Unified equation of state consistent with astrophysical, gravitational, high- and low- energy nuclear physics data

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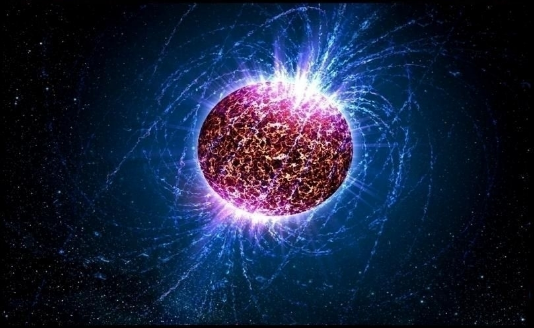
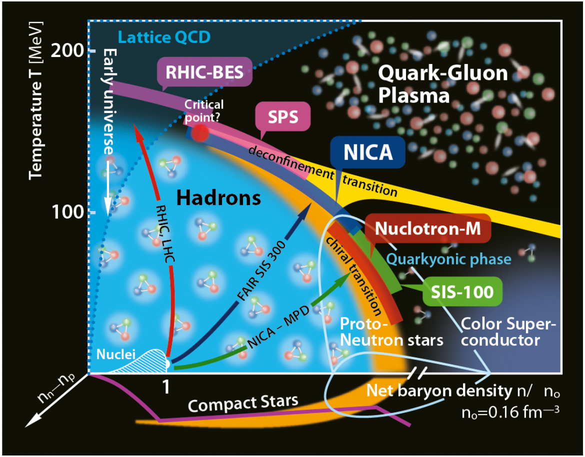


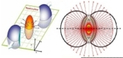
YITP workshop, 26 Sept. 2019

Strongly Interacting Matter Phase Diagram



Constraints on the EoS

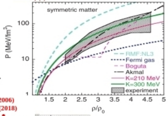
HEP



- **proton flow**

anisotropic expansion is caused by gradient of pressure, which gives an access to EoS

P. Danielewicz et al., Science 298, 1593 (2002)



- **hadron multiplicities**

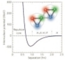
hard core radii of hadrons control the rate of their production in thermal medium: R= 0.3 - 0.5 fm

A. Andronic et al., Nucl. Phys. A 772, 167 (2006)
K. A. Bugaev et al., Nucl. Phys. A 970, 133 (2018)

- **nucleon-nucleon scattering**

hard core radius of nucleons extracted as a parameter of microscopic interaction potential: R = 0.5 fm

M. Naghdi, Phys. Part. Nucl. 5, 924 (2014)



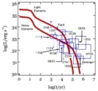
Astro

- **~ 2 M sun**

PSR J1614-2230 : $M = 1.97(4)M_\odot$
P. B. Demorest et al., Nature, 467, 1081 (2010)

PSR J0348-0432 : $M = 2.01(4)M_\odot$
J. Antoniadis et al., Science, 340, 448 (2013)

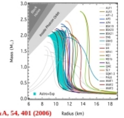
PSR J0740+6620: $M = 2.14^{+0.20}_{-0.18}M_\odot$
H. T. Cromartie et al. Nature, 996, 822 (2019)



- **NSs cooling**
- **observations of pulsars**

⇩

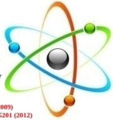
M-R relation



F. Ozel, P. Freire, A&A, 54, 401 (2006)

Nucl. Phys.

- **nuclear matter ground state**



- binding energy per nucleon at saturation density n_0:

$n_0 = 0.16 \pm 0.01\ \mathrm{fm}^{-3}$, $E(n_0)/A = -16.0 \pm 1.0$ MeV

- incompressibility at n_0:

K_0= 200 - 260 MeV

E. Khan, Phys.Rev. C, 80, 011307 (2009)
M. Dutra et al., Phys.Rev. C, 85, 035201 (2012)

- symmetry energy at n_0:

$S(n_0) \equiv J = 30 \pm 4$ MeV

Zhang, Z., Chen, L.-W., Phys. Lett. B, 726, 234 (2013)

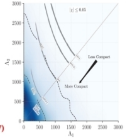
- symmetry energy slope at n_0: $L \equiv 3n_0 \left(\frac{\partial S(n_B)}{\partial n_B}\right)_{n_B=n_0}$ = 20 -115 MeV

Grav. Phys.

