Thermodynamics with continuous information flow

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Kyoto, Japan

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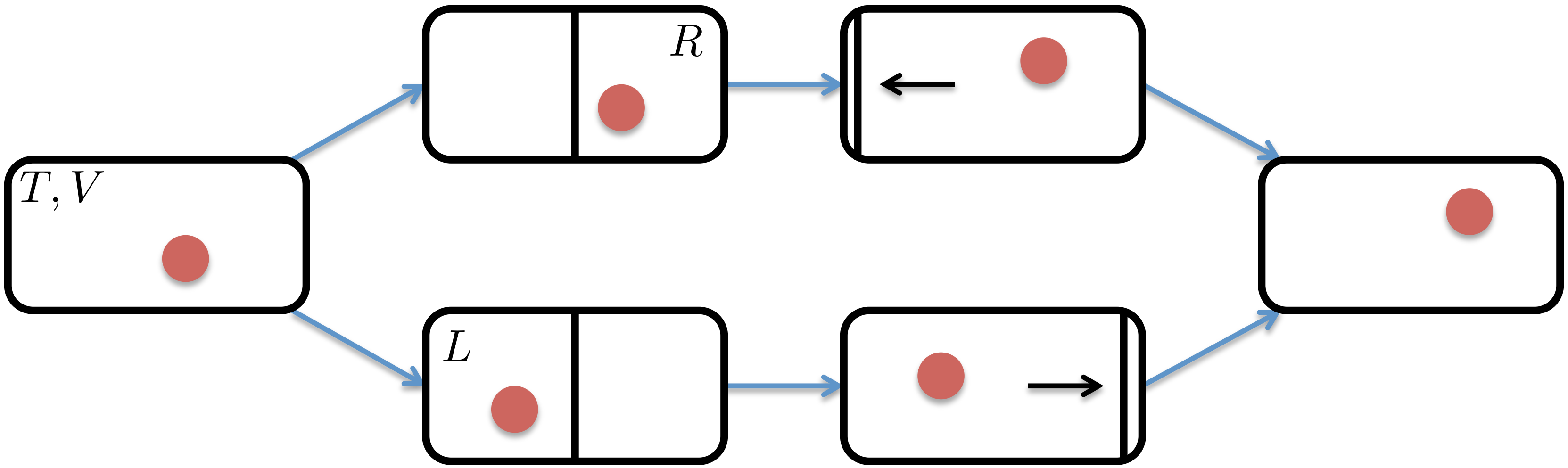
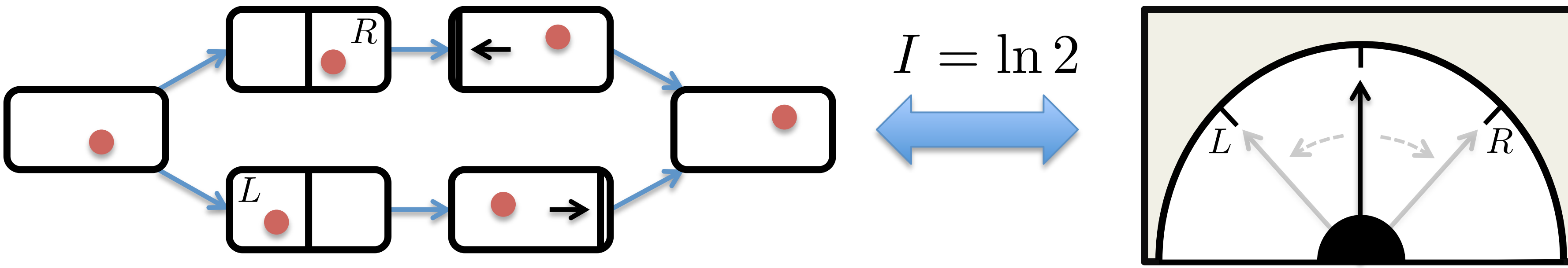
Technology

