Aug. 11th, 2015 Japan-France Joint Seminar
"New Frontiers in Non-equilibrium Physics of Glassy Materials"

Signatures of the full replica symmetry breaking in jamming systems under shear

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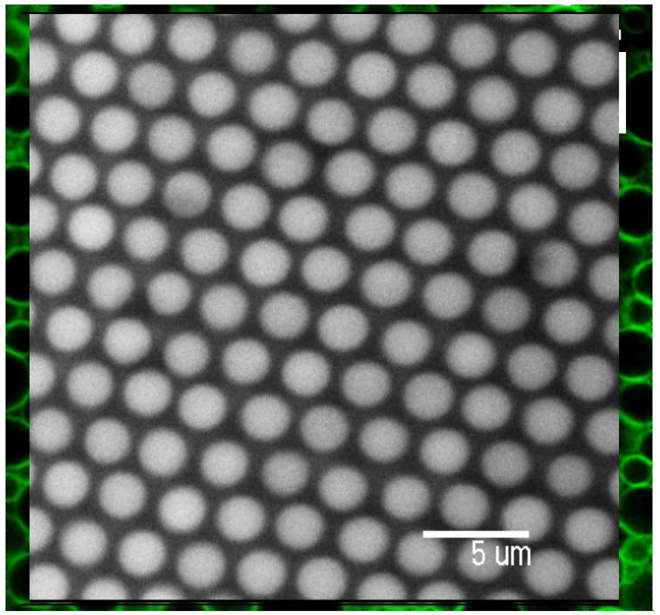
Synergy of Fluctuation and Structure :
Quest for Universal Laws in Non-Equilibrium Systems
2013-2017 Grant-in-Aid for Scientific Research on Innovative Areas, MEXT, Japan

JPS Core-to-Core program 2013-2015 Non-equilibrium dynamics of soft matter and information

Emulsions, colloids,...



E. R. Weeks,
in "Statistical Physics of Complex Fluids",
Eds. S Maruyama & M Tokuyama
(Tohoku University Press, Sendai, Japan, 2007).

hexagonal lattice

hexagonal close packing

volume fraction φ

Supercooled Liquid

Glass

“Jammed” Glass

$\varphi_{\rm m}$ ~ 0.5

$\varphi_{\rm g}$ ~ 0.58

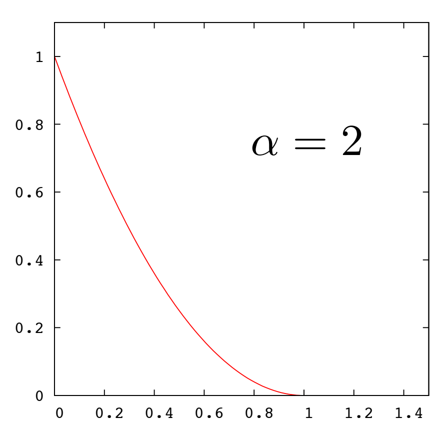
$\varphi_{\rm RCP}$ ~ 0.64 “random” close packing

0.74

$$k_{\rm B}T_{\rm room}/\epsilon \sim 10^{-5}$$

Model

$v(r)/\epsilon$



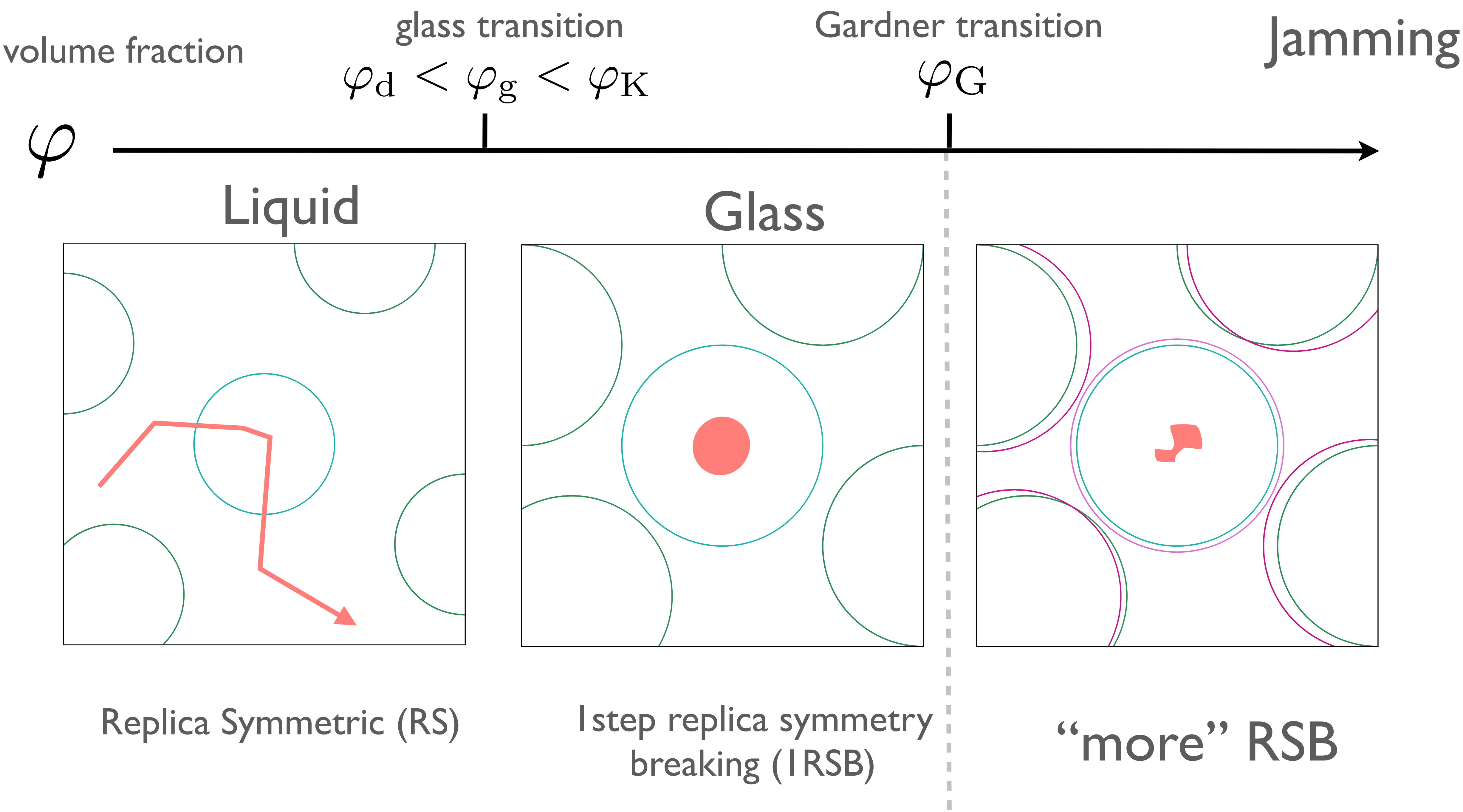
r/D

$$U = \sum_{\langle ij \rangle} v(r_{ij}) \qquad r_{ij} = |\mathbf{r}_i - \mathbf{r}_j|$$

$$v(r) = \epsilon(1 - r/D)^{\alpha}\theta(1 - r/D)$$

Mean-Field Picture on Glass transitions

F. Zamponi's talk



Kurchan-Parisi-Urbani-Zamponi, (2013).

$$\lambda_{\rm replicon} = 0$$

Almeida-Thouless (AT) instability much like the MF models of spin-glasses

Stress relaxation process

Okamura-Yoshino, unpublished (2013)

