Desert and the Naturalness

August 5 2013

Hikaru Kawai

(Kyoto Univ.)

Y. Hamada, K. Oda and HK: arXiv:1210.2358 (PRD), 1305.7055

HK: Int. J. of Mod. Phys. A vol. 28, nos. 3 & 4 (2013) 1340001

PART 1 Desert

LHC gave beautiful results

But in some sense, they indicate “the worst scenario”.

Higgs particle was discovered, but nothing else.

Especially, no signal of the SUSY.

$\Downarrow$

We need to reconsider the origin of the fine tuning.

The naturalness problem

Suppose the underlying fundamental theory, such as string theory, has the momentum scale m_s and the coupling constant g_s .

Then, by dimensional analysis and the power counting of the couplings, the parameters of the low energy effective theory are given as follows:

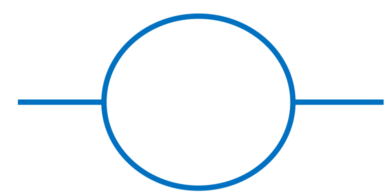
naturalness problem (cont.'d)

dimension -2 (Newton constant) $G_N \sim \dfrac{g_S{}^2}{m_S{}^2}.$

dimension 0 $g_1, g_2, g_3 \sim g_S,$

(gauge and Higgs couplings) $\lambda_H \sim g_S{}^2.$

dimension 2 (Higgs mass)

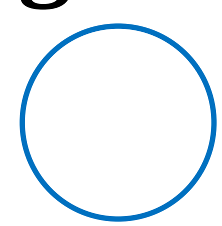


$m_H{}^2 \sim \overset{\text{tree}}{0} + g_S{}^2 m_S{}^2.$

unnatural ! $\rightarrow$ $m_H{}^2 \sim (100\,\text{GeV})^2 \ll g_S{}^2 m_S{}^2 \sim (10^{18}\,\text{GeV})^2$

dimension 4

(vacuum energy or cosmological constant)

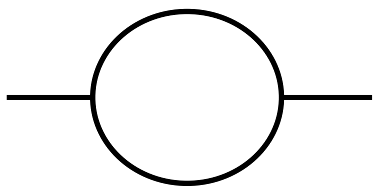
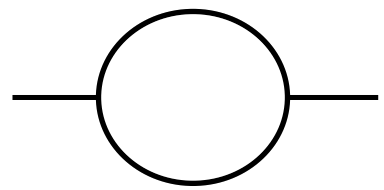


$\lambda \sim 0 \cdot g_s{}^{-2} + m_S{}^4.$

unnatural ! ! $\rightarrow$ $\lambda \sim (2 \sim 3\,\text{meV})^4 \ll m_S{}^4 \sim (10^{18}\,\text{GeV})^4$

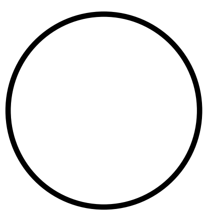
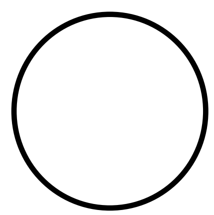
SUSY as a solution to the naturalness problem

Bosons and fermions cancel the UV divergences:

 +  $\Rightarrow \; m_H{}^2 = 0.$

$\Rightarrow \; m_H{}^2 \sim M_{\mathrm{SUSY}}{}^2.$

bosons **fermions**

 +  $\Rightarrow \; \lambda = 0.$

$\Rightarrow \; \lambda \sim M_{\mathrm{SUSY}}{}^4.$

However, SUSY must be spontaneously broken at some momentum scale M_{SUSY}, below which the cancellation does not work.

(cont'd)

Therefore, if M_{SUSY} is close to m_H , the Higgs mass is naturally understood, although the cosmological constant is still a big problem.

However, no signal of new particles is observed in the LHC below 1 TeV.

We have to think about other possibilities.

Possible explanation to the naturalness problem other than SUSY

1. We do not have to mind. We should simply take the parameters as they are.

2. Anthropic principle.

   The parameters should be such that we can exist.

   a) In some model, the wave function of the universe is a superposition of various worlds each of which has different low energy effective Lagrangians:

   $$\left|\Psi\right\rangle = \left|\Psi_1\right\rangle + \left|\Psi_2\right\rangle + \left|\Psi_3\right\rangle + \cdots.$$

   We are sitting in one of them. The parameters there must be such that we exist.

Anthropic principle. (cont.'d)

b) The universe has different parameters place by place. We are sitting at one place, where the parameters are such that we can exist.

3. The parameters are fixed by some non-perturbative effect of quantum gravity/string theory such as Coleman's baby universe mechanism.

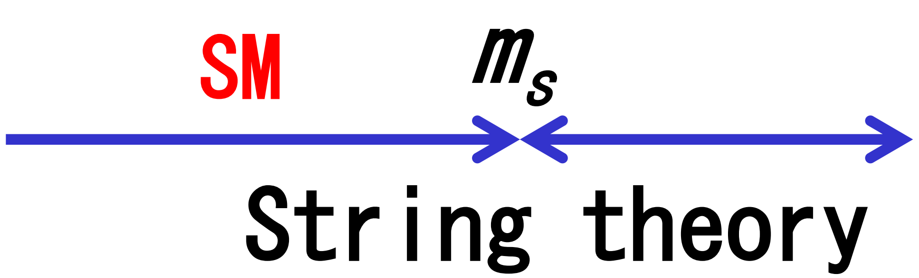
Although we do not understand the real reason, nature chooses the parameters as we observe.

Possibility of desert

It may not be right to doubt the SM in the high energy region by the reason that it is not natural.

The right attitude would be to examine simply whether the SM is valid to the string scale or some new physics is needed below the scale .

If it is the former case, there is a possibility for the desert, that is, we have only the SM below the string scale.



Can the SM valid to the Planck/string scale?

In order to answer the question, we consider the SM Lagrangian with cutoff momentum *Λ*,

$$\mathcal{L} = (D_\mu \phi_B)^\dagger (D^\mu \phi_B) - m_B^2 \phi_B^\dagger \phi_B - \lambda_B (\phi_B^\dagger \phi_B)^2 + \cdots .$$

and estimate its bare parameters in such a way that the observed low energy parameters are recovered.

If no inconsistency arises, it means that the SM can be valid to the energy scale *Λ*.

The bare coupling λ_B

As usual, the bare couplings can be approximated by the running couplings at Λ in a mass independent scheme such as MS bar.
The error can be evaluated once the cutoff scheme is specified, and is expected as small as the two-loop corrections.

$$\lambda^i{}_B = \lambda^i{}_{\overline{MS}}(\Lambda) + \sum_{j,k} b^{ijk} \lambda^j{}_{\overline{MS}}(\Lambda) \lambda^k{}_{\overline{MS}}(\Lambda)$$

$\lambda^i{}_B$: dimensionless couplings

(gauge, Yukawa, Higg sself couplings)

We can approximate $\lambda^i{}_B \simeq \lambda^i{}_{\overline{MS}}(\Lambda).$

The bare mass m_B^2

- **In general, the bare mass consists of quadratically divergent part and logarithmically divergent part:**

$$m_B{}^2 = a\Lambda^2 + m_{phys}{}^2\left(b_1 \log\left(\frac{\Lambda^2}{m_{phys}{}^2}\right) + \cdots\right).$$

- **Here we consider only the first part, or we simply assume**

$$m_{phys}{}^2 = 0.$$

- $m_B{}^2$ **is determined by an order by order perturbative calculation in the *bare* couplings demanding** $m_{phys}{}^2 = 0$:

$$m_B^2 = m_{B,\,\text{0-loop}}^2 + m_{B,\,\text{1-loop}}^2 + m_{B,\,\text{2-loop}}^2 + \cdots$$

Simple Φ^4 theory

No IR divergences

$$J_2 := \int \frac{d^4p_E}{(2\pi)^4}\frac{d^4q_E}{(2\pi)^4}\frac{1}{p_E^4 q_E^2} \propto \Lambda^2 \ln(\Lambda/\mu_{\rm IR})$$

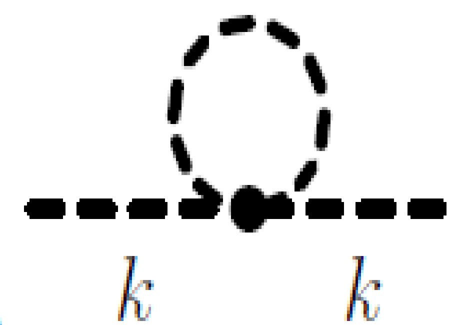
$$I_2 := \int \frac{d^4p_E}{(2\pi)^4}\frac{d^4q_E}{(2\pi)^4}\frac{1}{p_E^2 q_E^2 (p_E+q_E)^2} \propto \Lambda^2$$

$$I_1 := \int \frac{d^4p_E}{(2\pi)^4}\frac{1}{p_E^2} \propto \Lambda^2$$

$m_{phys}{}^2$

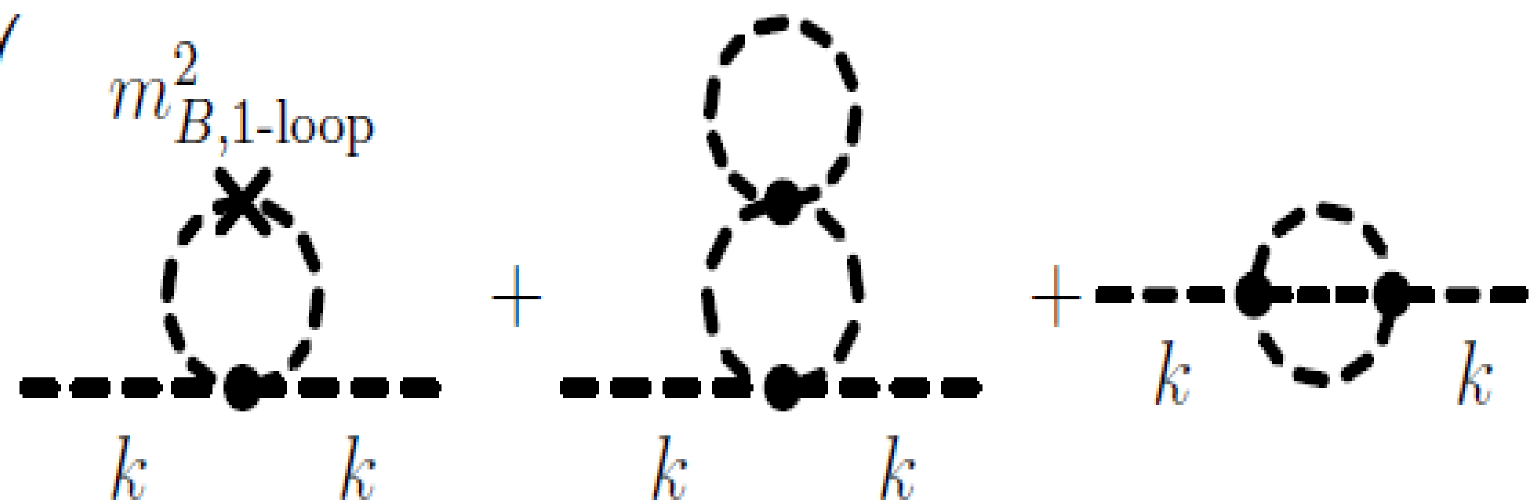
$$m^2_{B,\,0\text{-loop}} = 0$$

$m^2_{B,\,1\text{-loop}} + i\,\Big($



$\Big)\Big|_{k=0} = 0$

$m^2_{B,\,2\text{-loop}} + i\,\Big($



$\Big)\Big|_{k=0} = 0$

$$m^2_{B,\,\text{1-loop}} = -\frac{\lambda_B}{2} I_1$$

$$m^2_{B,\,\text{2-loop}} = -\frac{5}{72}\lambda_B^2 I_2$$

$$I_1 := \int \frac{d^4p}{(2\pi)^4}\frac{1}{p^2} \propto \Lambda^2$$

$$I_2 := \int \frac{d^4p}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}\frac{1}{p^2q^2(p+q)^2} \propto \Lambda^2$$

Ratio between I_1 and I_2

The ratio depends on the regularization, but its dependence is within a factor of 2~3.

If we introduce the proper time regularization

$$\int d^4k \frac{1}{k^2} = \int_\varepsilon^\infty d\alpha \int d^4k\, e^{-\alpha k^2}$$ **, we have**

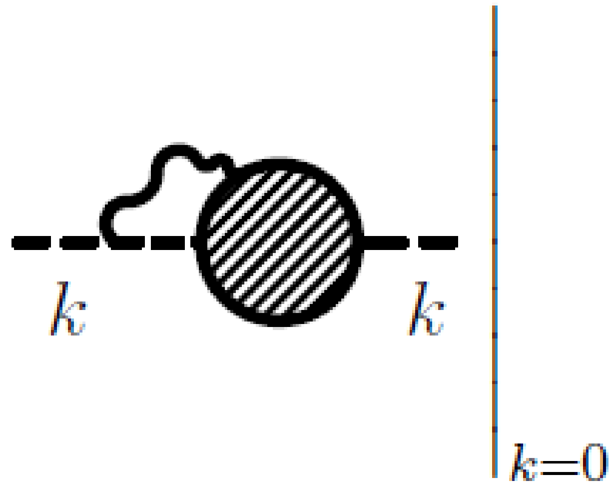
$$I_1 = \frac{1}{\varepsilon}\frac{1}{16\pi^2}, \quad I_2 = \frac{1}{\varepsilon}\frac{1}{(16\pi^2)^2}\ln\frac{2^6}{3^3} \simeq 0.005\, I_1 .$$

If we employ the momentum cutoff Λ, we have $I_1 = \dfrac{\Lambda^2}{16\pi^2}$ **, which indicates** $1/\varepsilon = \Lambda^2$ **.**

SM calculation

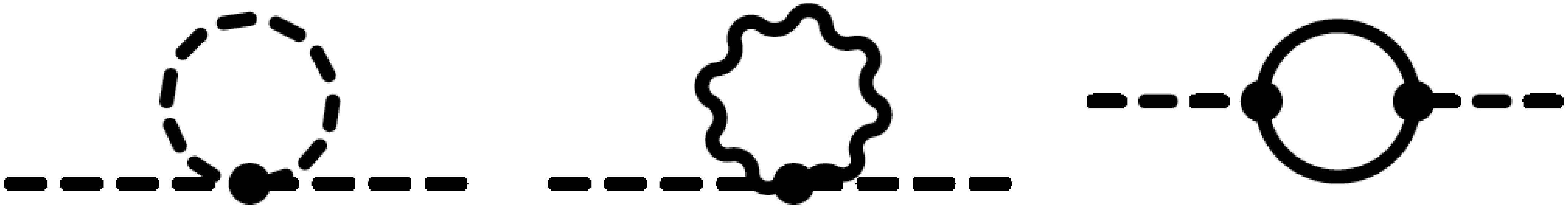
$$\mathcal{L} = (D_\mu \phi_B)^\dagger (D^\mu \phi_B) - m_B^2 \phi_B^\dagger \phi_B - \lambda_B (\phi_B^\dagger \phi_B)^2 + \cdots .$$

The calculation is simplified, if we consider the symmetric phase $\langle \varphi \rangle = 0$, and calculate in the Landau gauge:



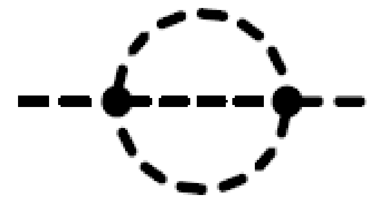
$= 0$

SM 1-loop

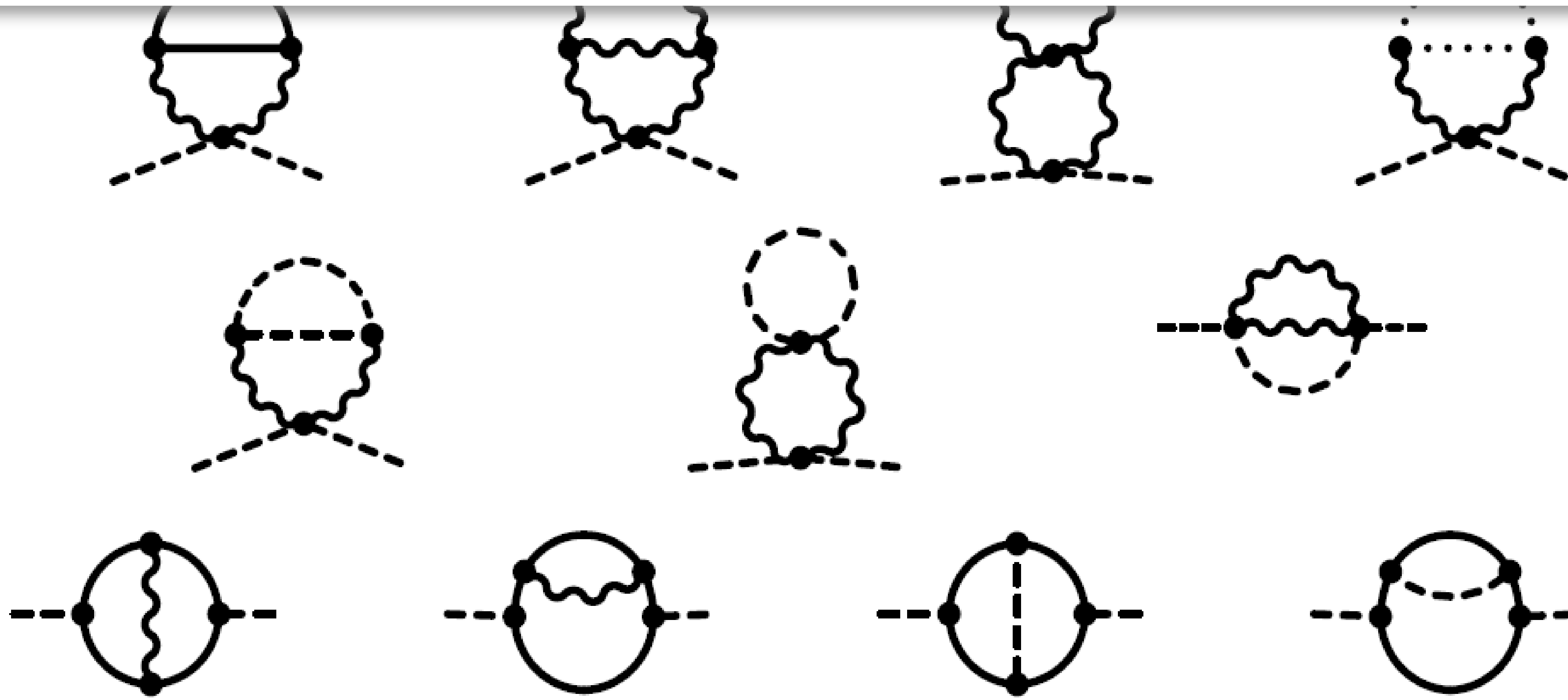


$$m^2_{B,\,\text{1-loop}} = -\left(6\lambda_B + \frac{3}{4}g^2_{Y\,B} + \frac{9}{4}g^2_{2B} - 6y^2_{tB}\right) I_1$$

SM 2-loop



$$m^2_{B,\,2\text{-loop}} = -\left\{9y_{tB}^4 + y_{tB}^2\left(-\frac{7}{12}g_{YB}^2 + \frac{9}{4}g_{2B}^2 - 16g_{3B}^2\right) + \frac{77}{16}g_{YB}^4 + \frac{243}{16}g_{2B}^4 \right.$$
$$\left. + \lambda_B\left(-18y_{tB}^2 + 3g_{YB}^2 + 9g_{2B}^2\right) - 10\lambda_B^2\right\} I_2.$$



Regularization dependence is small

$$I_2 = \frac{1}{\varepsilon} \frac{1}{(16\pi^2)^2} \ln \frac{2^6}{3^3} \simeq 0.005\, I_1$$

- **The ratio is regularization dependent, but it is about the order of 1/200.**

- **It turns out that 2-loop contribution is small in the case of the SM.**

Renormalization group equation

$$
\begin{aligned}
\frac{dg_Y}{dt} &= \frac{1}{16\pi^2}\frac{41}{6}g_Y^3 + \frac{g_Y^3}{(16\pi^2)^2}\left(\frac{199}{18}g_Y^2 + \frac{9}{2}g_2^2 + \frac{44}{3}g_3^2 - \frac{17}{6}y_t^2\right),\\
\frac{dg_2}{dt} &= -\frac{1}{16\pi^2}\frac{19}{6}g_2^3 + \frac{g_2^3}{(16\pi^2)^2}\left(\frac{3}{2}g_Y^2 + \frac{35}{6}g_2^2 + 12g_3^2 - \frac{3}{2}y_t^2\right),\\
\frac{dg_3}{dt} &= -\frac{7}{16\pi^2}g_3^3 + \frac{g_3^3}{(16\pi^2)^2}\left(\frac{11}{6}g_Y^2 + \frac{9}{2}g_2^2 - 26g_3^2 - 2y_t^2\right),\\
\frac{dy_t}{dt} &= \frac{y_t}{16\pi^2}\left(\frac{9}{2}y_t^2 - \frac{17}{12}g_Y^2 - \frac{9}{4}g_2^2 - 8g_3^2\right) + \frac{y_t}{(16\pi^2)^2}\Big(-12y_t^2 + 6\lambda^2 - 12\lambda y_t^2\\
&\quad + \frac{131}{16}g_Y^2 y_t^2 + \frac{225}{16}g_2^2 y_t^2 + 36g_3^2 y_t^2 + \frac{1187}{216}g_Y^4 - \frac{23}{4}g_2^4 - 108g_3^4 - \frac{3}{4}g_Y^2 g_2^2 + 9g_2^2 g_3^2 + \frac{19}{9}g_3^2 g_Y^2\Big),\\
\frac{d\lambda}{dt} &= \frac{1}{16\pi^2}\left(24\lambda^2 - 3g_Y^2\lambda - 9g_2^2\lambda + \frac{3}{8}g_Y^4 + \frac{3}{4}g_Y^2 g_2^2 + \frac{9}{8}g_2^4 + 12\lambda y_t^2 - 6y_t^4\right)\\
&\quad + \frac{1}{(16\pi^2)^2}\Big\{ -312\lambda^3 + 36\lambda^2(g_Y^2 + 3g_2^2) - \lambda\left(\frac{629}{24}g_Y^4 - \frac{39}{4}g_Y^2 g_2^2 + \frac{73}{8}g_2^4\right)\\
&\qquad + \frac{305}{16}g_2^6 - \frac{289}{48}g_Y^2 g_2^4 - \frac{559}{48}g_Y^4 g_2^2 - \frac{379}{48}g_Y^6 - 32g_3^2 y_t^4 - \frac{8}{3}g_Y^2 y_t^4 - \frac{9}{4}g_2^4 y_t^2\\
&\qquad + \lambda y_t^2\left(\frac{85}{6}g_Y^2 + \frac{45}{2}g_2^2 + 80g_3^2\right) + g_Y^2 y_t^2\left(-\frac{19}{4}g_Y^2 + \frac{21}{2}g_2^2\right)\\
&\qquad - 144\lambda^2 y_t^2 - 3\lambda y_t^4 + 30y_t^6\Big\},
\end{aligned}
$$

