Radius, Tidal Deformability and EoS Bounds with Different EoS Parameterizations

Tianqi Zhao, James Lattimer

Stony Brook University

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NS Core EoS

What people **hope to do**:

$$\textbf{Nuclear Theory} + \text{Experiment} \rightarrow \text{NS EoS}$$
$$\text{NS EoS} + \text{TOV Eqs} \rightarrow \text{NS properties}$$

NS properties can be related to observable,

Mass	− − − − − − − − −	Shapiro Delay + ⋯
Radius	− − − − − − − − −	X-ray binary + ⋯
Moment of inertia	− − − − − − − − −	Binary procession + ⋯
Binding energy	− − − − − − − − −	CC SNe neutrinos + ⋯
Tidal deformability	− − − − − − − − −	GW waveform + ⋯

NS Core EoS

What people **could do**:

$$\textbf{Effective Theory} + \text{Experiment} \rightarrow \text{NS EoS}$$
$$\text{NS EoS} + \text{TOV Eqs} \rightarrow \text{NS properties}$$

NS properties can be related to observable,

$$\begin{aligned} \text{Mass} &\quad - - - - - - - - - \quad \text{Shapiro Delay} + \cdots \\ \text{Radius} &\quad - - - - - - - - - \quad \text{X-ray binary} + \cdots \\ \text{Moment of inertia} &\quad - - - - - - - - - \quad \text{Binary procession} + \cdots \\ \text{Binding energy} &\quad - - - - - - - - - \quad \text{CC SNe neutrinos} + \cdots \\ \text{Tidal deformability} &\quad - - - - - - - - - \quad \text{GW waveform} + \cdots \end{aligned}$$

NS Core EoS

What **I am doing**:

$$\textbf{EOS Parameterizations} + \text{Experiment} \rightarrow \text{NS EoS}$$
$$\text{NS EoS} + \text{TOV Eqs} \rightarrow \text{NS properties}$$

NS properties can be related to observable,

Mass	– – – – – – – – –	Shapiro Delay + ⋯
Radius	– – – – – – – – –	X-ray binary + ⋯
Moment of inertia	– – – – – – – – –	Binary procession + ⋯
Binding energy	– – – – – – – – –	CC SNe neutrinos + ⋯
Tidal deformability	– – – – – – – – –	GW waveform + ⋯

Parameterizations of NS Core EoS

Pros:

- Independent of the effective theory.
- Allow wider range of EoS(more variability)
- Fewer parameters by removing unnecessary parameters.
- Great for prior in analysing neutron star observations.

Cons:

- Dependent on parameterizations.
  Solution: Use as many parameterizations as possible.
- Cannot relate to experiment directly.
  Solution: Using only bulk nuclear matter constrains come from experiment and theory.

Example

Nuclear Theory

Strong correlations of neutron star radii with the slopes of nuclear matter incompressibility and symmetry energy at saturation

N. Alam, B. K. Agrawal, M. Fortin, H. Pais, C. Providência, Ad. R. Raduta, A. Sulaksono

(Submitted on 20 Oct 2016)

We examine the correlations of neutron star radii with the nuclear matter incompressibility, symmetry energy, and their slopes, which are the key parameters of the equation of state (EoS) of asymmetric nuclear matter. The neutron star radii and the EoS parameters are evaluated using a representative set of 24 Skyrme-type effective forces and 18 relativistic mean field models, and two microscopic calculations, all describing $2M_{\odot}$ neutron stars. Unified EoSs for the inner-crust-core region have been built for all the phenomenological models, both relativistic and non-relativistic. Our investigation shows the existence of a strong correlation of the neutron star radii with the linear combination of the slopes of the nuclear matter incompressibility and the symmetry energy coefficients at the saturation density. Such correlations are found to be almost independent of the neutron star mass in the range $0.6\text{-}1.8M_{\odot}$. This correlation can be linked to the empirical

Astrophysics > High Energy Astrophysical Phenomena

Relativistic parameterizations of neutron matter and implications for neutron stars

Nadine Hornick, Laura Tolos, Andreas Zacchi, Jan-Erik Christian, Jürgen Schaffner-Bielich

(Submitted on 21 Aug 2018 (v1), last revised 14 Dec 2018 (this version, v2))

We construct parameter sets of the relativistic mean-field model fitted to the recent constraints on the asymmetry energy J and the slope parameter L for pure neutron matter. We find cases of unphysical behaviour, i.e.\ the appearance of negative pressures, for stiff parameter sets with low values of the effective mass m^*/m. In some cases the equation of state of pure neutron matter turns out to be outside the allowed band given by chiral effective field theory. The mass-radius relations of neutron stars for all acceptable parameter sets shows a maximum mass in excess of $2M_{\odot}$ being compatible with pulsar mass measurements. Given the constraints on the model in the low-density regime coming from chiral effective theory, we find that the radius of a $1.4M_{\odot}$ neutron star is nearly independent on the value of L. This is in contrast to some previous claims for a strong connection of the slope parameter with the radius of a neutron star. In fact, the mass-radius relation turns out to depend only on the isoscalar parameters of symmetric matter. The constraints of GW170817 on the tidal deformability and on the radius are also discussed.

Parameterizations of NS Core EoS

- Pure neutron matter expansion (*PNM-sat*): Express energy per baryon in series of $\frac{n-n_s}{3n_s}$.
- Pure neutron matter expansion (*PNM-tews*): Express energy per baryon in series of $(\frac{n}{n_s})^{\frac{1}{3}}$. (Tews 2016)
- Piecewise polytropic EoS in three fixed segments (*PP3*): Use three adiabatic index $\Gamma_{1,2,3} = \frac{d\log p}{d\log n}$, in three density segments. (Read 2008)
- Piecewise polytropic EoS in three unfixed segments (*PP3+1*): Add one parameter for *PP3* to make boundary of three segments flexible.
- Spectral decomposition (*Spectral4*): Express $\log(\Gamma)$ in series of log(pressure). (Lindblom 2010)
- Quarkyonic EoS (*Quarkyonic*): Add smooth transition from hadronic EoS to free quark gas for high density. Can be stiff at outer core and soft ($c_s^2 \to \frac{1}{3}$) at inner core. (McLerran 2018, Capano 2019)

Constrains on EoS

Universal constrain:

- Stability: Pressure is a non decreasing function of energy density.
- Causality: Sound speed should be less than light speed.
- Maximum mass of NS should reach at least $\approx 2\ \mathrm{M}_\odot$.
- Unitary gas constrain(Tews et al. 2016) is $E(u) > E_{UG}(u)$,

$$E_{UG} = \frac{3}{5}\xi_0 E_F \tag{1}$$

  $\xi_0 = 0.37$ is mesured in dilute neutron gas. E_F is Fermi energy.