First NS+NS merger GW170817

⇩

Love numbers and tidal polarizability are highly sensitive to EoS



LIGO and Virgo collaborations, PRL 119, 161101 (2017)

General Requirements

- causality
- thermodynamic consistency
- multicomponent character (n, p, e, ...)
- electric neutrality
- β-equilibrium
- realistic interaction between the constituents

EoS with hard core repulsion

- Hard core reduces volume available for motion of particles by $V_{excl} = Nb$

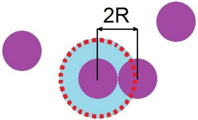
$$V \to V - V_{excl} \;\Rightarrow\; \overbrace{p = nT}^{ideal\ \ gas\ \ EoS} = \frac{NT}{V} \to \frac{NT}{V - V_{excl}} = \frac{NT}{V - Nb} = \overbrace{\frac{nT}{1 - nb}}^{Van\ \ der\ \ Waals\ \ EoS}$$

- Van der Waals EoS ($b = const$) in the Grand Canonical Ensemble

$$\begin{cases} p = p(T,\mu) \\ n = \frac{\partial p}{\partial \mu} \end{cases} \Rightarrow\quad p = T \int_{\vec{k}} \exp\left(\frac{\mu - pb - \sqrt{m^2 + k^2}}{T}\right) = p_{id}(T, \mu - pb)$$

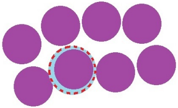
- Excluded volume (per particle) depends on density ($b \neq const$)

Low densities

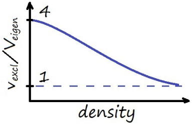


$b \simeq \frac{1}{2} \cdot \frac{4\pi}{3}(2R)^3 = 4v$

High densities



$b \simeq v$



The Induced Surface Tension (IST) EoS

- Quantitites of the Boltzmann ideal gas

$$p = nT, \quad n = \sum_i n_i^{id}, \quad n_i^{id} = \frac{p_i^{id}}{T}$$

- Virial expansion for one particle species

$$\frac{p}{T} = n + a_2 n^2 + a_3 n^3 + a_4 n^3 + \dots$$

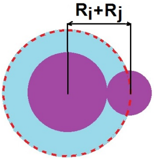
Ideal gas: $a_2 = 0,\ a_3 = 0,\ a_4 = 0,\ \dots$

Hard spheres: $a_2 = 4v,\ a_3 = 10v^2,\ a_4 = 18.365v^3,\ \dots$

R.K.Pathria, Statistical Mechanics, Pergamon Press, Oxford, 1972

- Excluded volume at low density ($v_i = \frac{4\pi}{3} R_i^3$ and $s_i = 4\pi R_i^2$)

$$a_2^{ij} = \frac{1}{2} \cdot \frac{4\pi}{3} (R_i + R_j)^3 = \frac{1}{2} \cdot (v_i + s_i R_j + R_i s_j + v_j)$$



- Virial expansion for many particle species

$$\frac{p}{T} = \overbrace{\sum_i n_i^{id}}^{\simeq n} - \overbrace{\sum_{i,j} a_2^{ij} n_i^{id} n_j^{id}}^{\simeq a_2 n^2} + \dots = \sum_i \frac{p_i^{id}}{T} \Big(1 - \overbrace{v_i \sum_j \frac{p_j^{id}}{T}}^{bulk\ \ term} - \overbrace{s_i \sum_j \frac{p_j^{id}}{T} R_j}^{surface\ \ term} \Big) + \dots$$

