HADRON INTERACTIONS FROM LATTICE QCD

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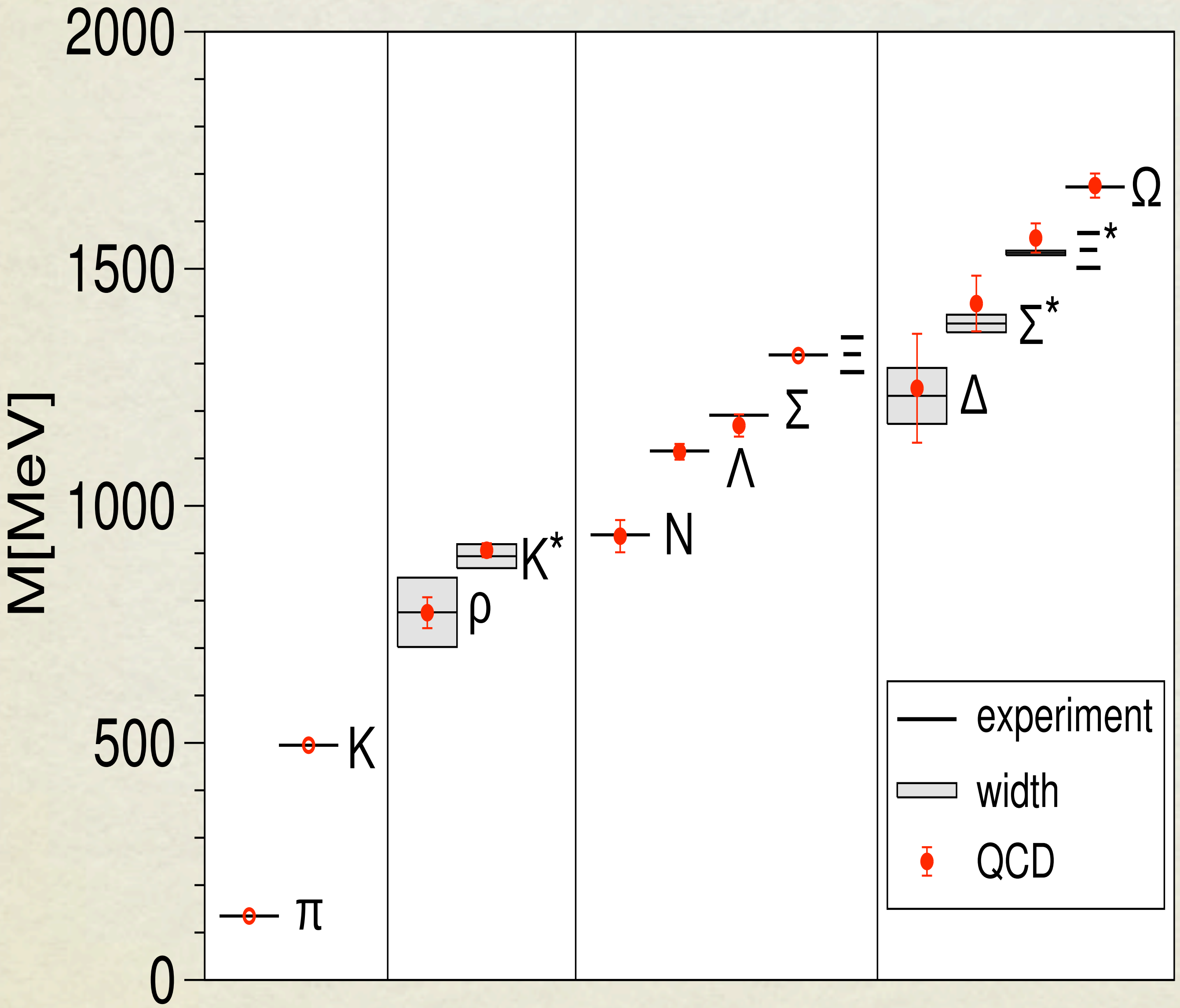
京都大学　基礎物理研究所

1. INTRODUCTION
CURRENT STATUS OF LATTICE QCD

Calculations of basic quantities are almost completed.

Hadron spectra



BMW collaboration
Sciences 322(2008)1224

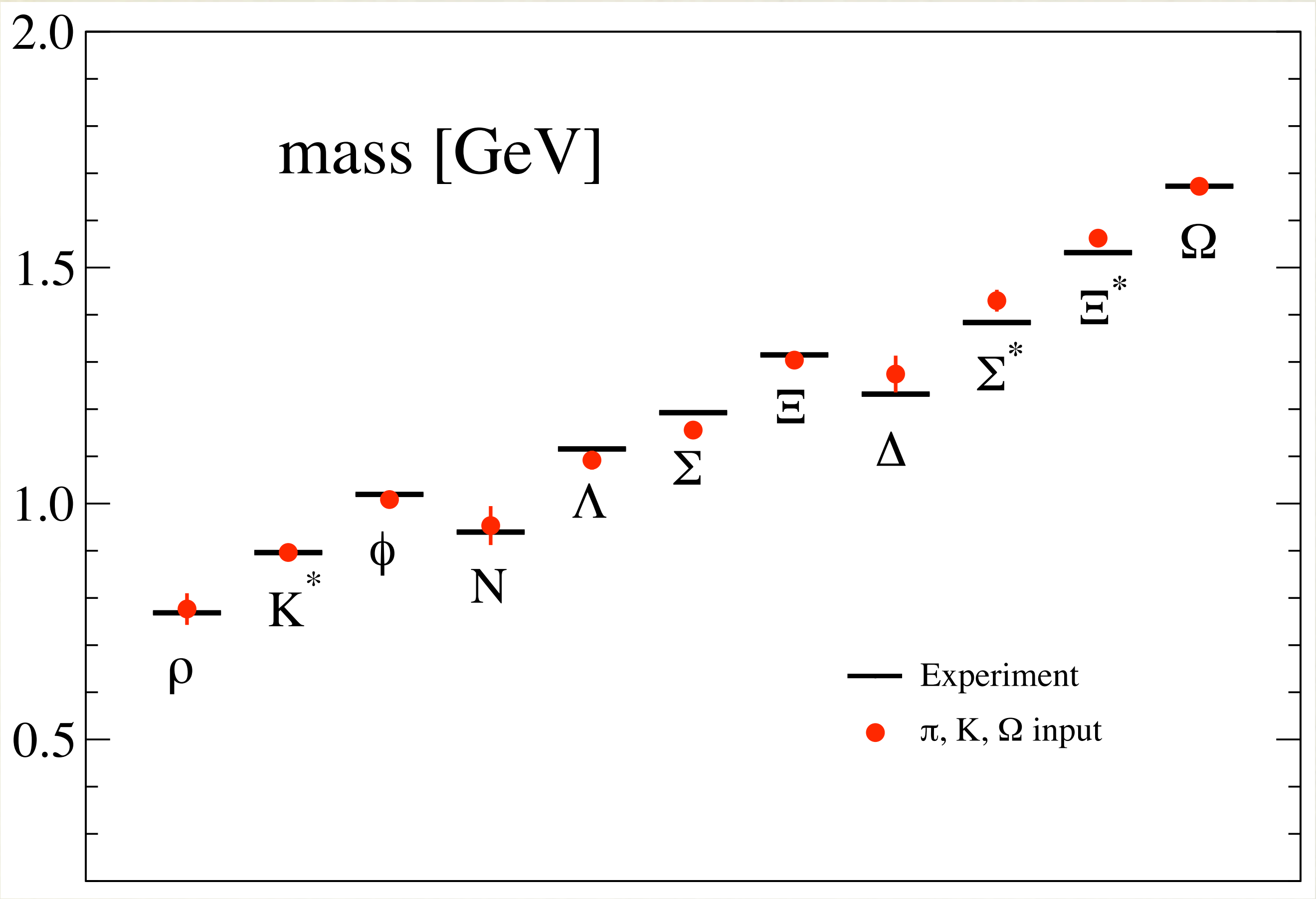
$a \to 0$ $m_\pi L \geq 4$

$m_\pi^{\mathrm{min.}} = 190 \text{ MeV}$

Agreement between lattice QCD and experiment is excellent !

Hadron spectra near physical point

PACS-CS Collaboration Phys. Rev. D79 (2009) 034503

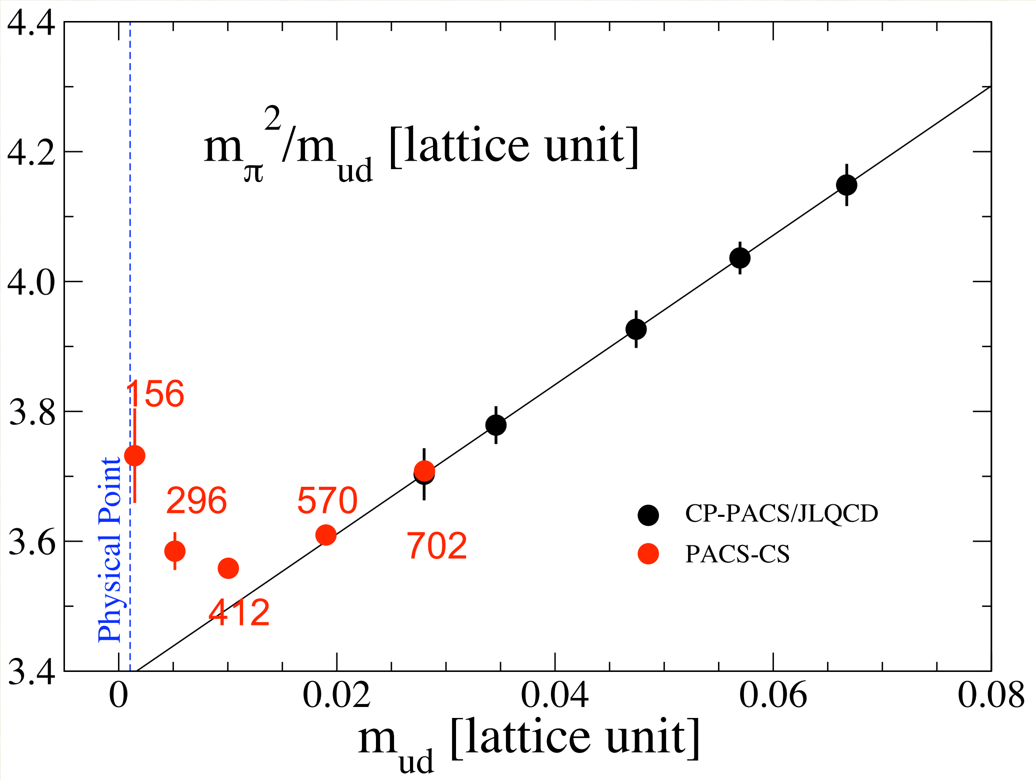


$a = 0.09$ fm $\quad L = 2.9$ fm

$m_\pi^{\text{min.}} = 156$ MeV $\quad m_\pi L = 2.3$

We are almost on the “physical point”.

Chiral extrapolation



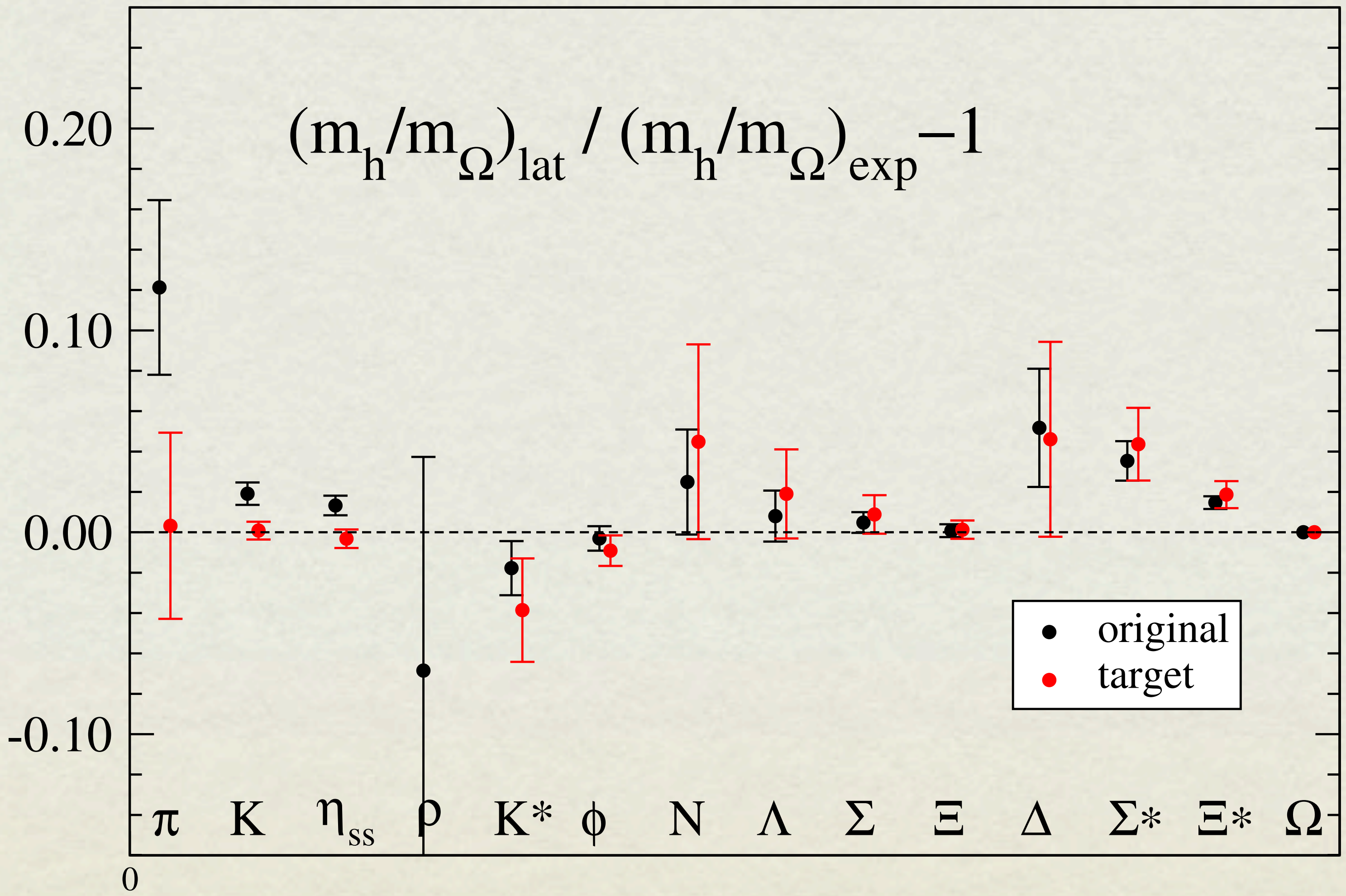
Reweighting to physical point

PACS-CS Collaboration, Phys. Rev. D81 (2010) 074503

Simulation at $m_\pi \simeq 140$ MeV by reweighting

$$\langle O[U]\rangle_m = \frac{\int \mathcal{D}U O(U) \det D_m(U) e^{-S_G(U)}}{\int \mathcal{D}U \det D_m(U) e^{-S_G(U)}} = \frac{\int \mathcal{D}U O(U) \frac{\det D_m(U)}{\det D_{m*}(U)} \det D_{m*}(U) e^{-S_G(U)}}{\int \mathcal{D}U \frac{\det D_m(U)}{\det D_{m*}(U)} \det D_{m*}(U) e^{-S_G(U)}}$$

