

# Ever More Physics in Gravitational Waveforms

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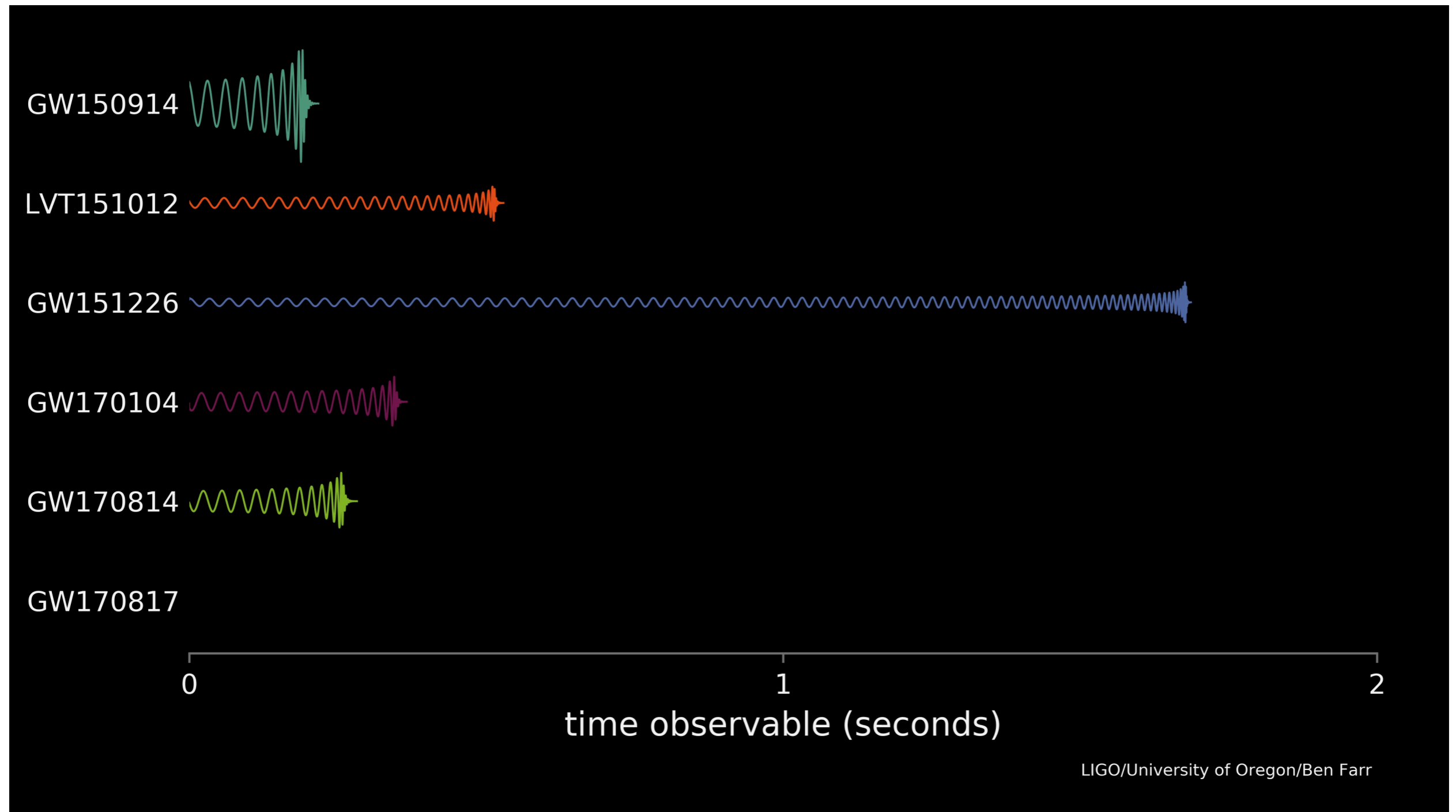


# Outline

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- **Observing gravitational waves and inferring astrophysical/physical information** hinge on our **ability** to make **precise predictions** of **two-body dynamics** and **gravitational radiation**.
- Are we **missing any gravitational-wave signal** with LIGO and Virgo detectors? Are **current inference studies** in any way **affected** by **modeling error**?
- Our **ability to infer** more **precise** cosmological, fundamental/nuclear physics and astrophysical **information**, and **carry out** more **stringent tests** of GR require **more accurate** waveforms and with **more physics**.
- **Status** of waveform modeling that includes **spin-precession**, **higher modes**, **matter** and **eccentricity**.
- **What** are the **highest priorities**, and **what** are the **challenges** in view of more sensitive detectors.

# Shape/length of signals detected by LIGO/Virgo in O1 & O2



(credit: B. Farr)

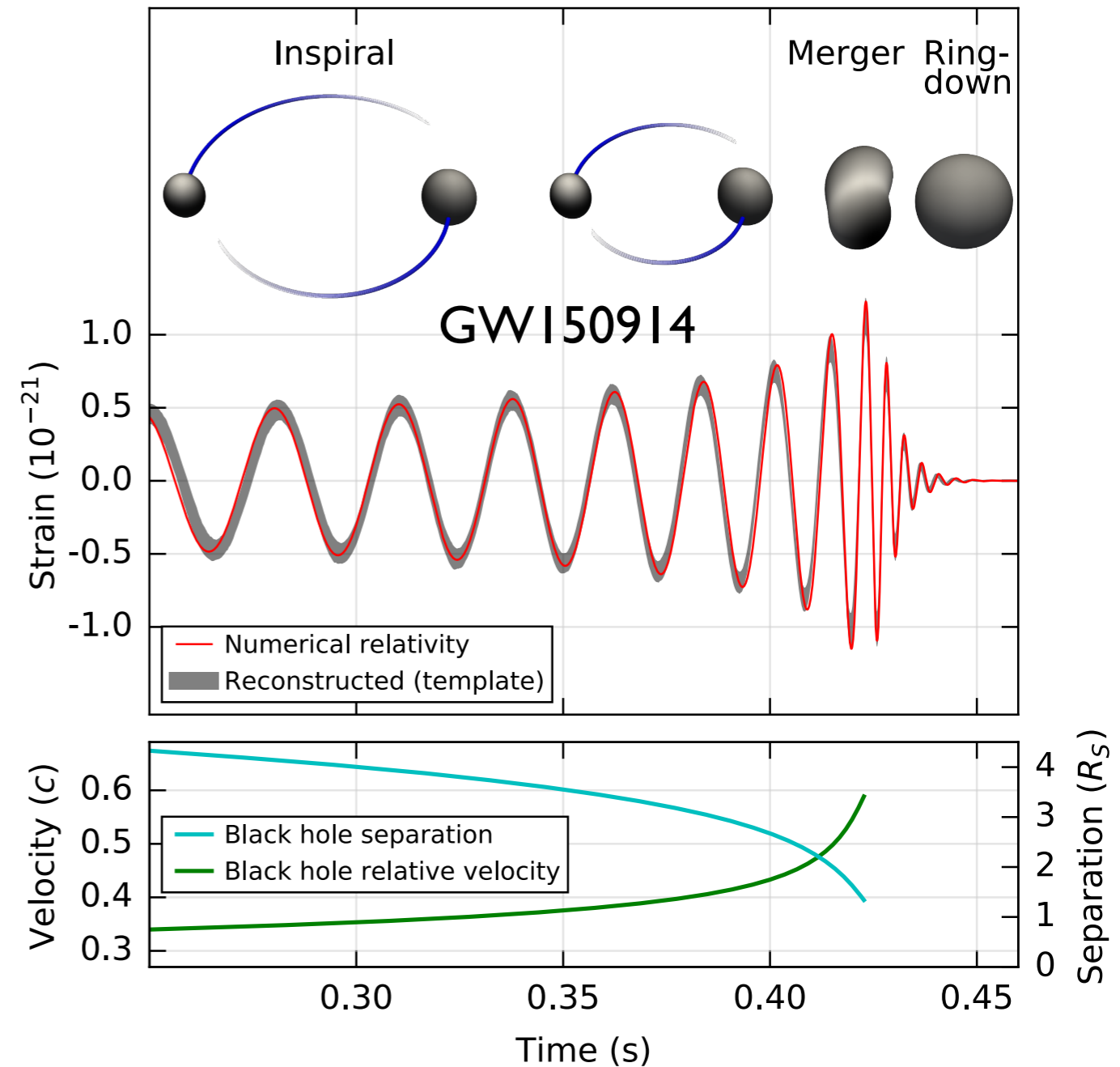
- **GW170608** and four **GWs** recently announced are **missing from video**.

# Characteristics of binary black-hole coalescence

- **Early inspiral:** low velocity & weak gravitational field.
- **Late inspiral/plunge:** high velocity & strong gravitational field.
- **Merger:** nonlinear & non perturbative effects; rapidly varying gravitational field
- **Ringdown:** excitation of quasi-normal modes/spacetime vibrations.

Phase/amplitude evolution **encodes unique information** about the source

(Abbott et al. PRL 116 (2016) 061102)



Black holes of radius of 90 km at separation of 350 km are making 75 orbits per second before merging.



# Some outstanding questions in physics and astrophysics

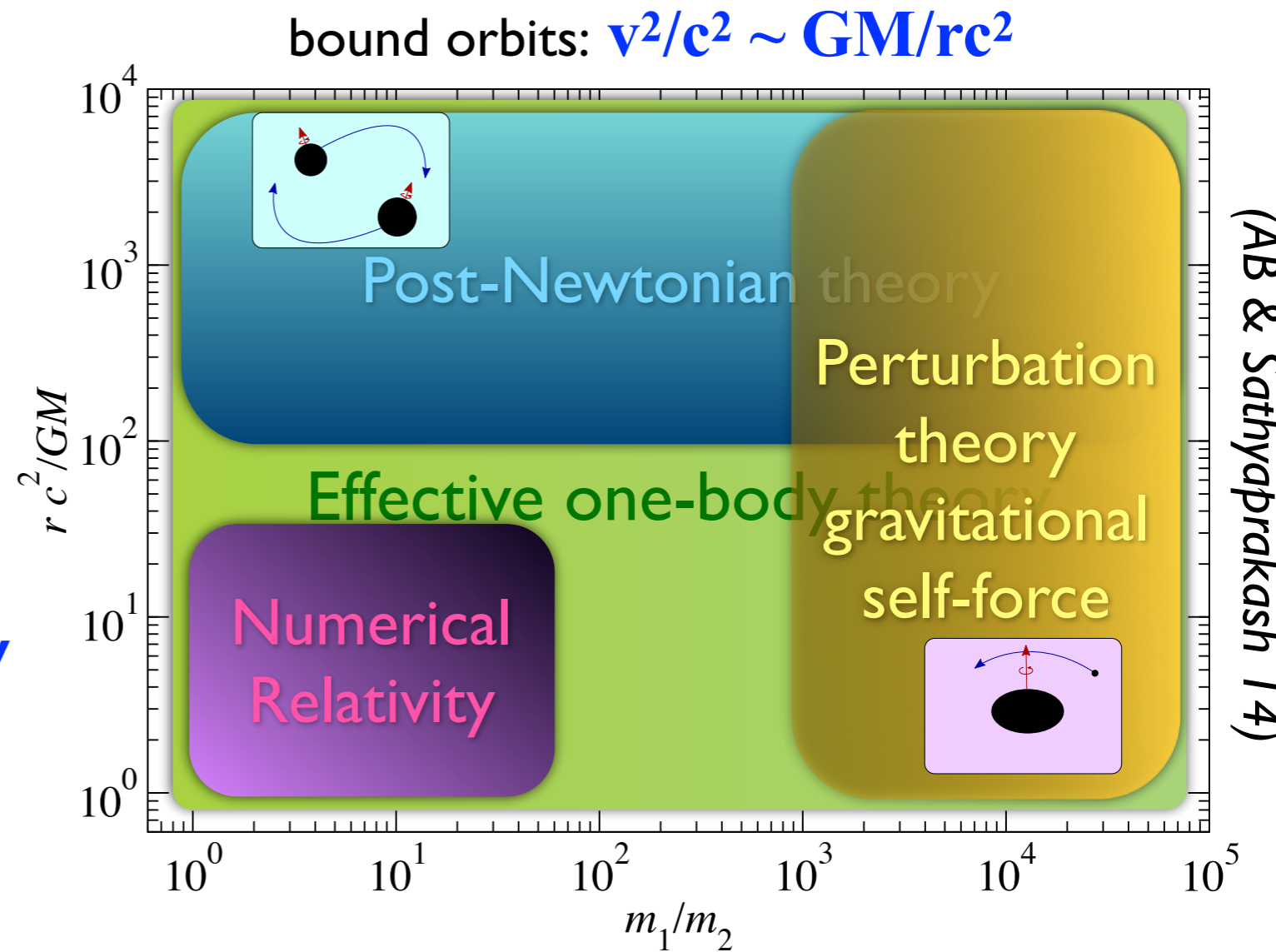
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- What are the **properties** of **dynamical spacetime** (gravitational waves)?
- Is **General Relativity still valid** in the highly dynamical, strong-field regime?
- Are **Nature's black holes** the black holes predicted in the **General** theory of **Relativity**?
- How **black holes** and **neutron stars form**, which is their **astrophysical environment**, and how do they **form binaries**?
- How **matter behaves** under **extreme density and pressure**? Can **dark matter** make compact objects?
- What's the **origin** of the **most energetic phenomena** in our Universe?
- Can we discover **new fundamental particles**, and **infer** the **cosmological model of our Universe** through gravitational-wave observations?

# Solving two-body problem in General Relativity

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu}$$

- **GR** is **non-linear theory**.
- Einstein's field equations can be solved:
  - **approximately**, but **analytically** (**fast way**)
  - **“exactly”**, but **numerically** on supercomputers (**slow way**)



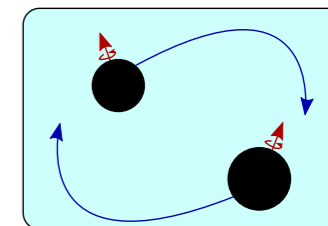
- **Synergy** between **analytical** and **numerical relativity** is **crucial**.

# Post-Newtonian approximation

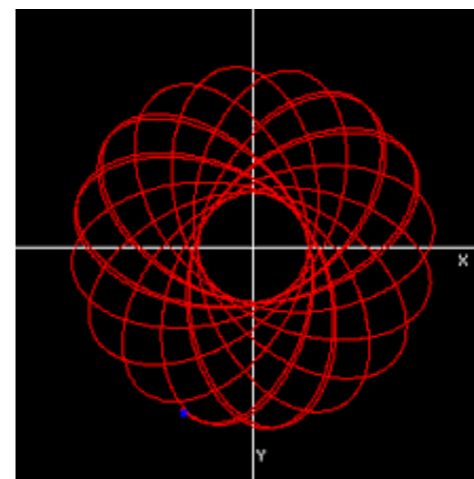
- **First introduced in 1917** (Droste & Lorentz 1917, ... Einstein, Infeld & Hoffmann 1938)

(Blanchet, Damour, Iyer, Faye, Bernard, Bohe', AB, Marsat; Jaranowski, Schaefer, Steinhoff; Will, Wiseman; Flanagan, Hinderer, Vines; Goldberger, Porto, Rothstein; Kol, Levi, Smolkin; Foffa, Sturani ...)

**Small parameter** is  $v/c \ll 1$ ,  $v^2/c^2 \sim GM/rc^2$   
large separation and slow motion, natural for **bound orbits**



$$H_N(\mathbf{p}, \mathbf{r}) = \frac{\mu}{2} \mathbf{p}^2 - \frac{GM\mu}{r}$$



$$H_{1PN}(\mathbf{p}, \mathbf{r}) = \frac{\mu}{8} (3\nu - 1) (\mathbf{p}^2)^2 - \frac{\mu}{2} [(3 + \nu) \mathbf{p}^2 + \nu (\mathbf{n} \cdot \mathbf{p})^2] \frac{GM}{r} + \mu \frac{(GM)^2}{2r^2}$$

- Compact object is **point-like body endowed** with time-dependent **multipole moments** (skeletonization).

# Small mass-ratio expansion/gravitational self-force formalism

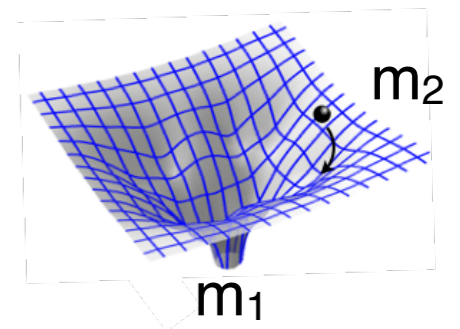
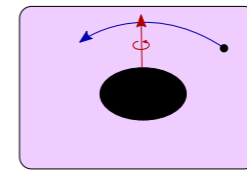
- **First works in 50-70s** (Regge & Wheeler 56, Zerilli 70, Teukolsky 72)

**Small parameter** is  $m_2/m_1 \ll 1$ ,  $v^2/c^2 \sim GM/rc^2 \sim 1$ ,  $M = m_1 + m_2$

scattering and bound orbits

Equation of gravitational perturbations in black-hole spacetime:

$$\frac{\partial^2 \Psi}{\partial t^2} - \frac{\partial^2 \Psi}{\partial r_{\star}^2} + V_{lm} \Psi = \mathcal{S}_{lm}$$

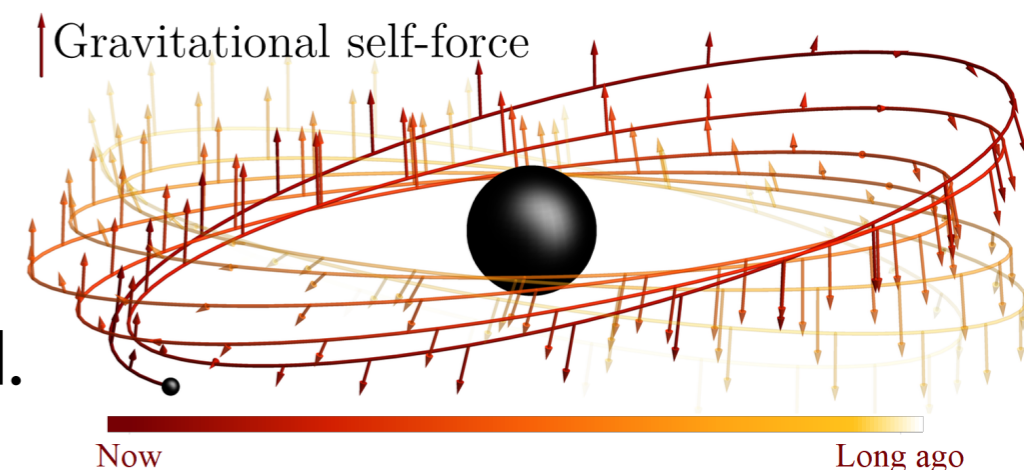


Computation of Green functions in Schwarzschild/Kerr spacetimes.

(Fujita, Poisson, Sasaki, Shibata, Khanna, Hughes, Bernuzzi, Harms, Nagar...)

- Accurate modeling of **relativistic dynamics of large mass-ratio** inspirals **requires** to include **back-reaction effects** due to interaction of small object with its own gravitational perturbation field.

(Deitweiler, Whiting, Mino, Poisson, Quinn, Wald, Sasaki, Tanaka, Barack, Ori, Pound, van de Meent, ...)

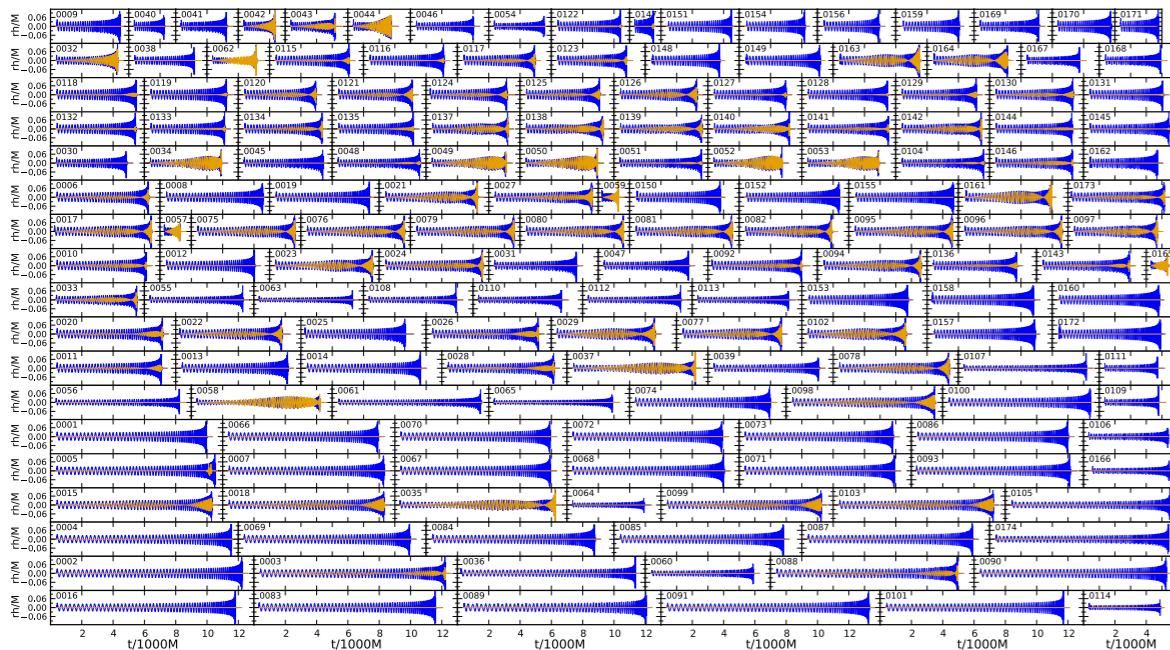


(credit: van de Meent)

# Numerical Relativity: catalogues of binary BH waveforms

- **Breakthrough** in 2005 (*Pretorius 05, Campanelli et al. 06, Baker et al. 06*)

(*Kidder, Pfeiffer, Scheel, Lindblom, Szilagy; Brüggmann; Hannam, Husa, Tichy; Laguna, Shoemaker ...*)



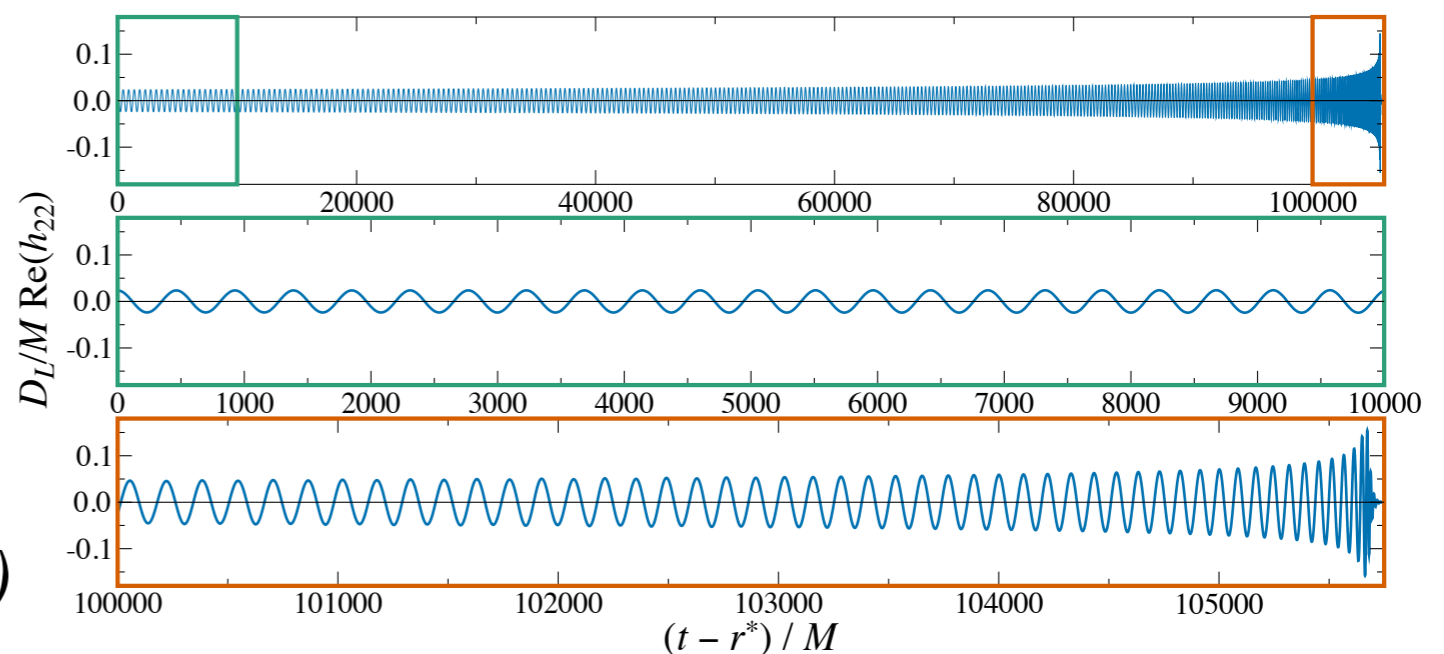
$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu}$$

- **376 GW cycles**, zero spins & mass-ratio 7 (8 months, few millions CPU-h)  
(*Szilagy, Blackman, AB, Taracchini et al. 15*)

- **Simulating eXtreme Spacetimes (SXS)** collaboration.  
(*Mroue et al. 13, Boyle et al. 18*)

- **Other NR catalogues**  
(*Husa et al. 15, Jani et al. 17, Healy et al. 17, 19*)

- **Numerical-Relativity & Analytical-Relativity** collaboration (*Hinder et al. 13*)



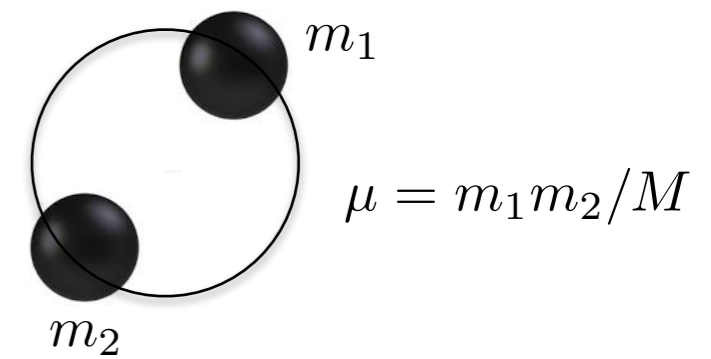
# Gravitational waveforms built from conservative & dissipative dynamics

- GW from time-dependent **quadrupole moment**:  $h_{ij} \sim \frac{G}{c^4} \frac{\ddot{Q}_{ij}}{D}$

$$h = \nu \left( \frac{GM}{c^2 D} \right) \frac{v^2}{c^2} \cos 2\Phi$$

$$\frac{v}{c} = \left( \frac{GM\omega}{c^3} \right)^{1/3}$$

$$\nu = \mu/M$$



- Center-of-mass energy:  $E(\omega)$

$$E(v) = -\frac{\mu}{2} v^2 + \dots$$

- GW luminosity:  $\mathcal{L}_{\text{GW}}(\omega) \equiv F(\omega)$

$$F(v) = \frac{32}{5} \nu^2 \frac{c^5}{G} \left( \frac{v}{c} \right)^{10} + \dots$$

- Balance equation:  $\frac{dE(\omega)}{dt} = -F(\omega) \rightarrow \dot{\omega}(t) = -\frac{F(\omega)}{dE(\omega)/d\omega}$

- Gravitational-wave **phase**:  $\Phi_{\text{GW}}(t) = 2\Phi(t) = \frac{1}{\pi} \int^t \omega(t') dt'$



# PN templates for compact-object binary inspirals

$$\begin{aligned}
 \varphi(f) = & \varphi_{\text{ref}} + 2\pi f t_{\text{ref}} + \frac{3}{128\nu} v^{-5} \left\{ 1 \right. \\
 & - \frac{5\hat{\alpha}^2}{336\omega_{\text{BD}}} v^{-2} - \frac{128}{3} \frac{\pi^2 D M \nu}{\lambda_g^2 (1+z)} v^2 \\
 & + \left( \frac{3715}{756} + \frac{55}{9} \nu \right) v^2 - 16\pi v^3 + 4\beta v^3 \\
 & + \left( \frac{15293365}{508032} + \frac{27145}{504} \nu + \frac{3085}{72} \nu^2 \right) v^4 - 10\sigma v^4 \\
 & \left. \dots - \frac{39}{2} \tilde{\Lambda}^t v^{10} + \dots \right\}
 \end{aligned}$$

dipole radiation  $\rightarrow$   $-\frac{5\hat{\alpha}^2}{336\omega_{\text{BD}}} v^{-2}$  (labeled -1PN)  
 $\frac{3}{128\nu} v^{-5}$  (labeled 0PN)  
 $-\frac{128}{3} \frac{\pi^2 D M \nu}{\lambda_g^2 (1+z)} v^2$  (labeled 1PN)  $\rightarrow$  graviton with non zero mass  
 $16\pi v^3$  (labeled 1.5PN)  $\rightarrow$  spin-orbit  
 $4\beta v^3$  (labeled 1.5PN)  
 $\left( \frac{3085}{72} \nu^2 \right) v^4$  (labeled 2PN)  
 $-10\sigma v^4$  (labeled spin-spin)  
 $-\frac{39}{2} \tilde{\Lambda}^t v^{10}$  (labeled 5PN)  $\rightarrow$  tidal

$\tilde{\Lambda}^t = f(m_1, m_2, \Lambda_1^t, \Lambda_2^t)$

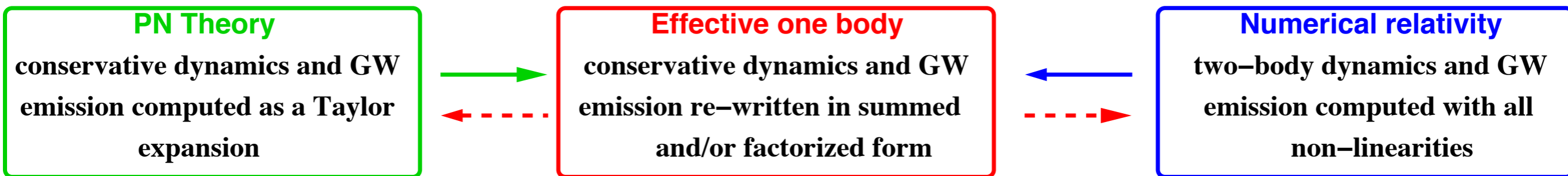
Depends on EOS & compactness  $\rightarrow$  it can be large  
 $\Lambda^t = \frac{2}{3} \kappa_2 \left( \frac{R_{\text{NS}}}{m_{\text{NS}}} \right)^5$

# The effective-one-body (EOB) approach

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- **EOB** approach introduced before **NR** breakthrough

(AB, Pan, Taracchini, Barausse, Bohe', Cotesta, Shao, Hinderer, Steinhoff, Vines; Damour, Nagar, Bernuzzi, Bini, Balmelli, Messina; Iyer, Sathyaprakash; Jaranowski, Schäfer)



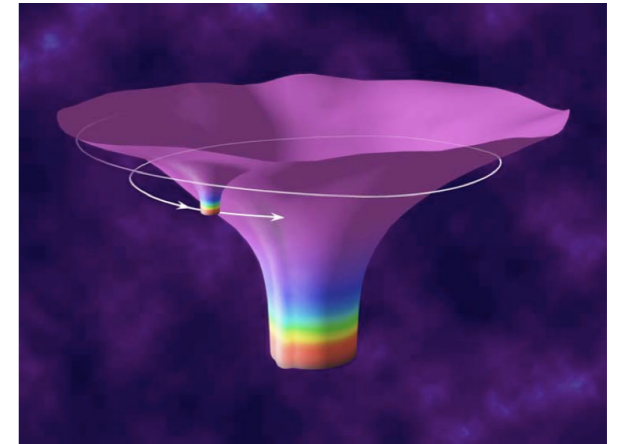
- **EOB** model uses best information available in PN theory, but **resums PN terms** in suitable way to describe accurately dynamics and radiation during inspiral and plunge (going beyond quasi-circular adiabatic motion).
- **EOB** assumes **comparable-mass** description is **smooth deformation of test-particle limit**. It employs non-perturbative ingredients and **models analytically merger-ringdown** signal.

