アクシオン暗黒物質の熱化過程の解析

Ken'ichi Saikawa
Tokyo Institute of Technology (Titech)

Collaborate with T. Noumi (RIKEN), R. Sato (KEK) and M. Yamaguchi (Titech)

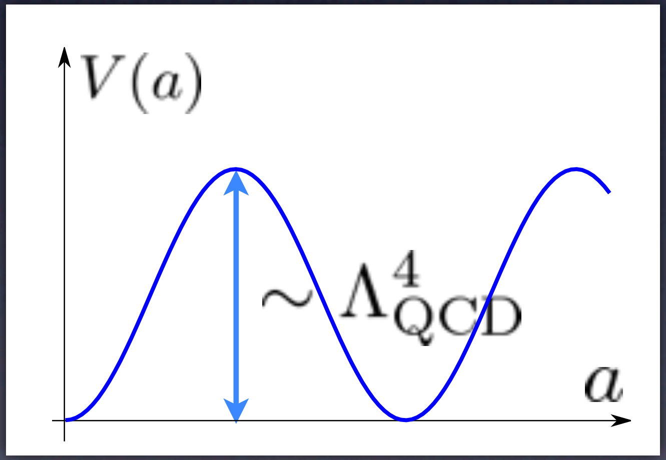
based on
[1] KS, M. Yamaguchi, hep-ph/1210.7080. [PRD87, 085010 (2013)]
[2] T. Noumi, KS, R. Sato, M. Yamaguchi, (in prep.)

Axion

- motivated as a solution of strong CP problem
- Spontaneous breaking of continuous Peccei-Quinn symmetry at $T \simeq F_a \simeq 10^{9-12}\mathrm{GeV}$ "axion decay constant"
  - Nambu-Goldstone theorem
    → emergence of the (massless) particle ≡ axion
    Weinberg(1978), Wilczek(1978)
- Axion has a small mass (QCD effect)
  → pseudo-Nambu-Golstone boson

$$m_a \sim \frac{\Lambda_{\mathrm{QCD}}^2}{F_a} \sim 6 \times 10^{-6}\mathrm{eV}\left(\frac{10^{12}\mathrm{GeV}}{F_a}\right)$$

$$\Lambda_{\mathrm{QCD}} \simeq \mathcal{O}(100)\mathrm{MeV}$$

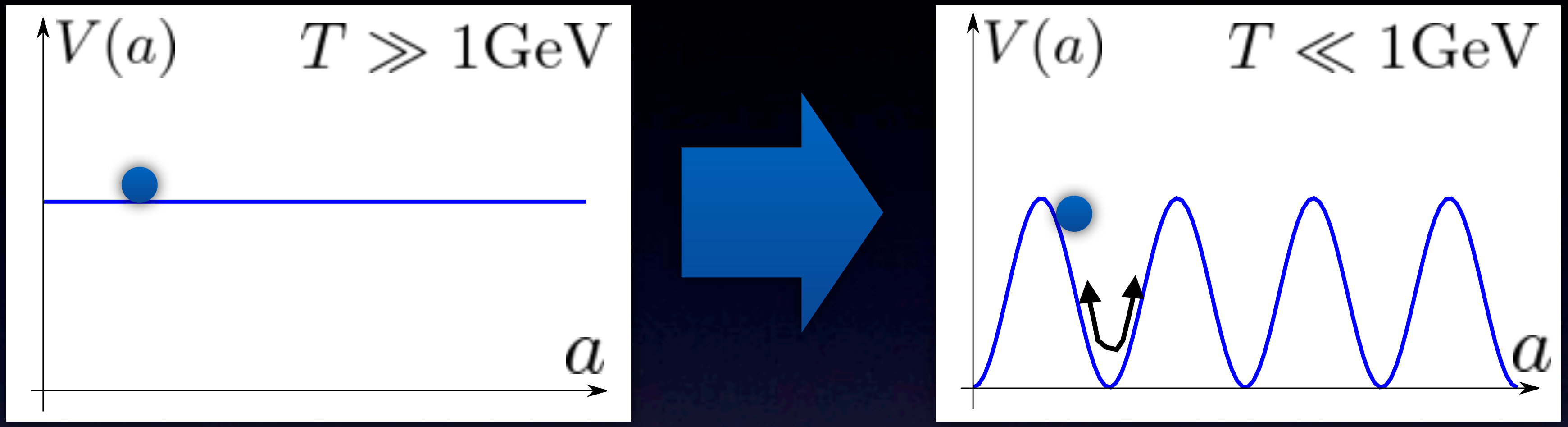


- Tiny coupling with matter + non-thermal production
  → good candidate of cold dark matter

