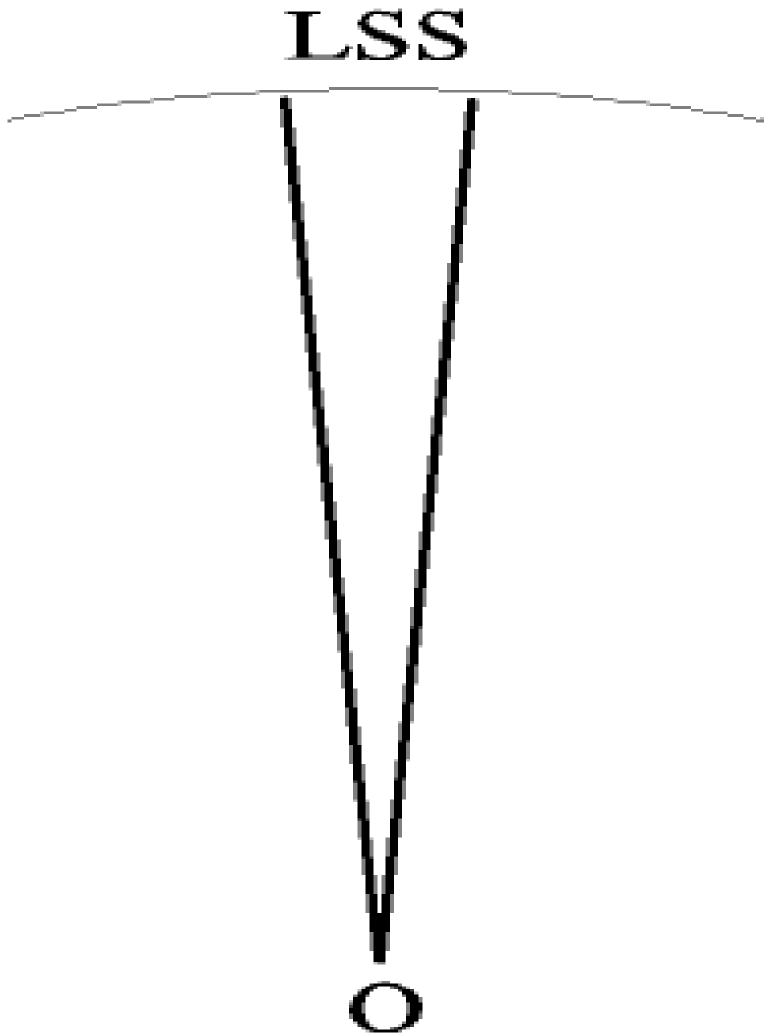
⋆ Dependence on spatial geometry

· For open space, the same physical length on LSS is sustained by a smaller angle.

Increase in $\Omega_\Lambda \approx$ decrease in Ω_{tot} $\Rightarrow$ slight degeneracy



$\Leftrightarrow$



Geometrical degeneracy

$\star$ CMB degeneracy of cosmological parameters

Agular diameter distance to LSS:

$$d_A(z) = \frac{1}{H_0}\frac{\sinh\sqrt{-k_0}R(z)}{(1+z)\sqrt{-k_0}}\,; \quad R(z) = \int_1^{1+z}\frac{dy}{\sqrt{\Omega_m\, y^3 + \Omega_\Lambda - k_0 y^2}}$$

$$\Omega_m = \Omega_b + \Omega_{CDM}\,, \quad k_0 \ \cdots \ \text{curvature parameter} : 1 + k_0 = \Omega_m + \Omega_\Lambda$$

$$\Rightarrow \quad \theta = \frac{r_{phys}}{d_A} \propto \frac{h}{f(\Omega_d, \Omega_{vac})}$$

· 4(+1) cosmological parameters: h, Ω_b, Ω_{CDM}, Ω_Λ, n_s (n_s $\cdots$ spectral index)

· Position of the first peak: $\theta_{\text{peak}\,1} \propto \dfrac{h}{f(\Omega_m, \Omega_\Lambda)}$

· Height of the first peak: height $\propto h_1(\Omega_b h^2, \Omega_{CDM} h^2)$

· Height of the second peak: height $\propto h_2(\Omega_b h^2, \Omega_{CDM} h^2)$