# One-body problem: test-particle orbiting non-spinning BH

- **Schwarzschild** metric and **Hamiltonian**:

$$ds^2 = - \left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

$$H_{\text{Schw}}(\mathbf{r}, \mathbf{p}) = \mu \sqrt{\left(1 - \frac{2M}{r}\right) \left[1 + \frac{\mathbf{p}^2}{\mu^2} - \frac{2M}{r} \frac{p_r^2}{\mu^2}\right]}$$

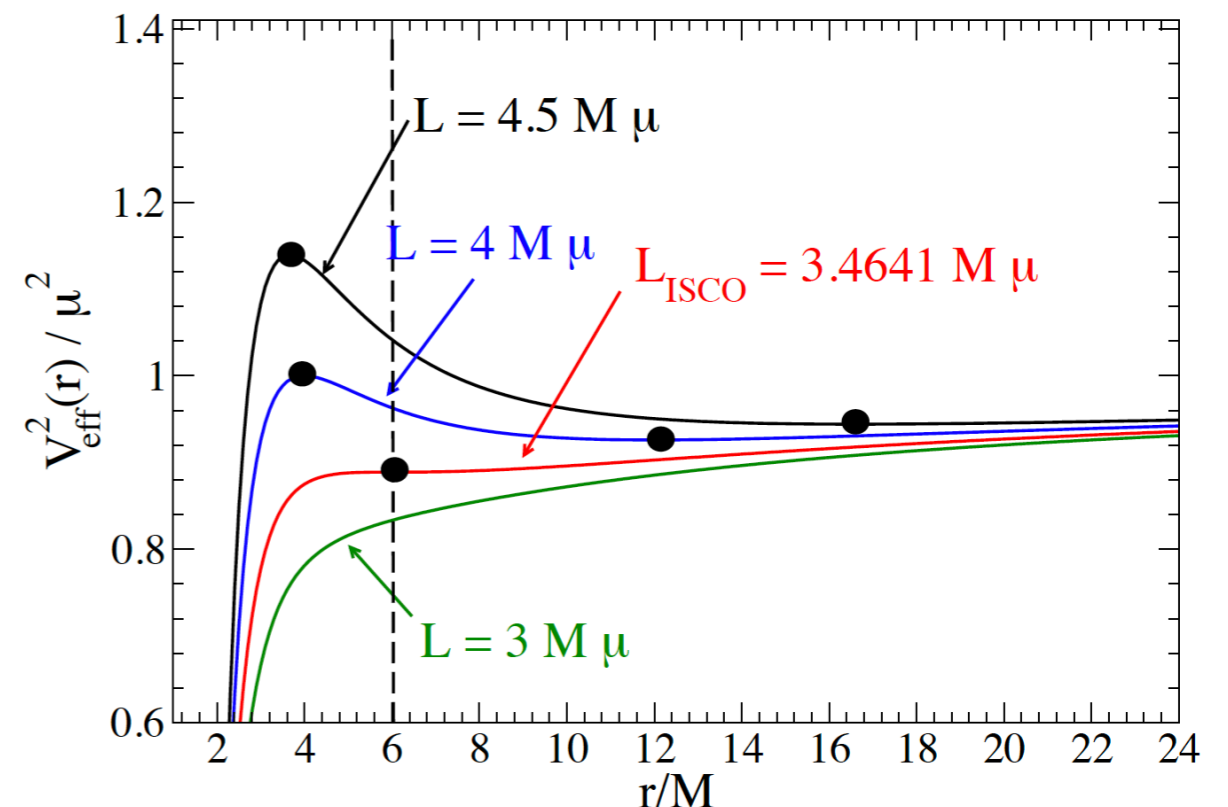


- $H_{\text{Schw}}(\mathbf{r}, \mathbf{p})$  describes a **test-particle of mass  $\mu$**  orbiting a BH of mass  $M$

- **Effective** (radial) **potential**:

$$\frac{V_{\text{eff}}^2(r)}{\mu^2} = \left(1 - \frac{2M}{r}\right) \left(1 + \frac{L^2}{\mu^2 r^2}\right)$$

- For  $L < L_{\text{ISCO}}$  **circular orbits no longer exist.**



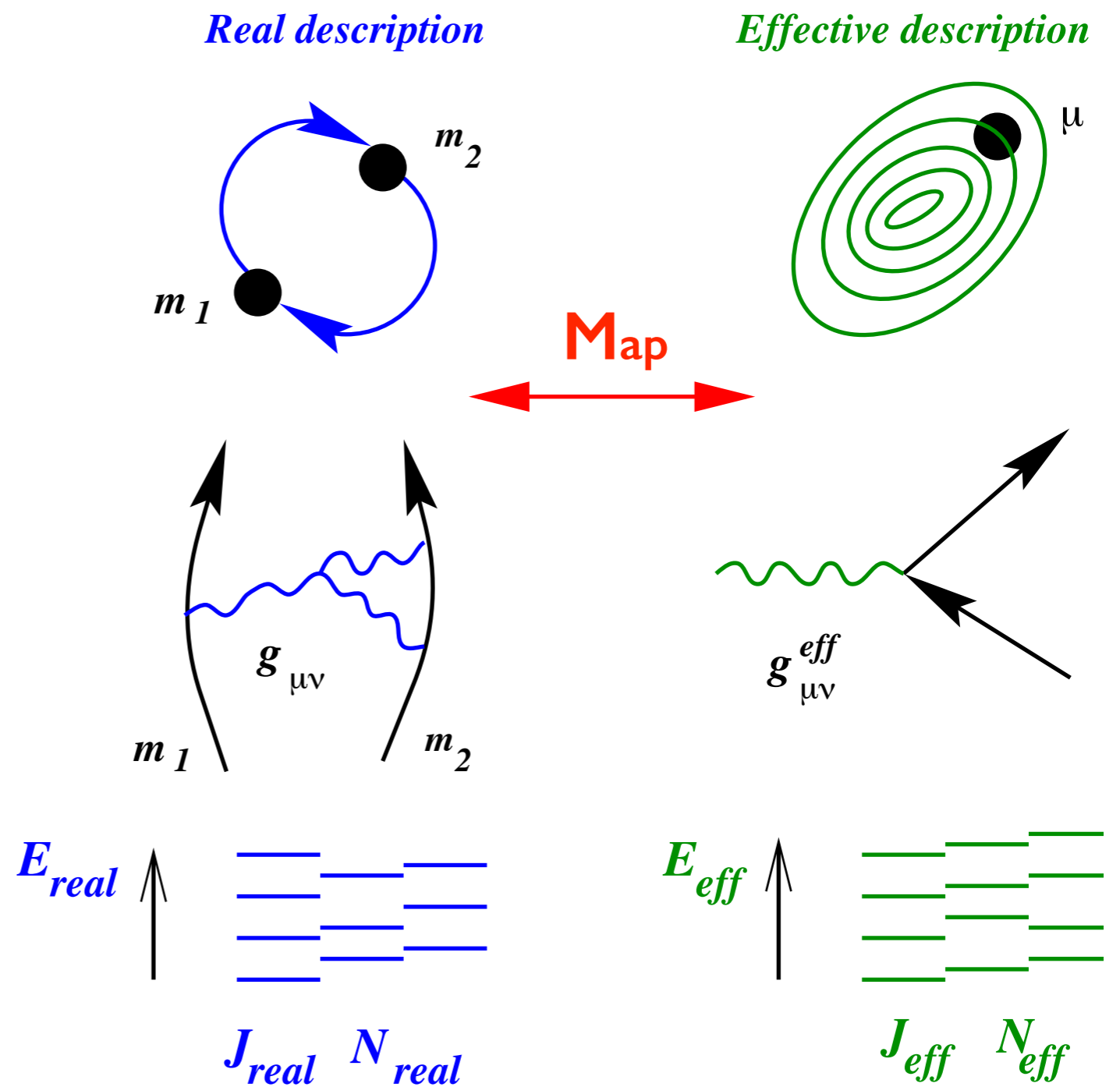
# The effective-one-body approach in a nutshell

$$\nu = \frac{\mu}{M} \quad 0 \leq \nu \leq 1/4$$

$$\mu = \frac{m_1 m_2}{M} \quad M = m_1 + m_2$$

- Two-body dynamics is mapped into dynamics of **one-effective body** moving **in deformed black-hole spacetime**, deformation being the mass ratio.

- Some key **ideas** of EOB model were **inspired by quantum field theory** when describing energy of comparable-mass charged bodies.



(AB & Damour 1999)

# EOB Hamiltonian: resummed conservative dynamics (@2PN)

- Real Hamiltonian

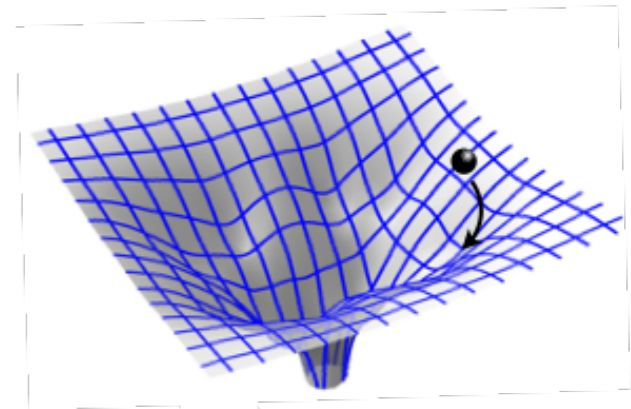
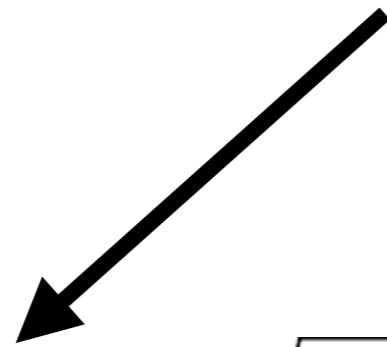


- Effective Hamiltonian

$$H_{\text{real}}^{\text{PN}} = H_{\text{Newt}} + H_{1\text{PN}} + H_{2\text{PN}} + \dots$$

$$H_{\text{eff}}^{\nu} = \mu \sqrt{A_{\nu}(r) \left[ 1 + \frac{\mathbf{p}^2}{\mu^2} + \left( \frac{1}{B_{\nu}(r)} - 1 \right) \frac{p_r^2}{\mu^2} \right]}$$

$$ds_{\text{eff}}^2 = -A_{\nu}(r) dt^2 + B_{\nu}(r) dr^2 + r^2 d\Omega^2$$



(credit: Hinderer)

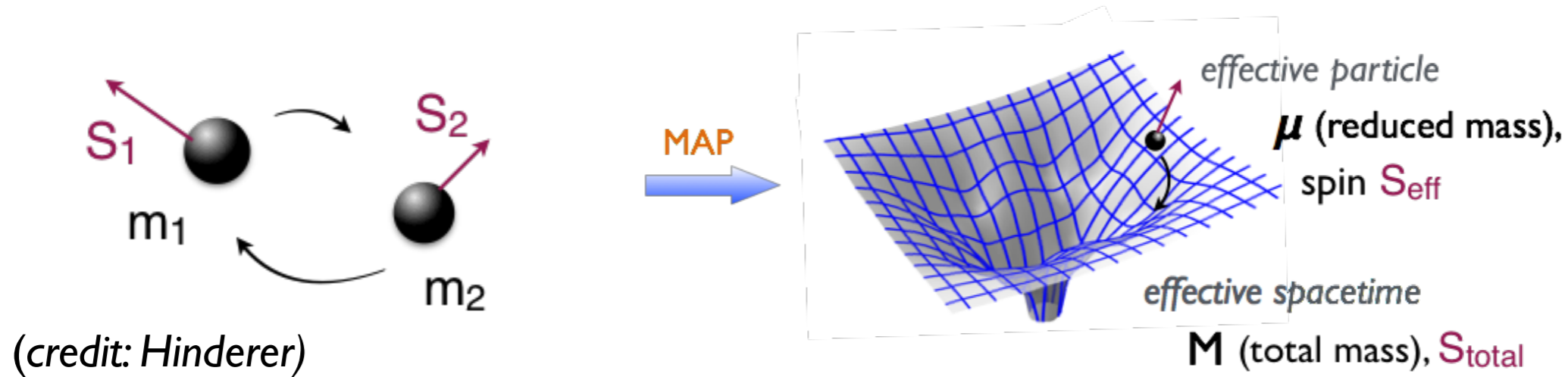
- EOB Hamiltonian:  $H_{\text{real}}^{\text{EOB}} = M \sqrt{1 + 2\nu \left( \frac{H_{\text{eff}}^{\nu}}{\mu} - 1 \right)}$

- Dynamics condensed  $A_{\nu}(r)$  and  $B_{\nu}(r)$

- $A_{\nu}(r)$ , which encodes the energetics of circular orbits, is quite simple:

$$A_{\nu}(r) = 1 - \frac{2M}{r} + \frac{2M^3\nu}{r^3} + \left( \frac{94}{3} - \frac{41}{32}\pi^2 \right) \frac{M^4\nu}{r^4} + \frac{a_5(\nu) + a_5^{\log}(\nu) \log(r)}{r^5} + \frac{a_6(\nu)}{r^6} + \dots$$

# EOB resummed spin dynamics & waveforms



$$H_{\text{real}}^{\text{EOB}} = M \sqrt{1 + 2\nu \left( \frac{H_{\text{eff}}^{\nu}}{\mu} - 1 \right)}$$

- $H_{\text{eff}}^{\nu}$  with spins, two EOB resumptions:  
(Barausse, Racine & AB 09; Barausse & AB 10, 11)

(Damour 01, Damour, Jaranowski & Schäfer 08; Damour & Nagar 14)

- **EOB equations of motion** (AB et al. 00, 05; Damour et al. 09):

$$\dot{\mathbf{r}} = \frac{\partial H_{\text{real}}^{\text{EOB}}}{\partial \mathbf{p}} \quad F \propto \frac{dE}{dt}, \quad \frac{dE}{dt} \propto \sum_{\ell m} |h_{\ell m}|^2$$

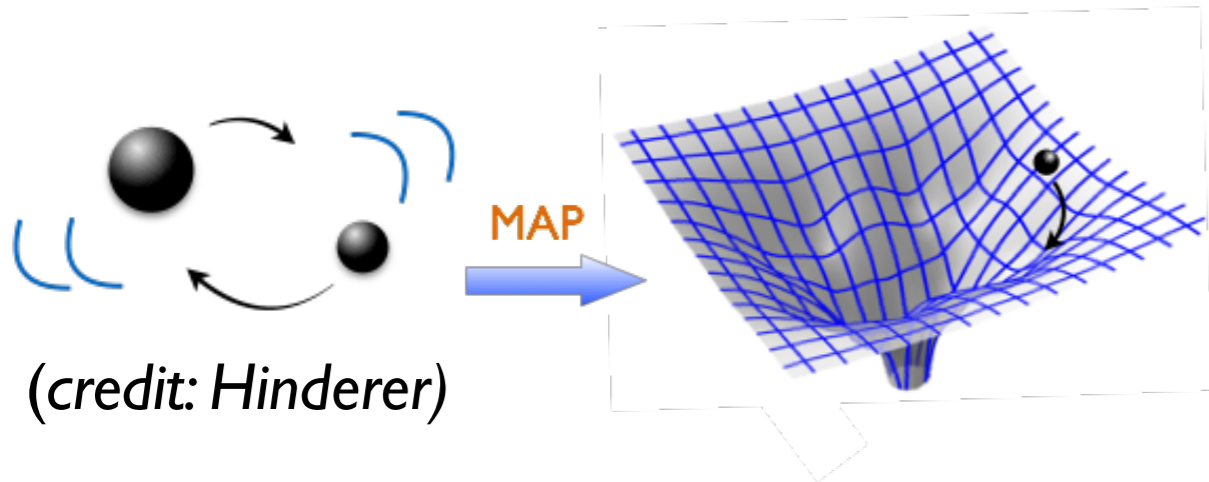
$$\dot{\mathbf{p}} = -\frac{\partial H_{\text{real}}^{\text{EOB}}}{\partial \mathbf{r}} + \mathbf{F} \quad \dot{\mathbf{S}} = \{ \mathbf{S}, H_{\text{real}}^{\text{EOB}} \}$$

- **EOB waveforms** (AB et al. 00; Damour et al. 09; Pan, AB et al. 11):

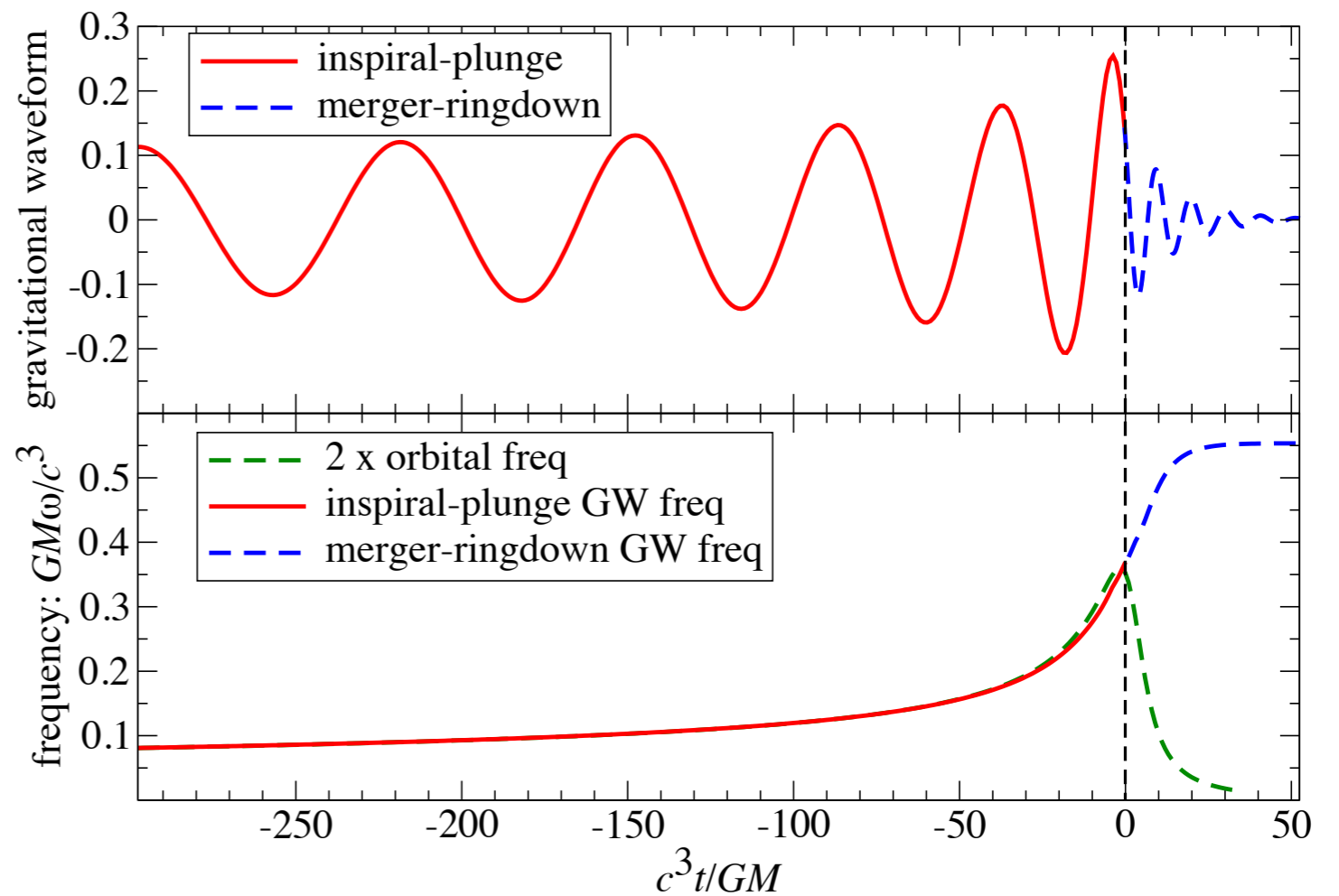
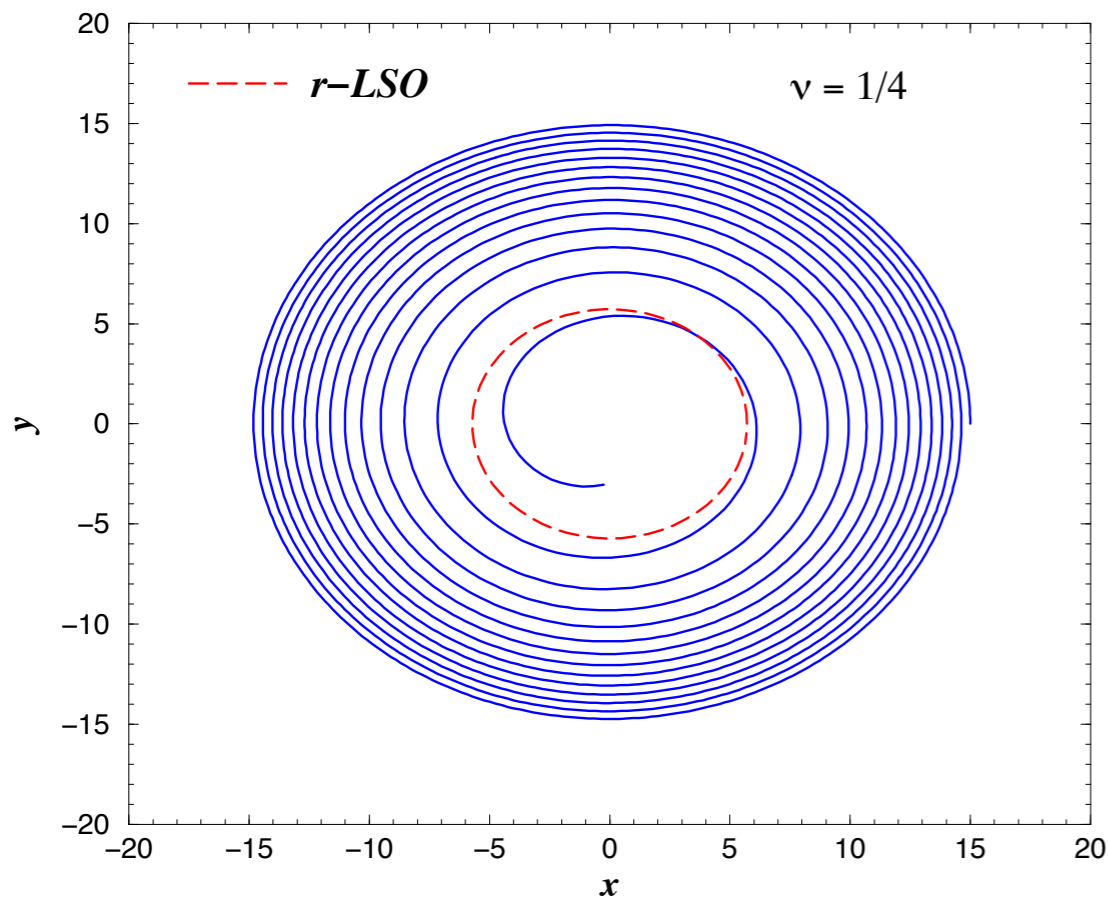
$$h_{\ell m}^{\text{insp-plunge}} = h_{\ell m}^{\text{Newt}} e^{-im\Phi} S_{\text{eff}} T_{\ell m} e^{i\delta_{\ell m}} (\rho_{\ell m})^{\ell} h_{\ell m}^{\text{NQC}}$$



# EOB inspiral-merger-ringdown analytic waveform



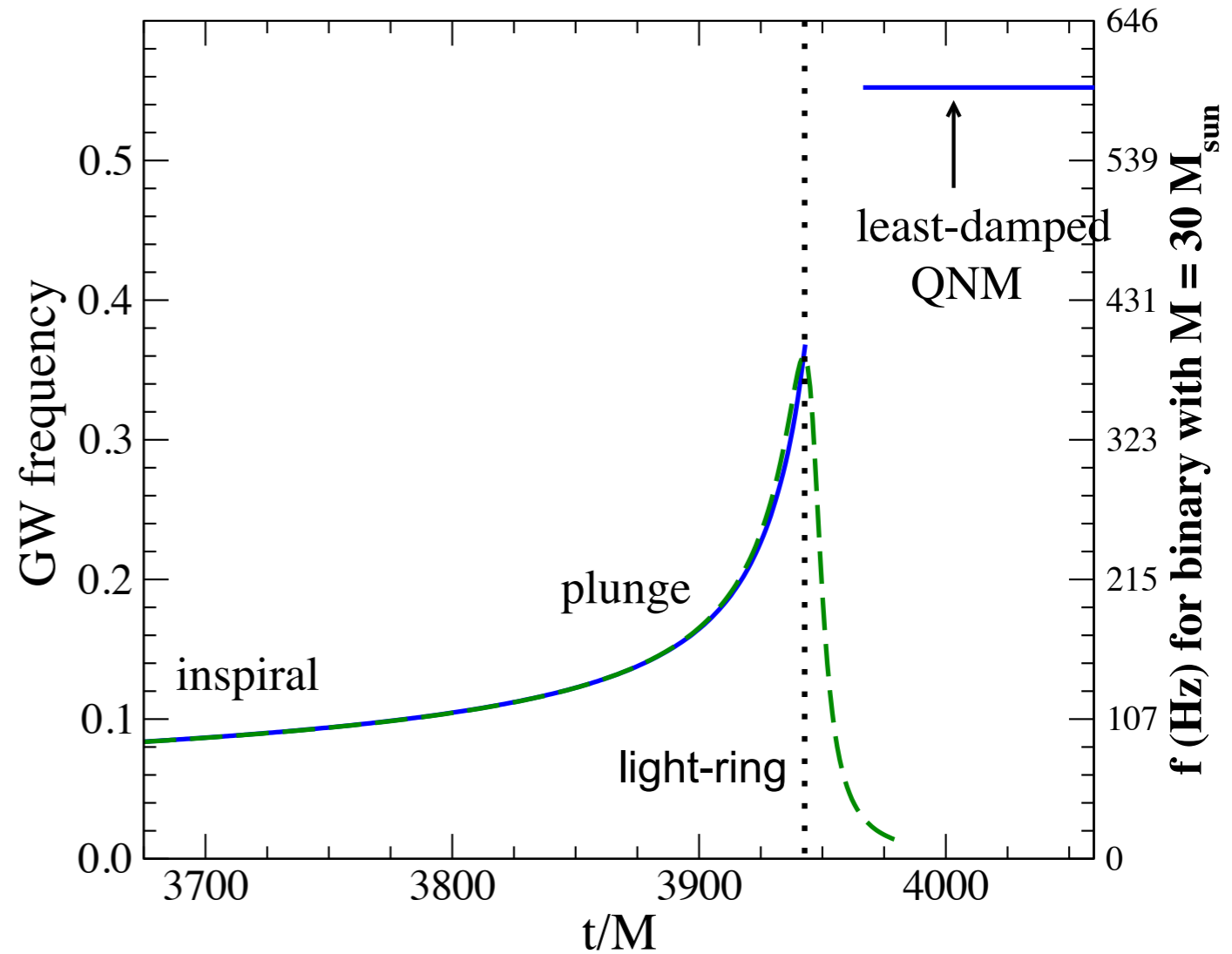
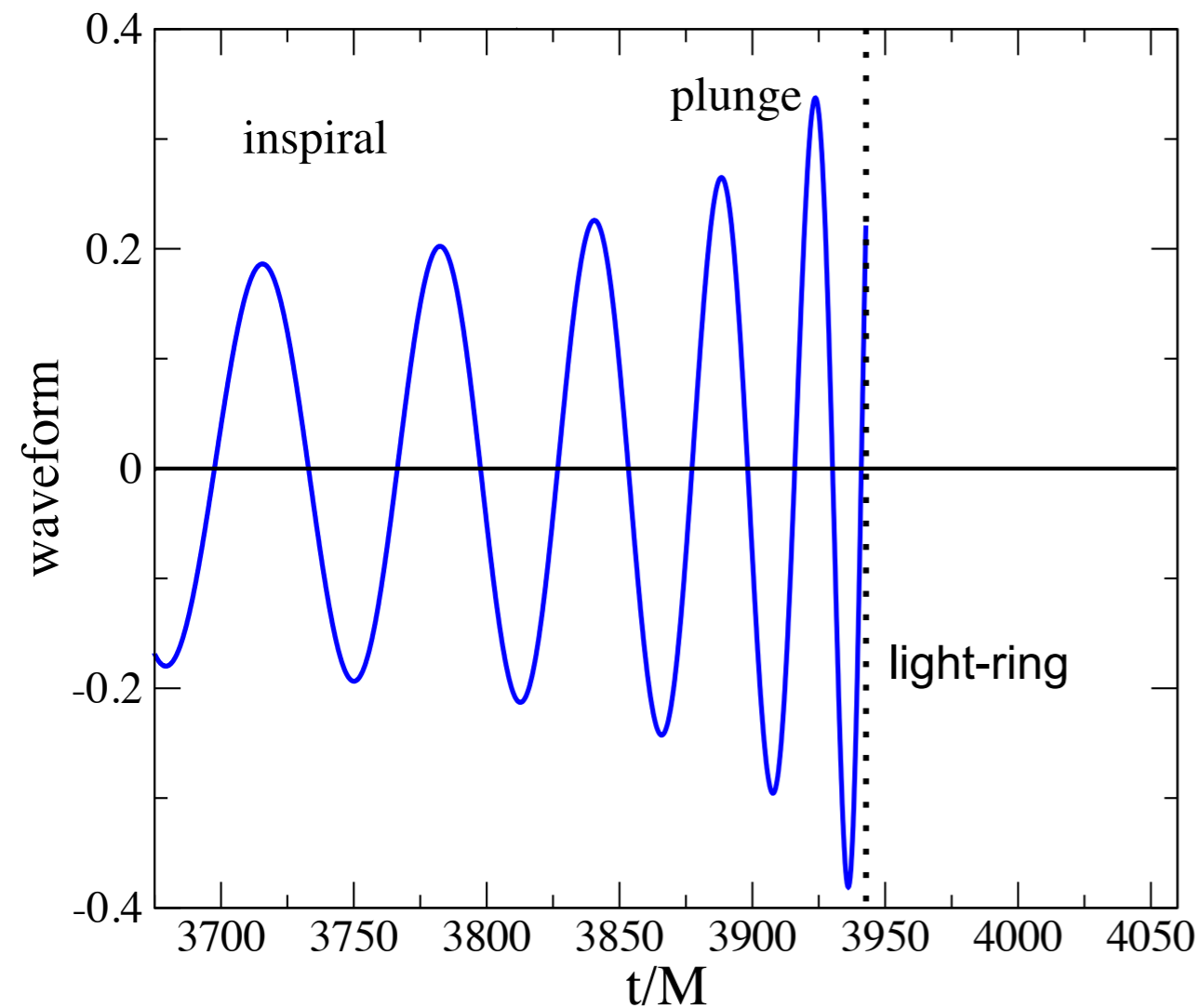
$$E_{\text{real}}^2 = m_1^2 + m_2^2 + 2m_1m_2 \left( \frac{E_{\text{eff}}}{\mu} \right)$$



(AB & Damour 00)

# EOB inspiral-plunge waveform & frequency

- Evolve **two-body dynamics up to light ring** (or photon orbit) and then ...

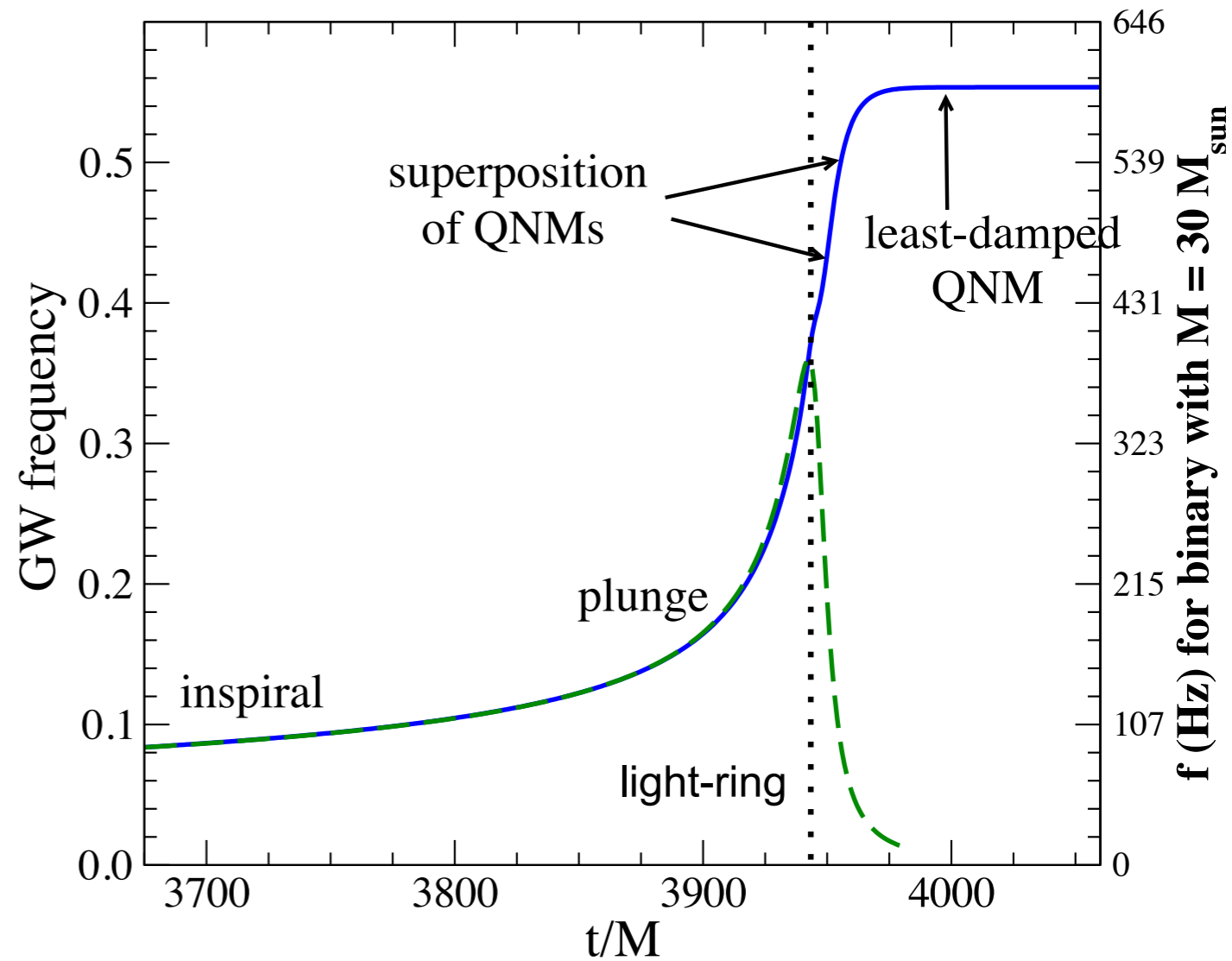
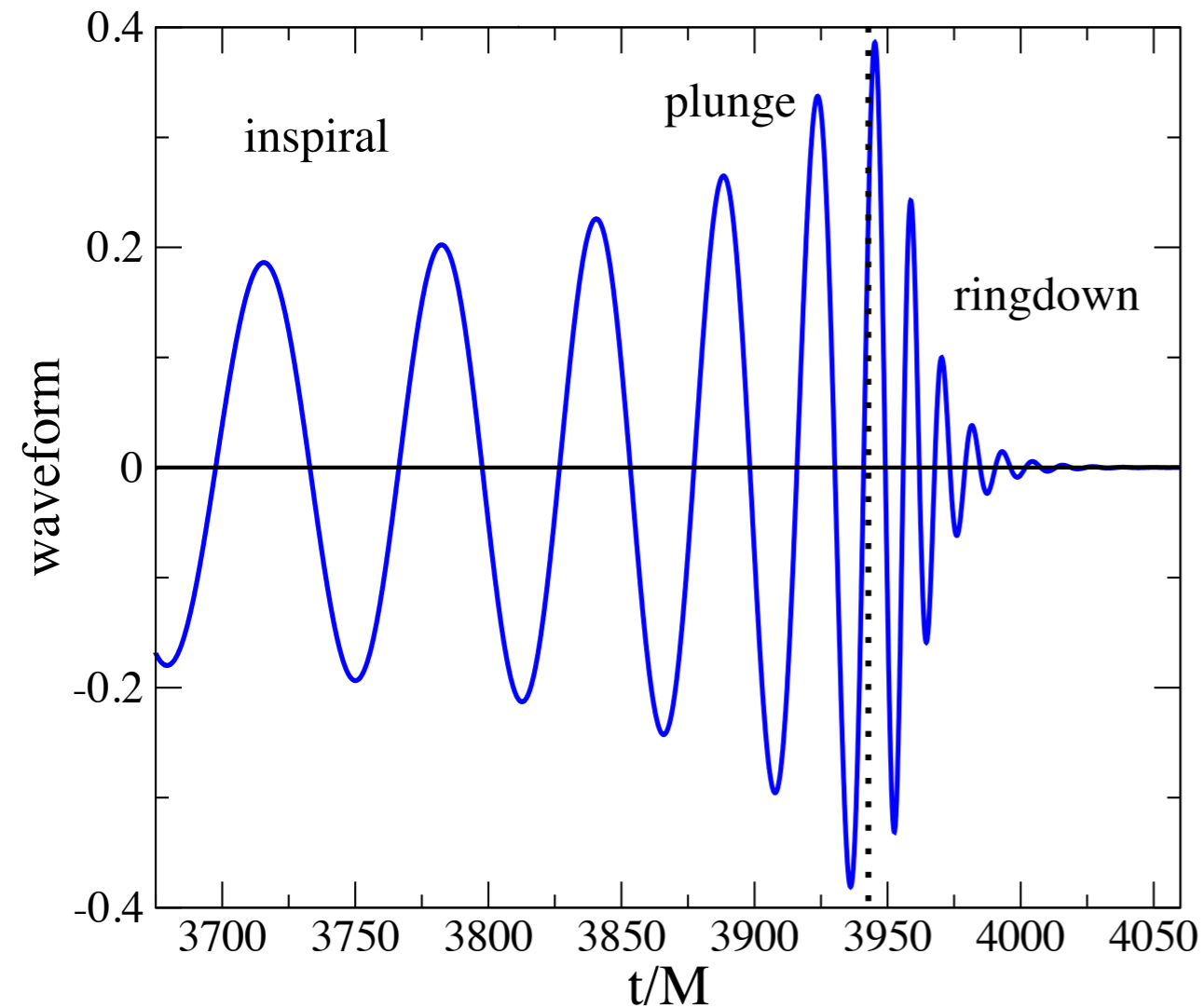


- **Quasi-normal modes** excited at **light-ring crossing**

(Goebel 1972, Davis et al. 1972, Ferrari et al. 1984, Damour et al. 07, Barausse et al. 11, Price et al. 15)

# EOB inspiral-merger-ringdown waveform & frequency

... attach **superposition of quasi-normal modes** of **remnant** black hole.