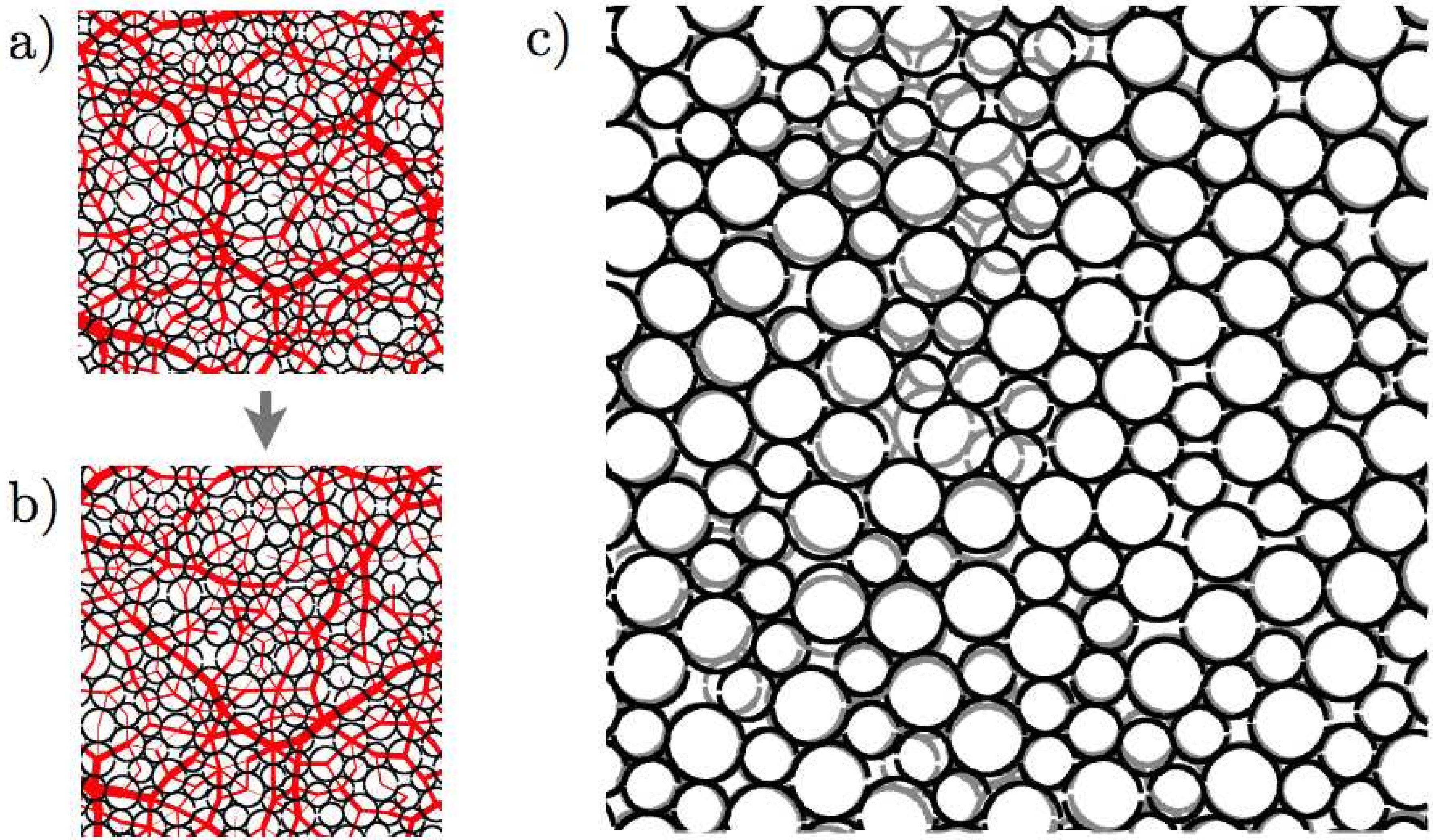


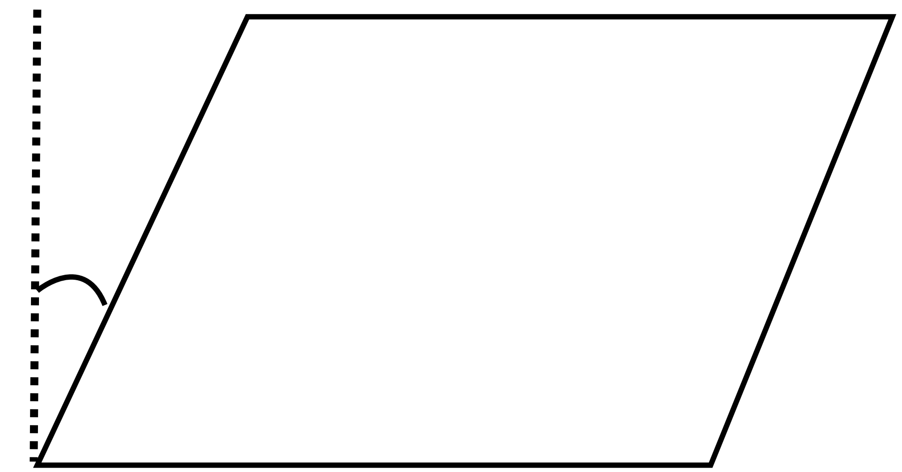
Figure 1: This figure show snaphots before/after a plastic event trigered by thermal noises. Here we used a 2-dimensional version of the model (for the purpose of a demonstration) at volume fraction $\phi = 0.85$ which is slightly above the jamming density $\phi_{\mathrm{J}} \sim 0.84$ (2-dim). The system is initially perturbed weakly by a shear-strain $\gamma = 0.05$ and let to relax at zero temperature by the conjugated gradient method which allows the system to relax using the harmonic modes. Then the thermal noise at (reduced) temperature $T = 10^{-6}$ is switched on. The configuration of particles are represented by the circles and that of the contact forces $f_{ij} = -dv_{ij}(r_{ij})/dr_{ij}$ are represented by bonds whose thickness is chosen to be proportional to f_{ij}. The panels a) and b) show the snapshots before/after a plastic event (which took about $10^4 t_{\mathrm{micro}}$ to complete). In panel c) the configuratoin of the particles before/after are overlaid : the one before the event is shown by the lighter color.

solids under shear

liquid theory + replica ⇨ mechanical properties of amorphous solids

Yoshino-Mezard, PRL 105, 015504 (2010), Yoshino, JCP 136, 214108 (2012)

Strain γ

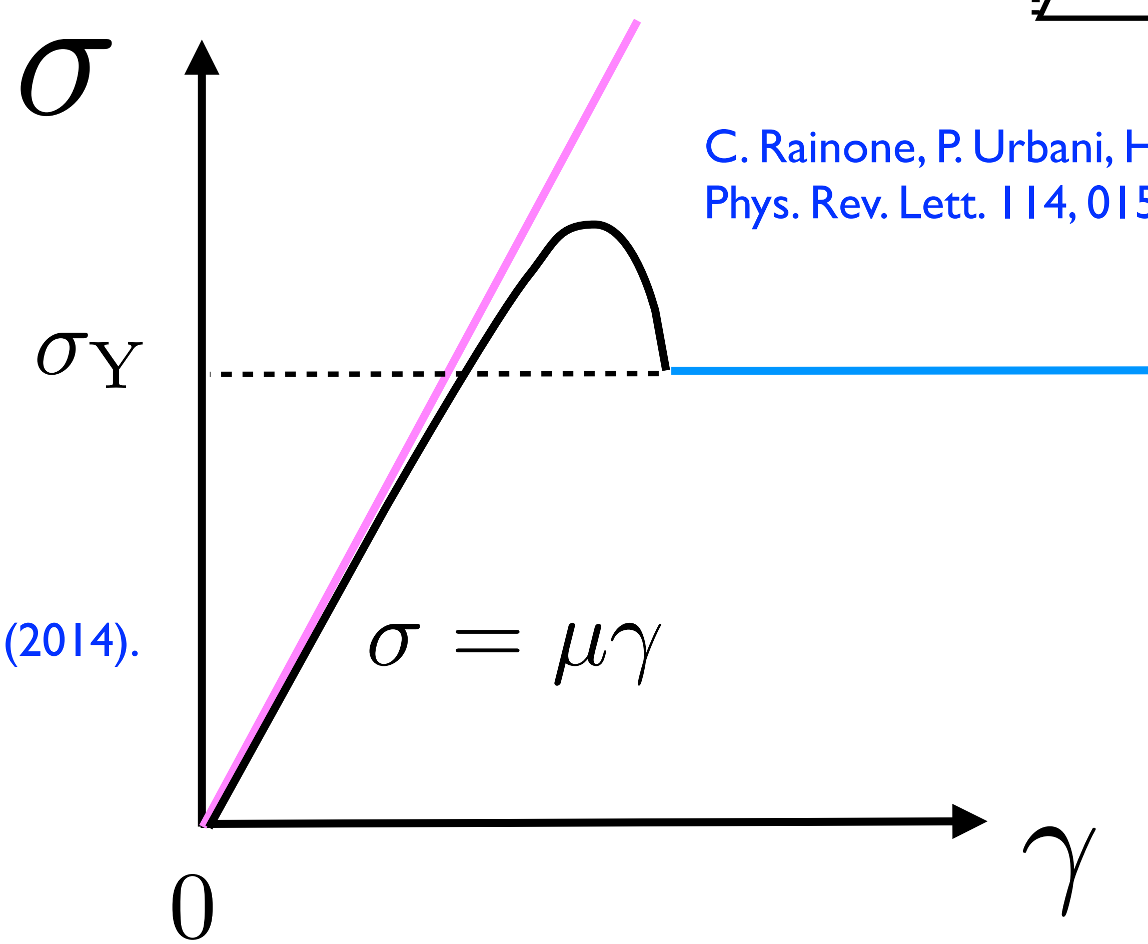


shear-stress (force/area)