Production mechanism

Preskill, Wise, Wilczek (1983); Abbott, Sikivie (1983); Dine, Fischler (1983)

- Misalignment mechanism



The axion mass "turns on" at $m_a(t_1) = H(t_1)$ $(T_1 \sim 1\mathrm{GeV})$

- EOM for homogeneous axion field

$$\left(\frac{d^2}{dt^2} + \frac{3}{2t}\frac{d}{dt} + m_a^2\right)\langle a\rangle = 0$$

$$\Rightarrow \quad m_a A^2 \propto R^{-3}(t) \quad , \quad \langle a\rangle = A(t)\cos(m_a t)$$

$R(t)$: scale factor of the universe

$$\Rightarrow \quad \rho_a(t) = \frac{1}{2}m_a^2\langle a\rangle^2 \propto R^{-3}(t)$$

behave like non-relativistic matter

Axion BEC dark matter ?

- Peculiarities of axion dark matter
  - Non-thermal production

$$H \lesssim m_a \quad (t = t_1) \qquad t_1 \sim 10^{-7}\mathrm{sec}$$

$$\delta v \sim \frac{\delta p}{m_a} \sim \frac{R(t_1)}{R(t_0)} \frac{1}{m_a t_1} \sim 3 \times 10^{-17} \left(\frac{F_a}{10^{12}\mathrm{GeV}} \right)^{0.81}$$

small velocity dispersion ("cold" dark matter)

  - Large occupation number

$$\mathcal{N} \sim n_a \frac{(2\pi)^3}{\frac{4\pi}{3}(m_a \delta v)^3} \sim 10^{61} \left(\frac{F_a}{10^{12}\mathrm{GeV}} \right)^{2.75}$$

($n_a \sim m_a F_a^2 (R(t_1)/R(t_0))^3$: number density of axions)

- A possibility that axions exist in the form of Bose-Einstein condensate (BEC) Sikivie, Yang, PRL103, 111301 (2009)
- Observable signatures (distinction between axions and WIMPs) ?
  - Effects on phase space structure of galactic halo (?)

    Sikivie, Phys. Lett. B695, 22 (2011)
  - Effects on cosmological parameters (?)

    Erken, Sikivie, Tam, Yang, PRL108, 061304 (2012)

Axions can form a BEC if...

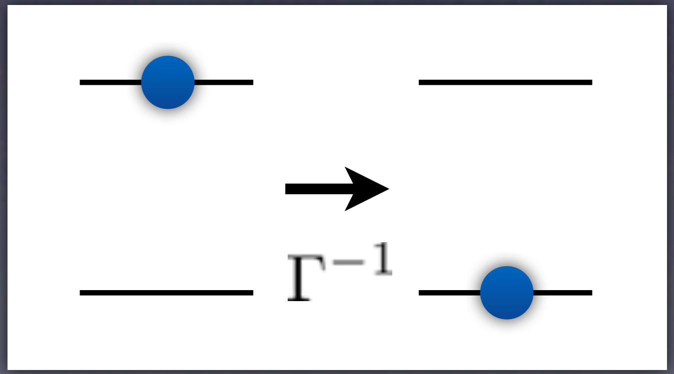
		For axions
1. Particles are bosons	→	satisfied
2. Number is conserved	→	satisfied
3. Large occupation number	→	satisfied
4. In thermal equilibrium	→	???