Szilard engine



$$W_{\text{ext}} = kT \ln 2$$

Szilard engine



$W_{\text{ext}} = kT \ln 2$

$W_{\text{meas}} = kT \ln 2$

$$W_{\text{ext}} = kTI = W_{\text{meas}}$$

Autonomous demons

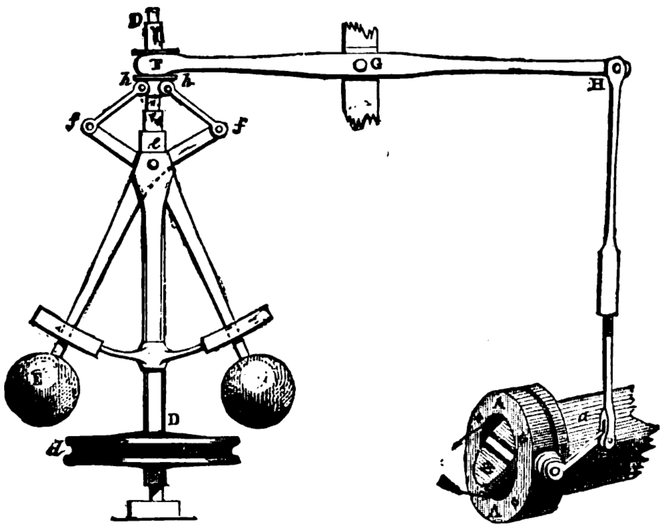
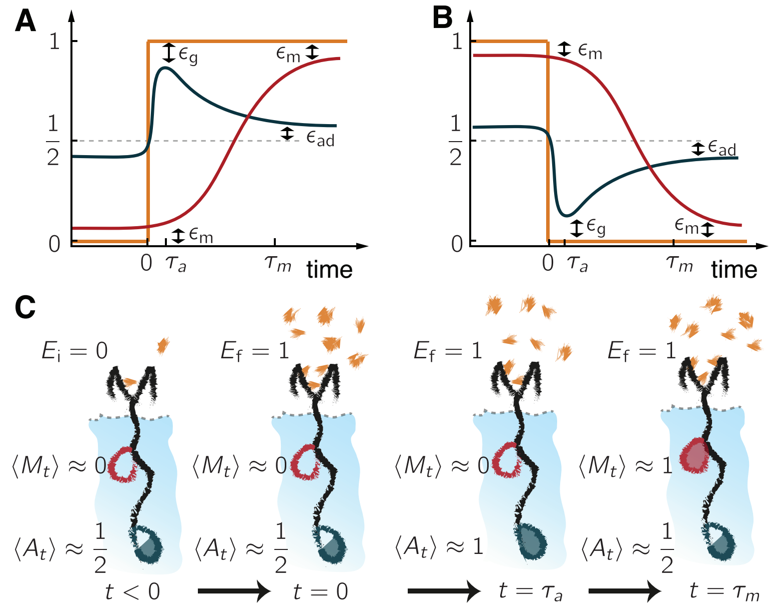
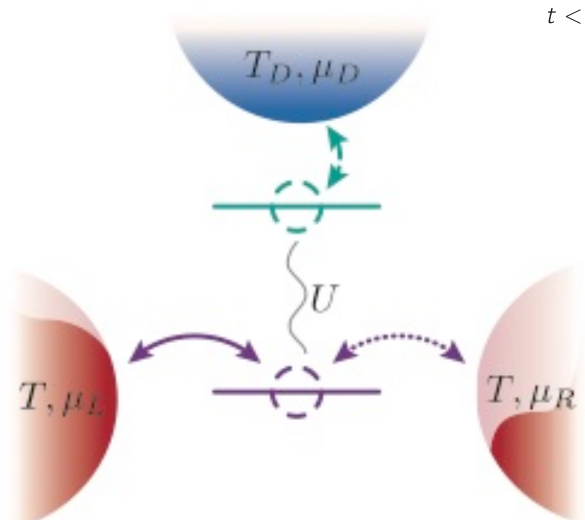


FIG. 4.—*Governor and Throttle-Valve.*

Watt Governor



Sensory Adaptation



Quantum dot Maxwell Demon

Information flow

Stochastic thermodynamics

Master equation

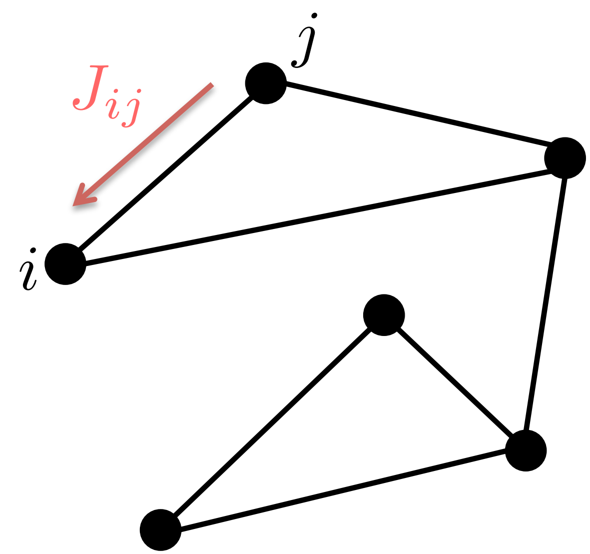
$$d_t p_i = \sum_j J_{ij}$$

Current

$$J_{ij} = W_{ij} p_j - W_{ji} p_i$$

Local detailed balance

$$\ln \frac{W_{ij}}{W_{ji}} = \beta q_{ij}$$



Second law

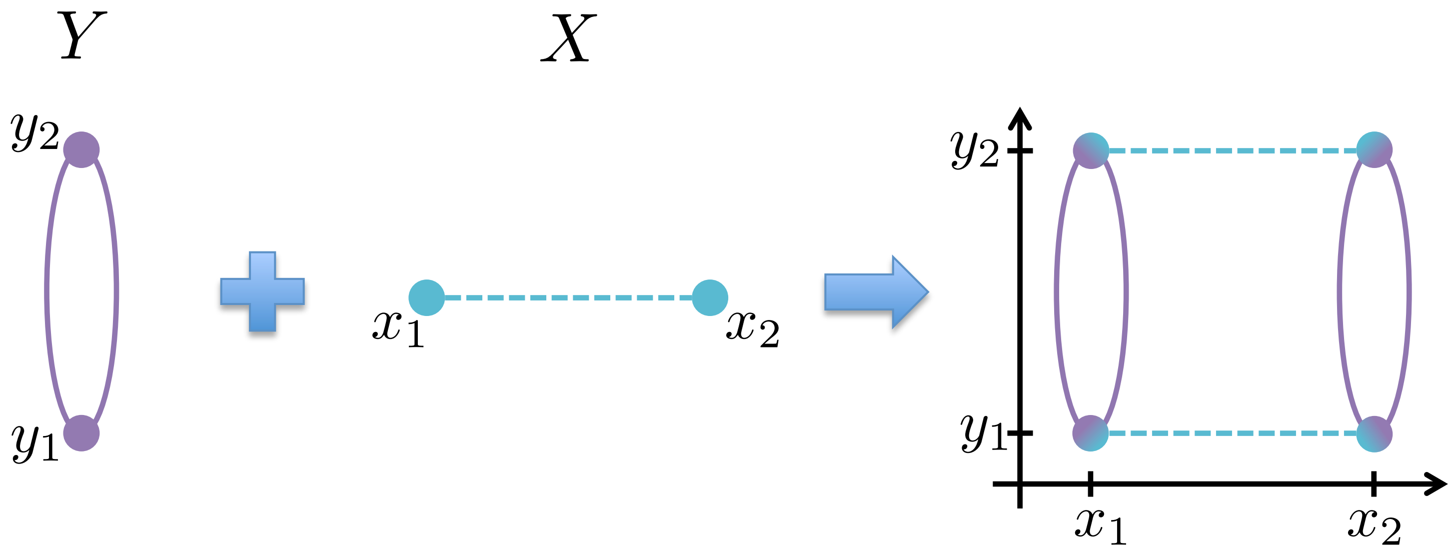
$$\dot{S}_{\mathbf{i}} = d_t S + \dot{S}_{\mathbf{r}} \geq 0$$