Initial values

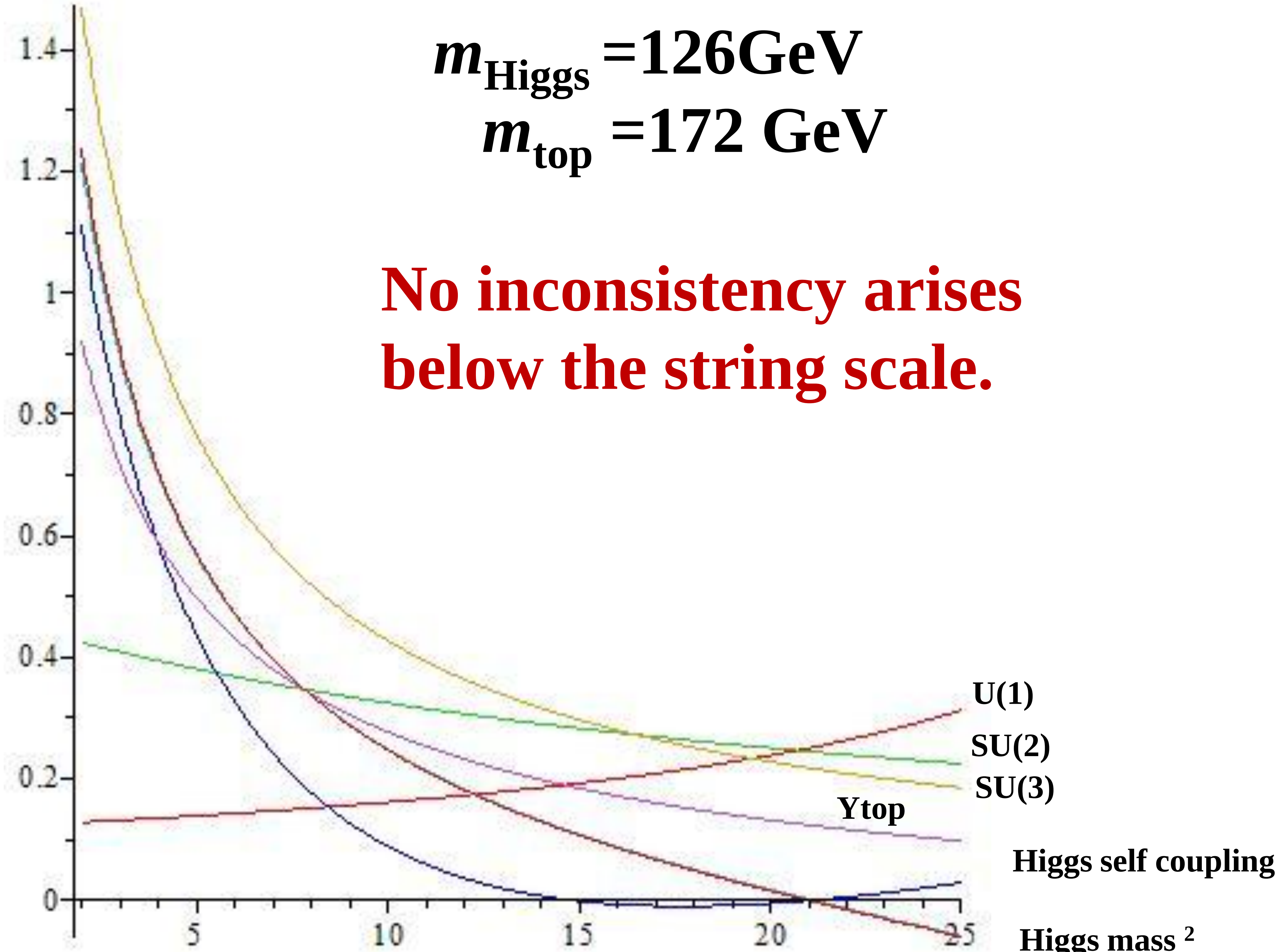
$$g_s(m_t^{\rm pole}) = 1.1645 + 0.0031\left(\frac{\alpha_s(m_Z) - 0.1184}{0.0007}\right) - 0.00046\left(\frac{m_t^{\rm pole}}{\rm GeV} - 173.15\right),$$

$$\lambda(m_t^{\rm pole}) = 0.12577 + 0.00205\left(\frac{m_H}{\rm GeV} - 125\right) - 0.00004\left(\frac{m_t^{\rm pole}}{\rm GeV} - 173.15\right) \pm 0.00140_{\rm th},$$

$$y_t(m_t^{\rm pole}) = 0.93587 + 0.00557\left(\frac{m_t^{\rm pole}}{\rm GeV} - 173.15\right) - 0.00003\left(\frac{m_H}{\rm GeV} - 125\right) - 0.00041\left(\frac{\alpha_s(m_Z) - 0.1184}{0.0007}\right) \pm 0.00200_{\rm th}.$$

G. Degrassi, S. Di Vita, J. Elias-Miro, J. R. Espinosa, G. F. Giudice, G. Isidori and A. Strumia,
Higgs mass and vacuum stability in the Standard Model at NNLO," JHEP 1208 (2012) 098 [arXiv:1205.6497 [hep-ph]].

Bare parameters of the cutoff theory (1)



m_{Higgs} =126GeV

m_{top} =172 GeV

No inconsistency arises below the string scale.

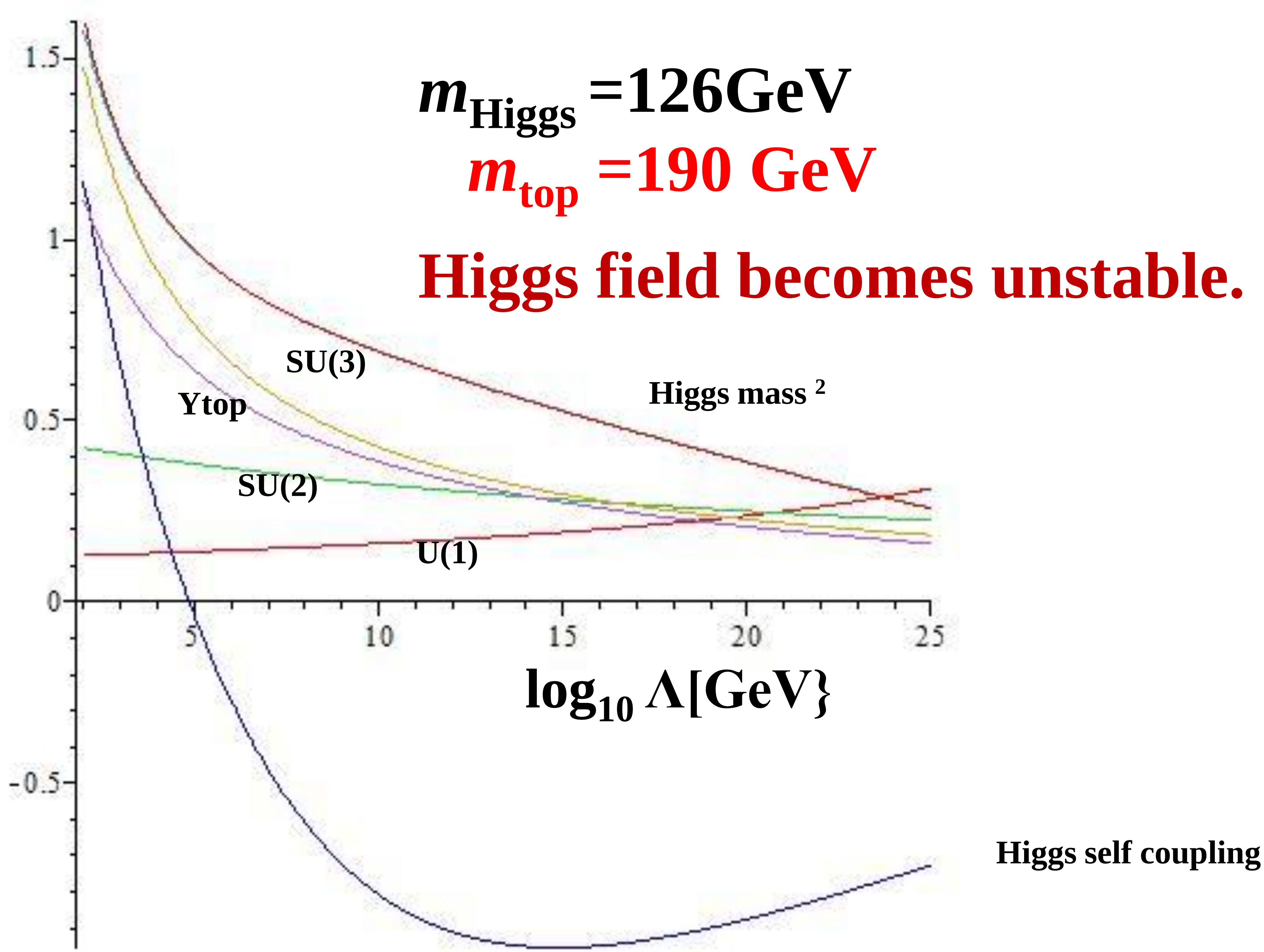
$\log_{10} \Lambda$[GeV}

Bare parameters of the cutoff theory (2)

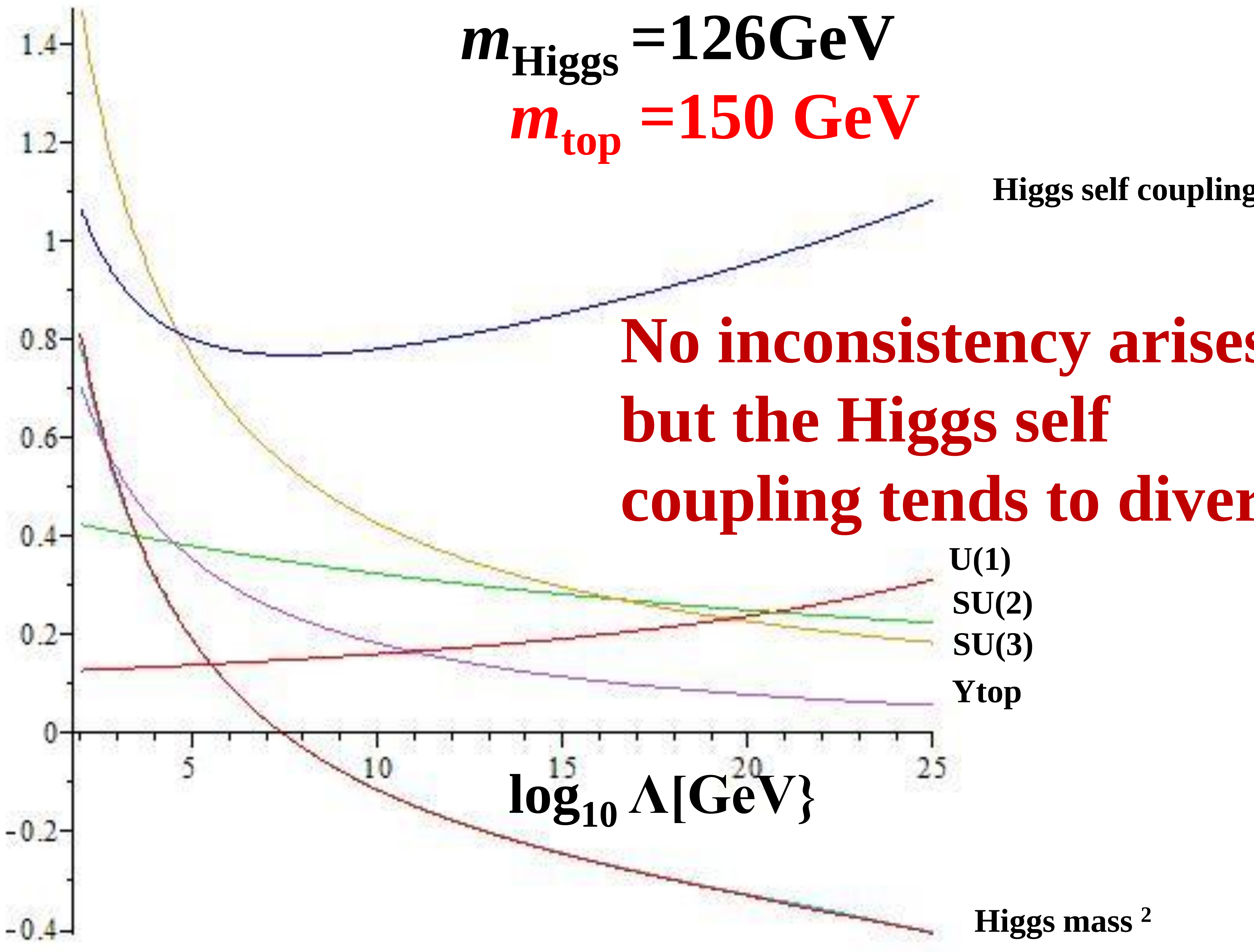
m_{Higgs} =126GeV

m_{top} =190 GeV

Higgs field becomes unstable.



Bare parameters of the cutoff theory (3)



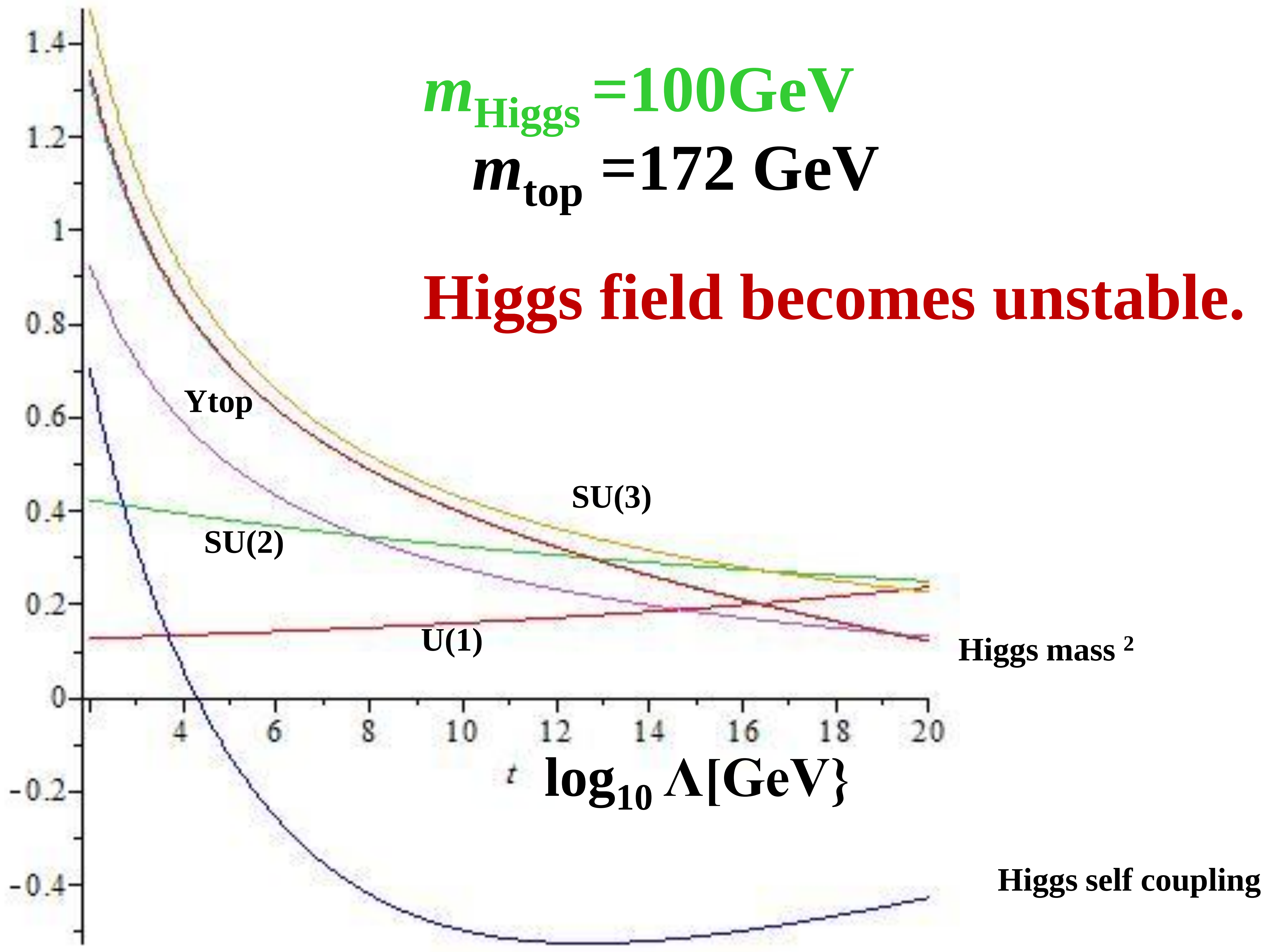
No inconsistency arises, but the Higgs self coupling tends to diverge.

Bare parameters of the cutoff theory (4)

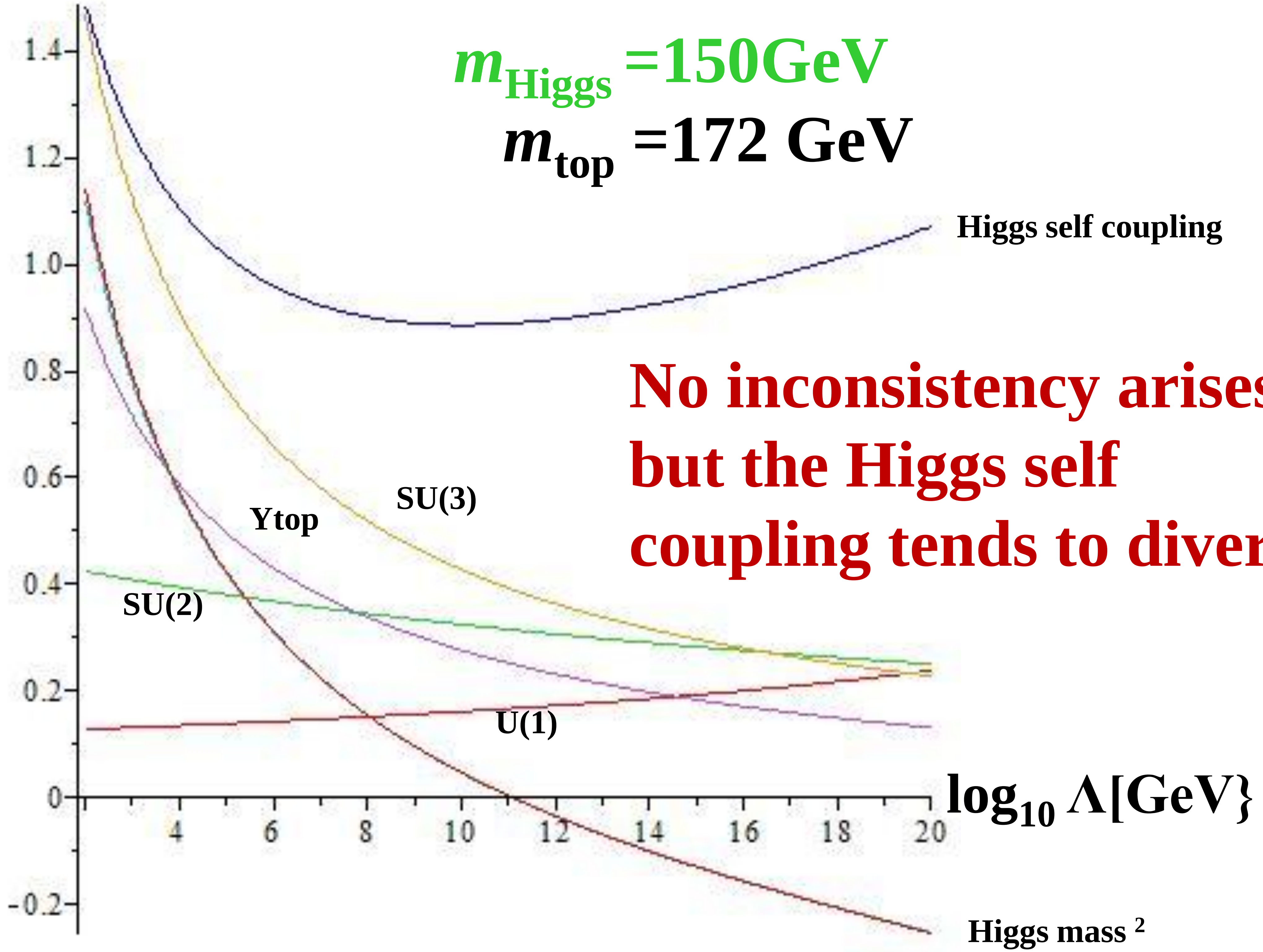
m_{Higgs} =100GeV

m_{top} =172 GeV

Higgs field becomes unstable.