C. Rainone, P. Urbani, HY, F. Zamponi, Phys. Rev. Lett. 114, 015701 (2015).

yield stress

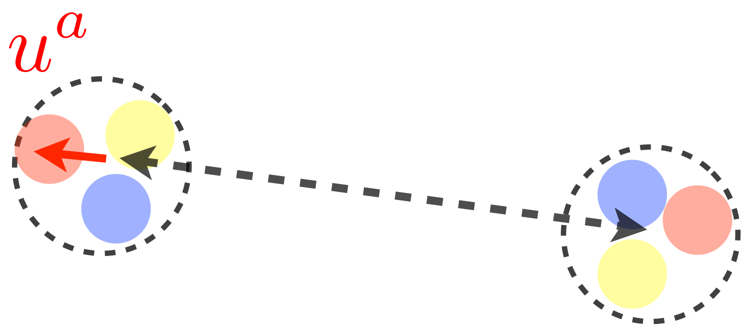
HY, F. Zamponi, PRE90, 015701(2014).

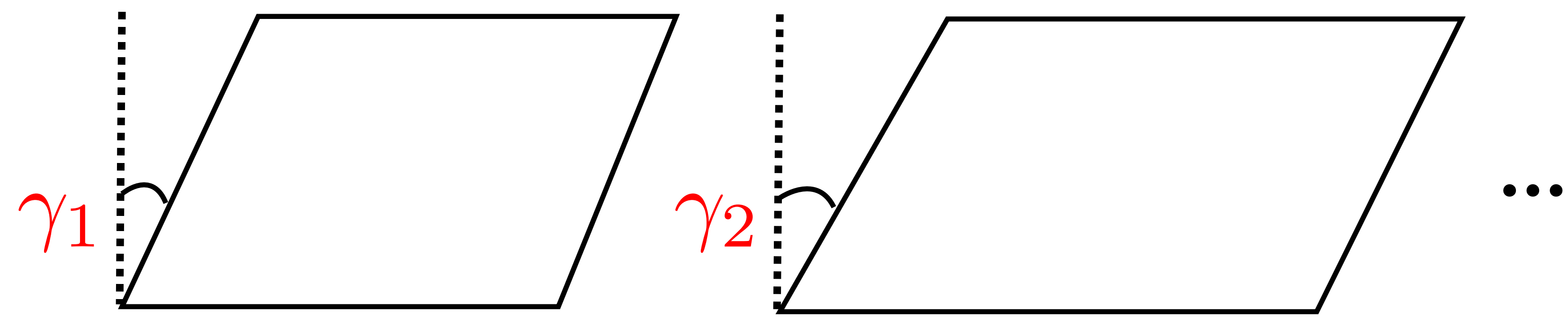


shear-strain

Twisting replicated hardsphere liquid $d \to \infty$

HY and F. Zamponi, Phys. Rev. E 90, 022302 (2014).





$$-\beta F(\{\gamma_a\}) = \int d\overline{x}\rho(\overline{x})[1-\log\rho(\overline{x})] + \frac{1}{2}\int d\overline{x}d\overline{y}\rho(\overline{x})\rho(\overline{y})f_{\{\gamma_a\}}(\overline{x},\overline{y})$$

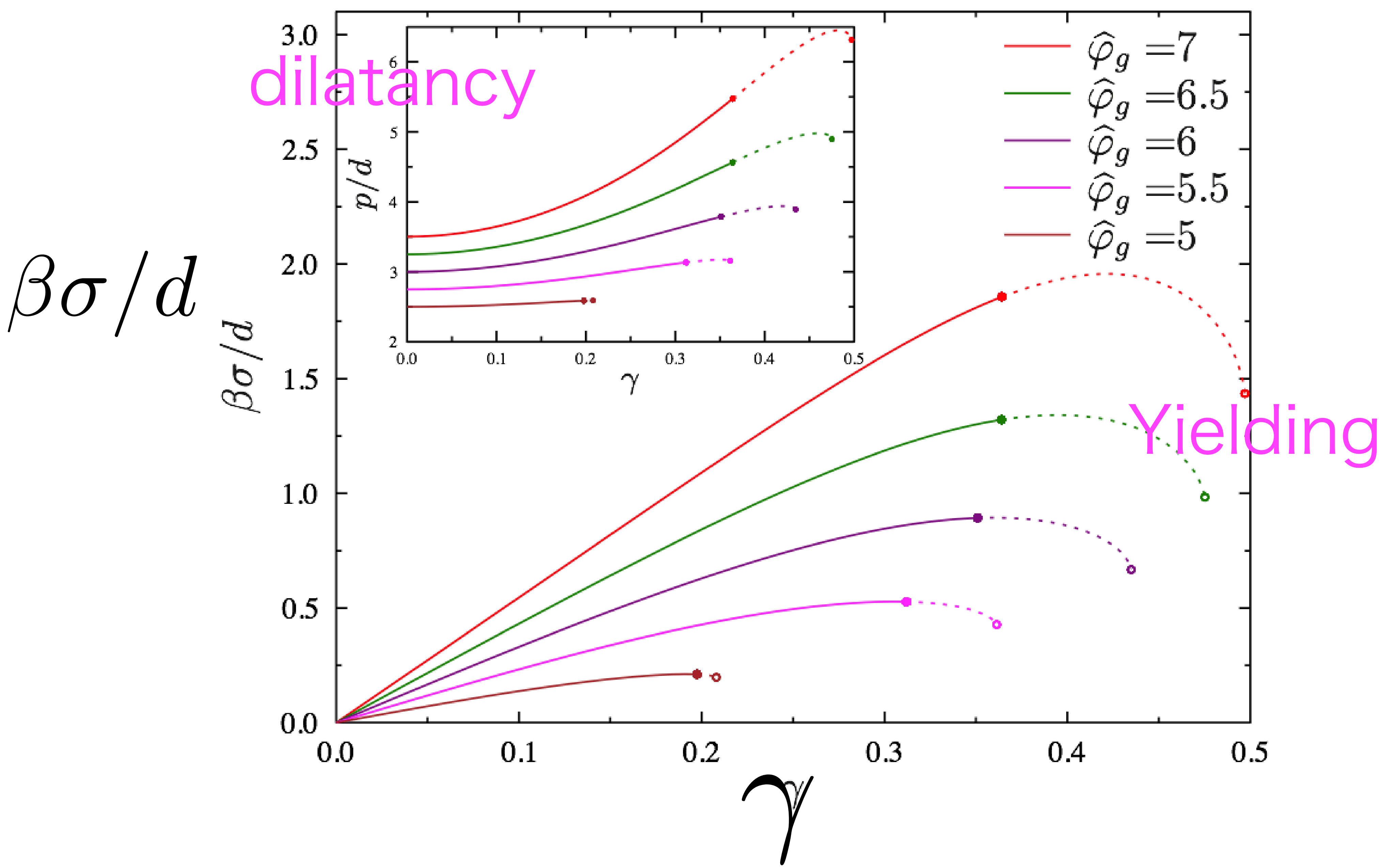
Replicated Mayer function (under shear)

$$f_{\{\gamma_a\}}(\overline{x},\overline{y}) = -1 + \prod_{a=1}^{m} e^{-\beta v(|S(\gamma_a)(x_a-y_a)|)} \qquad S(\gamma)_{\mu\nu} = \delta_{\mu\nu} + \gamma\delta_{\nu,1}\delta_{\mu,2}$$

$$-\beta F(\hat{\Delta},\{\gamma_a\})/N = 1 - \log\rho + d\log m + \frac{d}{2}(m-1)\log(2\pi e D^2/d^2) + \frac{d}{2}\log\det(\hat{\alpha}^{m,m}) - \frac{d}{2}\widehat{\varphi}\int\frac{d\lambda}{\sqrt{2\pi}}\mathcal{F}\left(\Delta_{ab} + \frac{\lambda^2}{2}(\gamma_a-\gamma_b)^2\right)$$

