General Holomorphic Pure Supergravity and Its Cosmology

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Sergei V. Ketov and Takahiro Terada, JHEP07(2013)127, arXiv:1304.4319[hep-th]

起

超重力理論の拡張

修正重力理論の超対称化

位置づけ

Simple

Supersymmetrization →

General

← Bosonic sector

R

- A. Einstein, 1915

$f(R)$ gravity

- A. De Felice and S. Tsujikawa, Living Rev. Rel. 13 (2010) 3 and references therein.

$f(R, C)$ gravity **NEW**

- Our work

$\mathcal{R}$

- D. Z. Freedman, P. van Nieuwenhuizen and S. Ferrara, Phys. Rev. D 13 (1976) 3214;
- S. Deser and B. Zumino, Phys. Lett. B 62 (1976) 335.

$F(\mathcal{R})$ supergravity

- S.J. Gates, Jr. and S.V. Ketov, Phys. Lett. B674 (2009) 59

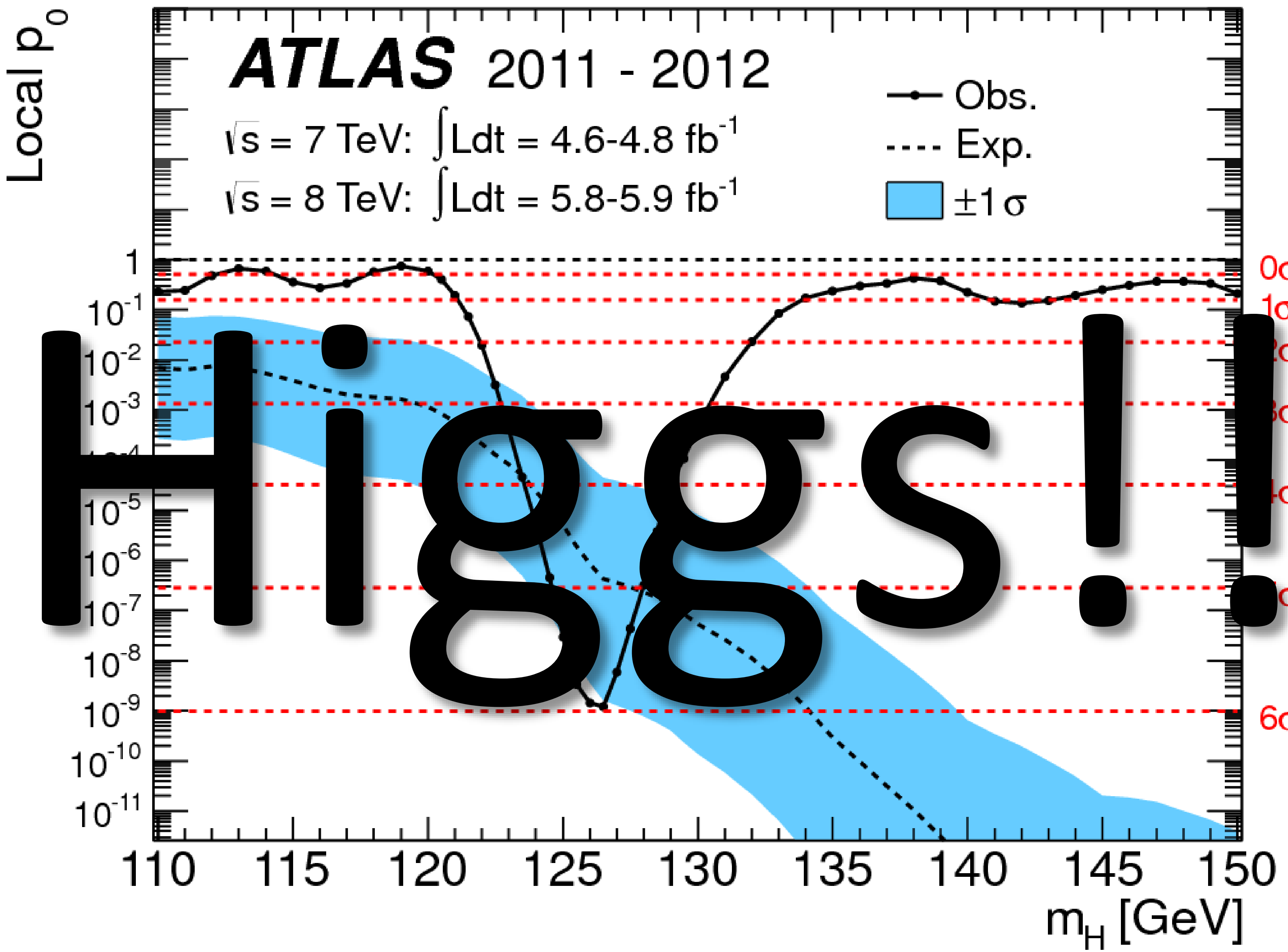
$F(\mathcal{R}, \mathcal{W})$ supergravity **NEW**

- Our work

どんな修正重力理論が
超対称性と整合的か？

超重力理論の拡張によって
どんな修正重力理論が導かれるか？


LHC!