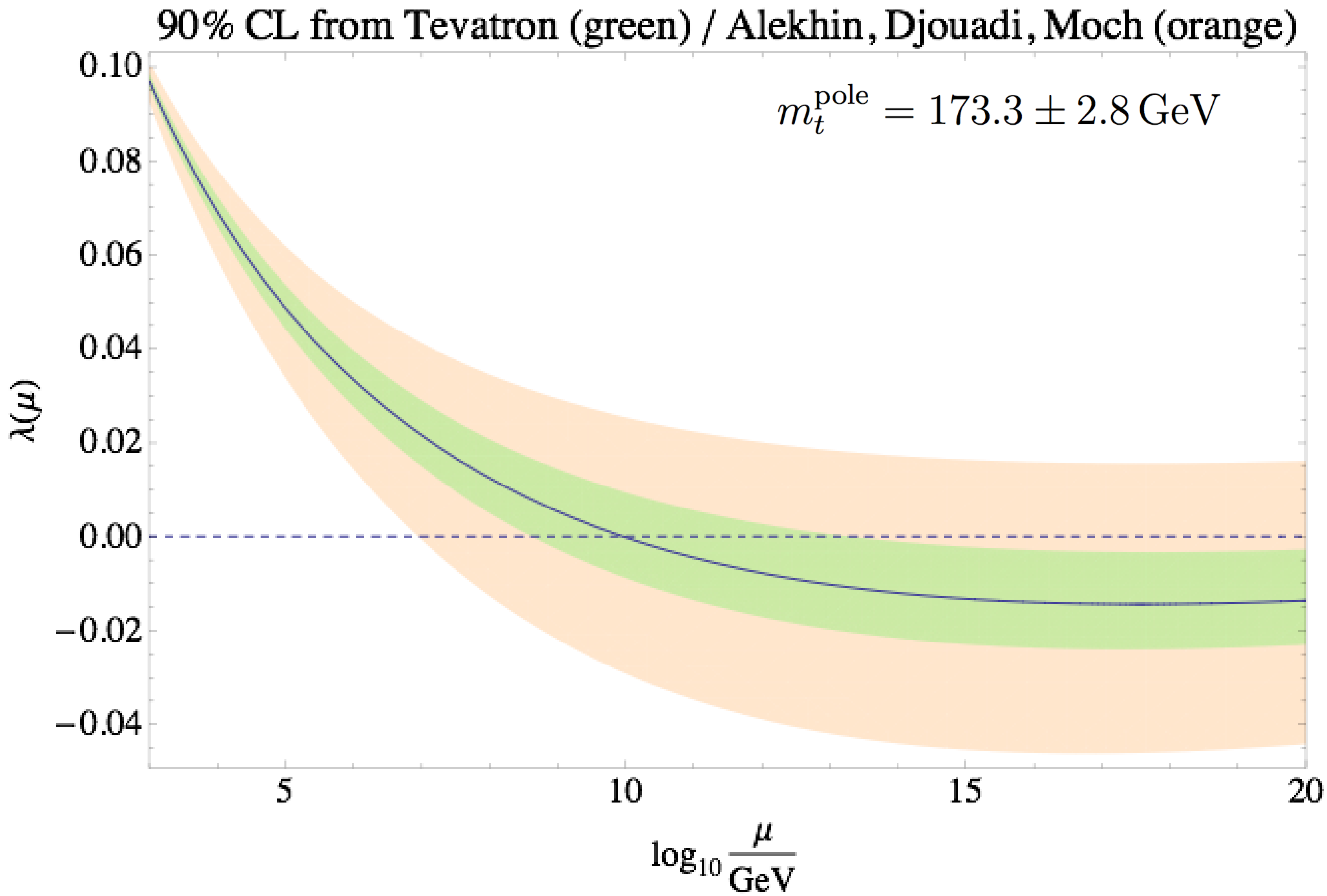


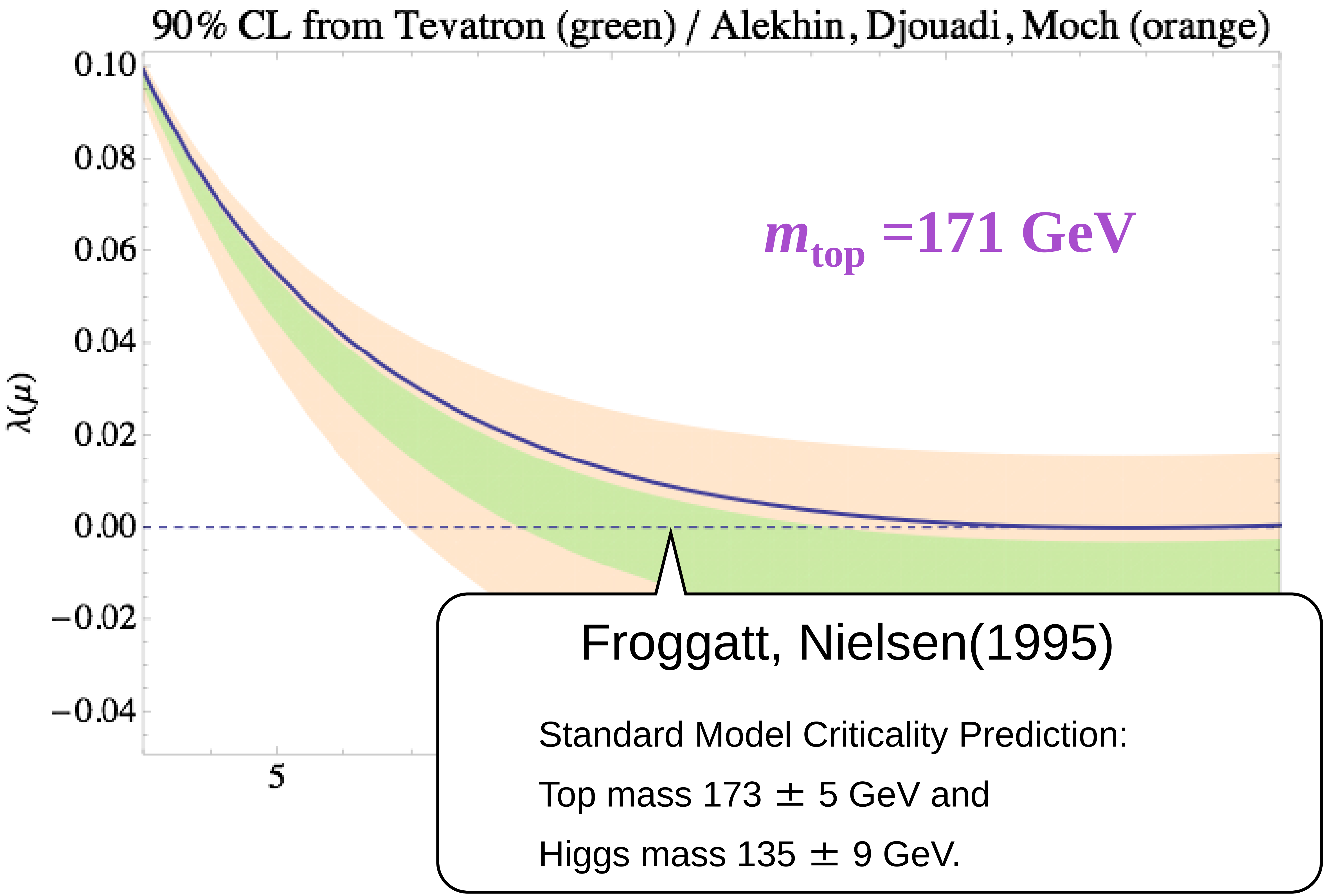
Bare parameters of the cutoff theory (5)



No inconsistency arises, but the Higgs self coupling tends to diverge.

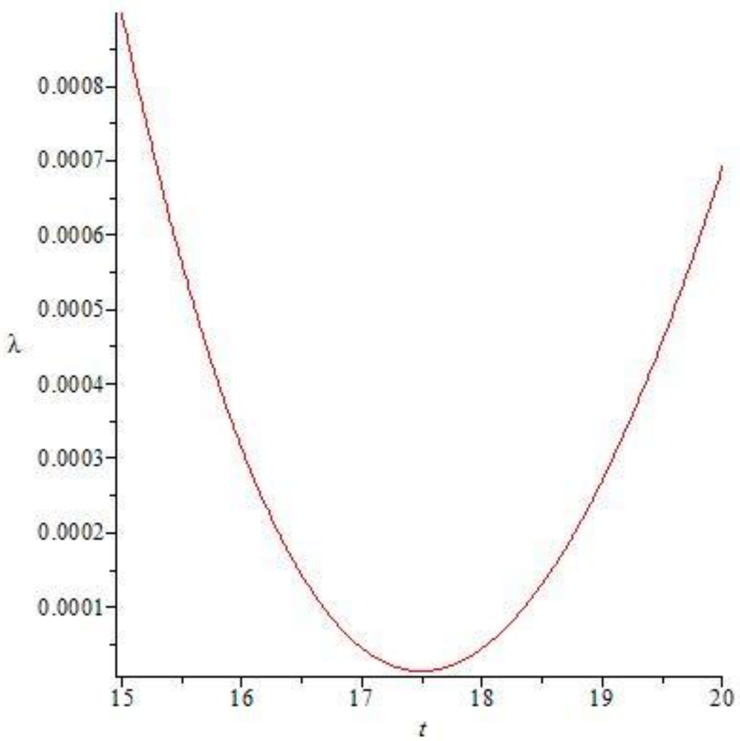
Stability of the potential

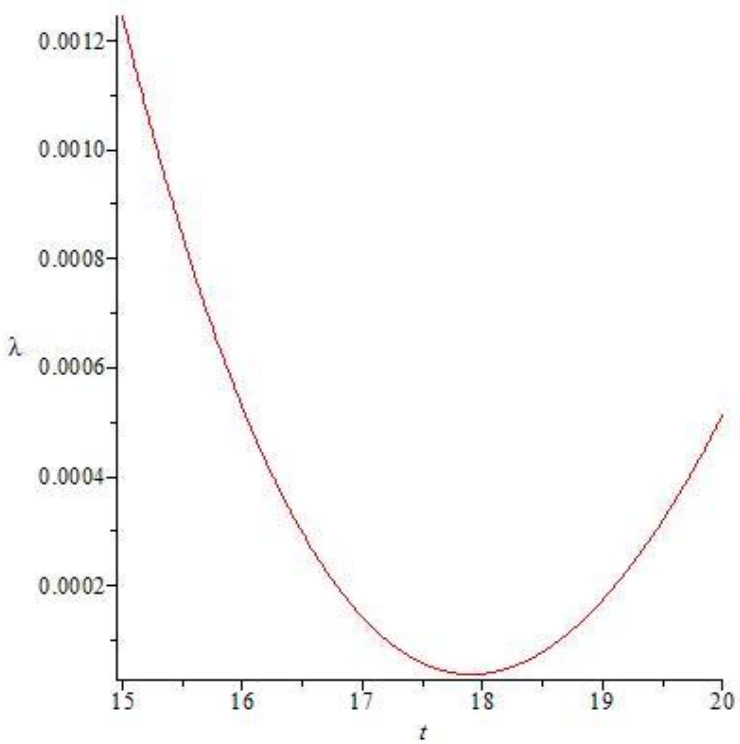


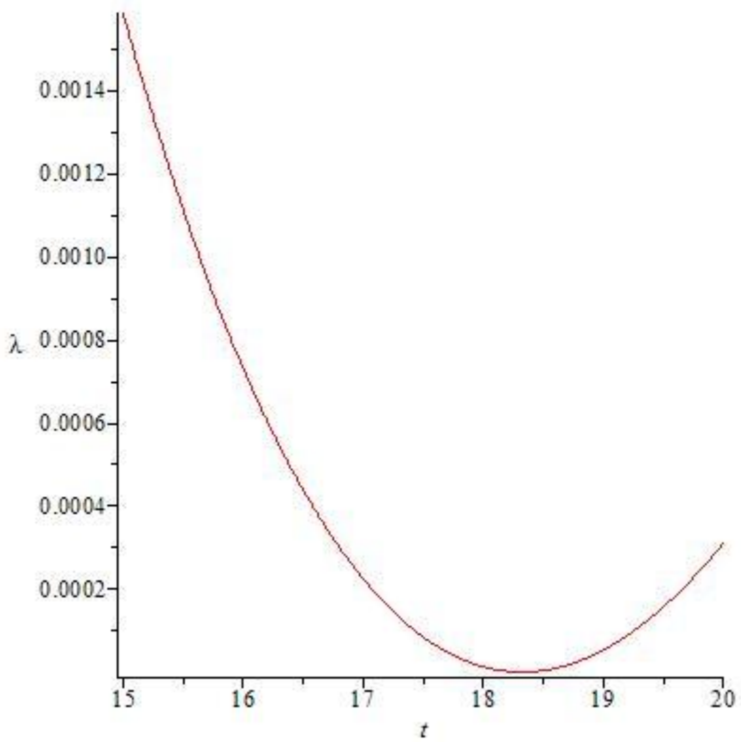


90% CL from Tevatron (green) / Alekhin, Djouadi, Moch (orange)
0.10
0.08
0.06
0.04
0.02
0.00
−0.02
−0.04
$\lambda(\mu)$
5
m_{top} =171 GeV
Froggatt, Nielsen(1995)
Standard Model Criticality Prediction:
Top mass 173 ± 5 GeV and
Higgs mass 135 ± 9 GeV.

Froggatt Nielsen by the recent values







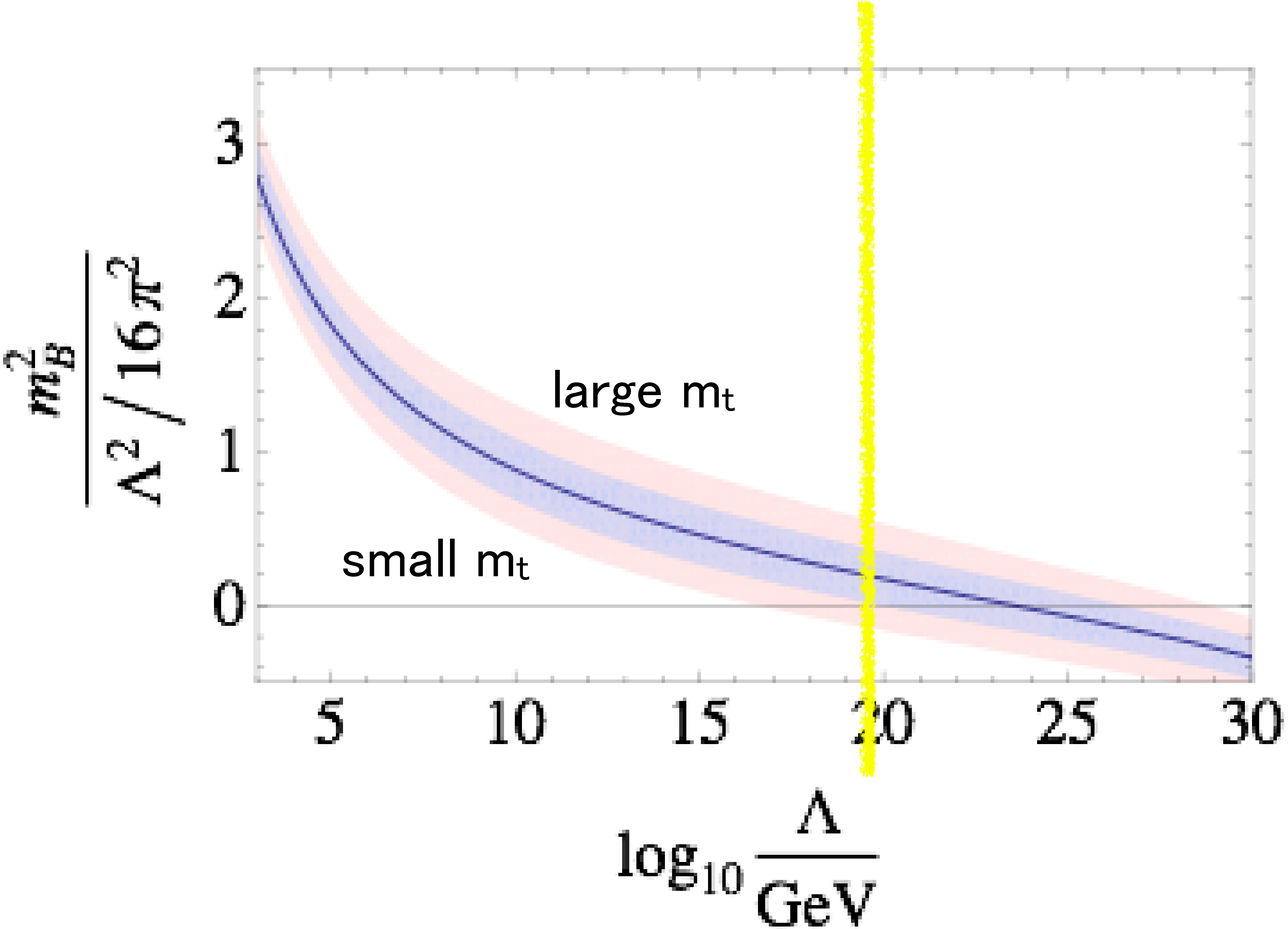
m_{Higgs} =125GeV
m_{top} =171.31 GeV

m_{Higgs} =127GeV
m_{top} =172.29 GeV

m_{Higgs} =129GeV
m_{top} =173.26 GeV

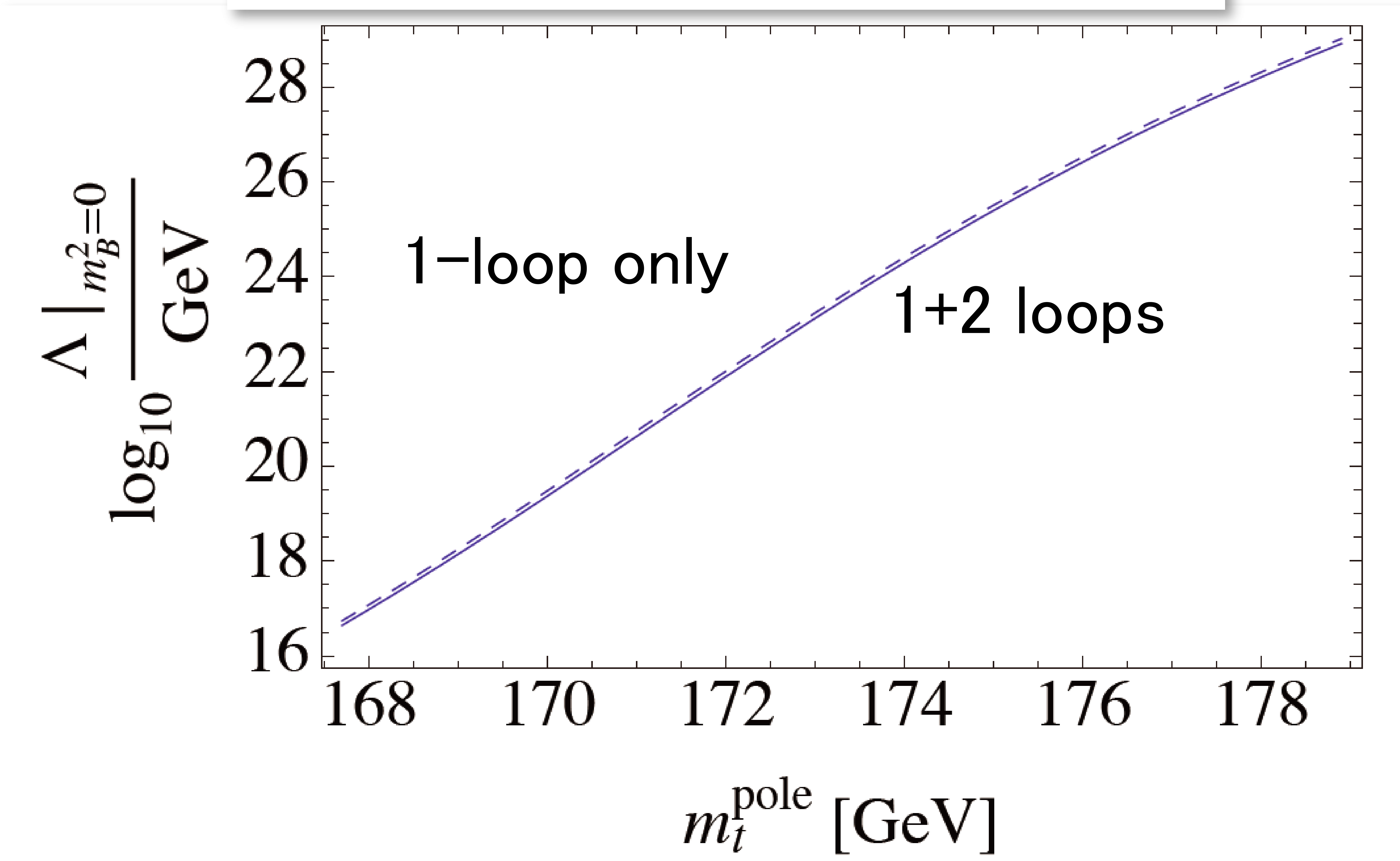
Cut off dependence of the bare mass

$$m_t^{\rm pole} = 173.3 \pm 2.8\,{\rm GeV}$$



Top mass dependence of the bare mass

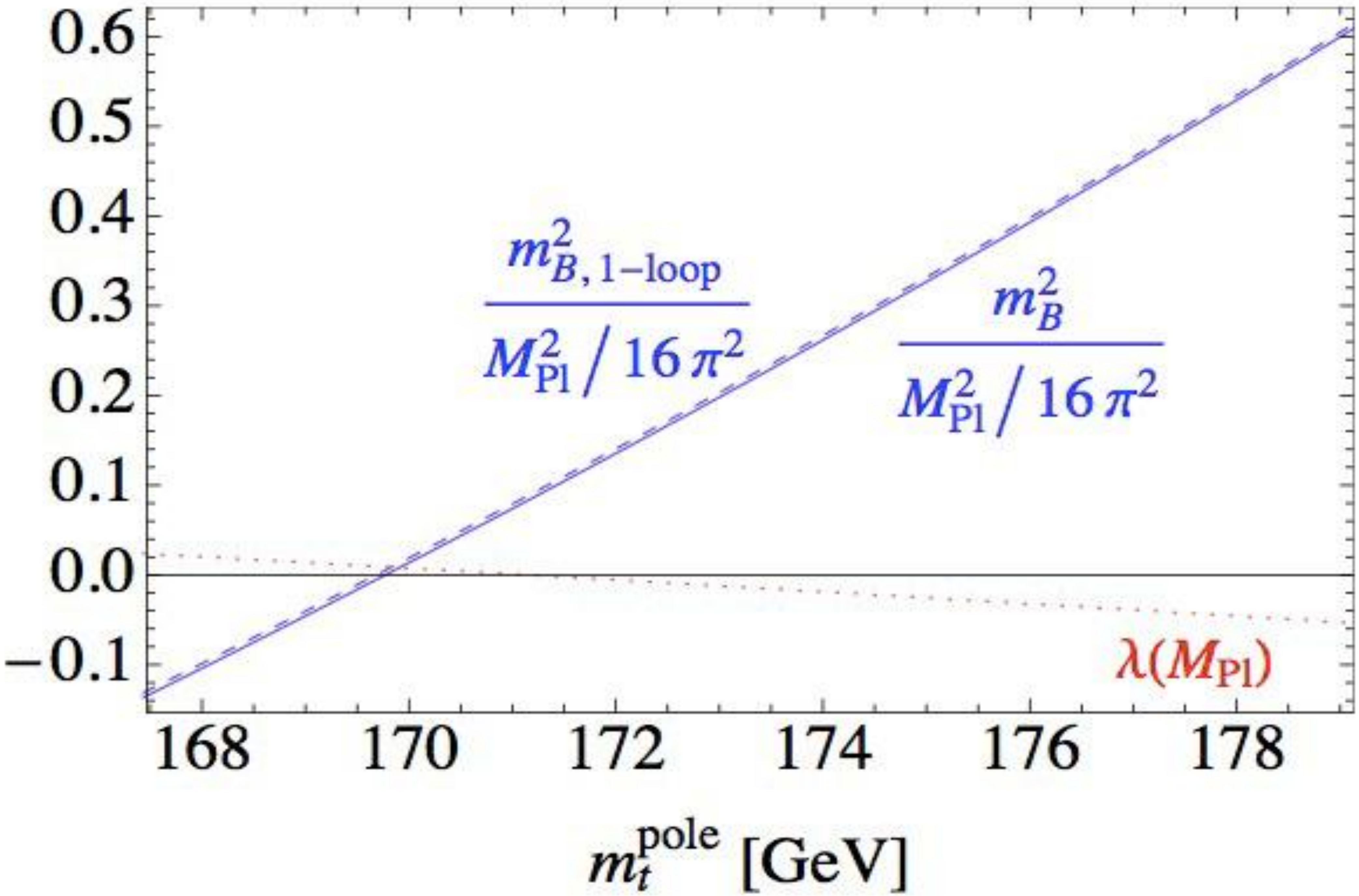
Bare Higgs mass vanishes
at Planck scale if m_t=170GeV



Both m_B^2 and λ vanish around the Planck scale

Bare Higgs mass becomes zero if m_t=170GeV.
Quadratic coupling vanishes if m_t=171GeV.

$\Lambda = M_{Pl}$



Triple coincidence

Three quantities,

$$\lambda_B,\ \beta_\lambda\left(\lambda_B\right),\ m_B$$

become close to zero around the Planck/string scale.

Summary of theHiggs bare parameters

- The SM can be valid to the string scale.

  Desert is possible.
- The experimental value of the Higgs mass seems to be just on the stability bound.

  Nature seems to like the marginal stability.
- The bare Higgs mass becomes close to zero at the string scale. It implies that SUSY is restored at the string scale. Actually there are many string vacua in which SUSY is broken at the string scale.
- The Higgs self coupling and the beta function also becomes close to zero at the string scale. It indicates that the Higgs potential becomes almost flat around the string scale, which opens the possibility that the Higgs field plays the roll of inflaton.
- It is important to know the top mass within 1% error.

PART 2 the Naturalness

We consider the possibility that the fine tunings result from not the conventional local field theory but something slightly beyond.