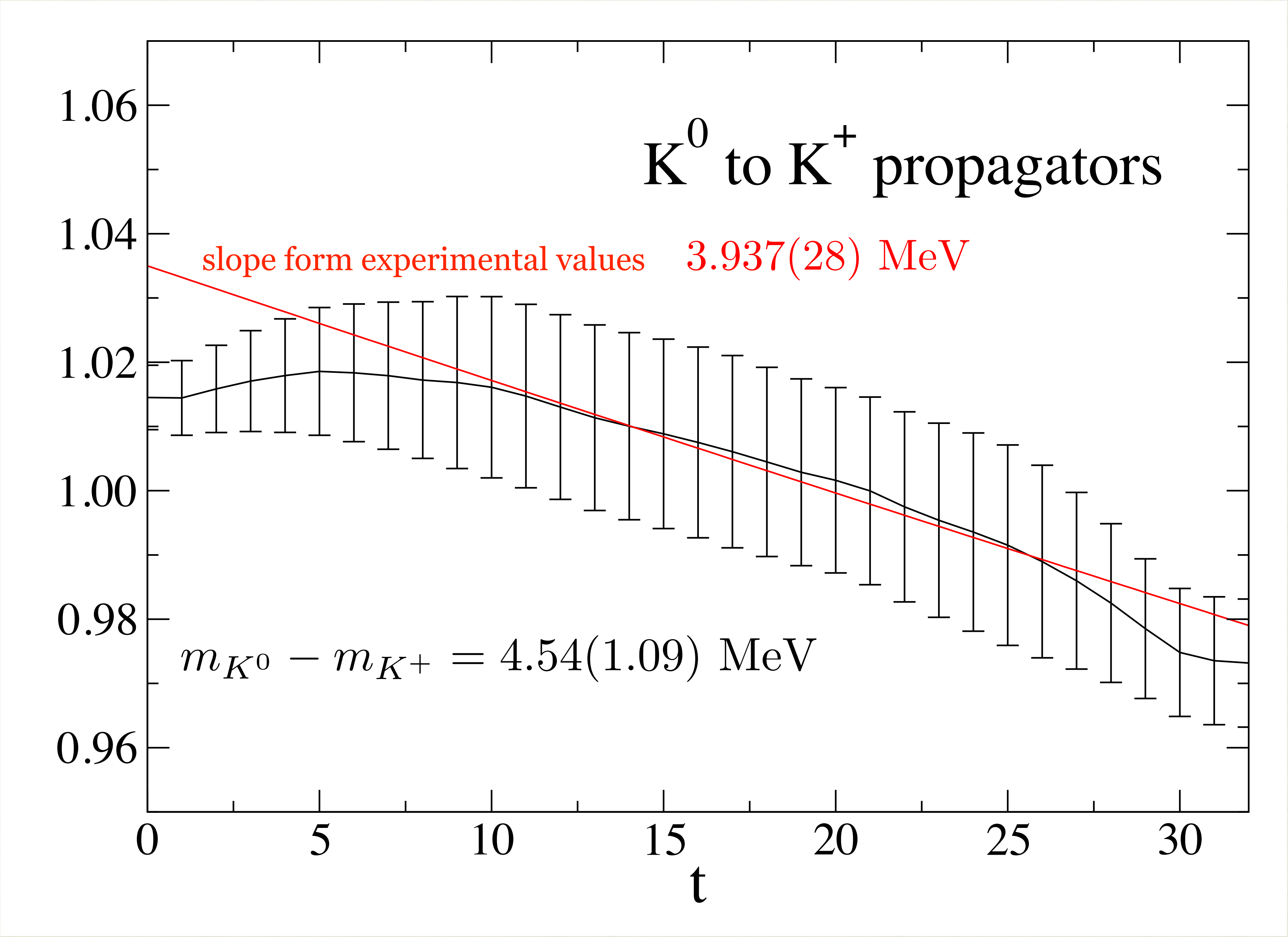
$$= \frac{\langle R_{m/m*}(U) O(U)\rangle_{m*}}{\langle R_{m/m*}(U)\rangle_{m*}} \qquad R_{m/m*}(U) = \frac{\det D_m(U)}{\det D_{m*}(U)}$$



1+1+1 flavor QCD+QED

PACS-CS Collaboration, Phys. Rev. D86 (2012) 034507

Reweighting for u-d quark mass difference and QED



Future directions of lattice QCD/gauge theories

- Heavy quark physics (charm, bottom)
  - CKM matrix, BSM physics
- Hadron structure
  - form factor, PDF of nucleon
- Finite temperature and density
  - phase transitions, EoS, heavy-ion collisions, neutron stars
- dynamical models for BSM
  - technicolor, extra-dimension
- Hadron interaction (This talk)
  - nuclear physics from QCD
  - Hyper-nuclei, J-PARC

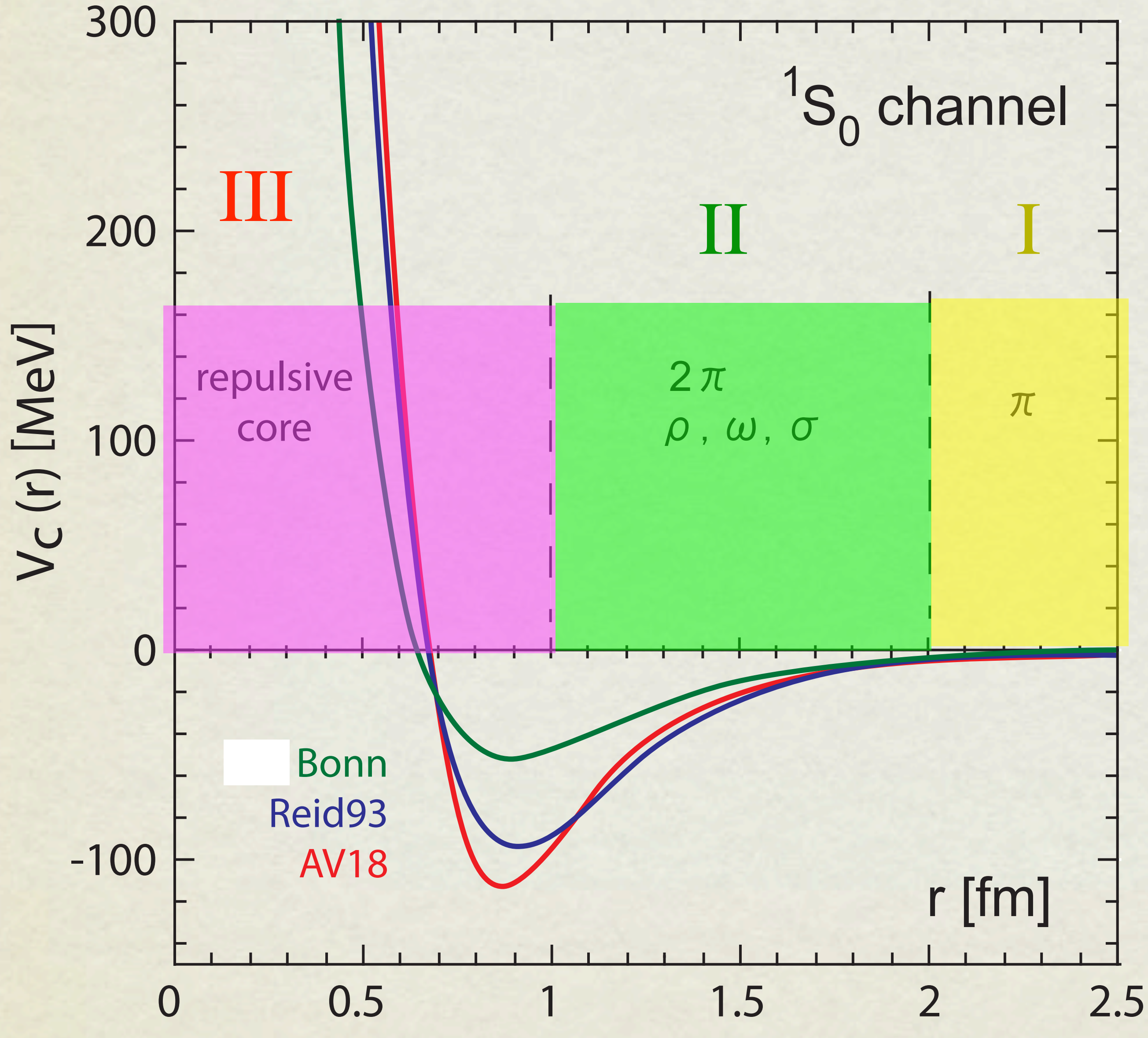
Plan of my talk

1. Introduction
2. Strategy
3. Nuclear potentials
4. Repulsive core: Hyperon Interactions
5. Extensions
6. Conclusion

2. STRATEGY

How can we extract hadronic interaction from lattice QCD ?

Ex. **Phenomenological NN potential**
(~40 parameters to fit 5000 phase shift data)



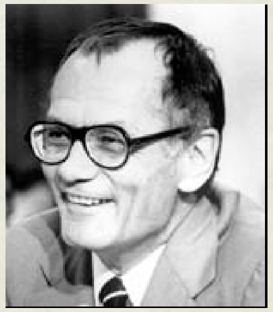
I One-pion exchange

II Multi-pions

III Repulsive core

Yukawa(1935)

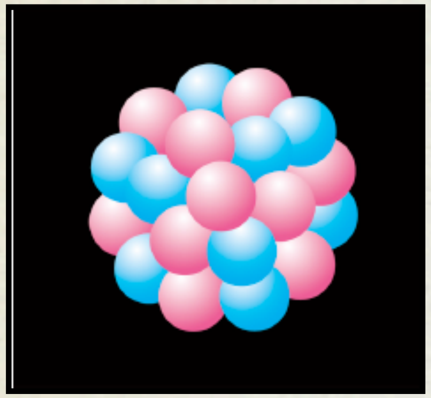
Taketani et al.(1951)

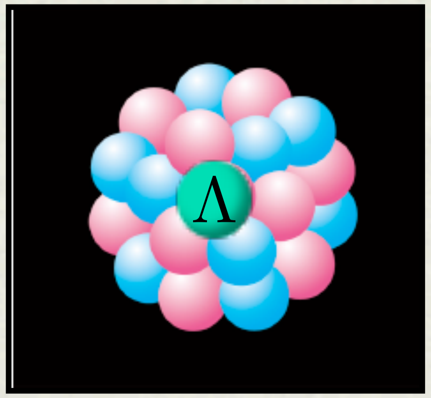


Jastrow(1951)

Nuclear force is a basis for understanding ...

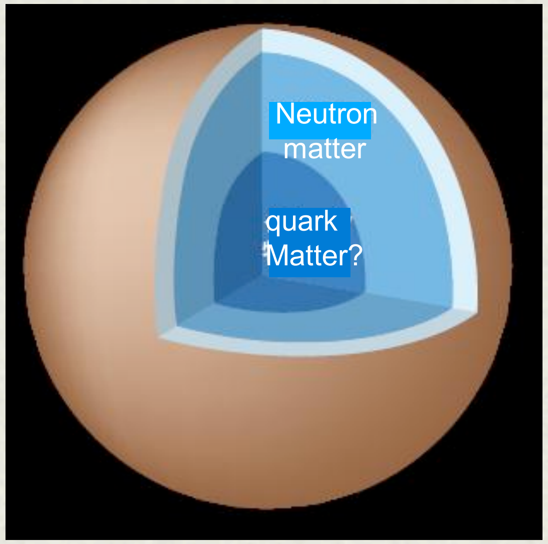
- Structure of ordinary and hyper nuclei



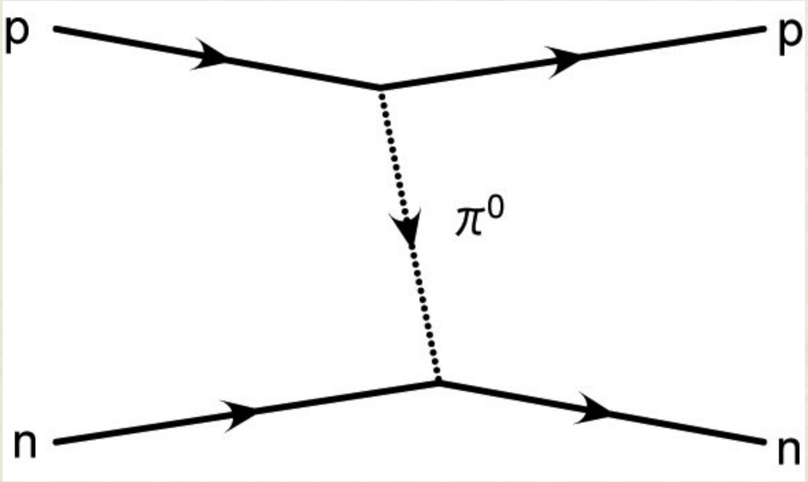


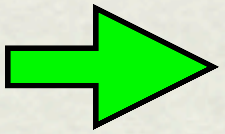
- Ignition of Type II SuperNova

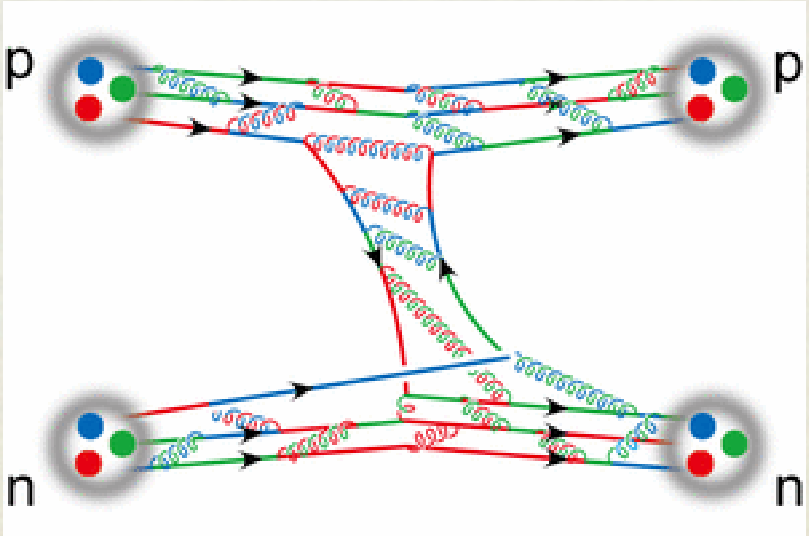
- Structure of neutron star



Can we extract a nuclear force in (lattice) QCD ?





