PS-D3 NQS2014@YITP

Symmetry protected topological phase in magnetization plateaux

NIMS Shintaro Takayoshi

Collaboration with

A. Tanaka (NIMS) and K. Totsuka (YITP)

Introduction

- Gapped systems

  Long range entangled state ･･･ anyonic excitation
  (Intrinsic topological order)

  Short Long range entangled state

  Local unitary transformation ↓ Without any symmetry

  Trivial (direct product) state

- Short Long range entangled state can be nontrivial if some symmetry is imposed.
  - Local order parameter → Ginzburg-Landau theory
  - No Local order parameter → Symmetry protected topological (SPT) phase

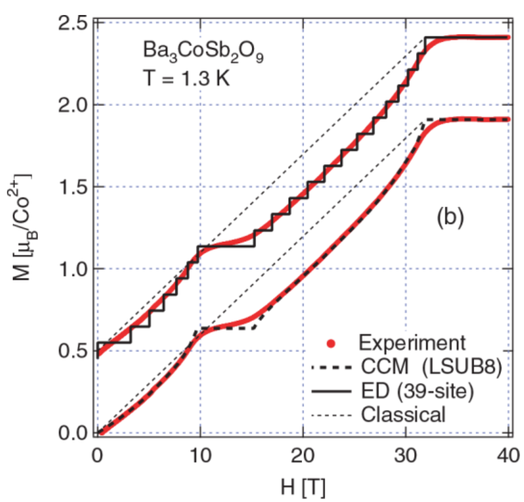
Results

Magnetization plateau states in 1D antiferromagnets are in an SPT phase protected by link-inversion symmetry if S-m = odd integer.

Size of spin (S) — Magnetization per site (m)

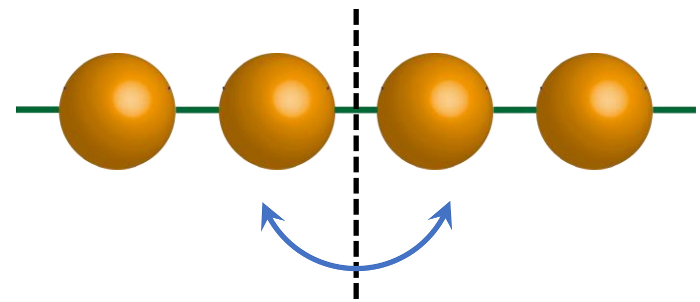
Magnetization plateau

– Region where M is unchanged with increasing H in magnetization curves



Shirata et al., 2012

Link-inversion



Methods

1. Field theories
   - Nonlinear sigma model,
     dual boson-vortex theory (sine-Gordon model)
2. Numerical calculations
   - Infinite time-evolving block decimation (iTEBD),
     entanglement spectra
3. Matrix product state (MPS) representation

Field theories

Oshikawa-Yamanaka-Affleck condition

Oshikawa-Yamanaka-Affleck, 1997

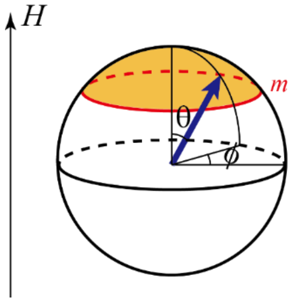
$$r(S-m)\in\mathbb{Z}$$

- r: Number of sites in a unit cell
- S: Size of spin
- m: Magnetization per site

Canted spin configuration

Tanaka-Totsuka-Hu, 2009

$$\boldsymbol{S}_j(\tau)=S\begin{pmatrix}(-1)^j\cos\phi_j(\tau)\sin\theta_0\\(-1)^j\sin\phi_j(\tau)\sin\theta_0\\\cos\theta_0\end{pmatrix}$$



$$\mathcal{S}=\mathcal{S}_{\text{kin}}+\mathcal{S}_{\text{BP}}^{\text{tot}}\qquad \mathcal{S}_{\text{kin}}\rightarrow\int dx d\tau\frac{\zeta}{2}\left\{\frac{1}{v^2}(\partial_\tau\phi)^2+(\partial_x\phi)^2\right\}$$