Shannon entropy rate: $$d_t S = \sum_{i \geq j} J_{ij} \ln \frac{p_j}{p_i}$$

Environment entropy rate: $$\dot{S}_{\mathbf{r}} = \sum_{i \geq j} J_{ij} \ln \frac{W_{ij}}{W_{ji}}$$

Entropy production: $$\dot{S}_{\mathbf{i}} = \sum_{i \geq j} J_{ij} \ln \frac{W_{ij} p_j}{W_{ji} p_i}$$

Bipartite systems



Flows

$$\mathcal{A}(J) = \sum_{x \geq x', y \geq y'} J_{xx'}^{yy'} A_{xx'}^{yy'} = \mathcal{A}^X + \mathcal{A}^Y$$

Information flow

Mutual information

$$I(X,Y) = \sum_{x,y} p(x,y) \ln \frac{p(x,y)}{p(x)p(y)}$$

Information flow

$$d_t I = \dot{I}^X + \dot{I}^Y$$

$$\dot{I}^X = \sum_{x \geq x', y} J^{y}_{x,x'} \ln \frac{p(y|x)}{p(y|x')}$$

$$\dot{I}^Y = \sum_{x, y \geq y'} J^{y,y'}_{x} \ln \frac{p(x|y)}{p(x|y')}$$

Second law

$$\dot{S}_{\mathbf{i}}^{X} = d_t S^{X} + \dot{S}_{\mathbf{r}}^{X} - \dot{I}^{X} \geq 0$$

$$\dot{S}_{\mathbf{i}}^{Y} = d_t S^{Y} + \dot{S}_{\mathbf{r}}^{Y} - \dot{I}^{Y} \geq 0$$

Horowitz/Esposito PRX 2014; Hartich/Barato/Seifert JSTAT 2014;
Allahverdyan/Janzig/Mahler JSTAT 2009

Second law for X

$$\sigma^X = d_t S^X + \dot{S}_{\mathbf{r}}^X \geq \dot{I}^X$$

Information resource

$$\dot{I}^X < 0 \rightarrow \sigma^X < 0$$

Measurement cost

$$\dot{I}^X > 0 \rightarrow \sigma^X > \dot{I}^X$$

Applications

Nonautonomous demons

Measurement

$$\Delta_{\mathbf{i}} S^{X}_{\mathrm{meas}} = \Delta S^{X} + \Delta_{\mathbf{r}} S^{X}_{\mathrm{meas}} - I \geq 0$$

$$\Delta_{\mathbf{i}} S^{Y}_{\mathrm{meas}} = 0$$

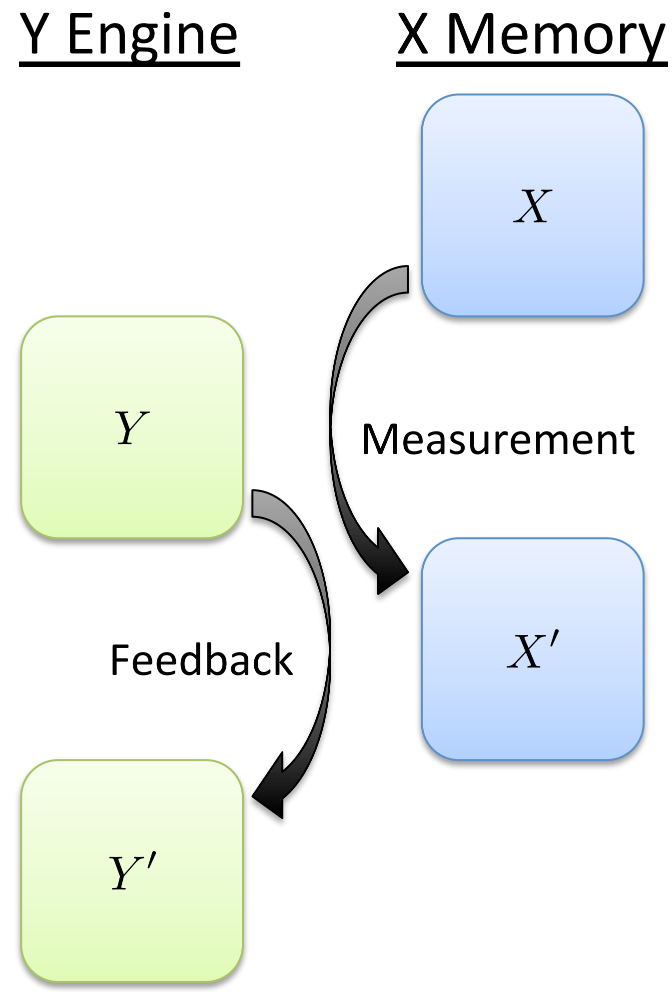
Feedback

$$\Delta_{\mathbf{i}} S^{X}_{\mathrm{fb}} = 0$$

$$\Delta_{\mathbf{i}} S^{Y}_{\mathrm{fb}} = \Delta S^{Y} + \Delta_{\mathbf{r}} S^{Y}_{\mathrm{fb}} + I \geq 0$$