■ Following glassy states under shear/compression

Corrado Rainone, Pierfrancesco Urbani, Hajime Yoshino, Francesco Zamponi,
Phys. Rev. Lett. 114, 015701 (2015)



Small strain expansion

$$F(\{\gamma_a\})/N = F(\{0\})/N + \sum_{a=1}^{m} \sigma_a \gamma_a + \frac{1}{2}\sum_{a,b}^{1,m} \mu_{ab}\gamma_a\gamma_b + \cdots$$

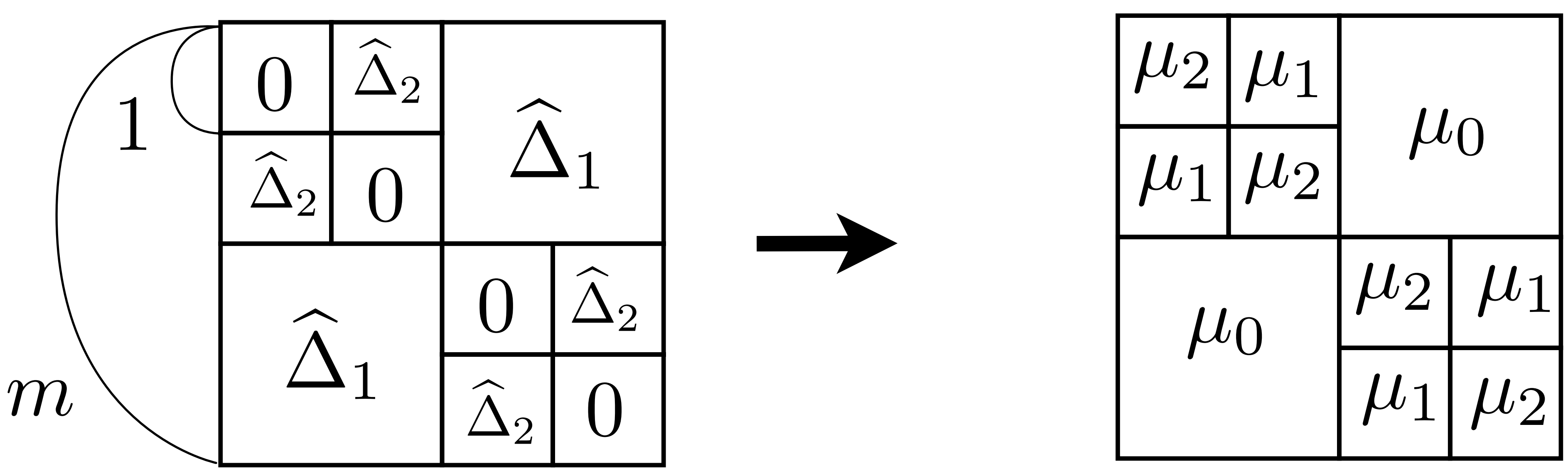
$$\beta\mu_{ab} = \frac{d}{2}\widehat{\varphi}\left[\delta_{ab}\sum_{c(\neq c)}\frac{\partial \mathcal{F}}{\partial \Delta_{ac}} - (1-\delta_{ab})\frac{\partial \mathcal{F}}{\partial \Delta_{ab}}\right]$$

translational invariance $\sum_{b} \mu_{ab} = 0$

$$\Delta(y) = \frac{\gamma(y)}{y} - \int_{y}^{1/m} \frac{dz}{z^2}\gamma(z)$$

$y = x/m$

$$\beta\hat{\mu}(y) = \frac{1}{m\gamma(y)}$$



$m < x < 1$

1RSB case : HY and M. Mezard (2010), HY (2012)

1 step RSB

HY and F. Zamponi, Phys. Rev. E 90, 022302 (2014).

$$\widehat{\varphi}_{\rm d} < \widehat{\varphi} < \widehat{\varphi}_{\rm Gardner}$$

$$\beta\widehat{\mu}_{ab} = \beta\widehat{\mu}_{\rm EA}\left(\delta_{ab} - \frac{1}{m}\right)$$

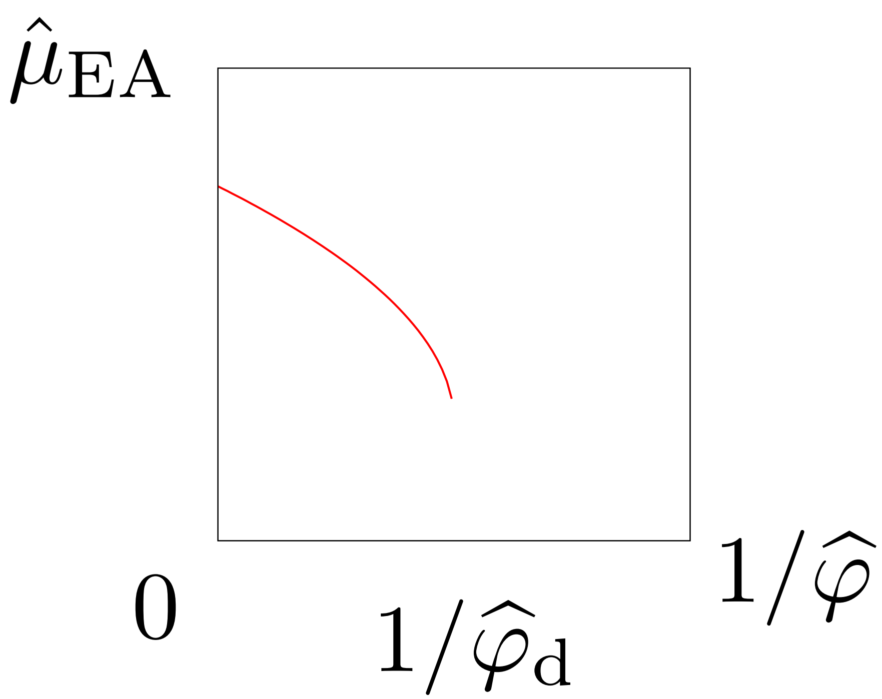
H. Yoshino and M. Me´zard, PRL **105**, 015504 (2010).

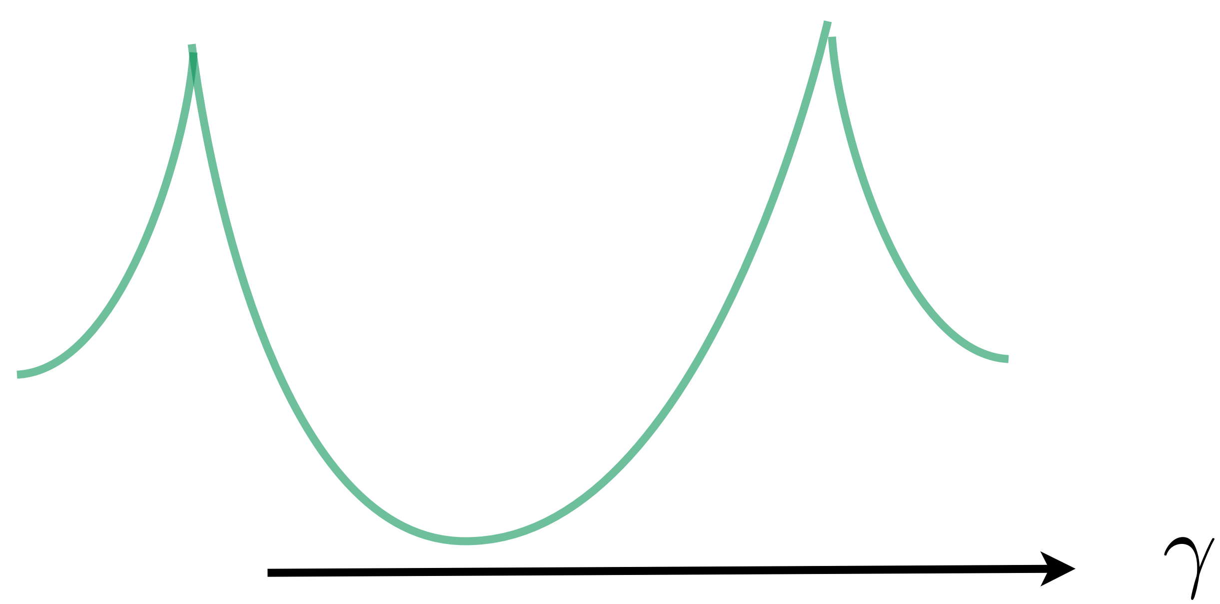
H. Yoshino, The Journal of Chemical Physics **136**, 214108 (2012).