Optional constrains:

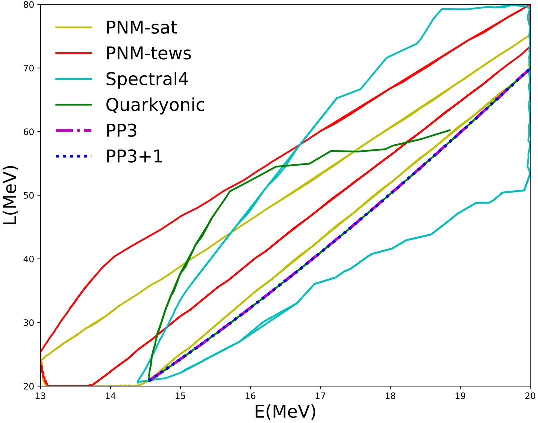
- PNM pressure at $n_1 = 1.85 n_s$. Neutron matter calculations (Drischler et al. 2016) such that $8.4\ \mathrm{MeV\ fm}^{-3} < p_1 < 20\ \mathrm{MeV\ fm}^{-3}$.
- Tighter maximum mass constrain, e.g. $2.14\ \mathrm{M}_\odot \leq M_{max} \leq 2.17\ \mathrm{M}_\odot$.
- Binary tidal deformability bound of given chirp mass, e.g. $\tilde{\Lambda}(M_{ch} = 1.186\ \mathrm{M}_\odot) < 720$

Constrains on EoS

Pure neutron matter(PNM) can be expanded around saturation as,

$$E(u) = E + \frac{L}{3}(u-1) + \frac{K}{18}(u-1)^2 + \frac{Q}{162}(u-1)^3 + \ldots \quad (2)$$

where $u = \frac{n}{n_s}$. 14 MeV $< E <$ 20 MeV and 20 MeV $< L <$ 80 MeV.



Application of EoS Parameterizations

- **Prior of Λ_1, Λ_2 in GW waveform analysis.**
- Bounds of M-R diagram
- Bounds of $R_{1.4}$-$\Lambda_{1.4}$-$\tilde{\Lambda}_{GW170817}$
- Bounds of I-love relation
- Bounds on EoS p(ε)

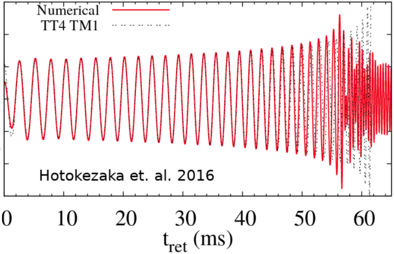
Tidal deformability in binary merger GW waveform

- Oscillating Quadruple moments of neutron star due to excitation of periodic tidal fields contributes to phase shift in GW form.
- Quadruple oscillating contribute to GW radiation reaction,

$$\dot{E}(\omega) = -\frac{1}{5} < \dddot{Q}^T_{ij} \dddot{Q}^T_{ij} >= -\frac{32}{5} M^{4/3}\mu^2\omega^{10/3}[1+g(\omega)] \qquad (3)$$

- By evaluating stable orbit, contribution of quadruple oscillating to total energy of the binary can be calculated,

$$E(\omega) = -M^{1/3}\omega^{-2/3}[1+f(\omega)] \qquad (4)$$



- Using formula $\frac{d^2\Phi}{d\omega^2} = 2(\frac{dE}{d\omega})/\dot{E}$, tidal phase correction can be derived,

$$\delta\Phi = -\frac{9}{16}\frac{\omega^{5/3}}{\mu M^{7/3}}[(\frac{12m_2+m_1}{m_1}\Lambda_1 + \frac{12m_1+m_2}{m_2}\Lambda_2] \quad (5)$$

Flanagan+ 2008

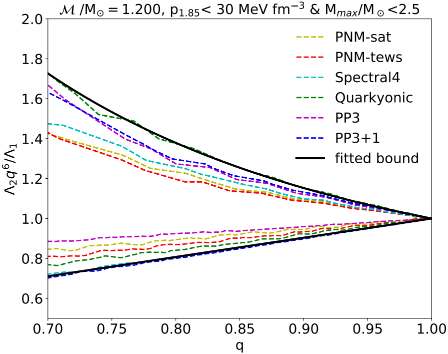
Prior of $\frac{\Lambda_2}{\Lambda_1}(\mathcal{M}, q)$

$$\Lambda \propto (\frac{M}{R})^6 \qquad (6)$$

Since two neutron star should have the same EoS, thus the same $\Lambda(M)$, we expect $\frac{\Lambda_2}{\Lambda_1} \approx (\frac{1}{q})^6$, if constant radius.

$$q^{n_-} \geq \Lambda_1/\Lambda_2 \geq q^{n_{0+}+qn_{1+}}, \qquad (7)$$

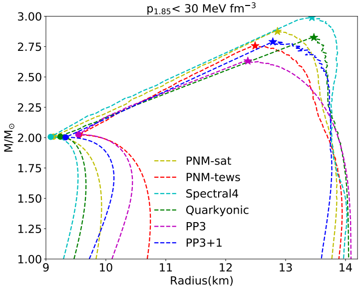
where n_-, n_{0+} and n_{1+} will vary with $\mathcal{M}$, tabulated in arxiv.1808.02858

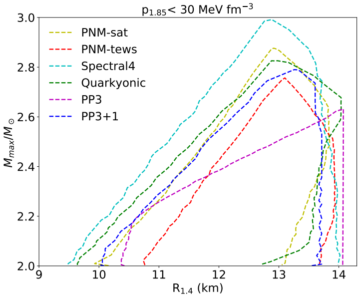


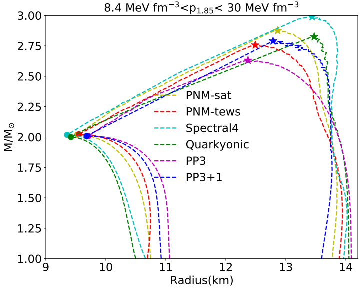
Application of EoS Parameterizations

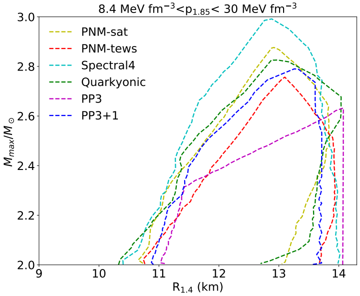
- Prior of Λ_1, Λ_2 in GW waveform analysis.
- **Bounds of M-R diagram**
- Bounds of $R_{1.4}$-$\Lambda_{1.4}$-$\tilde{\Lambda}_{GW170817}$ relations
- Bounds of I-love relation
- Bounds on EoS p(ε)