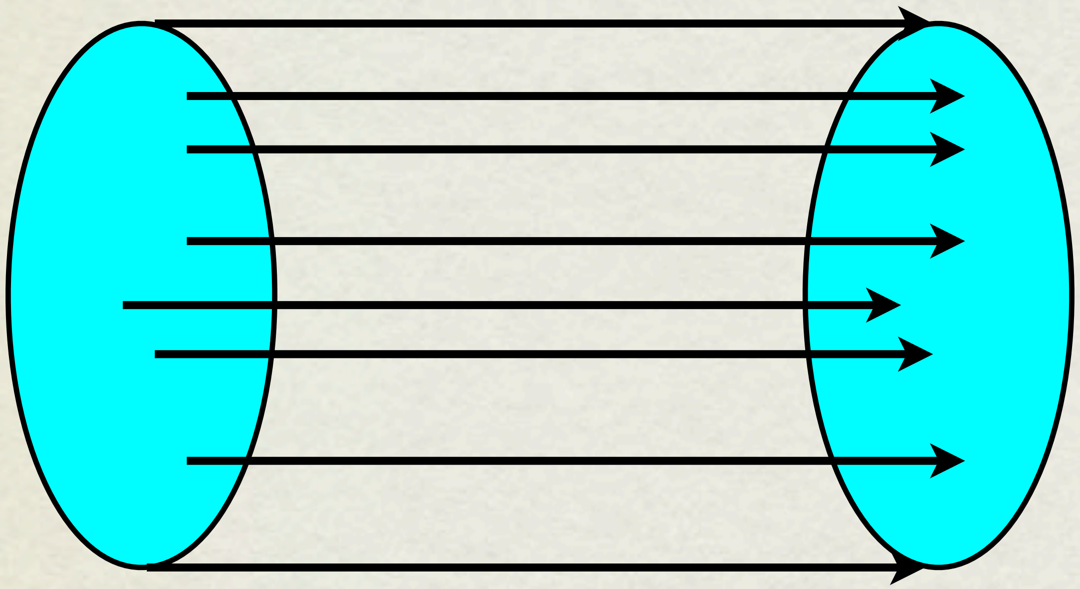


3 strategies to nuclear structure from lattice QCD

1. Extreme: calculate nuclear structure directly from lattice QCD

Ab-Initio but (almost) impossible, difficult to extract "physics" from results

difficult to apply results to other systems



$$\simeq e^{-m_A t} + \cdots$$

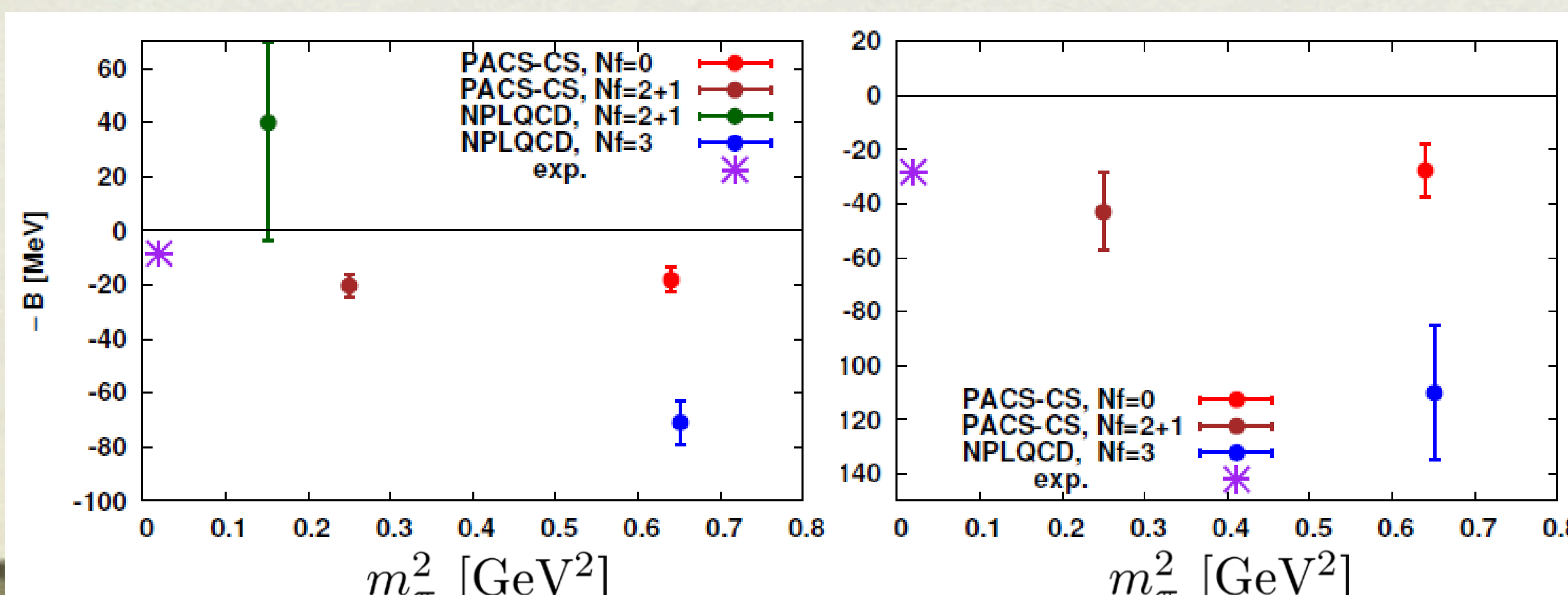
3A quark lines

A: atomic number

large number of contractions/very noisy

some reduction (Doi-Endres, CPC 184(2013)117)

$^3\mathrm{H}$ $(={}^3\mathrm{He})$ $^4\mathrm{He}$



2. Standard: calculate NN phase shift from lattice QCD

Ab-Initio for phase shift results can not be directly used to calculate nuclear structure

phase shift ⇨ nuclear potential ⇨ nuclear structure

Lüsher's finite volume method for the phase shift

two particles in the finite box $(V = L^3)$

$$E = 2\sqrt{\mathbf{k}^2 + m^2} \Rightarrow \mathbf{k} \neq \frac{2\pi}{L}\mathbf{n} \ (\mathbf{n} \in Z^3)$$

due to the interaction between two particles

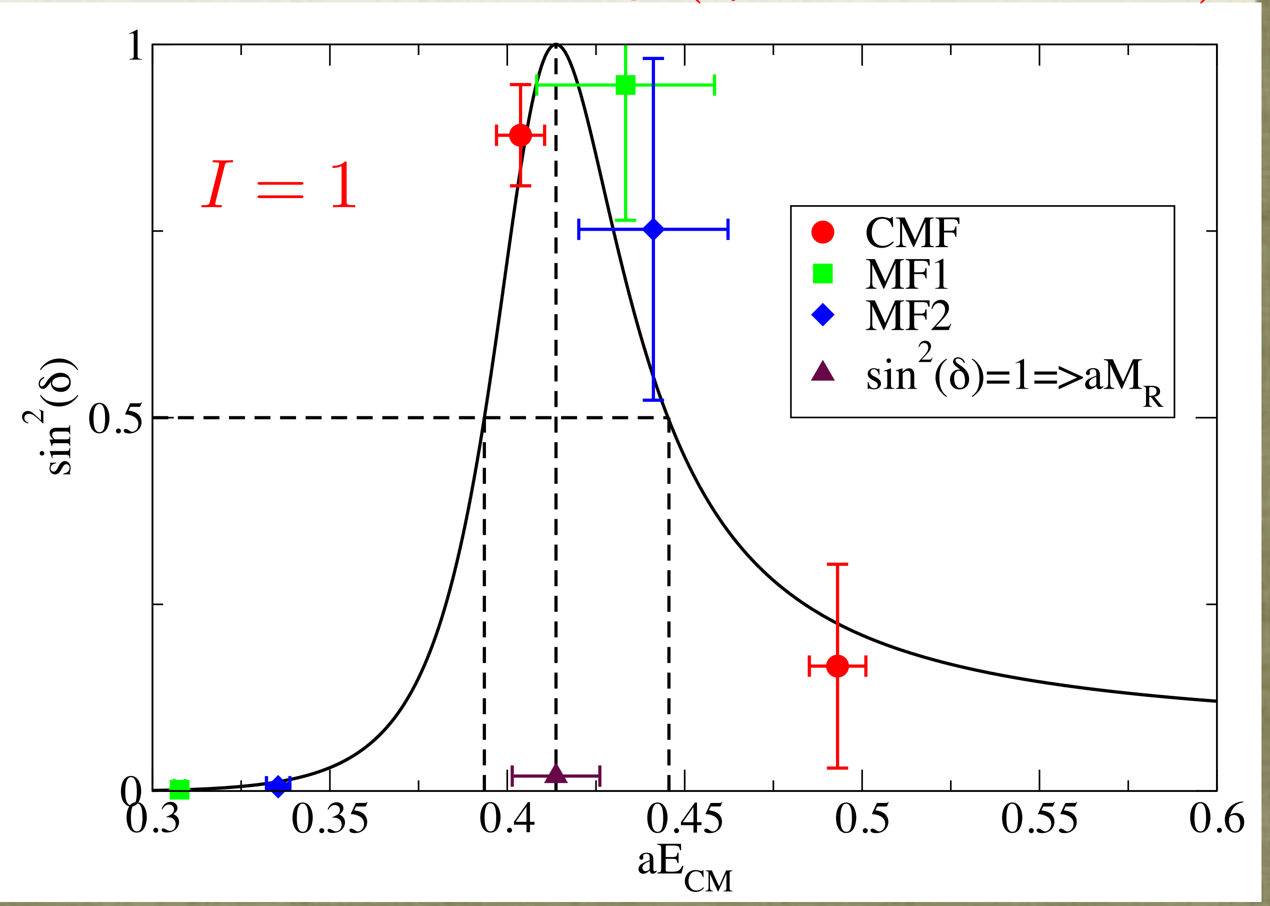
⇨ phase shift $\delta_l(k_n)$

Ex. $k \cot \delta_0(k) = \frac{2}{\sqrt{\pi}L} Z_{00}(1; q^2)$

$k = |\mathbf{k}|$ generalize zeta-function

$$q = \frac{kL}{2\pi} \qquad Z_{00}(s; q^2) = \frac{1}{\sqrt{4\pi}} \sum_{\mathbf{n} \in \mathbf{Z}^3} (\mathbf{n}^2 - q^2)^{-s}$$

$\pi^+\pi^-$ scattering (ρ meson width)



ETMC: Feng-Jansen-Renner, PLB684(2010)

3. Alternative: calculate "nuclear potential" from lattice QCD

our strategy

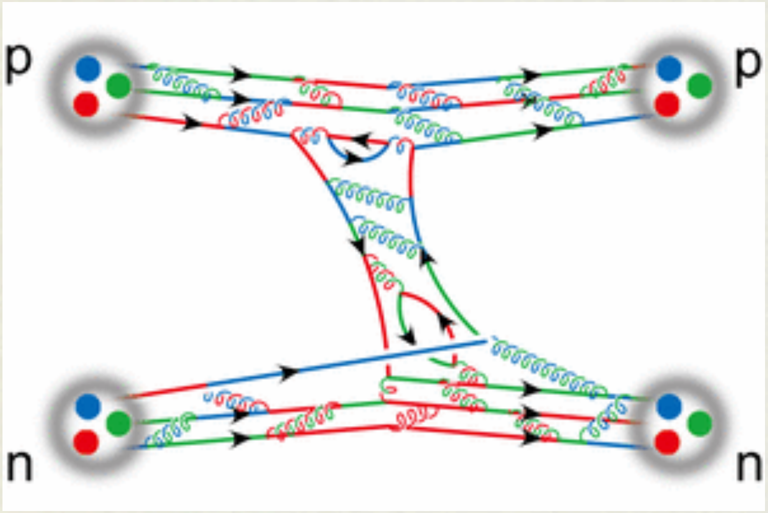
Ab-Initio for potential

"Physics" is clear

nuclear potential ⟹ nuclear structure

Difficulties for NN potentials

A. Interactions (2-body problem) are much more difficult than masses(1-body problem).



more complicated diagrams, larger volume,
more Monte-Carlo sampling, etc.

B. Definition of potential in quantum theories ?

classical $V(x)$ ⟹ quantum $V(x)$ potential is an input

no classical NN potentials QCD $V_{NN}(x)$? output from QCD

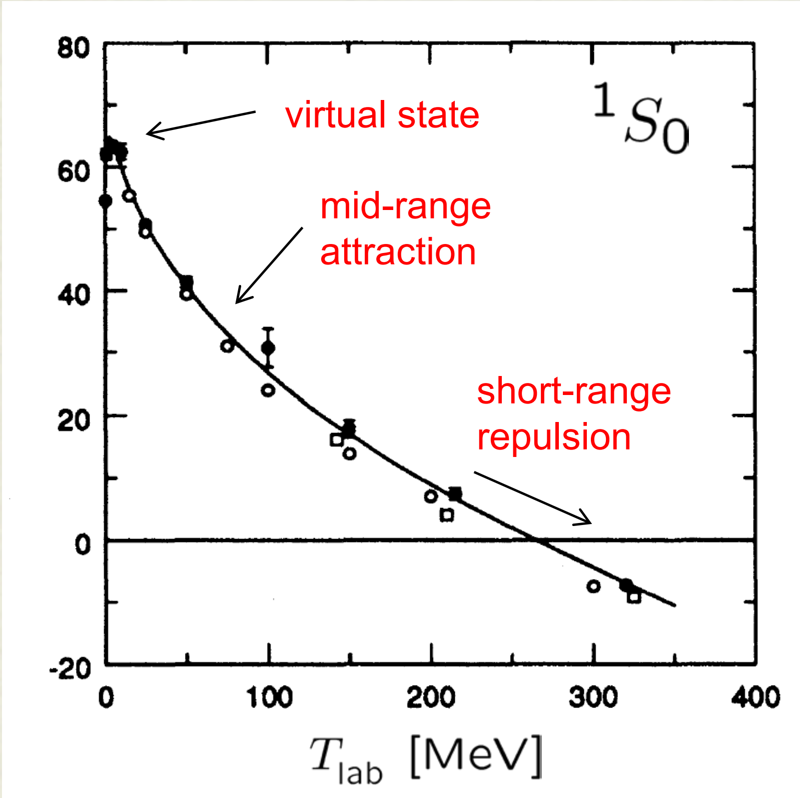
Potentials in QCD ?

What are "potentials" in quantum field theories such as QCD ?

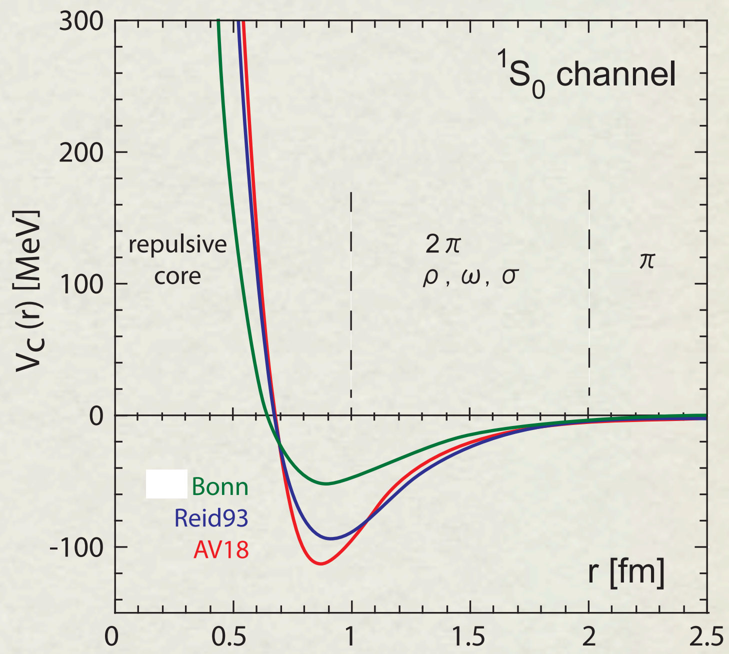
"Potentials" themselves can NOT be directly measured. analogy: running coupling in QCD

scheme dependent, Unitary transformation

experimental data of scattering phase shifts



potentials, but not unique



useful to "understand" physics

analogy: asymptotic freedom

"Potentials" are useful tools to extract observables such as scattering phase shift.

One may adopt a convenient definition of potentials as long as they reproduce correct physics of QCD.

Our strategy in lattice QCD

Full details: Aoki, Hatsuda & Ishii, PTP123(2010)89.

Consider "elastic scattering"

$NN \to NN$ ~~$NN \to NN +$ others $(NN \to NN + \pi, NN + \bar{N}N, \cdots)$~~

energy $W_k = 2\sqrt{\mathbf{k}^2 + m_N^2} < W_{\rm th} = 2m_N + m_\pi$ Elastic threshold

Quantum Field Theoretical consideration

- S-matrix below inelastic threshold. Unitarity gives $S = e^{2i\delta}$

Step 1 define (Equal-time) Nambu-Bethe-Salpeter (NBS) Wave function

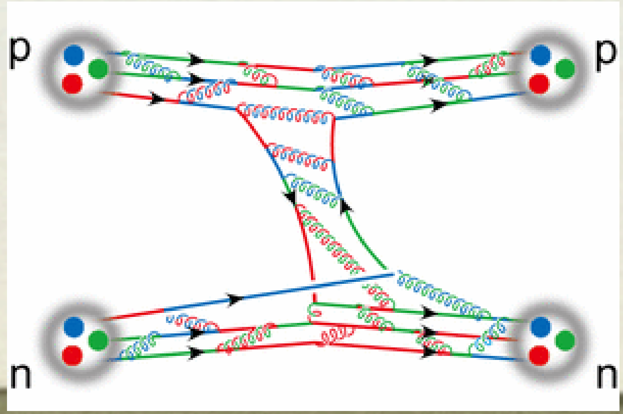
Spin model: Balog et al., 1999/2001

$$\varphi_{\mathbf{k}}(\mathbf{r}) = \langle 0 | N(\mathbf{x}+\mathbf{r}, 0) N(\mathbf{x}, 0) | NN, W_k \rangle$$

QCD eigen-state

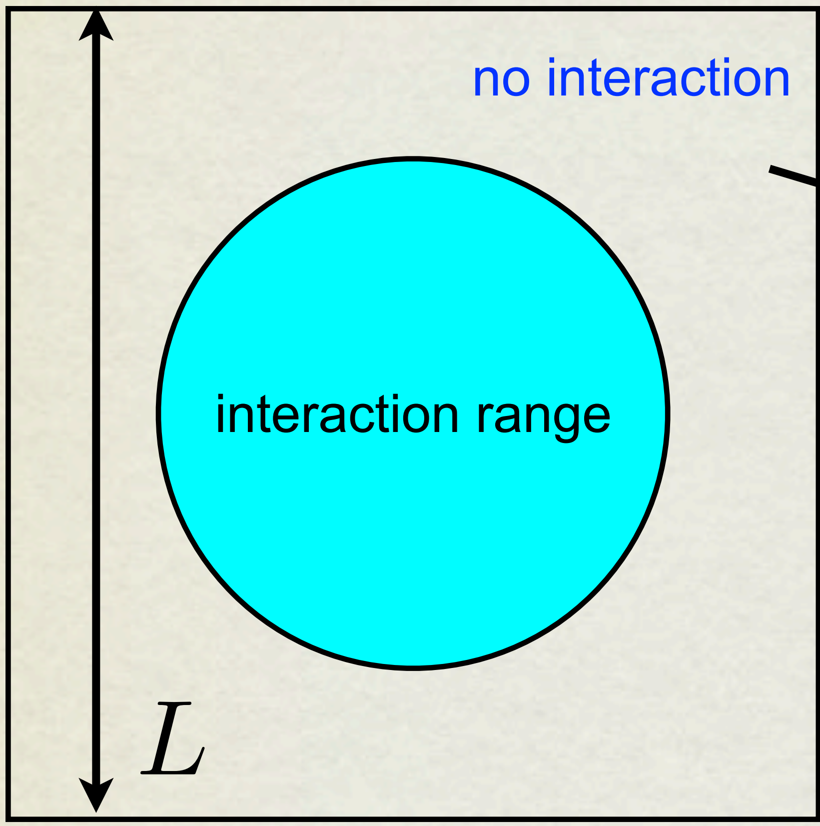
$N(x) = \varepsilon_{abc} q^a(x) q^b(x) q^c(x)$: local operator

"scheme"



Asymptotic behavior of NBS wave function

Lin et al., 2001; CP-PACS, 2004/2005



$$r = |\mathbf{r}| \to \infty$$

$$\varphi_{\mathbf{k}}^{l} \to A_l \frac{\sin(kr - l\pi/2 + \delta_l(k))}{kr}$$

partial wave

scattering phase shift (phase of the S-matrix by unitarity) in QCD !