- Can axions thermalize in the expanding universe ?
- Naive expectation :
  develop toward thermal equilibrium if
  the transition rate Γ exceeds the expansion rate H

$$\Gamma \sim \dot{\mathcal{N}}_{\mathbf{p}}/\mathcal{N}_{\mathbf{p}} > H$$

$\mathcal{N}_{\mathbf{p}}$: occupation number

(state labeled by three momentum $\mathbf{p}$)



Previous study

Erken, Sikivie, Tam, Yang, PRD85, 063520 (2012)

- Time evolution of quantum operators in the Heisenberg picture

$$H = \sum_i \omega_i a_i^\dagger a_i + \sum_{i,j,k,l} \frac{1}{4} \Lambda_{kl}^{ij} a_k^\dagger a_l^\dagger a_i a_j$$

l : label of the state (momentum)

$\mathcal{N}_l = a_l^\dagger a_l$

$$\dot{\mathcal{N}}_l = i[H, \mathcal{N}_l]$$

$$= i \sum_{i,j,k} \frac{1}{2} (\Lambda_{ij}^{kl} a_i^\dagger a_j^\dagger a_k a_l e^{-i\Omega_{ij}^{kl} t} - \text{H.c.})$$

$$+ \sum_{k,i,j} \frac{1}{2} |\Lambda_{ij}^{kl}|^2 [\mathcal{N}_i \mathcal{N}_j (\mathcal{N}_l + 1)(\mathcal{N}_k + 1)$$

$$- \mathcal{N}_l \mathcal{N}_k (\mathcal{N}_i + 1)(\mathcal{N}_j + 1)] \frac{2}{\Omega_{ij}^{kl}} \sin(\Omega_{ij}^{kl} t) + \ldots$$

$$\Omega_{ij}^{kl} \equiv \omega_k + \omega_l - \omega_i - \omega_j$$

Leading contribution in the condensed regime

$\Omega_{ij}^{kl} t \ll 1$

$\dot{\mathcal{N}}_l \sim \mathcal{O}(\Lambda_{ij}^{kl})$

reduce to Boltzmann eq. in the particle kinetic regime

$\Omega_{ij}^{kl} t \gg 1$

$\dot{\mathcal{N}}_l \sim \mathcal{O}(|\Lambda_{ij}^{kl}|^2)$

- Axions : condensed regime ($\Omega_{ij}^{kl} \sim m_a \delta v^2 < t^{-1}$)

→ enhancement of interaction rate $\Gamma \sim \dot{\mathcal{N}}/\mathcal{N} \sim \mathcal{O}(\Lambda)$

- What about the quantum-mechanical averages $\langle \dot{\mathcal{N}}_l(t) \rangle$?

Re-consider the thermalization process

KS, Yamaguchi, PRD87, 085010 (2013)

- Develop analytic methods to calculate the time evolution of the expectation value of the operator

$$\langle \mathrm{in}|\mathcal{O}(t)|\mathrm{in}\rangle = \langle \mathcal{O}\rangle + i\int_{t_0}^{t} dt_1 \langle [H_I(t_1), \mathcal{O}]\rangle + (i)^2 \int_{t_0}^{t} dt_2 \int_{t_0}^{t_2} dt_1 \langle [H_I(t_1), [H_I(t_2), \mathcal{O}]]\rangle + \dots$$

$\mathcal{O} = \mathcal{N}_n = \dfrac{a_n^\dagger a_n}{V}$: number operator $\qquad H_I(t)$: interaction Hamiltonian

- Specify the "in" (initial) state

V : volume of the 3-dim space

α_i : some complex value

For axions "wavy fields"

use a coherent state $|\mathrm{in}\rangle = |\{\alpha\}\rangle = \prod_i e^{-\frac{1}{2}|\alpha_i|^2} \sum_{n=0}^{\infty} \dfrac{\alpha_i^n}{n!\sqrt{V^n}} (a_i^\dagger)^n |0\rangle$

with $a_i|0\rangle = 0$

For other species "point particles"

use a number state $|\mathrm{in}\rangle = |\{\mathcal{N}\}\rangle = \prod_k \dfrac{1}{\sqrt{\mathcal{N}_k! V^{\mathcal{N}_k}}} (a_k^\dagger)^{\mathcal{N}_k} |0\rangle$