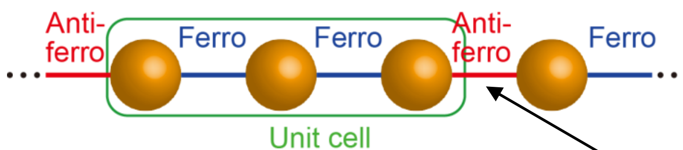
O(3) nonlinear sigma model

Also dual vortex theory approach

→ sine-Gordon theory

Numerical calculations

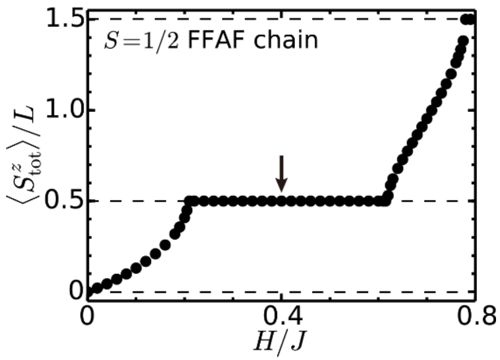
- Ferro-ferro-antiferro model
- Infinite-time evolving block decimation (iTEBD).
- Magnetization curves and entanglement spectra (ES).

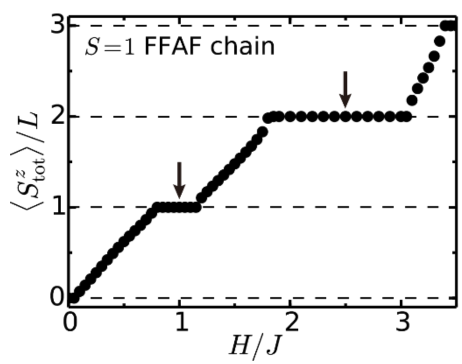


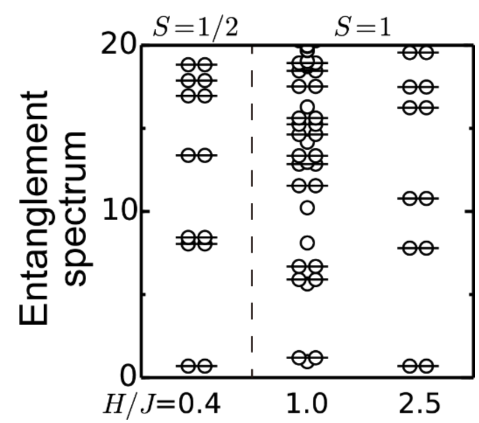
Bipartition at the AF bond

Schmidt decomposition $|\Psi\rangle = \sum_\alpha \lambda_\alpha |\Psi_\mathrm{A}\rangle_\alpha \otimes |\Psi_\mathrm{B}\rangle_\alpha$

ES $\{-\ln(\lambda_\alpha^2)\} \quad (\alpha = 1, \ldots, \chi)$



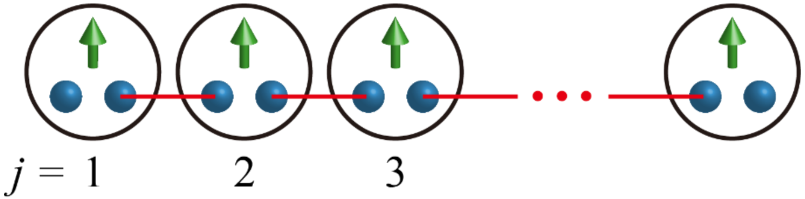




$S - m =$ odd even odd

MPS representation

VBS picture for m=1/2 plateau in S=3/2



Schwinger boson representation

$$|\Psi\rangle = \prod_j P_j a_j^\dagger (a_j^\dagger b_{j+1}^\dagger - a_j^\dagger b_{j+1}^\dagger) \bigotimes_j |0\rangle_j$$

up down

Canonical representation

$$|\Psi\rangle = \sum_{S_j^z=-3/2}^{3/2} \dots \Lambda\Gamma[S_{j-1}^z]\Lambda\Gamma[S_j^z]\Lambda\Gamma[S_{j+1}^z]\Lambda \dots \bigotimes_j |S_j^z\rangle$$

Link- Inversion $\mathcal{I}$ acts on MPS as $\Gamma^{\mathrm{T}} = e^{i\theta_{\mathcal{I}}} U_{\mathcal{I}}^\dagger \Gamma U_{\mathcal{I}}$

We can find that $U_{\mathcal{I}} = i\sigma_y = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

$U_{\mathcal{I}}^2 = -1$: Nontrivial

Conclusion

Magnetization plateau states in 1D antiferromagnets are in an SPT phase protected by link-inversion symmetry if $S-m$ = odd integer.

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2. Numerical calculations
3. Matrix product state (MPS) representation

✓Please visit PS-D3.

✓Discussions in free time are also welcome.