NBS wave function ⟺ scattering wave function in quantum mechanics

cf. Luescher's finite volume method   allowed k at L ⟹ $\delta_l(k_n)$

Step 2 — define non-local but energy-independent "potential" as

$\mu = m_N/2$

reduced mass

$$[\epsilon_k - H_0]\,\varphi_{\mathbf{k}}(\mathbf{x}) = \int d^3y\, U(\mathbf{x},\mathbf{y})\varphi_{\mathbf{k}}(\mathbf{y})$$

non-local potential

$$\epsilon_k = \frac{\mathbf{k}^2}{2\mu} \qquad H_0 = \frac{-\nabla^2}{2\mu}$$

(Trivial) proof of "existence"

We can construct a non-local but energy-independent potential easily as

inner product

$$U(\mathbf{x},\mathbf{y}) = \sum_{\mathbf{k},\mathbf{k}'}^{W_k, W_{k'} \le W_{\rm th}} [\epsilon_k - H_0]\,\varphi_{\mathbf{k}}(\mathbf{x})\eta^{-1}_{\mathbf{k},\mathbf{k}'}\varphi^{\dagger}_{\mathbf{k}'}(\mathbf{y})$$

$\eta^{-1}_{\mathbf{k},\mathbf{k}'}$: inverse of $\eta_{\mathbf{k},\mathbf{k}'} = (\varphi_{\mathbf{k}}, \varphi_{\mathbf{k}'})$

For ${}^\forall W_{\mathbf{p}} < W_{\rm th}$

$$\int d^3y\, U(\mathbf{x},\mathbf{y})\phi_{\mathbf{p}}(\mathbf{y}) = \sum_{\mathbf{k},\mathbf{k}'} [\epsilon_k - H_0]\,\varphi_{\mathbf{k}}(x)\eta^{-1}_{\mathbf{k},\mathbf{k}'}\eta_{\mathbf{k}',\mathbf{p}} = [\epsilon_p - H_0]\,\varphi_{\mathbf{p}}(x)$$

Remark

Non-relativistic approximation is NOT used. We just take the specific (equal-time) frame.

Step 3 expand the non-local potential in terms of derivative as

$$U(\mathbf{x}, \mathbf{y}) = V(\mathbf{x}, \nabla)\delta^3(\mathbf{x} - \mathbf{y})$$

$$V(\mathbf{x}, \nabla) = V_0(r) + V_\sigma(r)(\sigma_\mathbf{1} \cdot \sigma_\mathbf{2}) + V_T(r)S_{12} + V_{\mathrm{LS}}(r)\mathbf{L} \cdot \mathbf{S} + O(\nabla^2)$$

LO LO LO NLO NNLO

spins

tensor operator $S_{12} = \frac{3}{r^2}(\sigma_\mathbf{1} \cdot \mathbf{x})(\sigma_\mathbf{2} \cdot \mathbf{x}) - (\sigma_\mathbf{1} \cdot \sigma_\mathbf{2})$

This expansion is a part of our "scheme" for potentials.

Step 4 extract the local potential at LO as

$$V_{\mathrm{LO}}(\mathbf{x}) = \frac{[\epsilon_k - H_0]\varphi_\mathbf{k}(\mathbf{x})}{\varphi_\mathbf{k}(\mathbf{x})}$$

Step 5 solve the Schroedinger Eq. in the infinite volume with this potential.

phase shifts and binding energy below inelastic threshold

We can check a size of errors of the LO in the expansion. (See later).

3. NUCLEAR POTENTIALS

Extraction of NBS wave function

NBS wave function

$$\varphi_{\mathbf{k}}(\mathbf{r}) = \langle 0|N(\mathbf{x}+\mathbf{r},0)N(\mathbf{x},0)|NN,W_k\rangle$$

Potential

$$[\epsilon_k - H_0]\varphi_{\mathbf{k}}(\mathbf{x}) = \int d^3y\, U(\mathbf{x},\mathbf{y})\varphi_{\mathbf{k}}(\mathbf{y})$$

4-pt Correlation function

source for NN

$$F(\mathbf{r},t-t_0) = \langle 0|T\{N(\mathbf{x}+\mathbf{r},t)N(\mathbf{x},t)\}\overline{\mathcal{J}}(t_0)|0\rangle$$

complete set for NN

$$\begin{aligned} F(\mathbf{r},t-t_0) &= \langle 0|T\{N(\mathbf{x}+\mathbf{r},t)N(\mathbf{x},t)\}\sum_{n,s_1,s_2}|2N,W_n,s_1,s_2\rangle\langle 2N,W_n,s_1,s_2|\overline{\mathcal{J}}(t_0)|0\rangle + \cdots \\ &= \sum_{n,s_1,s_2} A_{n,s_1,s_2}\varphi^{W_n}(\mathbf{r})e^{-W_n(t-t_0)}, \quad A_{n,s_1,s_2} = \langle 2N,W_n,s_1,s_2|\overline{\mathcal{J}}(0)|0\rangle. \end{aligned}$$

ground state saturation at large t

$$\lim_{(t-t_0)\to\infty} F(\mathbf{r},t-t_0) = A_0\varphi^{W_0}(\mathbf{r})e^{-W_0(t-t_0)} + O(e^{-W_{n\neq 0}(t-t_0)})$$

NBS wave function

This is a standard method in lattice QCD and was employed for our first calculation.

Improved method

Ishii *et al.* (HALQCD), PLB712(2012) 437

normalized 4-pt function $R(\mathbf{r},t) \equiv F(\mathbf{r},t)/(e^{-m_N t})^2 = \sum_n A_n \varphi^{W_n}(\mathbf{r}) e^{-\Delta W_n t}$

$$\Delta W_n = W_n - 2m_N = \frac{\mathbf{k}_n^2}{m_N} - \frac{(\Delta W_n)^2}{4m_N}$$

$$-\frac{\partial}{\partial t} R(\mathbf{r},t) = \left\{ H_0 + U - \frac{1}{4m_N}\frac{\partial^2}{\partial t^2} \right\} R(\mathbf{r},t)$$

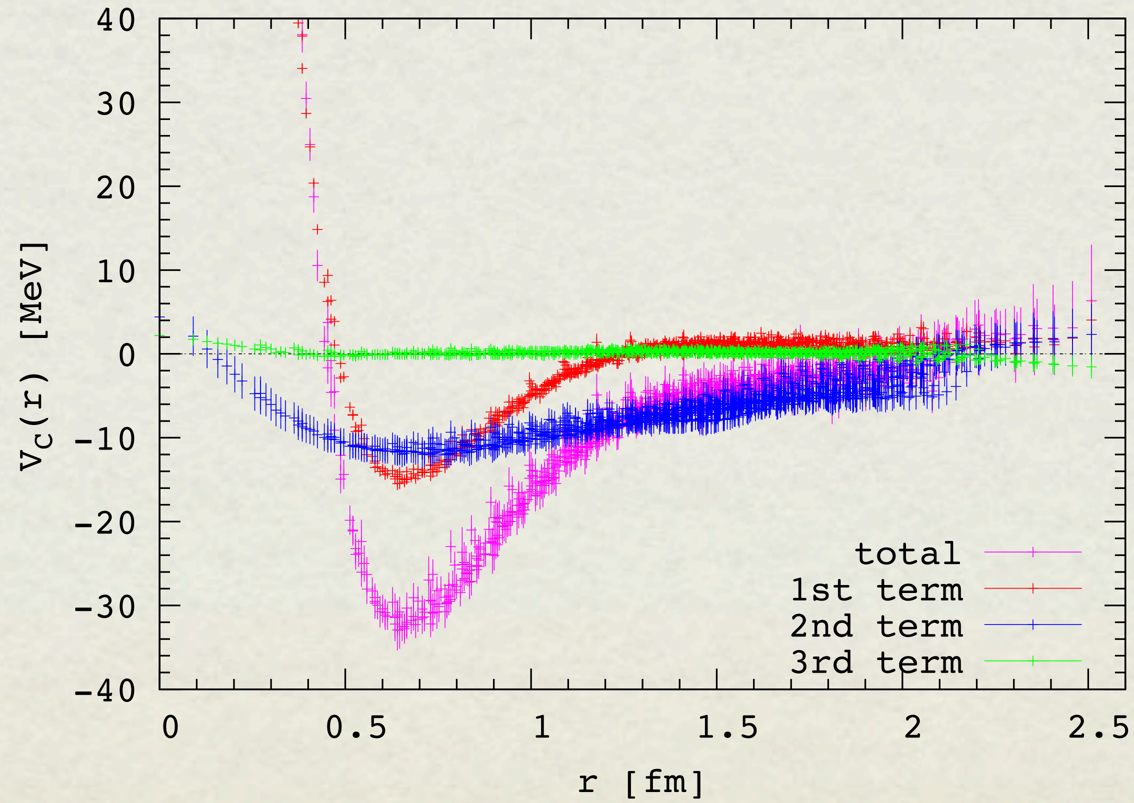
potential

Leading Order

$$\left\{ -H_0 - \frac{\partial}{\partial t} + \frac{1}{4m_N}\frac{\partial^2}{\partial t^2} \right\} R(\mathbf{r},t) = \int d^3r'\, U(\mathbf{r},\mathbf{r}') R(\mathbf{r}',t) = V_C(\mathbf{r}) R(\mathbf{r},t) + \cdots$$

1st **2nd** **3rd** **total**

3rd term(relativistic correction) is negligible.



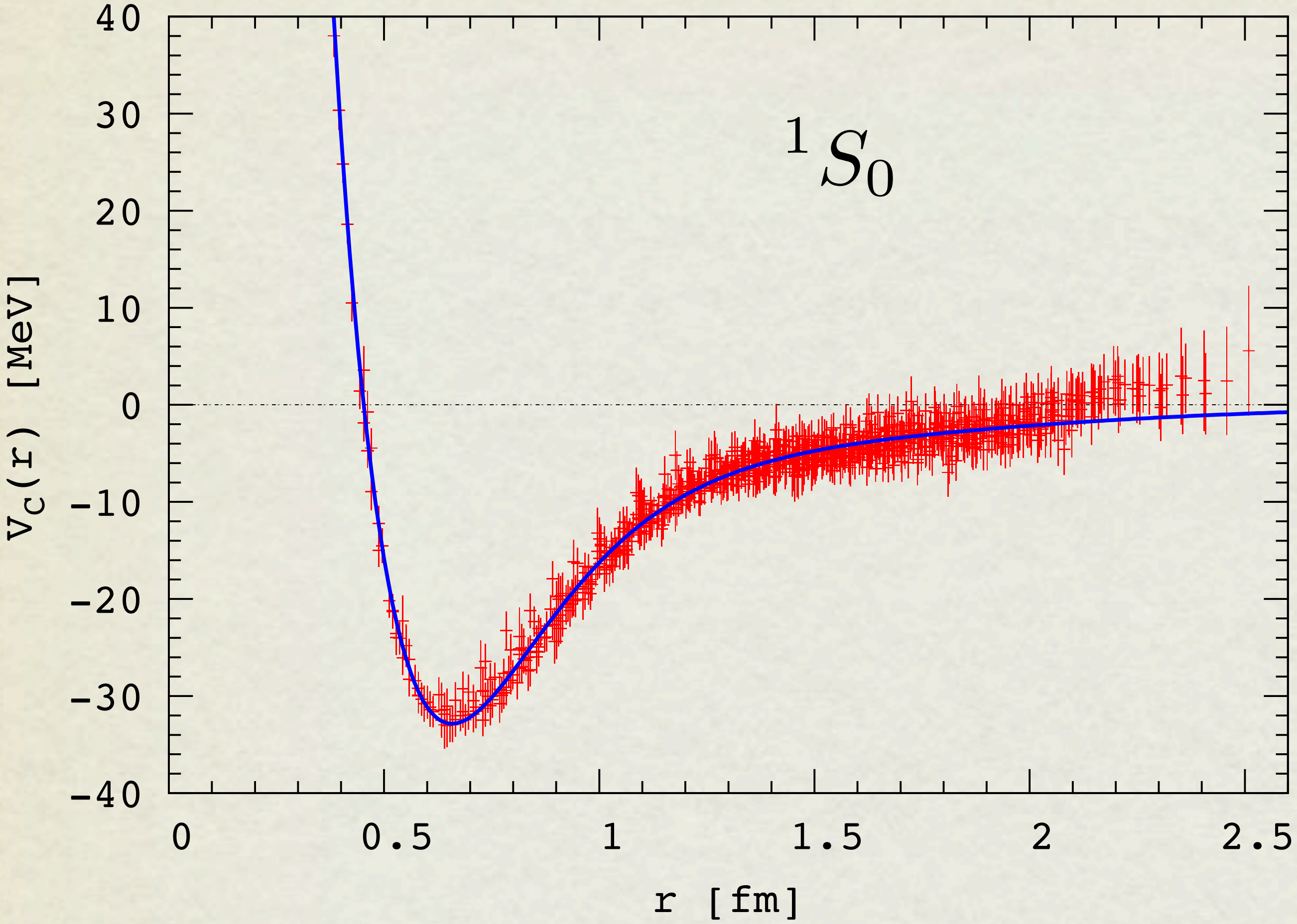
Ground state saturation is no more required ! (advantage over finite volume method.)