- Very **short/simple transition** plunge-merger-ringdown: “easy” to model!

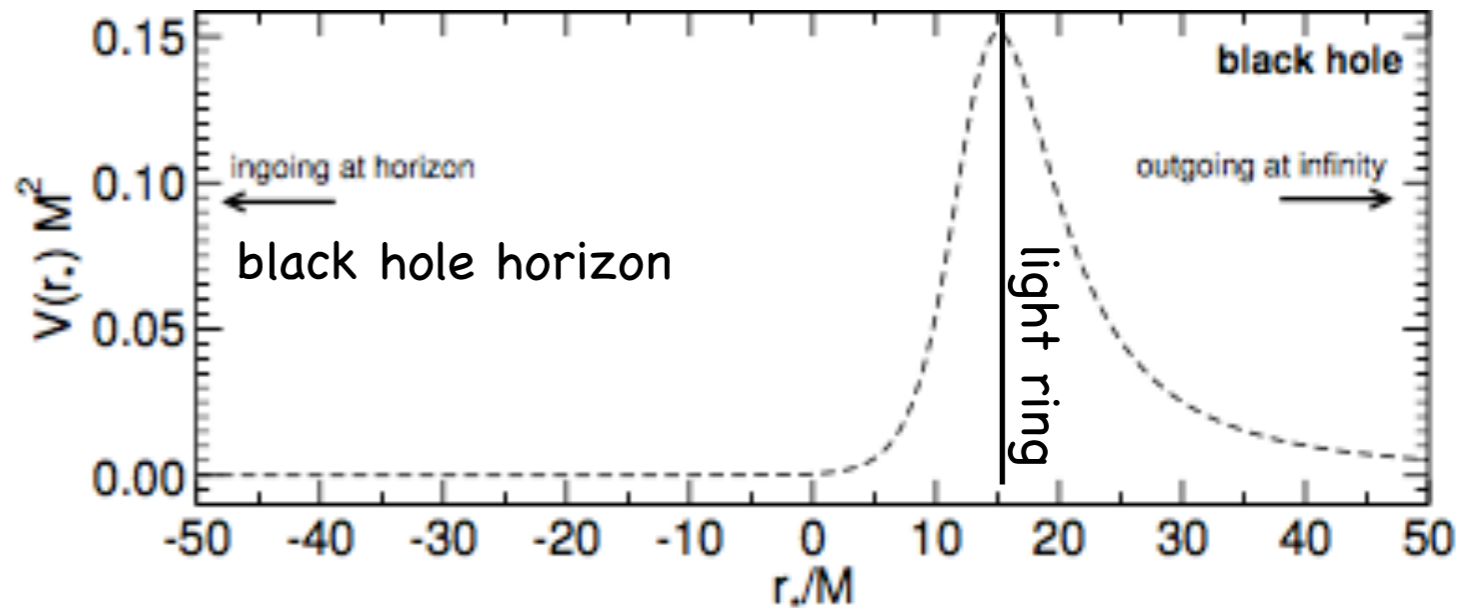
- Energy **quickly released** during merger: 2%-12% $M$

# On the simplicity of merger signal

- Equation of **gravitational perturbations** in black-hole spacetime

(Regge & Wheeler 56, Zerilli 70, Teukolsky 72)

$$\frac{\partial^2 \Psi}{\partial t^2} - \frac{\partial^2 \Psi}{\partial r_{\star}^2} + V_{\ell m} \Psi = \mathcal{S}_{\ell m}$$

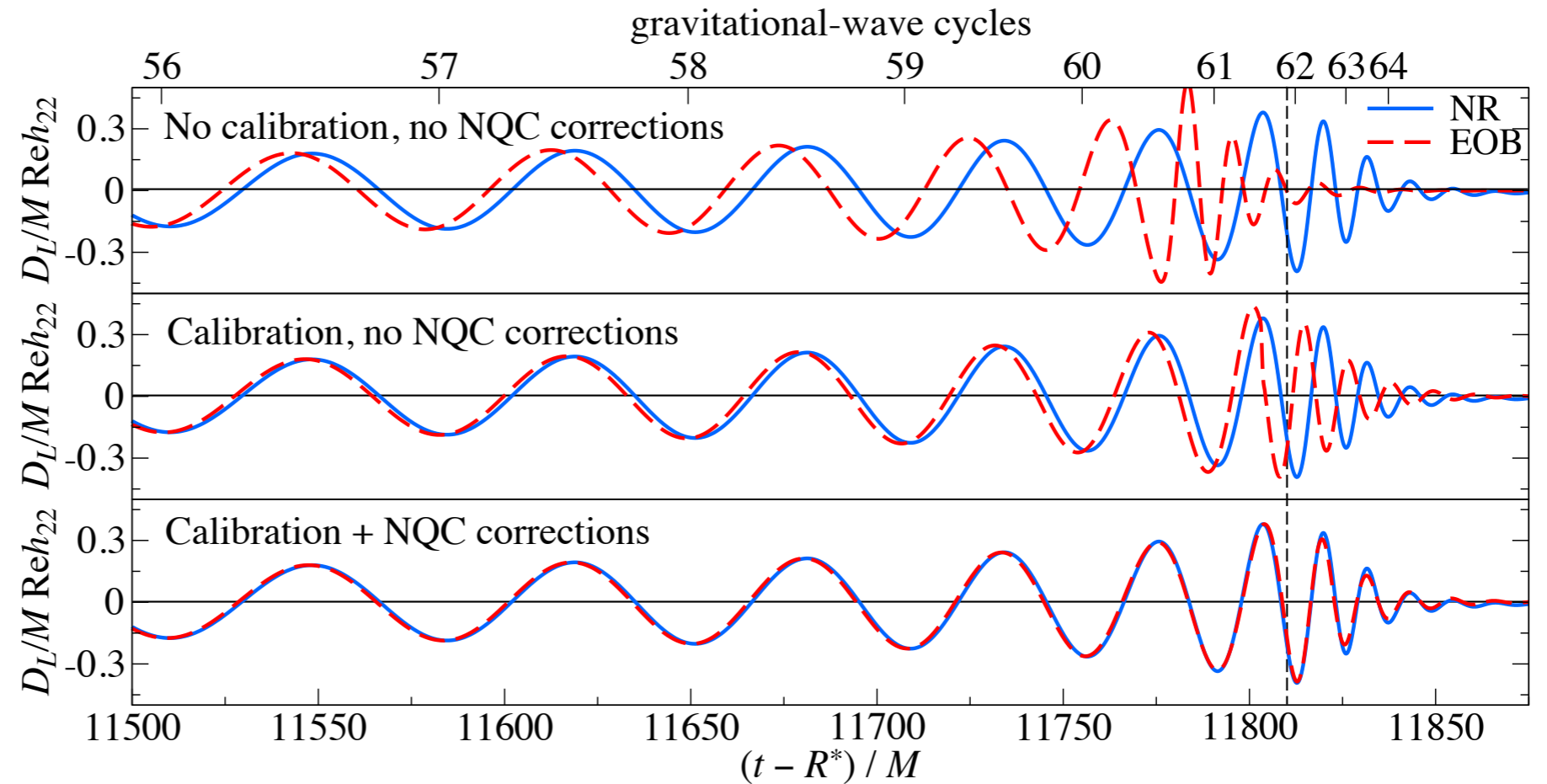


- **Peak** of black-hole potential **close to “light ring”**.
- Once particle is inside potential, **direct gravitational radiation** from its motion is **strongly filtered** by potential barrier (**high-pass filter**).
- Only **black-hole spacetime vibrations** (quasi-normal modes) **leaks out** black-hole potential.

(Goebel 1972, Davis et al. 1972, Ferrari & Mashhoon 1984)

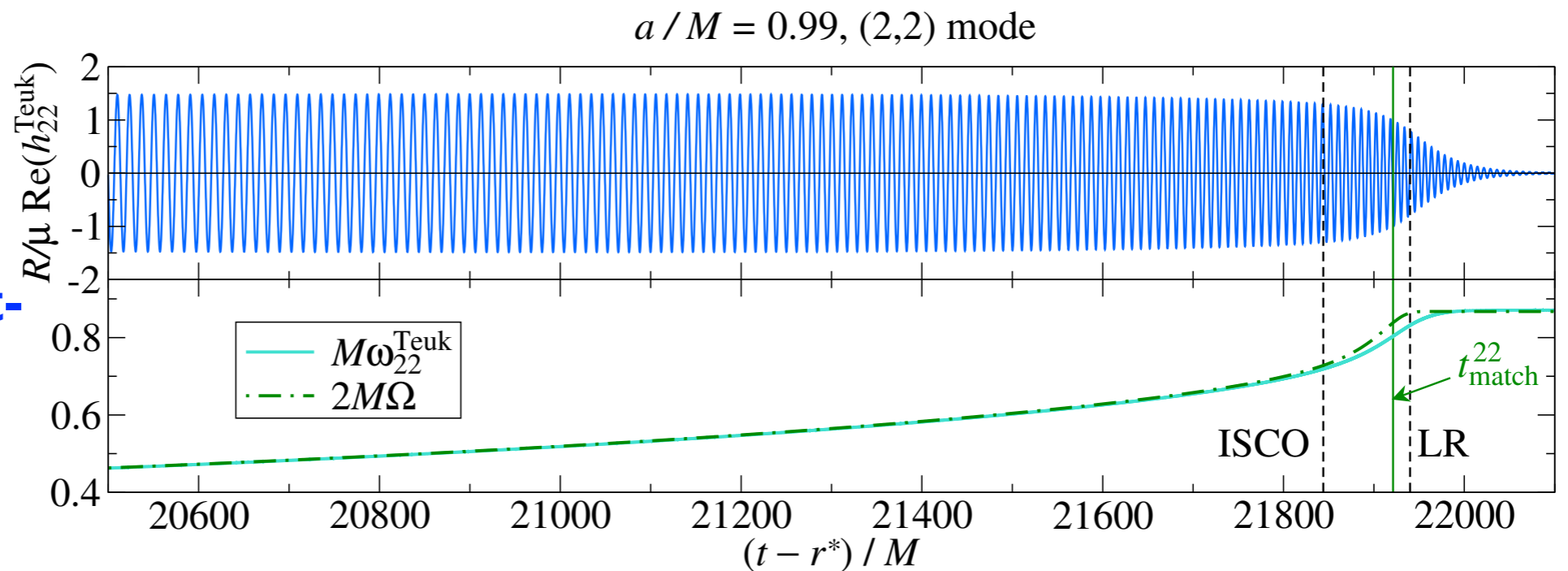
# Completing EOB waveforms using NR/perturbation theory information

- We calibrate to inspiral-merger-ringdown **NR** waveforms.



(credit: Taracchini)

- We calibrate to merger-ringdown waveforms in **test-body limit**.

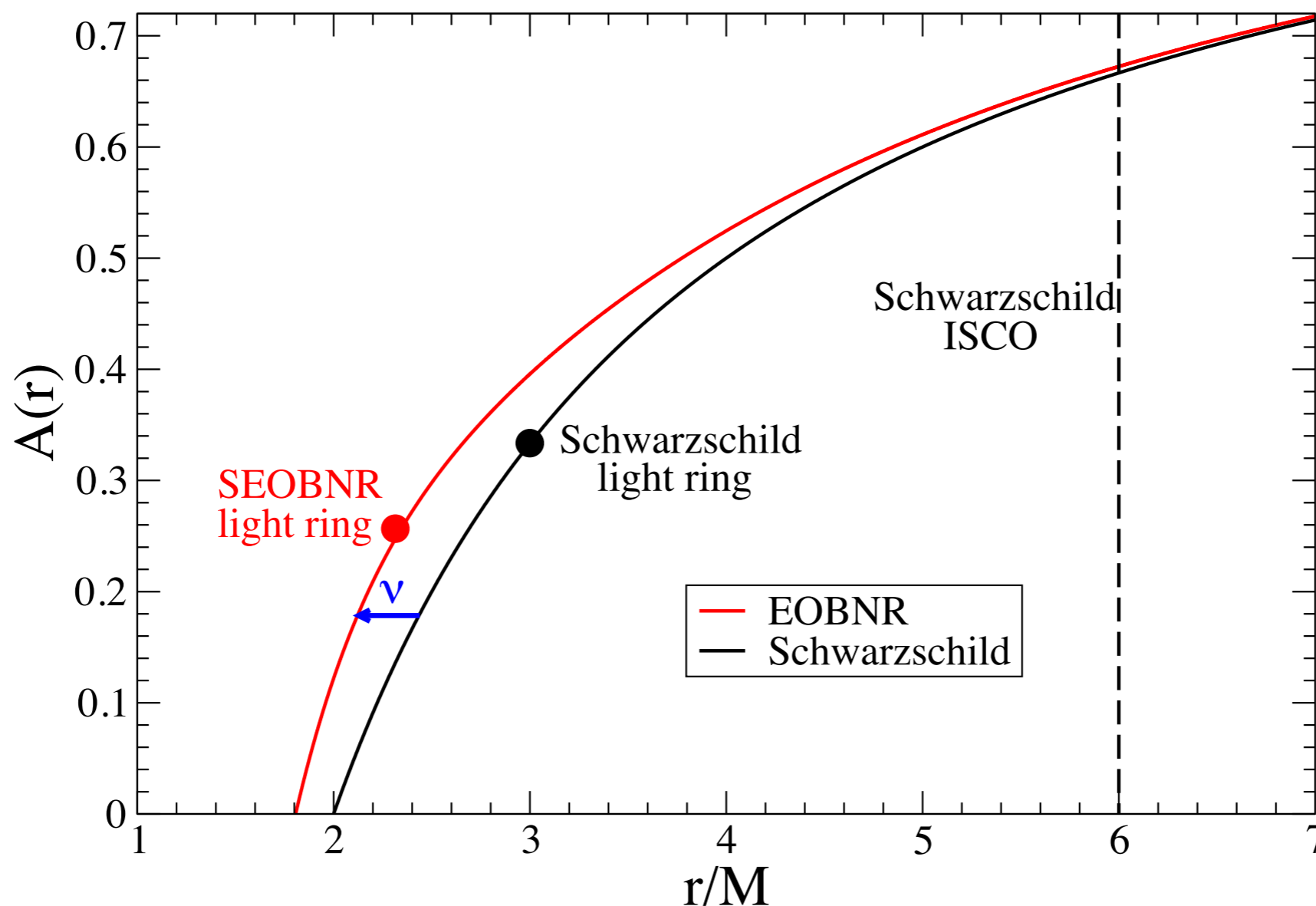


(credit: Taracchini)

# Strong-field effects in binary black holes included in EOB

Finite mass-ratio effects make gravitational interaction less attractive

(Taracchini, AB, Pan, Hinderer & SXS 14)



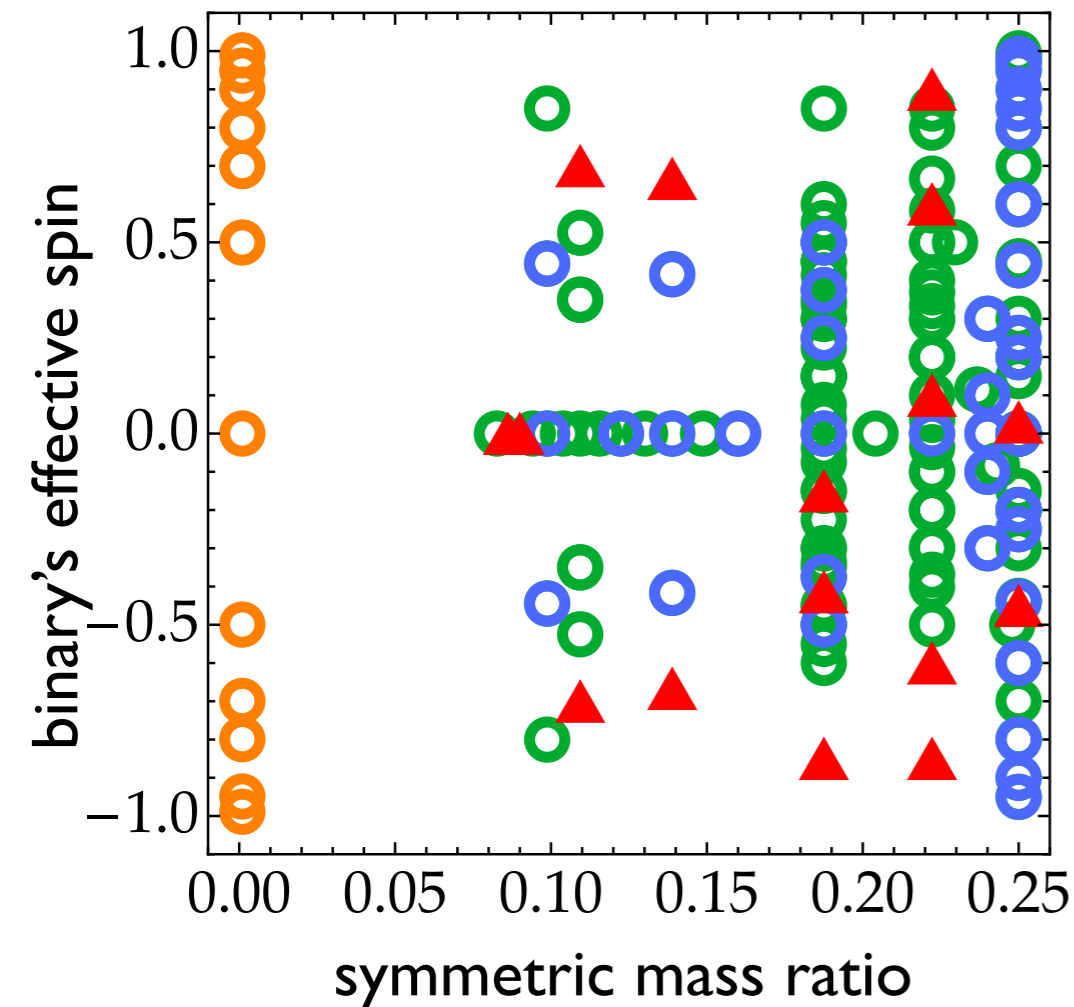
$$A_\nu(r) = 1 - \frac{2M}{r} + \frac{2M^3\nu}{r^3} + \left( \frac{94}{3} - \frac{41}{32}\pi^2 \right) \frac{M^4\nu}{r^4} + \frac{a_5(\nu) + a_5^{\log}(\nu) \log(r)}{r^5} + \frac{a_6(\nu)}{r^6} + \dots$$



# Waveforms for BBHs combining analytical & numerical relativity

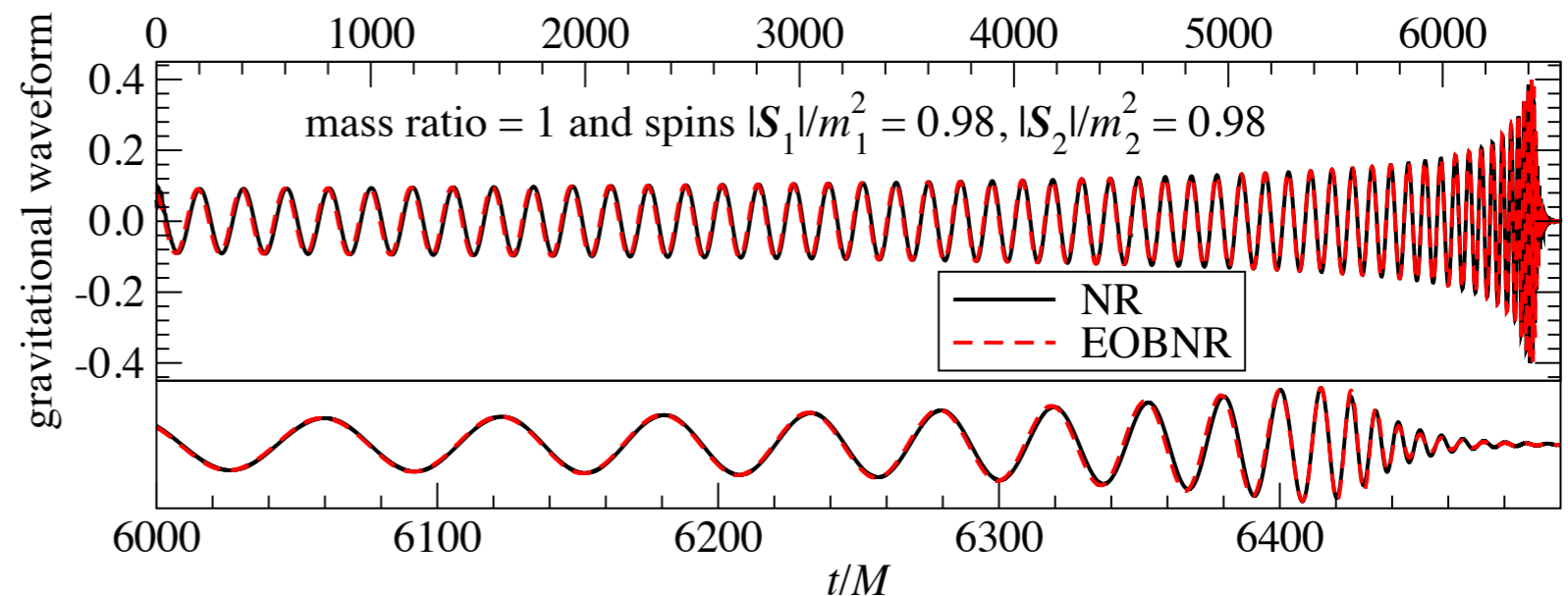
- **Effective-one-body** (EOB) theory & NR (EOBNR)

141 SXS simulations



(Bohe', Shao, Taracchini, AB & SXS 16)

(Taracchini, AB, Pan, Hinderer & SXS 14, Pürrer 15)



(see also Nagar et al. 18)

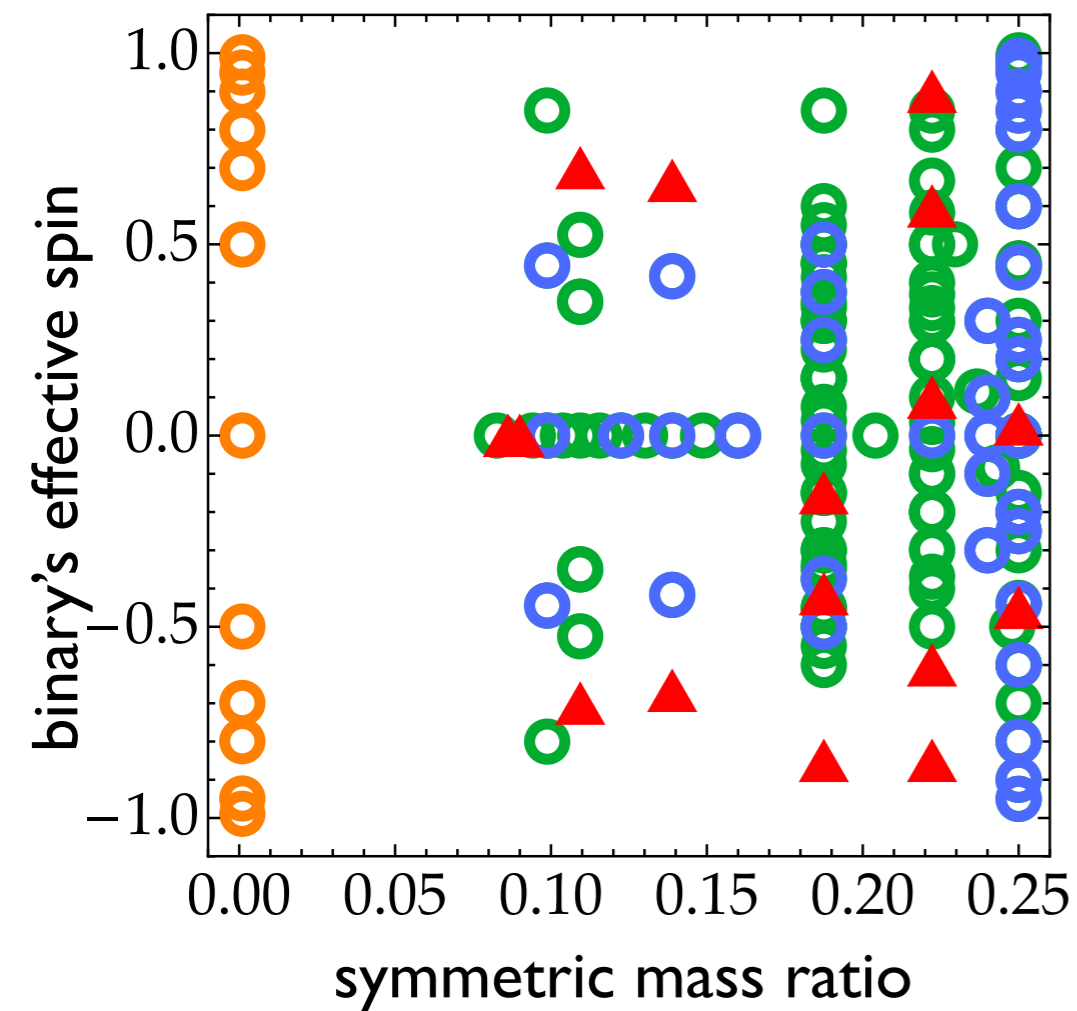
- **Inspiral-merger-ringdown phenomenological** waveforms fitting EOB & NR (IMRPhenom) (Khan et al. 16, Hannam et al. 16)

(If PN were used instead, accuracy will degrade, because of "gap" between PN and NR)

# Waveforms for BBHs combining analytical & numerical relativity

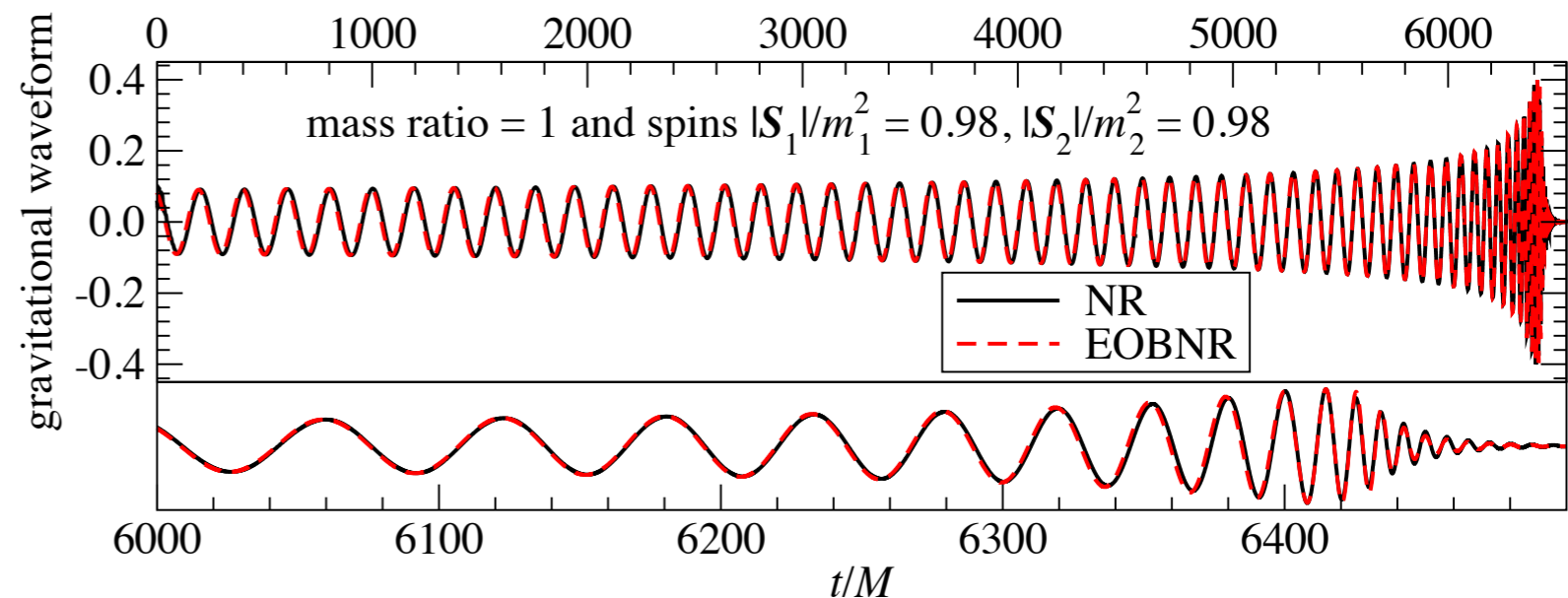
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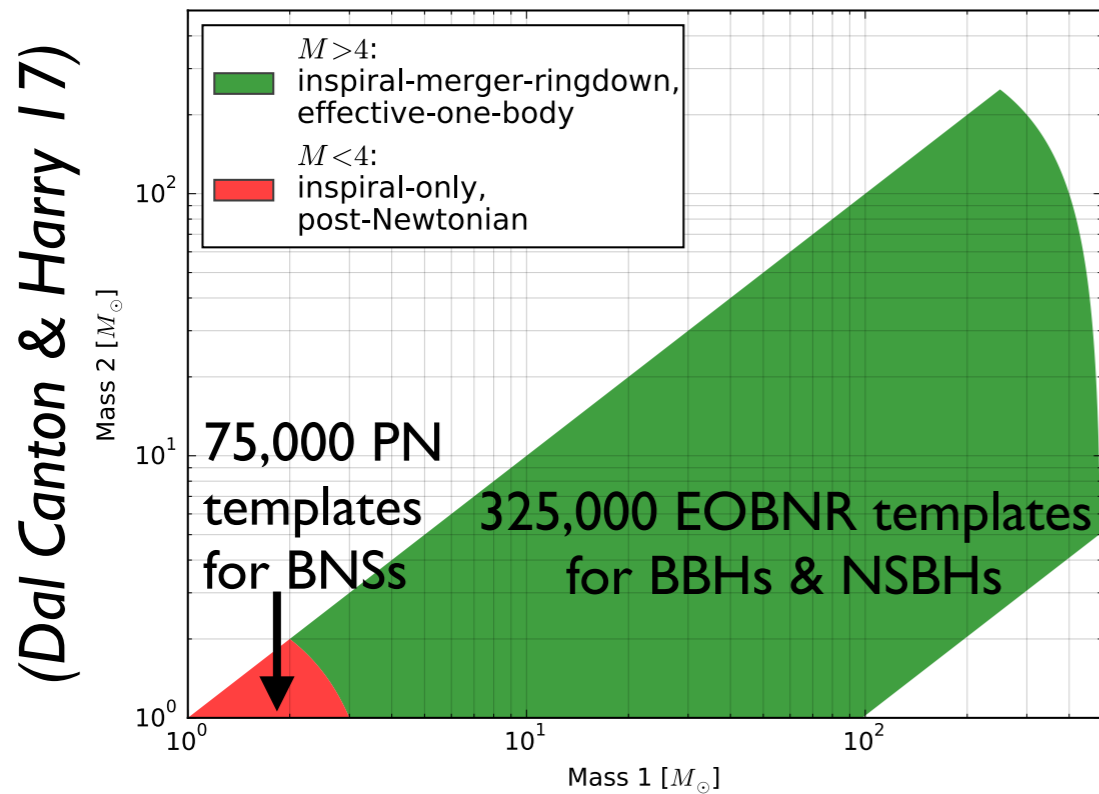
(see also Nagar et al. 18)

- **NR surrogate models** built **directly** interpolating **NR simulations**, which are **“selected”** in **parameter space** using analytical waveform models.