Bounds on M-R with additional $p_{1.85}$ constrain

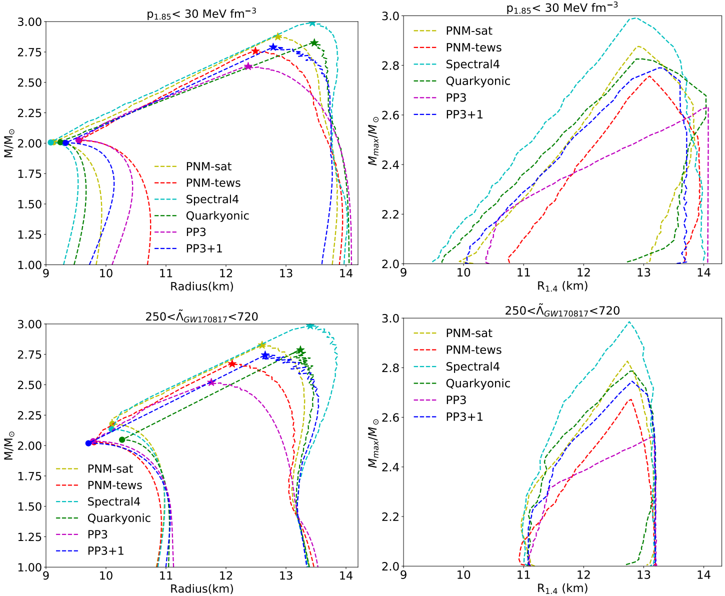








Bounds on M-R with additional $\Lambda_{GW170817}$ constrain



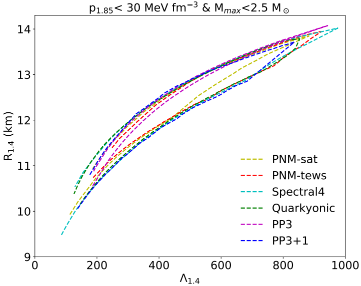
Application of EoS Parameterizations

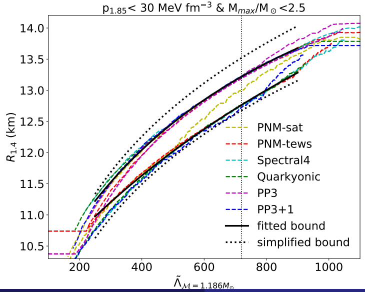
- Prior of Λ_1, Λ_2 in GW waveform analysis.
- Bounds of M-R diagram
- **Bounds of $R_{1.4}$-$\Lambda_{1.4}$-$\tilde{\Lambda}_{GW170817}$ relations**
- Bounds of I-love relation
- Bounds on EoS $p(\varepsilon)$

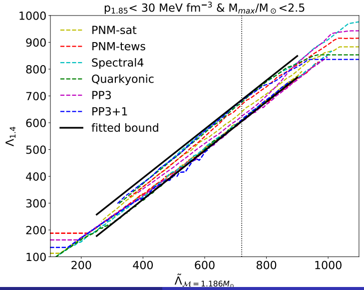
Bounds of $R_{1.4}$-$\Lambda_{1.4}$-$\tilde{\Lambda}_{GW170817}$ relations

$$\tilde{\Lambda} = a'\left(\frac{R_{1.4}c^2}{G\mathcal{M}}\right)^6 \quad (8)$$

where $a' = 0.0041 \pm 0.0016$ for $1.0M_\odot \leq \mathcal{M} \leq 1.4M_\odot$. In case of GW170817 $\mathcal{M} = 1.186$, the constant is $a' = 0.0042 \pm 0.008$







Application of EoS Parameterizations

- Prior of Λ_1, Λ_2 in GW waveform analysis.
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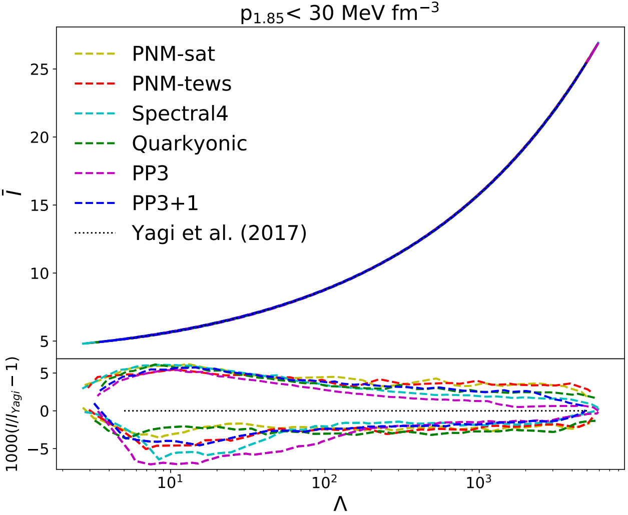
- **Bounds of I-love relation**
- Bounds on EoS p(ε)

I-love Relations

A correlation with 0.6% deviation with 1000000 hadronic EoS!!!

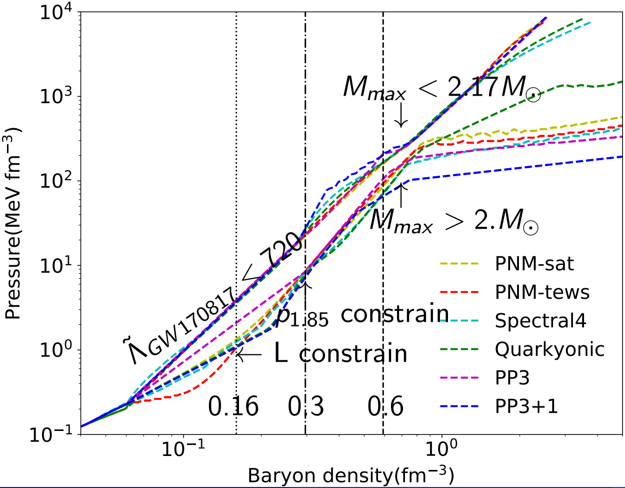


Application of EoS Parameterizations

- Prior of Λ_1, Λ_2 in GW waveform analysis.
- Bounds of M-R diagram
- Bounds of $R_{1.4}$-$\Lambda_{1.4}$-$\tilde{\Lambda}_{GW170817}$ relations
- Bounds of I-love relation
- **Bounds on EoS p(ε)**

Bounds on EoS

Apply universal constrains, neutron matter constrain on (E, L, $p_{1.85}$), tidal deformability constrain $\tilde{\Lambda}(M_{ch} = 1.186 M_\odot) < 720$ and maximum mass constrain 2 $M_\odot \leq M_{max} \leq 2.17$ $M_\odot$.

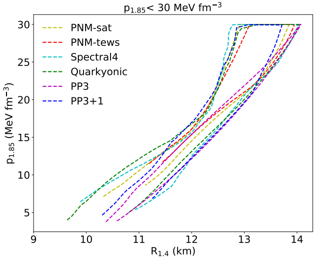


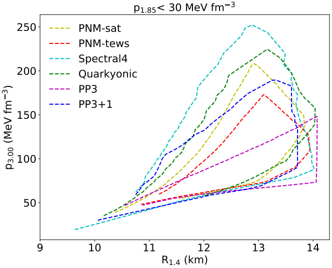
Bounds on $p(n)$-$R_{1.4}$

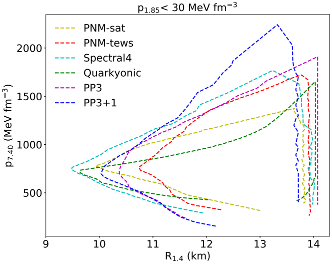
In natural units $c = G = 1$, there is nice unit scaling $[R^2] \approx [\frac{1}{p}]$ or $[\frac{1}{\varepsilon}]$,

$$\frac{\delta R}{R} = \frac{1}{2}\frac{\frac{\delta p(\varepsilon)}{p(\varepsilon)}}{\frac{d\log p}{d\log \varepsilon} - 1} \quad (9)$$

where $\delta p(\varepsilon) = \eta p(\varepsilon) + \eta\varepsilon \frac{dp(\varepsilon)}{d\varepsilon}$