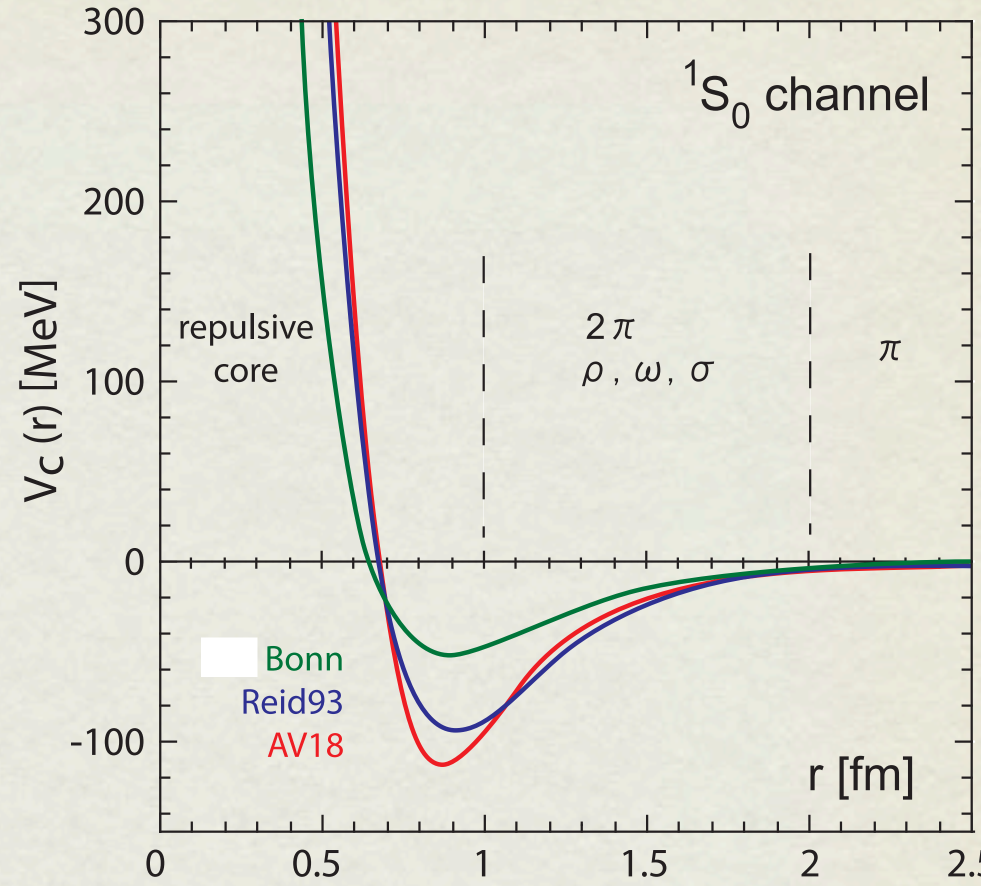
NN potential

2+1 flavor QCD, spin-singlet potential (PLB712(2012)437)

a=0.09fm, L=2.9fm $m_\pi \simeq 700$ MeV

phenomenological potential





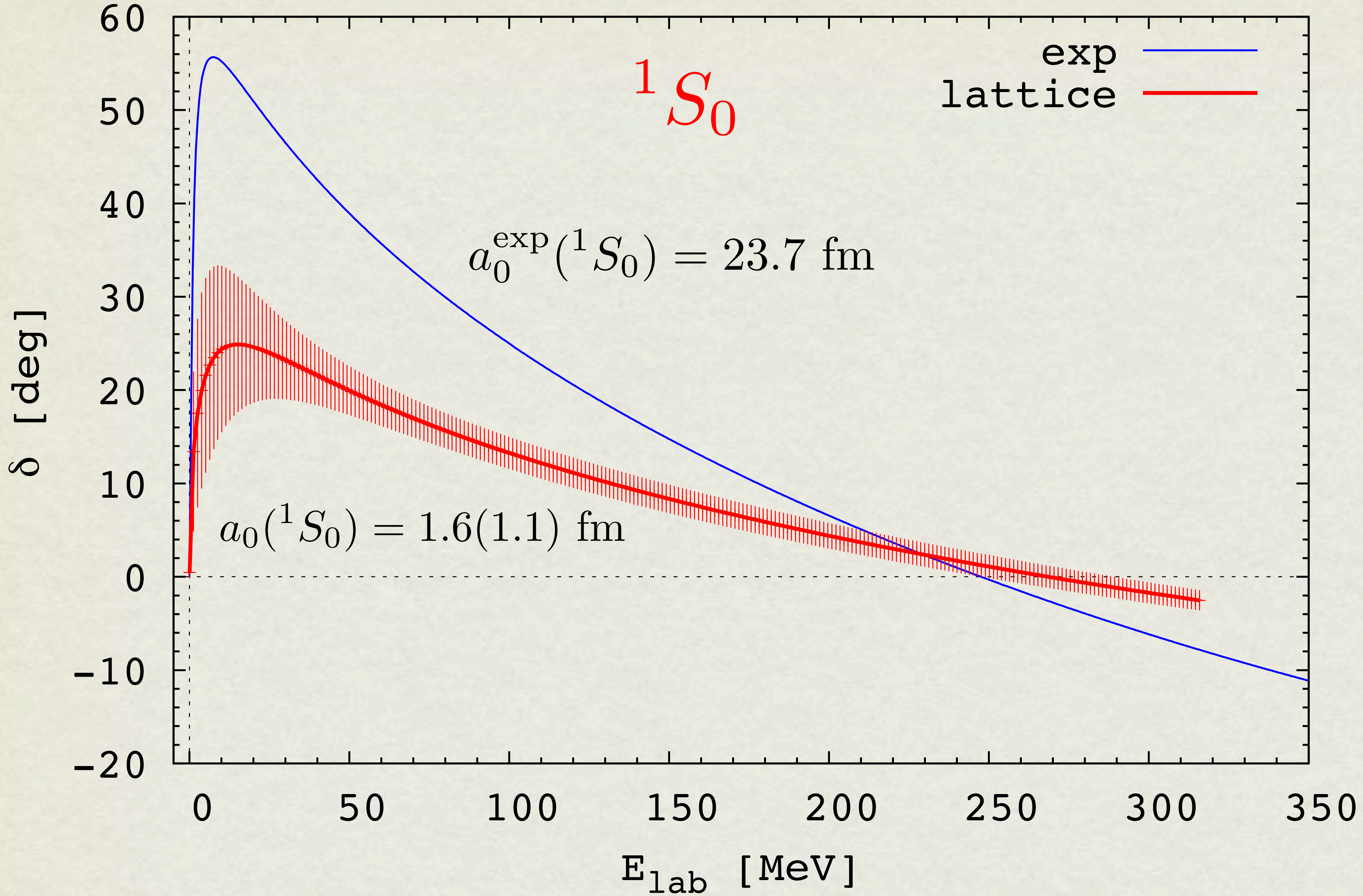
Qualitative features of NN potential are reproduced !

(1)attractions at medium and long distances
(2)repulsion at short distance(repulsive core)

1st paper(quenched QCD): Ishii-Aoki-Hatsuda, PRL90(2007)0022001

This paper has been selected as one of 21 papers in Nature Research Highlights 2007.
(One from Physics, Two from Japan, the other is on “iPS” by Sinya Yamanaka et al.)

NN potential → phase shift



It has a reasonable shape. The strength is weaker due to the heavier quark mass.

Need calculations at physical quark mass.

Convergence of velocity expansion

If the higher order terms are large, LO potentials determined from NBS wave functions at different energy become different.(cf. LOC of ChPT).

Numerical check in quenched QCD

$m_\pi \simeq 0.53$ GeV

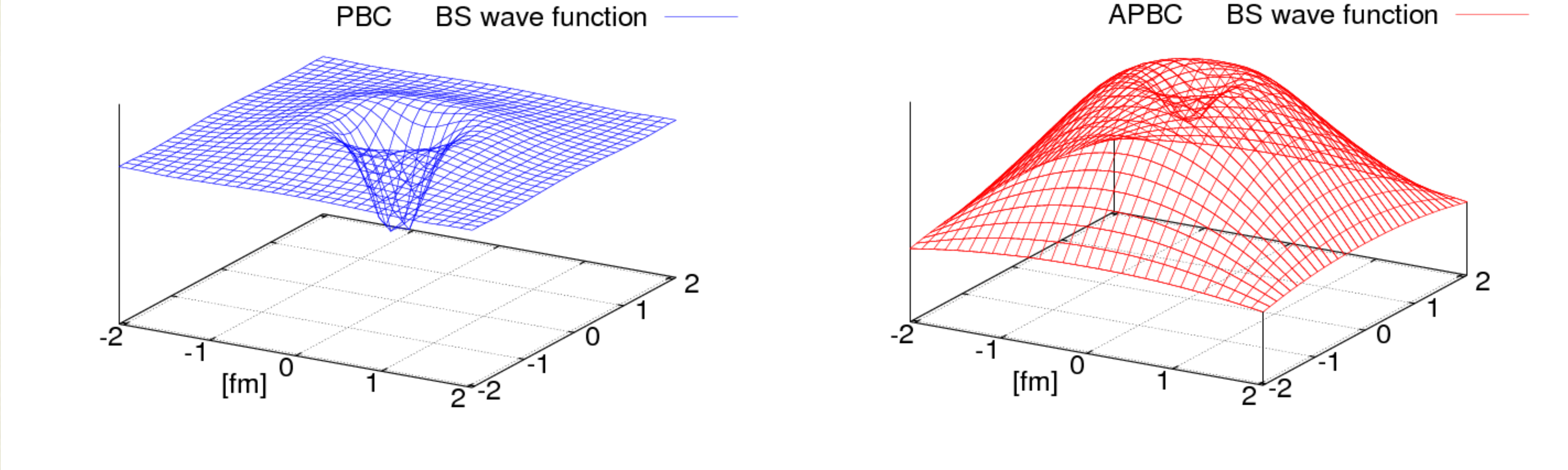
a=0.137fm, L=4.0 fm

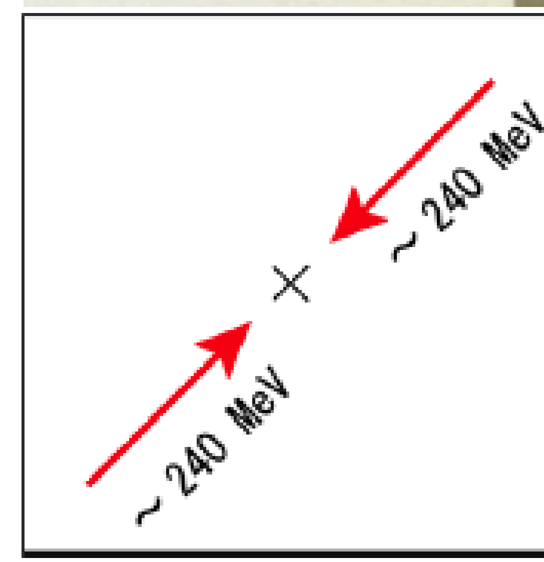
K. Murano, N. Ishii, S. Aoki, T. Hatsuda

PTP 125 (2011)1225.

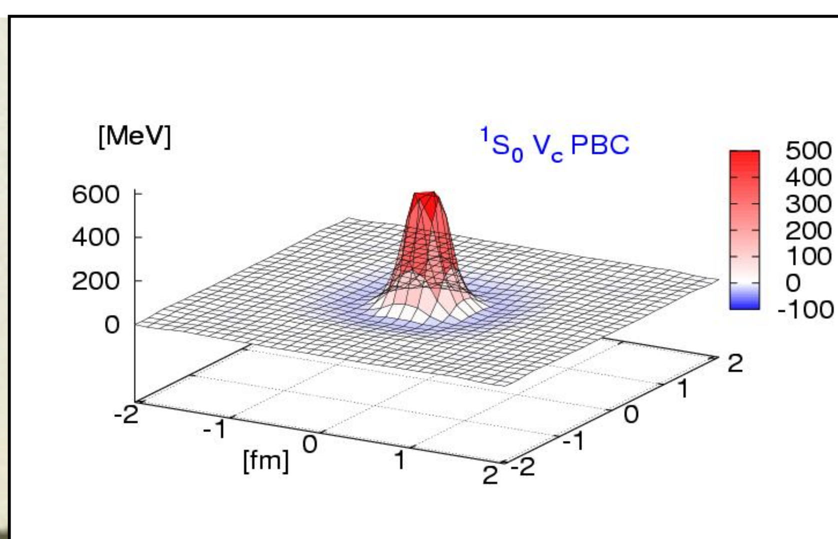
- PBC (E∼0 MeV)
- APBC (E∼46 MeV)