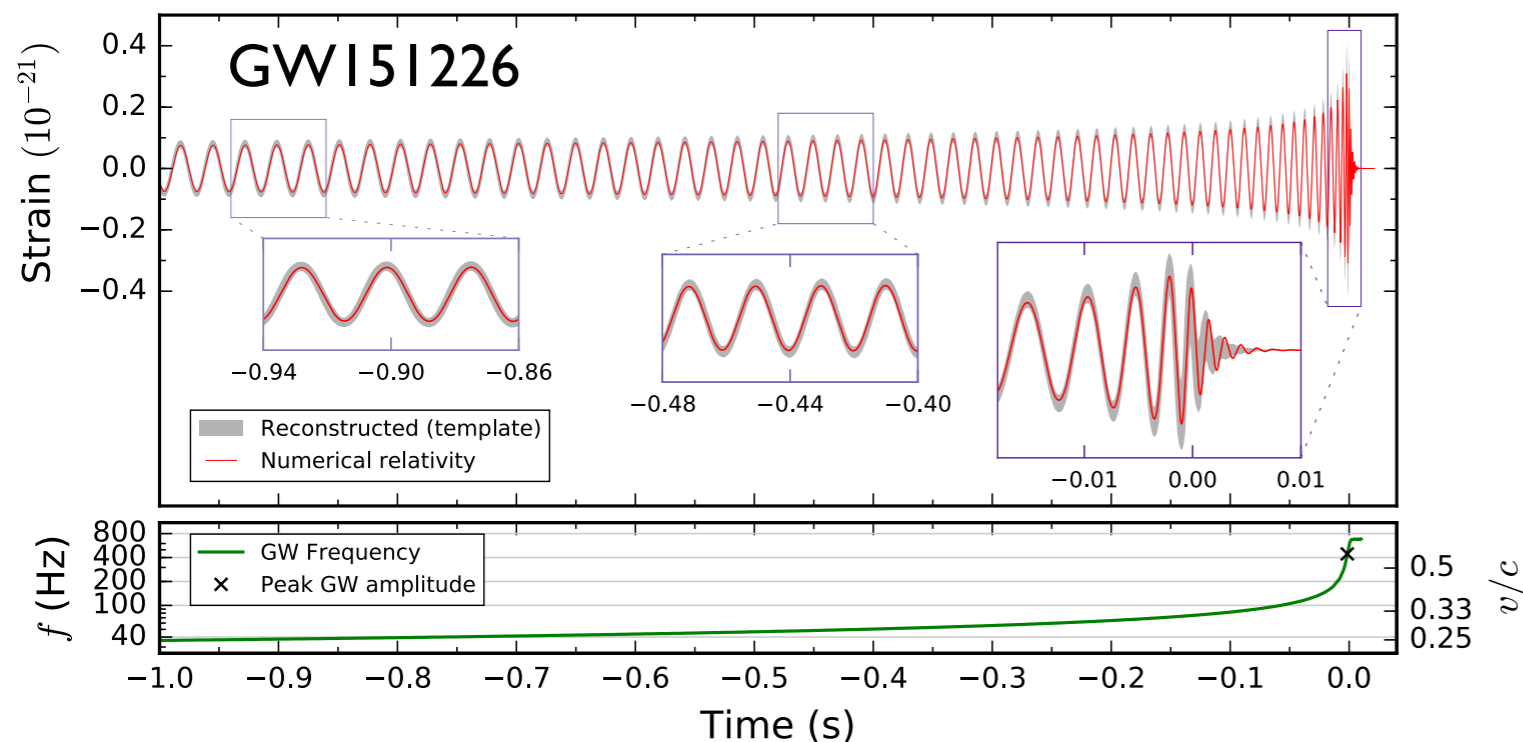
(Blackman et al. 17, Varma et al. 18, 19)

# Template bank for modeled search & possible systematics

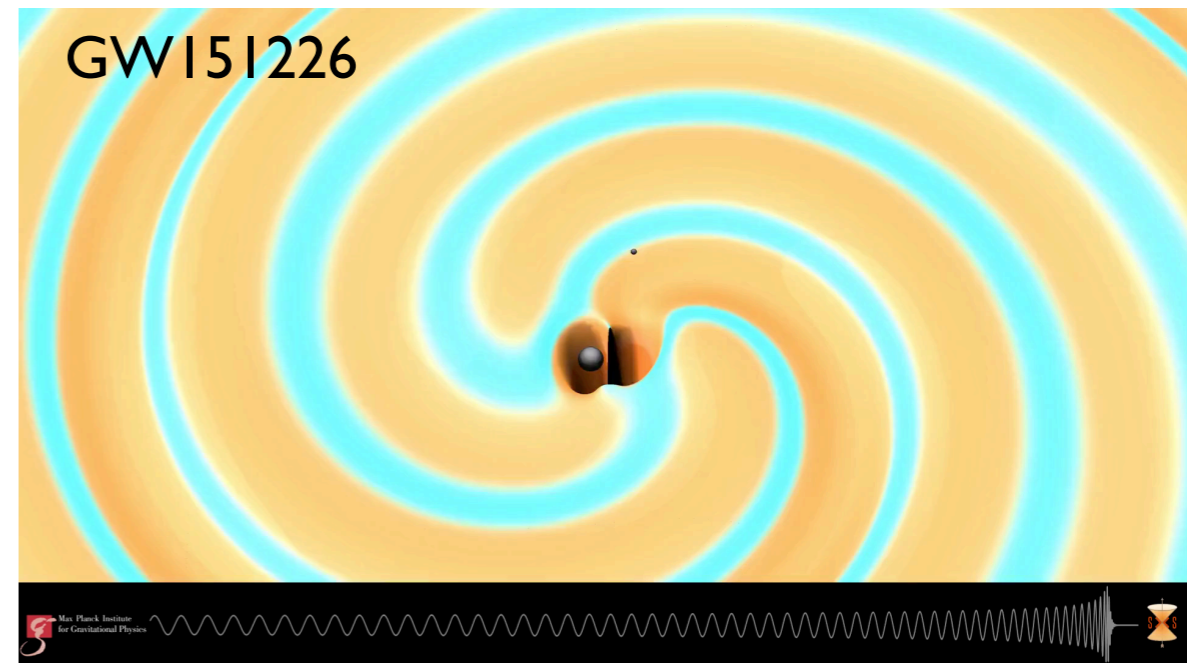
- **Matched filtering** employed



(Abbott et al. PRL 116 (2016) 241103)



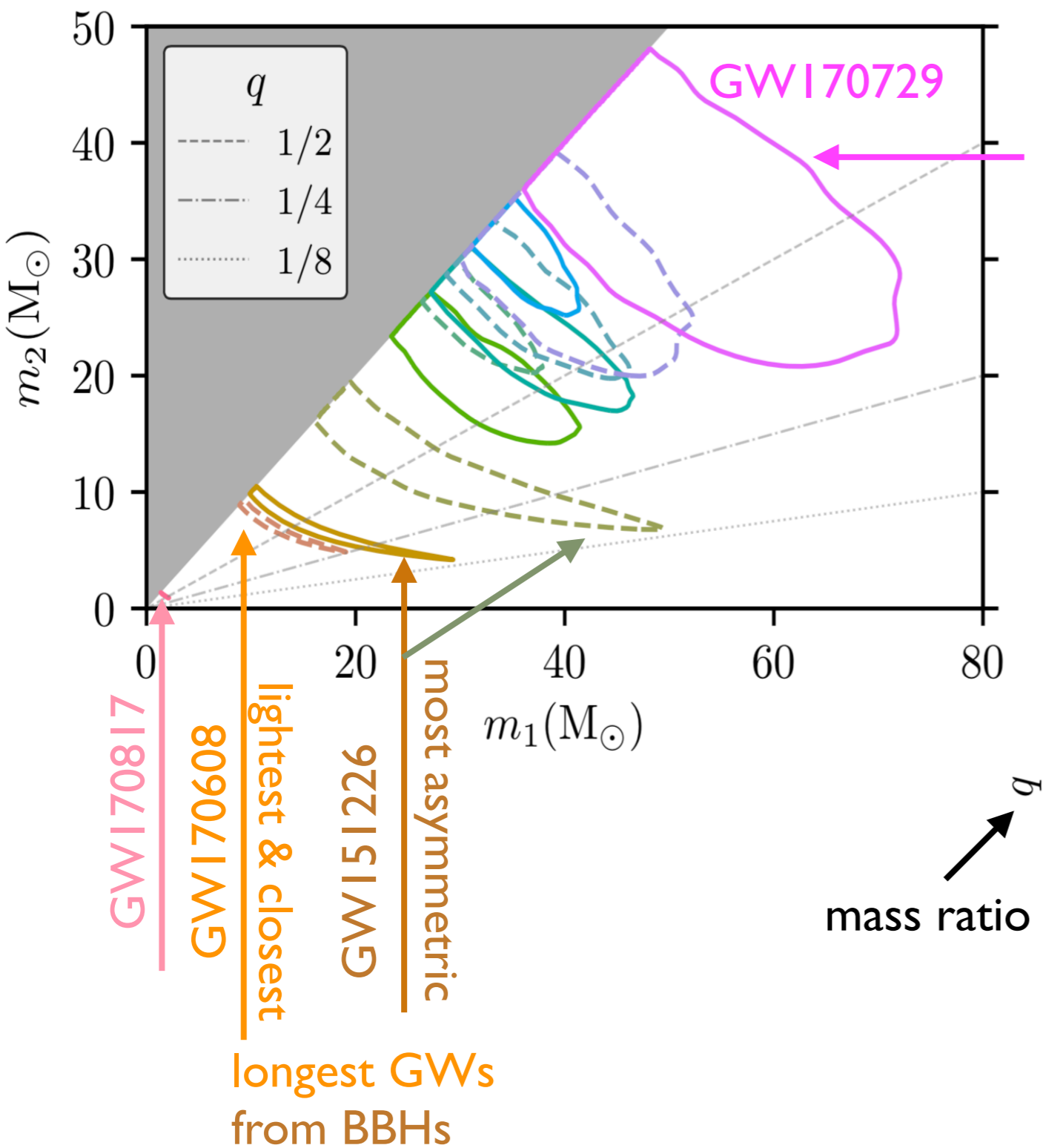
(visualization credit: Dietrich, Haas @AEI)  
(Ossokine, AB & SXS project)



- **Systematics** due to modeling are **smaller than statistical** errors for GW events observed in **O1 & O2 runs**.

(Abbott et al. CQG 34 (2017) 104002)

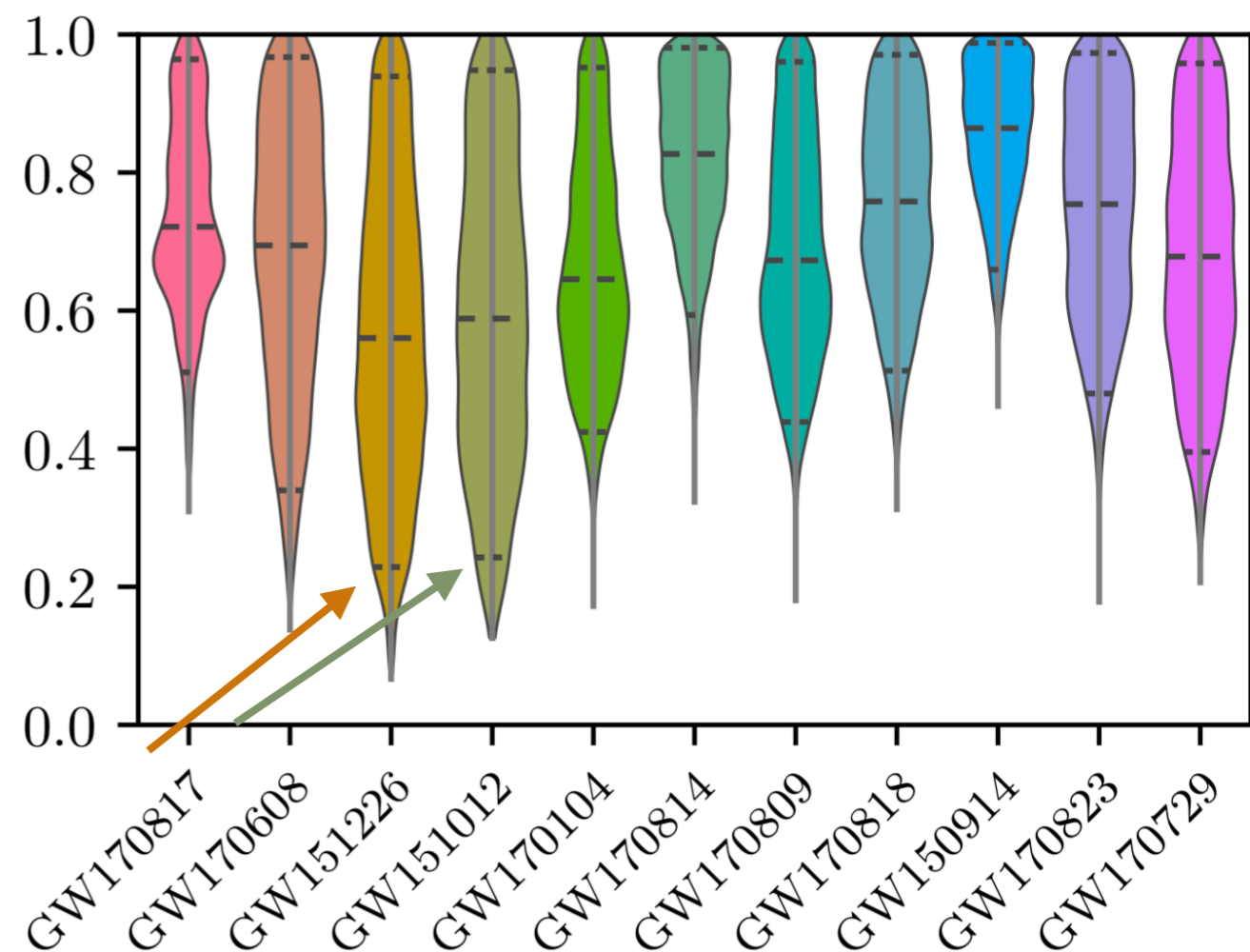
# Unveiling binary properties of GWTC-I: masses



(Abbott et al. PRX 9 (2019) 031040)

heaviest & most distant BBH

(see also Zackay et al. 2019, Venumadhav et al. 2019)



# Unveiling binary properties of GWTC-I: spins

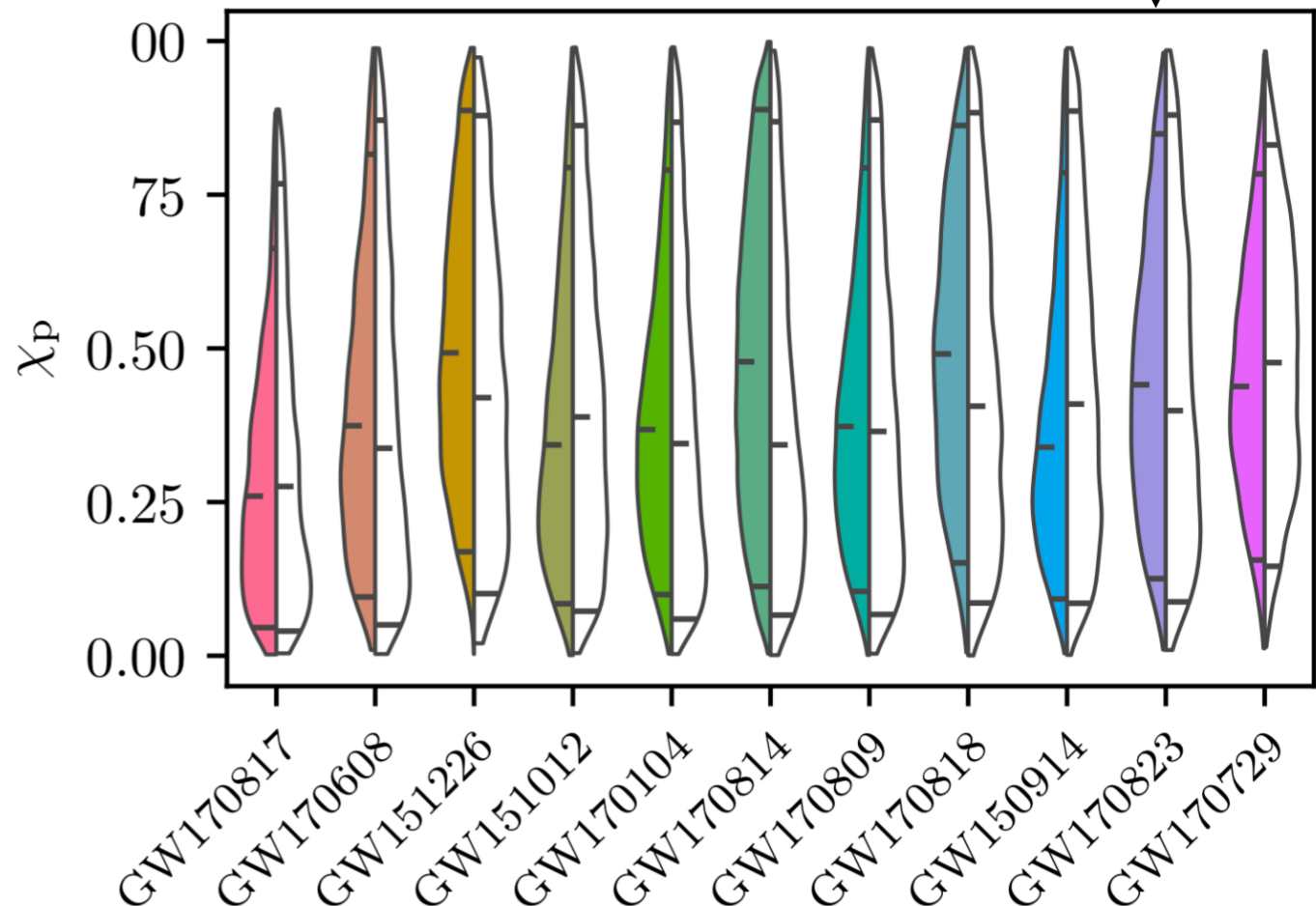
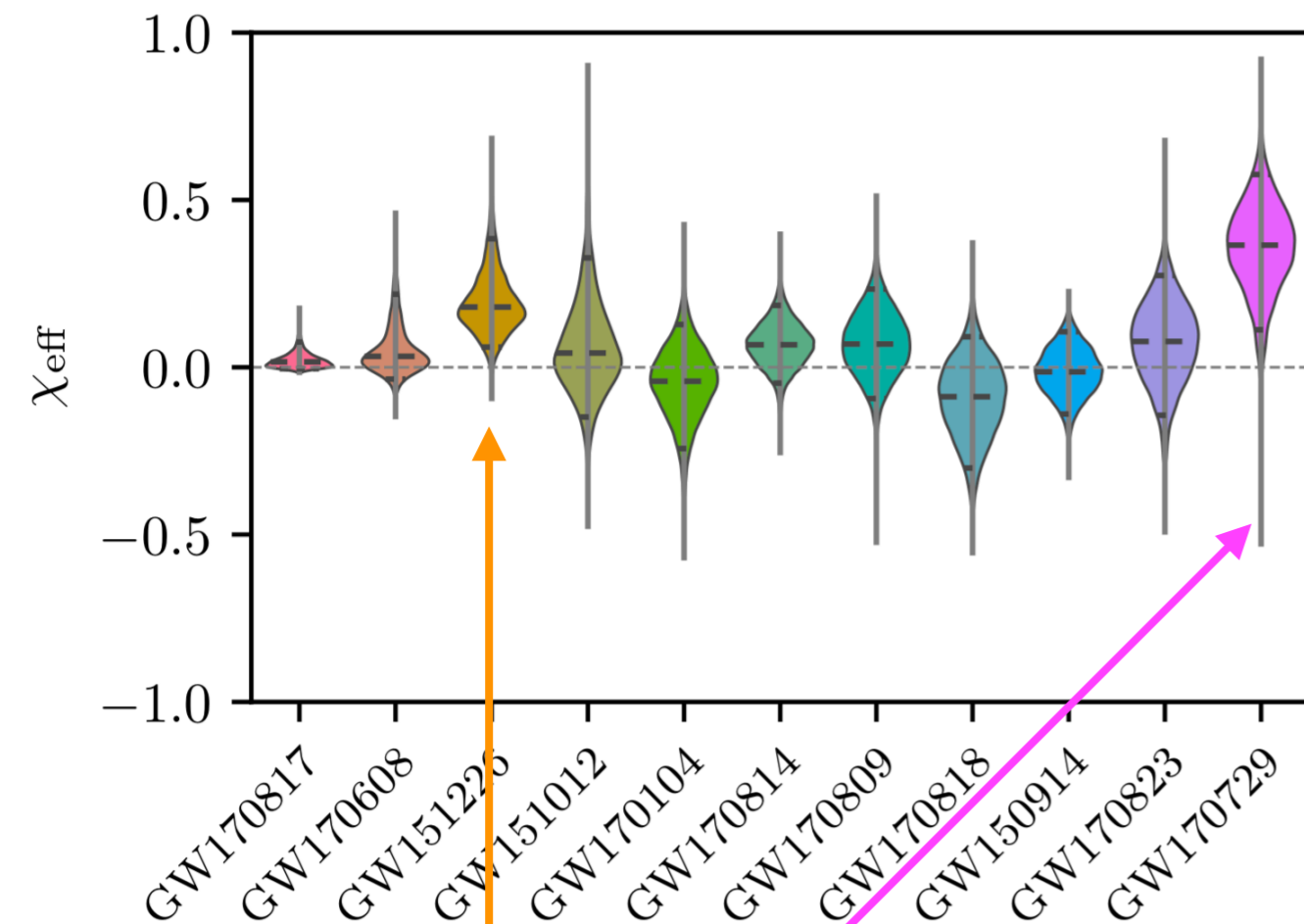
(Abbott et al. PRX 9 (2019) 031040)

$$\chi_{\text{eff}} = \frac{c}{GM} \left( \frac{\mathbf{S}_1}{m_1} + \frac{\mathbf{S}_2}{m_2} \right) \cdot \hat{\mathbf{L}}_N$$

measures spins perpendicular to orbital plane

$$\chi_p = \frac{c}{B_1 G m_1^2} \max(B_1 \mathbf{S}_{1\perp}, B_2 \mathbf{S}_{2\perp})$$

measures spins on the orbital plane



BH's spin larger than 0.2 at 99% confidence.

$\chi_{\text{eff}} = 0$  excluded at 90%

- **Moderate spins**, say  $< 0.6$ .

(see however Zackay et al. 2019)

- **No evidence for spin-precession.**

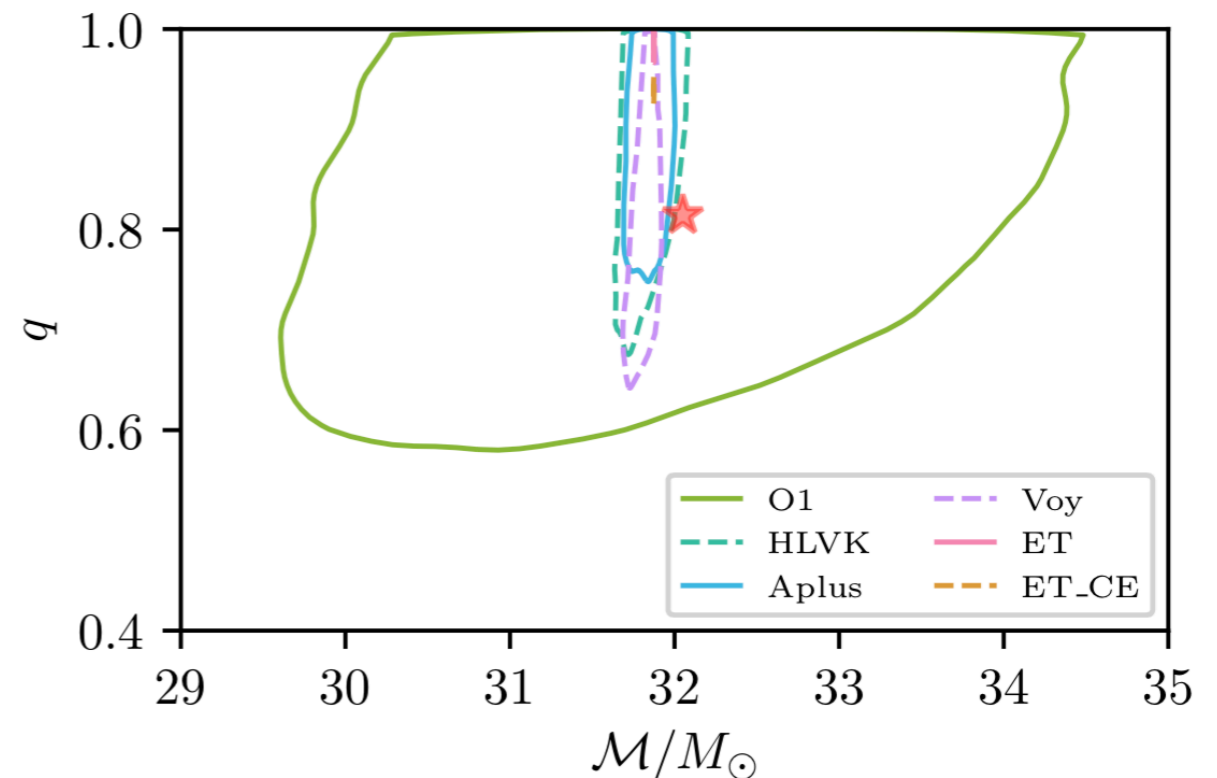
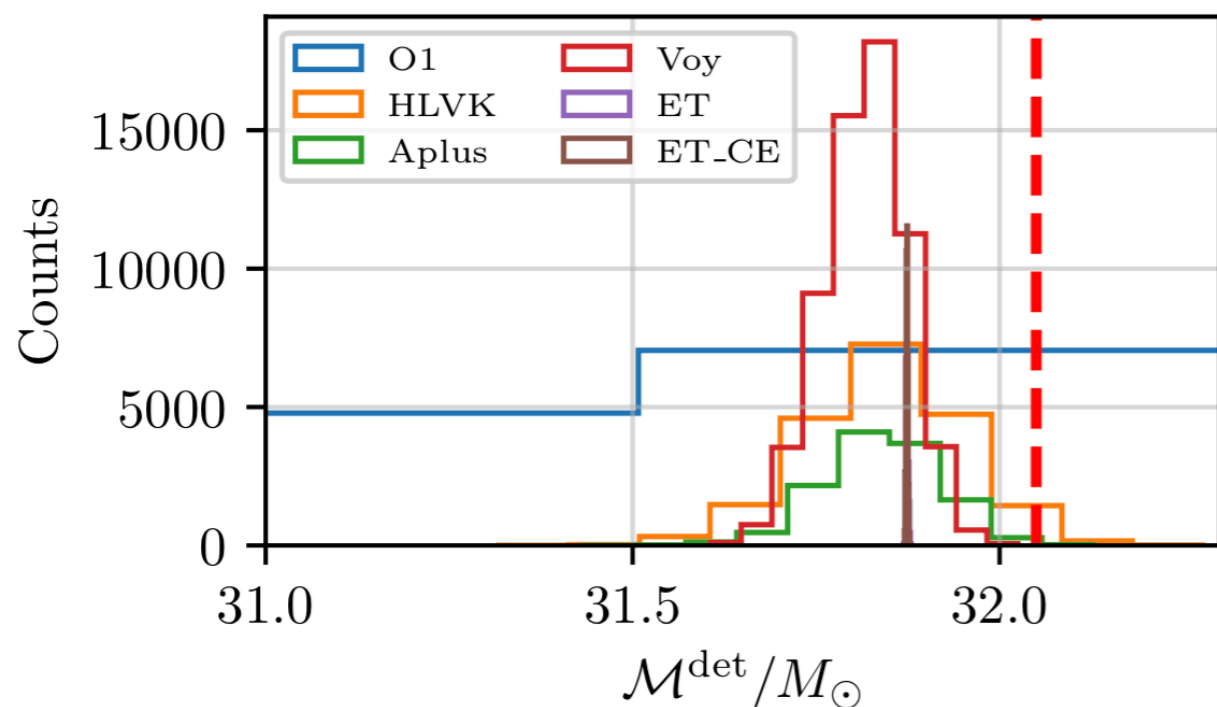


# Systematics due to modeling for GW150914-like event

- Synthetic GW signal of a **binary black hole** at **400 Mpc** is **injected** in Gaussian noise with **aLIGO design-sensitivity** noise-spectral density (**SNR  $\sim 70$** ).
- **Inference** with **one** of currently used waveform models (IMRPhenom).

(Pürrer & Haster in prep 19)

- **GW150914-like NR signal** is injected

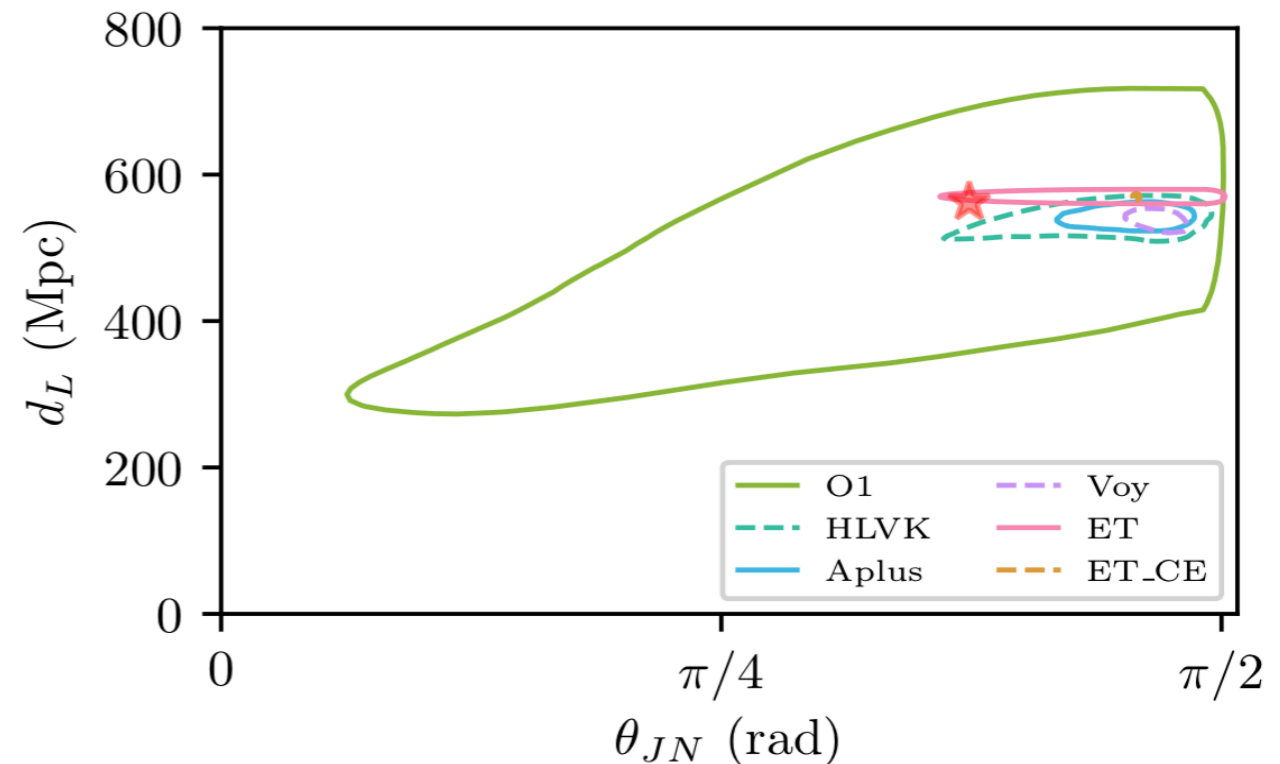
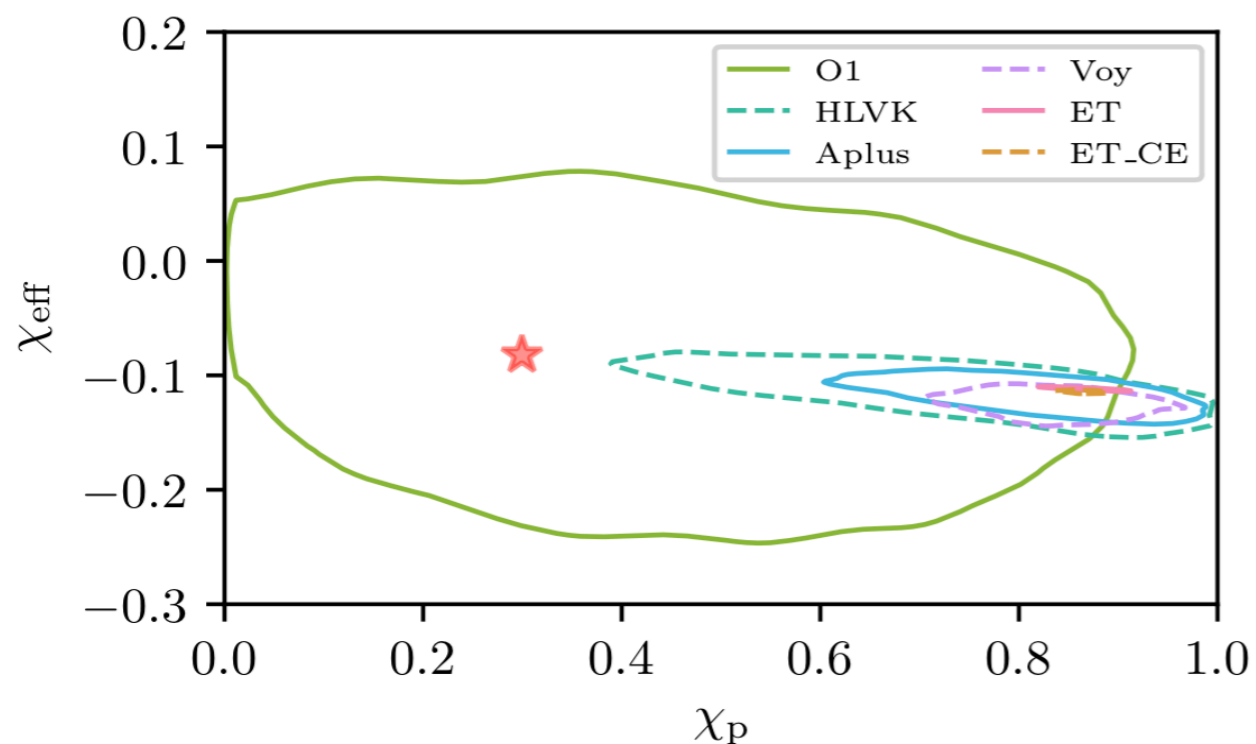




# Systematics due to modeling for GW150914-like event (contd.)

- Synthetic GW signal of a **binary black hole** at **400 Mpc** is **injected** in Gaussian noise with **aLIGO design-sensitivity** noise-spectral density (**SNR  $\sim 70$** ).
- **Inference** with **one** of currently used waveform models (IMRPhenom).