$$\beta\hat{\mu}_{\rm EA} = \widehat{\Delta}_{\rm EA}^{-1} \qquad \widehat{\Delta}_{\rm EA} \sim \widehat{\Delta}_{d} - C(\widehat{\varphi} - \widehat{\varphi}_{d})^{1/2}$$

in agreement with MCT

G. Szamel and E. Flenner, PRL **107**, 105505 (2011).





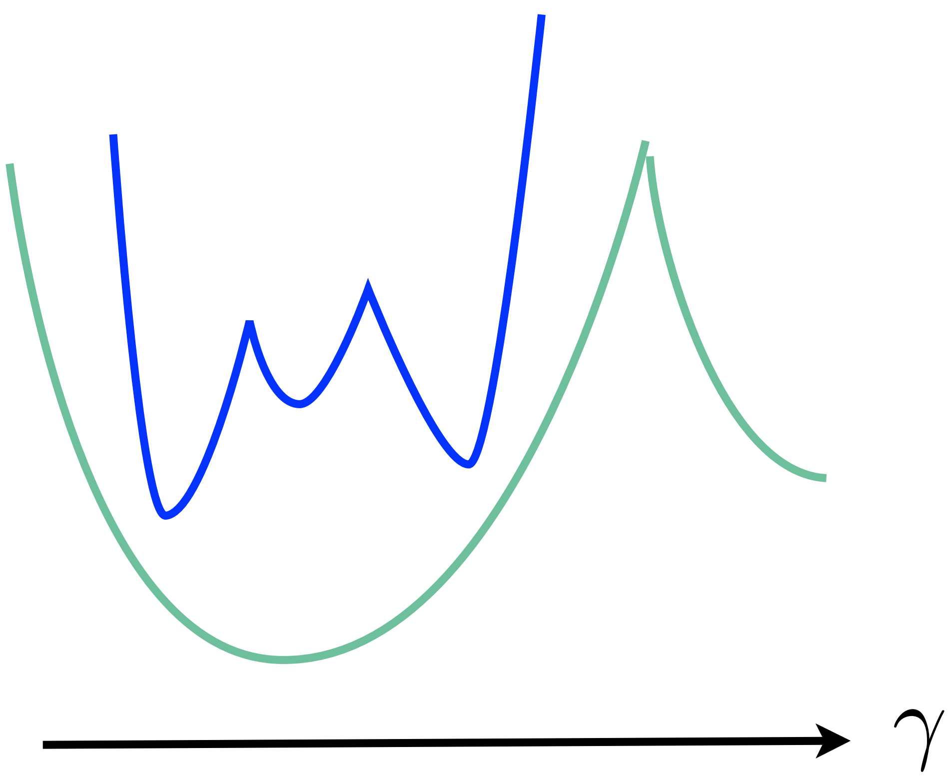
1+continuous RSB

$\widehat{\varphi}_{\mathrm{Gardner}} < \widehat{\varphi} < \widehat{\varphi}_{\mathrm{GCP}}$

$$\widehat{\varphi} \to \widehat{\varphi}_{\mathrm{GCP}}^{-} \qquad p \propto 1/m \to \infty$$

$$\gamma(y) \propto \gamma_\infty y^{-(\kappa-1)} \qquad \kappa = 1.41575$$

HY and F. Zamponi, Phys. Rev. E 90, 022302 (2014).



$$\beta\mu_{\mathrm{EA}} = 1/\Delta_{\mathrm{EA}} \propto m^{-\kappa} \propto p^{\kappa}$$

consistent with scaling argument + effective medium computation

E DeGiuli; E Lerner; C Brito; M Wyart, PNAS 111 (2014), 17054

"rigidity of inherent structures"

$$\beta\widehat{\mu}(1) = \frac{1}{m\gamma(1)} \propto p$$

"rigidity of metabasins"

Field Cooled/ Zero Field Cooled Susceptibilities in Spin-Glasses

Low-dc-field susceptibility of *Cu*Mn spin glass

Shoichi Nagata,* P. H. Keesom, and H. R. Harrison
Department of Physics, Purdue University, West Lafayette, Indiana 47907
(Received 7 August 1978)

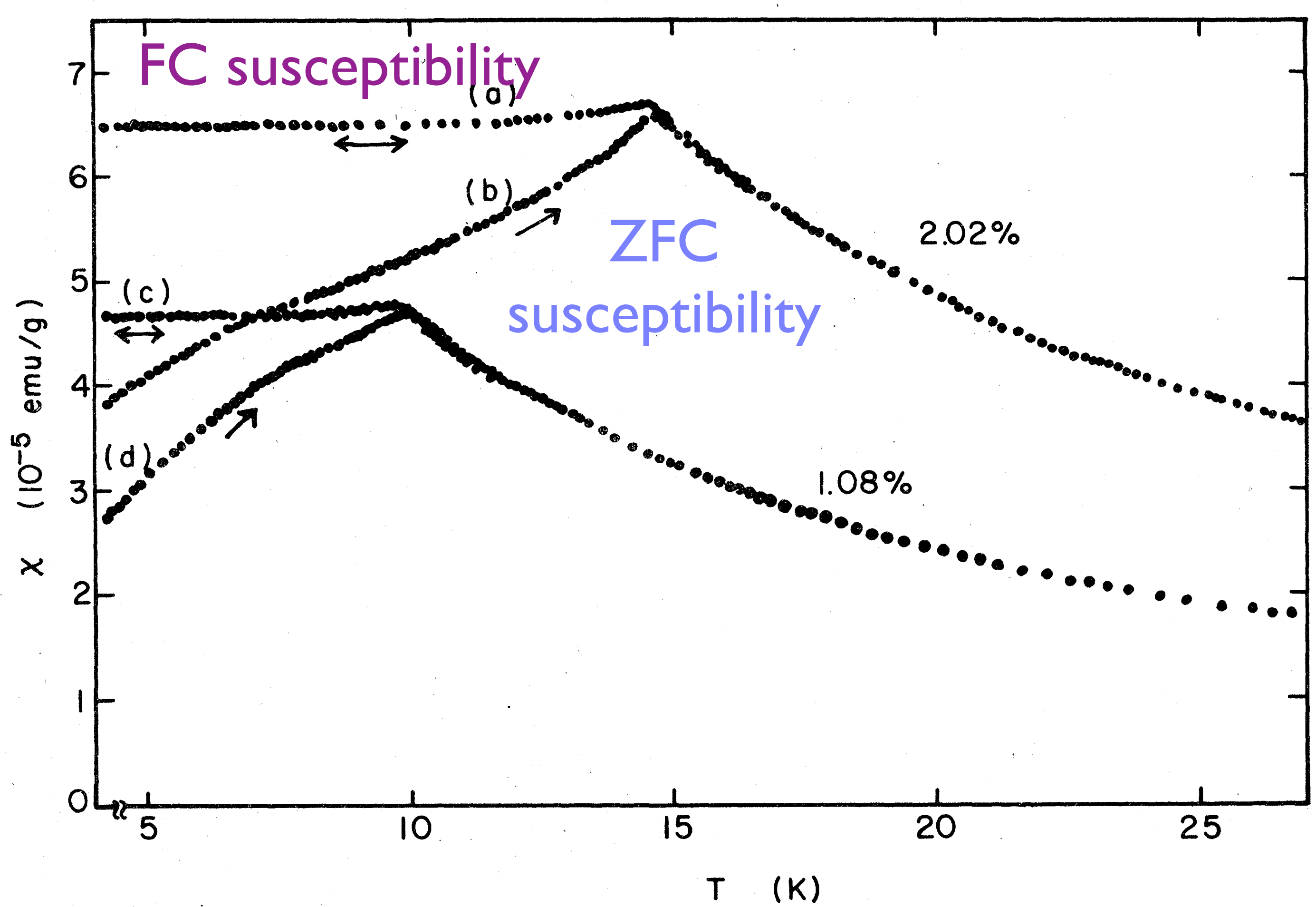


FIG. 1. Static susceptibilities of *Cu*Mn vs temperature for 1.08- and 2.02-at.% Mn. After zero-field cooling ($H < 0.05$ G), initial susceptibilities (b) and (d) were taken for increasing temperature in a field of $H = 5.90$ G. The susceptibilities (a) and (c) were obtained in the field $H = 5.90$ G, which was applied above T_g before cooling the samples.