Evolution of occupation number

KS, Yamaguchi, PRD87, 085010 (2013)

$$\langle \text{in}|\mathcal{N}_p(t)|\text{in}\rangle = \langle \mathcal{N}_p\rangle + i\int_{t_0}^{t} \langle [H_I(t_1), \mathcal{N}_p]\rangle + \mathcal{O}(H_I^2) + \dots$$

$$i\int_{t_0}^{t} dt_1 \langle [H_I(t_1), \mathcal{N}_p]\rangle \xrightarrow{t-t_0\to\infty} -\frac{1}{2V^2}\sum_j\sum_k\sum_l \left[\Lambda_{kl}^{pj}\frac{e^{-i\Omega_{kl}^{pj}t}}{\Omega_{kl}^{pj}}\alpha_k^*\alpha_l^*\alpha_j\alpha_p + \text{c.c.}\right]$$

for $\quad |\text{in}\rangle = \prod_i e^{-\frac{1}{2}|\alpha_i|^2}\sum_{n=0}^{\infty}\frac{\alpha_i^n}{n!\sqrt{V^n}}(a_i^\dagger)^n|0\rangle$

coherent state

$$i\int_{t_0}^{t} dt_1 \langle [H_I(t_1), \mathcal{N}_p]\rangle = 0$$

for $\quad |\text{in}\rangle = \prod_k \frac{1}{\sqrt{\mathcal{N}_k! V^{\mathcal{N}_k}}}(a_k^\dagger)^{\mathcal{N}_k}|0\rangle$

number state

- First order term is relevant if

  (1) condensed regime $\Omega_{kl}^{pj}t \ll 1$

  (c.f. $e^{-i\Omega_{kl}^{pj}t} \approx 0$ for particle kinetic regime $\Omega_{kl}^{pj}t \gg 1$)

  (2) coherent state representation $|\text{in}\rangle = |\{\alpha\}\rangle$

Transition rate

KS, Yamaguchi, PRD87, 085010 (2013)

- Transition rate of coherently oscillating components

$$\Gamma \equiv \frac{1}{\mathcal{N}_p(t)} \frac{d\mathcal{N}_p(t)}{dt} \simeq \Lambda n_a$$

n_a : number density of axions

$\Lambda^{kl}_{pj} = \Lambda V \delta_{k+l,p+j}$: coefficient in the interaction term

scalar self-coupling

$$H_I = -\int d^3x \frac{\lambda}{4!} a^4 \Rightarrow \Gamma_s \simeq \frac{\lambda n_a}{4m_a^2} \propto 1/R^3(t)$$

gravity

$$H_I = -\frac{G}{2}\int d^3x d^3x' \frac{\rho(\mathbf{x},t)\rho(\mathbf{x}',t)}{|\mathbf{x}-\mathbf{x}'|} \Rightarrow \Gamma_g \simeq \frac{4\pi G m_a^2 n_a}{(\delta p)^2} \propto 1/R(t)$$

$\rho(\mathbf{x},t)$: energy density of axions

$$\delta p \sim m_a \delta v \propto 1/R(t)$$

- Exceed the expansion rate at

$$\Gamma_g \gtrsim H \Rightarrow T \simeq 2\times 10^3 \mathrm{eV} \left(\frac{F_a}{10^{12}\mathrm{GeV}}\right)^{0.56}$$

Formation of BEC at $T \sim \mathrm{keV}$?

Effective interaction in general relativity

Noumi, KS, Sato, Yamaguchi, in prep.

- Calculations in previous slides : assumed Newtonian approximation
- Reformulate in general relativistic framework

$$S = \int d^4x \sqrt{-g} \Big[\underbrace{-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m^2 \phi^2}_{\text{axion}}$$

$$+ \frac{1}{2} M_{\rm Pl}^2 \mathcal{R} + \underbrace{M_{\rm Pl}^2 \dot{H} g^{00} - M_{\rm Pl}^2 (3H^2 + \dot{H})}_{\text{background fields (radiations)}} \Big]$$

- ADM decomposition

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

$$h_{ij} = R^2(t) \underbrace{e^{2\zeta} (e^\gamma)_{ij}}_{\text{fluctuations around FRW background}}$$