(Pürrer & Haster in prep 19) • **GW150914-like NR signal** is injected



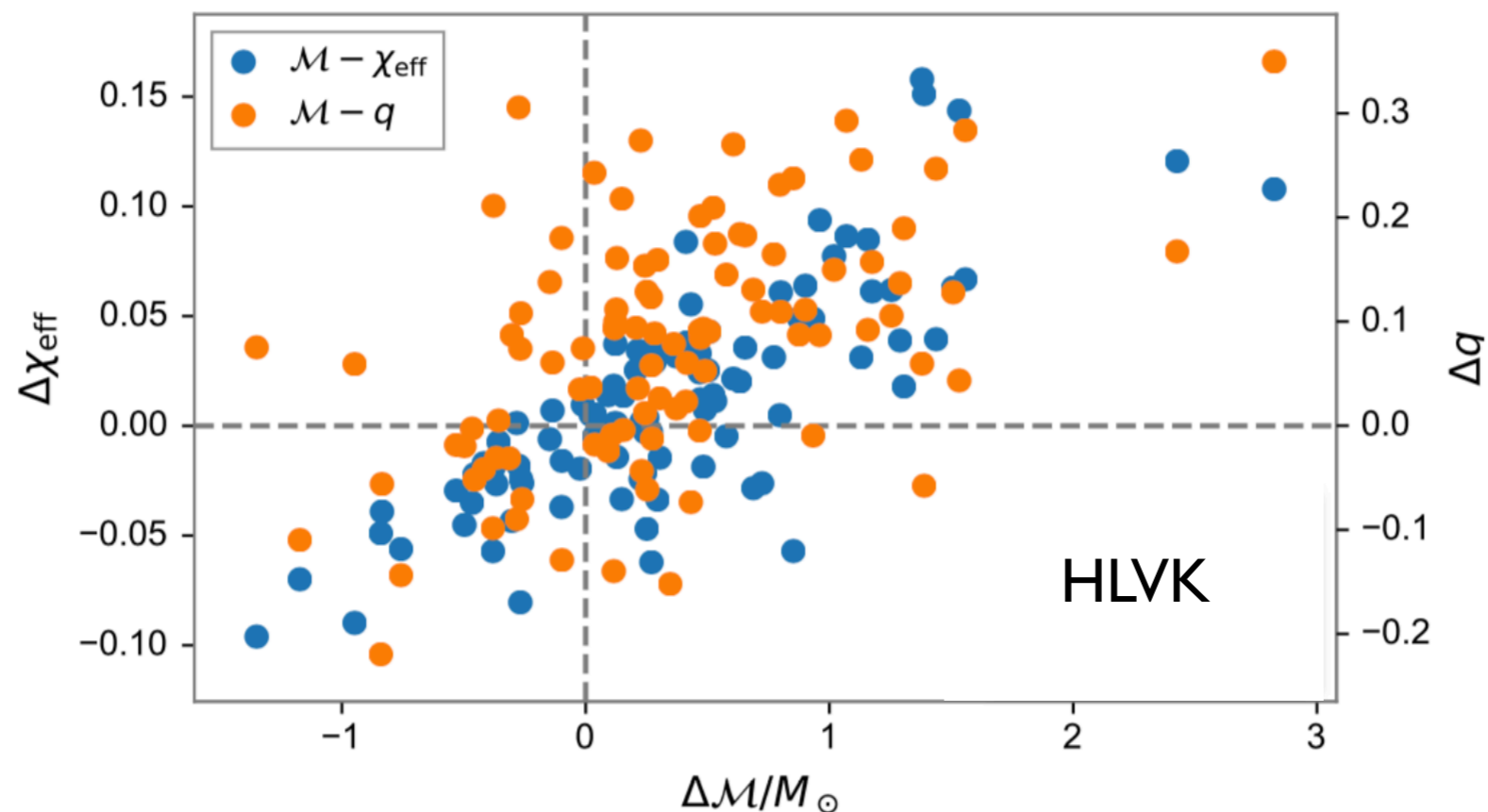
# Systematics due to modeling using population

---

- **Inference** with **one** of currently used waveform models (IMRPhenom).

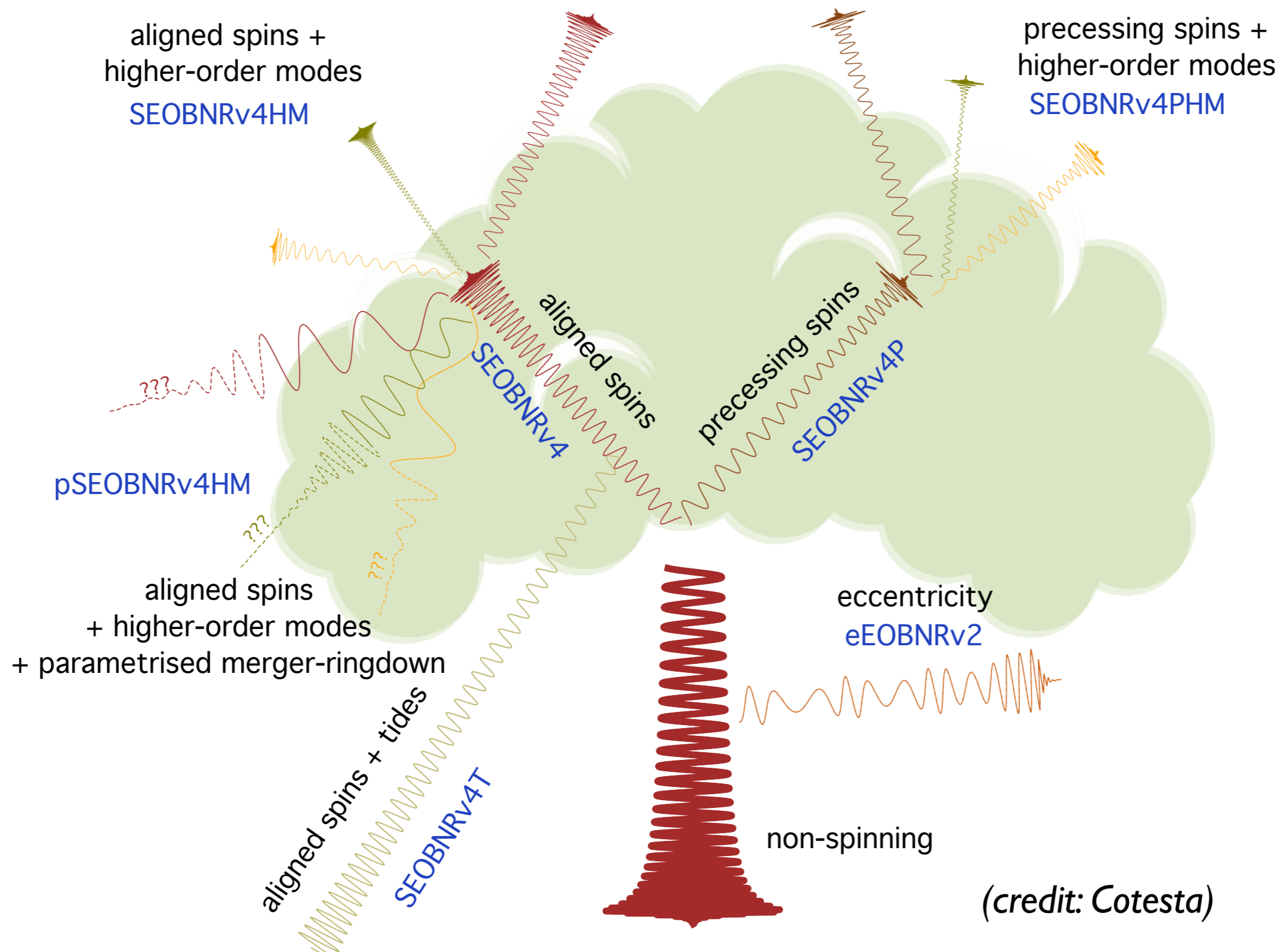
(Pürrer & Haster in prep 19) • **Population of 100 precessing NR signals** is injected

mass ratio = 1-2  
spins < 0.8



# Ever more physics in waveform models

Family tree of EOBNR waveforms

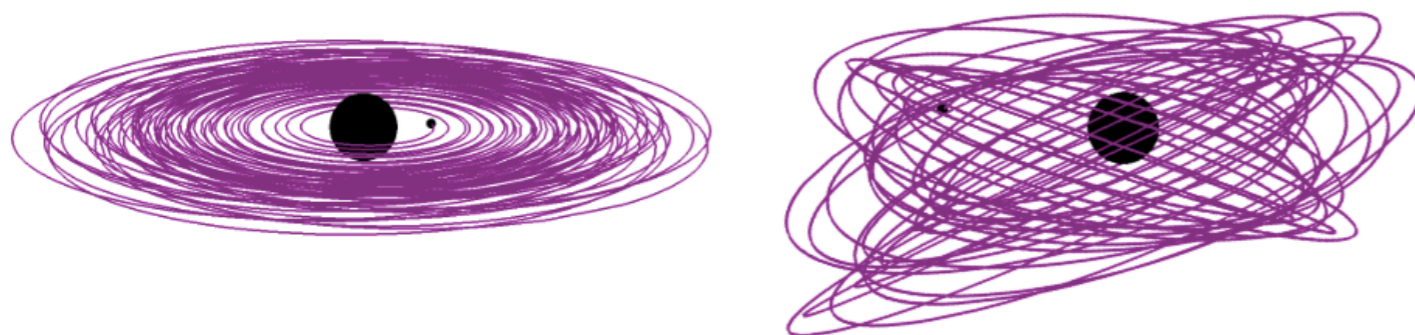


# Inferring best science by including more physical effects

- How to **discriminate** among binary's **formation scenarios**, and **probe astrophysical environment**? **Eccentricity** and **spin-precession** can disclose this information.

- **Eccentric** compact-object **binary**:

NR simulation



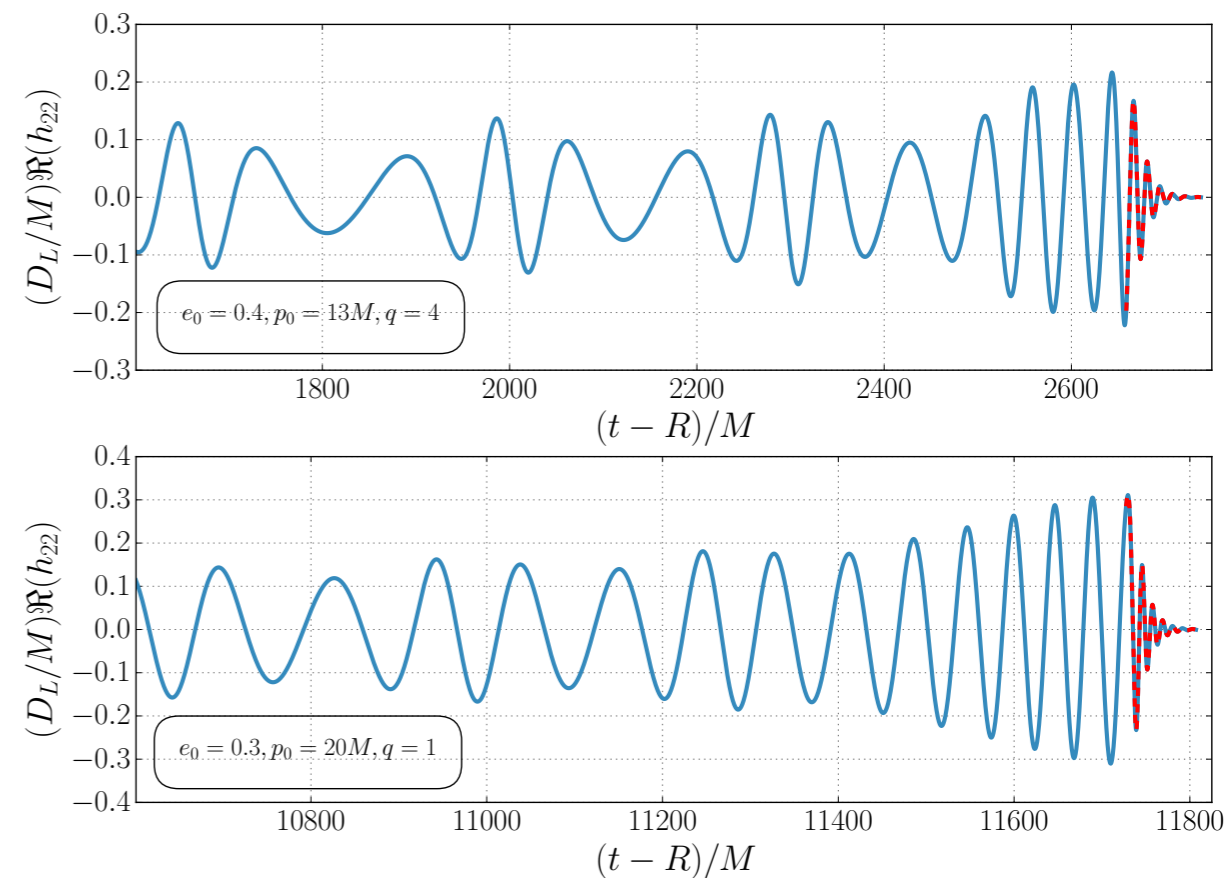
mass ratio = 7

(Lewis et al. 16)

(many PN papers; East et al. 13, Huerta et al. 14, 16, 18, Hinder et al. 17, Cao & Han 17, Loutrel & Yunes 16, 17, Huerta et al. 19, Ireland et al. 19, Moore & Yunes 19)

- **Current eccentric** waveform models **do not cover main physical effects** (e.g., spins and harmonics) **and all stages** of coalescence.

EOB waveform model



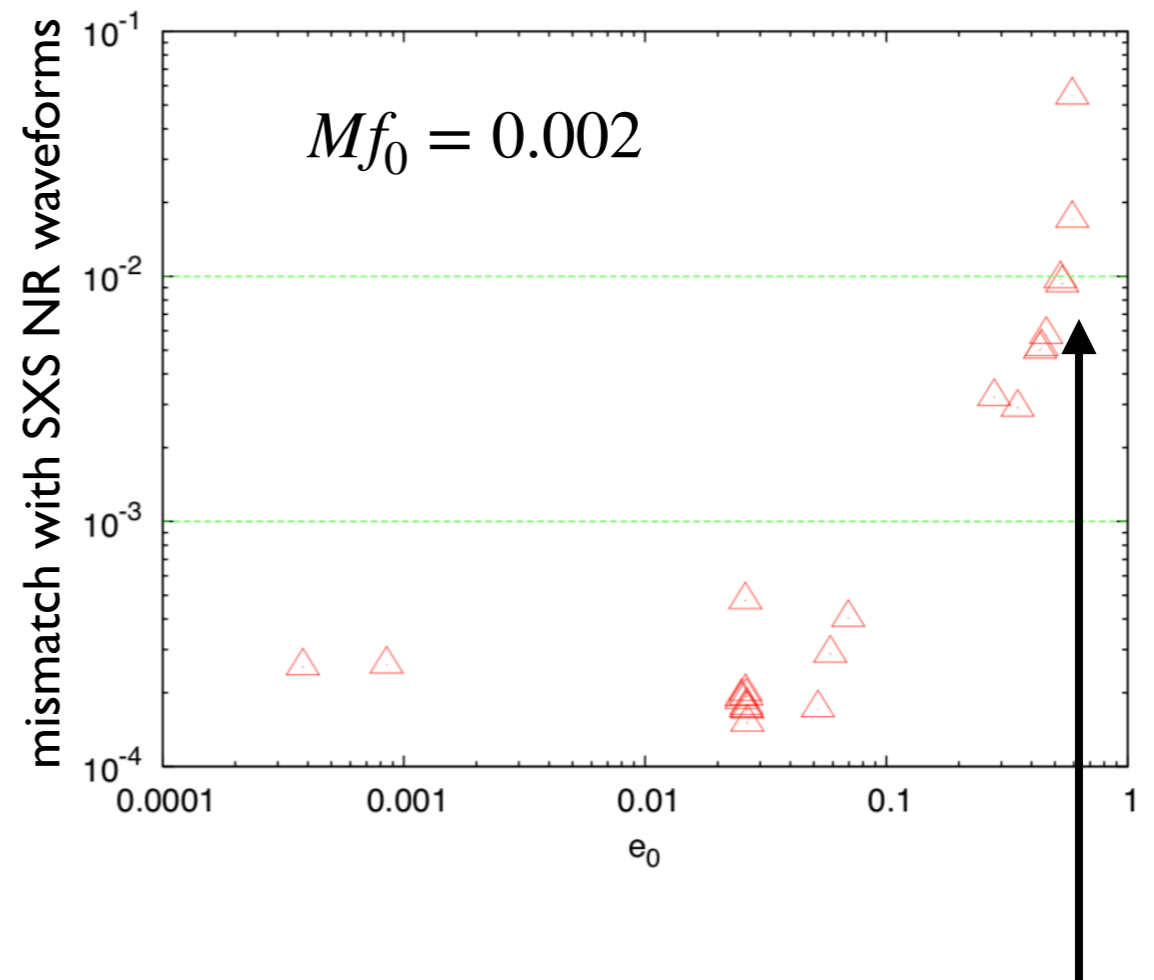
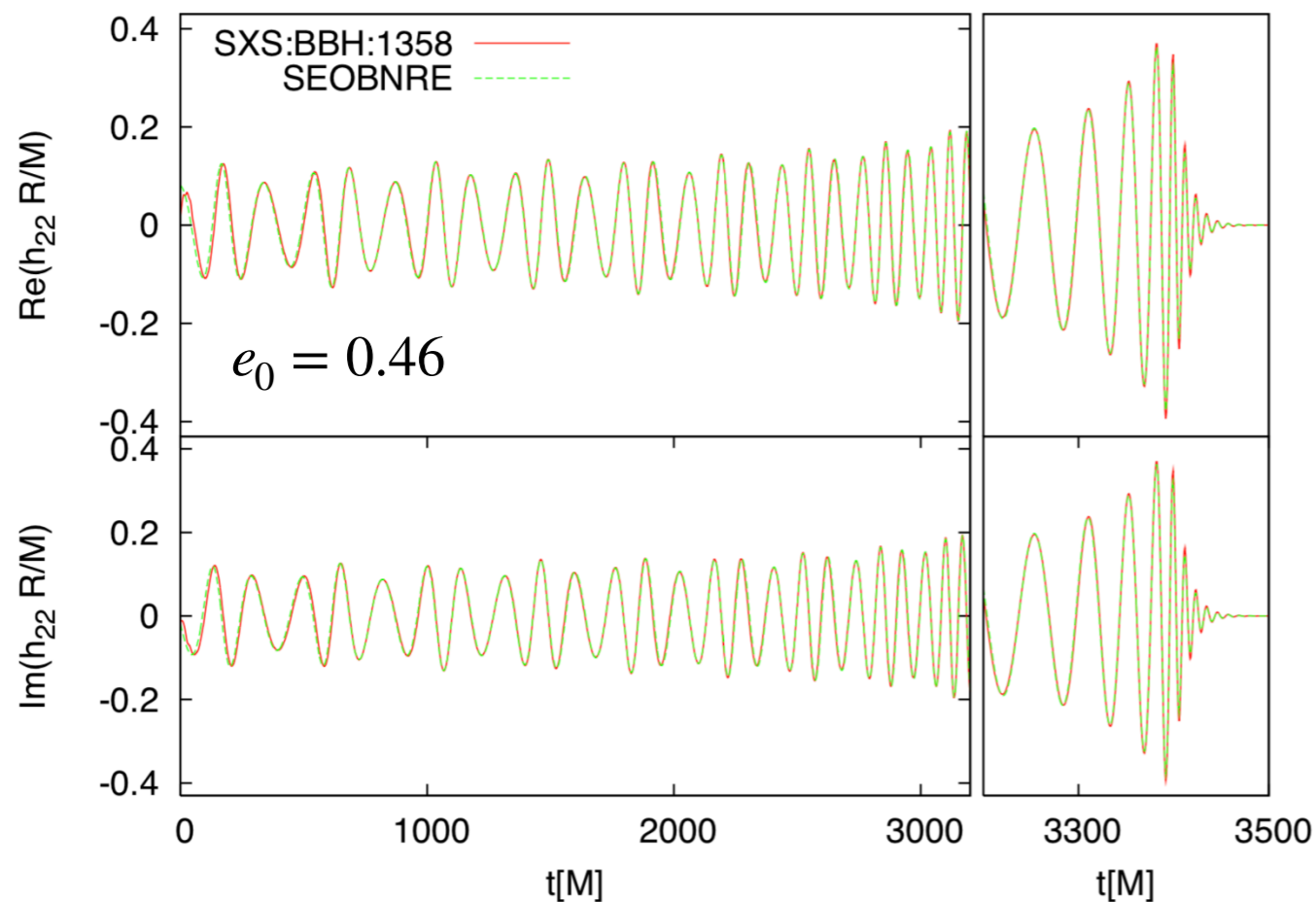
(Hinderer & Babak 17)

# Waveforms for eccentric BBHs using EOB formalism

- **Quasi-circular templates** can be used to match eccentric templates up to  $e_0 < 0.1$ .
- First **SEOBNR** model with **spins**, up to  $+0.6$ , and **eccentricity**.

(Cao et al. 17, Liu et al. 19)

SEOBNRE waveform model

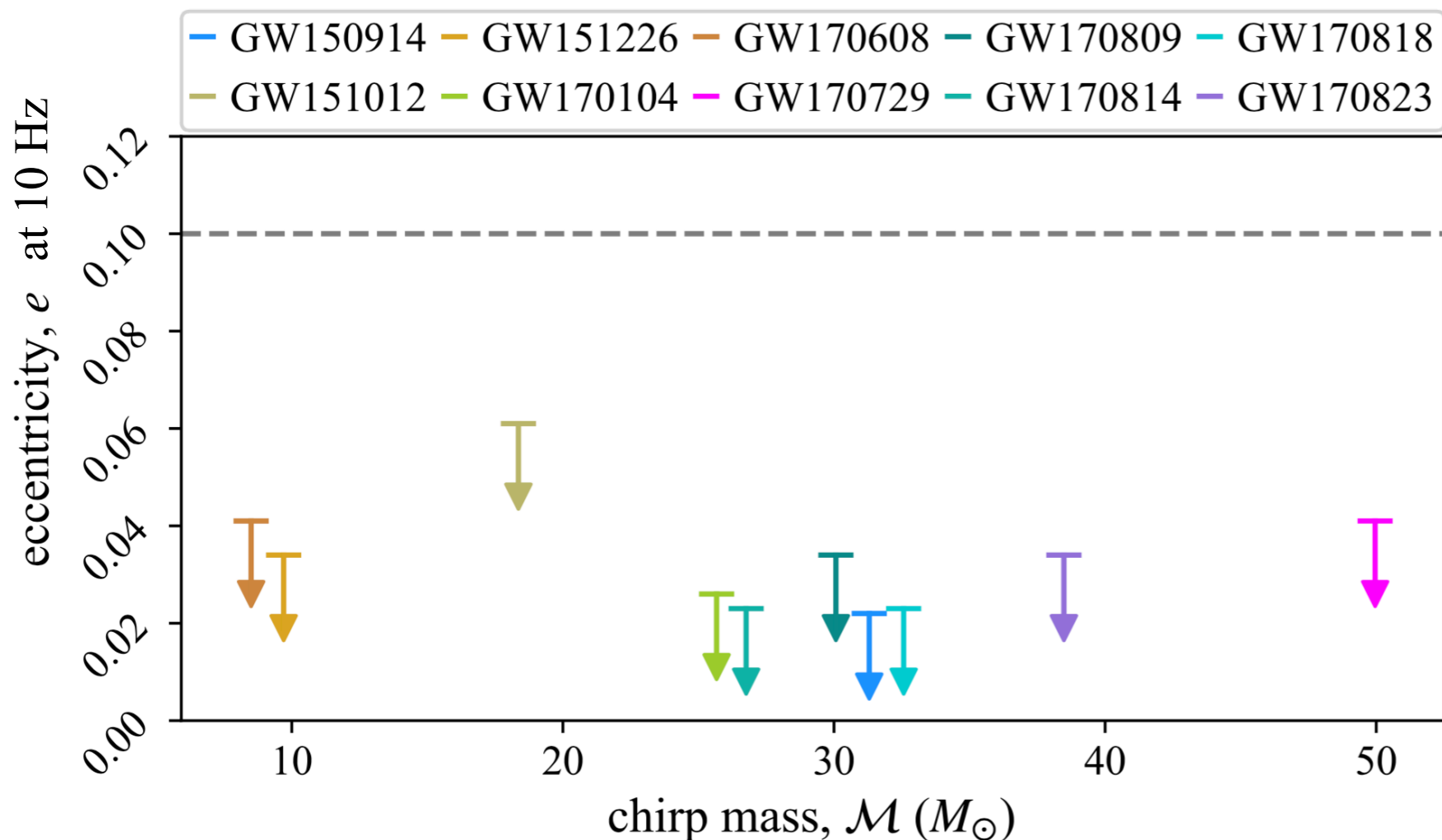


- **Mismatches** smaller than **1 %** up to  $e_0 < 0.55$ .

# SEOBNRE waveforms used to constrain eccentricity of GWTC-1

- **Isolated** binary **evolution** or **dynamical** formation?

(Romera-Shaw, Lasky & Thrane 19)



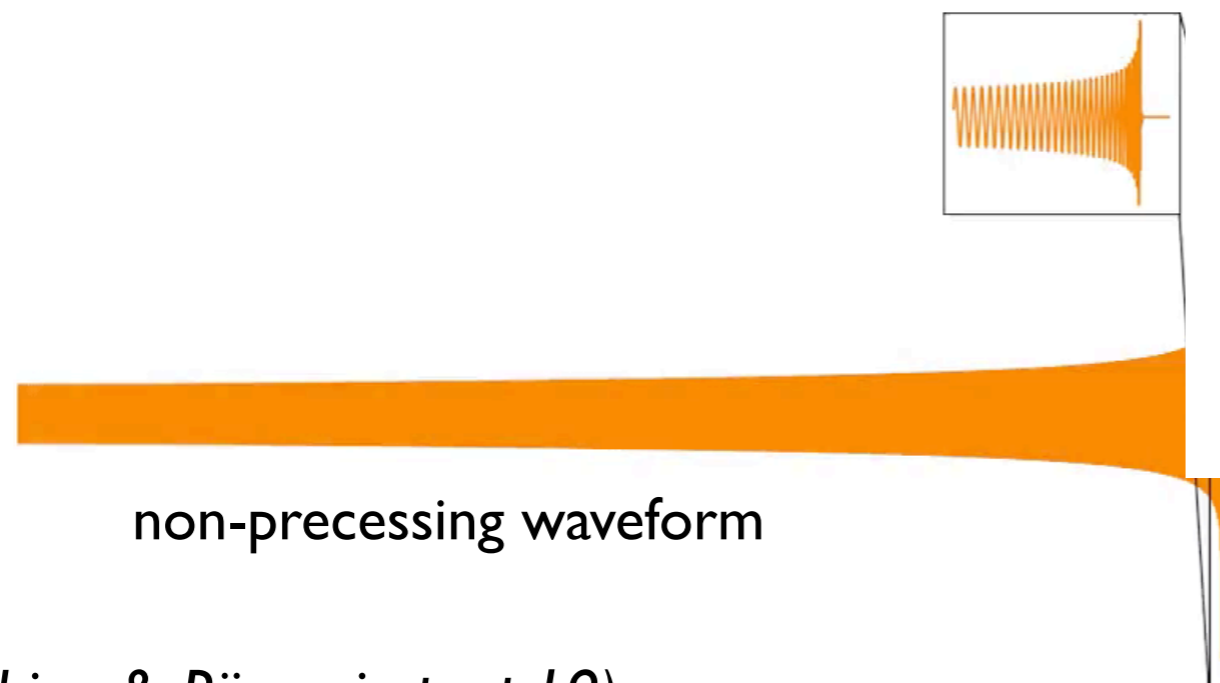
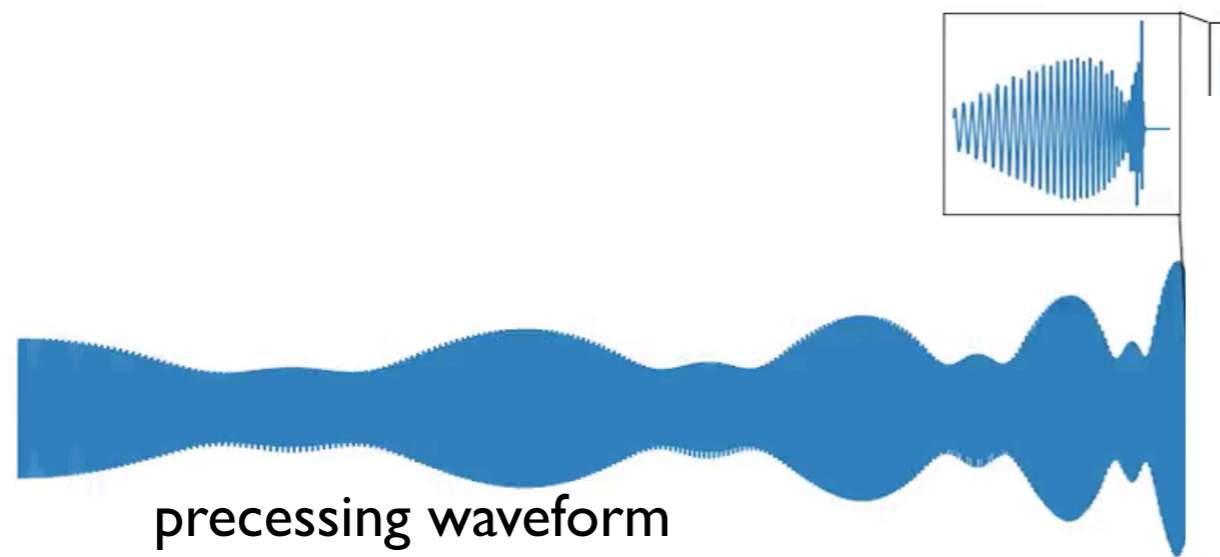
Event	$e_{\text{max}}^{90}$	$\ln \mathcal{B}$
GW150914	0.022	-1.64
GW151012	0.061	-0.96
GW151226	0.034	-1.63
GW170104	0.026	-1.52
GW170608	0.041	-1.47
GW170729	0.041	-0.53
GW170809	0.041	-1.47
GW170814	0.023	-1.48
GW170818	0.023	-1.47
GW170823	0.034	-1.58

- We should expect to observe  $\sim 5\%$  of all mergers from **globular clusters** with  $e_0 > 0.1$  at 10 Hz. (Rodriguez et al. 18, Samsing et al. 18)



# Characteristics of spin-precessing dynamics and waveform

(credit: Ossokine)



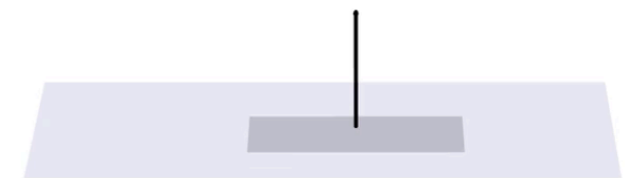
total mass = **29 Msun**

mass ratio = **5**

**574 GW** cycles, from 10 Hz

**5** precessional cycles

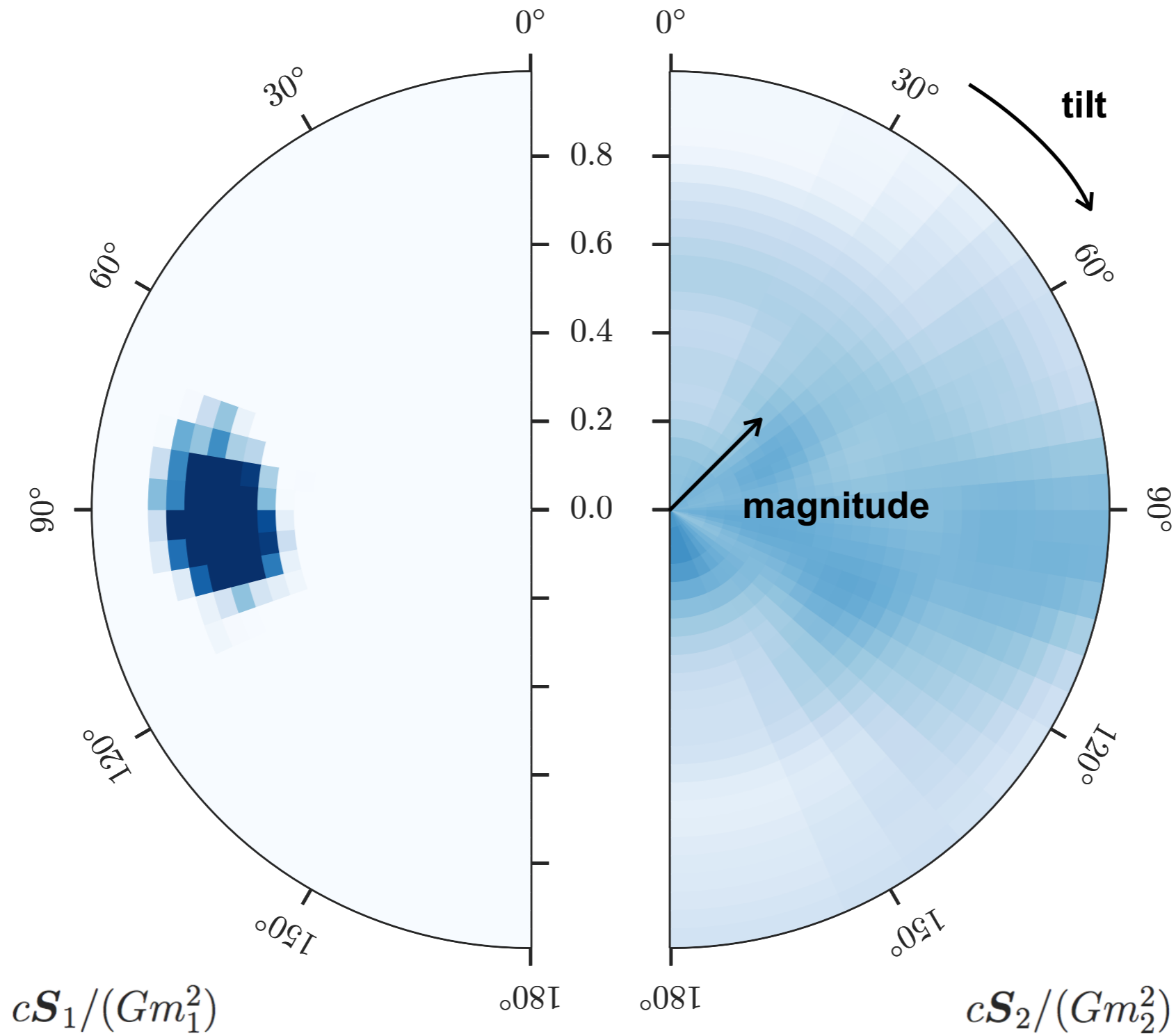
**40 sec** duration



binary orbital plane

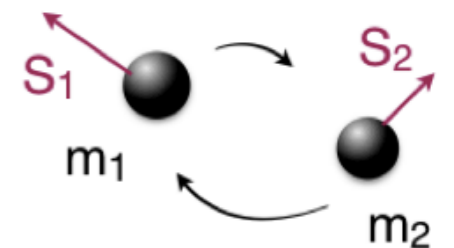
(Ossokine & Pürrer in prep 19)