2-1. Froggatt and Nielsen

Canonical and micro canonical ensembles

$$\int [d\varphi]\,\delta\big(H(\varphi)-E\big) \Leftrightarrow \int [d\varphi]\exp\big(-H(\varphi)/T\big)$$

We start with a micro canonical like path integral:

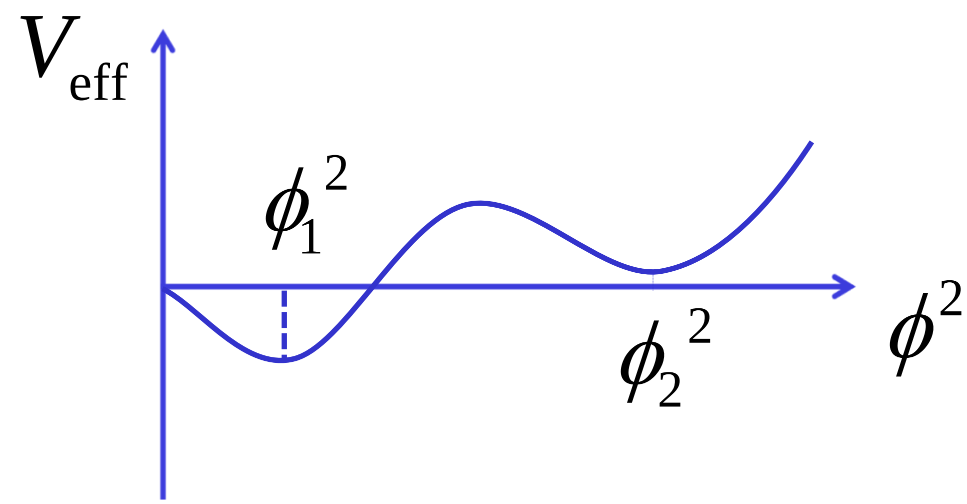
$$Z = \int [d\phi]\,\delta\Big(\int d^4x\,\phi^\dagger\phi - I_0\Big)\exp\big(iS[\phi]\big)$$
$$= \int dm^2 \int [d\phi]\exp\Big(i\Big(S[\phi]-m^2\int d^4x\,\phi^\dagger\phi + m^2 I_0\Big)\Big).$$

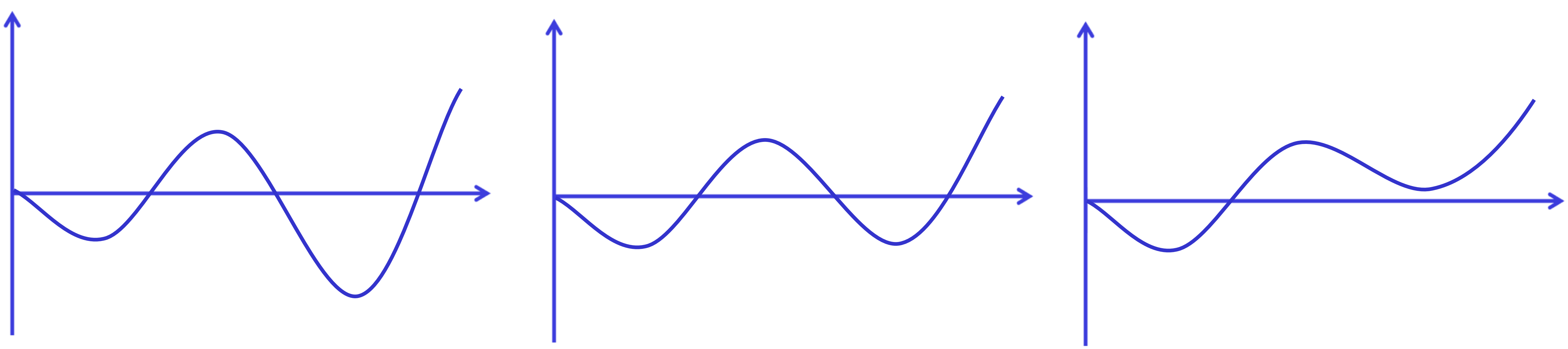
One value of m^2 **dominates in the RHS:**

$$Z = \int dm^2 \exp\big(-iVF(m^2)\big).$$

$$Z = \int [d\phi]\, \delta\left(\int d^4x\, \phi^\dagger \phi - I_0 \right) \exp\left(i\, S[\phi] \right)$$
$$= \int dm^2 \exp\left(i \left(S[\phi] - m^2 \int d^4x\, \phi^\dagger \phi + m^2 I_0 \right) \right).$$

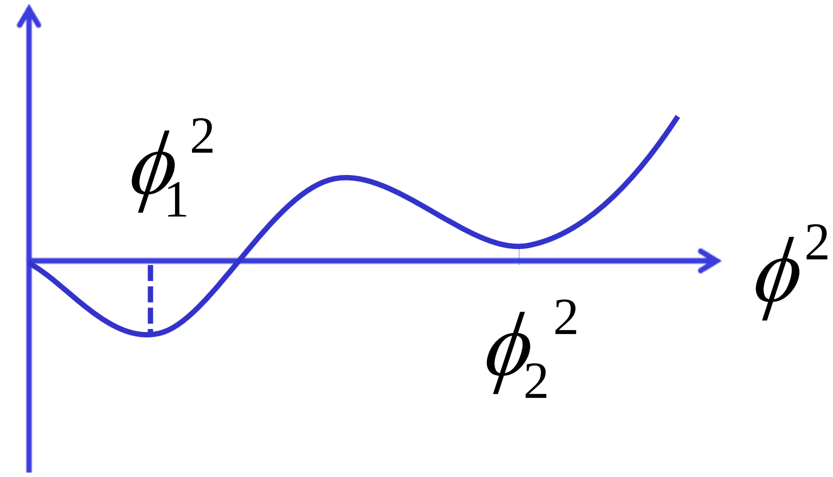
Assume that the effective potential for *S* has two minima.





$m^2 < m_c^{\ 2}$	$m^2 = m_c^{\ 2}$	$m^2 > m_c^{\ 2}$
$\langle \phi^2 \rangle \sim \phi_2^{\ 2}$	$\phi_1^{\ 2} \le \langle \phi^2 \rangle \le \phi_2^{\ 2}$	$\langle \phi^2 \rangle \sim \phi_1^{\ 2}$

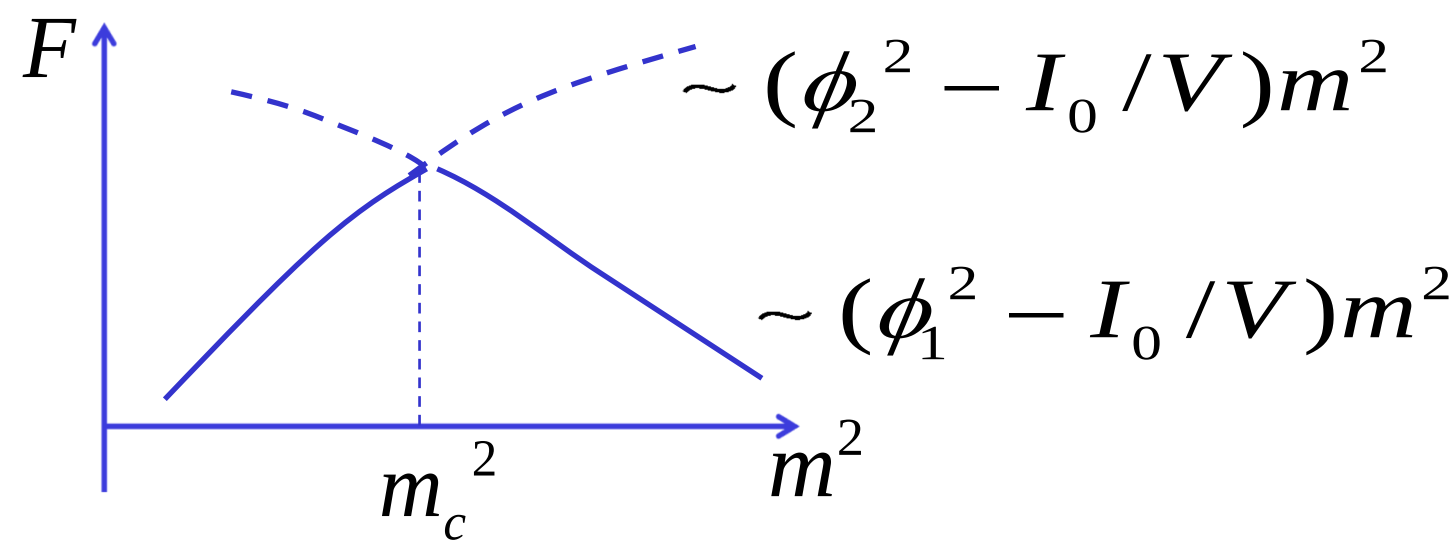
The original effective potential.



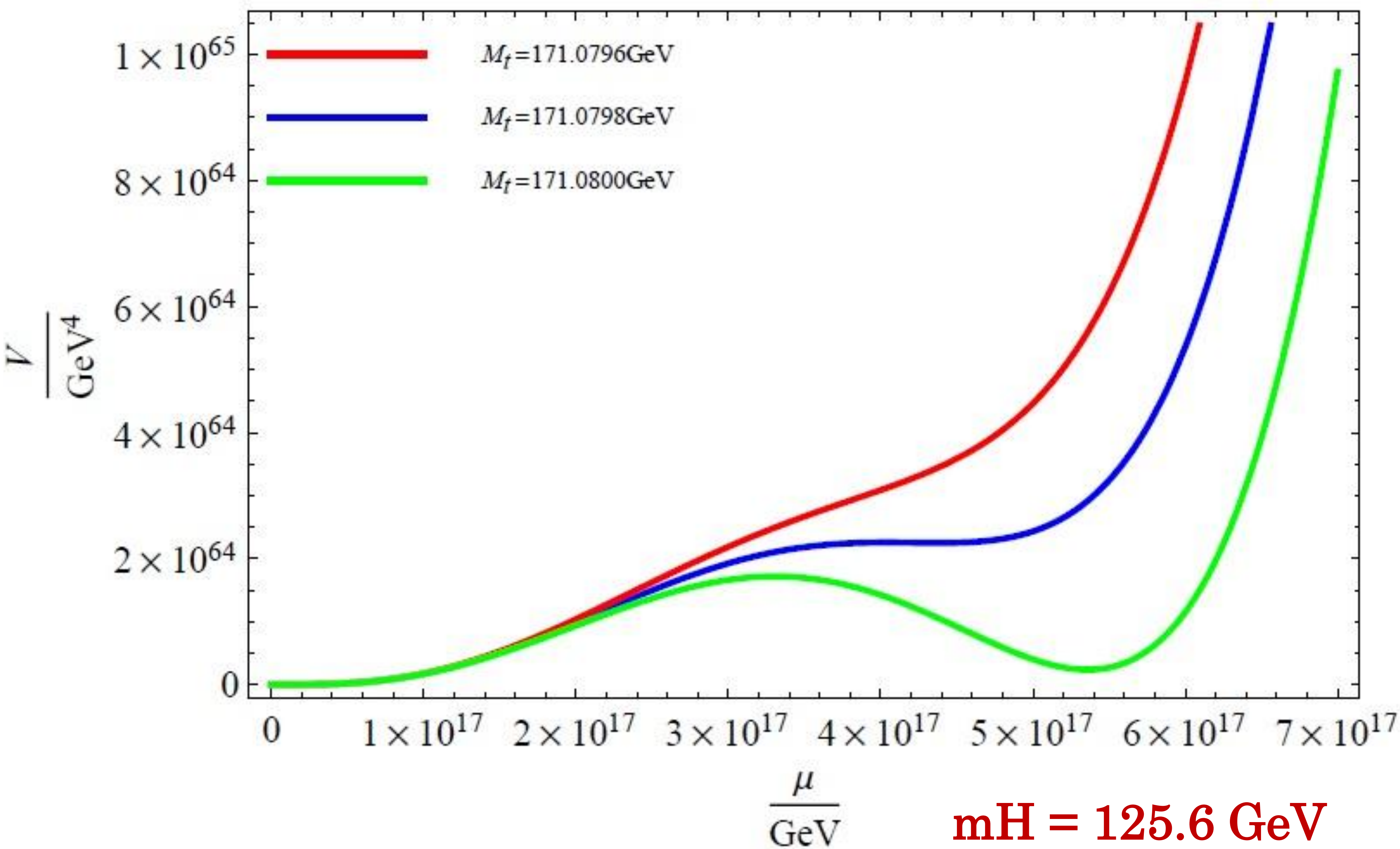
If $\phi_1^2 \leq I_0/V \leq \phi_2^2$, m^2 should be equal to m_c^2 in order for the vacuum to be a mixture of the two phases such that

$$\left\langle \int d^4x\, \phi^\dagger \phi \right\rangle = I_0.$$

In other words, F in $Z = \int dm^2 \exp\left(-iVF\left(m^2\right)\right)$ behaves as



The Higgs potential should have a degenerate minimum at a large value of the field.



mH = 125.6 GeV

generalization

The micro canonical like path integral can be generalized to

$$Z = \int [d\phi]\rho\left(\int d^4x\left(\phi^\dagger\phi - M^2\right)\right)\exp\left(iS[\phi]\right)$$

$$= \int dm^2 w\left(m^2\right)\int [d\phi]\exp\left(i\left(S[\phi] - m^2\int d^4x\left(\phi^\dagger\phi - M^2\right)\right)\right).$$

$M \sim$ Planck scale is natural.

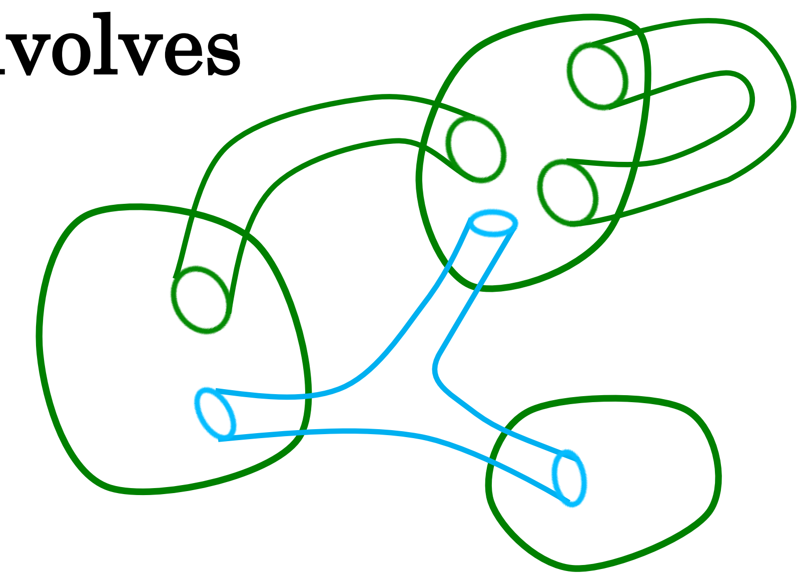
Again one value of m^2 dominates in the RHS:

$$Z = \int dm^2 w\left(m^2\right)\exp\left(-iVF\left(m^2\right)\right).$$

2-2. Coleman’s Baby Universe

Coleman ('88) an explicit mechanism to get the factorized action

Consider Euclidean path integral which involves the summation over topologies,



$$\sum_{\text{topology}} \int [dg] \exp(-S) \, .$$

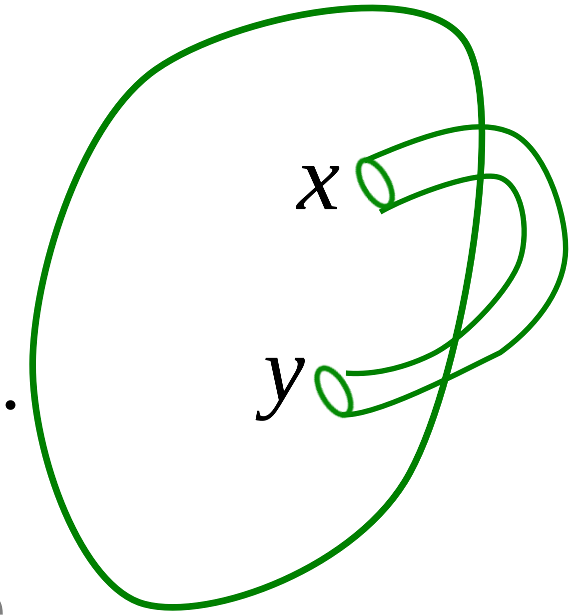
Then there should be a wormhole-like configuration in which a thin tube connects two points on the universe. Here, the two points may belong to either the same universe or the different universe.

If we see such configuration from the side of the large universe(s), it looks like two small punctures.

But the effect of a small puncture is equivalent to an insert ion of a local operator.

Therefore, a wormhole contribute to the path integral as

$$\int [dg] \sum_{i,j} c_{ij} \int d^4x\, d^4y \sqrt{g(x)}\sqrt{g(y)}\, O^i(x)\, O^j(y)\ \exp(-S)\ .$$



Summing over the number of wormholes, we have

$$\sum_{N=0}^{\infty} \frac{1}{n!} \left(\sum_{i,j} c_{ij} \int d^4x\, d^4y \sqrt{g(x)}\sqrt{g(y)}\, O^i(x)\, O^j(y) \right)^n$$

$$= \exp\left(\sum_{i,j} c_{ij} \int d^4x\, d^4y \sqrt{g(x)}\sqrt{g(y)}\, O^i(x)\, O^j(y) \right).$$

Thus wormholes contribute to the path integral as

$$\int [dg] \exp\left(-S + \sum_{i,j} c_{ij} \int d^4x\, d^4y \sqrt{g(x)}\sqrt{g(y)}\, O^i(x)\, O^j(y) \right).$$

bifurcated wormholes

$\Rightarrow$ cubic terms, quartic terms, ...

The effective action becomes a factorized form

$$S_{\text{eff}} = \sum_i c_i S_i + \sum_{ij} c_{ij} S_i S_j + \sum_{ijk} c_{ijk} S_i S_j S_k + \cdots,$$

$$S_i = \int d^D x \sqrt{g(x)} O_i(x).$$

By introducing the Laplace transform

$$\exp\left(-S_{\text{eff}}(S_1, S_2, \cdots)\right) = \int d\lambda\, w(\lambda_1, \lambda_2, \cdots) \exp\left(-\sum_i \lambda_i S_i\right),$$

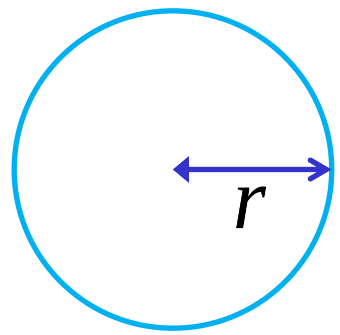
we can express the path integral as

$$Z = \int [d\phi] \exp(-S_{\text{eff}}) = \int d\lambda w(\lambda) \int [d\phi] \exp\left(-\sum_i \lambda_i S_i\right).$$

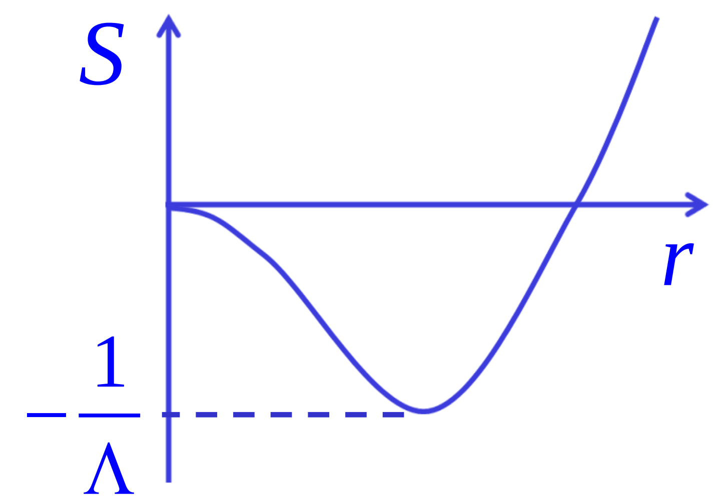
Coupling constants are not merely constant but to be integrated.

A solution to the cosmological constant problem

$$Z = \int d\Lambda\, w(\Lambda) \int [dg] \exp\left(-\int \sqrt{g}\, R - \Lambda \int \sqrt{g}\right).$$



S^4



$$\sim \int d\Lambda\, w(\Lambda) \int dr \exp\left(-\left(-r^2 + \Lambda r^4\right)\right)$$

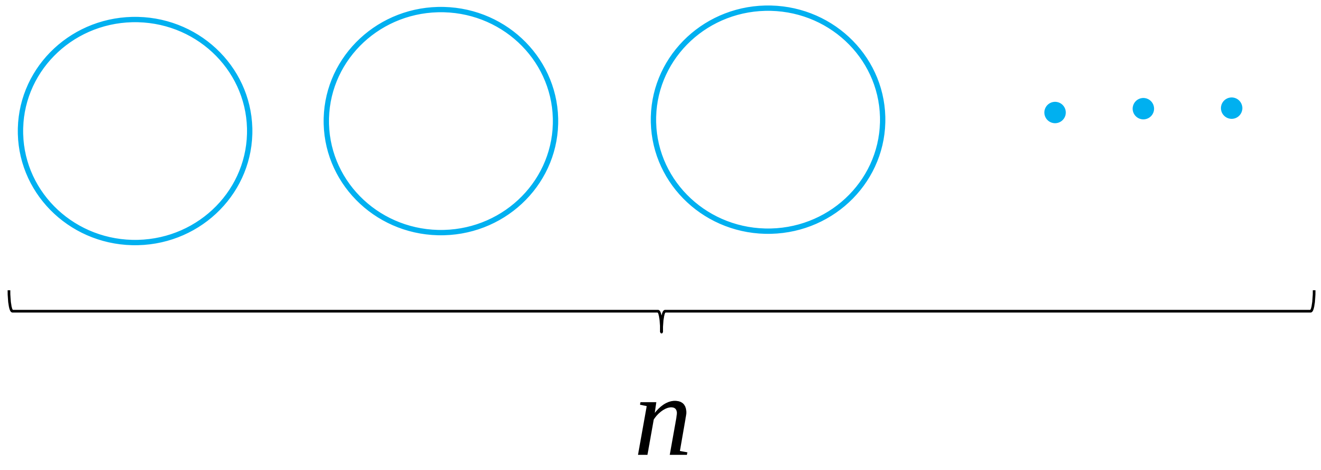
$$\sim \int d\lambda\, w(\Lambda) \begin{cases} \exp(1/\Lambda), & \Lambda > 0 \\ \text{no solution}, & \Lambda < 0 \end{cases}$$

$\Lambda \sim 0$ **dominates irrespectively of** $w(\Lambda)$.

including multiverse

$$Z = \int d\lambda\, w(\lambda) \int [d\phi] \exp\left(-S(\lambda)\right)$$

$$= \sum_{n=0}^{\infty} \frac{1}{n!} {Z_{\text{single}}}^{n} = \exp\left(Z_{\text{single}}\right).$$



Difficulty (1)

Problem of the Wick rotation

WDW eq. $$H_{\text{total}} \left| \Psi \right\rangle = 0$$

$$H_{\text{total}} = H_{\text{universe}} + H_{\text{matter}} + H_{\text{graviton}} + \cdots$$

$$H_{\text{universe}} = -\left(\frac{1}{2a} p^2 + \cdots \right)$$ **←wrong sign**

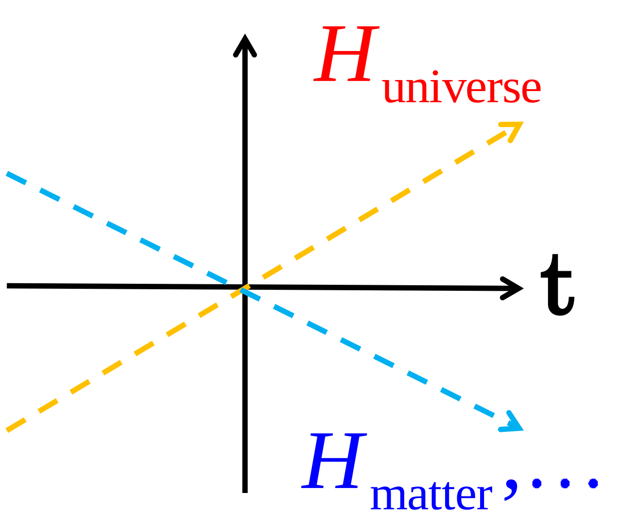
$\Downarrow$ a : radius of the universe

"Ground state" does not make sense.

Wick rotation is not well defined.

H_{matter} **is bounded from below.**

H_{universe} **is bounded from above.**



Difficulty (2)

Overall phase of the Partition function

The multiverse partition function

$$Z_{\text{multi}} \sim \int d\lambda\, w(\lambda) \exp\left(Z_{\text{single}}\right).$$

$$Z_{\text{single}} = \int [dg] \exp\left(-S_{\lambda}\right)$$

The overall phase of Z_{single} is important.