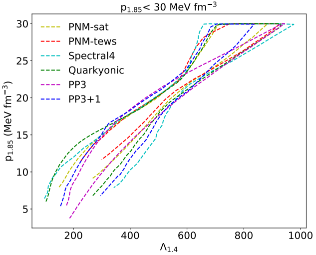


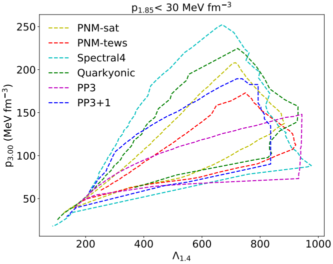
Bounds on $p(n)$-$\Lambda_{1.4}$

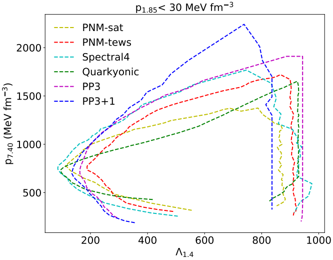
In natural units $c = G = 1$, there is nice unit scaling $[\Lambda] \approx [R^5]$,

$$\frac{\delta\Lambda}{\Lambda} = \frac{5}{2}\frac{\frac{\delta p(\varepsilon)}{p(\varepsilon)}}{\frac{d\log p}{d\log\varepsilon} - 1} \quad (10)$$

where $\delta p(\varepsilon) = \eta p(\varepsilon) + \eta\varepsilon\frac{dp(\varepsilon)}{d\varepsilon}$







Variation study of $\delta p(\varepsilon)$-$\delta R_{1.4}$

TOV equations are

$$\frac{dm}{dp} = -\frac{4\pi r^3 c^2 \epsilon (r - \frac{2Gm}{c^2})}{G(mc^2 + 4\pi r^3 p)(\epsilon + p)} = A(r, m, \epsilon, p) \tag{11}$$

$$\frac{dr}{dp} = -\frac{rc^4 (r - \frac{2Gm}{c^2})}{G(mc^2 + 4\pi r^3 p)(\epsilon + p)} = B(r, m, \epsilon, p) \tag{12}$$

We linearize above Eqs by $m \to m + \delta m$, $r \to r + \delta r$ and $\varepsilon \to \varepsilon + \delta\varepsilon$,

$$\frac{d\delta m}{dp} = \frac{\partial A}{\partial m}\delta m + \frac{\partial A}{\partial r}\delta r + \frac{\partial A}{\partial \varepsilon}\delta\varepsilon \tag{13}$$

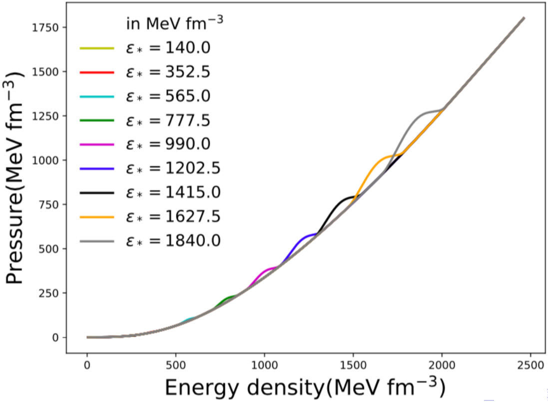
$$\frac{d\delta r}{dp} = \frac{\partial B}{\partial m}\delta m + \frac{\partial B}{\partial r}\delta r + \frac{\partial B}{\partial \varepsilon}\delta\varepsilon \tag{14}$$

Variation study of $\delta p(\varepsilon)$

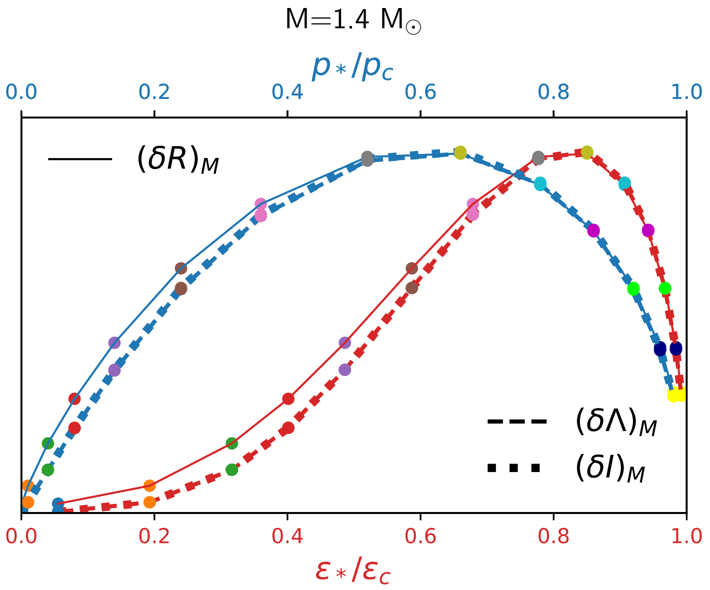
We choose the EoS variation to be,

$$\frac{\delta p(\varepsilon)}{p} = a[(\frac{\varepsilon - \varepsilon_*}{b\varepsilon_*})^2 - 1]^2 \quad (15)$$

where, a and b are dimensionless amplitude and range of variation, ε_* is the energy density where variation located. Scaling relation: $\delta r^2 \propto a \times b$.



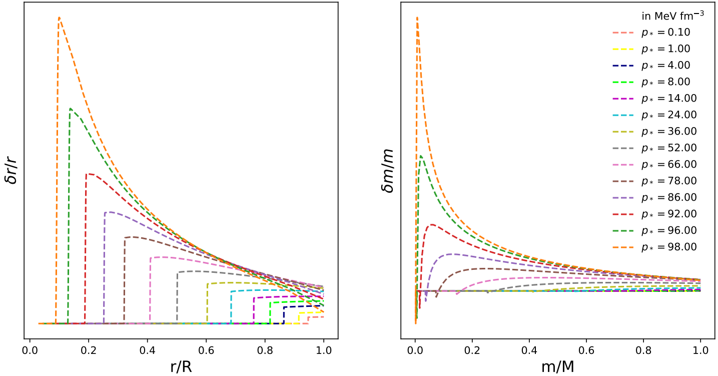
Variation study of $\delta p(\varepsilon)$



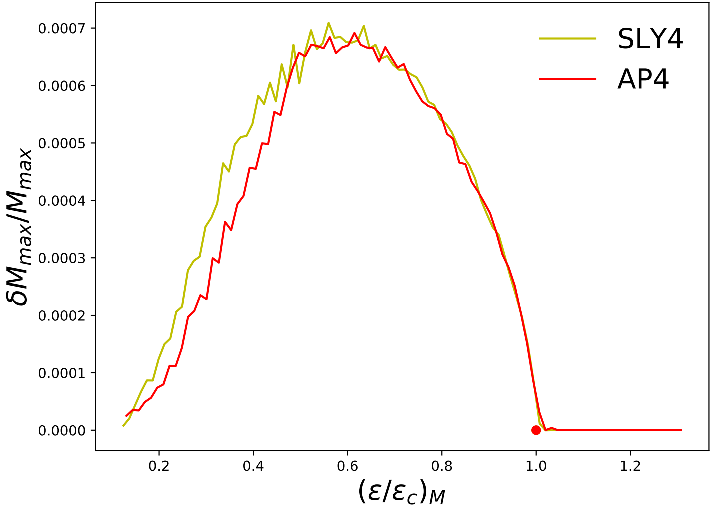
Summary

- Allow wider range of EoS(more variability), Great for study prior distribution of properties, correlation of proprieties and EoS constrain.
- Upgrade PP3 to PP3+1 can reach higher maximum mass.
- $\tilde{\Lambda}_{GW170817}$ gives comparable constrain as from neutron matter calculations.
- Lower bound in radius and low density EoS are strongly dependent on EoS parameterization.
- L is corelated with neutron star radius , but not the direct cause of radius.