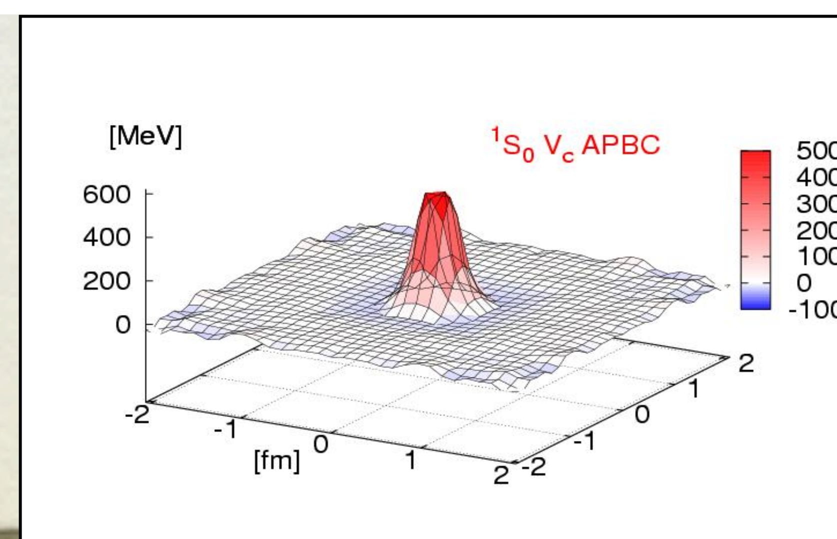
NBS wave functions

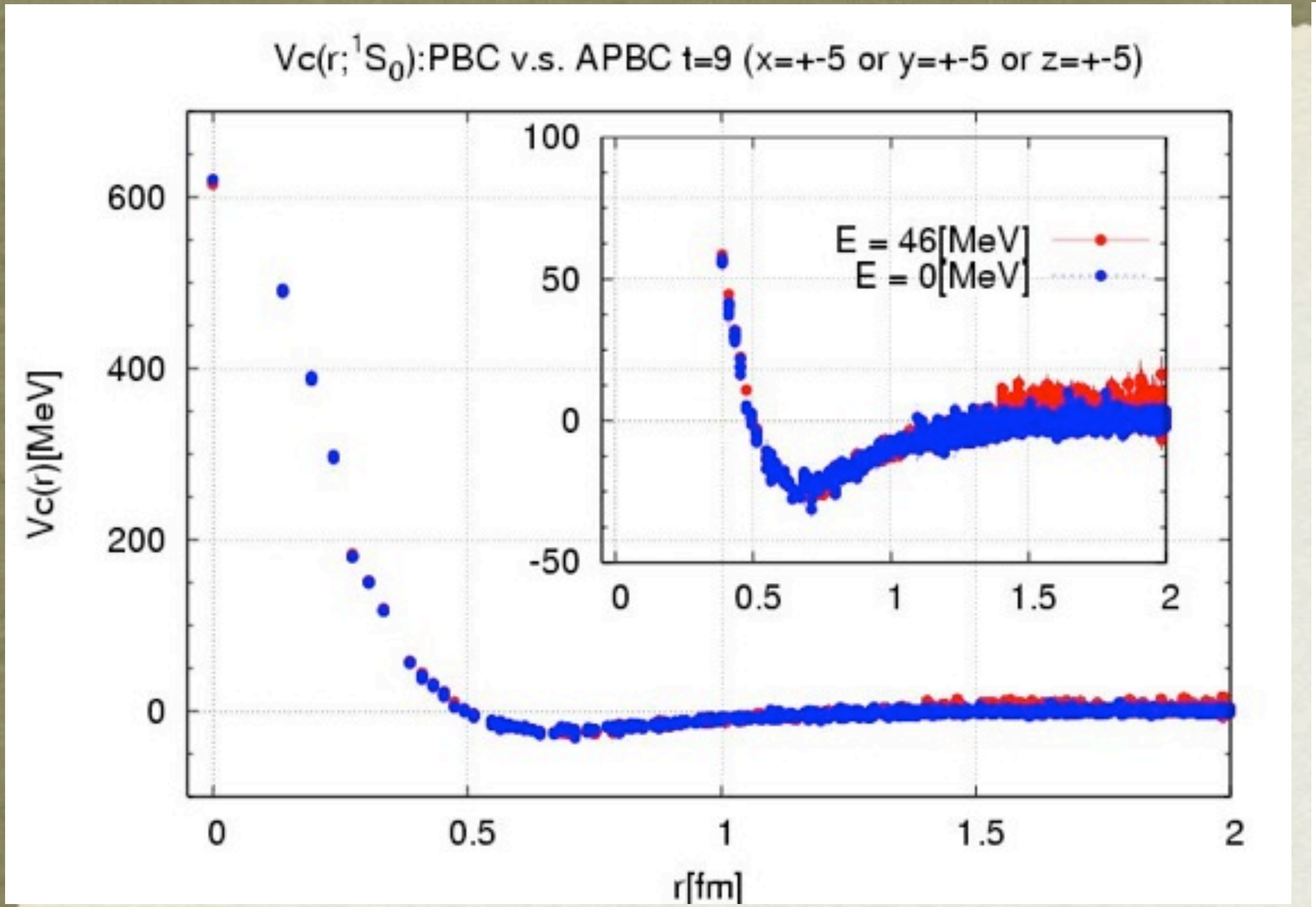


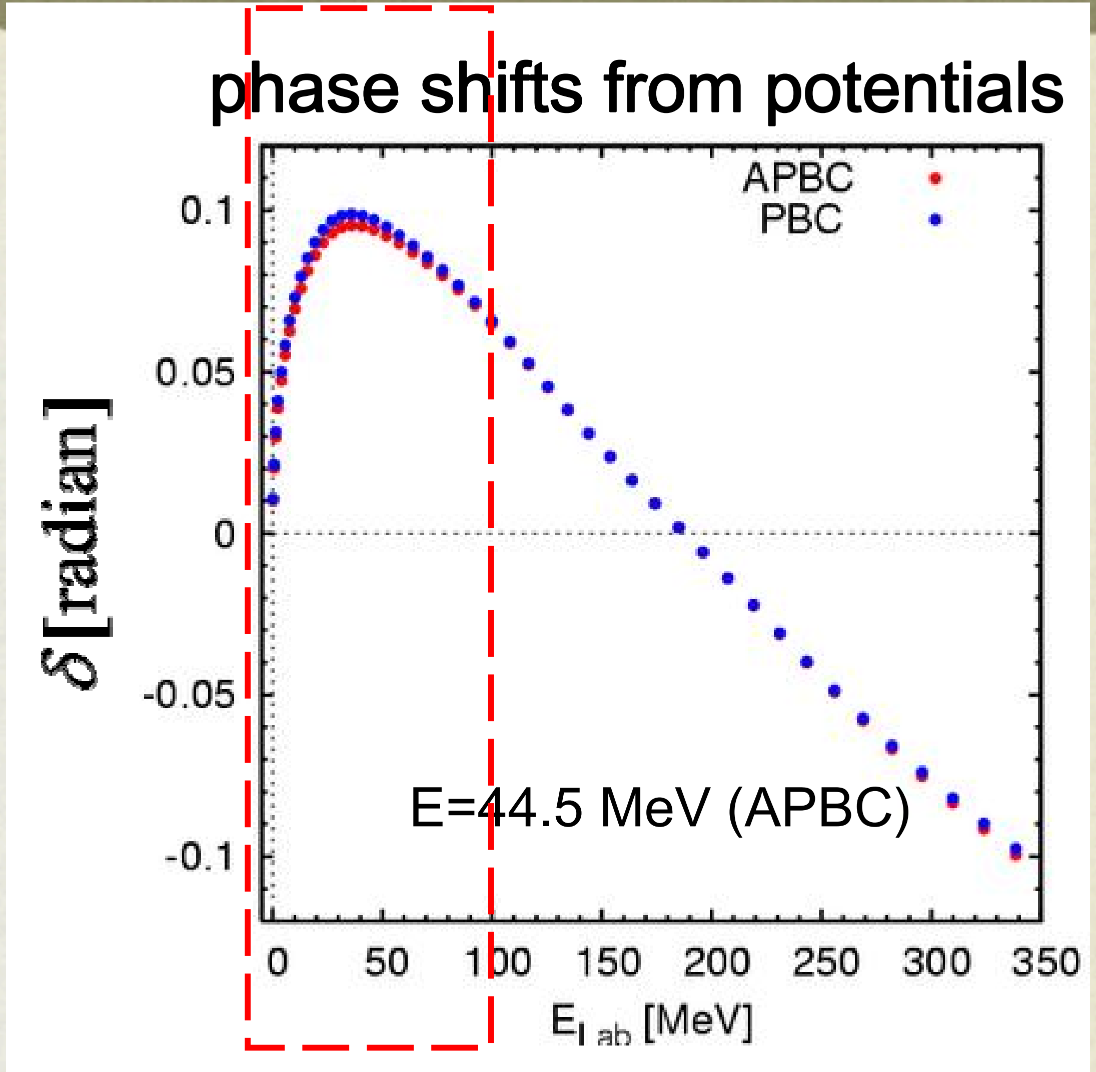


potentials









Higher order terms turn out to be very small at low energy in our scheme.

Need to be checked at lighter pion mass in 2+1 flavor QCD.

Note: convergence of the velocity expansion can be checked within this method.

(in contrast to convergence of ChPT, convergence of perturbative QCD)

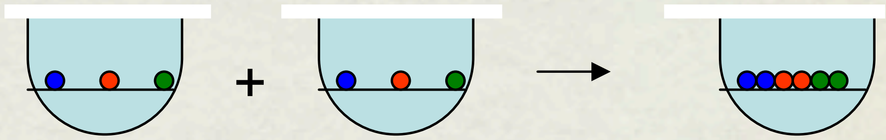
4. REPULSIVE CORE HYPERON INTERACTIONS

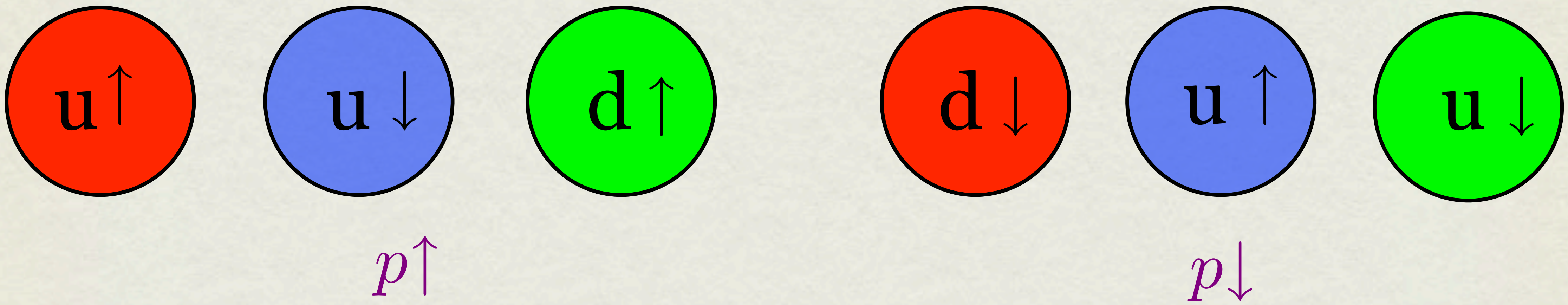
Origin of the repulsive core ?

quarks are "fermion" → two can not occupy the same position. ("Pauli principle")

they have 3 colors(red,blue,green), 2 spin(↑ ↓), 2 flavors(up,down)

→ 6 quark can occupy the same position



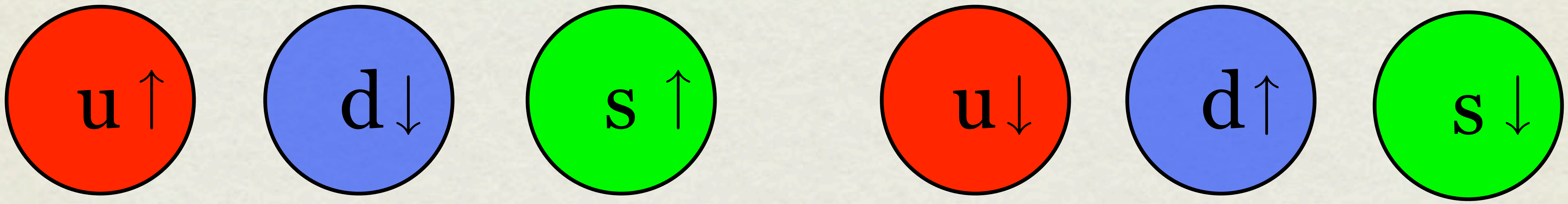


but allowed color combinations are limited + interaction among quarks

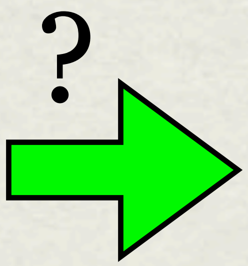
? → repulsive core ?

What happen if strange quarks are added ?

$\Lambda(uds)$ - $\Lambda(uds)$ interaction



all color combinations are allowed



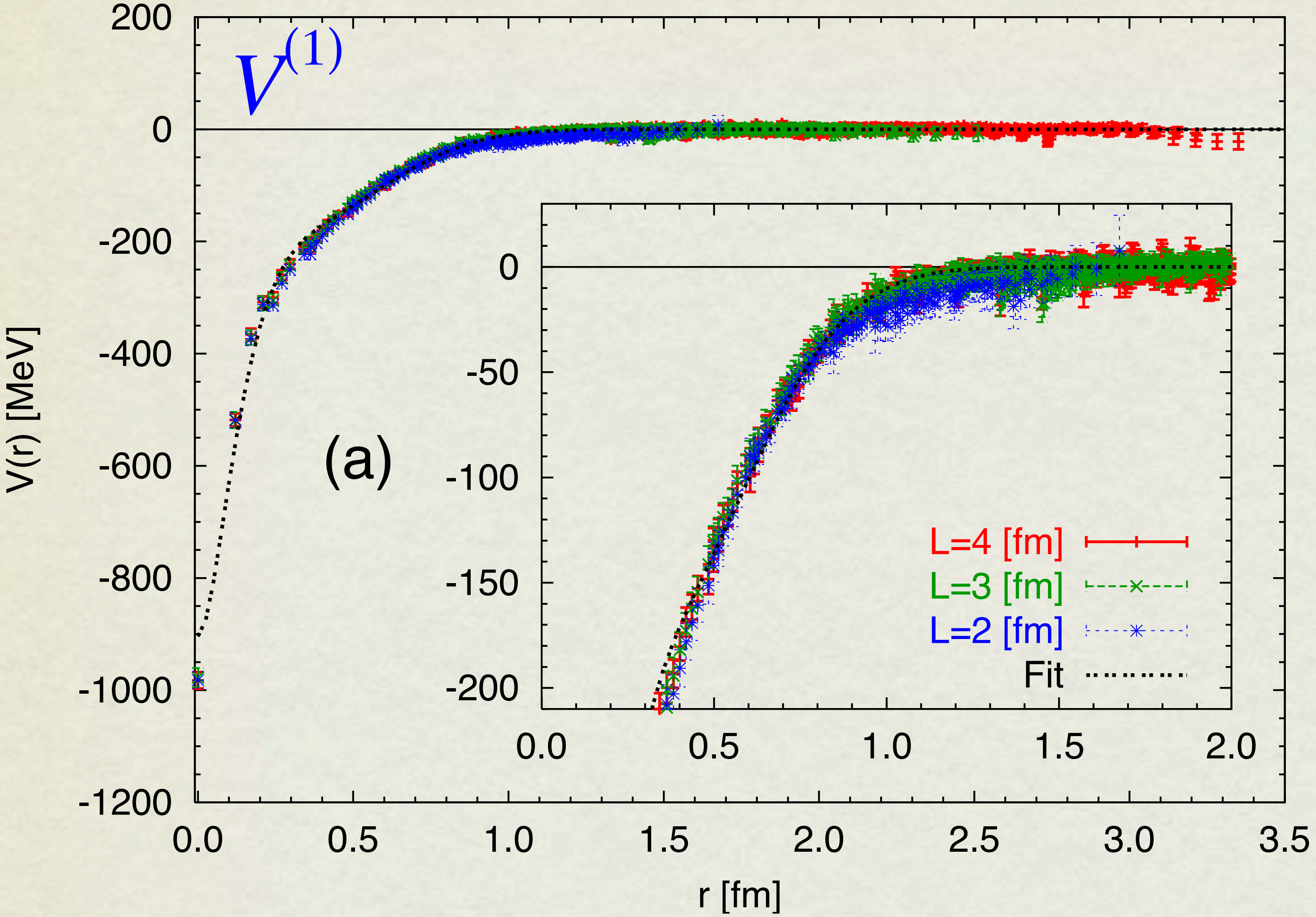
no repulsive core ?

Our lattice QCD result

Inoue *et al.* (HAL QCD Coll.), Progress of Theoretical Physics 124(2010)591

flavor SU(3) limit

$$m_u = m_d = m_s$$



Indeed, attractive instead of repulsive core appears.

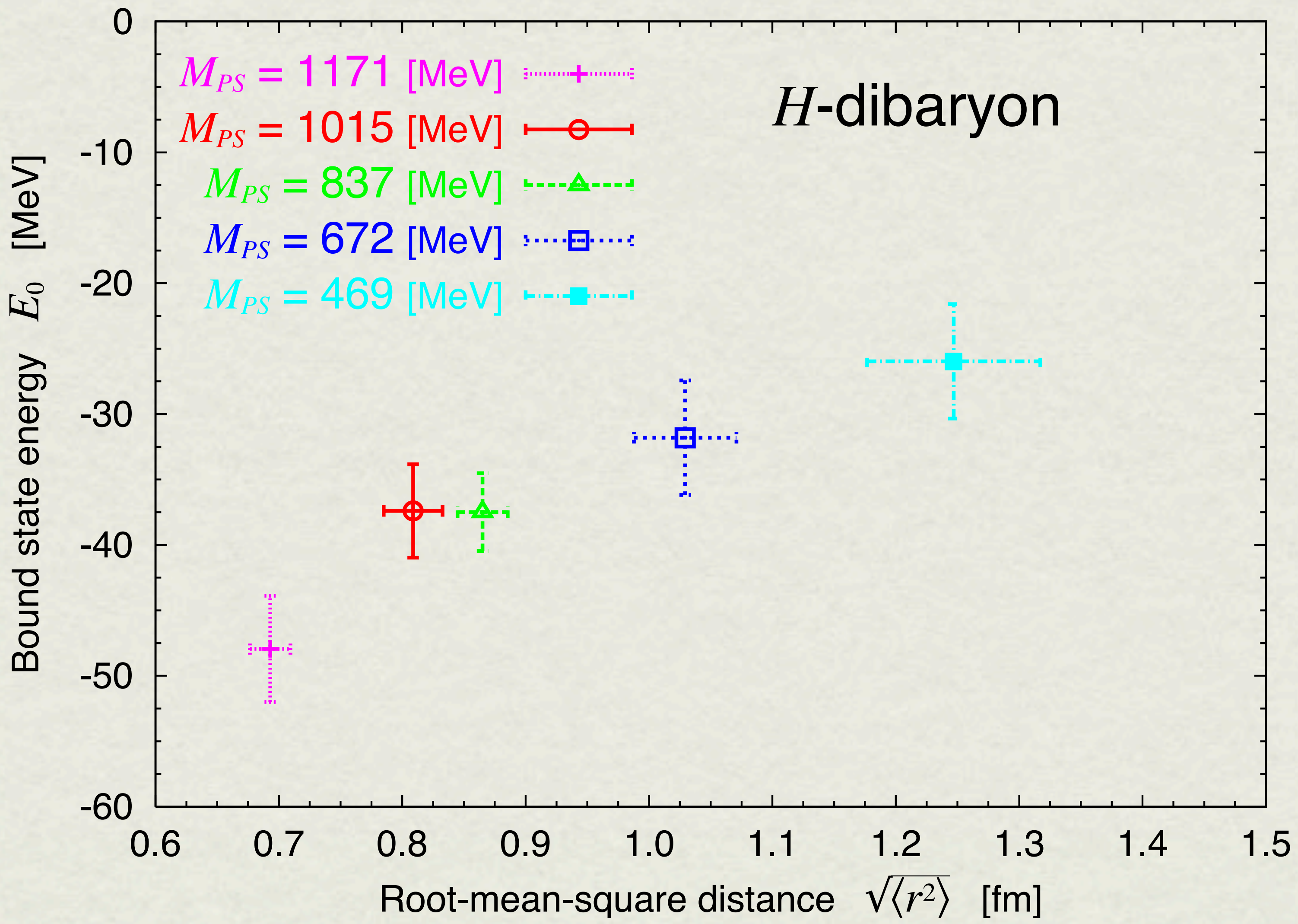
This suggests that “Pauli principle” is important for the repulsive core.