Cycle

$$\Delta_{\mathbf{i}} S = \Delta S^{X} + \Delta S^{Y} + \Delta_{\mathbf{r}} S^{X}_{\mathrm{meas}} + \Delta_{\mathbf{r}} S^{Y}_{\mathrm{fb}} \geq 0$$



Autonomous demons

$$\dot{S}_{\mathbf{i}}^{X} = d_t S^{X} + \dot{S}_{\mathbf{r}}^{X} - \dot{I}^{X} \geq 0$$

$$\dot{S}_{\mathbf{i}}^{Y} = d_t S^{Y} + \dot{S}_{\mathbf{r}}^{Y} - \dot{I}^{Y} \geq 0$$

Autonomous demons

$$\dot{S}_{\mathbf{i}}^{X} = d_t S^X + \dot{S}_{\mathbf{r}}^{X} - \dot{I}^X \geq 0$$

$$\dot{S}_{\mathbf{i}}^{Y} = d_t S^Y + \dot{S}_{\mathbf{r}}^{Y} - \dot{I}^Y \geq 0$$

$$\dot{\mathcal{I}} = \dot{I}^X = -\dot{I}^Y$$

Autonomous demons

$$\dot{\mathcal{S}}_{\mathbf{i}}^{X} = \dot{\mathcal{S}}_{\mathbf{r}}^{X} - \dot{\mathcal{I}} \geq 0$$

$$\dot{\mathcal{S}}_{\mathbf{i}}^{Y} = \dot{\mathcal{S}}_{\mathbf{r}}^{Y} + \dot{\mathcal{I}} \geq 0$$

Autonomous demons

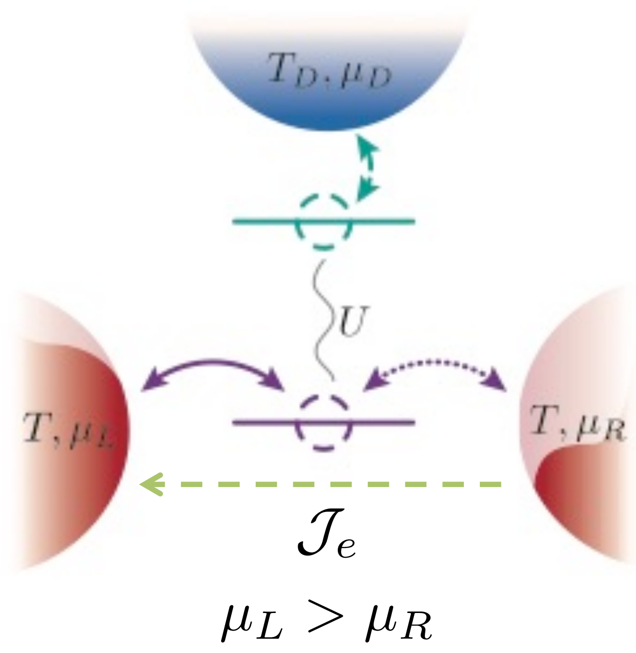
$$\dot{\mathcal{S}}_{\mathbf{r}}^{X} \geq \dot{\mathcal{I}}$$

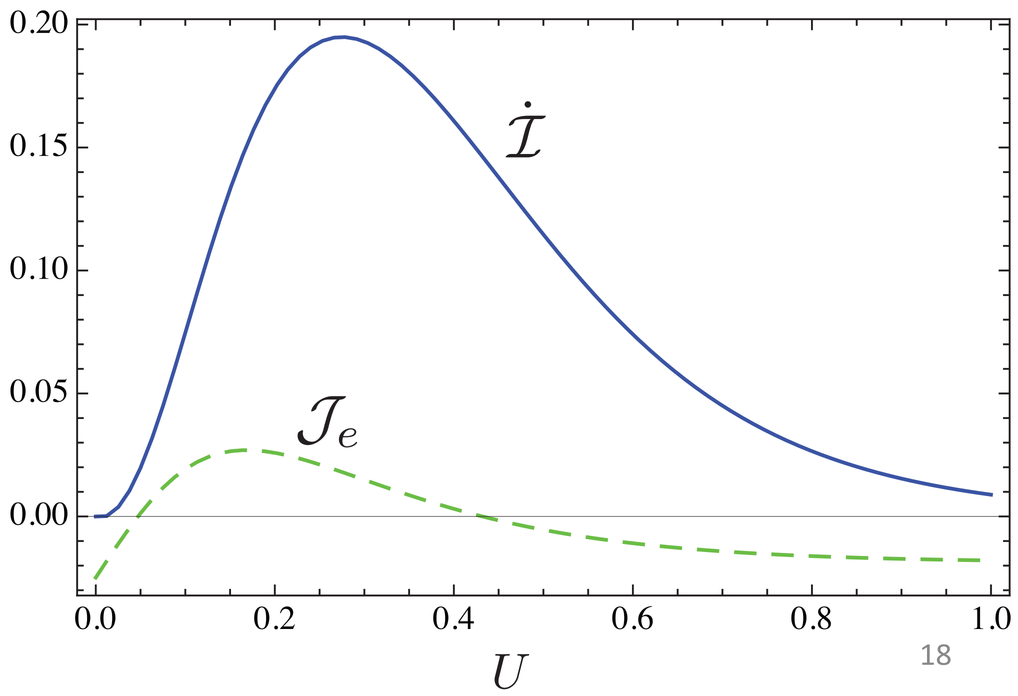
$$-\dot{\mathcal{S}}_{\mathbf{r}}^{Y} \leq \dot{\mathcal{I}}$$