We need subtle analyses.

2-3. Lorentzian Path integral of the factorized action

It is natural to imagine that the low energy effective action of a theory including gravity has the same structure as Coleman's:

e.g. IIB matrix model
Y. Asano, A. Tsuchiya, HK

$$S_{\text{eff}} = \sum_i c_i S_i + \sum_{ij} c_{ij} S_i S_j + \sum_{ijk} c_{ijk} S_i S_j S_k + \cdots,$$

$$S_i = \int d^D x \sqrt{g(x)} O_i(x).$$

Then the coupling constants are determined by the state

$$S_{\text{eff}} \simeq \sum_i c_i S_i + \sum_{i,j} 2c_{ij} \langle S_i \rangle S_j + \sum_{i,j,k} 3c_{ijk} \langle S_i \rangle \langle S_j \rangle S_k + \cdots.$$

More precisely, the path integral is given by

$$Z = \int [d\phi] \exp(i S_{\text{eff}}) = \int d\lambda\, w(\lambda) \int [d\phi] \exp\left(i \sum_i \lambda_i S_i \right).$$

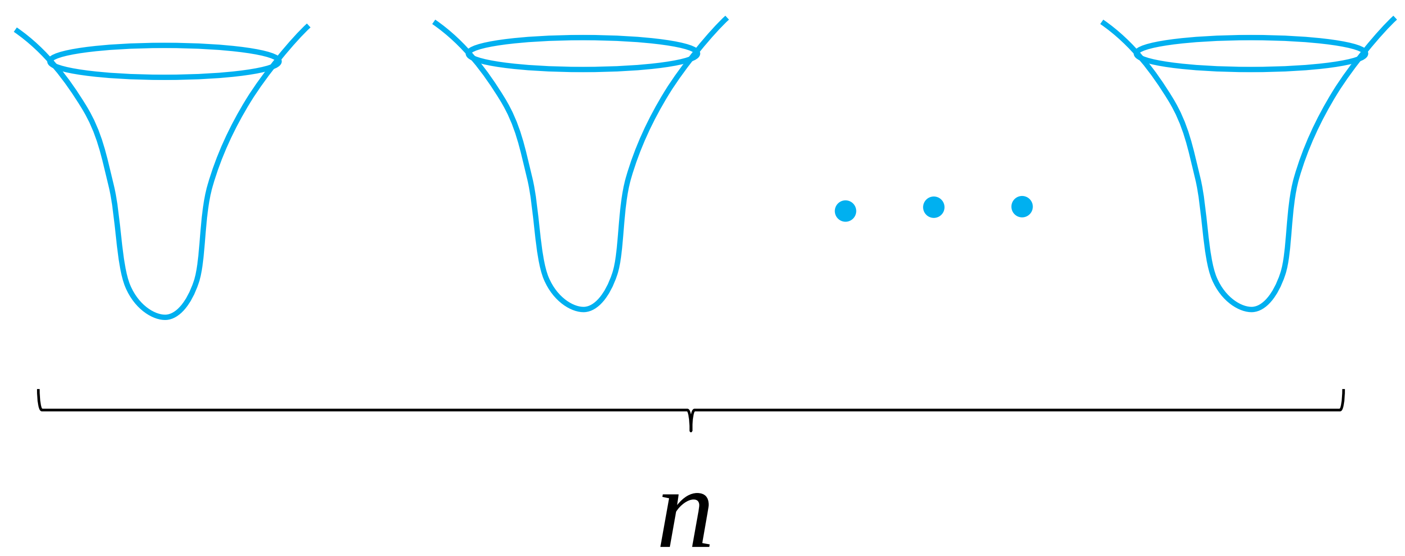
Coupling constants are not merely constant but to be integrated.

It is natural to apply this action to the multiverse.

$$Z=\int[d\phi]\exp\left(iS_{\mathrm{eff}}\right)=\int d\lambda\, w(\lambda)\int[d\phi]\exp\left(i\sum_i \lambda_i S_i\right).$$

$$\int[d\phi]\exp\left(i\sum_i \lambda_i S_i\right)=\sum_{n=0}^{\infty}\frac{1}{n!}Z_1{}^n$$

$$Z_1=\int[d\phi]_{\text{single universe}}\exp\left(i\sum_i \lambda_i S_i\right)$$



$$=\int d\lambda\, w(\lambda)\exp\left(Z_1(\lambda)\right).$$

2-4. Partition function of a single universe

Basic problem

Define and evaluate the partition function of a single universe:

$$Z_1(\lambda) = \int [d\phi] \exp\left(i \sum_i \lambda_i S_i \right).$$

The path integral of a universe

If the initial and final states are given, the path integral is evaluated as follows.

$$Z_1(\lambda) = \int [d\phi] \exp(i S)$$

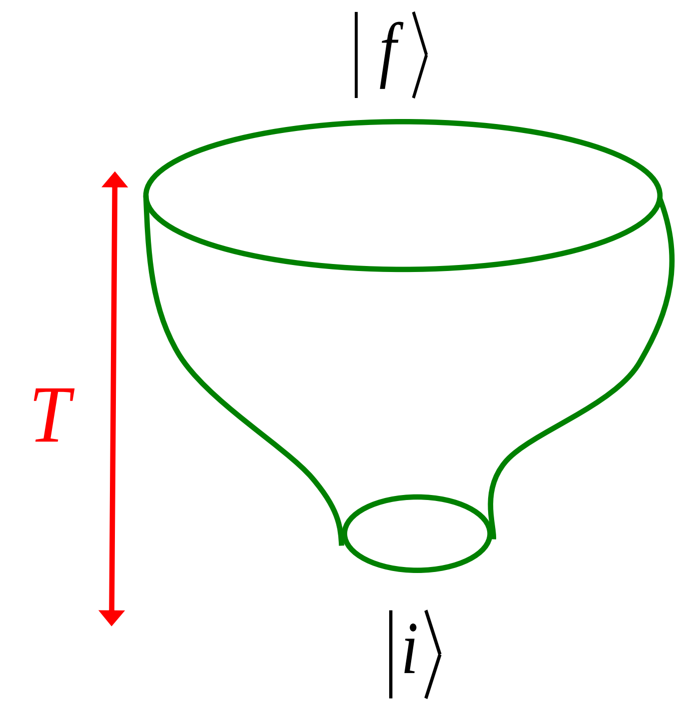
$$= \langle f | \int [da\,dp\,dN] \exp\left(i \int_0^1 dt \left(p\dot{a} - N H_\lambda \right) \right) | i \rangle$$

$$= \langle f | \int_{-\infty}^{\infty} dT \exp\left(-iT\hat{H}_\lambda \right) | i \rangle$$

$$= \langle f | \delta\left(\hat{H}_\lambda \right) | i \rangle$$

$$= \langle f | \phi_{E=0} \rangle \langle \phi_{E=0} | i \rangle$$

$$\langle \phi_E | \phi_{E'} \rangle = \delta\left(E - E' \right)$$



$$\hat{H}_\lambda = -\frac{1}{2} \frac{1}{\sqrt{a}} p^2 \frac{1}{\sqrt{a}} - a^3 U(a)$$

$$U(a) = \frac{1}{a^2} - \Lambda - \frac{C_{matt}}{a^3} - \frac{C_{rad}}{a^4}$$

a : radius of the universe

Question:

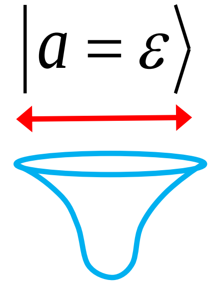
Is there a natural choice for them?

Initial state

For the initial state, we assume that the universe emerges with a small size ε.

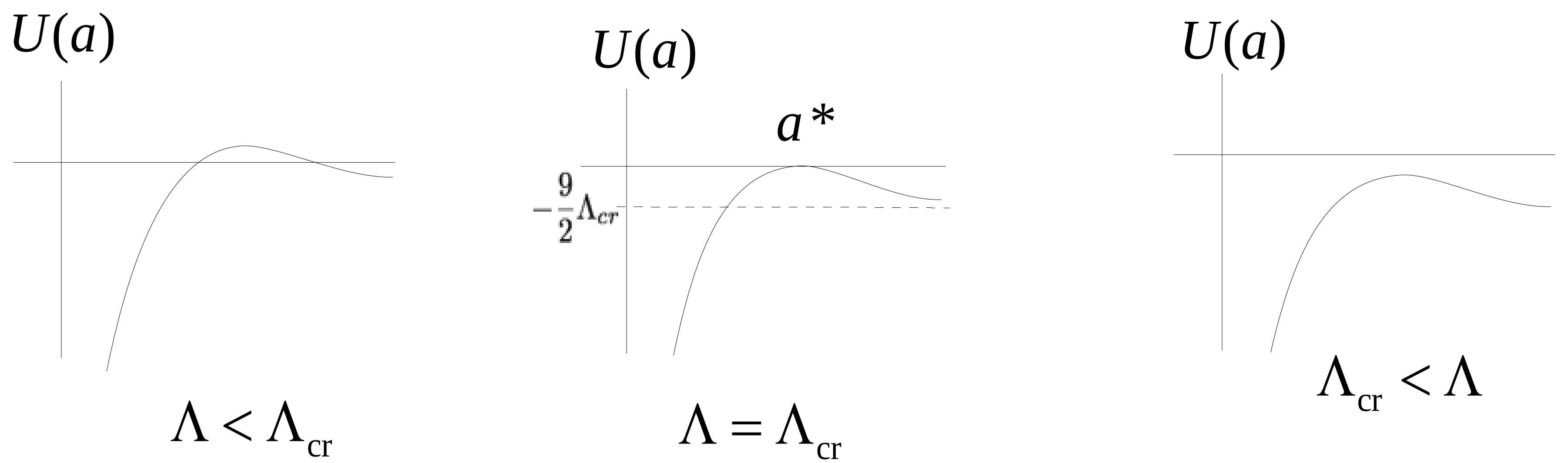
$$\left|i\right\rangle = \mu\left|a=\varepsilon\right\rangle \otimes \left|matter\cdots\right\rangle,$$

μ : probability amplitude of a universe emerging.



Evolution of the universe

S^3 topology



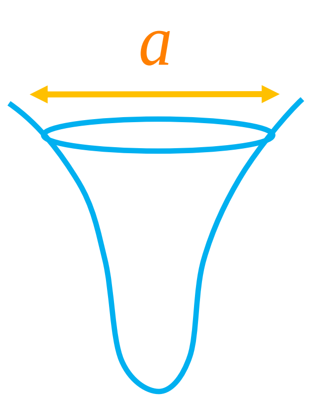
Λ ~curvature

~energy density

WKB solution

$$S = \int d^4x\sqrt{-g}\left(R - \Lambda + matter\right)$$

$$\phi_{E=0}(a,\lambda) \sim \frac{1}{\sqrt{a^{-1}p(a,\lambda)}}\sin\left(\boxed{\int_0^z da' p(a',\lambda)} + \alpha\right)$$



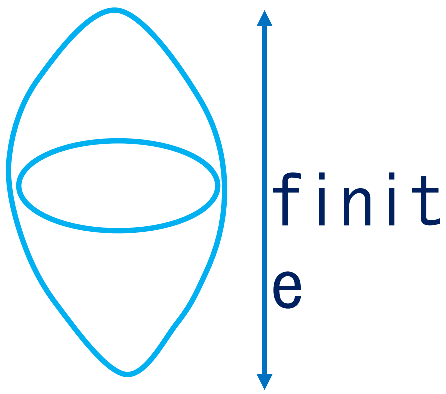
with $p(a,\lambda) = \sqrt{-2a^4 U(a)}.$

$$U(a) = \frac{1}{a^2} - \Lambda - \frac{C_{matt}}{a^3} - \frac{C_{rad}}{a^4}$$

For the final state, we have two possibilities.

Final state: case 1

$\Lambda < \Lambda_{cr}$



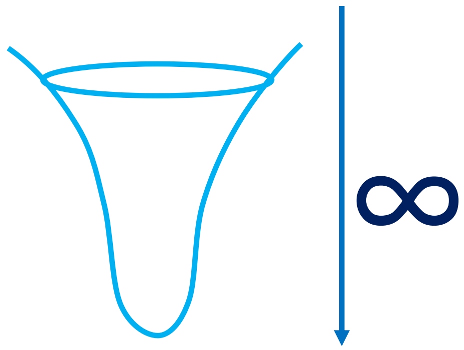
The universe is closed.
We assume the final state is

$$|f\rangle = \mu' |a=\varepsilon\rangle \otimes |matter \cdots\rangle.$$

The partition function

$$Z_1(\lambda) = \langle f | \delta\left(\hat{H}_\lambda\right) | i \rangle$$
$$\sim const \left|\phi_{E=0}(\varepsilon)\right|^2 .$$

Final state: case 2 $\Lambda > \Lambda_{cr}$

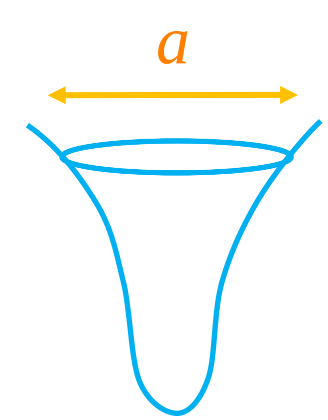


The universe is open.
It is not clear how to define the path integral for the universe:

$$Z_1(\lambda) = \int [d\phi] \exp(iS) \ .$$

As an ad hoc assumption **we consider**

$$|f\rangle = \lim_{a_{IR} \to \infty} c\sqrt{a_{IR}}\,|a_{IR}\rangle \otimes |matter \cdots\rangle.$$



Final state: case 2 contd

Then the partition function becomes

$$Z_1(\lambda) = \mu\, c\sqrt{a_{IR}}\,\phi_{E=0}(a_{IR})\,\phi^*{}_{E=0}(\varepsilon)$$

$$\sim \mu\, c\sqrt{a_{IR}}\,\frac{1}{\sqrt{a_{IR}}\sqrt[4]{\Lambda}}\sin\left(a_{IR}{}^3\sqrt{\Lambda}+\alpha'\right)\phi^*{}_{E=0}(\varepsilon)$$

$$\sim \mu\, c\,\frac{1}{\sqrt[4]{\Lambda}}\sin\left(a_{IR}^3\sqrt{\Lambda}+\alpha'\right)\phi^*{}_{E=0}(\varepsilon).$$

$$\phi_{E=0}(a) \sim \frac{1}{\sqrt{a^{-1}p(a)}}\sin\left(\int_0^z da' p(a')+\alpha\right)$$

$$p(a,\lambda) = \sqrt{-2a^4 U(a)} \qquad U(a) = \frac{1}{a^2} - \Lambda - \frac{C_{\mathrm{mat}}}{a^3} - \frac{C_{\mathrm{rad}}}{a^4}$$

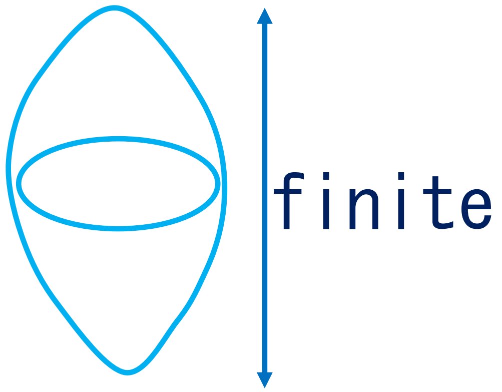
The result does not depend on a_{IR} except for the phase which should come from the classical action.

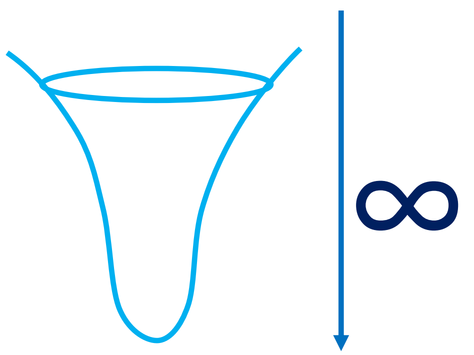
Under the ad hoc assumption, we have

the partition function for a universe $Z_1(\lambda)$

for $\Lambda < \Lambda_{\mathrm{cr}}$ $\qquad$ $const$ of order 1,

for $\Lambda > \Lambda_{\mathrm{cr}}$ $\qquad$ $const \dfrac{1}{\sqrt[4]{\Lambda}} \sin\left(a_{IR}{}^3 \sqrt{\Lambda} + \alpha' \right).$





Then the λ **integration for the multiverse partition**

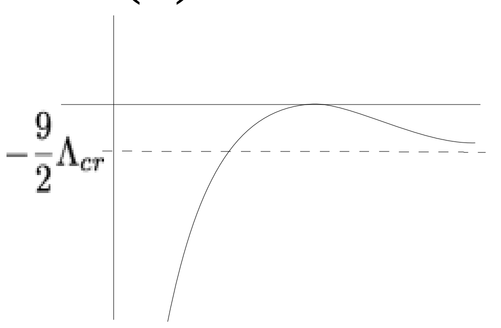
$$Z = \int d\lambda\, w(\lambda) \exp\left(Z_1(\lambda)\right).$$

has a large peak at $\Lambda(\lambda) \sim \Lambda_{\mathrm{cr}}$**, which means that the cosmological constant at the late stages of the universe almost vanishes.**

2-5. Naturalness and Big Fix

Big Fix

For simplicity we assume the S^3 topology of the space and that all matters decay to radiation at the late stages.



$\Lambda = \Lambda_{\mathrm{cr}}$

$$U(a) = \frac{1}{a^2} - \Lambda - \frac{C_{\mathrm{rad}}}{a^4}$$

$$\Lambda_{\mathrm{cr}} \sim 1/C_{\mathrm{rad}}$$

Then the multiverse partition function is given by

$$Z = \int d\lambda w(\lambda) \exp\left(Z_1(\lambda)\right)$$

$$\sim \exp\left(\mathrm{const}\frac{1}{\sqrt[4]{\Lambda_{cr}}}\right) \sim \exp\left(\mathrm{const}\sqrt[4]{C_{\mathrm{rad}}(\lambda)}\right).$$

BIG FIX

The low energy couplings are determined in such a way that the entropy at the late stages of the universe is maximized.

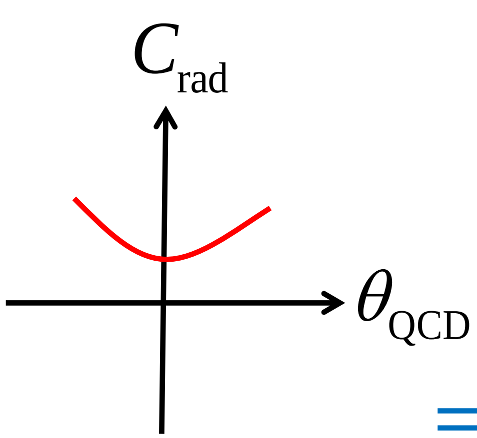
Examples of the Big Fix (1)

If the cosmological evolution is completely understood, we can calculate $C_{\text{rad}}(\lambda)$ theoretically, and all of the renormalized couplings are in principle determined.

At present, we do not have enough knowledge about the very early and late stages of the universe, especially the origin of inflation, dark energy and dark matter.

However, some of the couplings can be determined without knowing the details of the cosmological evolution.

case 1. Symmetry example θ_{QCD} Nielsen, Ninomiya

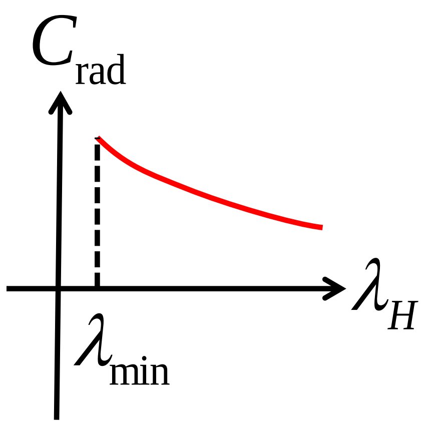


1. It becomes important only after the QCD phase transition.
2. The mass spectrum of hadrons is invariant under $\theta_{\text{QCD}} \to -\theta_{\text{QCD}}$.

$\Rightarrow C_{\text{rad}}$ is minimum or maximum at $\theta_{\text{QCD}} = 0$ at least locally.

Examples of the Big Fix (2)

case 2. End point **example** **Higgs coupling** λ_H



1. **Some (renormalized) couplings are bounded.**
2. C_{rad} **can be monotonic in them.**

$\Rightarrow$ C_{rad} **is maximized at the end point.**

A scenario for λ_H .

Fix v_h to the observed value and vary λ_H.

assuming the leptogenesis

$\lambda_H \searrow$ $\Rightarrow$ sphaleron process $\nearrow$

$\Rightarrow$ baryon number $\nearrow$

$\Rightarrow$ radiation from baryon decay $\nearrow$

$\Rightarrow$ **Higgs mass is at its lower bound.**

Summary 2

In wide classes of quantum gravity or string theory, the low energy effective action has the factorized form:

$$S_{\mathrm{eff}} = \sum_i c_i S_i + \sum_{ij} c_{ij} S_i S_j + \sum_{ijk} c_{ijk} S_i S_j S_k + \cdots.$$

The multiverse appears universally, and it becomes a superposition of states with various values of the coupling constants.

It is important to investigate the consequences of such action, but at present we do not fully understand their physics. It seems that we do not have even the right definition of the path integral for such action.

Although it is not conclusive, the naturalness problem might be solved by the dynamics of such action. In the most optimistic case, the Big Fix occurs, and all the low energy coupling constants would be determined in a predictable way.

Conclusion

- It seems we have nothing other than a minor modification of the SM below the string scale.
- It is possible that the fine tunings result from not the conventional local field theory but something (slightly) beyond.
- For example, we can consider the possibility that the couplings are fixed to maximize the entropy of the universe. This can be checked for some couplings, if they do not play crucial roles in the early or late universe.

Appendices

Higgs field as inflaton (1)

If we allow a fine tuning of the parameters of the SM, the Higgs field can play the role of inflaton.

[with Y. Hamada and K. Oda]

The effective potential of the Higgs field is given by

$$V_{\text{eff}}\left(\phi\right) = \lambda\left(c\phi\right)\phi^4 + \frac{\lambda_6}{m_{\text{P}}{}^2}\phi^6 + \frac{\lambda_8}{m_{\text{P}}{}^4}\phi^8 + \cdots.$$

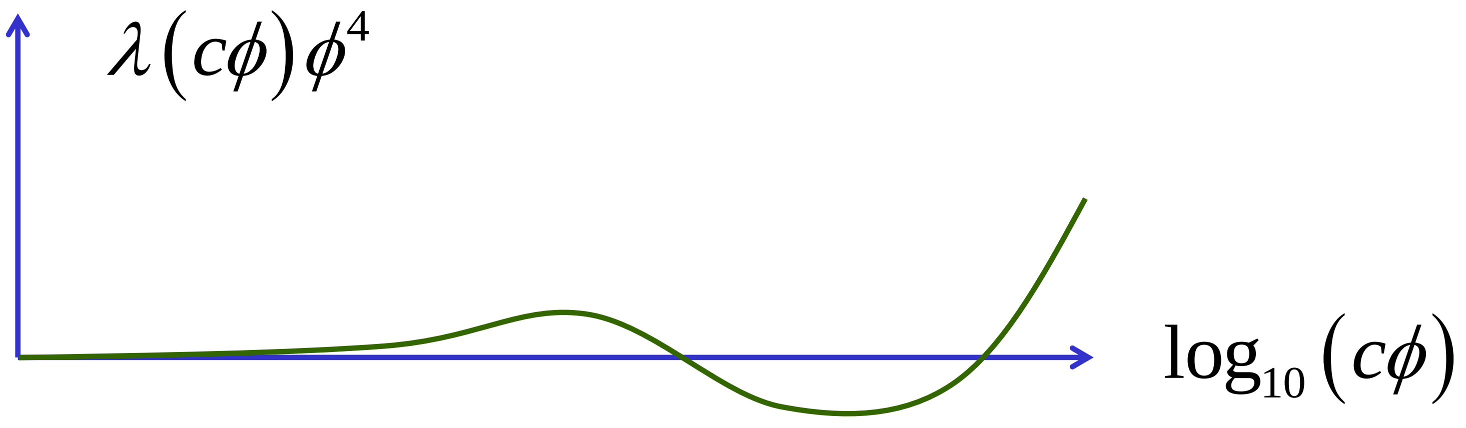
Here the first term on the RHS is determined by the low energy renormalizable theory, that is the standard model, and the other terms are so called Plank suppressed terms that depend on the underlying microscopic theory.