EoS with induced surface tension (IST)

$$p = \sum_i p_i^{id}\Big(1 - \overbrace{\underbrace{\frac{v_i}{T}\sum_j p_j^{id}}_{p+\mathcal{O}(n^2)}}^{bulk\ \ term} - \overbrace{\underbrace{\frac{s_i}{T}\sum_j p_j^{id} R_j}_{\Sigma+\mathcal{O}(n^2)}}^{surface\ \ term}\Big) + \dots$$

- Σ is conjugated to s_i – **induced surface tension (IST)**

$$\begin{cases} p = \sum_i p_i^{id}\left(1 - \frac{p v_i}{T} - \frac{\Sigma s_i}{T}\right) \\ \Sigma = \sum_i p_i^{id}\left(1 - \frac{p v_i}{T} - \frac{\alpha \Sigma s_i}{T}\right) R_i \end{cases}$$

α accounts for not uniqueness of extrapolation to high densities

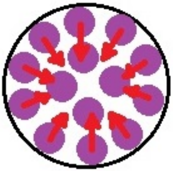
- High density extrapolation (gives exponentials)

$$\begin{cases} p = \sum_i p_i^{id} \exp\left(-\frac{p v_i + \Sigma s_i}{T}\right) \\ \Sigma = \sum_i p_i^{id} \exp\left(-\frac{p v_i + \alpha \Sigma s_i}{T}\right) R_i \end{cases} \rightarrow \begin{cases} p = \sum_i p_i^{id}(\mu_i - p v_i - \Sigma s_i) \\ \Sigma = \sum_i p_i^{id}(\mu_i - p v_i - \alpha \Sigma s_i) R_i \end{cases}$$

VS, A. Ivanytskyi, K. Bugaev, I. Mishustin, NPA 924, 24 (2014)

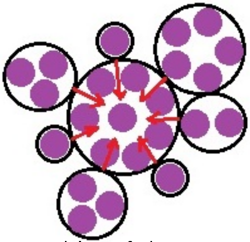
Physical Origin of the Induced Surface Tension

Vacuum



attraction of constituents
⇒ **eigen surface tension**

Medium



repulsion of clusters
⇒ **induced surface tension**

- Hard core repulsion only in part is accounted by eigen volume
- The rest corresponds to surface tension and curvature tension
  Curvature tension can be accounted explicitly or implicitly
- Physical clusters tend to have spherical (in average) shape

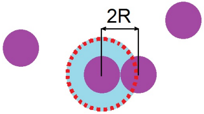
Determination of α

- One component EoS with IST and $\alpha > 1$

$$\begin{cases} p = p^{id}\exp\left(-\frac{pv+\Sigma s}{T}\right) \\ \Sigma = p^{id}\exp\left(-\frac{pv+\alpha\Sigma s}{T}\right)R \end{cases} \Rightarrow \begin{cases} p = p^{id}\exp\left(-\frac{pb}{T}\right)R \\ \Sigma = pR\exp\left(\frac{(1-\alpha)\Sigma s}{T}\right) \end{cases}$$

VVS, A. I. Ivanytskyi, K. A. Bugaev, I. N. Mishustin, Nucl. Phys. A, 924, 24 (2014)

Excluded volume: $\frac{b}{v} = 1 + 3e^{\frac{(1-\alpha)\Sigma s}{T}} \to \begin{cases} 4, & \Sigma \to 0 \\ 1, & \Sigma \to \infty \end{cases}$



$\alpha > 1$ **switches different regimes of excluded volume**

- Virial expansion of one component EoS with IST

Second virial coefficient: $a_2 = 4V$ is reproduced always

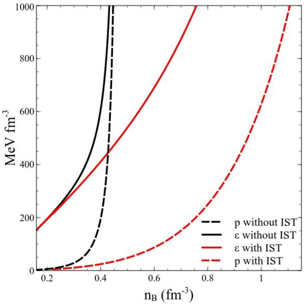
Third virial coefficient: $a_3 = 10V^2 \Rightarrow \alpha = \frac{4}{3}$
a_4 is not reproduce