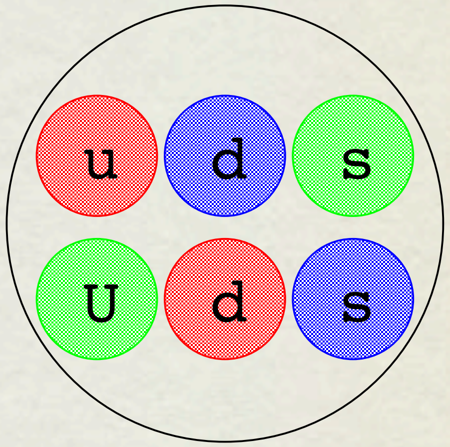
Force is attractive at all distances. Bound state ?

H-dibaryon:
a possible six quark state(uuddss)
predicted by the model but not observed yet.

This potential

One bound state (H-dibaryon) exists !





An H-dibaryon exists in the flavor SU(3) limit.
Binding energy = 25-50 MeV at this range of quark mass.
A mild quark mass dependence.

5. EXTENSION

Limitations of the potential method

1. Only for two particle scattering $NN \rightarrow NN$

2. Only for elastic scattering $W < W_{\rm th}$

In order to remove these limitations and extend the HAL QCD method to inelastic and/or multi-particle scatterings, we have to show

Key Property 1

Asymptotic behaviors of NBS wave functions for more than 2 particles

Key Property 2

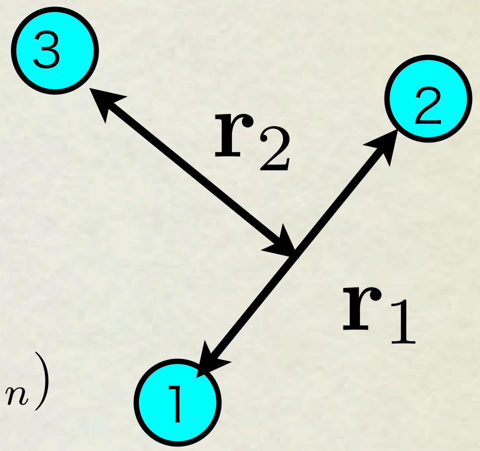
Existence of energy independent potentials above inelastic thresholds

Key Property 1

NBS wave functions for multi-particles

For simplicity,
(1) we consider scalar particles with "flavors"
(2) we assume no bound state exists.

Sinya Aoki, et al., arXiv.1303.2210 [hep-lat], to appear in PRD.



Unitarity constraint $T^\dagger - T = iT^\dagger T.$

Jacobi momenta

parametrization $_0\langle [\boldsymbol{p}^A]_n | T | [\boldsymbol{p}^B]_n \rangle_0 \equiv \delta(E^A - E^B)\delta^{(3)}(\boldsymbol{P}^A - \boldsymbol{P}^B) T([\boldsymbol{q}^A]_n, [\boldsymbol{q}^B]_n)$

$$\boldsymbol{Q}_X = (\boldsymbol{q}^X{}_1, \boldsymbol{q}^X{}_2, \cdots, \boldsymbol{q}^X{}_{n-1})$$

momentum in D=3(n-1) dim.

$$T([\boldsymbol{q}^A]_n, [\boldsymbol{q}^B]_n) \equiv T(\boldsymbol{Q}_A, \boldsymbol{Q}_B) = \sum_{[L],[K]} T_{[L][K]}(Q_A, Q_B) Y_{[L]}(\Omega_{\boldsymbol{Q}_A}) \overline{Y_{[K]}(\Omega_{\boldsymbol{Q}_B})}$$

hyper-spherical harmonic function $\hat{L}^2 Y_{[L]}(\Omega_{\boldsymbol{s}}) = L(L+D-2) Y_{[L]}(\Omega_{\boldsymbol{s}})$

solution to the unitarity constraint with non-relativistic approximation

$$T_{[L][K]}(Q,Q) = \sum_{[N]} U_{[L][N]}(Q) T_{[N]}(Q) U^\dagger_{[N][K]}(Q),$$

$$T_{[L]}(Q) = -\frac{2n^{3/2}}{mQ^{3n-5}} e^{i\delta_{[L]}(Q)} \sin\delta_{[L]}(Q),$$ "phase shift" $\delta_{[L]}(Q)$

Lippmann-Schwinger equation in QFT

$$|\alpha\rangle_{\rm in} = |\alpha\rangle_0 + \int d\beta \frac{|\beta\rangle_0 T_{\beta\alpha}}{E_\alpha - E_\beta + i\varepsilon},$$

$$T_{\beta\alpha} = {}_0\langle\beta|V|\alpha\rangle_{\rm in},$$

off-shell

$${}_0\langle\beta|T|\alpha\rangle_0 = 2\pi\delta(E_\alpha - E_\beta) T_{\alpha\beta}.$$

on-shell   off-shell

$$(H_0 + V)|\alpha\rangle_{\rm in} = E_\alpha |\alpha\rangle_{\rm in},$$ full

$$H_0|\alpha\rangle_0 = E_\alpha|\alpha\rangle_0.$$ free

NBS wave functions

n-scalar fields with different flavors

$$\Psi_\alpha^n([\boldsymbol{x}]) = {}_{\rm in}\langle 0|\varphi^n([\boldsymbol{x}],0)|\alpha\rangle_{\rm in},$$

$$\varphi^n([\boldsymbol{x}],t) = T\{\prod_{i=1}^n \varphi_i(\boldsymbol{x}_i,t)\},$$

$$\Psi_\alpha^n([\boldsymbol{x}]) = \frac{1}{Z_\alpha}{}_0\langle 0|\varphi^n([\boldsymbol{x}],0)|\alpha\rangle_0 + \int d\beta \frac{1}{Z_\beta}\frac{{}_0\langle 0|\varphi^n([\boldsymbol{x}],0)|\beta\rangle_0 T_{\beta\alpha}}{E_\alpha - E_\beta + i\varepsilon}.$$

$${}_0\langle 0|\varphi^n([\boldsymbol{x}],0)|[\boldsymbol{k}]_n\rangle_0 = \left(\frac{1}{\sqrt{(2\pi)^3}}\right)^n \prod_{i=1}^n \frac{1}{\sqrt{2E_{k_i}}} e^{i\boldsymbol{k}_i\boldsymbol{x}_i}$$

D-dimensional hyper-coordinates

$$\Psi^n(\boldsymbol{R}, \boldsymbol{Q}_A) = C\left[e^{i\boldsymbol{Q}_A\cdot R} + \frac{2m}{2\pi n^{3/2}}\int d^D Q\, \frac{e^{i\boldsymbol{Q}\cdot R}}{Q_A^2 - Q^2 + i\varepsilon} T(\boldsymbol{Q}, \boldsymbol{Q}_A)\right]$$

Expansion in terms of hyper-spherical harmonic function

$$e^{i\boldsymbol{Q}\cdot\boldsymbol{R}} = (D-2)!!\frac{2\pi^{D/2}}{\Gamma(D/2)}\sum_{[L]} i^L\, \underline{j_L^D(QR)}\, Y_{[L]}(\Omega_{\boldsymbol{R}})\, \overline{Y_{[L]}(\Omega_{\boldsymbol{Q}})},$$

hyper-spherical Bessel function

$$\Psi^n(\boldsymbol{R}, \boldsymbol{Q}_A) = \sum_{[L],[K]} \Psi^n_{[L],[K]}(R, Q_A) Y_{[L]}(\Omega_{\boldsymbol{R}}) \overline{Y_{[K]}(\Omega_{\boldsymbol{Q}_A})},$$

Asymptotic behavior of NBS wave functions $R \to \infty$

$$\Psi^n_{[L],[K]}(R, Q_A) \simeq C i^L \frac{(2\pi)^{D/2}}{(Q_A R)^{\frac{D-1}{2}}} \sum_{[N]} U_{[L][N]}(Q_A) e^{i\delta_{[N]}(Q_A)} U^\dagger_{[N][K]}(Q_A)$$

$$\times \sqrt{\frac{2}{\pi}}\, \underline{\sin\left(Q_A R - \Delta_L + \delta_{[N]}(Q_A)\right)}$$

$$\Delta_L = \frac{2L_D - 1}{4}\pi.$$

scattering wave with “phase shift” !

Key Property 2

Sinya Aoki, et al., Phys. Rev. D87(2013)34512

Energy-independent potential above inelastic thresholds

Let us consider $NN \to NN, NN\pi$ energy $W \in \Delta_1$

$W_{\rm th}^2 = 2m_N + 2m_\pi$

$\Delta_1 = [W_{\rm th}^1, W_{\rm th}^2)$

$W_{\rm th}^1 = 2m_N + m_\pi$

$\Delta_0 = [W_{\rm th}^0, W_{\rm th}^1)$

$W_{\rm th}^0 = 2m_N$

2 operators × 2 states

$N(x)N(y)$ $\quad |NN, W, c_0\rangle$

$N(x)N(y)\pi(z)$ $\quad |NN+\pi, W, c_1\rangle$

other quantum numbers (c_0, c_1)

4 NBS wave functions

$$
\begin{aligned}
Z_N \varphi^{00}_{W,c_0}(\boldsymbol{x}_0) &= \langle 0|T\{N(\boldsymbol{x},0)N(\boldsymbol{x}+\boldsymbol{x}_0,0)\}|NN,W,c_0\rangle_{\rm in},\\
Z_N Z_\pi^{1/2} \varphi^{10}_{W,c_0}(\boldsymbol{x}_0,\boldsymbol{x}_1) &= \langle 0|T\{N(\boldsymbol{x},0)N(\boldsymbol{x}+\boldsymbol{x}_0,0)\pi(\boldsymbol{x}+\boldsymbol{x}_1,0)\}|NN,W,c_0\rangle_{\rm in},\\
Z_N \varphi^{01}_{W,c_1}(\boldsymbol{x}_0) &= \langle 0|T\{N(\boldsymbol{x},0)N(\boldsymbol{x}+\boldsymbol{x}_0,0)\}|NN+\pi,W,c_1\rangle_{\rm in},\\
Z_N Z_\pi^{1/2} \varphi^{11}_{W,c_1}(\boldsymbol{x}_0,\boldsymbol{x}_1) &= \langle 0|T\{N(\boldsymbol{x},0)N(\boldsymbol{x}+\boldsymbol{x}_0,0)\pi(\boldsymbol{x}+\boldsymbol{x}_1,0)\}|NN+\pi,W,c_1\rangle_{\rm in},
\end{aligned}
$$

$\varphi^{ij}_{W,c_j}([\mathbf{x}]_i)$ $\quad i(j)$: number of π's in the operator(state) $\quad [\boldsymbol{x}]_0 = \boldsymbol{x}_0 \quad [\boldsymbol{x}]_1 = \boldsymbol{x}_0, \boldsymbol{x}_1$

coupled channel equation

$$(E_W^k - H_0^k)\varphi_{W,c_i}^{ki} = \sum_{l=0,1} \int \prod_{n=0}^{l} d^3 y_n\, U^{kl}([\boldsymbol{x}]_k, [\boldsymbol{y}]_l)\varphi_{W,c_i}^{li}([\boldsymbol{y}]_l), \quad k, i \in (0,1)$$

non-local potential matrix

We can prove an existence of non-local potential matrix using non-relativistic approximation.

$$E_W^n = \frac{\boldsymbol{p}_1^2}{2m_N} + \frac{\boldsymbol{p}_2^2}{2m_N} + \sum_{i=1}^{n} \frac{\boldsymbol{k}_i^2}{2m_\pi}$$

kinetic energy

non-relativistic approx. for n=1

$$W = \sqrt{m_N^2 + \boldsymbol{p}_1^2} + \sqrt{m_N^2 + \boldsymbol{p}_2^2} + \sum_{i=1}^{n} \sqrt{m_\pi^2 + \boldsymbol{k}_i^2}$$

total energy

$\boldsymbol{p}_1 + \boldsymbol{p}_2 + \sum_{i=1}^{n} \boldsymbol{k}_i = 0.$

The construction of U can easily be generalized to $NN + n\pi \to NN + k\pi$

and to $\Lambda\Lambda \to \Lambda\Lambda, N\Xi, \Sigma\Sigma$

Non-local potential U describes all QCD processes.

QFT(QCD) at given energy ⟺ Quantum mechanics with energy-independent non-local coupled channel potentials for stable particles

$W_{\rm total}$		$N, \bar{N}, \pi, \cdots$
$\Delta, \rho, \cdots$	resonance	$N\pi, \pi\pi, \cdots$
deuteron, H,...	bound-state ?	$NN, \Lambda\Lambda, \cdots$ $D, H, \cdots$

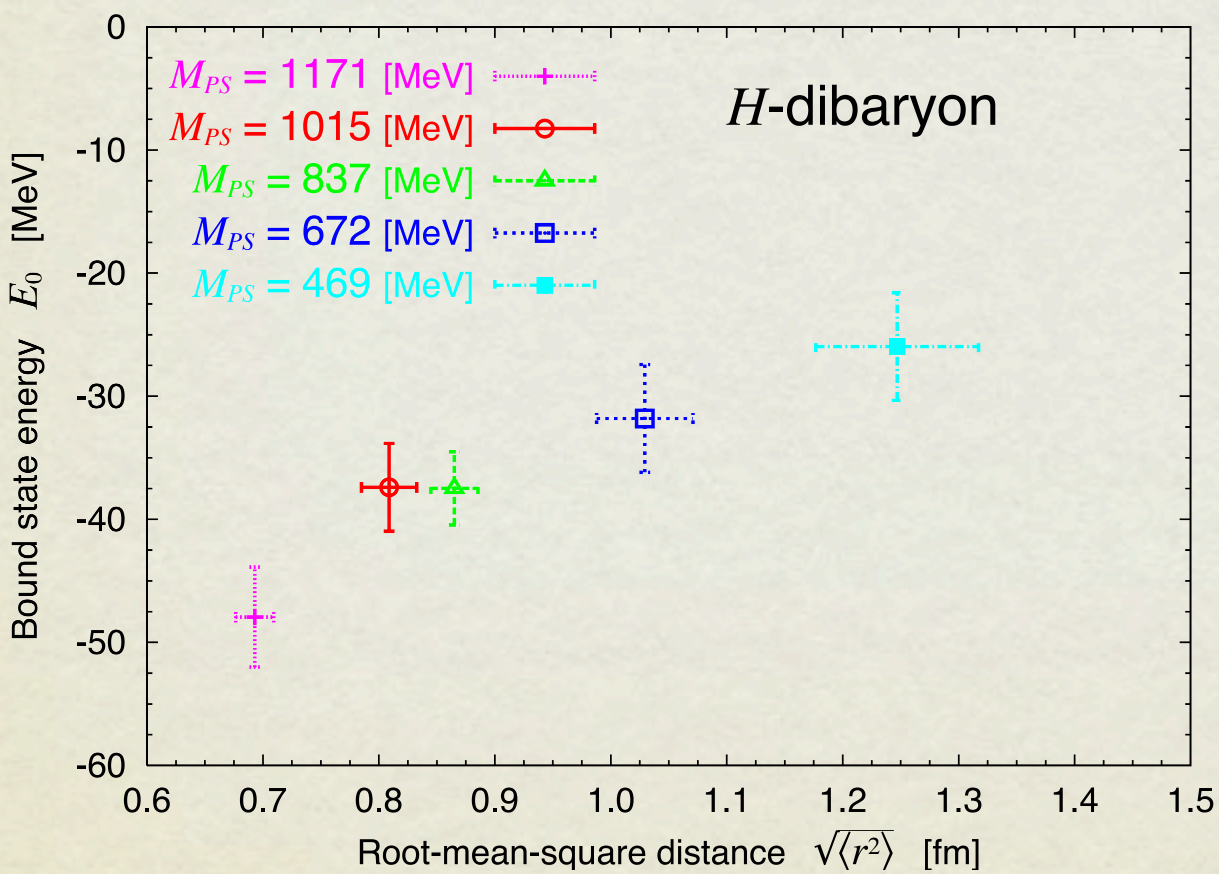
H-dibaryon with the flavor SU(3) breaking

SU(3) limit

$\Lambda\Lambda - N\Xi - \Sigma\Sigma$

25-50 MeV

H



Real world $m_u = m_d \neq m_s$

$\Sigma\Sigma$

2386 MeV

129 MeV

$N\Xi$

2257 MeV

H ?