# Measuring spin-precession from collision of BHs



SNR ~ 25  
aLIGO/Virgo

(credit: Pürrer)

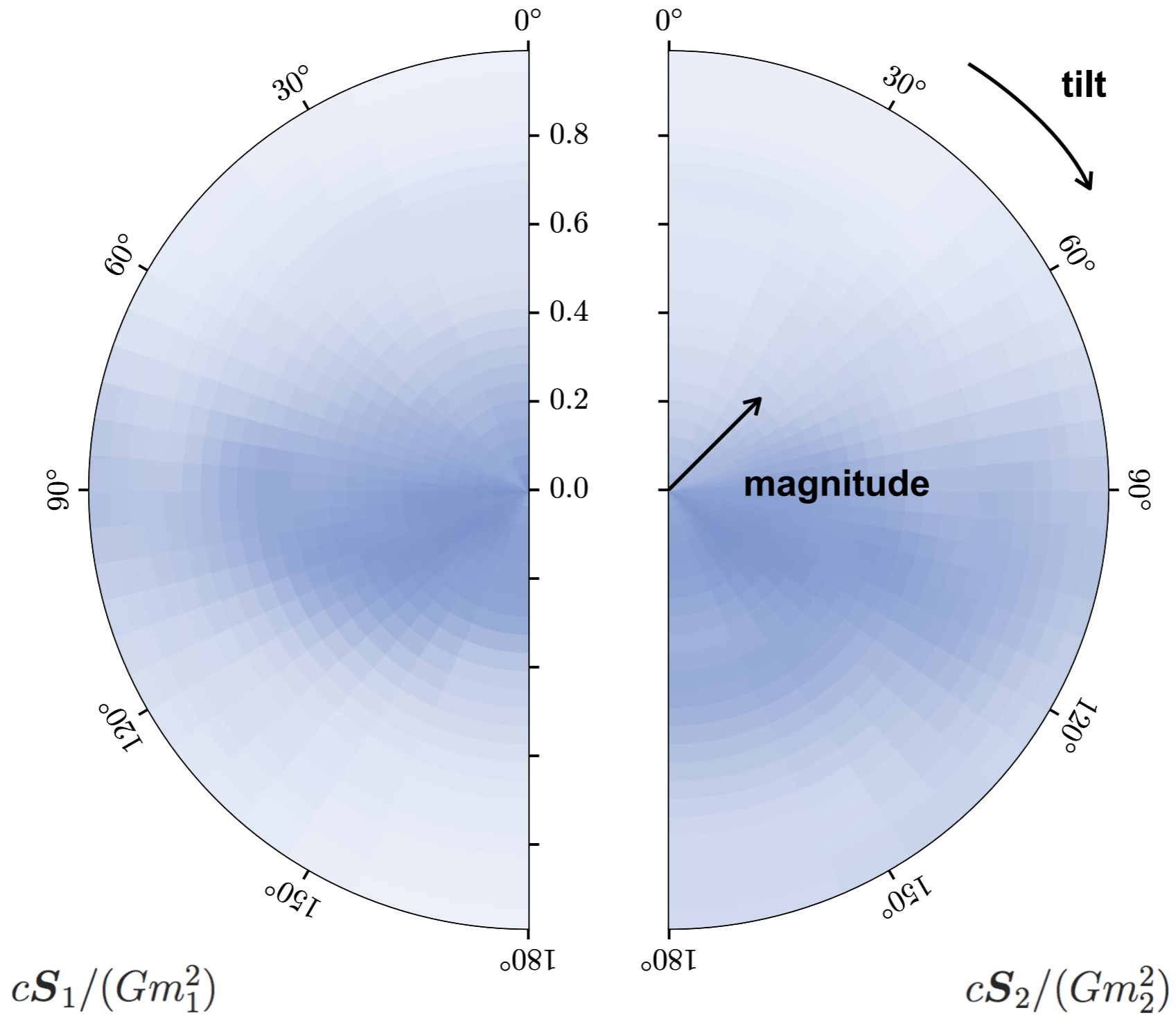


(Ossokine & Pürrer in prep 19)

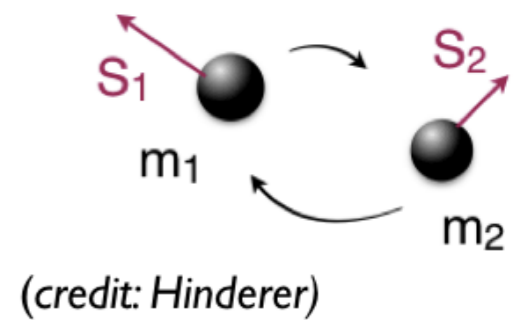
(credit: Hinderer)

# Measuring spin-precession with GW150914

(Abbott et al. PRL 116 (2016) 061102)

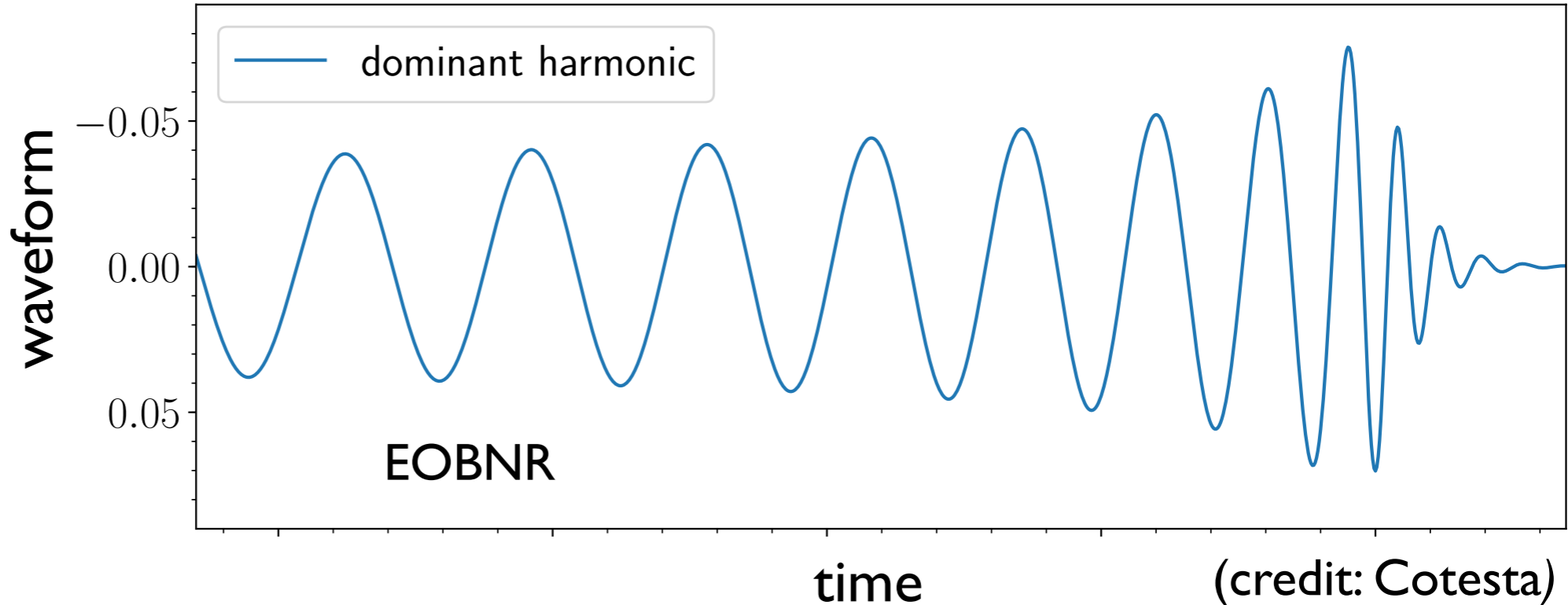


(credit: Pürrer/LIGO/Virgo)

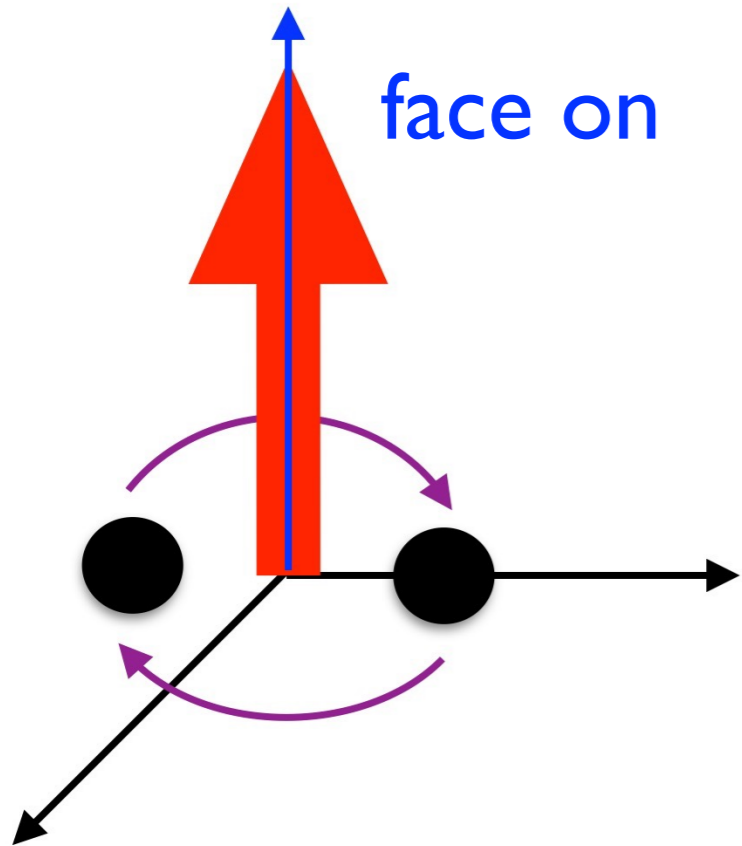


(credit: Hinderer)

# Enriching the GW symphony by tuning higher harmonics



face on

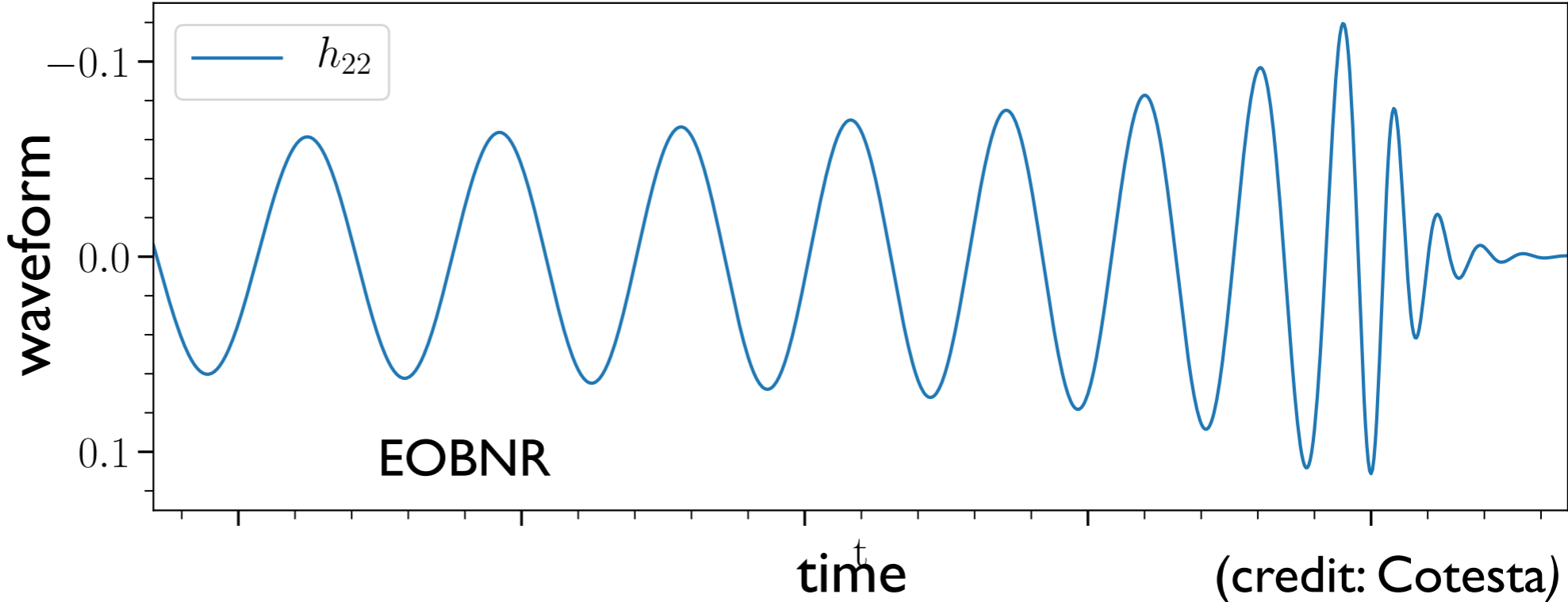


- So far, LIGO/Virgo **observed GW events mostly face-on/face-off.**
- **Face-on/face-off orientation** suppress **higher harmonics.**

# Enriching the GW symphony by tuning higher harmonics (contd.)

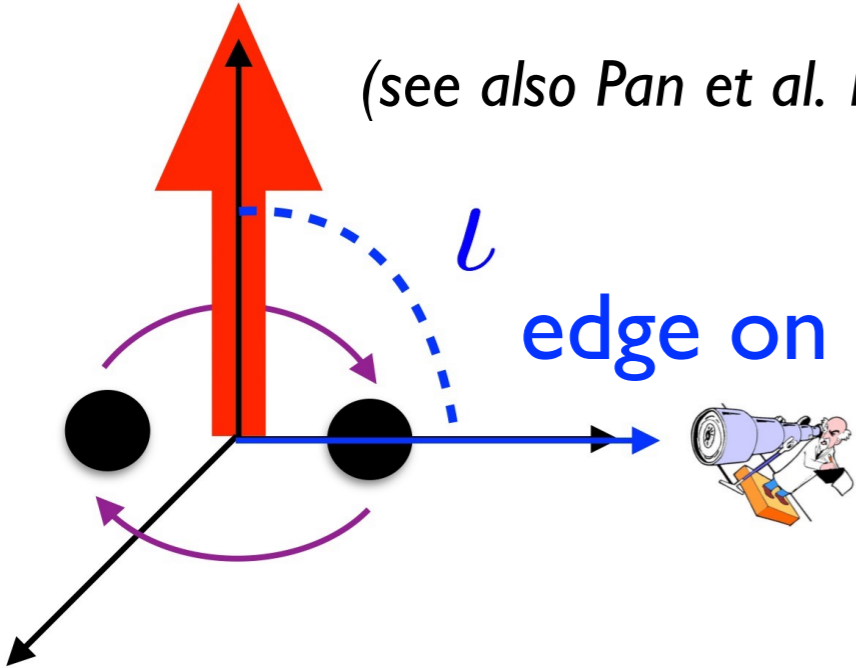


fundamental harmonic



(Cotesta, AB, Bohe, Taracchini, Hinder, Ossokine 18)

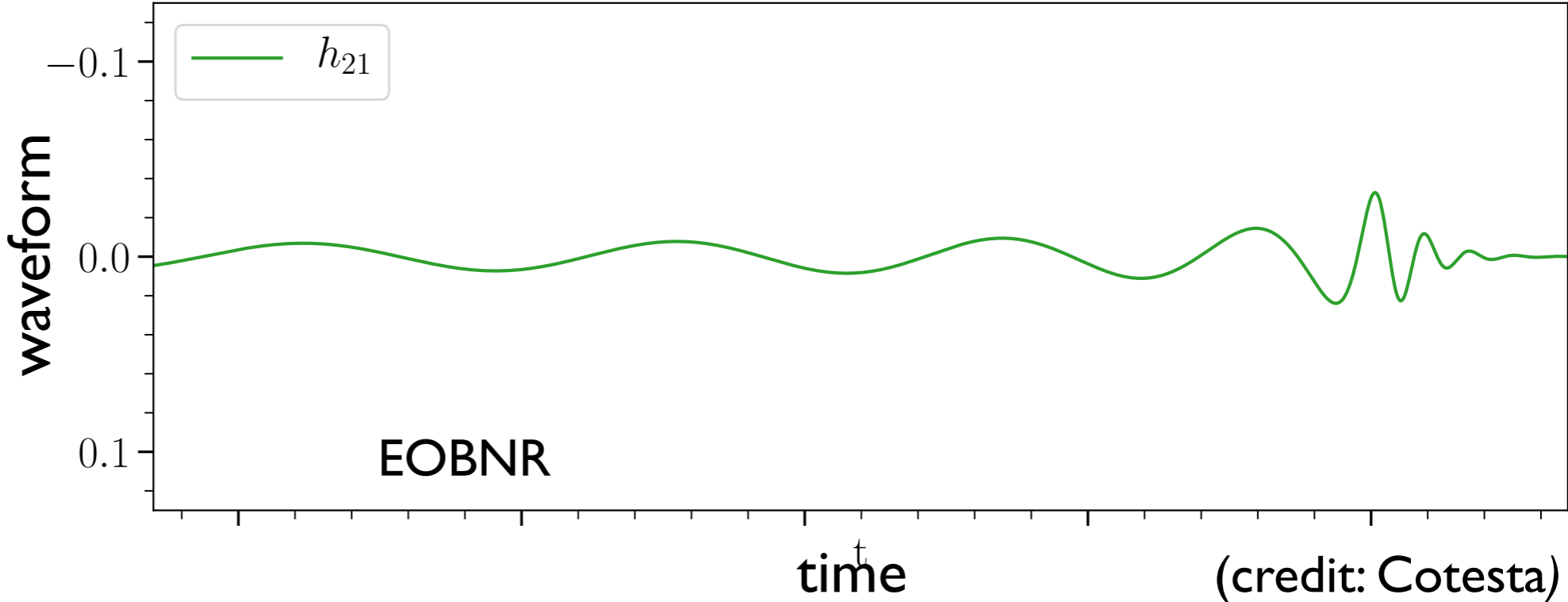
(see also Pan et al. 11, London et al. 17, Mehta et al. 17, 19, Nagar et al. 19)



# Enriching the GW symphony by tuning higher harmonics (contd.)

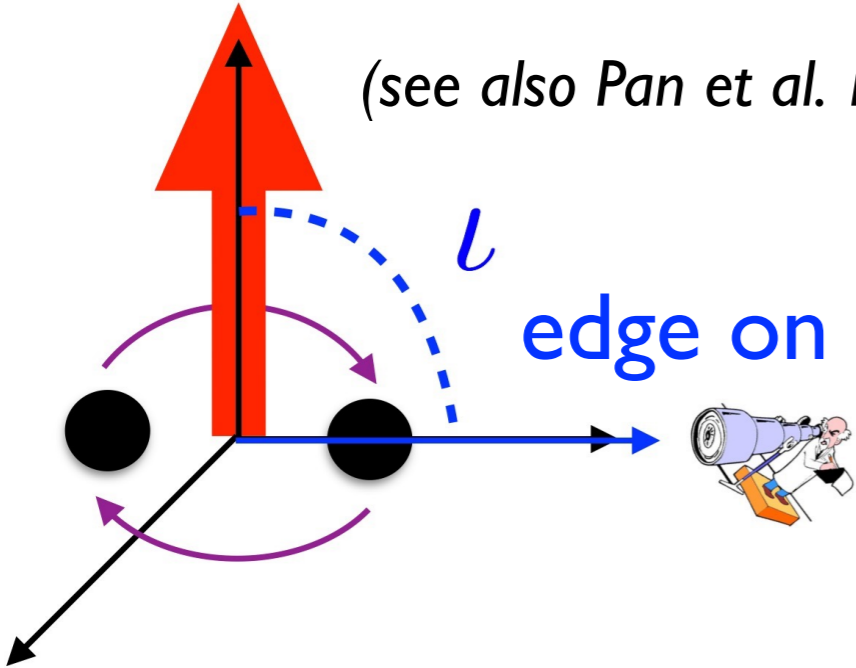


first harmonic



(Cotesta, AB, Bohe, Taracchini, Hinder, Ossokine 18)

(see also Pan et al. 11, London et al. 17, Mehta et al. 17, 19, Nagar et al. 19)

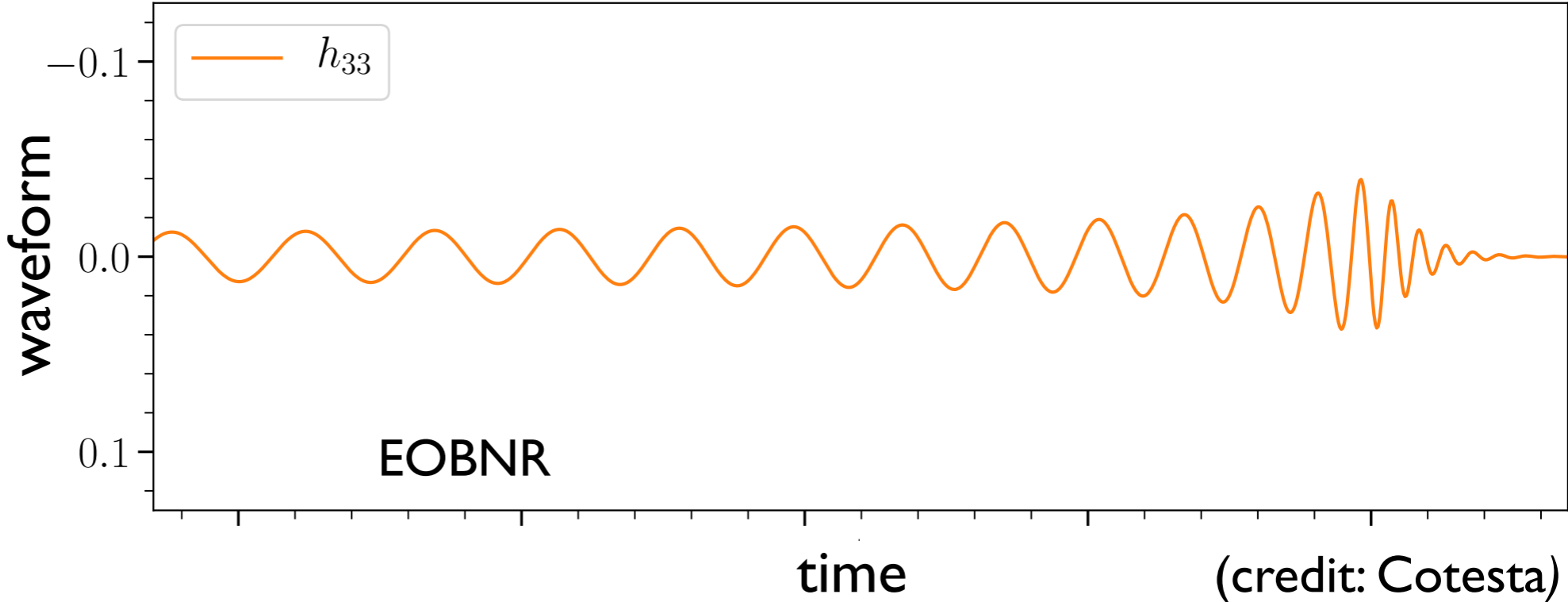




# Enriching the GW symphony by tuning higher harmonics (contd.)

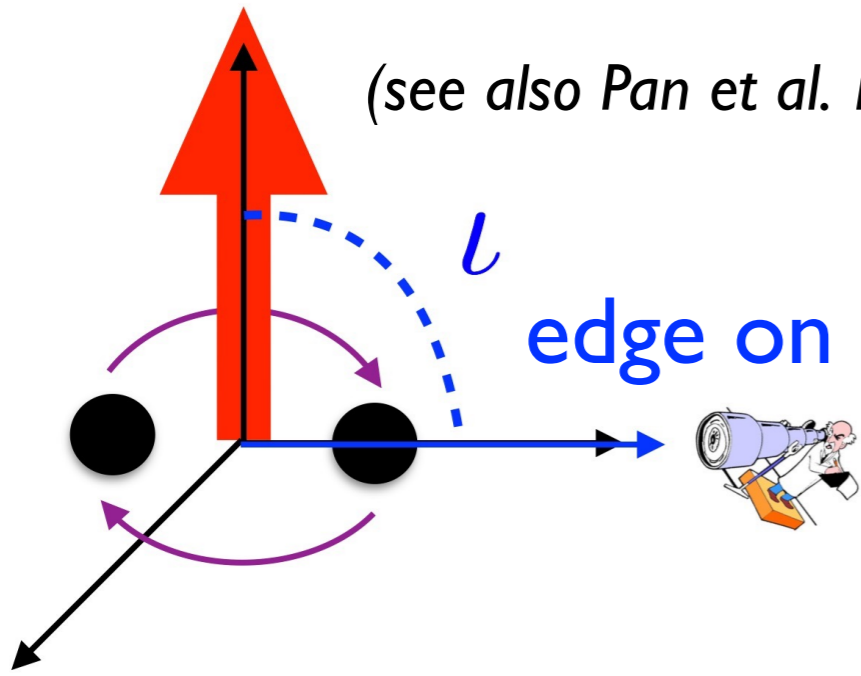


third harmonic



(Cotesta, AB, Bohe, Taracchini, Hinder, Ossokine 18)

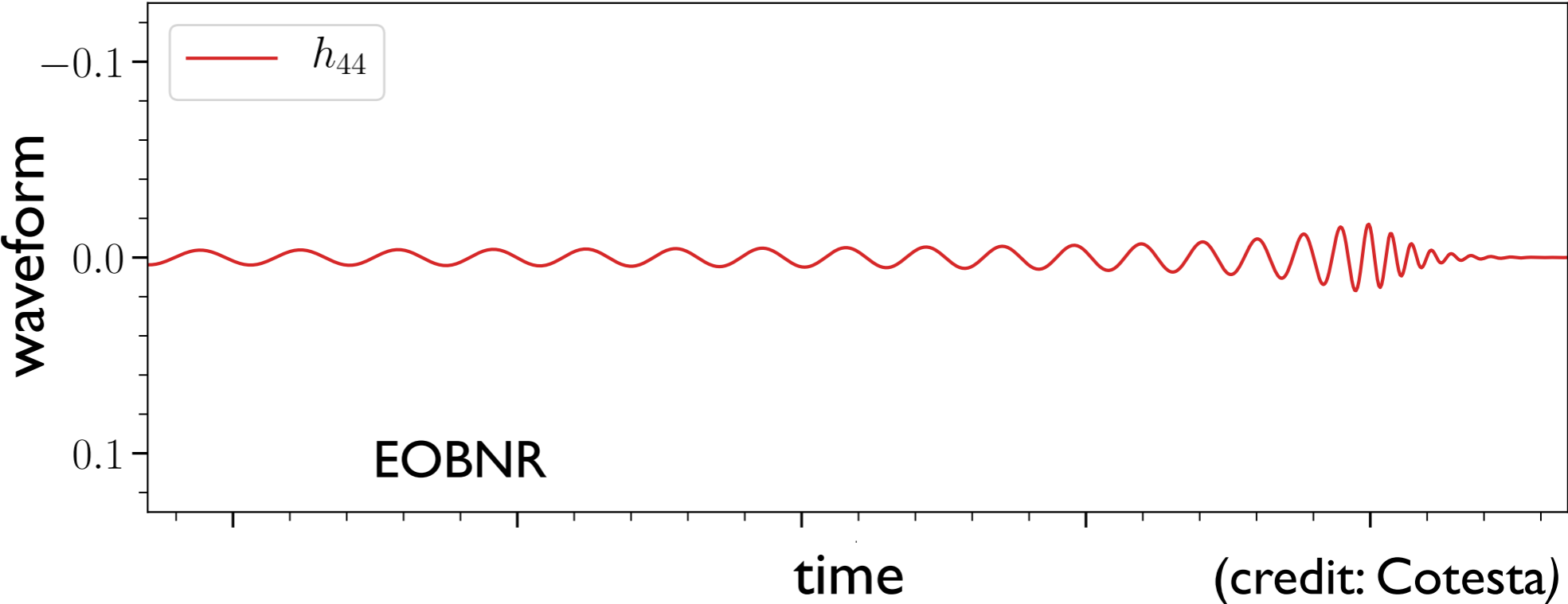
(see also Pan et al. 11, London et al. 17, Mehta et al. 17, 19, Nagar et al. 19)



# Enriching the GW symphony by tuning higher harmonics (contd.)

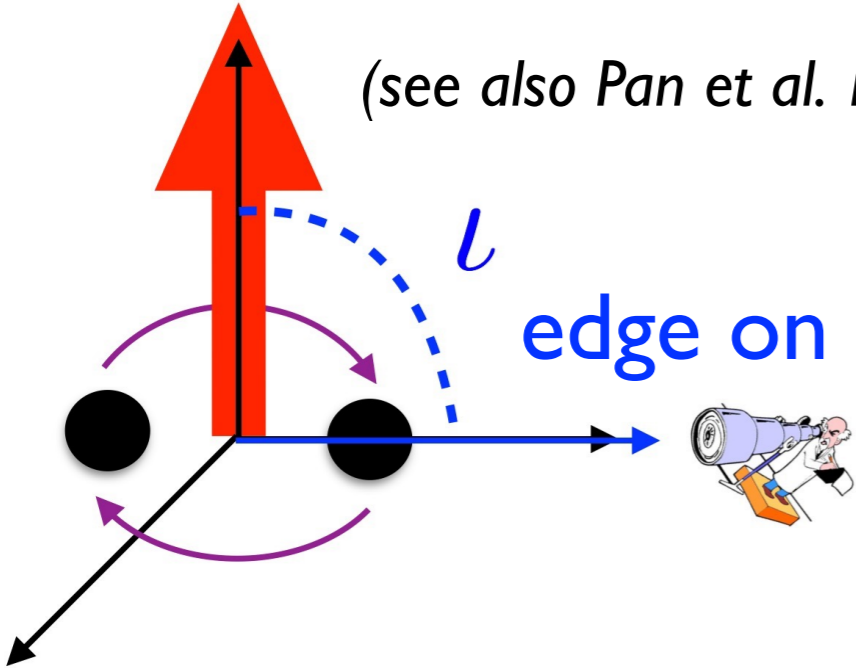


fourth harmonic



(Cotesta, AB, Bohe, Taracchini, Hinder, Ossokine 18)

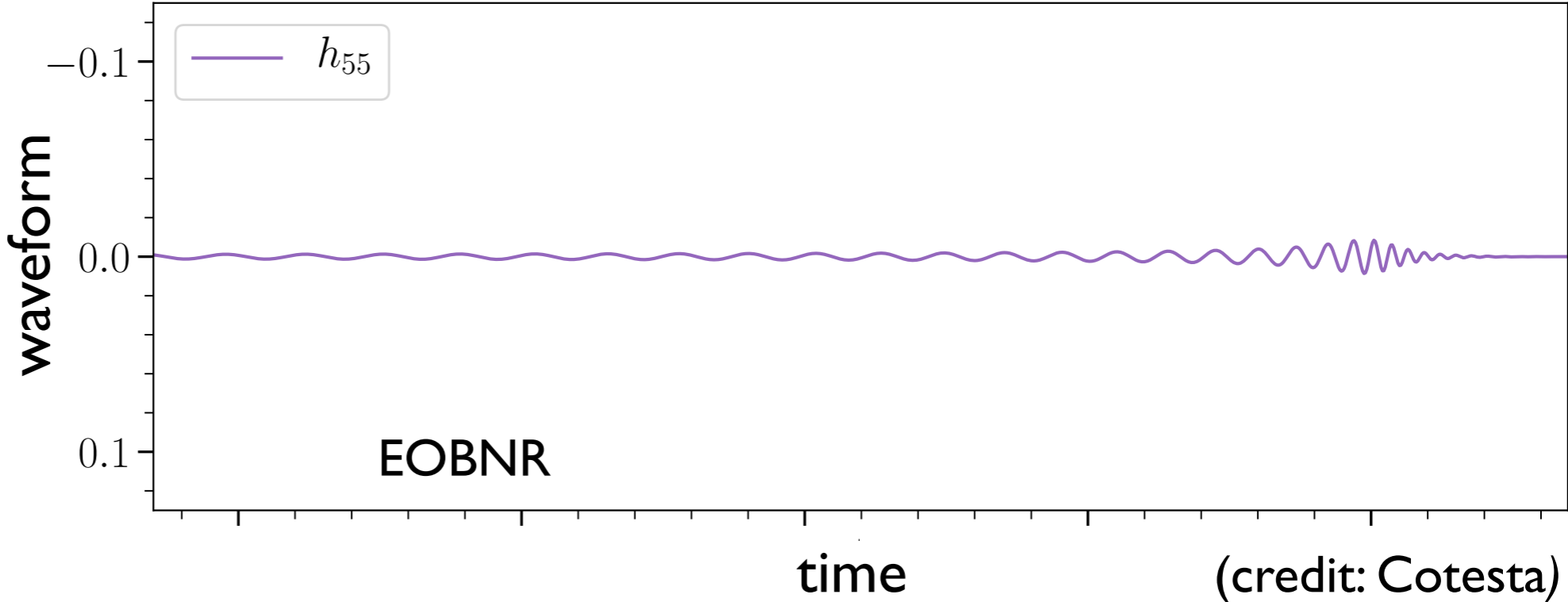
(see also Pan et al. 11, London et al. 17, Mehta et al. 17, 19, Nagar et al. 19)