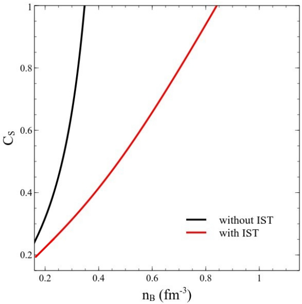
Fourth virial coefficient: $a_4 \simeq 18.365V^3 \Rightarrow \alpha \simeq 1.245$
a_3 – reproduced with 16% accuracy

$\alpha > 1.245$ **reproduces two (3rd and 4th) virial coefficients**

Effect of the IST

Thermodynamic parameters with and without IST





Nuclear Matter Properties Near the (3)CEP

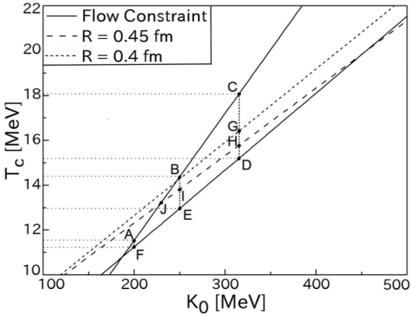


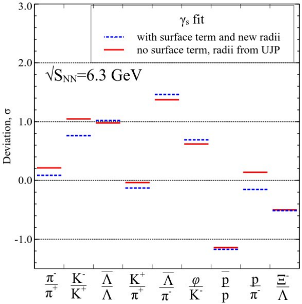
FIG. 3. Values of incompressibility constant K_0 and critical temperature T_C, which obey the proton flow constraint are located between the lines ABC and FED. The lines ABC and FED are, respectively, generated by the lower and upper branches of the proton flow constraint. The vertical lines AF, BE, and CD correspond to K_0 values 200 MeV, 250 MeV, and 315 MeV, respectively.

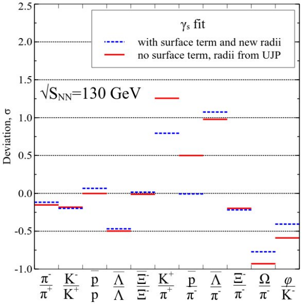
VVS, et al., Nucl. Phys. A, 924, 24 (2014)

A. Ivanytskyi et al., PRC 97, 064905 (2018)

Hadron Resonance Gas Model

- Hadrons with masses $\leq$ 2.5 GeV (widths, strong decays, zero strangeness)
- 111 independent particle ratios measured at 14 energies (from 2.7 GeV to 200 GeV)
- 14×4 local parameters ($T, \mu_B, \mu_{I3}, \gamma_s$) + 5 global parameters (hard core radii)





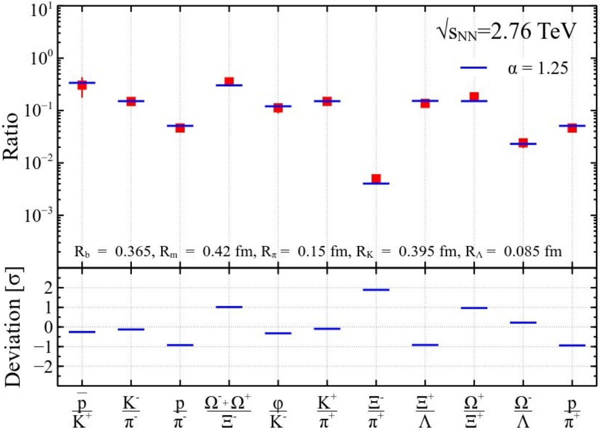
$R_b = 0.365\ fm,\ R_m = 0.42\ fm,\ R_\pi = 0.15\ fm,\ R_K = 0.395\ fm,\ R_\Lambda = 0.085\ fm$