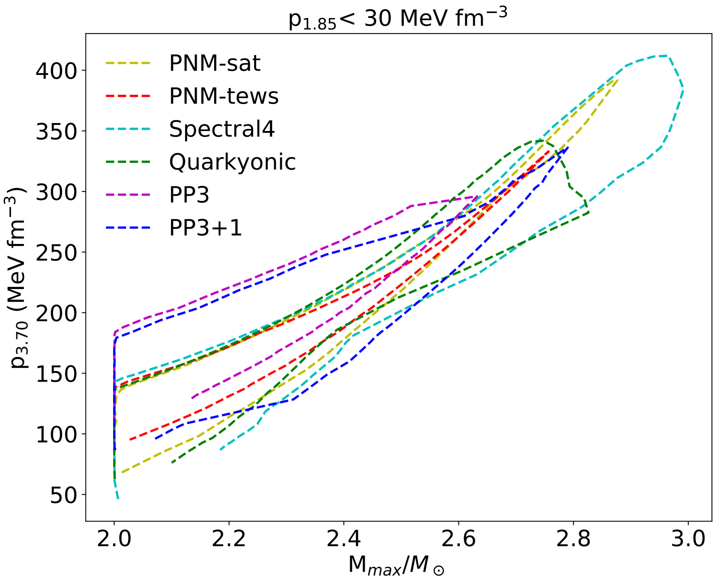
Variation on R and Λ at M=1.4 $M_\odot$



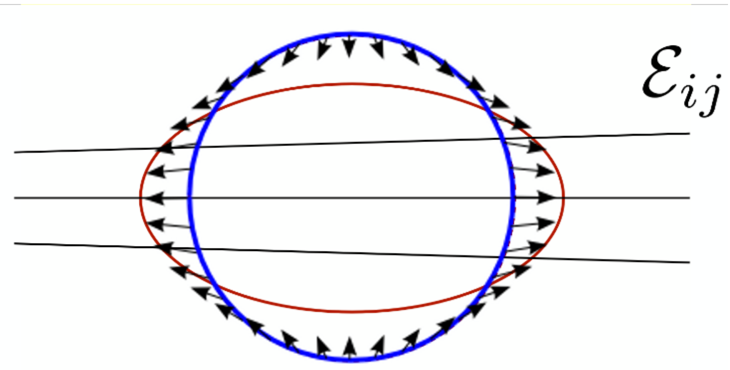
Variation on M_{max}



Variation on M_{max}



Tidal field



$$Q_{ij} = -\Lambda \mathcal{E}_{ij}$$

Tidal deformability(Newtonian)

- Tidal field potential and quadruple moment in Newtonian physics are,

$$\Phi_{tidal} = \frac{1}{2}\varepsilon_{ij}x^i x^j \quad (16)$$

$$Q_{ij} = \int d^3x\delta\rho(x_i x_j - \frac{1}{3}\delta_{ij}) \quad (17)$$

- Quadruple moment is induced by tidal field linearly,

$$Q_{ij} = -\lambda\varepsilon_{ij} \quad (18)$$

where λ is tidal deformability with a unit of (mass)(length)^2/(time)^2.

- Two dimensionless parameters are defined from λ,

$$\Lambda = \lambda M^{-5} \quad \text{dimensionless tidal deformability} \quad (19)$$

$$k_2 = \frac{3}{2}\lambda R^{-5} \quad \text{tidal Love number} \quad (20)$$

where $c = G = 1$.

Tidal deformability(GR generized)

- Total gravitational potential in Newtonian physics are,

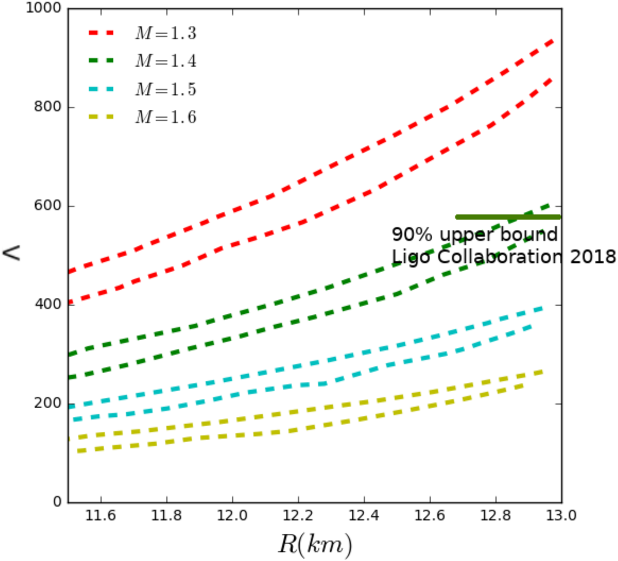
$$\Phi = \frac{1}{2}\varepsilon_{ij}x^i x^j - \frac{M}{r} - \frac{3}{2}\frac{Q_{ij}x^i x^j}{r^5} \quad (21)$$

- We know general relativity reduce to Newtonian gravity at weak(far) field. And tt component of metric plays the exact role of Newtonian potential. Thus is should be expanded in powers of r to match Newtonian definition of quadruple moment and tidal deformability.

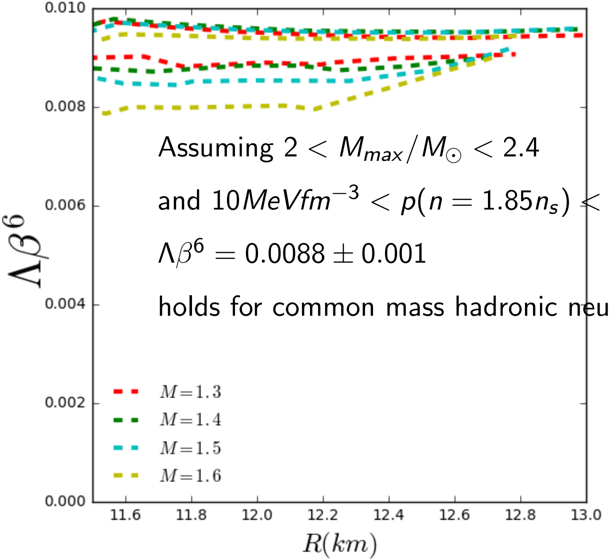
$$-\frac{1+g_{00}}{2} = \frac{1}{2}\varepsilon_{ij}x^i x^j - \frac{M}{r} - \frac{3}{2}\frac{Q_{ij}x^i x^j}{r^5} + \sum C_n r^n \quad (n \neq 2, -1, -3) (22)$$

- g_{00} can be solved by introducing a linear $Y_{l=2}^{m=0}(\theta, \phi)$ perturbation on spherical symmetric metric(TOV metric). And the ratio between its (n=2) order and (n=-3) order coefficient defines tidal deformability.