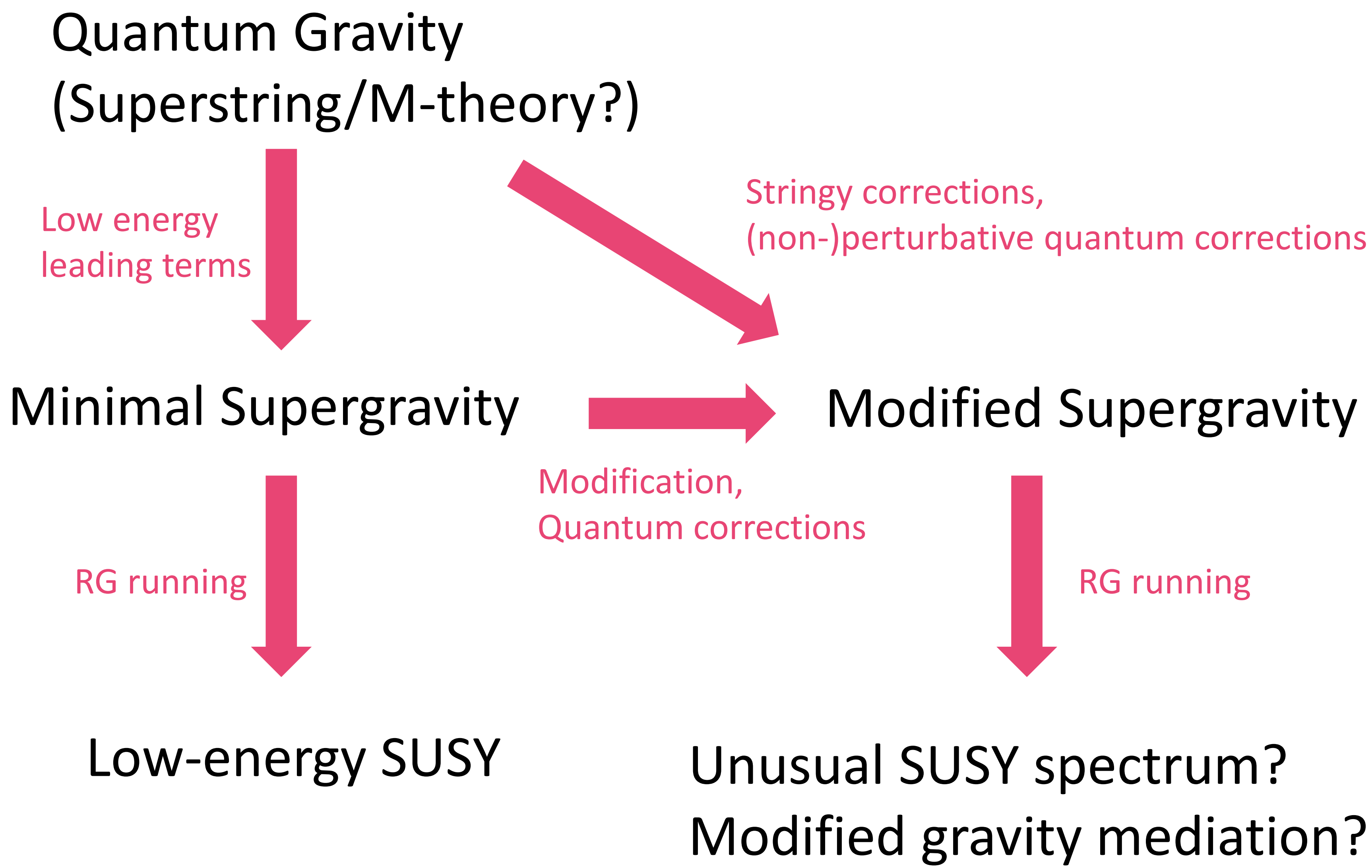

Higgs!!
ATLAS 2011 - 2012
√s = 7 TeV: ∫Ldt = 4.6-4.8 fb⁻¹
√s = 8 TeV: ∫Ldt = 5.8-5.9 fb⁻¹
Obs.
Exp.
±1 σ
Local p0
mH [GeV]
0σ
1σ
6σ
110
115
120
125
130
135
140
145
150

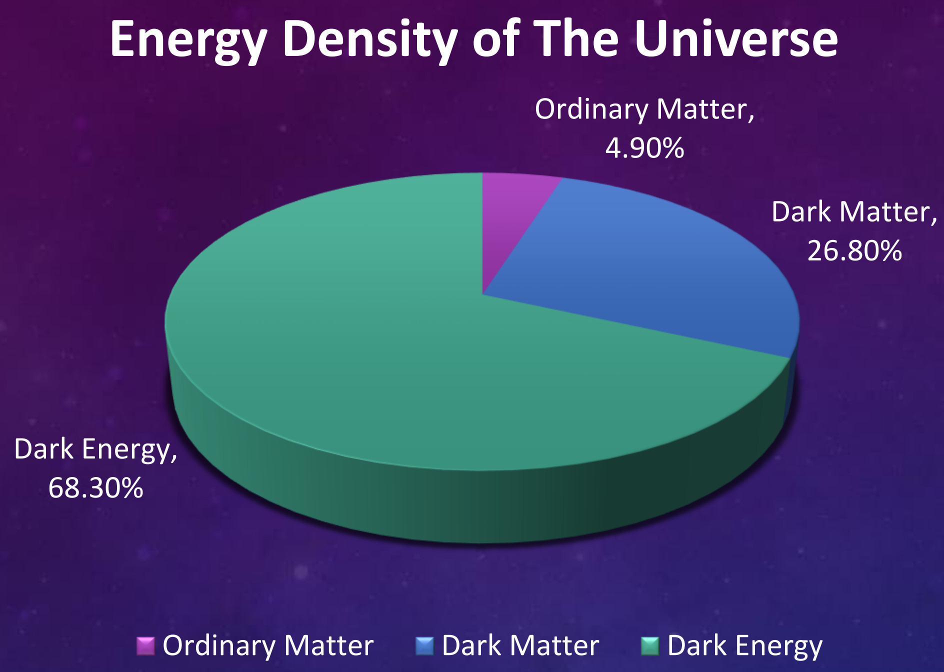

SUSY!!!


SUSY!


SUSY


…… SUSY?


Quantum Gravity
(Superstring/M-theory?)
Low energy
leading terms
Stringy corrections,
(non-)perturbative quantum corrections
Minimal Supergravity
Modified Supergravity
Modification,
Quantum corrections
RG running
RG running
Low-energy SUSY
Unusual SUSY spectrum?
Modified gravity mediation?



$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

1. General Relativity + Dark Matter & Dark Energy
2. Modified Gravity (+ Dark Matter)

What is the relation between the phenomenological modified gravity theory and more fundamental theory?