$Overall\ \chi^2/dof \simeq 1.038$

K.A. Bugaev, et al., NPA 970, p. 133-155, (2018)
VVS, et al., Eur. Phys. J. A 54, No 6, p. 16 (2018)

Hadron Resonance Gas at ALICE Energies

- 11 independent particle yields, 6 parameters (temperature + 5 hard core radii)
- Overal $\chi^2/dof \simeq 0.89$
- Freeze out temperature $T_{FO} = 148 \pm 7$ MeV



K.A. Bugaev, et al., NPA 970, p. 133-155, (2018)

The Induced Surface Tension (IST) EoS

$$\begin{cases} p = \sum_i^{all\ particles} \left[p_{id}(T, \mu_i - pV_i - \Sigma S_i + U_{at} \pm U_{sym}) + p_{id}(\mu_e) - p_{at} + p_{sym} \right] \\ \Sigma = \sum_i^{all\ particles} p_{id}(T, \mu_i - pV_i - \alpha \Sigma S_i + U_0) R_i \end{cases}$$

p_{id} – pressure of the ideal gas for quantum statistics
Σ – induced surface tension
U_0, α – model parameters

$$Thermodynamic\ consistency\ of\ the\ model: \quad \frac{\partial p_{int}}{\partial n_{id}} = n_{id} \frac{\partial U(n_{id})}{\partial n_{id}}$$

$$Parametrization\ of\ the\ mean\ field\ potential: U_{at} = -C_d^2 n_{id}^{\kappa}$$

VVS, et al., Nucl. Phys. A, 924, 24 (2014)
A. Ivanytskyi et al., PRC 97, 064905 (2018)

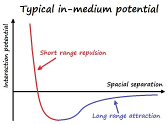
Mean field interaction for nuclear matter

- Thermodynamic consistency provides identity $\frac{\partial p}{\partial \mu} = n$

$$p(\mu) = p^{id}(\mu - U(x)) + p_{int}(x), \quad p_{int}(x) = \int_0^x dx' x' \frac{\partial U(x')}{\partial x'}$$

x – any quantity (density, asymmetry parameter, ...)

K. A. Bugaev and M. I. Gorenstein, Z. Phys. C 43, 261 (1989)
D.H. Rischke, et al. Z. Phys. C 51, 485 (1991)



Long range attraction (negative contribution to pressure)

$$p_{at}(x) = -\frac{\kappa}{\kappa+1} C_d^2 x^{1+\kappa}, \quad x = n_n^{id} + n_p^{id}$$

$\kappa < 1,\ C_d^2$ – fitted to flow constraint and properties of ground state

A. Ivanytskyi et al., PRC 97, 064905 (2018)

- Repulsion due to symmetry energy (positive contribution to pressure)

binding energy of stable nuclei: $E_{sym} = a_{sym}\frac{(N-Z)^2}{A},\ a_{sym} = 30 \pm 4\ MeV$

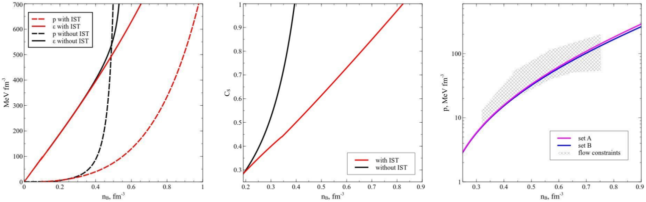
$$p_{sym}(x) = \frac{A^{sym} x^2}{[1+(B^{sym}x)^2]^2}, \quad x = n_n^{id} - n_p^{id}$$

$A^{sym},\ B^{sym}$ – fitted to a_{sym} and slope of symmetry energy at ground state