Tidal deformability of hadronic NS



Tidal deformability of hadronic NS



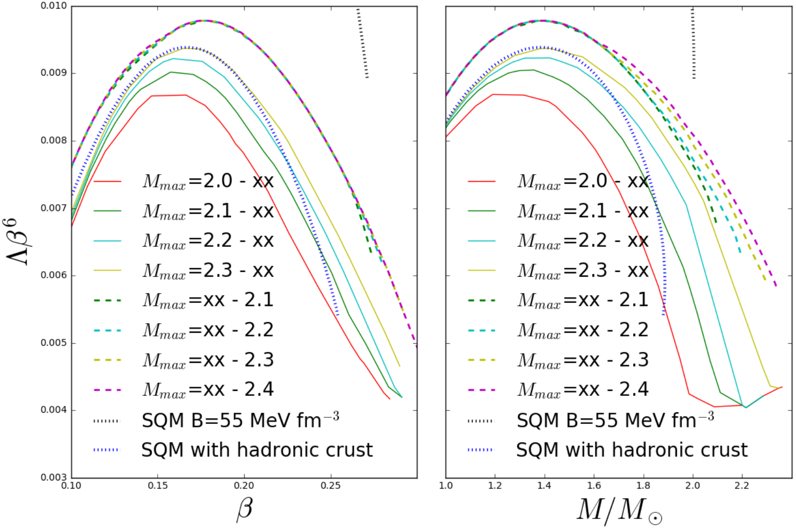
Assuming $2 < M_{max}/M_{\odot} < 2.4$

and $10 MeV fm^{-3} < p(n = 1.85 n_s) < 20 MeV fm^{-3}$

$\Lambda\beta^6 = 0.0088 \pm 0.001$

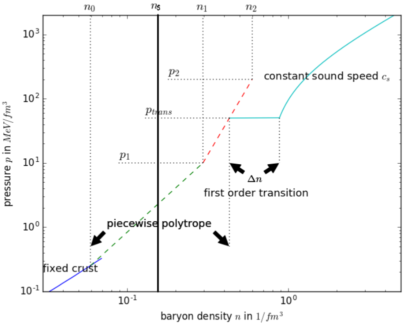
holds for common mass hadronic neutron star

Tidal deformability of hadronic NS



Hybrid star with first order phase transition

- BPS as fixed crust EoS up to $n_0 \approx n_s/2.7$
- Three piecewise polytropic EoS devided by $n_1 = 1.85 n_s, n_2 = 3.74 n_s$ (J.S. Read 2008)
- Constant sound speed(CSS) is used for quark core.
- A first order transition is assumed to happened between n_s and $3.74 n_s$, or $p_{trans} < 250 MeV/fm^3$. Chemical equilibrium is assumed at boundary,

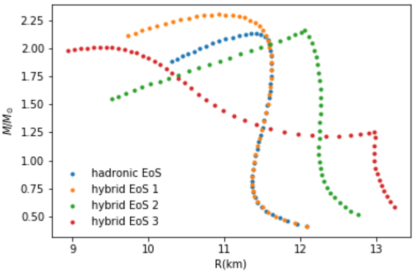


$$p_{hardron} = p_{quark}, \; \mu_{hardron} = \mu_{quark} \qquad (23)$$

$$\epsilon(p) = \begin{cases} \epsilon_{poly}(p) & \text{if } p < p_{trans} \\ \epsilon_{poly}(p_{trans}) + \Delta\epsilon + \frac{p - p_{trans}}{c_s^2} & \text{if } p < p_{trans} \end{cases} \qquad (24)$$

Hybrid star with first order phase transition

- $p_1 = p(n_1)$ is closely related with radius of a typical neutron star, and is constrained by neutron matter calculation.
- $p_2 = p(n_2)$ define stiffness below transition, bounded by causality at transition.
- Sound speed c_s^2 affect maximum mass of hybrid star.
- Energy discontinuity $\Delta\varepsilon > 0$(stability), is bounded by requiring $M_{max} > 2.01 M_\odot$.



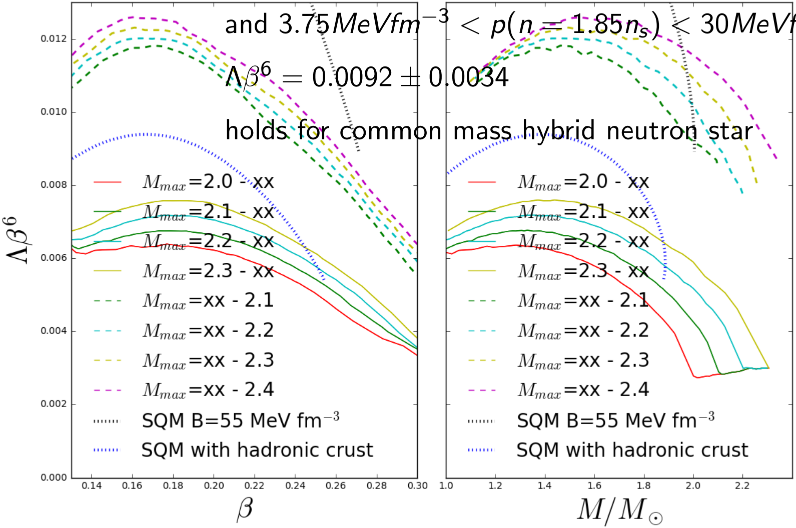
Tidal Deformability of Hybrid NS

Assuming $2 < M_{max}/M_{\odot} < 2.4$, $n_{trans} > n_s$

and $3.75 MeVfm^{-3} < p(n = 1.85 n_s) < 30 MeVfm^{-3}$,

$\Lambda\beta^6 = 0.0092 \pm 0.0034$

holds for common mass hybrid neutron star



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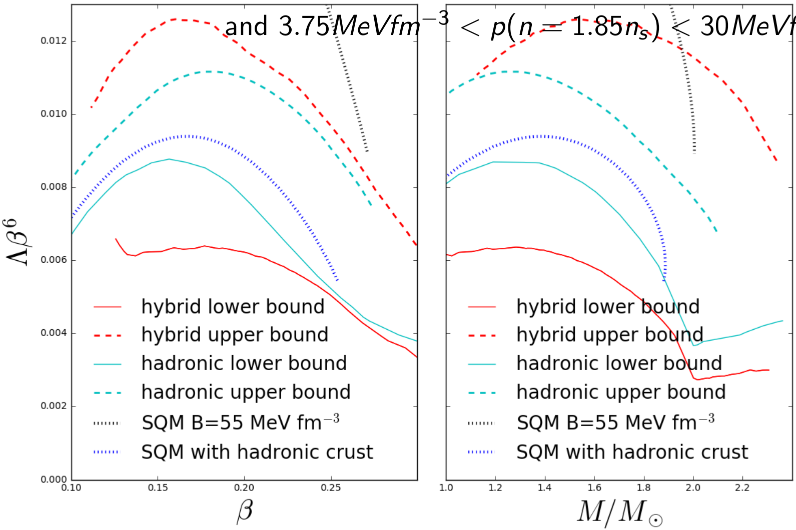