# Enriching the GW symphony by tuning higher harmonics (contd.)

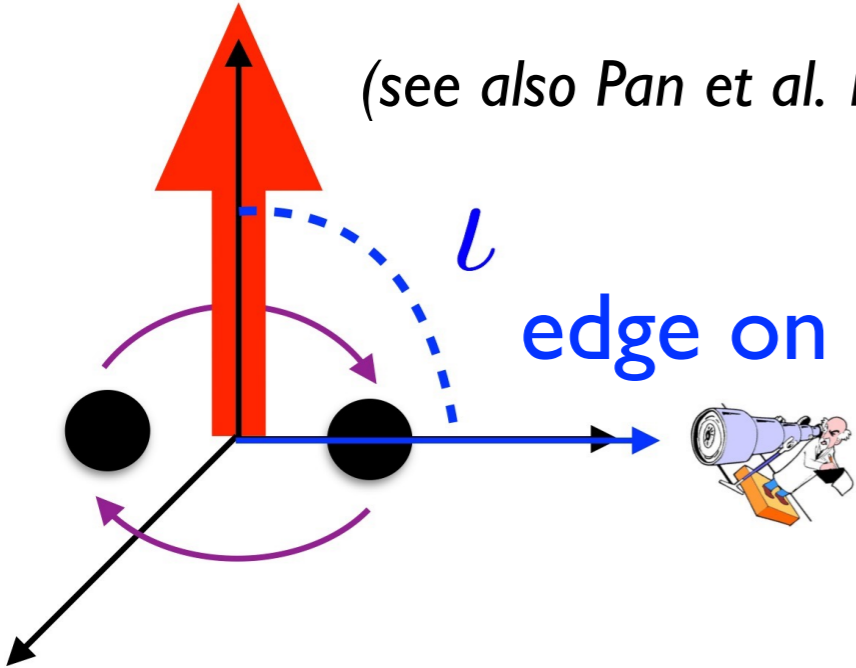


fifth harmonic



(Cotesta, AB, Bohe, Taracchini, Hinder, Ossokine 18)

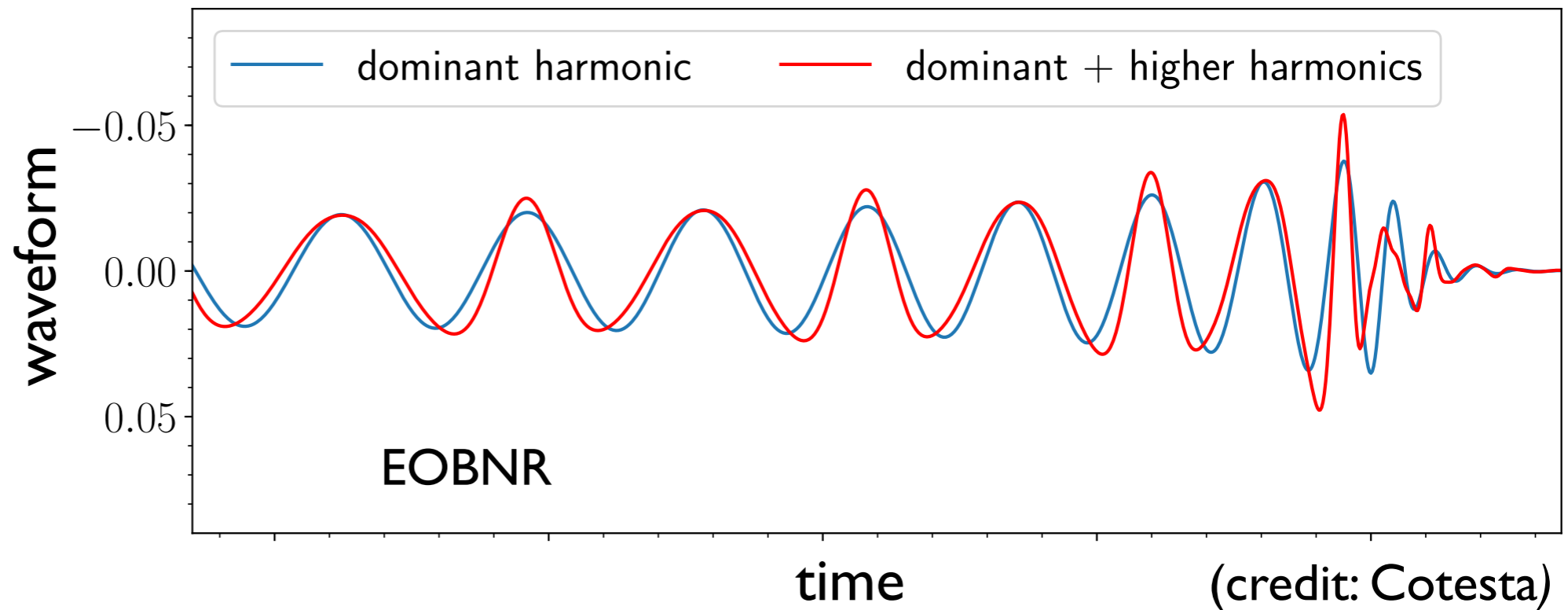
(see also Pan et al. 11, London et al. 17, Mehta et al. 17, 19, Nagar et al. 19)



# Enriching the GW symphony by tuning higher harmonics (contd.)

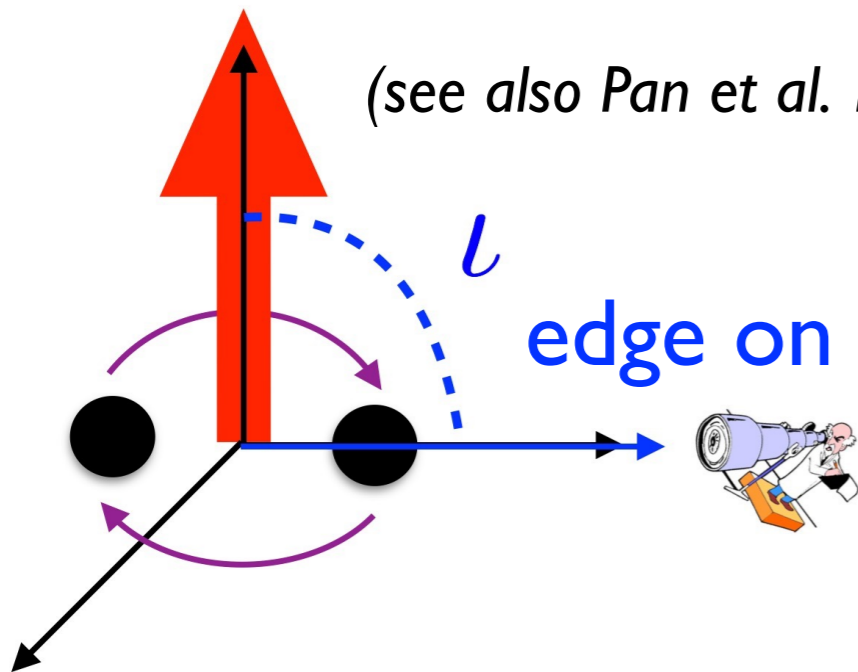


adding all five harmonics



(Cotesta, AB, Bohe, Taracchini, Hinder, Ossokine 18)

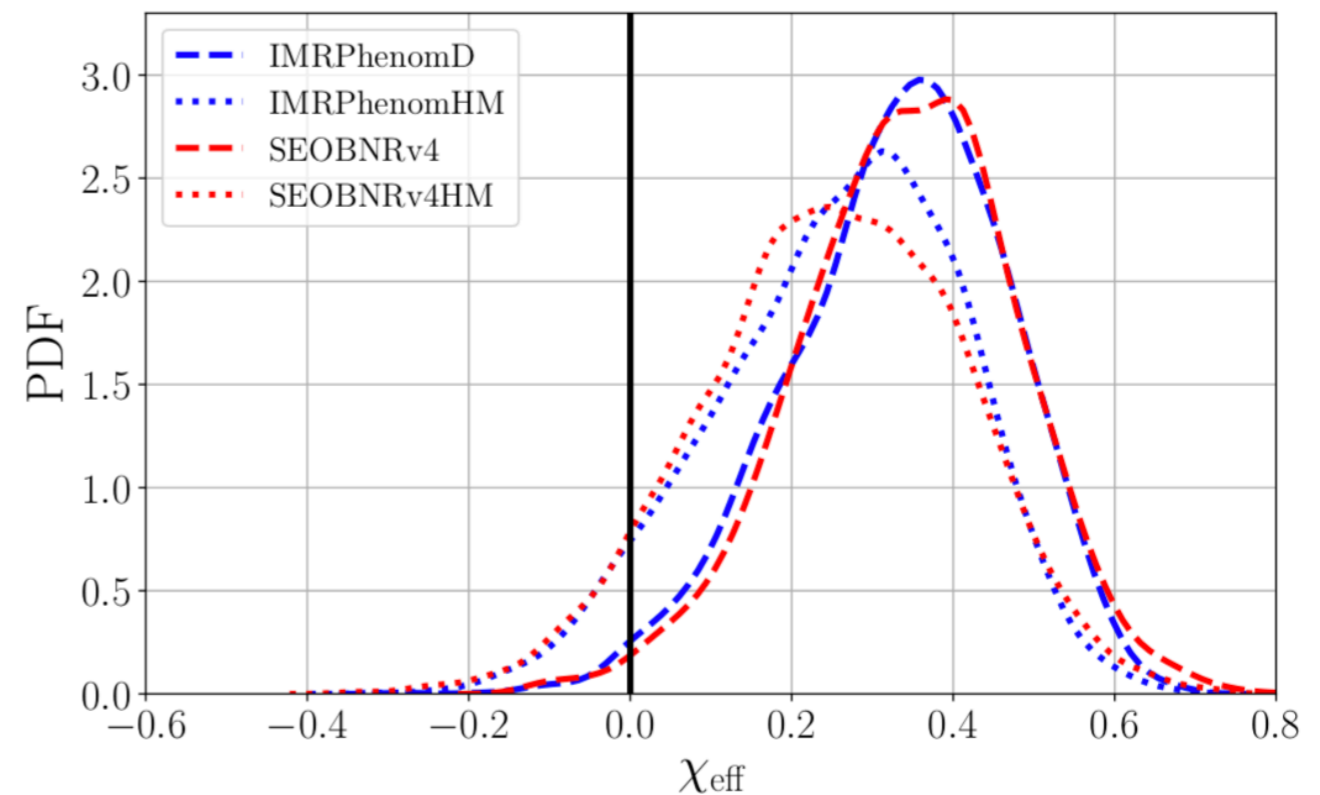
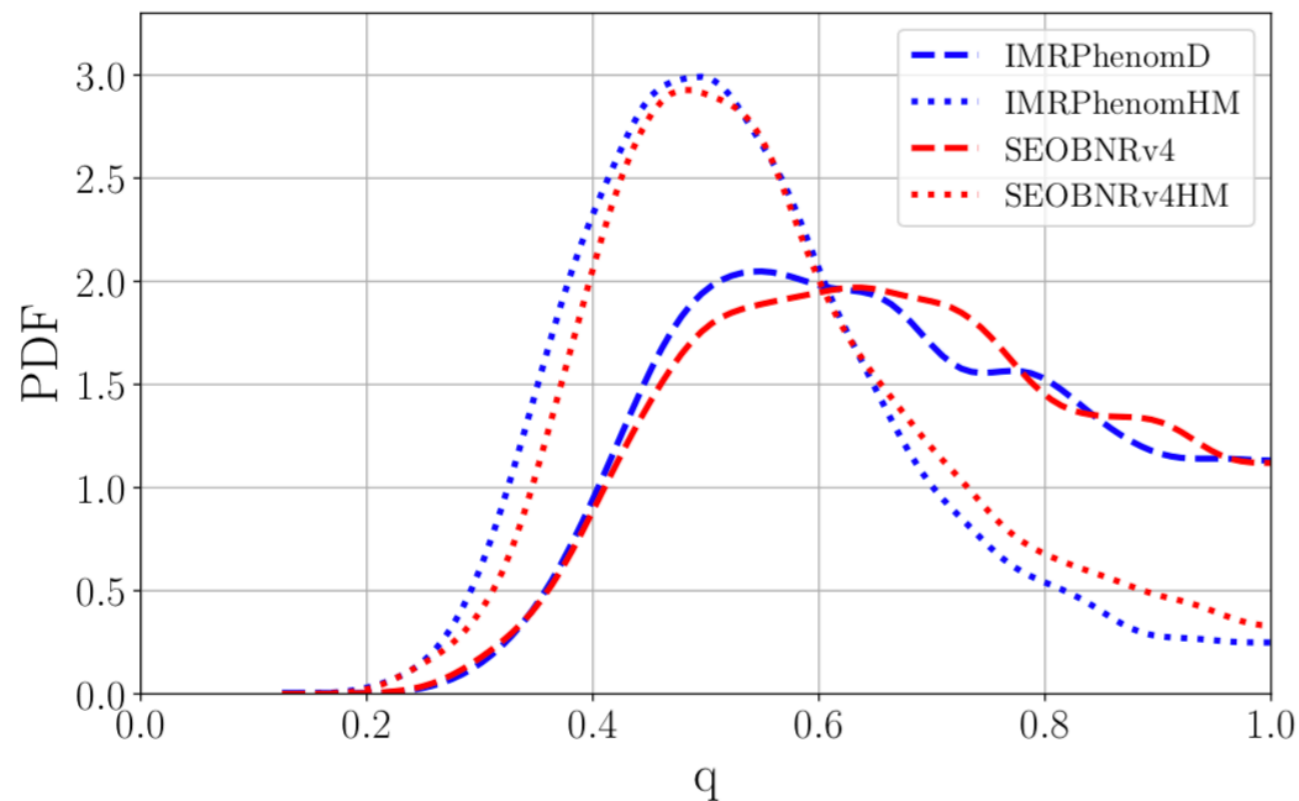
(see also Pan et al. 11, London et al. 17, Mehta et al. 17, 19, Nagar et al. 19)



- We can **detect more rare** GW events.
- We can **infer source's properties more accurately**.
- We can **perform more stringent tests** of GR (e.g., BH spectroscopy)

# Inference of GW I70729 with higher-mode waveform models

(Chatziioannou, ... AB ..., 19)



- **Improved estimate** for mass ratio: **(0.3 – 0.8)** at 90%. Measurement **excludes equal masses** at 90%.

# Accuracy of multipolar precessing SEOBNR model against NR

- **SEOBNRv4PHM**:  
new spin-precessing waveform model

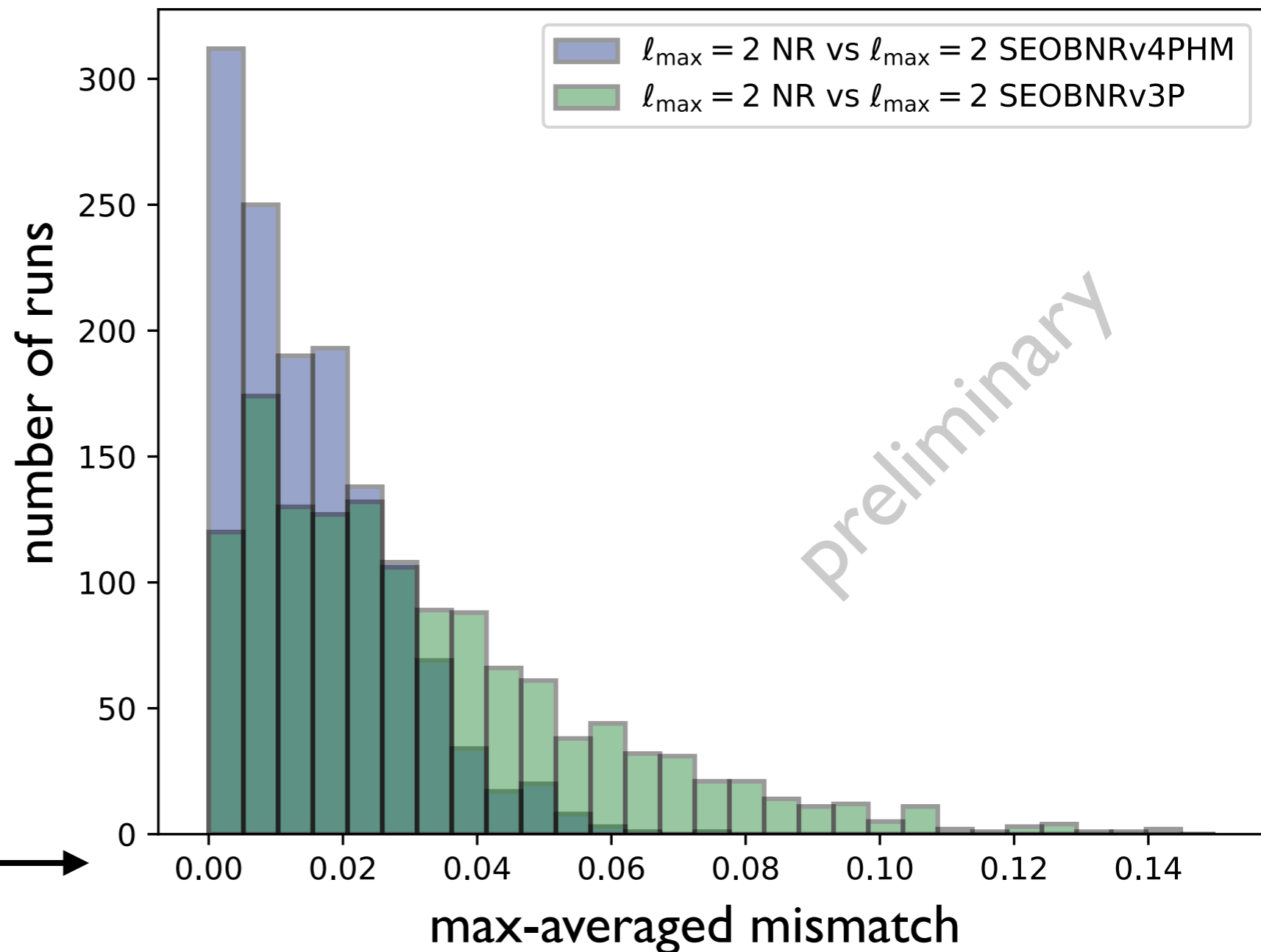
- **SEOBNRv3P**: old spin-precessing waveform model, **without HMs**, used in **O1 & O2**.

(Pan, AB et al. 13, Babak, ... AB 17)

- Mismatch against **public SXS** NR catalog (1344) plus **non-public SXS** NR waveforms (141).

(Boyle et al. 19)

(Ossokine, Marsat, AB & Cotesta in prep 19)



binary's inclination:  $i = \pi/3$



# Accuracy of multipolar precessing SEOBNR model against NR

- **SEOBNRv4PHM**:  
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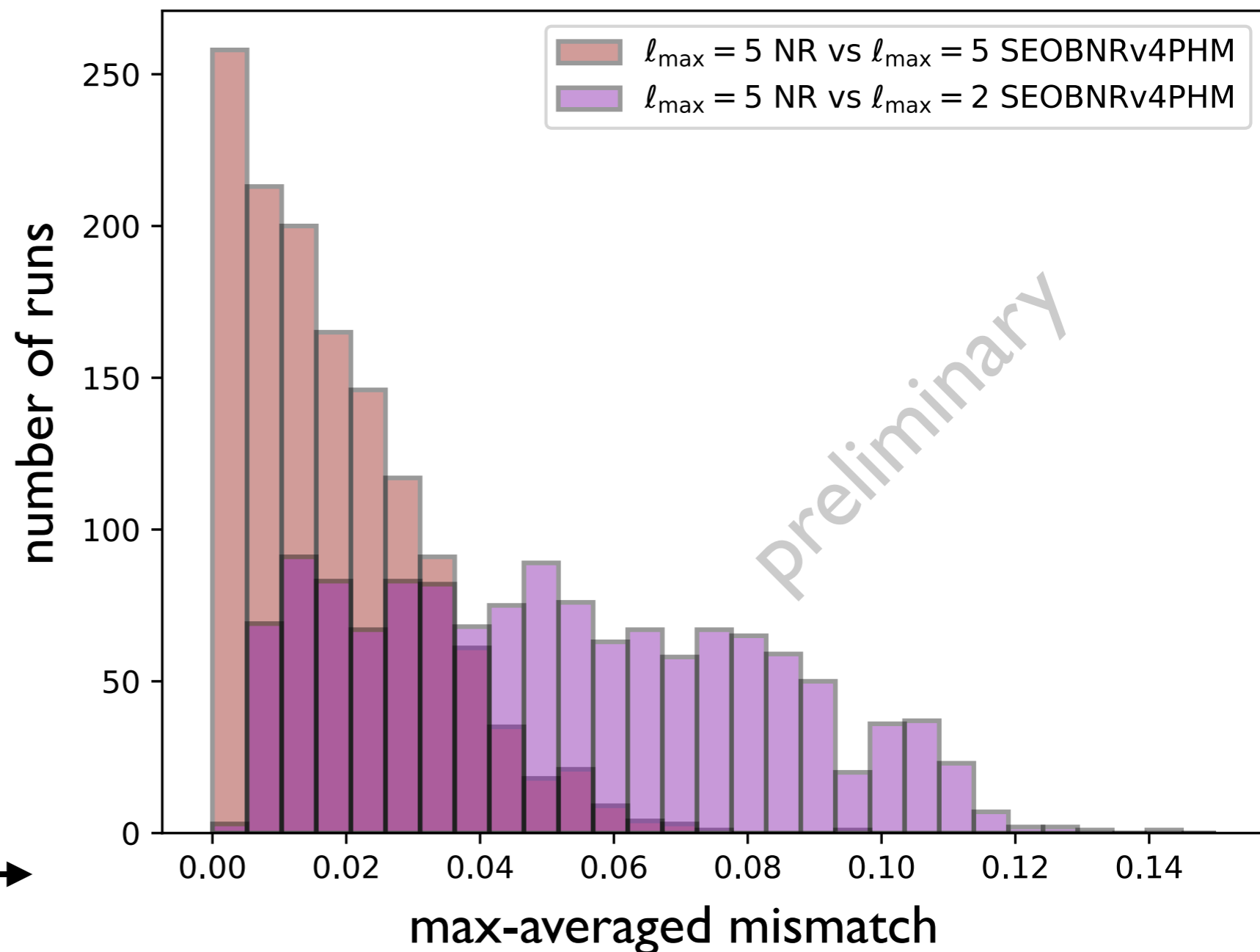
- **SEOBNRv3P**: **old** spin-precessing waveform model, **without HMs**, used in **O1 & O2**.

(Pan, AB et al. 13, Babak, ... AB 17)

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(Boyle et al. 19)

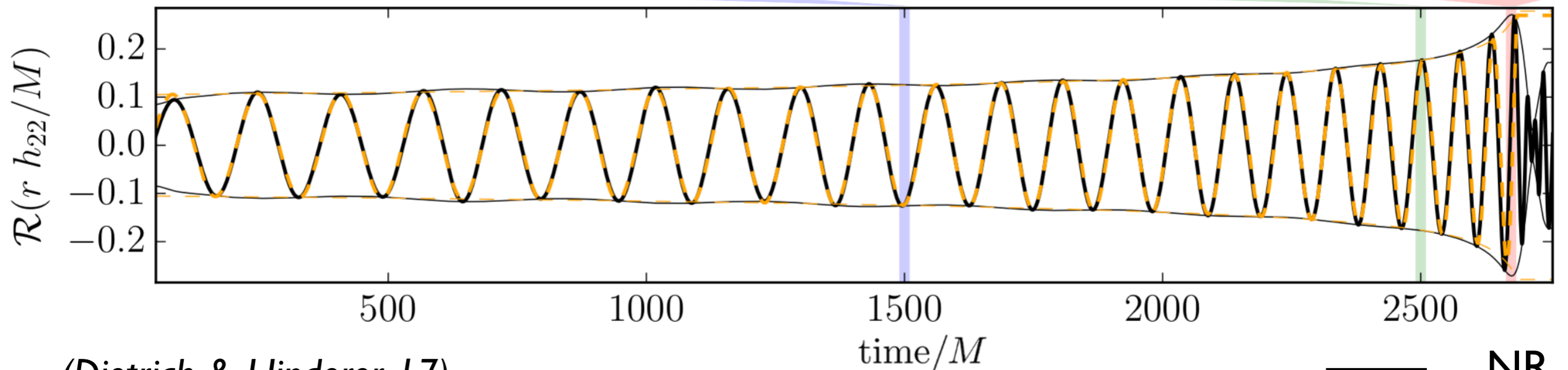
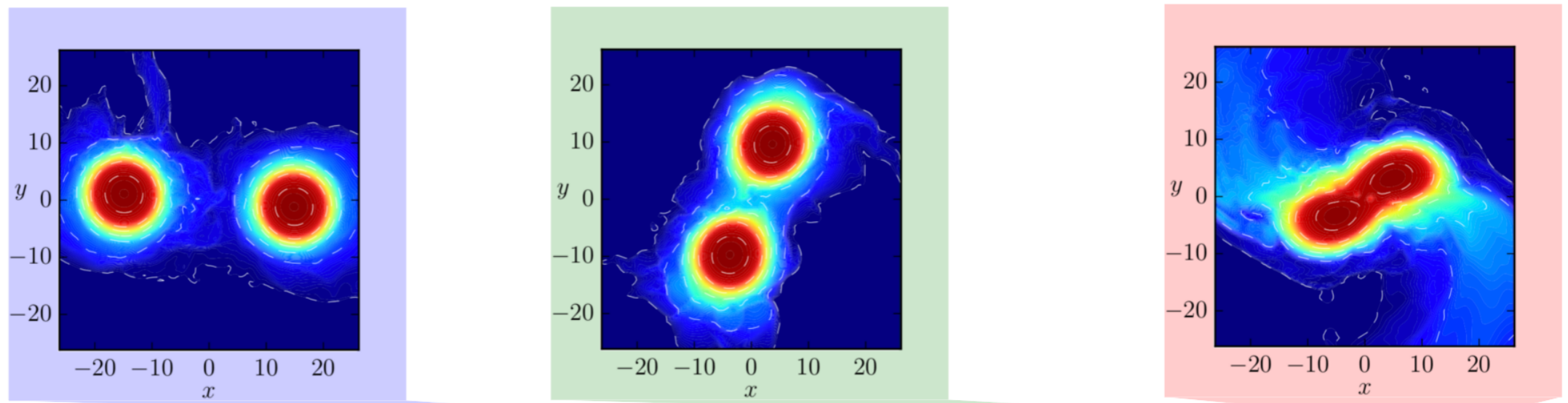
(Ossokine, Marsat, AB & Cotesta in prep 19)



binary's inclination:  $i = \pi/3$

# Waveforms for BNS combining analytical & numerical relativity

- Synergy between **analytical** and **numerical work** is **crucial**.

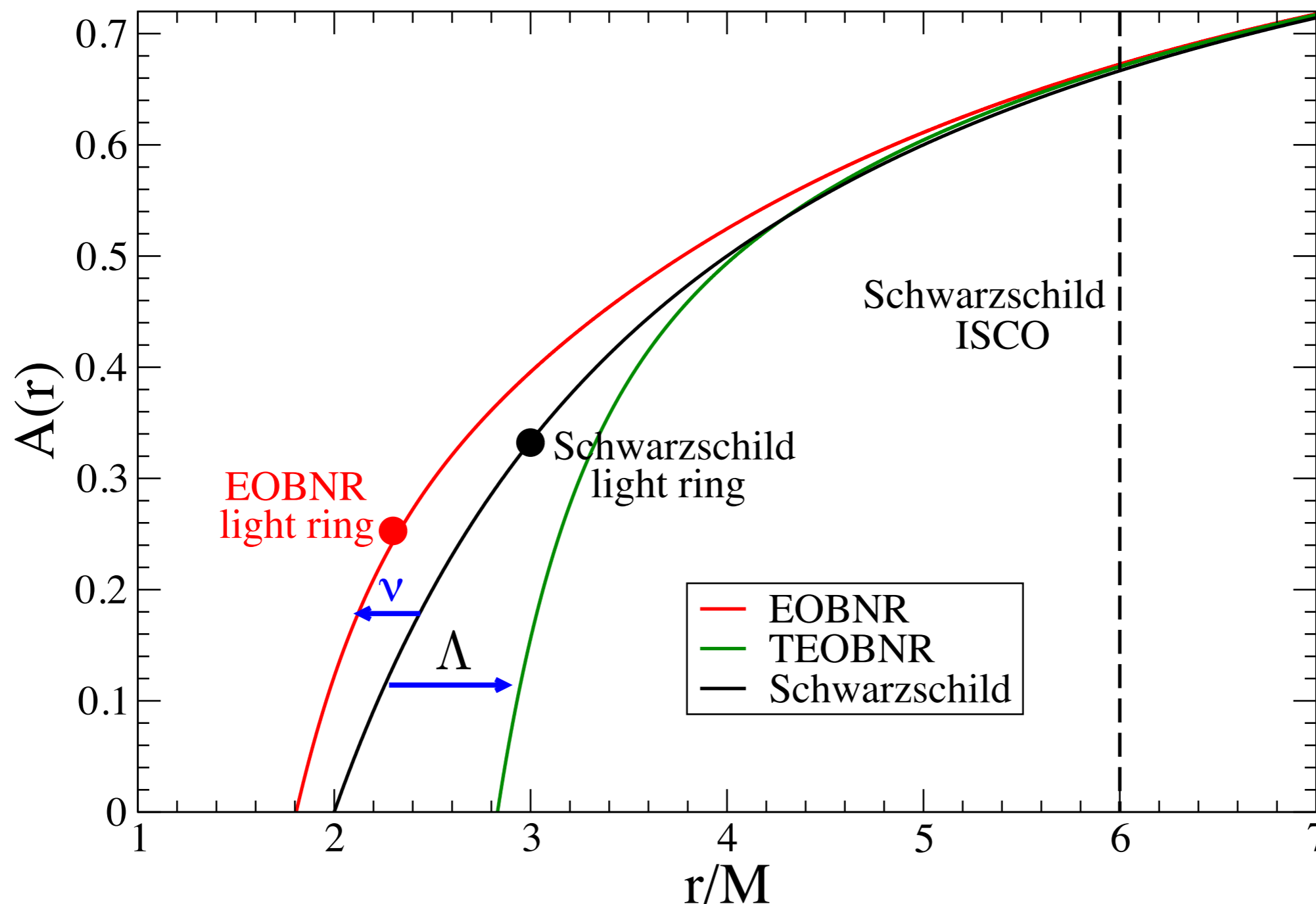


(Dietrich & Hinderer 17)

(Damour 1983, Flanagan & Hinderer 08, Binnington & Poisson 09, Vines et al. 11, Damour & Nagar 09, 12, Bernuzzi et al. 15, Hinderer, ...AB ... et al. 16, Steinhoff, ... AB ... et al. 16, Dietrich et al. 17-19, Nagar et al. 18)

# Strong-field effects in presence of matter in EOB theory

$$A(r) = A_\nu(r) + A_{\text{tides}}(r)$$



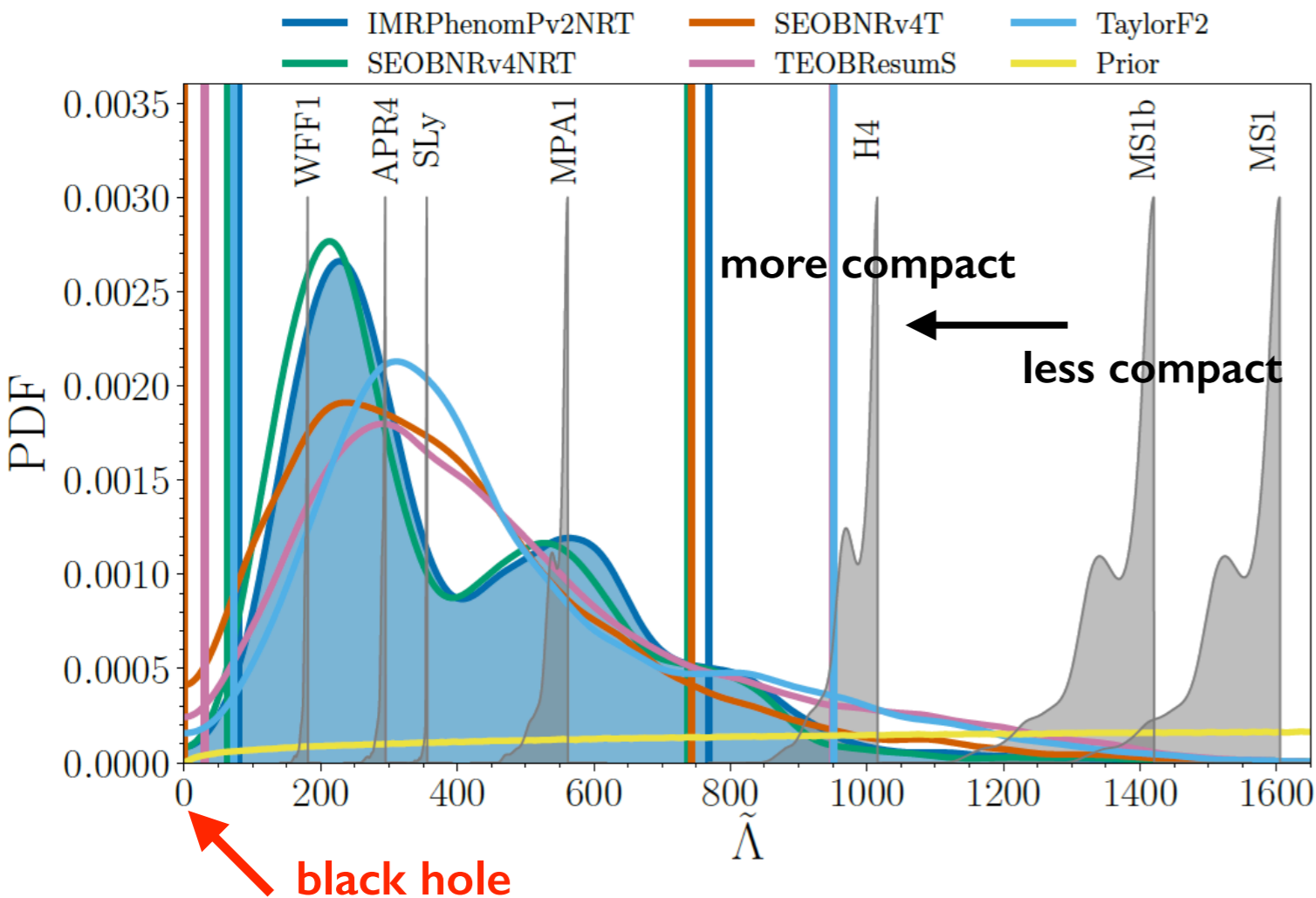
(Hinderer et al. 2016, Steinhoff et al. 2016,  
see also Bernuzzi et al. 15)

**Tides** make **gravitational** interaction **more attractive**

# Constraining NS equation of state with GW170817

(Abbott et al. PRX 9 (2019) 031040)

$$|\chi| \leq 0.05$$



Depends on EOS & compactness

$$\Lambda = \frac{\lambda}{m_{\text{NS}}^5} = \frac{2}{3} k_2 \left( \frac{R_{\text{NS}} c^2}{G m_{\text{NS}}} \right)^5$$

- **Effective tidal deformability** enters **GW phase at 5PN order:**

$$\tilde{\Lambda} = \frac{16(m_1 + 12m_2)m_1^4\Lambda_1 + (m_2 + 12m_1)m_2^4\Lambda_2}{(m_1 + m_2)^5}$$

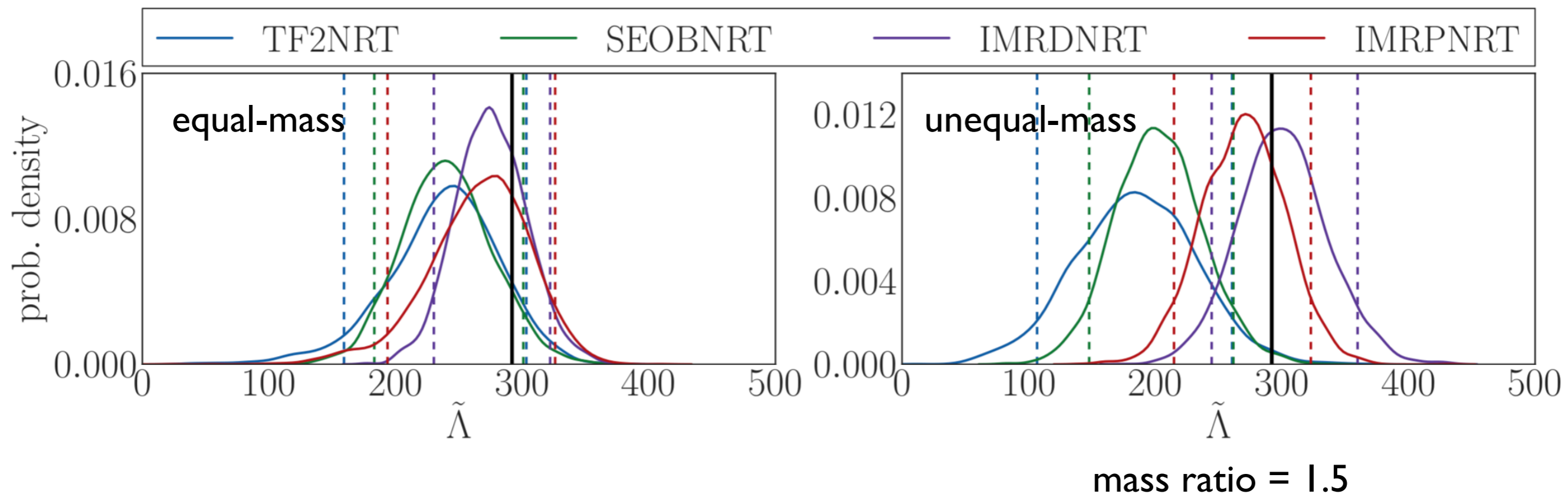
- Current **measurements** of tidal effects **dominated by statistical** error, but **inference** with **PN inspiral-only** waveform somewhat **stands out**.