EoS for NS interiors

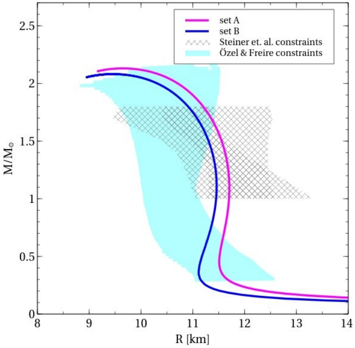
$$\begin{cases} p = \sum_{i}^{all\ \ particles} [p_{id}(T, \mu_i - pV_i - \Sigma S_i + U_{at} \pm U_{sym}) + p_{id}(\mu_e) - p_{at} + p_{sym}] \\ \Sigma = \sum_{i}^{all\ \ particles} p_{id}(T, \mu_i - pV_i - \alpha\Sigma S_i + U_0) R_i \end{cases}$$

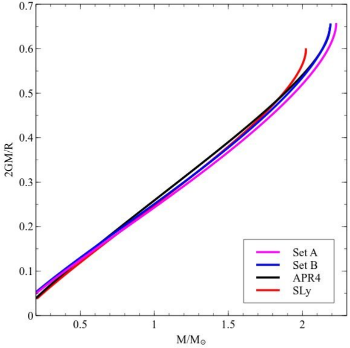
- **Degrees of freedom:** neutrons, protons, electrons
- **Quantum statistics** is accounted by construction of p_{id} and n_{id}
- **Realistic interaction**: HC repulsion, MF attraction, symmetry energy repulsion
- **Virial coefficients** of classic hard spheres are reproduced
- **Causal behaviour** up to densities where QGP is expected and **Flow constraint**



VVS, I. Lopes, A. Ivanytskyi, APJ, 871, 157 (2019)
VVS, I. Lopes, ApJ, 850, 75 (2017)

M-R relation and compactness of NS





Set	R_{nucl} (fm)	α –	κ –	B^{sym} (fm^3)	A^{sym} $(MeV \cdot fm^3)$	C_d^2 $(MeV \cdot fm^{3\kappa})$	U_0 (MeV)	K_0 (MeV)	J (MeV)	L (MeV)	M_{max} $(M_\odot)$
A (magenta curve)	0.477	1.245	0.254	14.0	111.87	145.90	157.35	202.36	30.0	96.05	2.15
B (magenta curve)	0.463	1.245	0.25	16.0	138.30	146.30	162.87	201.02	30.0	93.19	2.08

The NS core was modelled within the IST EoS, while its crust was described via the polytropic EoS with $\gamma = \frac{4}{3}$

VVS, I. Lopes, A. Ivanytskyi, APJ, 871, 157 (2019)
VVS, G. Panotopoulos, I. Lopes, submitted to PRD (2019)

Conclusions

IST approach was successfully applied to the description of

- compressible nuclear matter properties near the (3)CEP

  VVS, et al., Nucl. Phys. A, 924, 24 (2014)
- heavy-ion collision data between $\sqrt{S_{NN}} = 2.7$ GeV – 2.76 TeV

  K.A. Bugaev, et al., NPA 970, p. 133-155, (2018)
- NS properties at T=0

  VVS, I. Lopes, A. Ivanytskyi, APJ, 871, 157 (2019)

Advantages of the IST EoS

- can be easily generalized to any number of particle species
- can be formulated to finite temperatures $\Rightarrow$ proto-neutron stars
- provide a unified description of hadron and nuclear matter

Future prospects

- will be available soon on CompOSE
- add hyperons and heavy leptons
- formulate a hybrid star model with QGP core

Dark Matter Admixed NS

DM admixed NSs

3 NSs with mass above $2M_\odot$

- PSR J1614-2230: $M = 1.97^{+0.04}_{-0.04} M_\odot$ (Demorest et al.'10)
- PSR J0348-0432: $M = 2.01^{+0.04}_{-0.04} M_\odot$ (Antoniadis et al.'13)
- PSR J0740+6620: $M = 2.14^{+0.20}_{-0.18} M_\odot$ (Cromartie et al.'19)