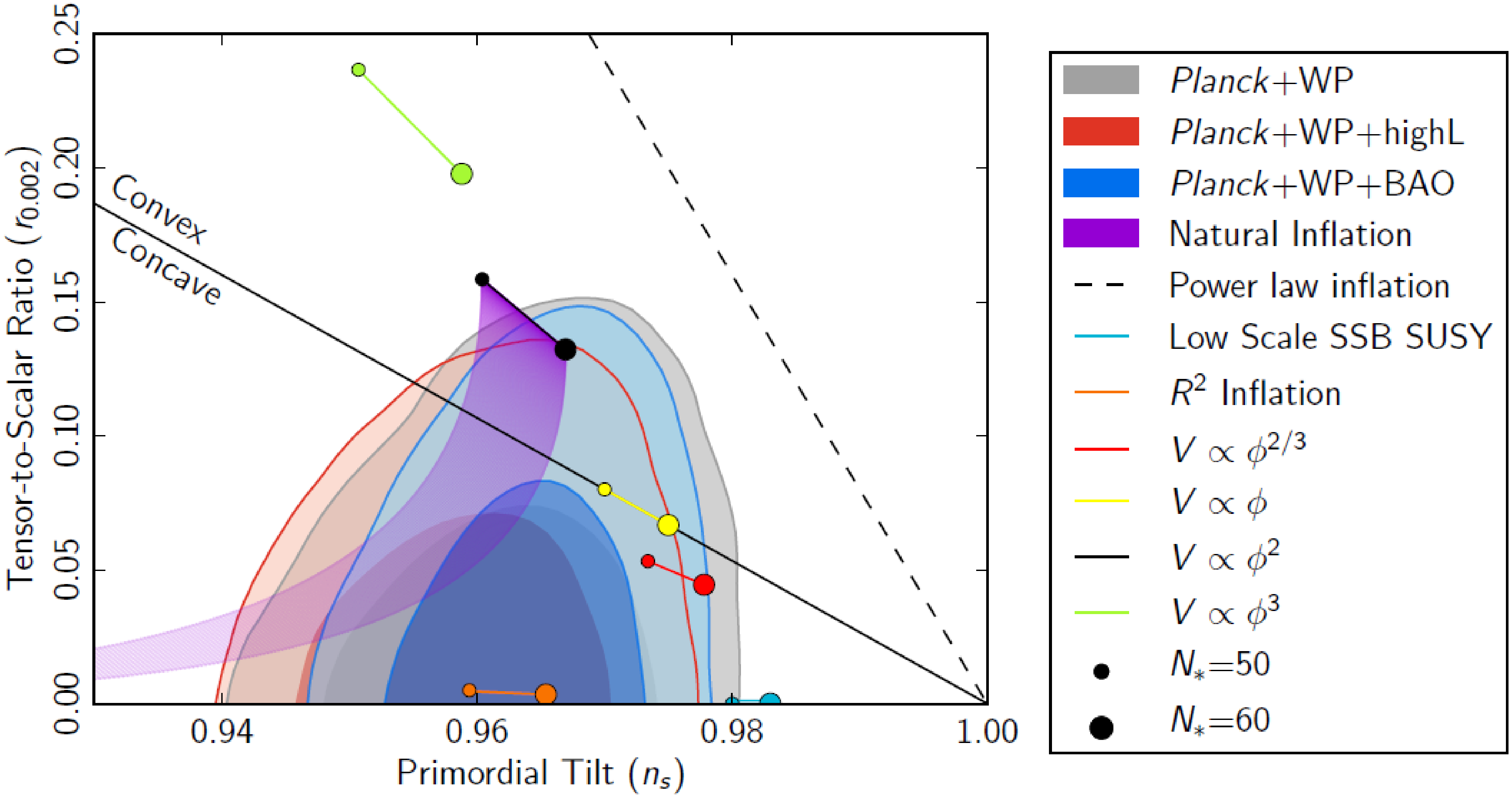


Fig. 1. Marginalized joint 68% and 95% CL regions for n_s and $r_{0.002}$ from *Planck* in combination with other data sets compared to the theoretical predictions of selected inflationary models.

Planck collaboration, arXiv:1303.5082

承

Review of the old-minimal formulation of supergravity in superspace (1/3)

J. Wess and J. Bagger, Supersymmetry and supergravity, Princeton University Press, Princeton U.S.A. (1992).

Notation

- Greek: spinor; Latin: tensor
- External (Einstein) indexes: $M = m; \mu; \dot{\mu}$, $N = n; \nu; \dot{\nu}$, etc.
- Internal (Lorentz) indexes: $A = a; \alpha; \dot{\alpha}$, $B = b; \beta; \dot{\beta}$, etc.

Basic ingredients

- Vierbein, ${E_M}^A(x, \theta, \bar{\theta})$
- Connection of the local Lorentz symmetry, ${\phi_{MA}}^B(x, \theta, \bar{\theta})$

Review of the old-minimal formulation of supergravity in superspace (2/3)

Torsion and Curvature

- Torsion 2-form $T^A = \mathcal{D}E^A$
- Curvature 2-form $R_A{}^B = d\phi_A{}^B + \phi_A{}^C \phi_C{}^B$

These satisfy the Bianchi identities.

$$\mathcal{D}T^A = E^B R_B{}^A, \qquad \mathcal{D}R_A{}^B = 0$$

To obtain the minimal gravitational multiplet:

graviton $e_m{}^a$, gravitino $\psi_m{}^\alpha$, auxiliary fields b_m and M,

we put constraints on the torsion.

$$T_{cb}{}^a = T_{c\underline{\beta}}{}^a = T_{\gamma\beta}{}^a = T_{\dot{\gamma}\dot{\beta}}{}^a = T_{\underline{\gamma\beta}}{}^{\alpha} = 0, \qquad T_{\gamma\dot{\beta}}{}^a = 2i\sigma^a_{\gamma\dot{\beta}}$$

“Solve” the Bianchi identities under these constraints.

Review of the old-minimal formulation of supergravity in superspace (3/3)

All components of the torsion and curvature are written in terms of three Lorentz-irreducible superfields.
See Wess-Bagger (*) Chap. 15 [6].

$\mathcal{R}$: The chiral superfield containing Ricci scalar R.
$\mathcal{G}_a$: The real superfield containing Ricci tensor R_{mn}.
$\mathcal{W}_{\alpha\beta\gamma}$: The chiral superfield containing Weyl tensor C_{klmn}.

They satisfy the following constraints.

$$\bar{\mathcal{D}}\mathcal{R} = \bar{\mathcal{D}}\mathcal{W}_{\alpha\beta\gamma}, \qquad \mathcal{W}_{\alpha\beta\gamma} = \mathcal{W}_{(\alpha\beta\gamma)}$$

$$\mathcal{D}^{\beta}\mathcal{G}_{\beta\dot{\alpha}} = \bar{\mathcal{D}}_{\dot{\alpha}}\mathcal{R}, \qquad \mathcal{D}^{\rho}\mathcal{W}_{\rho\alpha\beta} = i\mathcal{D}_{(\alpha}{}^{\dot{\rho}}\mathcal{G}_{\beta)\dot{\rho}}$$

(*) J. Wess and J. Bagger, Supersymmetry and supergravity, Princeton University Press, Princeton U.S.A. (1992).