25 MeV

$\Lambda\Lambda$

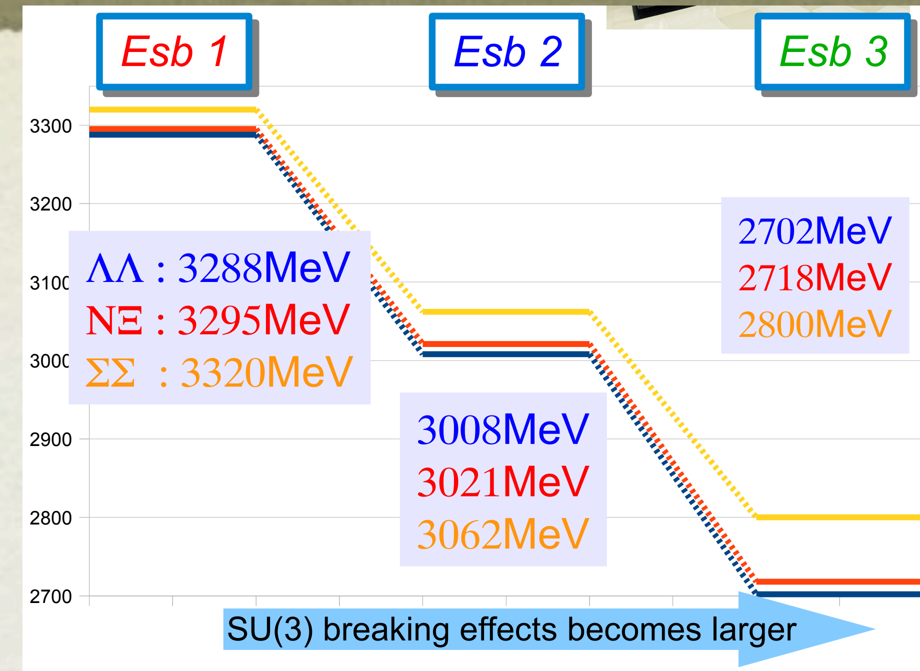
2232 MeV

H ?

In unit of MeV	Esb 1	Esb 2	Esb 3
π	701±1	570±2	411±2
K	789±1	713±2	635±2
m_π/m_K	0.89	0.80	0.65
N	1585±5	1411±12	1215±12
Λ	1644±5	1504±10	1351± 8
Σ	1660±4	1531±11	1400±10
Ξ	1710±5	1610± 9	1503± 7

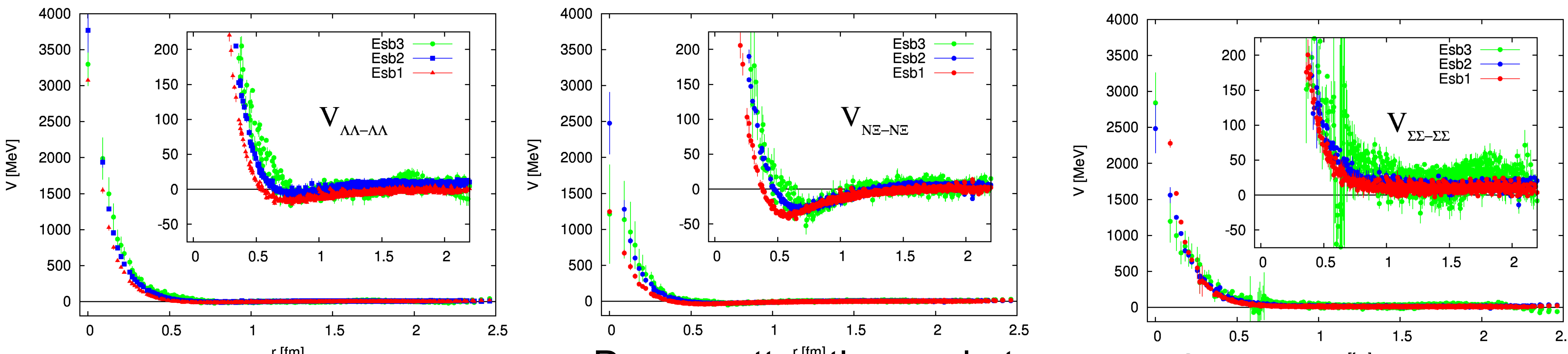
u,d quark masses lighter

Gauge ensembles



coupled channel 3x3 potentials

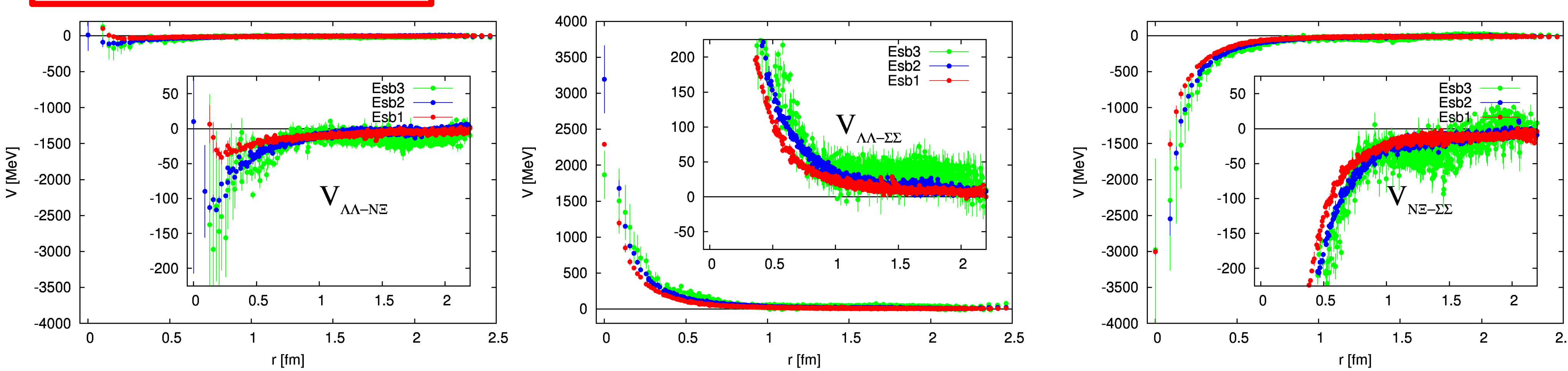
Diagonal elements



shallow attractive pocket — Deeper attractive pocket — Strongly repulsive

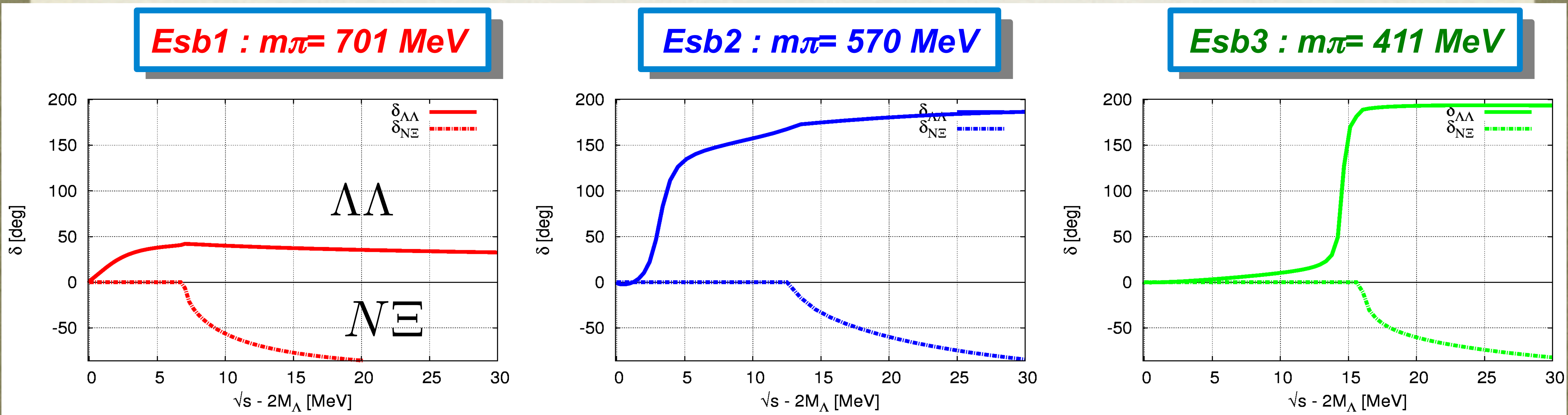
All channels have repulsive core

Off-diagonal elements



$\Lambda\Lambda$ and $N\Xi$ phase shift

Preliminary !



Bound H-dibaryon

Resonance H

Resonance H

This suggests H-dibaryon becomes resonance at physical point.

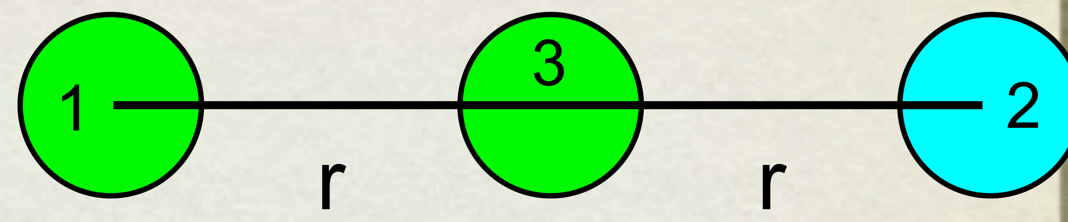
Three nucleon force (TNF)

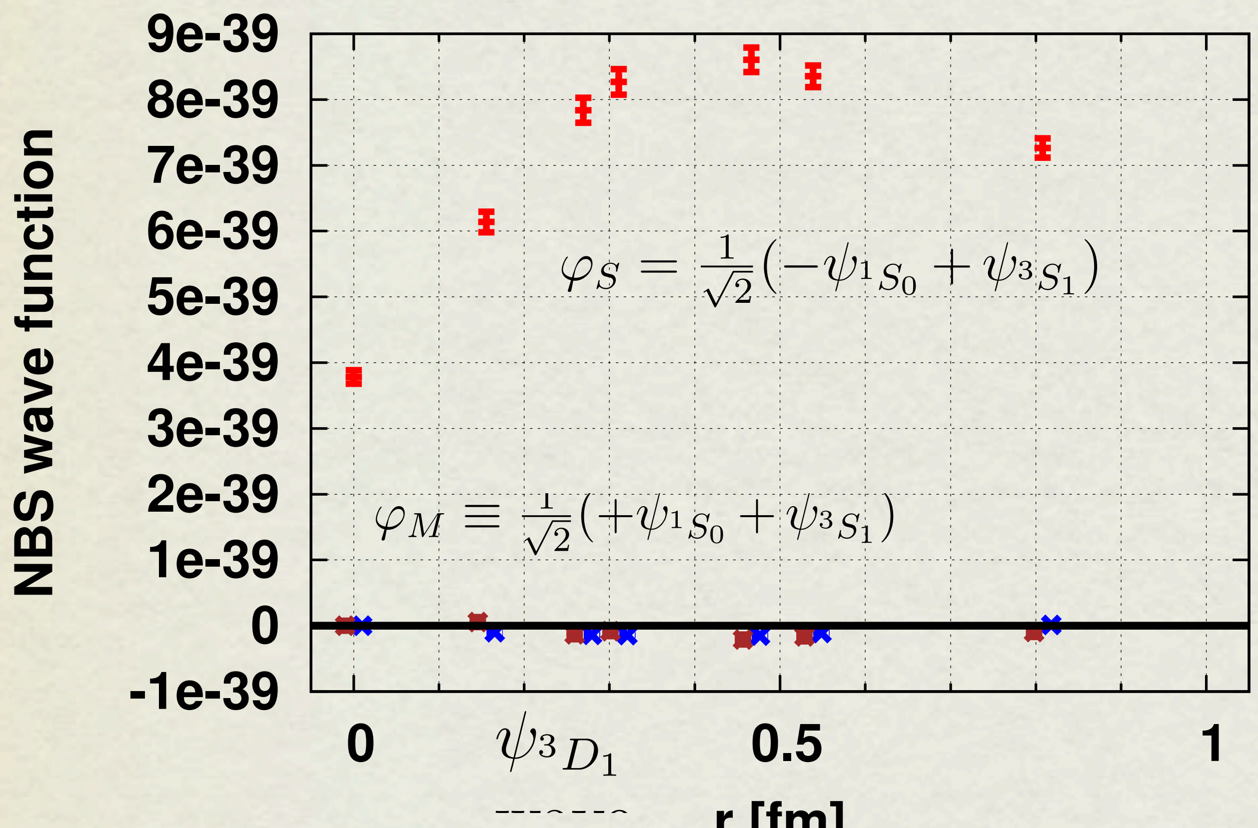
Doi et al. (HAL QCD), PTP 127 (2012) 723

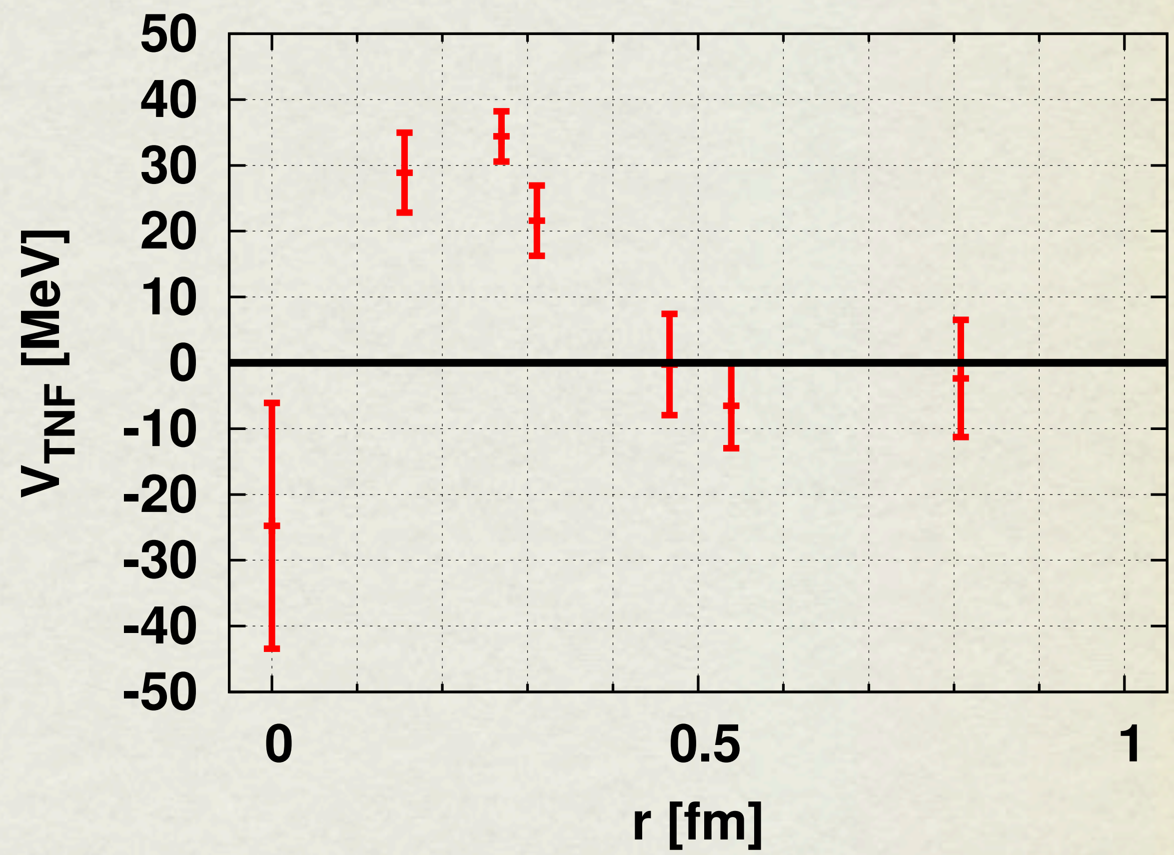
(1,2) pair $\quad {}^1S_0,\ {}^3S_1,\ {}^3D_1 \quad$ S-wave only

$$\mathrm{Triton}(I = 1/2,\ J^P = 1/2^+)$$

Linear setup







scalar/isoscalar TNF is observed at short distance.

further study is needed to confirm this result.

6. CONCLUSION

- the potential method (HAL QCD method) is new but very useful to investigate not only the nuclear force but also general baryonic interactions in (lattice) QCD.
- some understanding of repulsive cores
- the method can be easily applied also to meson-baryon and meson-meson interactions.

Our strategy

Potentials from lattice QCD → Nuclear Physics with these potentials → Neutron stars Supernova explosion