Dark matter EoS

- **Asymmetric dark matter**
  relativistic Fermi gas of noninteracting particles with the spin 1/2

A. Nelson, S. Reddy, D. Zhou, arXiv:1803.032668(2019)

Baryon matter EoS

- **EoS with induced surface tension (IST EoS)**
  consistent with:
  nuclear matter ground state properties,
  proton flow data,
  heavy-ion collisions data,
  astrophysical observations,
  tidal deformability constraint from the NS-NS merger (GW170817)

VS, I. Lopes, A. Ivanytskyi, ApJ, 871, 157 (2019)

VS, A. Ivanytskyi, K. Bugaev, et al., Nucl. Phys. A, 924, 24 (2014)

TOV equations

2 TOV equations:

$$\frac{dp_B}{dr} = -\frac{(\epsilon_B + p_B)(M + 4\pi r^3 p)}{r^2\,(1 - 2M/r)}$$

$$\frac{dp_D}{dr} = -\frac{(\epsilon_D + p_D)(M + 4\pi r^3 p)}{r^2\,(1 - 2M/r)}$$

BM and DM are coupled only through gravity, and their energy-momentum tensors are conserved separately

total pressure $p(r) = p_B(r) + p_D(r)$

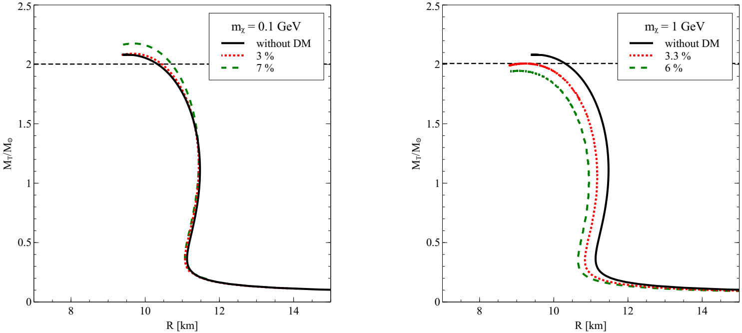
gravitational mass $M(r) = M_B(r) + M_D(r)$, where $M_j(r) = 4\pi \int_0^r \epsilon_j(r')r'^2 dr'$ (j=B,D)

Fraction of DM inside the star:

$$f_\chi = \frac{M_D(R_D)}{M_T}$$

$M_T = M_B(R_B) + M_D(R_D)$ - total gravitational mass

Mass-Radius diagram of the DM admixed NSs

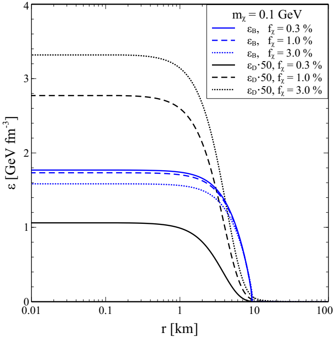


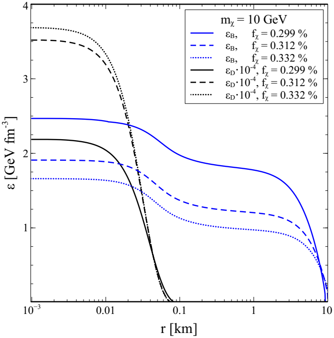
$M_{max} > 2\ M_\odot$ for any f_χ

for $f_\chi = 3.3$ % M_{max} equals to 2 $M_\odot$ further increase of the DM fraction leads to $M_{max} < 2\ M_\odot$