超重力理論の最小の作用

$$\mathcal{L} = \int \mathrm{d}^2\Theta 2\mathcal{E}\, 3\mathcal{R} + \mathrm{H.c.}$$

$$e^{-1}\mathcal{L} = -\frac{1}{2}R + \frac{1}{2}\epsilon^{klmn}(\bar{\psi}_k\bar{\sigma}_l\mathcal{D}_m\psi_n - \psi_k\sigma_l\mathcal{D}_m\bar{\psi}_n) - \frac{1}{3}M^*M + \frac{1}{3}b^a b_a$$

Einstein-Hilbert action for graviton

massless gravitino kinetic terms

complex auxiliary scalar field (compensator F-term)

real auxiliary vector field (gauge field for $U(1)_R$-symmetry)

(復習)

最小超重力多重項

重力子（四脚場）	$e_m^{\ a}$
重力微子	$\psi_m^{\ \alpha}$
複素スカラー補助場	M
実ベクトル補助場	b_a

超時空	$Z = (x, \theta^\alpha, \bar{\theta}_{\dot{\alpha}})$
多脚場・接続場	$E_M^{\ \ A}, \phi_{MA}^{\ \ \ \ B}$
捩率・曲率	$T_{MN}^{\ \ \ \ A}, R_{MNA}^{\ \ \ \ \ \ B}$

$\mathcal{R}$

Ricci スカラー R
を含むカイラル超場

$\mathcal{W}_{\alpha\beta\gamma}$

Weyl テンソル C_{klmn}
を含むカイラル超場

$\mathcal{G}_a$

Ricci テンソル R_{mn}
を含むリアル超場

$$\mathcal{L} = \int \mathrm{d}^2\Theta\, 2\mathcal{E}\, F(\mathcal{R}, \mathcal{W}) + \mathrm{H.c.}$$

超重力理論の新しい作用

$$\mathcal{L} = \int \mathrm{d}^2\Theta 2\mathcal{E}\, F(\mathcal{R}, \mathcal{W}) + \mathrm{H.c.}$$

where F is a holomorphic function.

$$e^{-1}\mathcal{L} = -\frac{1}{4}\left(-\frac{1}{3}R - \frac{2}{3}ie_a^m\mathcal{D}_m b^a + \frac{4}{9}M^*M + \frac{2}{9}b^a b_a\right)\frac{\partial F}{\partial \mathcal{R}}\bigg| - M^* F\bigg|$$

$$-\frac{1}{2304}\epsilon^{\rho\lambda}\left(R^{\dot\rho}_{\lambda\dot\rho\gamma\alpha\beta} + i\epsilon_{\lambda\alpha}\sigma^m_{\beta\dot\rho}\mathcal{D}_m b^{\dot\rho}_{\gamma}\right)\left(R^{\dot\sigma}_{\rho\dot\sigma\delta\epsilon\phi} + i\epsilon_{\rho\delta}\sigma^n_{\epsilon\dot\sigma}\mathcal{D}_n b^{\dot\sigma}_{\phi}\right)\frac{\partial^2 F}{\partial W_{\delta\epsilon\phi}\partial W_{\alpha\beta\gamma}}\bigg|$$

+H.c.

関数Fの例

$$\mathcal{L} = \int \mathrm{d}^2\Theta 2\mathcal{E}\, F(\mathcal{R},\mathcal{W}) + \mathrm{H.c.}$$

$$F(\mathcal{R},\mathcal{W}) = -3\mathcal{R} + 4g\mathcal{R}\mathcal{W}^{\alpha\beta\gamma}\mathcal{W}_{\alpha\beta\gamma}$$

$$e^{-1}\mathcal{L}_b$$

$$= -\frac{1}{2}R + \frac{1}{3}b^a b_a$$

$$+\frac{g^2}{432}\left[\left(\frac{9}{4}\left(C_{klmn}C^{klmn}\right)^2 - \frac{9}{2}C_{mnrs}C^{rskl}C_{kltu}C^{tumn} + 9C_{mnrs}C^{rs}{}_{kl}C^{mk}{}_{tu}C^{tunl}\right.\right.$$

$$- 3C^{mnrs}C_{mnrs}F^{kl}F_{kl} - 6F^{mn}C_{mnrs}C^{rskl}F_{kl} + 12F^{mk}F^{nl}C_{mnrs}C_{kl}{}^{rs}$$

転

U67Q

$f(R,C)$ gravity

F(R,W) 超重力のボソン作用はかなり複雑…

f(R,C) 重力もそれ自体新しい提案！

f(R,C) 重力のトイモデルで、インフレーションに関する示唆を探る。

$$S = -\frac{1}{2}\int d^4x\,\sqrt{-g}\,f(R,C)$$

Weyl tensor は FLRW metric の時ゼロ。

⇒ 初期宇宙の背景のダイナミクスには寄与しない。

FLRW からの摂動を議論する場合や、ブラックホール時空の場合等に寄与する。

Move to Einstein frame

f(R) gravity	⟷ equivalent	Scalar-tensor gravity
f(R,C) gravity	⟷ equivalent	scalar-tensor gravity with Weyl tensor

$$S = -\frac{1}{2}\int d^4x\,\sqrt{-g}\,f(R,C)$$
$$= -\frac{1}{2}\int d^4x\,\sqrt{-g}\,[f'(\phi,C)(R-\phi)+f(\phi,C)]$$

Field dependent local Weyl transformation $\tilde{g}_{\mu\nu} = f'(\phi,C)g_{\mu\nu} = \exp\left(\sqrt{2/3}\sigma\right) g_{\mu\nu}$

$$S = \int d^4x\,\sqrt{-\tilde{g}}\,\left[-\frac{1}{2}\tilde{R}+\frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - V(\sigma,\tilde{C})\right]$$

$$V(\sigma,\tilde{C}) = \frac{1}{2}e^{-2\sqrt{\frac{2}{3}}\sigma} f\left(\phi(\sigma,\tilde{C}), e^{-\sqrt{\frac{2}{3}}\sigma}\tilde{C}\right) - \frac{1}{2}e^{-\sqrt{\frac{2}{3}}\sigma}\phi(\sigma,\tilde{C})$$

Implication for Inflation

Starobinsky model + correction term ($\propto b$) as a simple non-trivial example:

$$f(R,C) = R - \frac{R^2}{6M^2} + bRC^{\mu\nu\rho\sigma}C_{\mu\nu\rho\sigma}$$

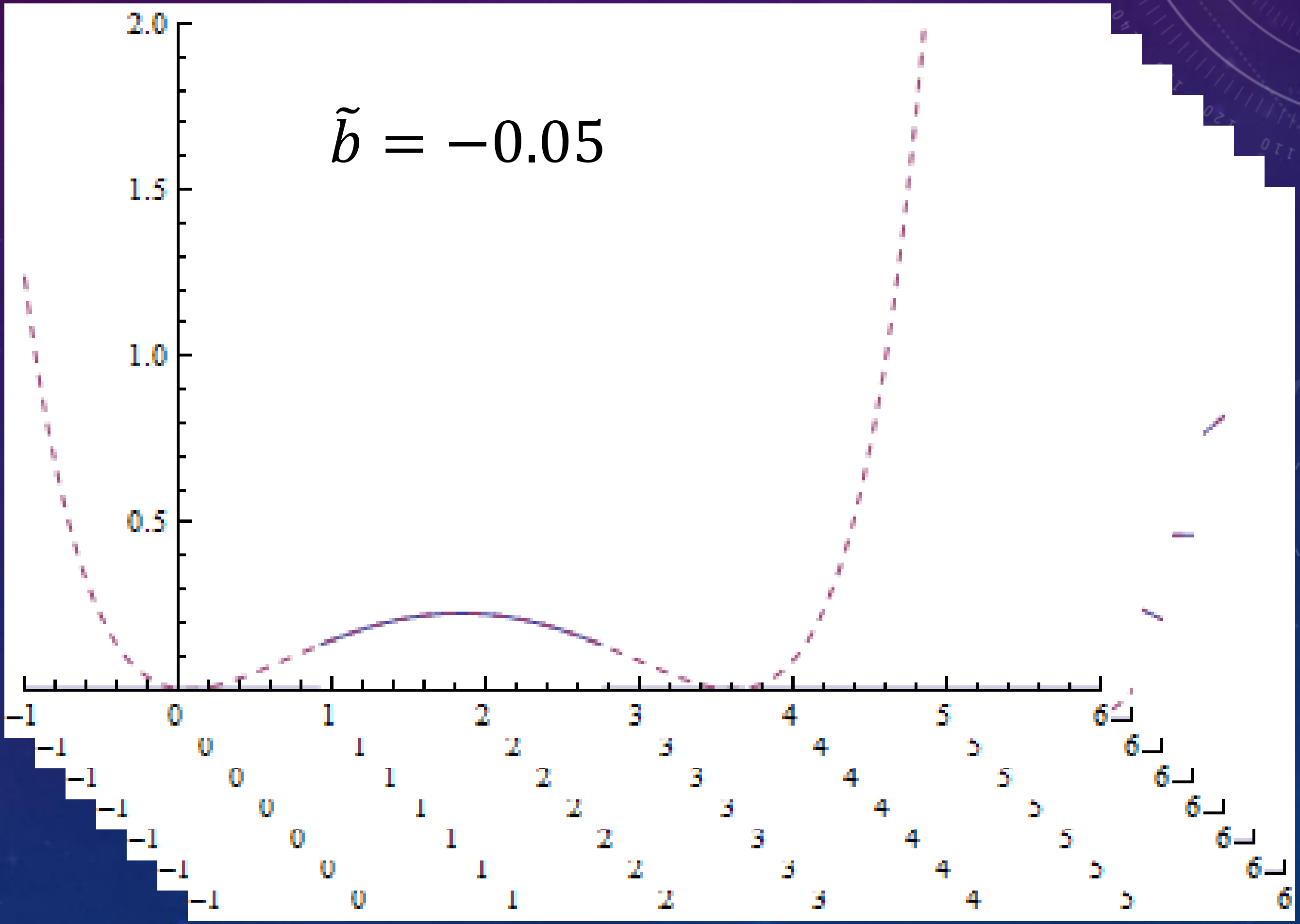
where M is the inflaton mass and b is a coupling constant.

$$V = \frac{3}{4}M^2\left(1 - e^{-\sqrt{2/3}\sigma} + bC^{\mu\nu\rho\sigma}C_{\mu\nu\rho\sigma}e^{\sqrt{2/3}\sigma}\right)^2$$

Weyl テンソルの非一様性があると、Starobinsky インフレーションが不安定になる。
e-foldings が非一様になる可能性がある。

$$V = \frac{3}{4} M^2 \left(1 - e^{-\sqrt{2/3}\sigma} + \tilde{b} e^{\sqrt{2/3}\sigma}\right)^2$$

$$\tilde{b} \equiv b C^{\mu\nu\rho\sigma} C_{\mu\nu\rho\sigma}$$



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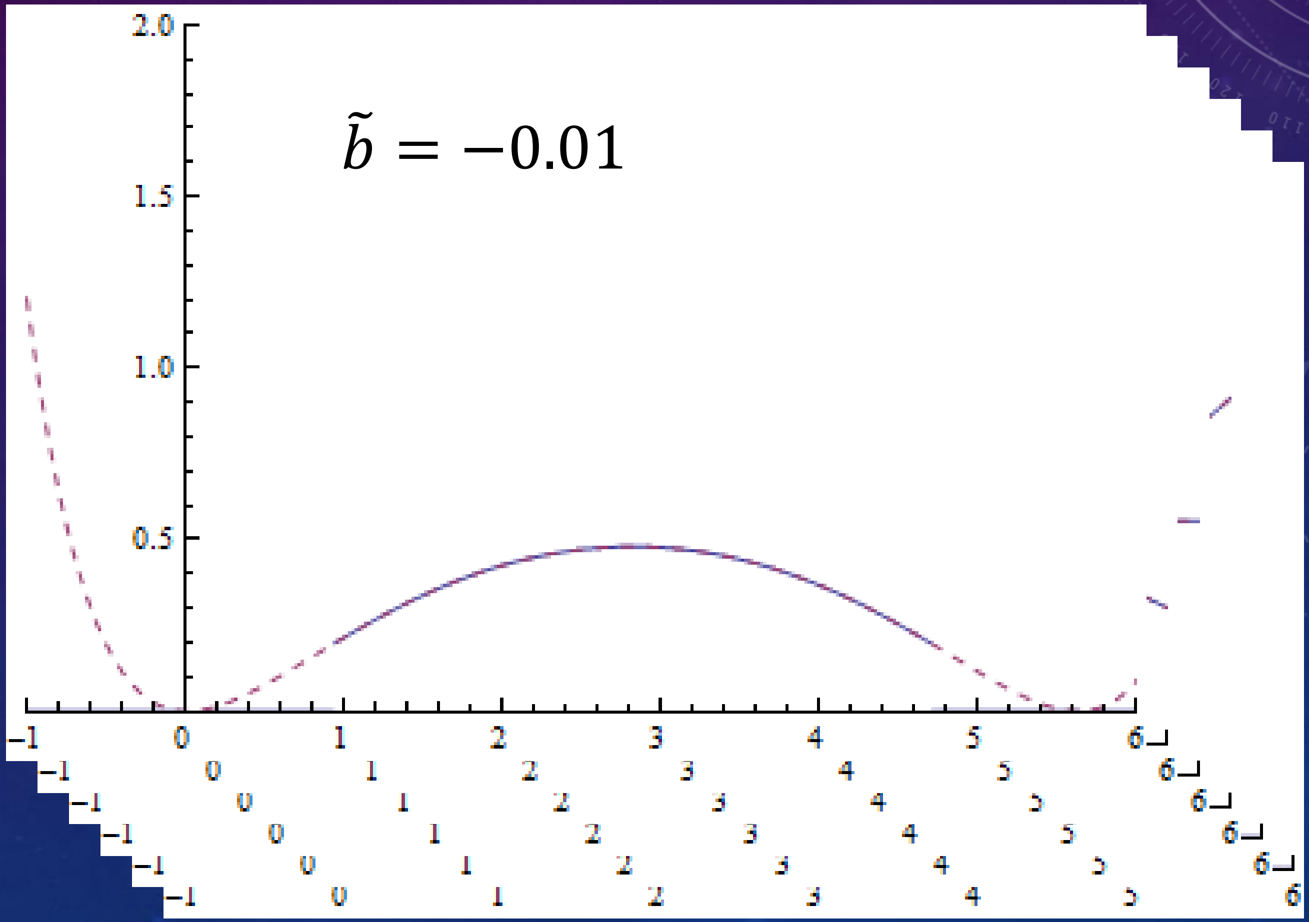