Information efficiency $(\dot{\mathcal{I}} > 0)$

$$\varepsilon^{X} = \frac{\dot{\mathcal{I}}}{\dot{\mathcal{S}}_{\mathbf{r}}^{X}} \qquad \varepsilon^{Y} = \frac{|\dot{\mathcal{S}}_{\mathbf{r}}^{Y}|}{\dot{\mathcal{I}}}$$

Quantum dot





Horowitz/Esposito PRX 2014;
Strasberg et al. PRL 2013

Diffusion processes

Bipartite diffusions have *uncorrelated* noise

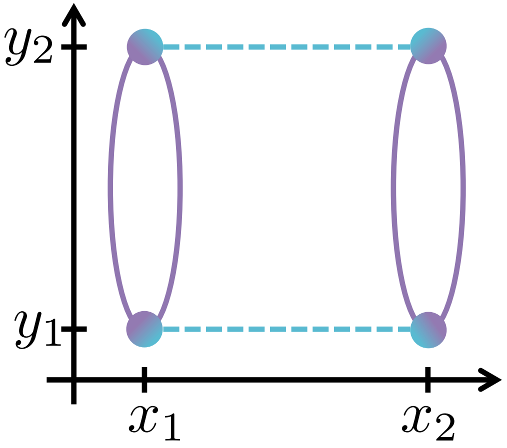
$$d_t p = -\partial_x J^x - \partial_y J^y$$

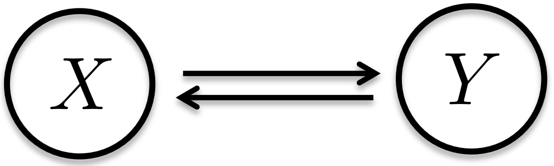
$$J^k = \mu_k F_k(x, y) - \mu_k T \partial_k p$$

Allahverdyan/Janzig/Mahler JSTAT 2009;
Horowitz JSTAT P03006 (2015)