O. Ivanytskyi, VS, I. Lopes, submitted to PRL (2019)

Internal structure of the stars

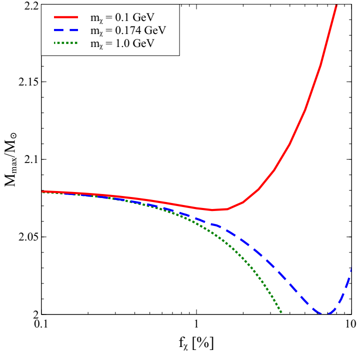




$R_D = 9.4$ km for $f_\chi = 0.3\%$
$R_D = 21.2$ km for $f_\chi = 1.0$ %
$R_D = 135.2$ km for $f_\chi = 3.0$ %

Large values of R_D relate to the existence of dilute and extended halos of DM around a baryon core of NS

Maximal mass of NS as a function of the DM fraction

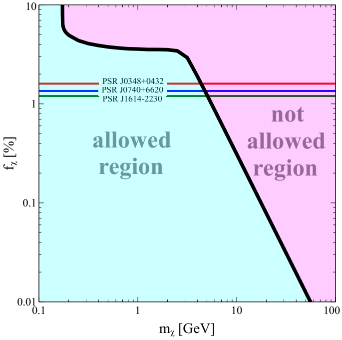


for $m_\chi = 0.174$ GeV M_{max} is 2 $M_\odot$

DM particles with $m_\chi \leq 0.174$ GeV are consistent with the 2 $M_\odot$ constraint for any f_χ

For heavier DM particles the NS mass can reach 2 $M_\odot$ only if f_χ is limited from above

Constraint on the mass of DM particles



Pulsar	distance to the GC	f_χ^*
PSR J0348+0432	9.9 kpc	1.6 ± 0.4 %
PSR J0740+6620	8.6 kpc	1.35 ± 0.35 %
PSR J1614-2230	7.0 kpc	1.2 ± 0.3 %

Navarro-Frenk-White distribution for DM:

$$\rho_\chi(d) = \rho_c \cdot \frac{d_c}{d} \cdot \left(1 + \frac{d}{d_c}\right)^{-2} \quad (1)$$

$\rho_c = 5.22 \pm 0.46\, 10^7\, M_\odot\, \mathrm{kpc}^{-3}$ **and** $d_c = 8.1 \pm 0.7$ kpc

H.-N. Lin, X. Li, arXiv:1906.08419 (2019)

BM distribution in a stellar disc:

$$\rho_B(d) = \rho_{dc} e^{-\frac{d}{d_{dc}}} \quad (2)$$

$\rho_{dc} = 15.0\, M_\odot\, \mathrm{pc}^{-3}$ **and** $d_{dc} = 3.0$ **kpc**

Y. Sofue, Publ. Astr. Soc. Jap., 65, 118 (2013)

Assuming that the DM fraction inside the NS is not lower than the one in the surrounding medium

Conclusions

- Using the observational fact of existence of the three heaviest known NSs (i.e., PSR J0348+0432, PSR J0740+6620, PSR J1614-2230) with the masses exceeding the two solar ones, we present a novel upper constraint on the mass of DM particles.
- We demonstrated that DM lighter than 0.2 GeV can create an extended halo around the NS leading not to decrease but to increase of the NS total (gravitational) mass.
- By using recent results on the distribution of DM and BM in Milky Way, we argue that particles of ADM can not be more massive than 5 GeV.
- We expect to have more NSs observations and measurements of their masses with higher precision from the following telescopes:

  radio telescopes
  - the Karoo Array Telescope (MeerKAT)
  - the Square Kilometer Array (SKA)
  - the Next Generation Very Large Array (ngVLA)

  space telescopes
  - the Neutron Star Interior Composition Explorer Mission (NICER)
  - the Advanced Telescope for High Energy Astrophysics (ATHENA)
  - the enhanced X-ray Timing and Polarimetry mission (eXT)
  - the Spectroscopic Time-Resolving Observatory for Broadband Energy X-rays (STROBE-X)


thank you for your attention!