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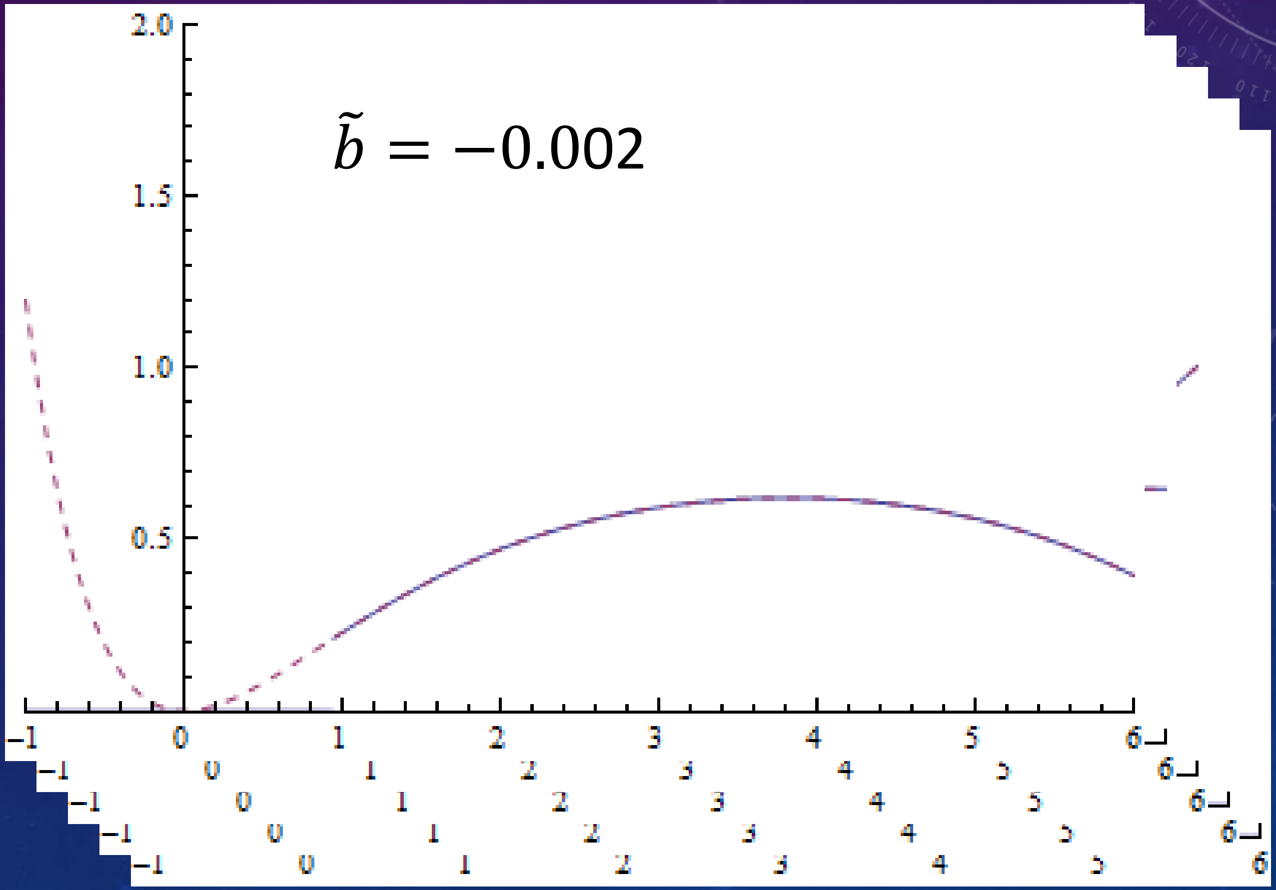

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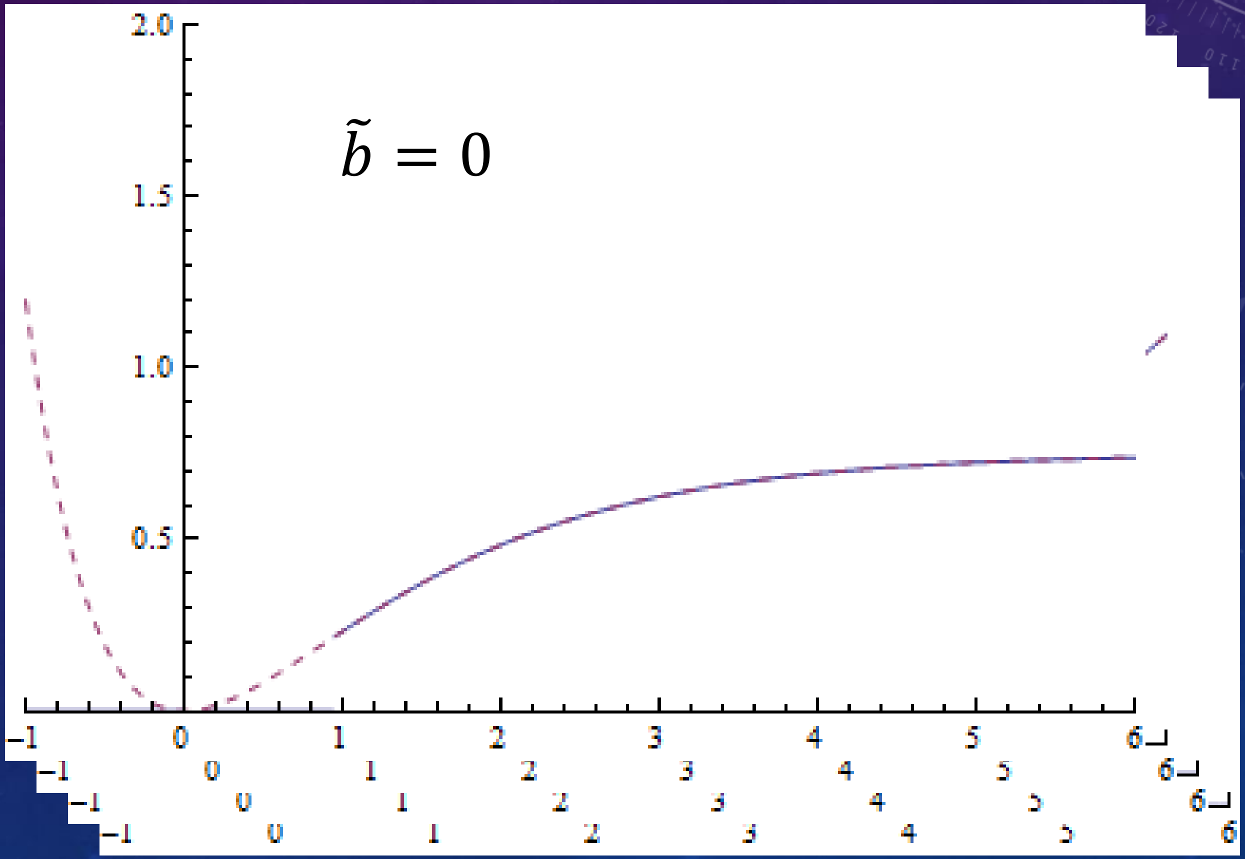