Full RSB solution of the Sherrington-Kirkpatrick (SK) model (exact solution of the Edwards-Anderson spin-glass model in the $d \to \infty$ limit)

$$\chi_{\rm FC} = \beta[1 - \int_0^1 dx Q(x)]$$

$$\chi_{\rm ZFC} = \beta[1 - Q(1)]$$

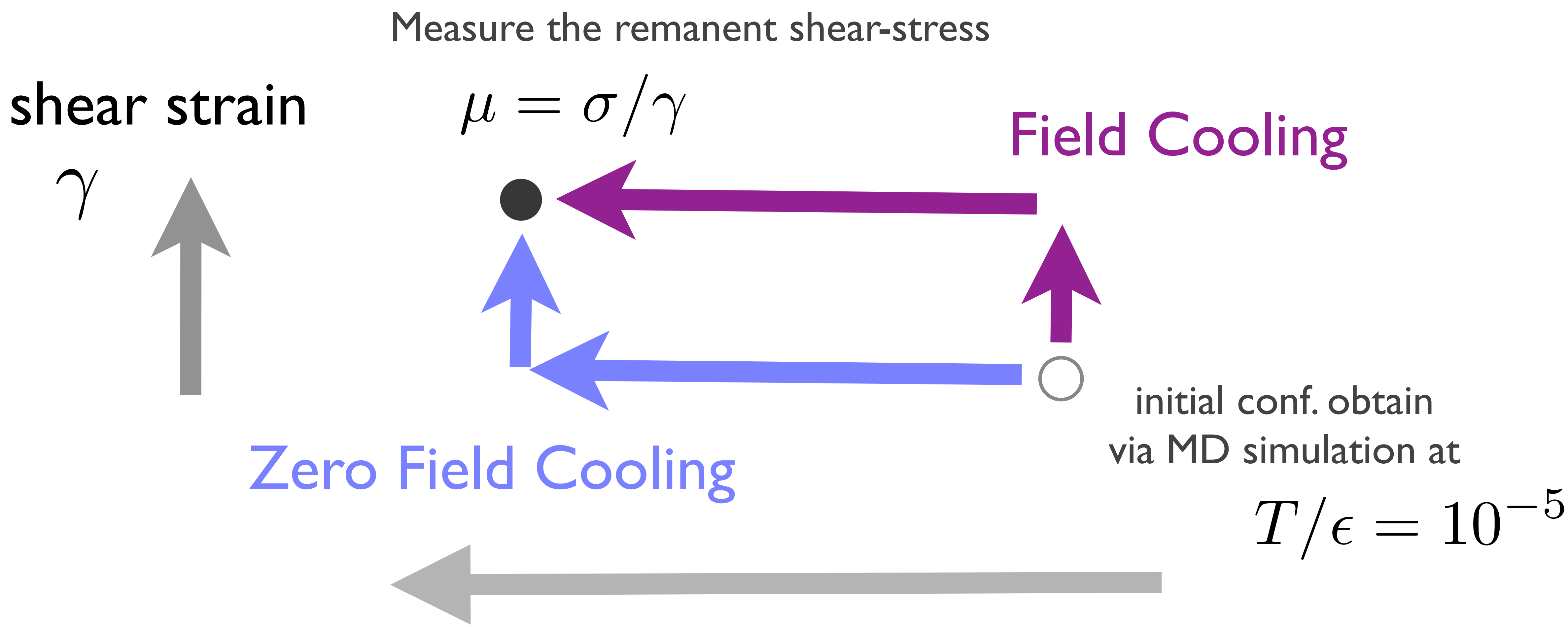
(NOTE) spin-wave rigidity of spin-glass is also hierarchical reflecting RSB

G. Kotliar, H. Sompolinsky, and A. Zippelius PRB 35, 311 (1987)

H. Yoshino, JCP 136, 214108 (2012)

FC/ZFC shear response of glasses ?

Nakayama-Yoshino-Zamponi, in progress



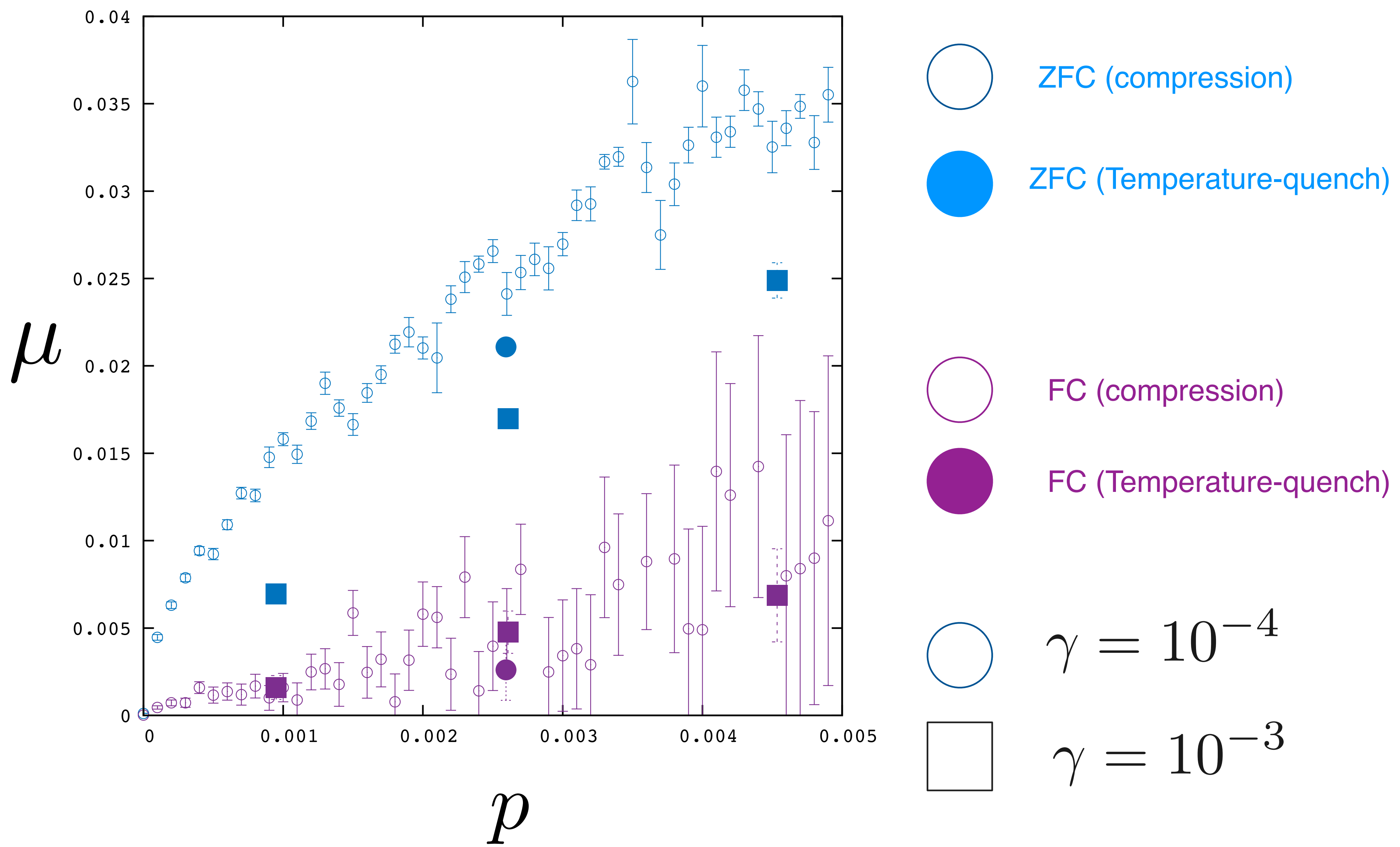
(1) temperature quench to T=0 (working at $\varphi_{\rm target}$)
(2) compression (working at T=0)

Energy minimization : conjugated gradient method

Simulation of densely packed soft-spheres in 3 dim.