$\zeta, \ \gamma_{ij}$: dynamical fields

$N, \ N^i$: Lagrange multipliers

Effective quatic interactions

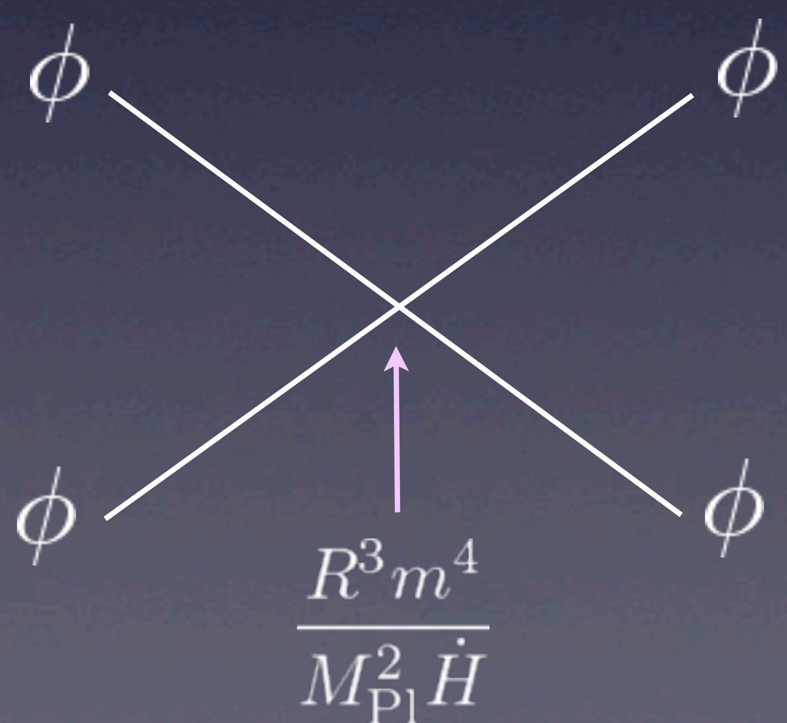
Noumi, KS, Sato, Yamaguchi, in prep.

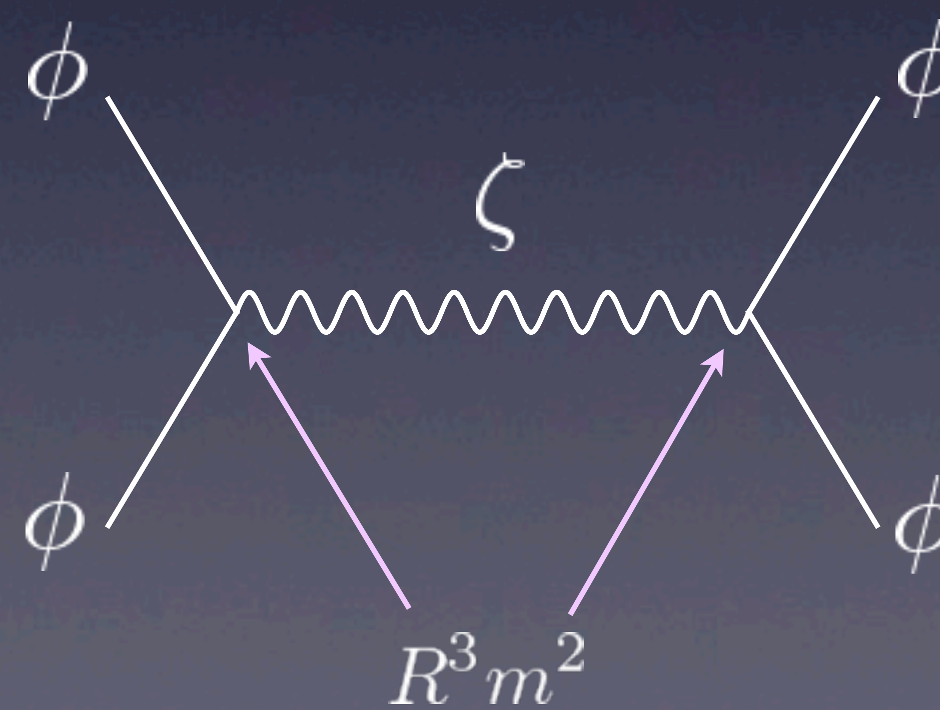
- Eliminating auxiliary fields $N, \ N^i$