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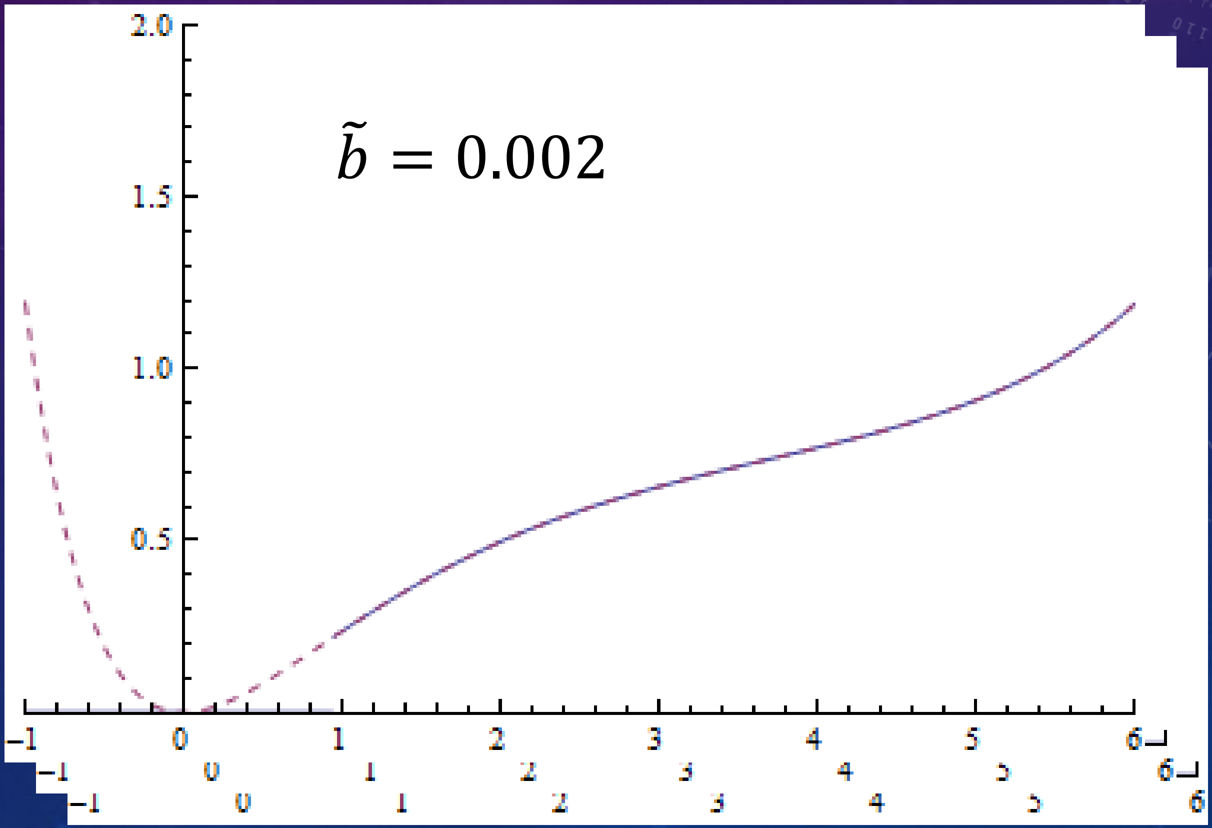

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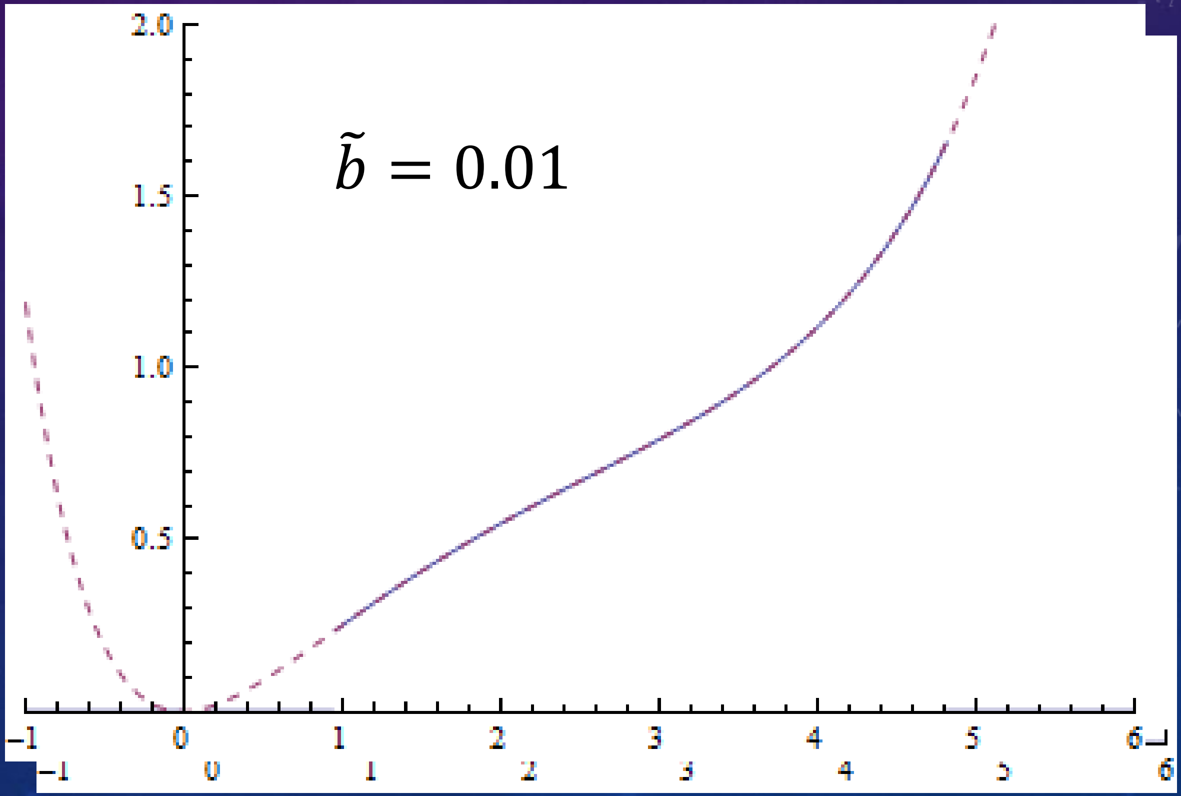