# Systematics due to modeling for GW170817-like event

- Synthetic GW signal of a **binary neutron star** at **50 Mpc** is **injected** in Gaussian noise with **aLIGO design-sensitivity** noise-spectral density (**SNR  $\sim 87$** ).
- **Inference** with waveform models that have **same matter effects**, but **baseline point-mass model is different**.

(Samajdar & Dietrich 18)

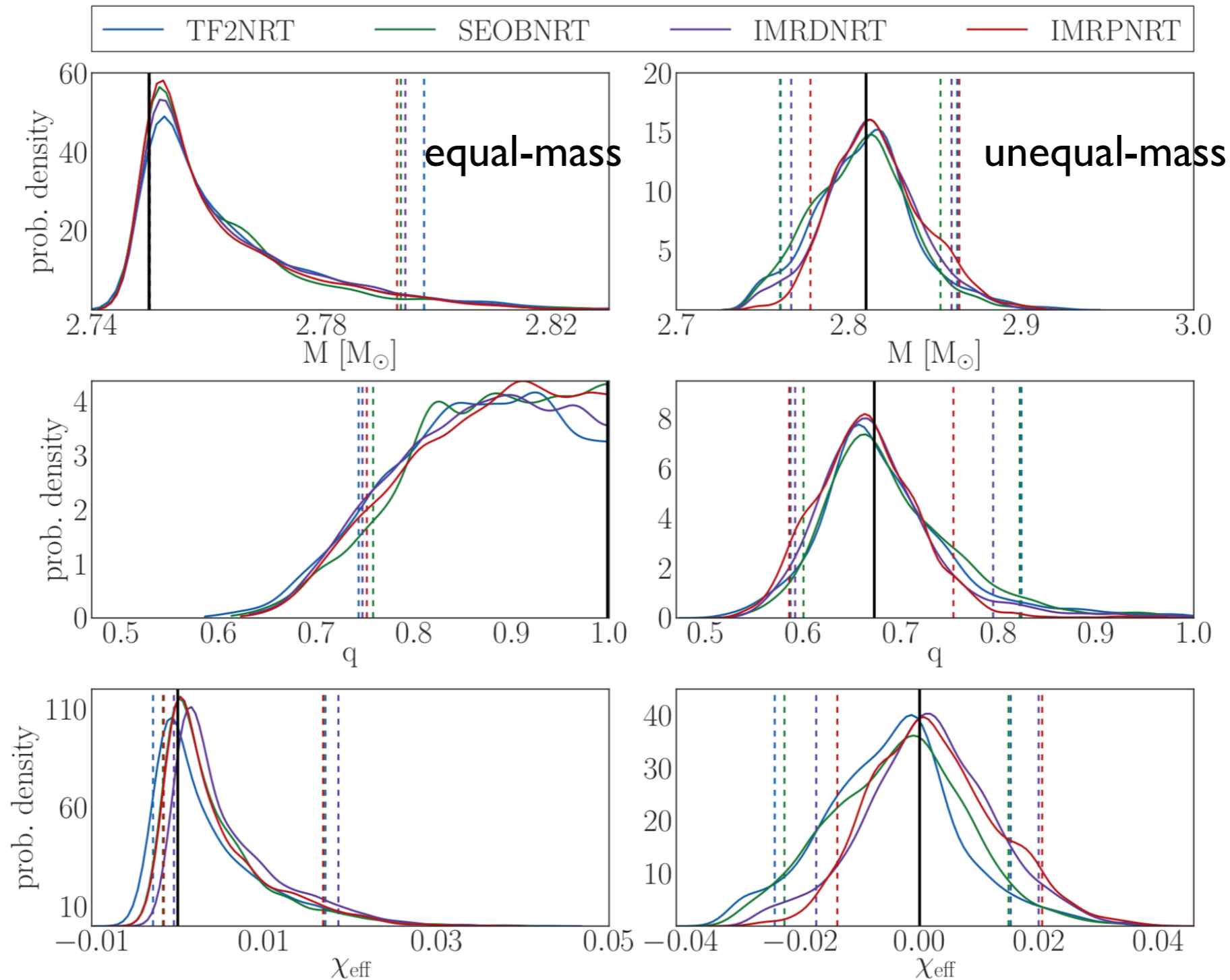
- **IMRPNRT** is injected



# Systematics due to modeling for GW170817-like event (contd.)

(Samajdar & Dietrich 18)

• **IMRPNRT** is injected



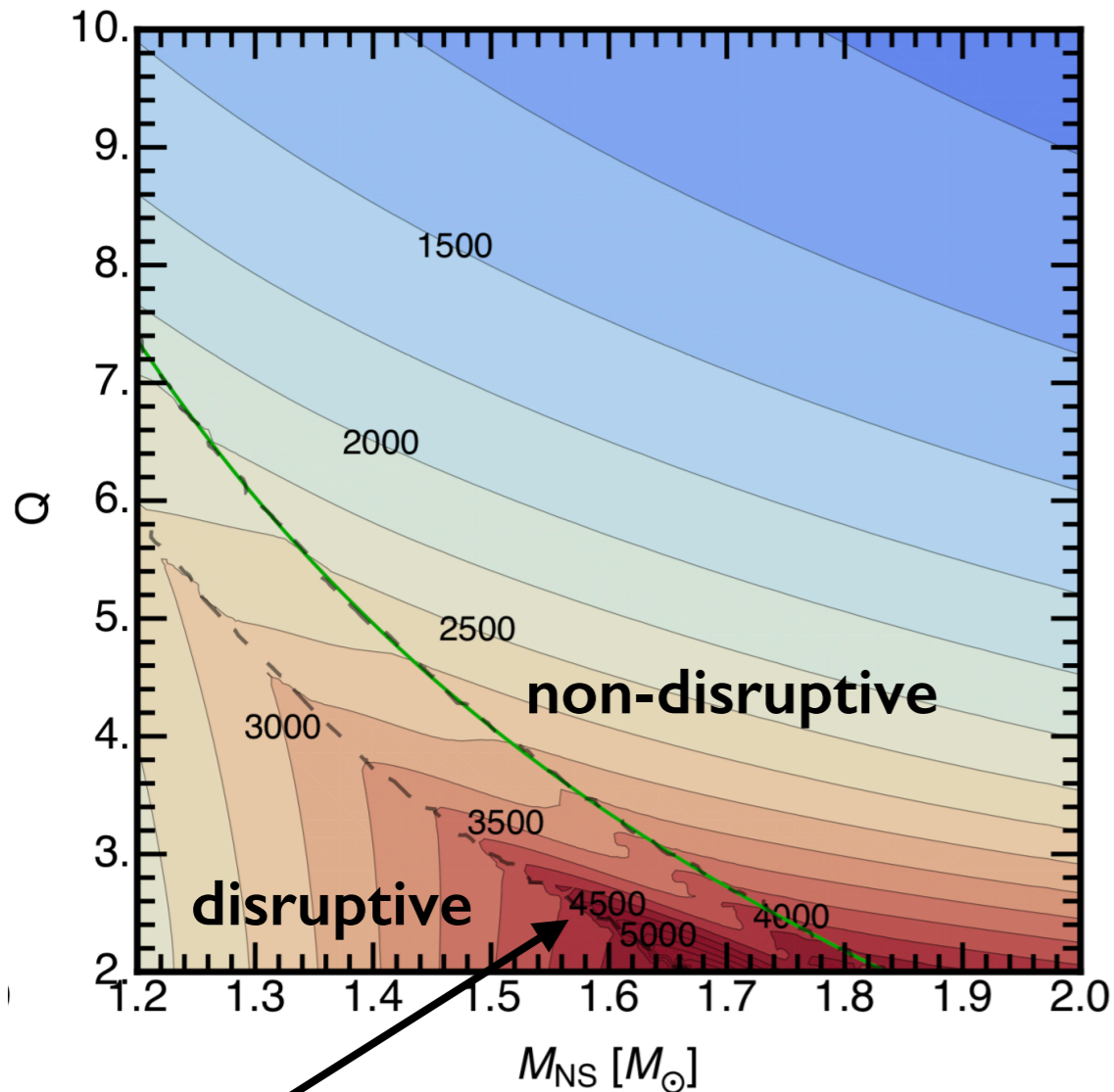
- For highly spinning BNS, **spin-related EOS effects** must be included to avoid biases. (Harry & Hinderer 18)



# Waveforms for NSBH combining analytical & numerical relativity

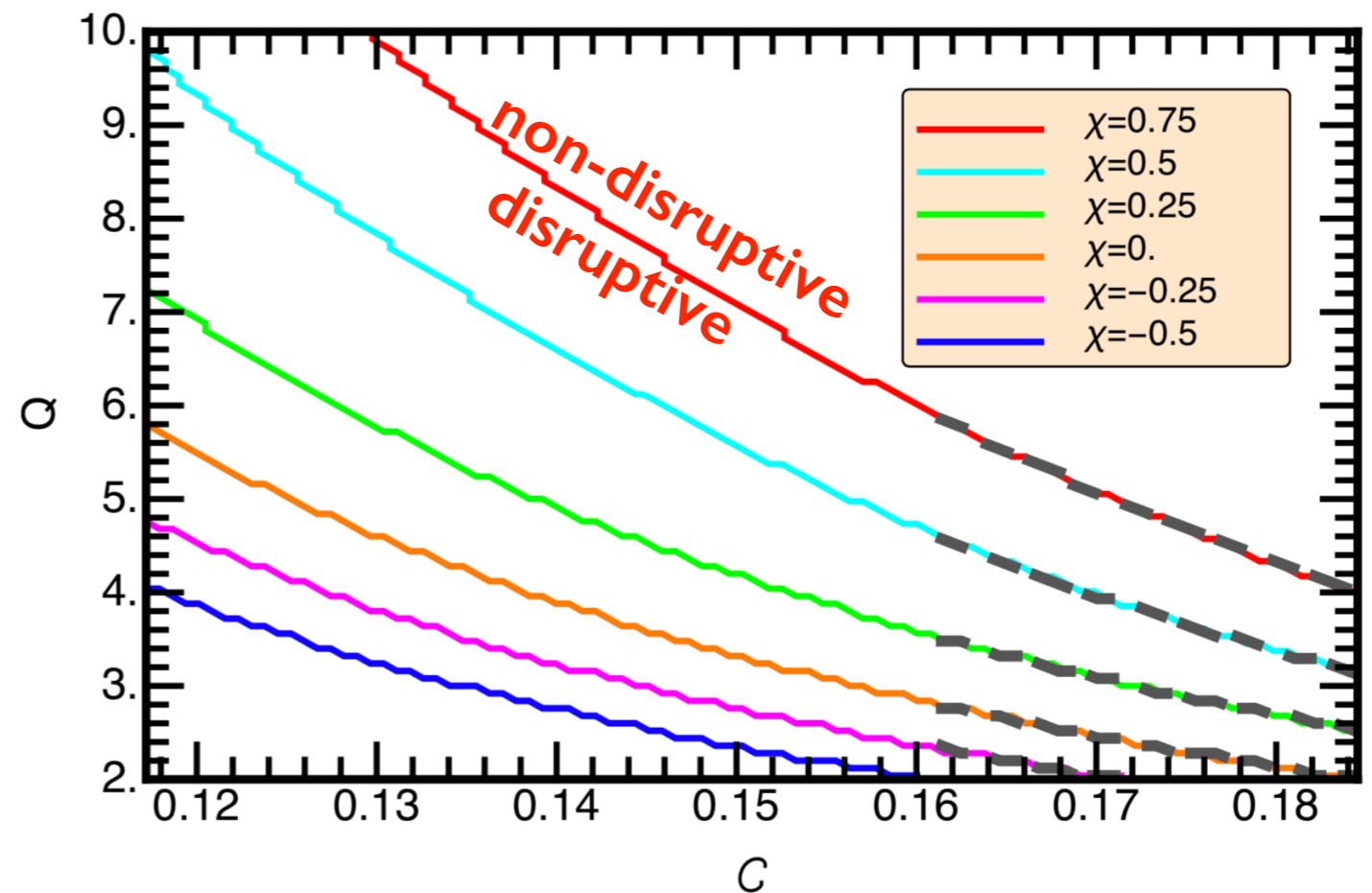
- Synergy between **analytical** and **numerical work** is **crucial**.
- **Current** waveform **models for NSBHs** are **not sufficiently accurate** to extract tidal effects (*Lackey et al. 14, Pannarale et al. 16, Pürrer et al. 17, Chakravarti et al. 17*)

EOS B,  $\chi=0.75$



$$f_{\text{cut}} \quad e f_{\text{cut}} h(f_{\text{cut}}) = f_{\text{max}} h(f_{\text{max}})$$

(Pannarale et al. 16)



NSBH is disrupted whenever  $Q < Q_D(C, \chi)$

# Waveforms for NSBH: disruptive case (contd.)

(Matas, AB, Dietrich, Hinderer & Pürrer in prep 19)

$$\tilde{h}_{\text{NSBH}}(f) = \mathcal{A}_{\text{NSBH}}(f) e^{i\varphi_{\text{NSBH}}(f)}$$

with **tidal effects**  
(Dietrich et al. 17-18)

$$\mathcal{A}_{\text{NSBH}}(f) = w_{\text{D}}(f) w_{\text{ND}}(f) \mathcal{A}_{\text{SEOBNRT}}(f)$$

- Disruptive:**

$$w_{\text{D}}(f) = w_{f_{\text{D}}, \sigma_{\text{D}}}^{-}(f)$$

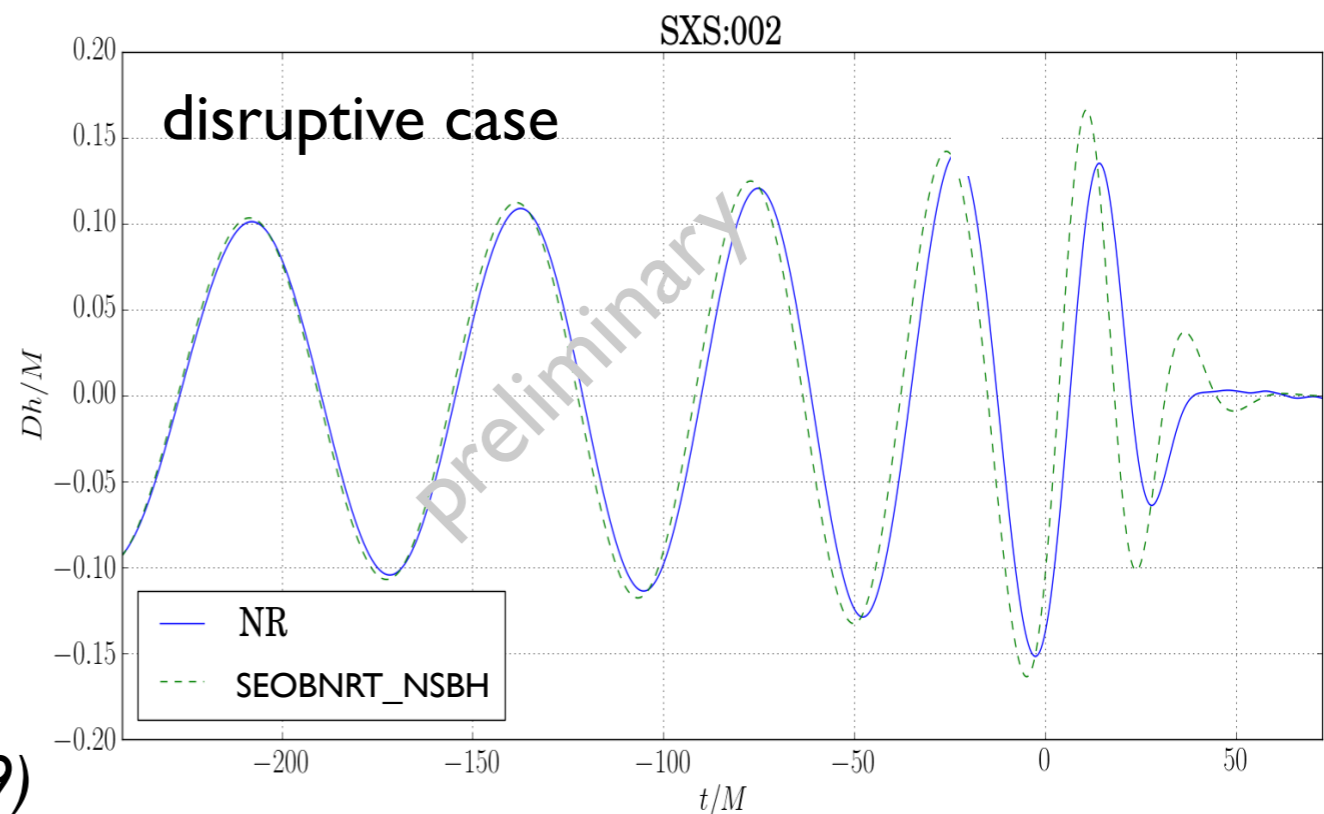
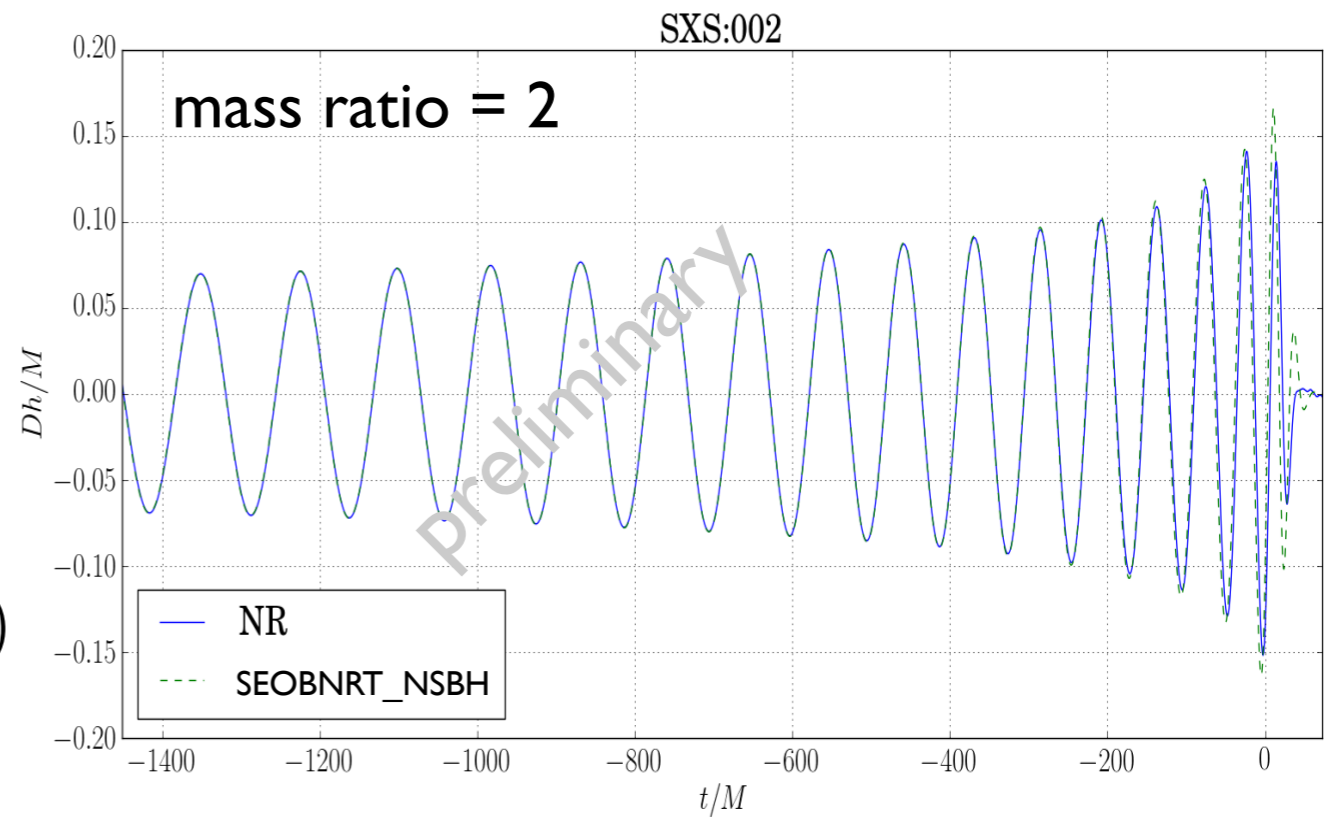
← waveform cut off  
at tidal frequency

- Non-disruptive:**

$$w_{\text{ND}}(f) = w_{f_{\text{ND}}, \sigma_{\text{ND}}}^{-}(f) + \epsilon w_{f_{\text{ND}}, \sigma_{\text{ND}}}^{+}(f)$$

$$w_{f_0, \sigma}^{\pm} = \frac{1}{2} \left[ 1 \pm \tanh \left( \frac{4(f - f_0)}{\sigma} \right) \right]$$

- Final mass and spin of NSBHs** (Zappa et al. 19)



# Waveforms for NSBH: disruptive case

(Matas, AB, Dietrich, Hinderer & Pürrer in prep 19)

$$\tilde{h}_{\text{NSBH}}(f) = \mathcal{A}_{\text{NSBH}}(f) e^{i\varphi_{\text{NSBH}}(f)}$$

with **tidal effects**  
(Dietrich et al. 17-18)

$$\mathcal{A}_{\text{NSBH}}(f) = w_{\text{D}}(f) w_{\text{ND}}(f) \mathcal{A}_{\text{SEOBNRT}}(f)$$

- Disruptive:**

$$w_{\text{D}}(f) = w_{f_{\text{D}}, \sigma_{\text{D}}}^{-}(f)$$

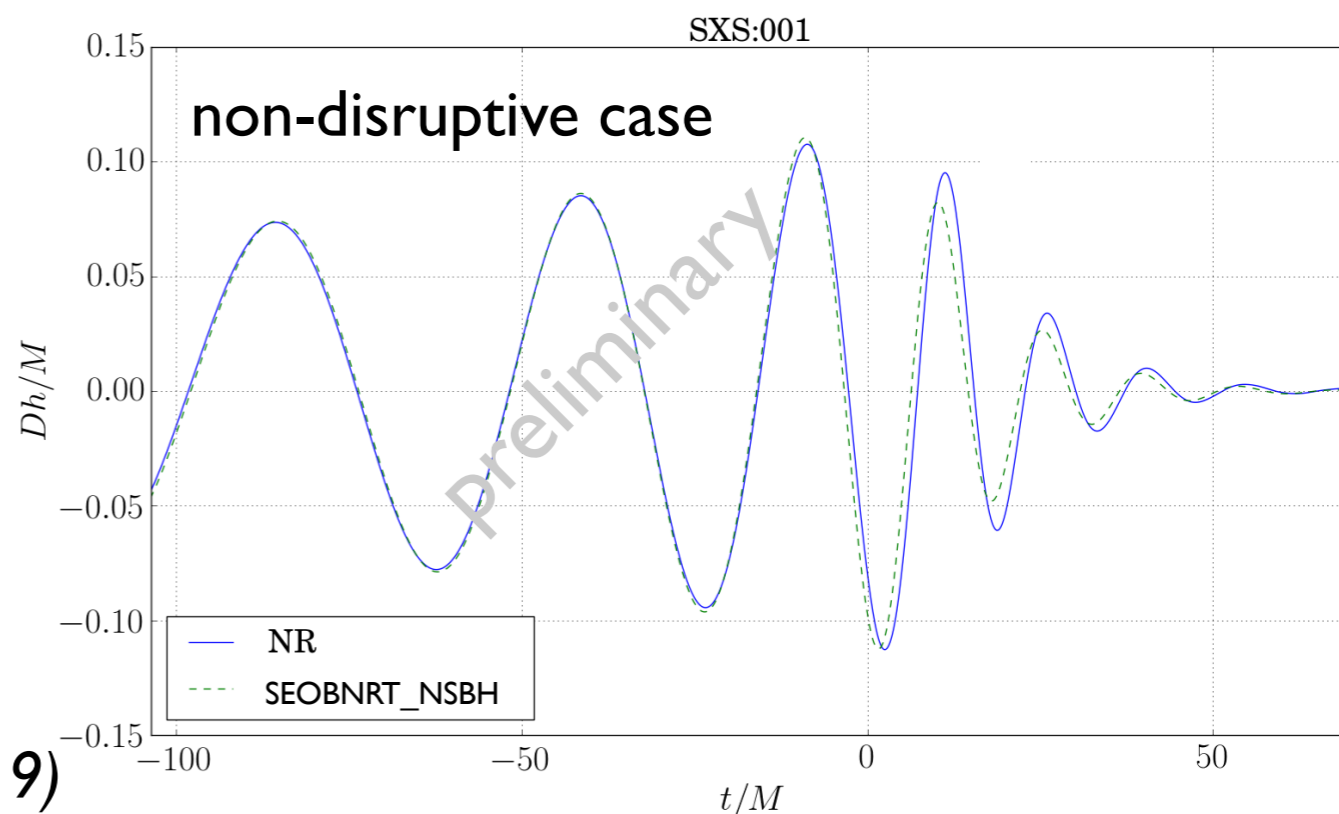
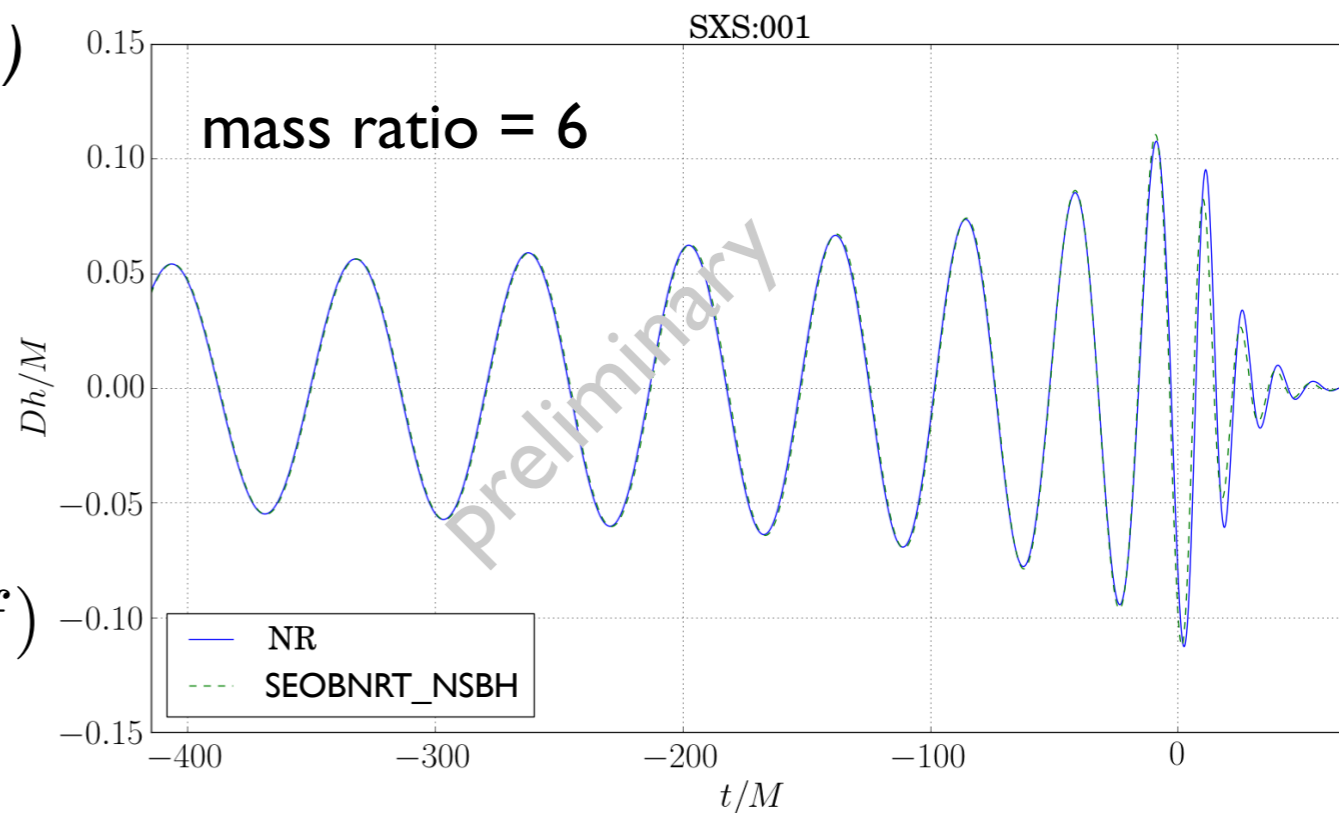
← waveform cut off  
at tidal frequency

- Non-disruptive:**

$$w_{\text{ND}}(f) = w_{f_{\text{ND}}, \sigma_{\text{ND}}}^{-}(f) + \epsilon w_{f_{\text{ND}}, \sigma_{\text{ND}}}^{+}(f)$$

$$w_{f_0, \sigma}^{\pm} = \frac{1}{2} \left[ 1 \pm \tanh \left( \frac{4(f - f_0)}{\sigma} \right) \right]$$

- Final mass and spin of NSBHs** (Zappa et al. 19)



# Mismatches of waveform model with NR: no hybridization

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(Matas, AB, Dietrich, Hinderer & Pürrer in prep 19)

- **NSBH amplitude model** fitted to **117 spinning NR** waveforms from **Shibata's group** and **5 non-spinning SXS NR** waveforms.

	$M_{\text{NS}}$	$Q$	$\chi_{\text{BH}}$	$\Lambda_{\text{NS}}$	Mismatch
SXS:001	1.4	6	0	791	$4.0 \times 10^{-3}$
SXS:003	1.35	3	0	624	$9.9 \times 10^{-3}$
SXS:002	1.4	2	0	791	$4.9 \times 10^{-3}$
SXS:006	1.4	1.5	0	791	$5.0 \times 10^{-3}$
SXS:004	1.4	1	0	791	$1.7 \times 10^{-2}$

# Mismatches of waveform model with NR: hybridization

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(Matas, AB, Dietrich, Hinderer & Pürrer in prep 19)

- **NSBH amplitude model** fitted to **117 spinning NR** waveforms from **Shibata's group** and **5 non-spinning SXS NR** waveforms.

	$M_{\text{NS}}$	$Q$	$\chi_{\text{BH}}$	$\Lambda_{\text{NS}}$	Mismatch
SXS:001	1.4	6	0	791	$2.0 \times 10^{-4}$
SXS:003	1.35	3	0	624	$8.0 \times 10^{-4}$
SXS:002	1.4	2	0	791	$2.1 \times 10^{-4}$
SXS:006	1.4	1.5	0	791	$3.0 \times 10^{-4}$
SXS:004	1.4	1	0	791	$7.8 \times 10^{-4}$

# Toward the era of precision gravitational-wave astrophysics

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- We have **not missed “loud” events**. For **sub-threshold events**, it might be **critical** to use waveform models with **more physics**.
- So far, **inference from GW** observations is **dominated by statistical** instead of modeling **error**.
- **Highest priorities:**
  - **NSBH** modeling, analytically and numerically
  - inclusion of **eccentricity** and **precessing spins** in **IMR waveforms**
  - NR simulations with **large mass ratios** ( $> 4$ ) and **large spins** ( $> 0.8$ ), with **larger number** of GW **cycles** ( $> 50$ )
- More **extensive** studies to **assess *real* biases** of waveform models are needed, comparing models **among themselves** and **against NR**.