***c* is a constant of order the coupling constants:**

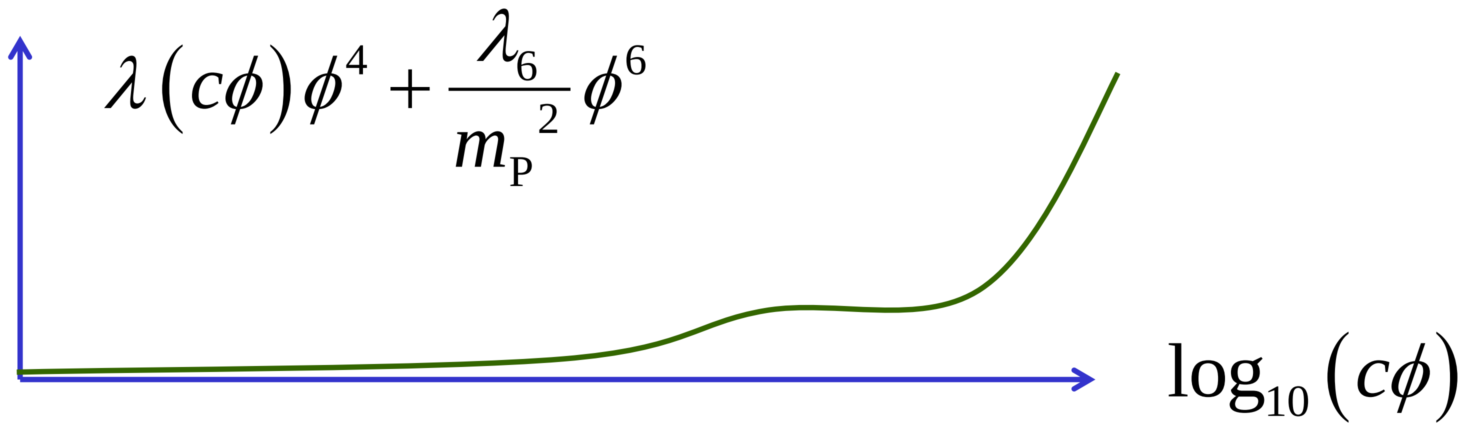
$$c \sim 0.1 .$$

Higgs field as inflaton (2)

Then the first term of V_{eff} looks as follows.



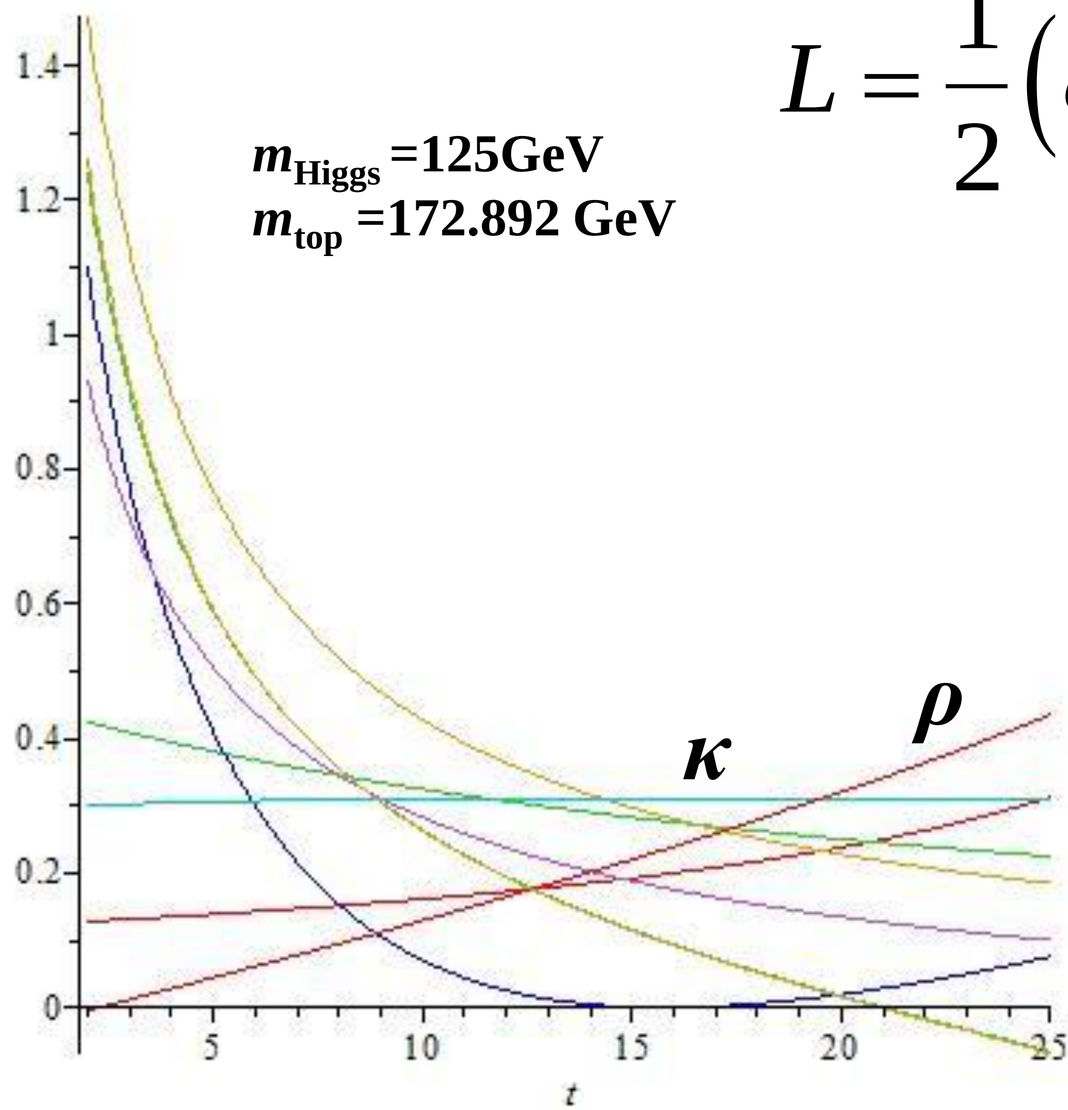
Therefore if we add the second term, we can obtain a saddle point by tuning one parameter.



Room for extra fields

There is a room for introducing small modification. For example, if we introduce a scalar field, the bare coupling constants change As below.

$$L = \frac{1}{2}\left(\partial_\mu \sigma\right)^2 - \frac{\rho}{4!}\sigma^4 - \frac{\kappa}{2}\sigma^2\phi^\dagger\phi + \mathrm{SM}$$



Neutrino Yukawa couplings

- If we assume the see–saw mechanism,
- Our analysis corresponding to the case where M_R is small:

$$m_\nu \sim y_D^2 v^2 / M_R \sim 0.1\,\mathrm{eV}$$

$$y_D \lesssim 10^{-2}$$

$$M_R \lesssim 10^{10}\,\mathrm{GeV}$$

- The case where M_R large is also interesting.

2-1. The IIB matrix model

IIB Matrix Model

Ishibashi, HK, Kitazawa, Tsuchiya

$$S = -\frac{1}{g^2} Tr\left(\frac{1}{4}[A^\mu, A^\nu]^2 + \frac{1}{2}\overline{\Psi}\gamma^\mu[A^\mu, \Psi]\right)$$

A candidate of the constructive definition of string theory.

Evidences

(1) World sheet regularization

Green-Schwartz action in the Schild Gauge

$$S = \int d^2\xi \left(\frac{1}{4}\{X^\mu, X^\nu\}^2 + \frac{1}{2}\overline{\Psi}\gamma^\mu\{X^\mu, \Psi\}\right)$$

Regularization by matrix

$$\{\ ,\ \} \to [\ ,\ ]$$

$$\int \to Tr$$

$$\downarrow$$

$$S = -\frac{1}{g^2} Tr\left(\frac{1}{4}[A^\mu, A^\nu]^2 + \frac{1}{2}\overline{\Psi}\gamma^\mu[A^\mu, \Psi]\right)$$

(2) Loop equation and string field

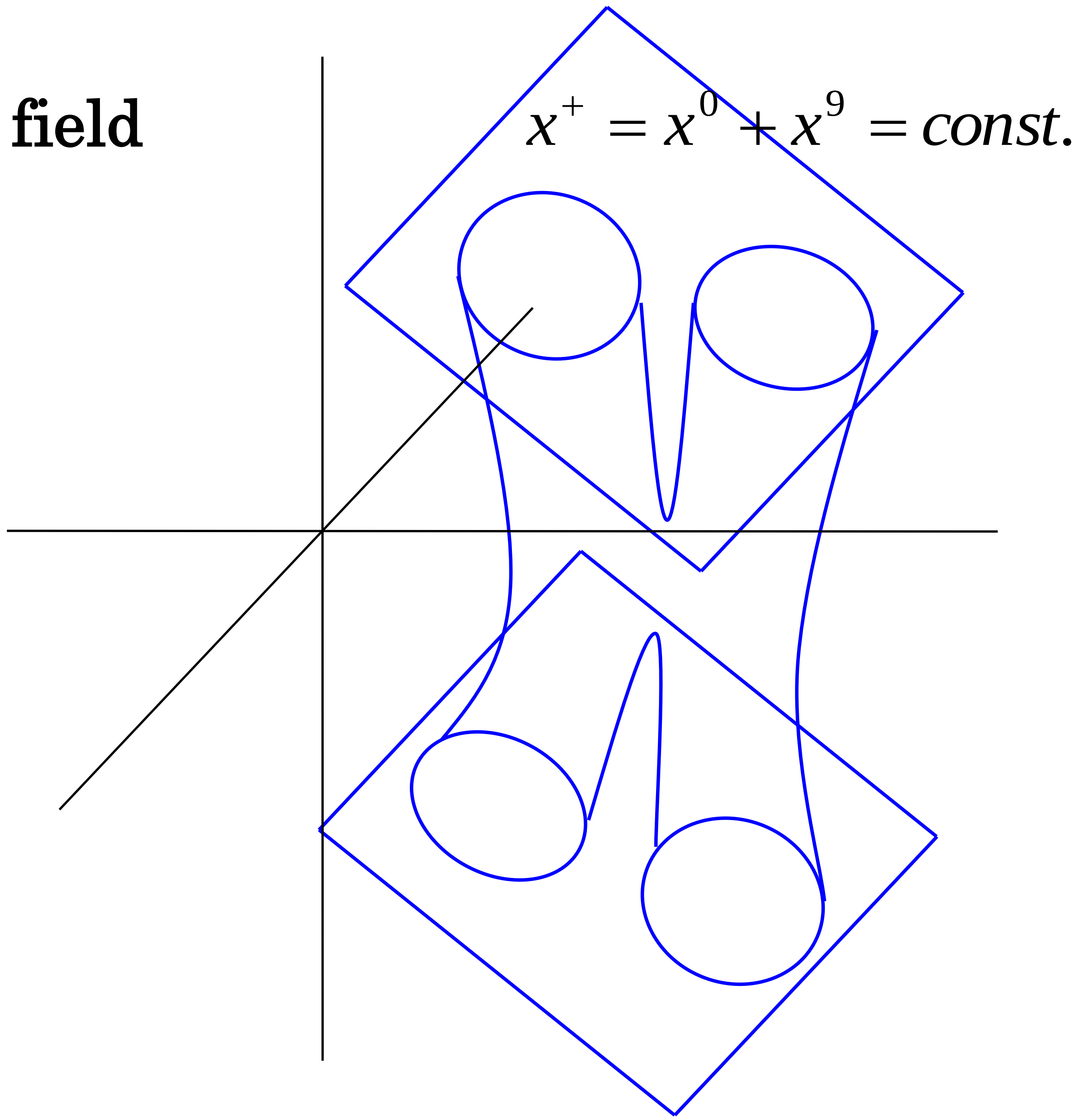
Wilson loop = string field

$$w(k_\mu(\cdot\cdot)) = Tr(P\exp(i\oint d\sigma\, k_\mu(\sigma) A^\mu + fermion))$$

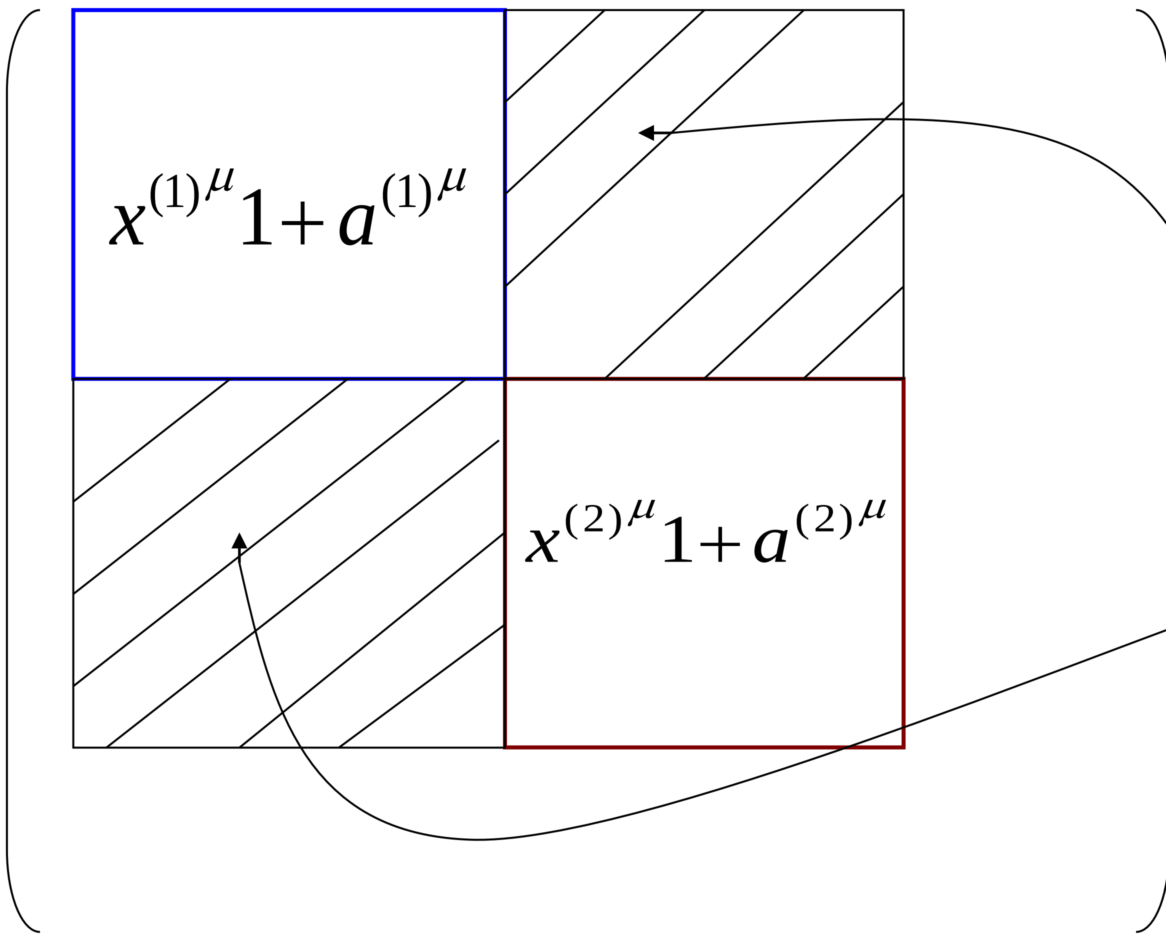
$\Leftrightarrow$ creation annihilation operator of $|k_\mu(\cdot\cdot)>$

loop equation $\rightarrow$ light-cone string field

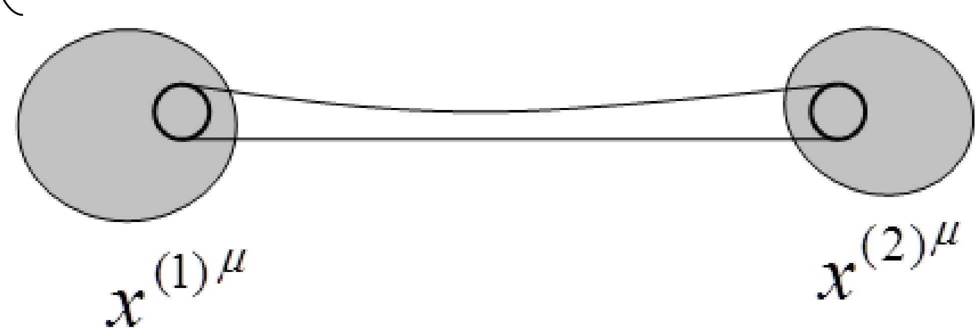
This can be shown with some assumptions .



(3) effective Lagrangian and gravity



Integrate out this part.



The loop integral gives the exchange of graviton and dilaton.

$$S_{eff} = -\frac{1}{(x^{(1)} - x^{(2)})^8}\{const \cdot tr(f^{(1)}{}_{\mu\lambda} f^{(1)}{}_{\nu\lambda})\, tr(f^{(2)}{}_{\mu\lambda} f^{(2)}{}_{\nu\lambda})$$

$$-const \cdot tr(f^{(1)}{}_{\mu\nu} f^{(1)}{}_{\mu\nu})\, tr(f^{(2)}{}_{\lambda\rho} f^{(2)}{}_{\lambda\rho}) + \cdots\}$$

Space-time in the IIB matrix model

$$S = -\frac{1}{g^2} Tr \left(\frac{1}{4} [A^\mu, A^\nu]^2 + \frac{1}{2} \overline{\Psi} \gamma^\mu [A^\mu, \Psi] \right)$$

Various possibilities for the emergence of space-time $\Rightarrow$ various interpretations of A_μ

(1) A_μ as the space-time coordinates

mutually commuting A_μ $\Rightarrow$ space-time

(2) A_μ as non-commutative space-time

non-commutative A_μ $\Rightarrow$ NC space-time

(3) A_μ as derivatives

in the naive large-N reduction

2-2. Derivative interpretation

Derivative interpretation of the IIB matrix model

Hanada, HK, Kimura

If the covariant derivative acts on the regular representation field, its action can be decomposed into D scalars.

φ_α : regular representation field on D-dim manifold M

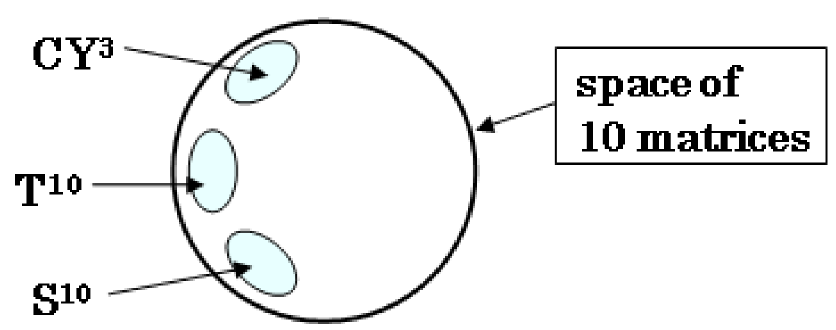
$$(A_a\varphi)_\alpha = C_{(a)\alpha}{}^{b,\beta} \nabla_b \varphi_\beta$$

$$V_{\text{vector}} \otimes V_r \simeq V_r \oplus \cdots \oplus V_r$$ r : regular representation

$C_{(a)\alpha}{}^{b,\beta}$, $(a = 1,..,D)$: the Clebsh-Gordan coefficients

embedding in ten matrices

$$(A_a)_\alpha{}^\beta = \begin{cases} C_{(a)\alpha}{}^{b,\beta} \nabla_b & (a = 1..D) \\ 0 & (a = D+1..10) \end{cases}$$



Clebsh-Gordan coefficients

$G = spin(9,1)$

The space of the regular representation is the function space on G:

$$V_{\mathrm{reg}} = \left\{ f : G \to \mathbb{C} \right\}.$$

G acts on it as the left multiplication:

$$f(z) \underset{g \in G}{\longmapsto} f\left(g^{-1} z\right).$$

An element of $V_{\mathrm{vector}} \otimes V_{\mathrm{reg}}$ is expressed as $\upsilon_a(z)$, where a is the vector index.

G act on it as

$$\upsilon_a(z) \underset{g \in G}{\longmapsto} R_a{}^b(g)\, \upsilon_b\left(g^{-1} z\right).$$

If we define $\upsilon_{(a)}(z) = R_{(a)}{}^{b}(z^{-1})\upsilon_b(z)$,

for each (a), $\upsilon_{(a)}(z)$ is the regular representation:

$$\upsilon_{(a)}(z)$$

$$\underset{g\in G}{\longmapsto} R_{(a)}{}^{c}(z^{-1})R_{c}{}^{b}(g)\upsilon_b(g^{-1}z) = R_{(a)}{}^{b}(z^{-1}g)\upsilon_b(g^{-1}z)$$

$$= \upsilon_{(a)}(g^{-1}z), \qquad (a = 1,\cdots,D).$$

If we regard z as a kind of continuous index,
the Clebsh-Gordan coefficients for the decomposition

$$V_{\text{vector}} \otimes V_{\text{reg}} \cong V_{\text{reg}} \oplus \cdots \oplus V_{\text{reg}}$$

can be written as

$$C_{(a)z}{}^{b,z'} = R_{(a)}{}^{b}(z^{-1})\delta(z,z').$$

$\delta(z,z')$:delta function on G

field of the regular representatation

A field of the regular representation means that we have a function on G at each point on M.

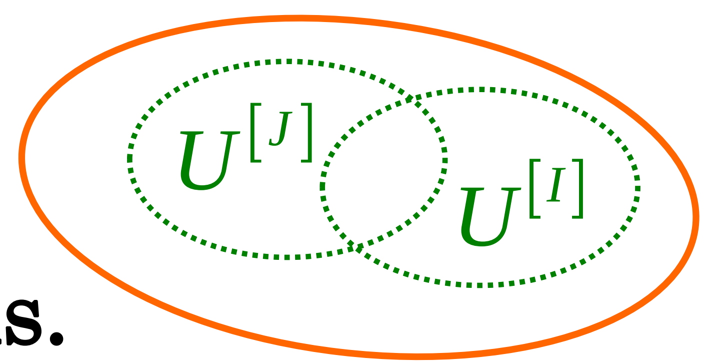
Locally it can be written as a function of x and z

$$\varphi(x,z),\ \left(x\in M, z\in G\right).$$

We then glue the patches by the left multiplication of the transition function

$$\varphi^{[I]}(x,z^{[I]})=\varphi^{[J]}(x,g^{[I,J]}\left(x\right)^{-1}z^{[J]}),\ x\in U^{[I]}\cap U^{[J]}.$$

In other words, φ is a global section of the principal G-bundle E_{prin} associated with the transition functions.



Therefore the space of the regular representation field V is identical to $V\cong C^{\infty}\left(E_{\mathrm{prin}}\right)$.

Endomorphic covariant derivative

Now we can explicitly perform the procedure to convert the covariant derivative to endomorphisms.