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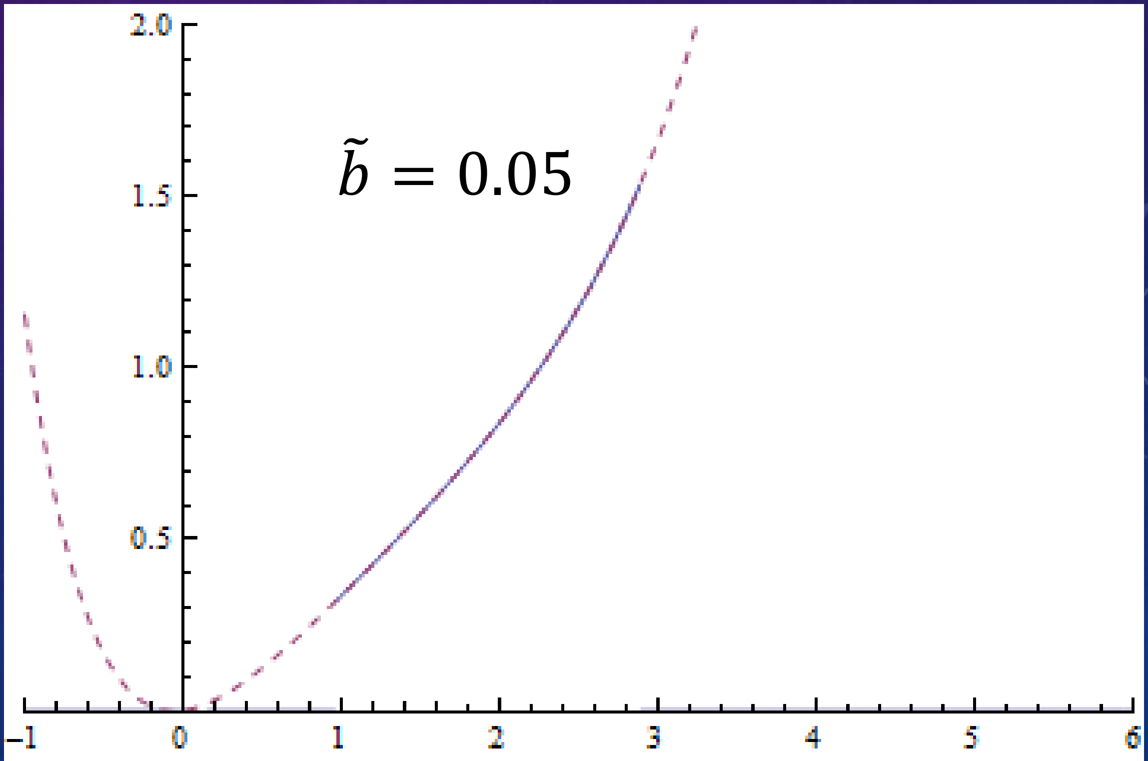

“Ghost” from Weyl tensor

K. Stelle, Phys. Rev. D 16 (1977) 953

$$S = -\int d^4x\sqrt{-g}\left(\frac{1}{2}R + \alpha R_{\mu\nu}R^{\mu\nu} - \beta R^2\right)$$

The theory is Renormalizable.
However,
there is a massive spin-2 particle with either
Negative Energy or Negative Norm.

No Problem?

no-ghost theorem for a toy model: C. M. Bender and P. D. Mannheim, PRL 100, 110402 (2008)
mini-review containing criticism: A. V. Smilga, SIGMA 5 017 (2009)
more recent review: P. D. Mannheim, Found. Phys. 42 (2012) 388

No longer auxiliary

$F(\mathcal{R})$ SUGRA で補助場だった場が、$F(\mathcal{R},\mathcal{W})$ SUGRA ではダイナミカルな場になる。

> *While for the effective action of 4D, N=1 supersymmetric QCD we are not able to make definitive argument that propagating auxiliary fields are "bad", for compactified 4D, N=1 superstrings and heterotic strings there is a potential for making such arguments for their low-energy effective actions. The point in these theories is that* ***the spectrum of the effective action is strictly controlled by string theory.*** *Propagating auxiliary fields would have to correspond to higher mass (m>0) modes of the string.* ***If the spectrum of the string cannot accommodate the states described by the propagating auxiliary fields, that is reason to rule them out.***
>
> — S.J. Gates, Jr., Phys. Lett. B 365(1996)132

SUSY が自発的に破れていれば、"補助場"が質量を持てるので必ずしも矛盾しない？

结

まとめ

超重力理論の拡張(＝修正重力理論の超対称化)

- 純理論的な興味
- 現象論的な修正重力の基礎理論への埋め込み
- 非標準的な重力媒介、超対称質量スペクトル？

修正(超)重力理論の新しい作用

- $f(R, C)$ gravity with a real vector field
- $F(\mathcal{R}, \mathcal{W})$ supergravity
  最も一般的な、物質場無しのカイラルな超重力理論の作用
- インフレーションの不安定性、非物理的状態の存在、補助場の伝播
  →$\mathcal{W}(C)$依存性の無い作用が尤もらしいと考えられる