3 dim Harmonic-sphere(binary)

$N = 320, 1000$ # of samples $O(10^3) - O(10^4)$



Reminder: theory $d \to \infty$ $\mu_{\mathrm{ZFC}} \propto \sqrt{p}$ $\mu_{\mathrm{FC}} \propto p$

Divergence of non-linear susceptibility at spin-glass transition

Journal of the Physical Society of Japan
Vol. 52, No. 12, December, 1983, pp. 4323–4330

Linear and Non-Linear Susceptibilities in Canonical Spin Glass AuFe (1.5 at.%Fe)

Toshio TANIGUCHI, Hideo MATSUYAMA,
Susumu CHIKAZAWA† and Yoshihito MIYAKO

Department of Physics, Faculty of Science, Hokkaido University, Sapporo 060
†Department of Applied Material Science, Muroran Institute of Technology, Muroran 050

(Received June 30, 1983)

$$\chi_2 \propto \left(1 - \frac{T}{T_{\rm SG}}\right)^{-\gamma}$$

$$m = \chi_0 h + \chi_2 h^3 + \ldots$$

$$\delta q_{\rm EA} = \chi_{\rm SG} h^2 + \ldots$$

Edwards-Anderson Order parameter

$$q_{\rm EA} = \frac{1}{N}\sum_i \langle S_i \rangle^2$$

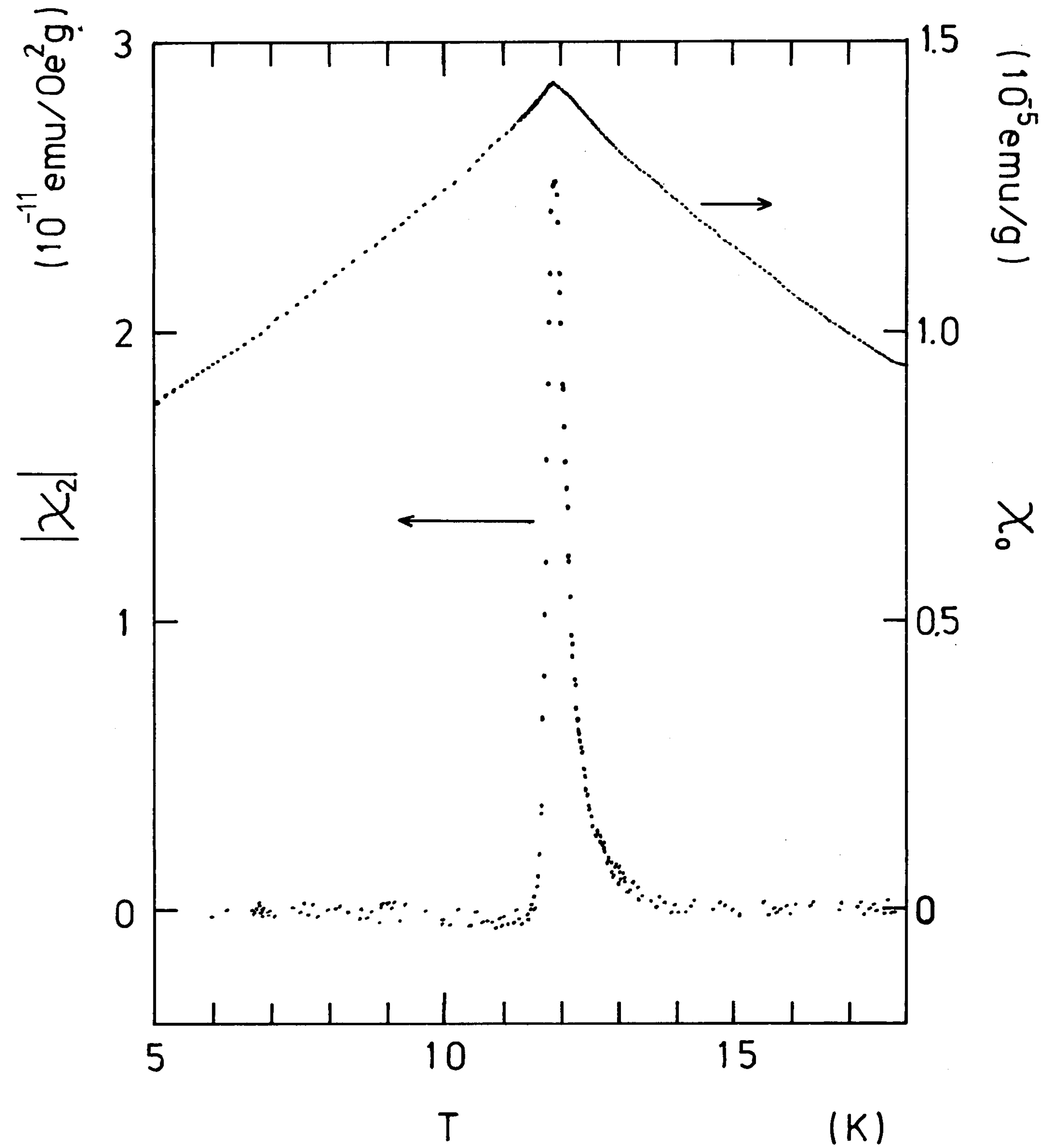
Spinglass susceptibility

$$\chi_{\rm SG} = \frac{1}{N}\sum_{ij}[\langle S_i S_j\rangle^2 - \langle S_i\rangle^2\langle S_j\rangle^2]$$

Non-linear susceptibility and SG susceptibility

$$\chi_2 = -\beta\chi_{\rm SG} \qquad (T > T_{\rm SG})$$

Fig. 2. Linear χ_0 and nonlinear, χ_2, susceptibilities as a function of temperature T for AuFe (1.5 at%Fe).



Non-linear shear-modulus

H.Yoshino, in progress (2015)

$$\mathcal{F}(\hat{\Delta},\{\gamma\}) \quad = \quad \mathcal{F}_{\text{entropic}}(\hat{\Delta}) + \mathcal{F}_{\text{interaction}}(\hat{\Delta},\{\gamma\})$$

fluctuation around the saddle point

$$\hat{\Delta} \to \hat{\Delta} + \delta\hat{\Delta} \qquad H_{ab,cd} = \frac{\partial^2 \mathcal{F}(\hat{\Delta}),\{\gamma=0\}}{\partial\Delta_{ab}\partial\Delta_{cd}}$$

$$\begin{aligned} -\beta F/N &= N^{-1}\ln\int\sum_{a<b} d\Delta_{ab} e^{-\beta\mathcal{F}(\hat{\Delta}^*+\delta\hat{\Delta},\{\gamma\})} \\ &= -\beta\left(F(0)/N + \frac{\gamma^2}{2}\mu_0 + \ldots\right) + \frac{1}{2}\gamma^4 \mathrm{Tr}(cH^{-1}c) + \ldots \end{aligned}$$

$$c_{ab} = \frac{1}{2}\sum_{c(\in\text{slave}),d(\in\text{reference})} \frac{\partial^2\mathcal{F}_{\text{int}}}{\partial\Delta_{ab}\partial\Delta_{cd}}$$