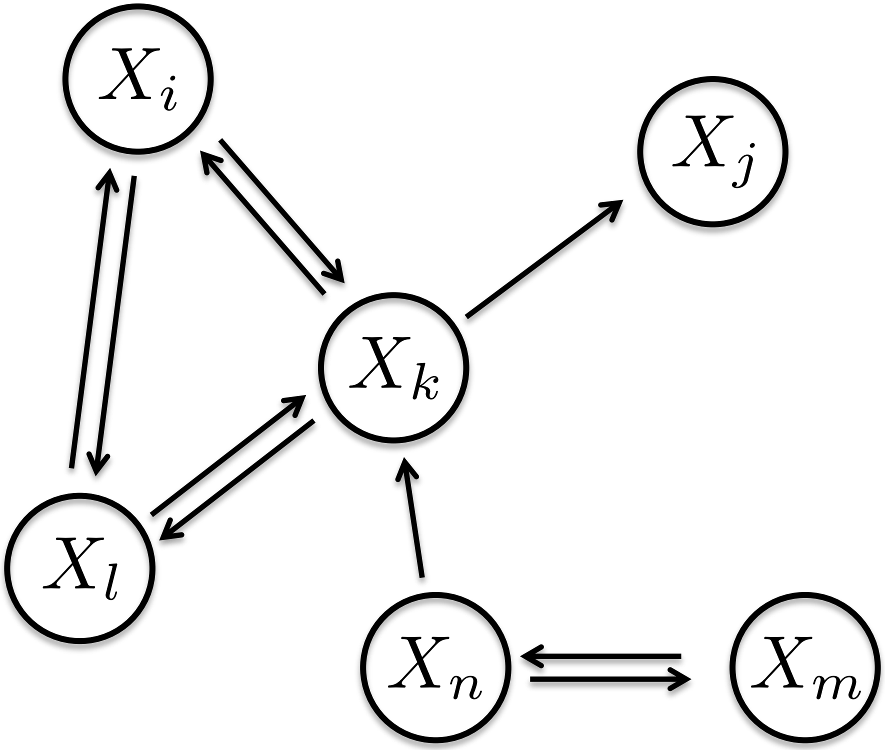
Multipartite Information Flow

Influence network

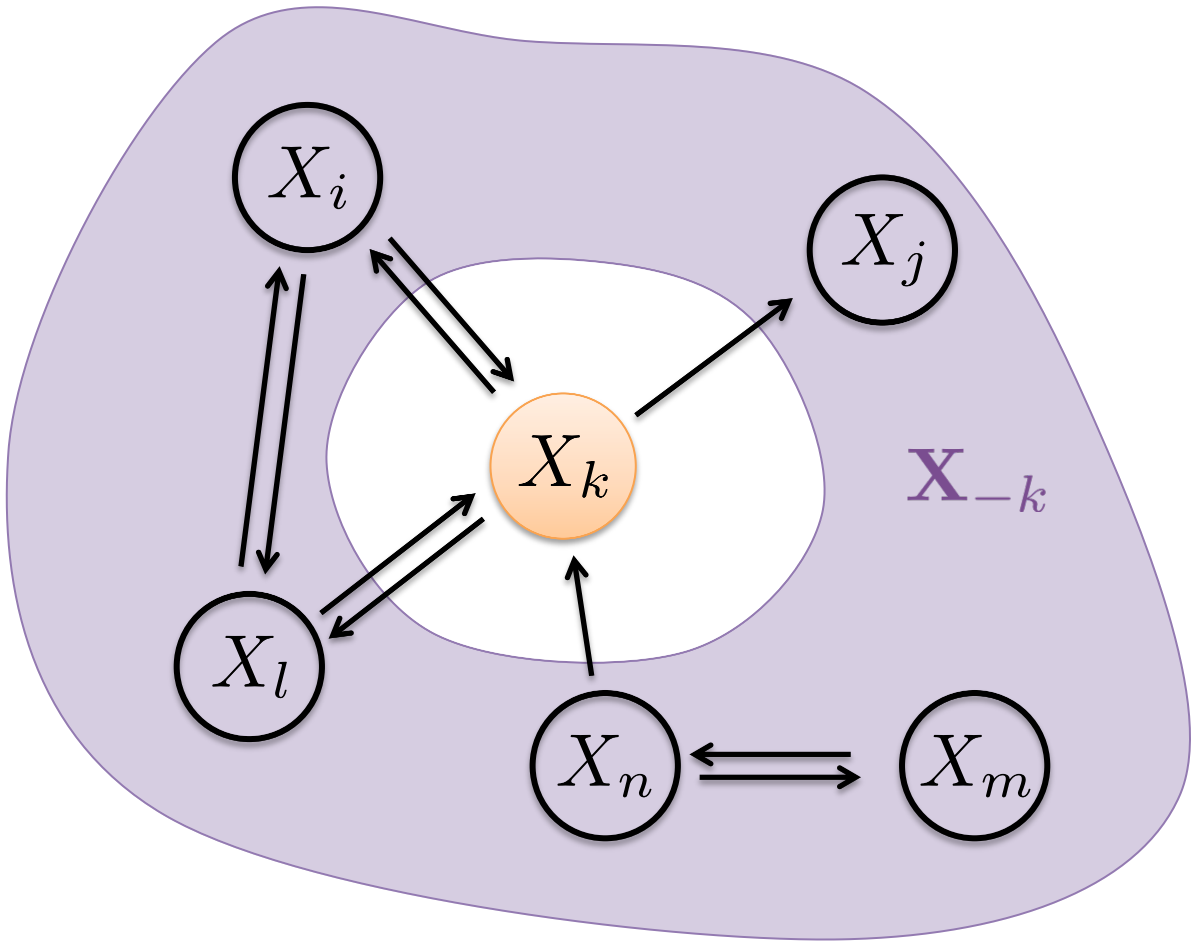




Multipartite systems

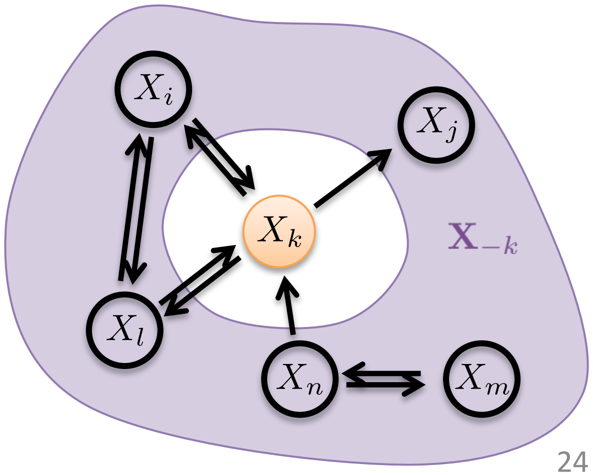


Multipartite systems



Second law

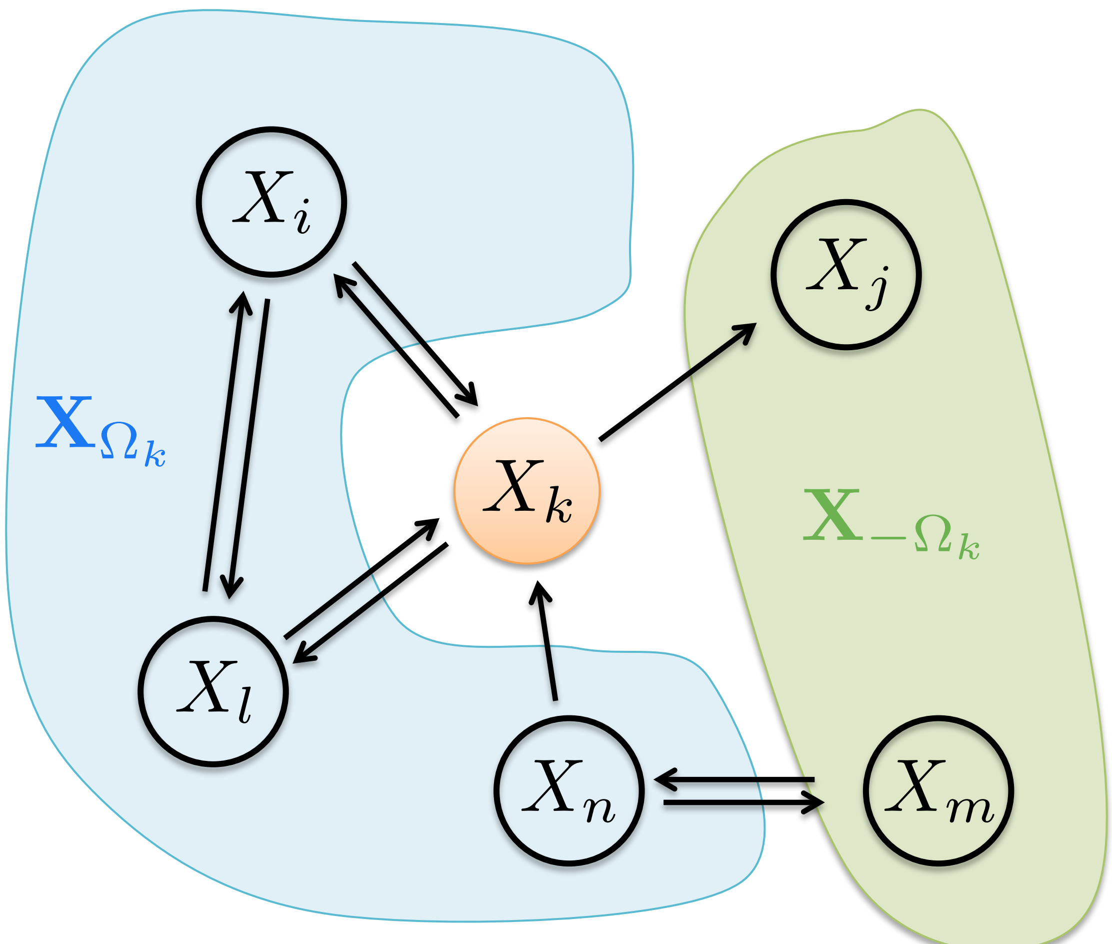
$$\dot{S}_{\mathrm{i}}^{k} = d_t S(X_k) + \dot{S}_{\mathrm{r}}^{k} - \dot{I}^{k}(X_k;\mathbf{X}_{-k}) \geq 0$$