$$H_{I,\zeta\phi^2} \simeq \int d^3x R^3 \left[\frac{1}{2H} \dot{\zeta}(\dot{\phi}^2 + m^2\phi^2) - \frac{3}{2}\zeta(\dot{\phi}^2 - m^2\phi^2) \right] + \mathcal{O}(H^2\zeta\phi^2)$$

$$H_{I,\phi^4} \simeq \int d^3x R^3 \left[- \frac{1}{16 M_{\rm Pl}^2 \dot{H}} (\dot{\phi}^2 + m^2\phi^2)^2 \right] + \mathcal{O}(m^2\phi^4/M_{\rm Pl}^2)$$

$$H_{I,\gamma\phi^2} = \int d^3x R^3 \left[-\frac{1}{2}\gamma_{ij}\frac{\partial_i\phi\partial_j\phi}{R^2} \right] \sim \mathcal{O}(H^2\gamma\phi^2)$$ (subdominant)





Contributions for $\langle \mathcal{N}_{\mathbf{p}}(t) \rangle$

Noumi, KS, Sato, Yamaguchi, in prep.

$$\langle \mathcal{N}_{\mathbf{p}}(t) \rangle = \langle \mathcal{N}_{\mathbf{p}}(t_0) \rangle + \underbrace{i \int_{t_0}^{t} dt_1 \langle [H_{I,\phi^4}(t_1), \mathcal{N}_{\mathbf{p}}] \rangle}_{(I)}$$

$$\underbrace{+ i^2 \int_{t_0}^{t} dt_2 \int_{t_0}^{t_2} dt_1 \langle [H_{I,\zeta\phi^2}(t_1), [H_{I,\zeta\phi^2}(t_2), \mathcal{N}_{\mathbf{p}}]] \rangle + \ldots}_{(II)}$$

$$(I) \equiv \int \frac{d^3k_1}{(2\pi)^3} \int \frac{d^3k_2}{(2\pi)^3} V \alpha^*_{\mathbf{k}_1} \alpha^*_{\mathbf{k}_2} \alpha_{|\mathbf{k}_1+\mathbf{k}_2-\mathbf{p}|} \alpha_{\mathbf{p}} I_{\phi^4} + \ldots$$

$$(II) \equiv \int \frac{d^3k_1}{(2\pi)^3} \int \frac{d^3k_2}{(2\pi)^3} V \alpha^*_{\mathbf{k}_1} \alpha^*_{\mathbf{k}_2} \alpha_{|\mathbf{k}_1+\mathbf{k}_2-\mathbf{p}|} \alpha_{\mathbf{p}} I_{\zeta\phi^2} + \ldots$$

- For modes inside the horizon ($|\mathbf{k}_1 - \mathbf{p}|\tau \gg 1$)

$d\tau = dt/R$: conformal time

$$I_{\zeta\phi^2} \to -\frac{9im^2}{16M_{\rm Pl}^2} \frac{\tau}{|\mathbf{k}_1 - \mathbf{p}|^2}$$

→ reproduces transition rate in Newtonian approx.

$$\Gamma \sim \frac{m^2 n_a}{M_{\rm Pl}^2 (\delta p)^2}$$

(I_{ϕ^4} is suppressed by $\mathcal{O}((\delta p)^4/H^2m^2)$)

- For modes outside the horizon ($|\mathbf{k}_1 - \mathbf{p}|\tau \ll 1$)

$I_{\phi^4},\ I_{\zeta\phi^2} \to$ suppressed as $\mathcal{O}(|\mathbf{k}_1 - \mathbf{p}|^2\tau^2)$

Summary

- Estimate transition rate of axions in coherent state
- Gravitational self-interaction of cold axions becomes relevant at $T \sim \mathcal{O}(1)\mathrm{keV}$

  → formation of axion BEC (?)
- Reanalyze in general relativistic framework
  - Derive effective quartic interactions of massive scalar fields
  - Modes inside the horizon can contribute to the interactions (and thermalization process)