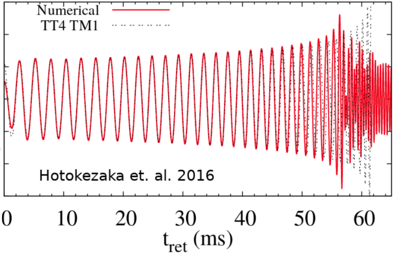
Tidal deformability in binary merger GW waveform

- Oscillating Quadruple moments of neutron star due to excitation of periodic tidal fields contributes to phase shift in GW form.
- Quadruple oscillating contribute to GW radiation reaction,

$$\dot{E}(\omega) = -\frac{1}{5} < \dddot{Q}^T_{ij} \dddot{Q}^T_{ij} >= -\frac{32}{5} M^{4/3} \mu^2 \omega^{10/3} [1 + g(\omega)] \quad (25)$$

- By evaluating stable orbit, contribution of quadruple oscillating to total energy of the binary can be calculated,

$$E(\omega) = -M^{1/3} \omega^{-2/3} [1 + f(\omega)] \quad (26)$$



- Using formula $\frac{d^2\Phi}{d\omega^2} = 2(\frac{dE}{d\omega})/\dot{E}$, tidal phase correction can be derived,

$$\delta\Phi = -\frac{9}{16} \frac{\omega^{5/3}}{\mu M^{7/3}} [(\frac{12m_2 + m_1}{m_1} \lambda_1 + \frac{12m_1 + m_2}{m_2} \lambda_2] \quad (27)$$

Flanagan+ 2008

Binary tidal deformability

- At leading order, phase shift of GW is proportional to the binary tidal deformability,

$$\bar{\Lambda} = \frac{16}{13}\frac{(12q+1)\Lambda_1 + (12+q)q^4\Lambda_2}{(1+q)^5} \tag{28}$$

where $q = m_2/m_1 < 1$.

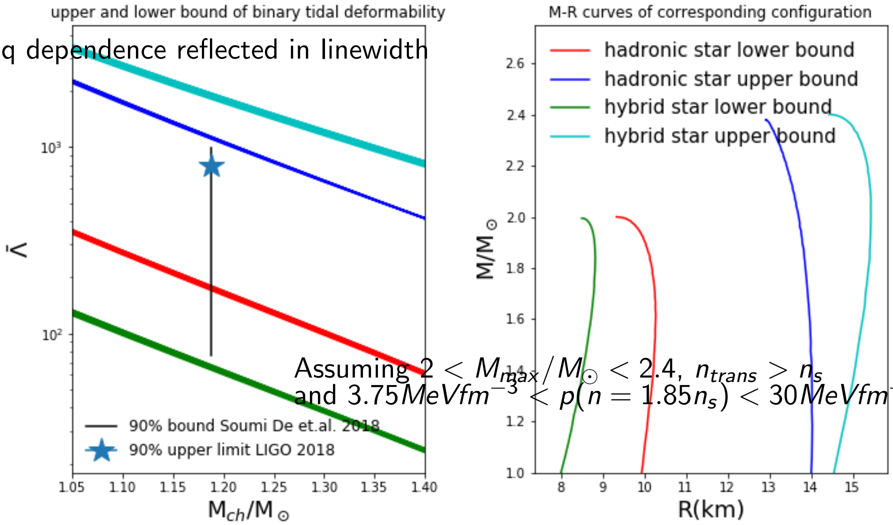
- Statistics are required in order to make any conclusion for binary tidal deformability. Thus, prior distribution of $\tilde{\Lambda}, \Lambda_1, \Lambda_2$ is important.
- We set up bounds of $\tilde{\Lambda}$ and Λ_2/Λ_1 as a function of chirp mass M_{ch} and q, in the scenario of hadronic star and hybrid star respectively.

$$\tilde{\Lambda}_{lower}(M_{ch}, q) \quad < \tilde{\Lambda} < \quad \tilde{\Lambda}_{upper}(M_{ch}, q) \tag{29}$$

$$(\Lambda_2/\Lambda_1)_{lower}(M_{ch}, q) \quad < (\Lambda_2/\Lambda_1) < \quad (\Lambda_2/\Lambda_1)_{upper}(M_{ch}, q) \tag{30}$$

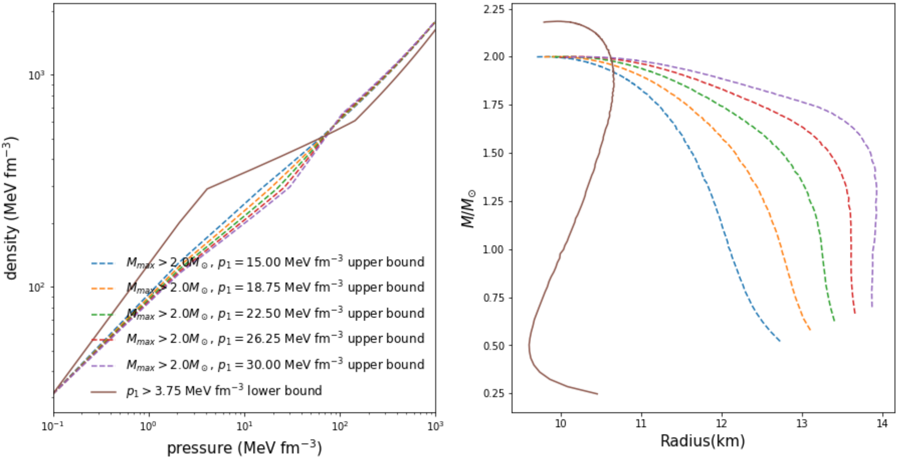
Bounds of binary tidal deformability $\tilde{\Lambda}$