Horowitz JSTAT 2014

Neighbors

$$\dot{I}^k(X_k;\mathbf{X}_{-k}) = \dot{I}^k(X_k;\mathbf{X}_{\Omega_k}) + \dot{I}^k(X_k;\mathbf{X}_{-\Omega_k}|\mathbf{X}_{\Omega_k})$$



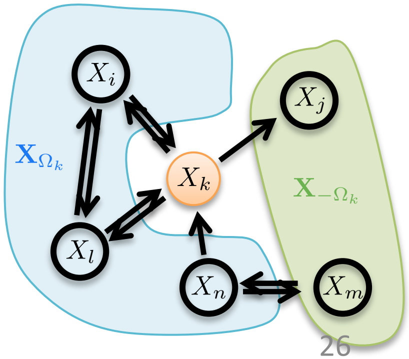
Refined second law

Neighbors are useful

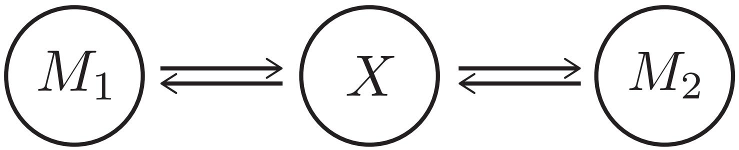
$$d_t S(X_k) + \dot{S}_{\mathrm{r}}^k - \dot{I}^k(X_k; \mathbf{X}_{\Omega_k}) \geq 0$$

Non-neighbor information decreases

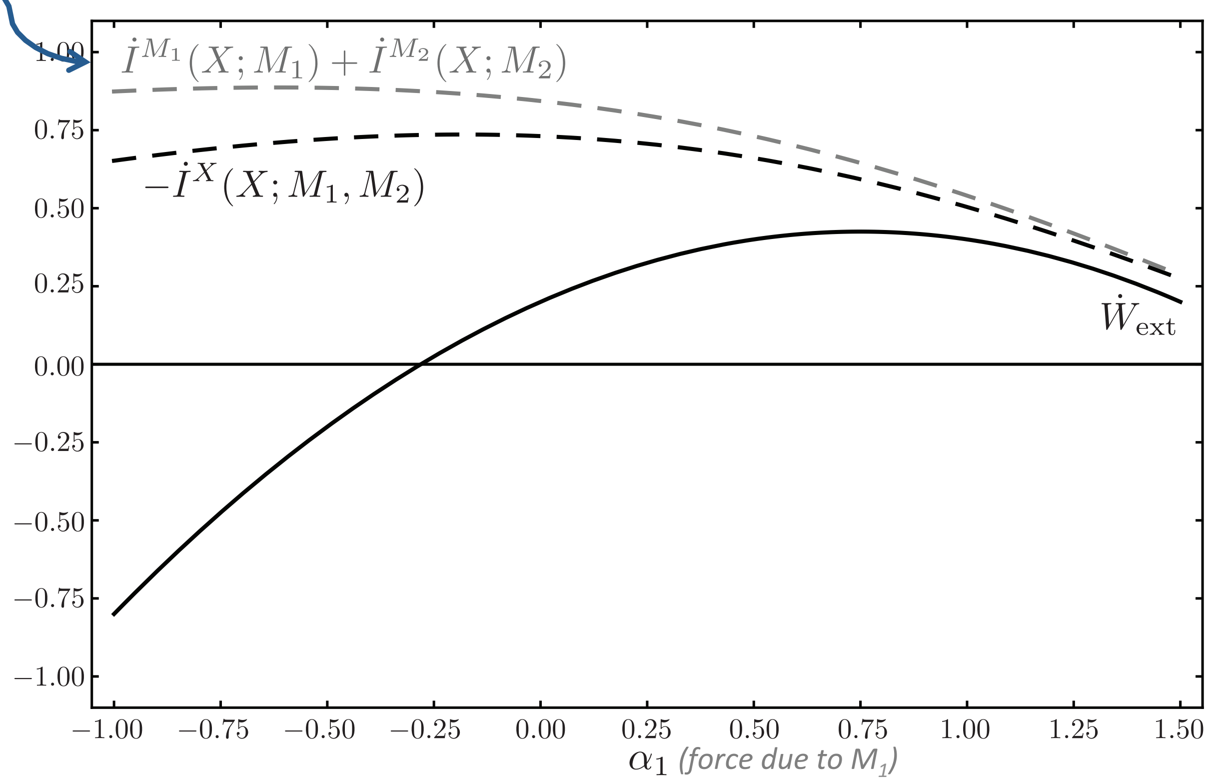
$$\dot{I}^k(X_k; \mathbf{X}_{-\Omega_k} | \mathbf{X}_{\Omega_k}) \leq 0$$