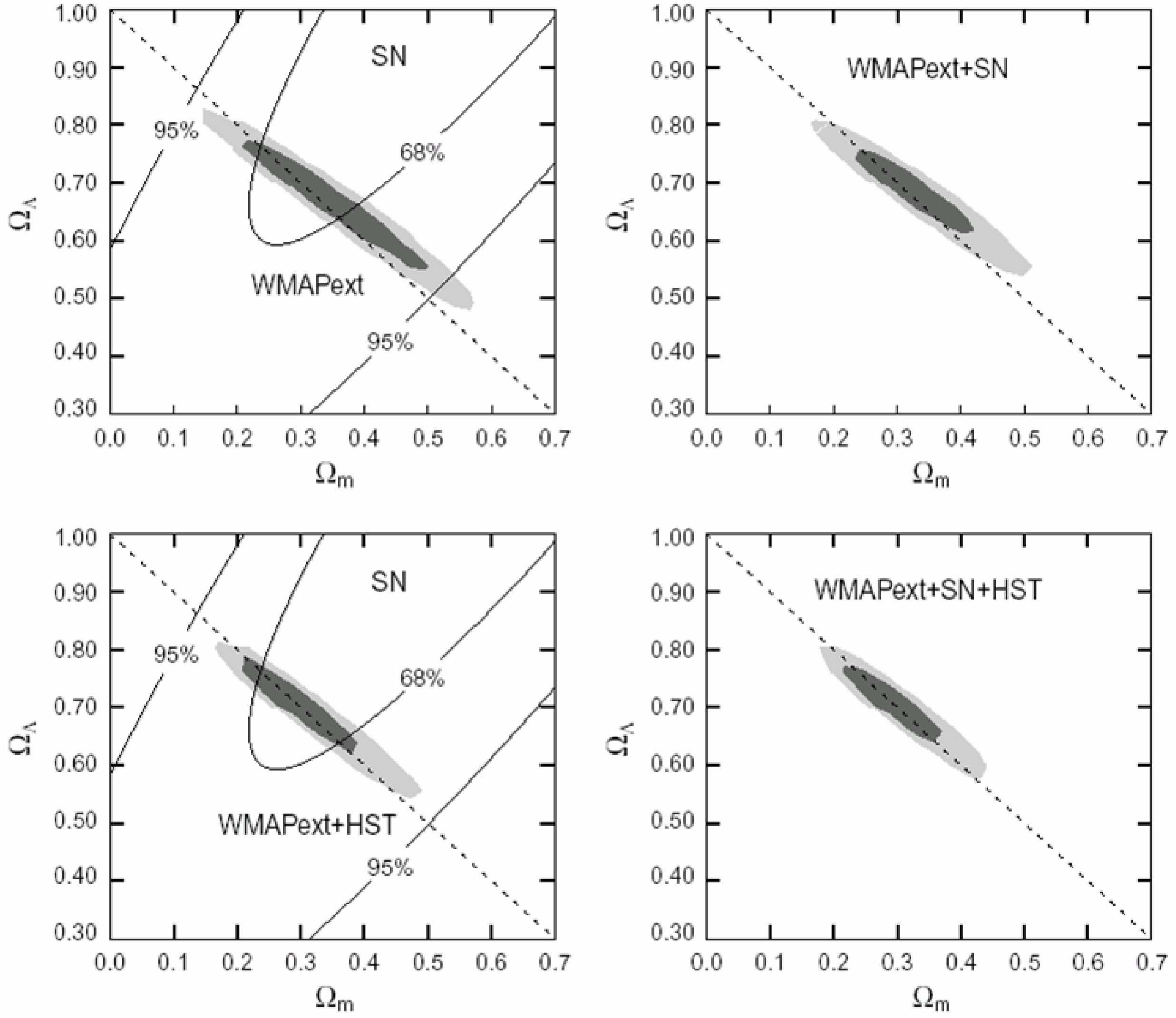
· Global temperature anisotropy spectral shape: determination of n_s.

WMAP/Planck determined $\Omega_b h^2$, $\Omega_{CDM} h^2$ and n_s.

$$\Omega_b h^2 = 0.02205 \pm 0.00028, \quad \Omega_{CDM} h^2 = 0.1199 \pm 0.0027, \quad n_s = 0.9603 \pm 0.0073$$

5 parameters − 4 observational constraints = 1 degree of degeneracy

Degeneracy line projected on $(\Omega_m, \Omega_\Lambda)$ plane by WMAP



Degeneracy line projected on $(\Omega_m, \Omega_\Lambda)$ plane by Planck