q dependence reflected in linewidth

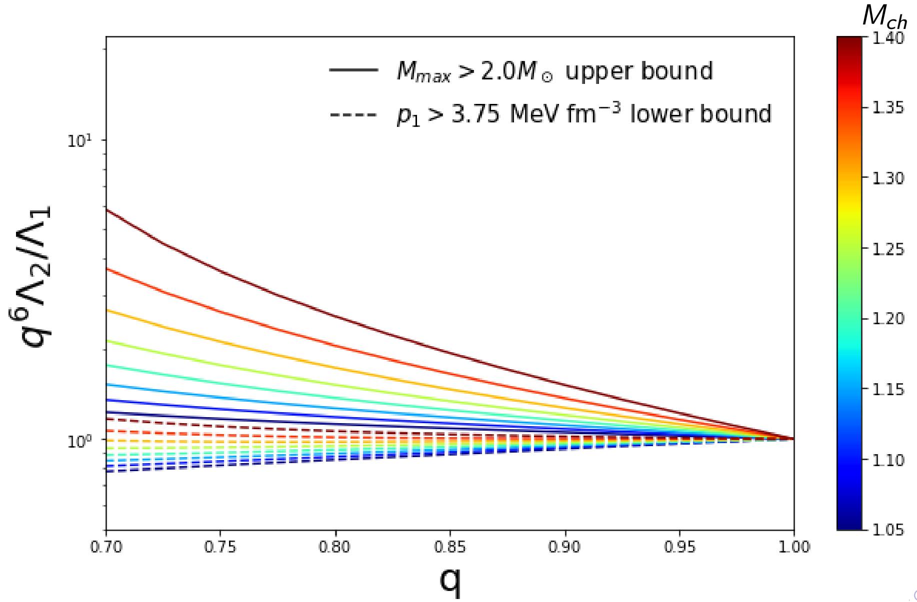


Assuming $2 < M_{max}/M_\odot < 2.4$, $n_{trans} > n_s$ and $3.75 MeV fm^{-3} < p(n = 1.85 n_s) < 30 MeV fm^{-3}$,

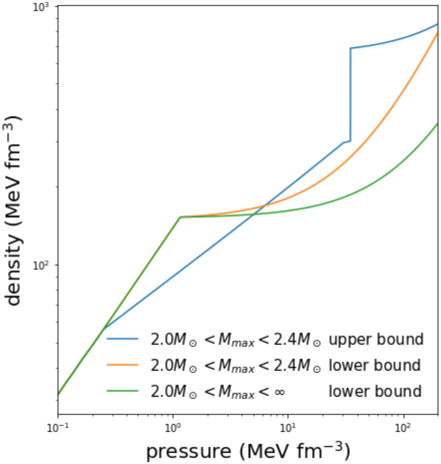
EoS and M-R curves for hadronic bounds of Λ_2/Λ_1

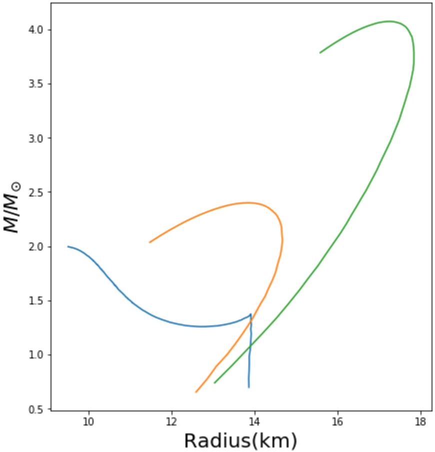


Hadronic bounds of binary tidal deformability ratio $\tilde{\lambda}_2/\tilde{\lambda}_1$

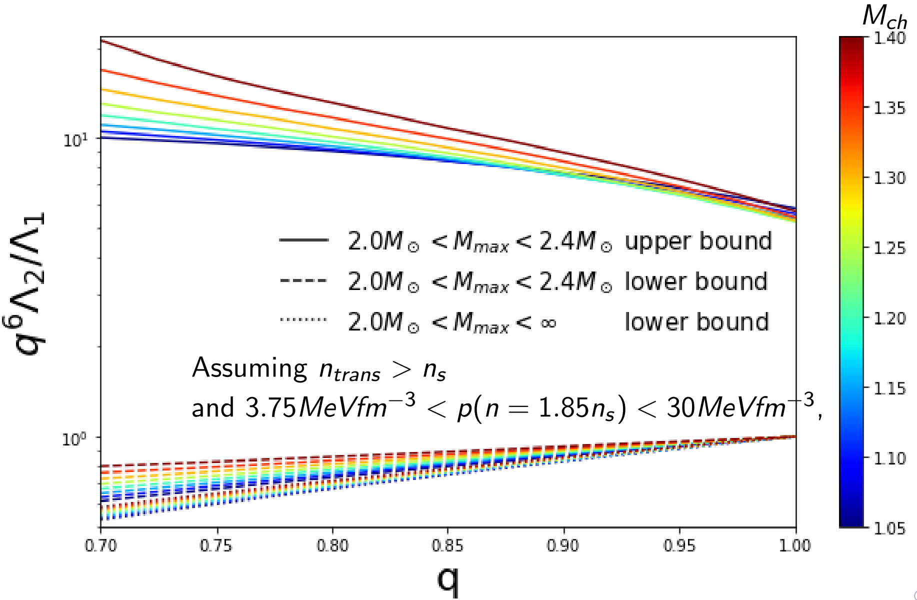


Hybrid star bounds of binary tidal deformability ratio $\tilde{\lambda}_2/\tilde{\lambda}_1$

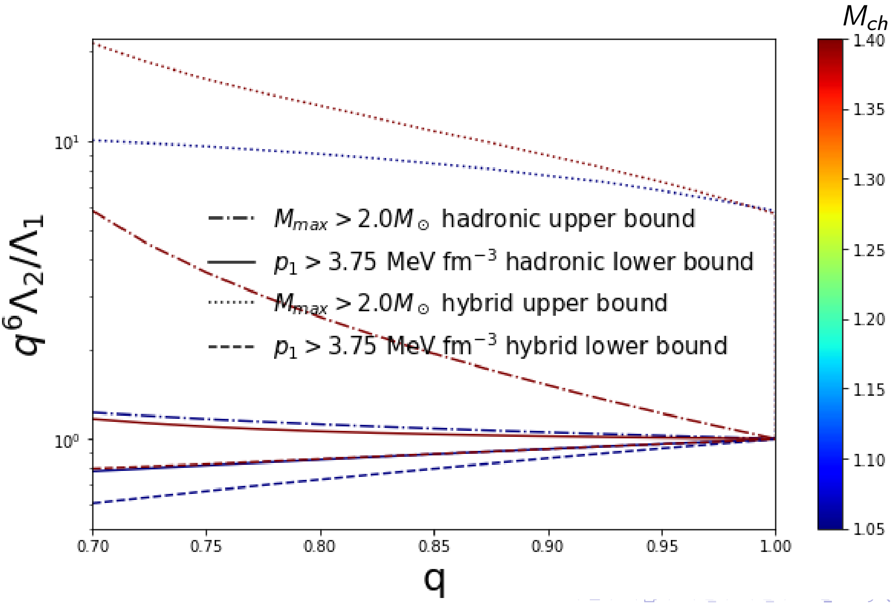




Hybrid star bounds of binary tidal deformability ratio $\tilde{\lambda}_2/\tilde{\lambda}_1$



Bounds of binary tidal deformability ratio $\tilde{\lambda}_2/\tilde{\lambda}_1$



Summary

- Tidal deformability is a measure of compactness.
- Tidal deformability-compactness relation is broaden when hybrid neutron star was taken into account.
- Binary tidal deformability is a 'average' tidal deformability of the two star with more weight on the massive one. It appear as perturbation of phase shift in GW observation.
- Together with mass knowledge, tidal deformability will provide us radius of neutron star, eventually the information of EoS around $n = 1 - 2n_s$