We start with the covariant derivative acting on the regular representation field:

$$\nabla_a = e_b^{\mu}(x)\left(\partial_\mu + \omega_\mu^{cd}(x)\hat{O}_{cd}\right)$$

$$\varepsilon^{ab}\hat{O}_{ab}\varphi(x,z) = \varphi(x,(1-\varepsilon^{ab}\tau_{ab})z) - \varphi(x,z)$$

As is discussed, if we multiply the CG coefficients,

$$\nabla_{(a)} \equiv R_{(a)}{}^{b}\left(z^{-1}\right)e_b^{\mu}(x)\left(\partial_\mu + \omega_\mu^{cd}(x)\hat{O}_{cd}\right),$$

each of $\nabla_{(a)}\ (a=1,..,D)$ is an endomorphism on V.

Therefore if we introduce UV and IR cutoff to the space V, each of $\nabla_{(a)}$ is expressed by a matrix.

Classical EOM of the derivative interpretation

The classical EOM of the IIB matrix model is

$$\left[A_a\left[A_a, A_b\right]\right] = 0.$$

If we impose the Ansatz

$$A_a = \begin{cases} \nabla_{(a)} & (a = 1..D) \\ 0 & (a = D+1..10). \end{cases}$$

it becomes

$$0 = \left[\nabla_{(a)}, \left[\nabla_{(a)}, \nabla_{(b)}\right]\right] \Leftrightarrow 0 = \left[\nabla_a, \left[\nabla_a, \nabla_b\right]\right] = (\nabla_a R_{ab}{}^{cd})O_{cd} - R_{ab}{}^{ca}\nabla_c$$

$$\Leftrightarrow \nabla_a R_{ab}{}^{cd} = 0\,,\ R_{ab} = 0 \Leftrightarrow R_{ab} = 0\,.$$

The Einstein equation follows from the EOM of the IIB matrix model

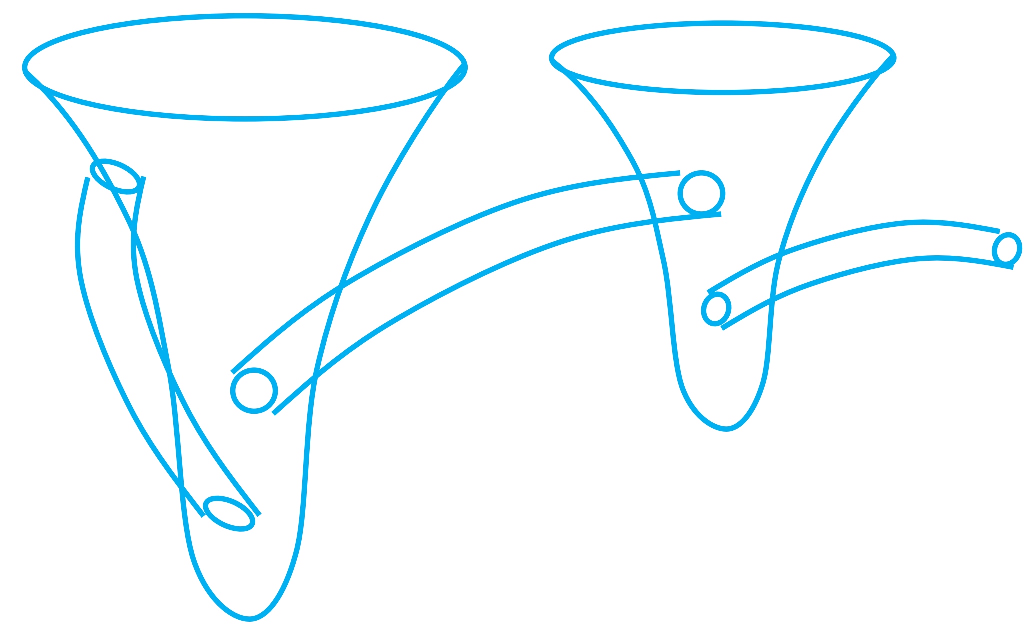
multiverse in the matrix model

Multiverse appears naturally in the derivative interpretation.

$$\begin{pmatrix} \boxed{C_{(a)\alpha}{}^{b,\beta}\nabla_b} & 0 & \\ 0 & \boxed{C_{(a)\alpha}{}^{b,\beta}\nabla_b'} & \\ & & \ddots \end{pmatrix}$$

Each block represents a universe.

matrix model



quantum gravity

2-3. Low energy effective action

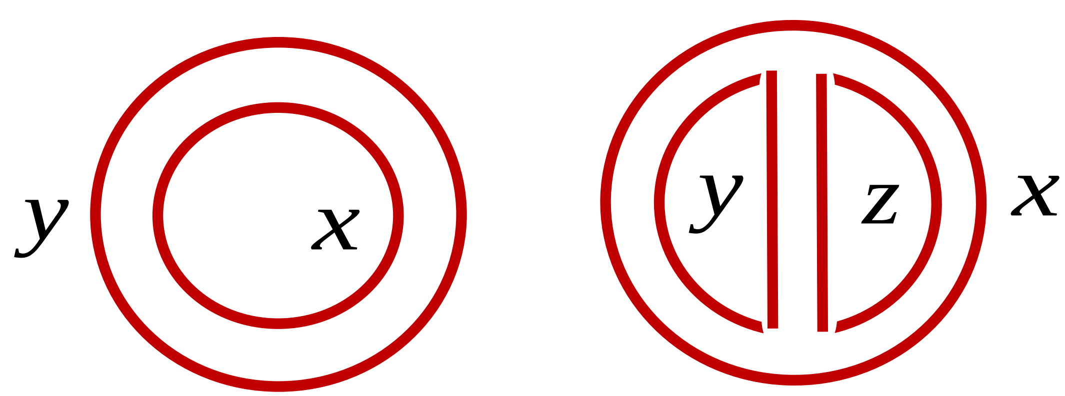
Factorized action from IIB matrix model

Y. Asano, A Tsuchiya, HK

We can calculate the low energy effective action by using the background field method, and we obtain

$$S_{\text{eff}} = \sum_i c_i\, S_i + \sum_{i\,j} c_{i\,j}\, S_i S_j + \sum_{i\,jk} c_{i\,jk}\, S_i S_j S_k + \cdots,$$

$$S_i = \int d^D x \sqrt{g(x)} O_i(x).$$



in the Lorentzian space time

Background field method

In the derivative interpretation, matrices are identified with endomorphisms on $V \cong C^\infty\left(E_{prin}\right)$.

If we introduce a coordinate basis of V

$$\left|x, g\right\rangle, \text{where } \left(x, g\right) \in E_{\text{prin}},$$

they are expressed as bilocal fields on E_{prin} :

$$A_a\left(x, g; y, h\right) = \left\langle x, g\right| A_a \left|y, h\right\rangle.$$

(y,h)

(x,g)

We decompose them to the background and fluctuation:

$$A_a\left(x, g; y, h\right) = A^0{}_{(a)}\left(x, g; y, h\right) + \phi_{(a)}\left(x, g; y, h\right).$$

We further expand the background around flat space:

$$A^0{}_{(a)}\left(x, g; y, h\right) = \left[i\partial_{(a)} + B_{(a)}(x, g) + \frac{1}{2}\left\{h^b_{(a)}(x, g), i\partial_b\right\} + \frac{1}{4}\left\{\omega^{bc}_{(a)}(x, g), O_{bc}\right\} + \cdots \right] \delta\left(x - y\right)\delta_{gh}$$

We can further expand the background by the Peter-Weyl theorem:

$$B_{(a)}(x,g)=\sum_{r:\text{irr.rep}}\sum_{i,j}R_{\langle r\rangle\, i}{}^{(j)}(g)\boxed{B_{(a)\langle r\rangle(j)}{}^{i}(x),}$$

$$h^{b}_{(a)}(x,g)=\sum_{r:\text{irr.rep}}\sum_{i,j}R_{\langle r\rangle\, i}{}^{(j)}(g)\boxed{h^{b}_{(a)\langle r\rangle(j)}{}^{i}(x),}$$

$$\vdots$$

background fields

There are infinite towers of higher spin fields.

We do not expand $\phi_{(a)}$, but treat it as a bi-local field.

(We also decompose the fermionic field similarly.)

Then the action becomes

$$S=\frac{1}{4}Tr\Big(\big[A^0_{(a)},A^0_{(b)}\big]^2+4\big[A^0_{(a)},A^0_{(b)}\big]\big[A^0_{(a)},\phi_{(b)}\big]$$
$$+2\big[A^0_{(a)},\phi_{(b)}\big]^2+\big[A^0_{(a)},A^0_{(b)}\big]\big[\phi_{(a)},\phi_{(b)}\big]-2\big[A^0_{(a)},\phi_{(b)}\big]\big[A^0_{(b)},\phi_{(a)}\big]$$
$$+4\big[A^0_{(a)},\phi_{(b)}\big]\big[\phi_{(a)},\phi_{(b)}\big]+\big[\phi_{(a)},\phi_{(b)}\big]^2+\text{fermion}\Big).$$

The 0-th order term

$$S_0=\frac{1}{4}Tr\Big(\big[A^0_{(a)},A^0_{(b)}\big]^2\Big)$$

can be evaluated by the heat kernel method, which gives a local action:

$$S_{\text{eff}}{}^{(\text{tree})}=S_0=\sum_i c_i S_i,\ \ S_i=\int\sqrt{-g}\,O_i(x).$$

$O_i(x)$ are local field consisting of the background fields.

The one-loop contribution is obtained by the Gaussian integral of the quadratic part.
For simplicity, we consider one hermitian matrix ϕ instead of A_a and ψ, because the mechanism of the factorization is completely captured by this case.

We consider

$$S = -\frac{1}{2} Tr\left(\left[A_a, \phi \right]^2 \right) + \text{interactions},$$

whose quadratic part is given by

$$S_{\phi^2} = -\frac{1}{2} Tr\left(\left[A^{(0)}{}_a, \phi \right]^2 \right).$$

(y,h)

(x,g)

In terms of the bi-local field,

$$S_{\phi^2} = -\frac{1}{2} Tr\left(\left[A^{(0)}{}_a, \phi\right]^2\right)$$

$$= \frac{1}{2}\int d^D x\, d^D y\, dg\, dh \left| \left(\underbrace{R_{(a)}{}^b(g^{-1})\frac{\partial}{\partial x^b} + R_{(a)}{}^b(h^{-1})\frac{\partial}{\partial y^b}}_{\text{Square gives the propagator.}} - i\tilde{A}_{(a)}(x,g;y,h) \right) \phi(x,g;y,h) \right|^2,$$

where

$$\tilde{A}_{(a)}(x,g;y,h) = \tilde{A}_{(a)L}(x,g) + \tilde{A}_{(a)R}(y,h),$$

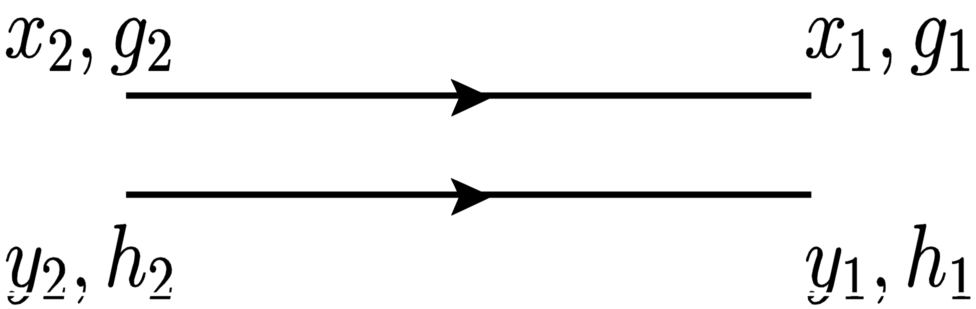
$$\tilde{A}_{(a)L}(x,g) = B_{(a)}(x,g) + \frac{1}{2}\left\{h^b_{(a)}(x,g),\, i\frac{\partial}{\partial x^b}\right\} + \frac{1}{4}\left\{\omega^{bc}_{(a)}(x,g),\, O^{[g]}{}_{bc}\right\} + \cdots,$$

$$\tilde{A}_{(a)R}(y,h) = -B_{(a)}(y,h) + \frac{1}{2}\left\{h^b_{(a)}(y,h),\, i\frac{\partial}{\partial h^b}\right\} + \frac{1}{4}\left\{\omega^{bc}_{(a)}(y,h),\, O^{[h]}{}_{bc}\right\} + \cdots.$$

$\tilde{A}_{(a)L}(x,g)$ and $R_{(a)}{}^b(g^{-1})\frac{\partial}{\partial x^b}$

$\tilde{A}_{(a)R}(y,h)$ and $R_{(a)}{}^b(h^{-1})\frac{\partial}{\partial y^b}$ **are Lorentz scalars.**

Then the propagator is given by



$$
\begin{aligned}
&G\left(x_1, g_1; y_1, h_1 \mid x_2, g_2; y_2, h_2\right) \\
&= \left\langle \phi\left(x_1, g_1; y_1, h_1\right) \phi\left(x_2, g_2; y_2, h_2\right) \right\rangle \\
&= D\left(x_1 - x_2\right) \delta\left(R^{(a)}{}_b (g_1^{-1}) \left(x_1{}^b - x_2{}^b \right) - R^{(a)}{}_b (h_1^{-1}) \left(y_1{}^b - y_2{}^b \right) \right) \delta_{g_1 g_2} \delta_{h_1 h_2} .
\end{aligned}
$$

$D(x)$: scalar propagator

This is invariant under

(1) independent translation on each index line:

$$x_i{}^a \to x_i{}^a + a^a ,$$

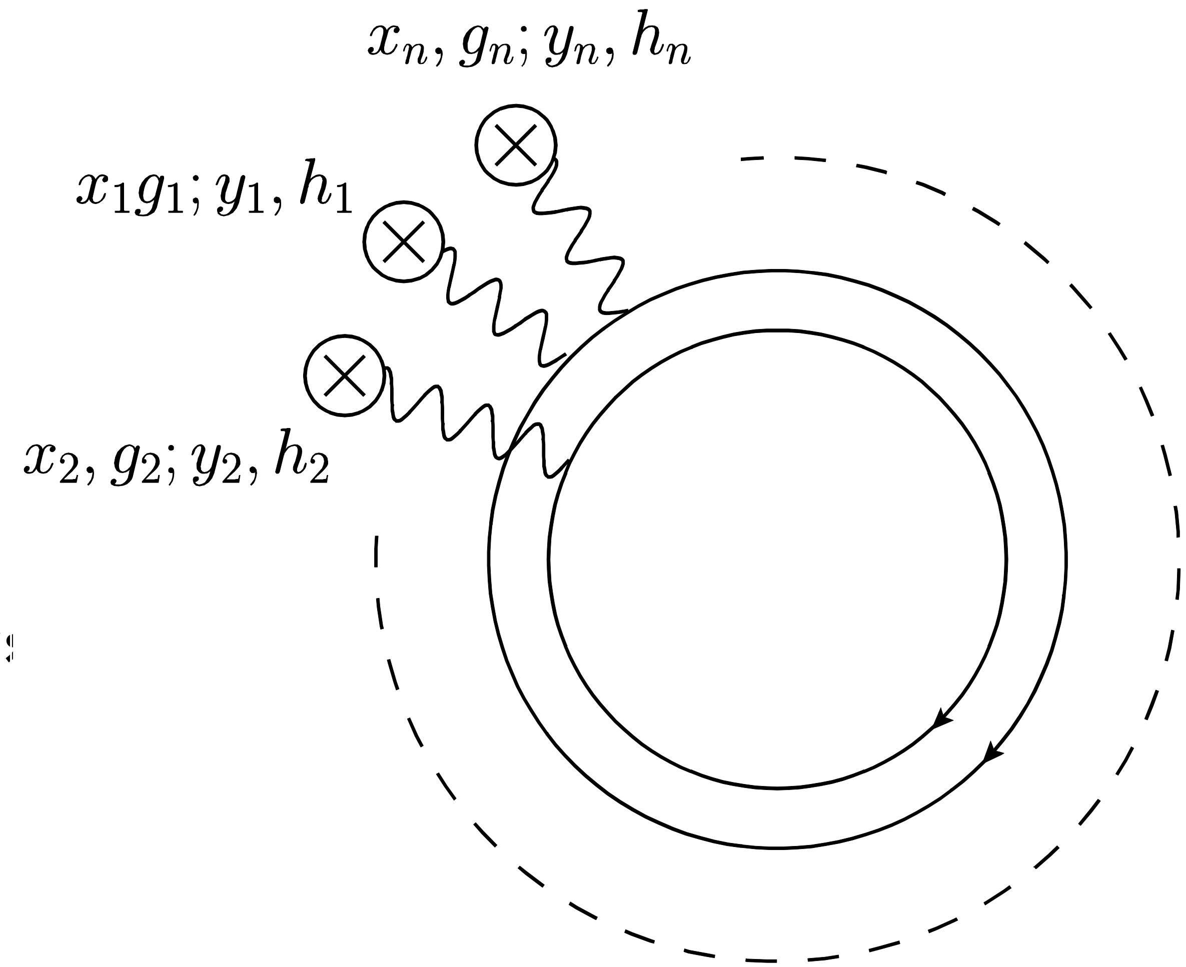
$$y_i{}^a \to y_i{}^a + b^a . \quad (i = 1, 2)$$

(2) independent Lorentz tr. on each index line:

$$x_i{}^a \to R^a{}_b(u) x_i{}^b , \; g_i \to u g_i ,$$

$$y_i{}^a \to R^a{}_b(v) y_i{}^b , \; h_i \to v h_i . \quad (i = 1, 2)$$

The general one-loop diagrams look like the figure, and are given by a sum of terms of the form



$$I = \int d^D x_1 \cdots d^D x_n d^D y_1 \cdots d^D y_n dg_1 \cdots dg_n dh_1 \cdots dh_n \prod_{i=1}^{n} P_i,$$

$$P_i = F_i\left(\left\{A\left(x_j\right), g_j, \frac{\partial}{\partial x_j}, O^{[g_j]}\right\}\right) F_i'\left(\left\{A\left(y_j\right), h_j, \frac{\partial}{\partial y_j}, O^{[h_j]}\right\}\right) G\left(x_i, g_i; y_i, h_i \mid x_{i+1}, g_{i+1}; y_{i+1}, h_{i+1}\right).$$

$A(x)$: differential polynomials of the backgroung fields at x

F_i is a Lorentz scalar because it comes from insertions of scalars $\tilde{A}_{(a)L}(x, g)$ and $R_{(a)}{}^{b}(g^{-1})\dfrac{\partial}{\partial x^b}$. So is F_i' .

We expand the back ground fields on x_i around $x_n (= x)$ such as

$$B_{(a)}(x_i) = \sum_s \frac{1}{s!}\left(x_i^{a_1} - x^{a_1}\right)\cdots\left(x_i^{a_s} - x^{a_s}\right)\partial_{a_1}\cdots\partial_{a_s} B_{(a)}(x).$$

Then F_i becomes a sum of the terms like

$$A_i^{I}(x) F_{i\,I}\left(\left\{\left(x_j - x\right), g_j, \frac{\partial}{\partial x_j}, O^{[g_j]}\right\}\right)$$

Here $A_i^{I}(x)$ is a differential polynomial of the background fields with Lorentz indices I. Similarly for F_i'.

Thus I becomes the sum of the terms

$$I = \int d^D x\, d^D y \int d^D x_1 \cdots d^D x_{n-1} d^D y_1 \cdots d^D y_{n-1} dg_1 \cdots dg_n dh_1 \cdots dh_n$$
$$\cdot A^{I}(x) A^{J}(y) K_{I\,J}\left(\left\{x_j - x, g_j, y_j - y, h_j\right\}\right)$$

Because of the invariance of the propagators , K_{IJ} is

invariant under translation on each index line:

$$x_i{}^a \to x_i{}^a + a^a,$$

$$y_i{}^a \to y_i{}^a + b^a. \quad (i = 1,..,n)$$

covariant under Lorentz tr. on each index line:

$$x_i{}^a \to R^a{}_b(u) x_i{}^b,\ g_i \to u g_i,$$

$$y_i{}^a \to R^a{}_b(v) y_i{}^b,\ h_i \to v h_i. \quad (i = 1,..,n)$$

From this it follows

$$\int d^D x_1 \cdots d^D x_{n-1} d^D y_1 \cdots d^D y_{n-1} dg_1 \cdots dg_n dh_1 \cdots dh_n\, K_{IJ}\left(\left\{x_j - x, g_j, y_j - y, h_j\right\}\right)$$
$$= C_I C'_J,$$

where C_I and C'_J are invariant constant tensors.