Competing Demons



Total work to measure



Information measures

Second-law-like inequalities

$$\sigma^X \geq \dot{I}$$

Information flow: $\dot{I}_{\mathrm{flow}}$

rate of change of mutual information

Transfer entropy rate: $\dot{I}_{\mathrm{trans}}$

information rate between measurement trajectory and system state

Trajectory mutual information: $\dot{I}_{\mathrm{traj}}$

information rate between system and measurement trajectories

Entropy pumping: $\dot{I}_{\mathrm{pump}}$

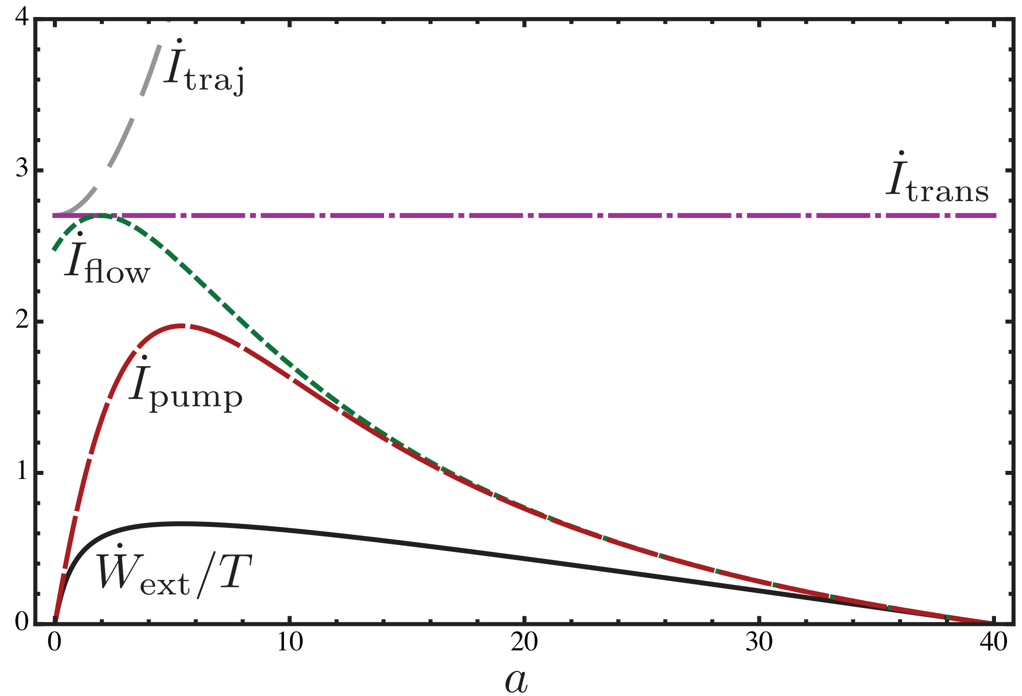
phase space compression due to feedback

Information hierarchy

$$\dot{I}_{\text{pump}} \leq \dot{I}_{\text{flow}} \leq \dot{I}_{\text{trans}} \leq \dot{I}_{\text{traj}}$$

Velocity damping:

$$m\dot{v}_t = -\gamma v_t - a y_t + \xi_t$$
$$\tau \dot{y}_t = -(y_t - v_t - \eta_t)$$

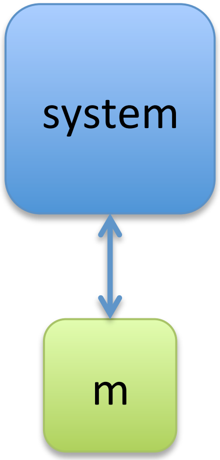


Horowitz/Sandberg NJP 2014; Hartich/Barato/Seifert JSTAT 2014;
Allahverdyan/Janzig/Mahler JSTAT 2009; Munakata/Rosinberg JSTAT 2012

Measurement cost

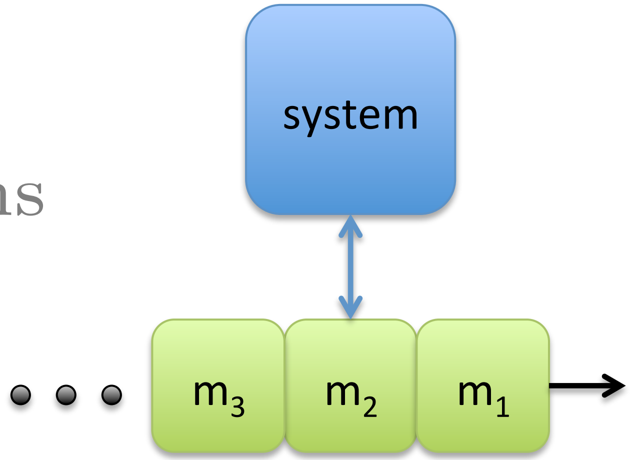
Rewriting *one* memory

$$\sigma^{m}_{\text{meas}} \geq \dot{I}_{\text{flow}}$$



Tape of memories

$$\sigma^{m_1 m_2 \cdots}_{\text{meas}} \geq \dot{I}_{\text{trans}}$$



Horowitz/Sandberg NJP 2014

Second-law-like inequalities

$$\dot{S}_{\mathrm{i}} = d_t S + \dot{S}_{\mathrm{env}}$$
$$= d_t S + \dot{S}_{\mathrm{r}} - \dot{I} \geq 0$$

Environmental entropy flow includes information

Horowitz/Esposito PRX 2014; Hartich/Barato/Seifert JSTAT 2014; Deffner/Jarzynski PRX 2014

Summary

Information flow

quantifies the thermodynamically exploitable correlations for interacting multipartite systems

Information cost to measure

information bounds the entropy flow to the memory device