shear stress

$$\sigma = \mu_0\gamma + \frac{1}{3!}\mu_2\gamma^3 + \ldots$$

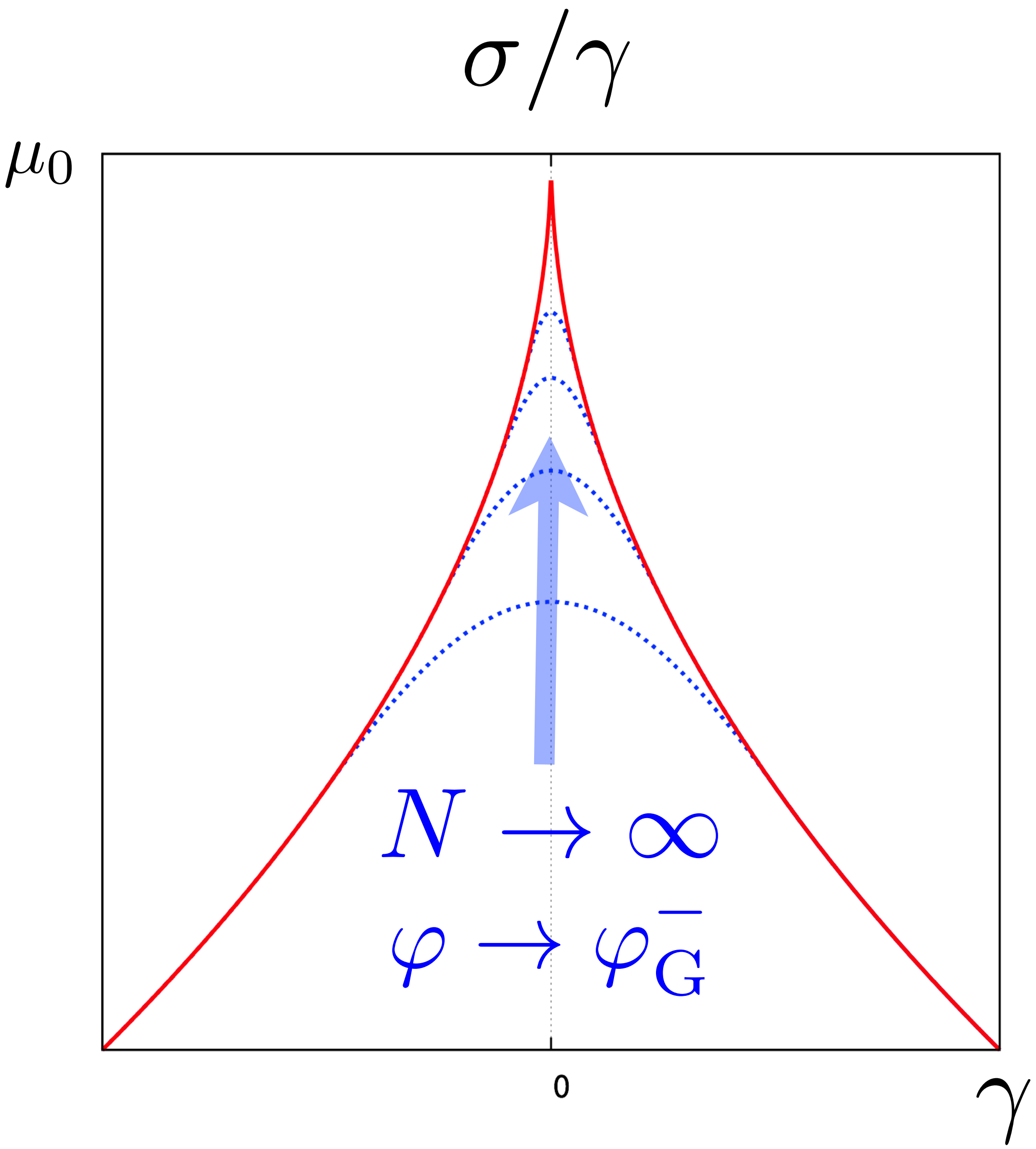
non-linear shear modulus

$$\beta\mu_2 = \frac{1}{4!}\frac{1}{N}\frac{\partial^4\beta F}{\partial\gamma^4} = -\frac{1}{2}\sum_\lambda \frac{c(\lambda)^2}{\lambda} \to -\infty$$

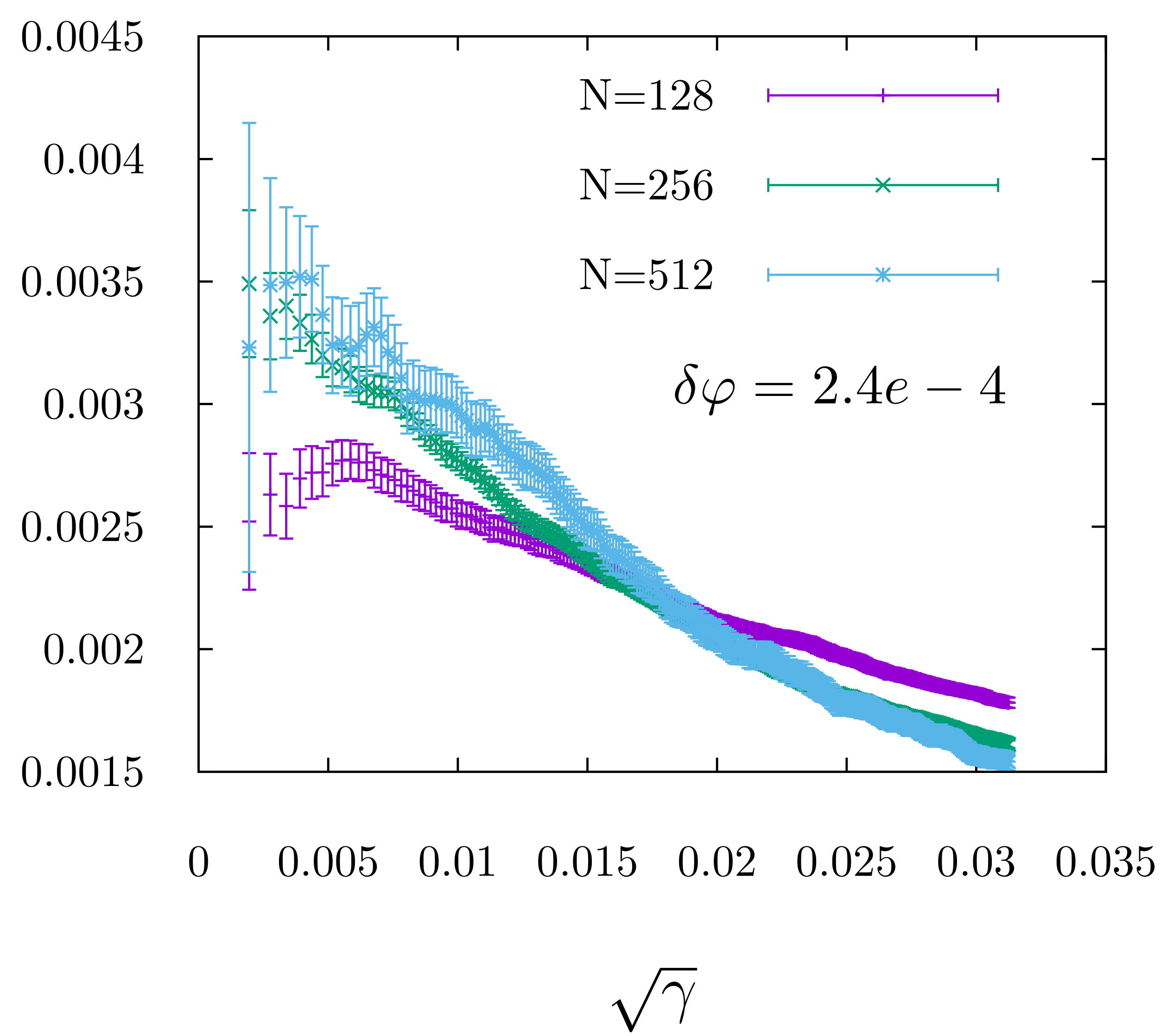
Implication of "negatively" diverging non-linear shear-modulus

$$\frac{\sigma}{\gamma} = \mu_0 + \frac{1}{3!}\mu_2\gamma^2 + \dots$$

Vanishing linear response regime
in the Gardner's phase



D. Nakayama, H. Yoshino, in progress (2015)



see also Otsuki-Hayakawa, PRE 90, 042202 (2014)

Summary

Response to shear of a hard-sphere glass in $d \to \infty$

1. Exact free-energy functional **under shear**

2. Analysis of shear-modulus

   ✳ 1RSB - jump + square-root singularity at $\widehat{\varphi}_{\rm d}$

   ✳ 1+continuous RSB

   (1) **rigidities of inherent structures/metabasin**

   (2) **jamming scaling** as $\widehat{\varphi} \to \widehat{\varphi}_J$

   H. Yoshino and F. Zamponi, Phys. Rev. E 90, 022302 (2014).

3. State following under shear/compression : jamming, melting, yielding

   C. Rainone, P. Urbani, H. Yoshino, F. Zamponi, Phys. Rev. Lett. 114, 015701 (2015).

Numerical simulations of a 3-dim soft-particle system

1) FC/ZFC under shear

   Nakayama-Yoshino-Zamponi, in progress

2) non-linear response (ZFC) under shear

   Nakayama-Yoshino, in progress