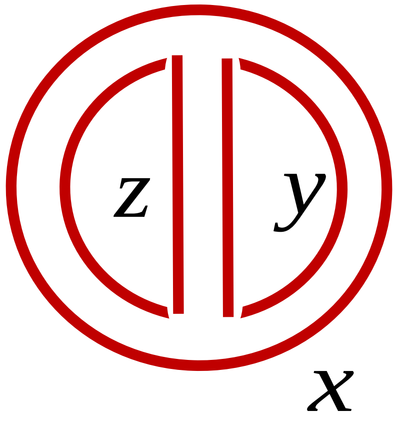
Thus we find that I is factorized into two scalars

$$I = \int d^D x\, d^D y\, A(x) A(y).$$

Finally, because of the diffeomorphism invariance, the terms of the effective action should be combined to

$$S_{\mathrm{eff}}^{\text{1-loop}} = \sum_{i\,j} c_{i\,j} S_i S_j , \quad S_i = \int d^D x \sqrt{g(x)} O_i(x).$$

In the two loop order, from the planar diagrams we have



$$S_{\mathrm{eff}}^{\text{2-loop}} = \sum_{i,j,k} c_{i\,jk} S_i S_j S_k \ ,$$

while non-planar diagrams give

$$S_{\mathrm{eff}}^{\text{2-loop NP}} = \sum_i c_i{}' S_i .$$

Probabilistic interpretation

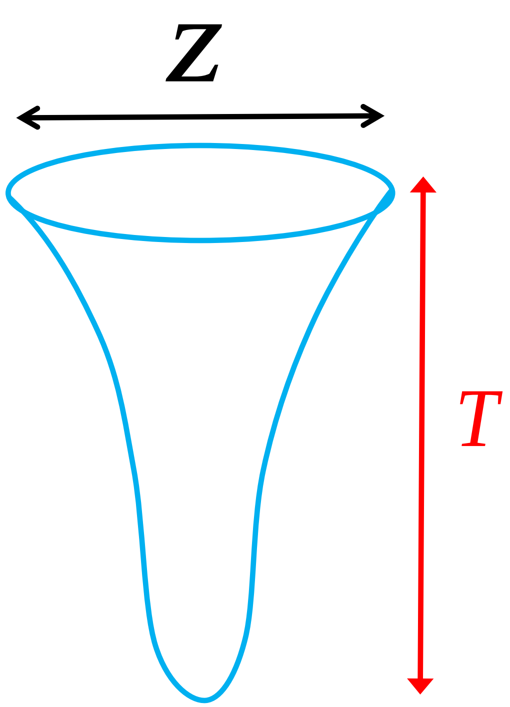
Probabilistic interpretation (1)

postulate $\psi(z) = \mu\,\phi_{E=0}(z)$

$$|\psi(z)|^2 dz \propto \text{probability of finding a universe of size } z$$

meaning of this measure

$$\int dz\,|\phi_{E=0}(z)|^2 \sim \int dz\,\frac{1}{z\,p(z)} \qquad \phi_{E=0}(z) \sim \frac{1}{\sqrt{z\,p(z)}}\exp\left(i\int^{z} dz'\,p(z')\right)$$



$$= \int dz\,\frac{1}{\dot{z}} = \int dT \qquad H = z\left(-\frac{1}{2}p^2 + \cdots\right) \to \dot{z} = \frac{\partial H}{\partial p} = -zp$$

T : age of the universe
the time that has passed
after the universe is created

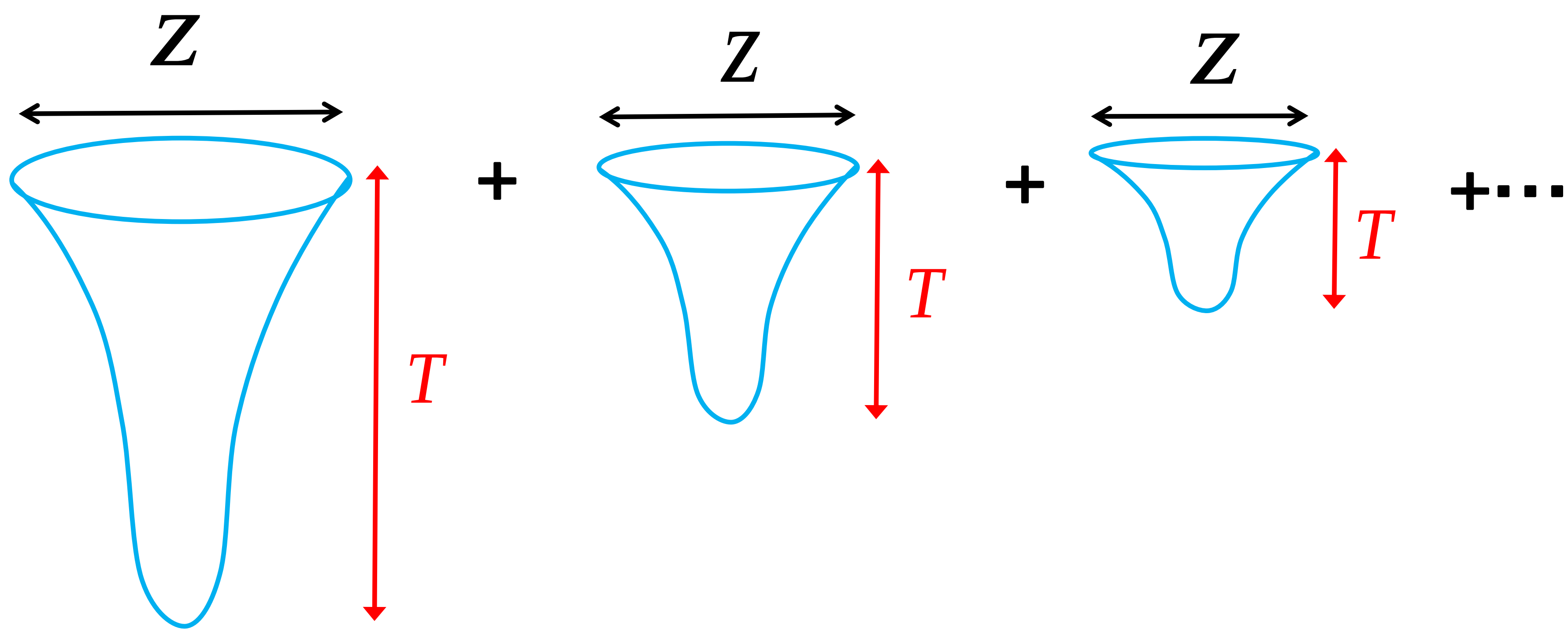
$$\Rightarrow\ |\psi(z)|^2 dz \sim |\mu|^2 dT$$

$|\mu|^2 =$ probability of a universe emerging in unit time

Probabilistic interpretation (2)

$|\psi\rangle$ **is a superposition of the universe with various age,**

$$|\psi\rangle =$$



$|\psi(z)|^2 dz \sim |\mu|^2 dT$ **gives the probability of finding a universe of age** $T \sim T + dT$ **.**

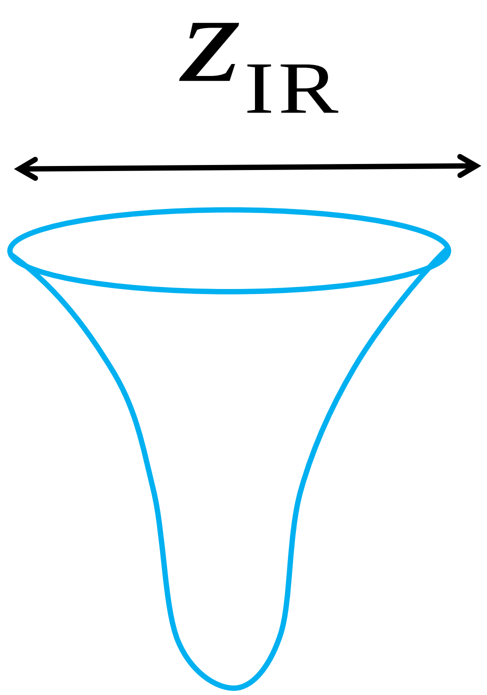
Lifetime of the universe

dimensionless

$$\int |\psi(z)|^2 dz \sim |\mu|^2 \int dT = |\mu|^2 \times (\text{life time of the universe})$$

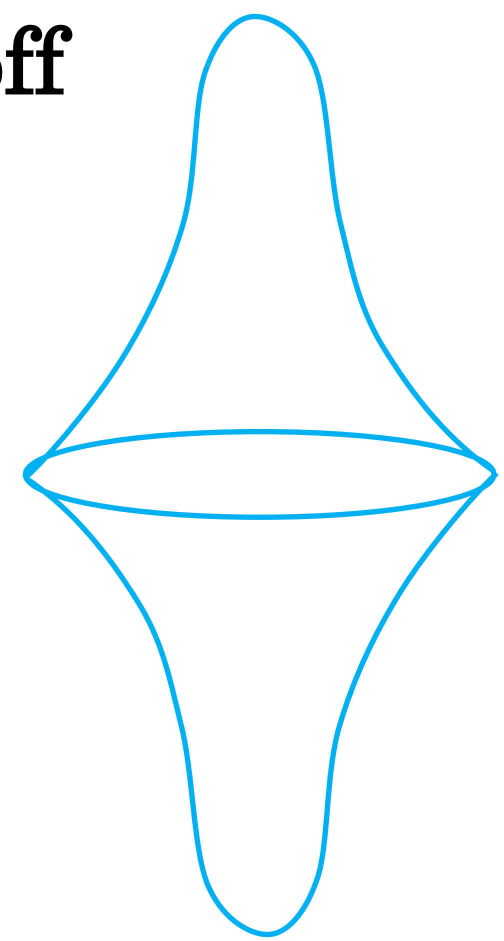
infrared cutoff

We introduce an infrared cutoff for the size of universes.



ceases to exist

or

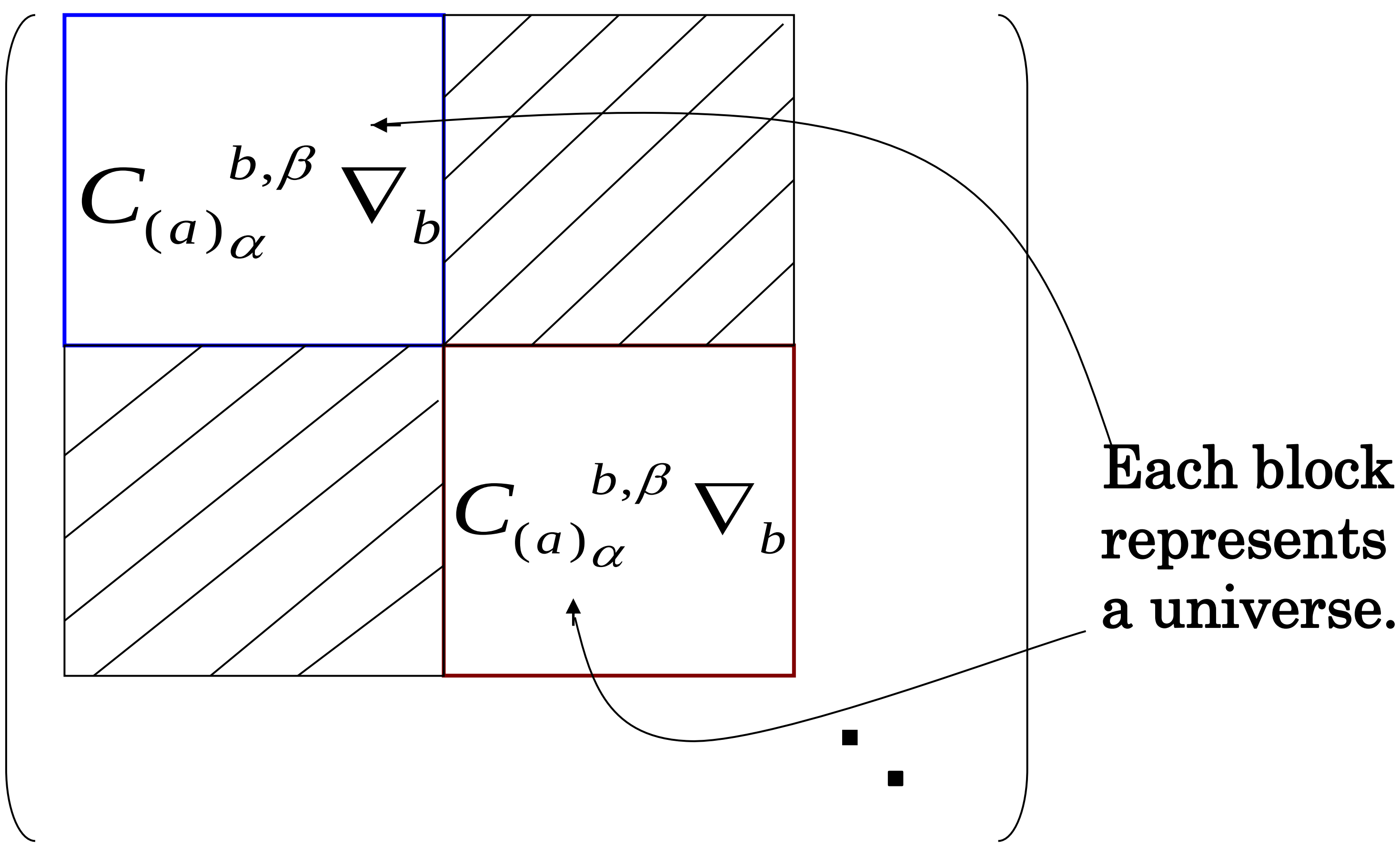


bounces back

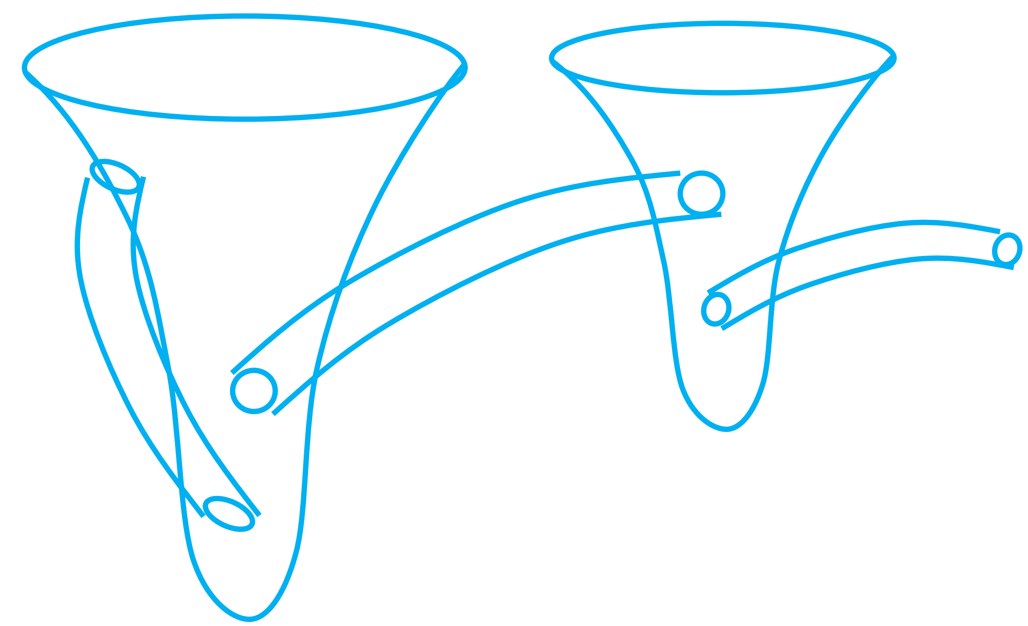
Wave Function of the multiverse (1)

Okada, HK

Multiverse appears naturally in quantum gravity / string theory.



matrix model



quantum gravity

Wave Function of the multiverse (2)

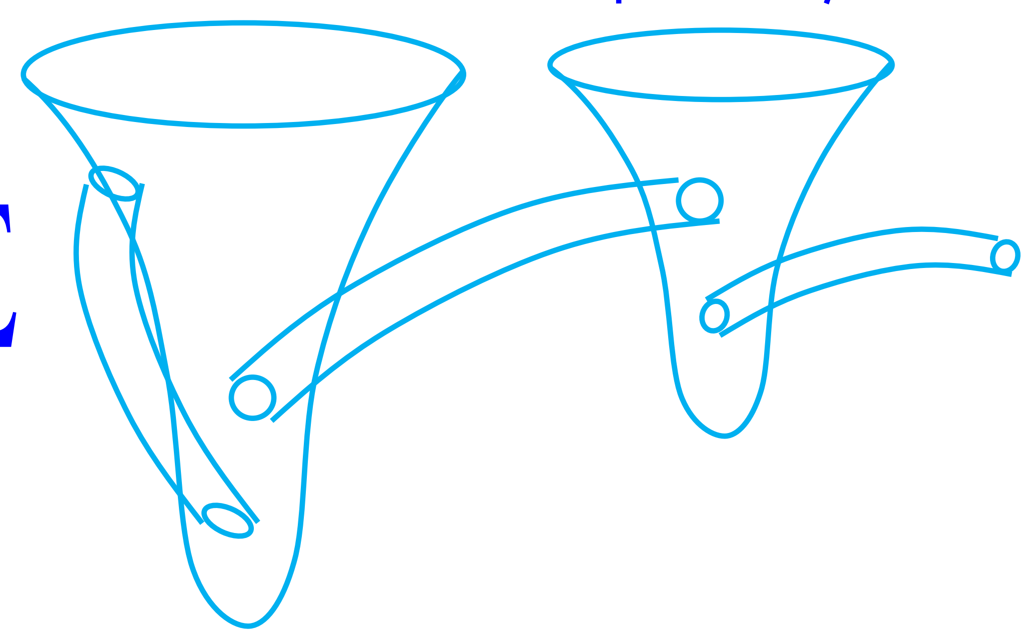
The multiverse sate is a superposition of N-verses.

$$\left|\Psi_{\mathrm{multi}}\right\rangle = \int d\lambda\, w(\lambda) \sum_{N=0}^{\infty} \left|\Psi_N, \lambda\right\rangle \otimes \left|\lambda\right\rangle \quad \leftarrow Z = \int d\lambda w(\lambda) \int [d\phi] \exp\left(i \sum_i \lambda_i S_i \right)$$

$$\Psi_N \left(z_1, \cdots, z_N, \lambda\right) = \psi\left(z_1, \lambda\right) \cdots \psi\left(z_N, \lambda\right)$$

$$\left|\psi, \lambda\right\rangle \otimes \left|\psi, \lambda\right\rangle \otimes \cdots$$

$$\left|\Psi_{\mathrm{multi}}\right\rangle = \int d\lambda\, w(\lambda) \sum_N$$



Wave Function of the multiverse (3)

Probabilistic interpretation

$$\left|\Psi_{\text{multi}}\right\rangle = \int d\lambda\, w(\lambda) \sum_{N=0}^{\infty} \left|\Psi_N, \lambda\right\rangle \otimes \left|\lambda\right\rangle$$

$$\Psi_N(z_1, \cdots, z_N, \lambda) = \psi(z_1, \lambda) \cdots \psi(z_N, \lambda)$$

$\left|\Psi_N(z_1, \cdots, z_N, \lambda)\right|^2 dz_1 \cdots dz_N \left|w(\lambda)\right|^2 d\lambda$ **represents**

the probability of finding N universes with size

$$z_1 \sim z_1 + dz_1,\ \cdots,\ z_N \sim z_N + dz_N$$

and finding the coupling constants in

$$\lambda \sim \lambda + d\lambda.$$

Probability distribution of λ

$$P(\lambda) = \sum_{N=0}^{\infty} \int \frac{dz_1 \cdots dz_N}{N!} \left| \Psi_N(z_1, \cdots, z_N, \lambda) \right|^2 \left| w(\lambda) \right|^2$$

$$= \exp\left(\int dz \left| \psi(z, \lambda) \right|^2 \right) \left| w(\lambda) \right|^2 \qquad \leftarrow \Psi_N(z_1, \cdots, z_N, \lambda) = \psi(z_1, \lambda) \cdots \psi(z_N, \lambda)$$

$$= \exp\left(\left| \mu \tau(\lambda) \right|^2 \right) \left| w(\lambda) \right|^2 \qquad \leftarrow \left| \psi \right\rangle = \mu \left| \phi_{E=0} \right\rangle$$

$$\tau(\lambda) = \int dz \left| \phi_{E=0}(z, \lambda) \right|^2 \sim \text{(life time of the universe)}$$

$\tau(\lambda)$ **can be very large.**

$\Downarrow$

λ **is chosen in such a way that** $\tau(\lambda)$ **is maximized, irrespectively of** $w(\lambda)$.

We have seen

the coupling constants are chosen in such a way that the lifetime of the universe becomes maximum.

<u>the question</u>

What values of the coupling constants make the lifetime maximum?

Big Fix in prob. Int.

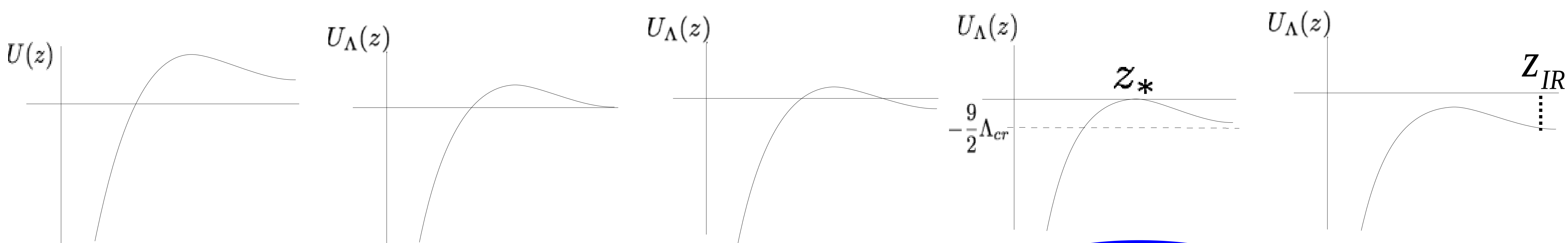
Cosmological constant

What value of Λ maximizes $\int dz \left|\mu\,\phi_{E=0}(z,\lambda)\right|^2$?

WKB sol $\phi_{E=0}(z,\lambda) \sim \dfrac{1}{\sqrt{z\,p(z,\lambda)}}$ with $p(z,\lambda) = \sqrt{-2U(z)}.$ S^3 topology

$$U(z) = \frac{1}{z^{2/3}} - \Lambda - \frac{C_{matt}}{z} - \frac{C_{rad}}{z^{4/3}}$$

assuming all matters decay to radiation



$\Lambda < 0$ | $\Lambda = 0$ | $0 < \Lambda < \Lambda_{cr}$ | $\Lambda = \Lambda_{cr}$ | $\Lambda_{cr} < \Lambda$

The cosmological constant in the far future is predicted to be very small.

Λ ~curvature~energy density

$\Lambda_{cr} \sim 1/C_{rad}$ (extremely small)

The other couplings (Big Fix)

$$P(\lambda) = \exp\left(\left|\mu\tau(\lambda)\right|^2\right)\left|w(\lambda)\right|^2 \qquad \leftarrow \tau(\lambda) = \int dz \left|\phi_{E=0}(z,\lambda)\right|^2$$

The exponent is divergent, and regulated by the IR cutoff :

$$\int dz \left|\phi_{E=0}(z,\lambda)\right|^2 \sim \int_0^{z_{IR}} \frac{1}{z\sqrt{\Lambda_{\mathrm{cr}}}} \sim \sqrt{C_{\mathrm{rad}}}\log z_{IR}. \qquad \leftarrow \Lambda_{\mathrm{cr}} \sim 1/C_{\mathrm{rad}}$$

assuming all matters decay to radiation

BIG FIX

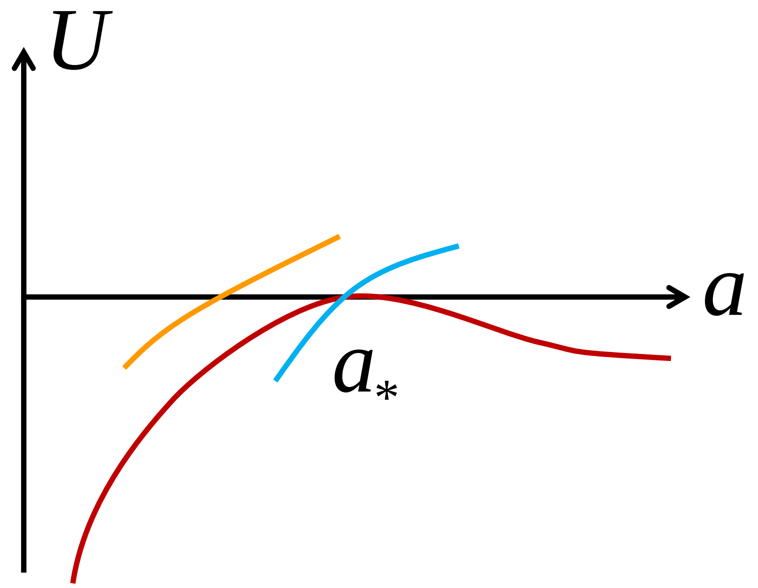
λ are determined in such a way that $C_{\mathrm{rad}}(\lambda)$
is maximized.

Example of the Big Fix

<u>non-trivial example</u> QCD coupling or proton mass m

We assume that dark matters decay faster than protons, and do not consider matter dominant era by leptons after the protons decay.

$$U(a) = \frac{1}{a^2} - \Lambda - \frac{C_{\mathrm{matt}}}{a^3} - \frac{C_{\mathrm{rad}}}{a^4}$$



If the curvature term balances with matter before the proton decay, the universe bounce back when the protons decay.

The earlier the protons decay, the less C_{rad} remains.

C_{rad} is maximized if the curvature term balances with the energy density when the protons decay.

Example of the Big Fix (cont'd)

The curvature term balances with the energy density when the protons decay.

$$\frac{1}{a_*^{\,2}} = \frac{GM}{a_*^{\,3}}, \; M = N_B m.$$

$$a_* = \left(GM\tau^2\right)^{\frac{1}{3}}$$

N_B : total baryon number

m : proton mass

τ : proton life time

$$\Rightarrow \quad \tau = GM$$

$$\tau = \frac{m_{GUT}^{\;4}}{g^4 m^5}, \; GM = \frac{N_B m}{m_P^{\;2}}$$

$$\Rightarrow \quad m^6 = \frac{m_P^{\;2} m_{GUT}^{\;4}}{g^4 N_B}$$

$$N_B = \frac{m_P^{\;2} m_{GUT}^{\;4}}{g^4 m^6} \sim 10^{105}$$

Cf. 10^{78} protons in $\left(10^{10}\ \mathrm{ly}\right)^3$ **in our universe**

$$\Rightarrow \quad a_{\text{present}} = 10^9 \times 10^{10}\ \mathrm{ly}$$

Reasonable?