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ENTANGLEMENT-INDUCED QUANTUM RADIATION

Kazuhiro Yamamoto (Hiroshima University)

Based on the collaborations with

A. Higuchi, S. Iso, N. Oshita, R. Tatsukawa, K. Ueda, S. Zhang

S. Iso, R. Tatsukawa, K. Ueda, K. Y., arXiv:1704.03616

S. Iso, N. Oshita, R. Tatsukawa, K. Y., S. Zhang, PRD 95 023512 (2017)

N. Oshita, K. Y., S. Zhang, PRD 93 085016 (2016)

N. Oshita, K. Y., S. Zhang, PRD 92 045027 (2015)

S. Iso, K. Y., S. Zhang, PTEP 063B01 (2013)

Contents of talk

- 1. Introduction
- 2. Unruh effect and entanglement of quantum field
- 3. Quantum radiation in Unruh de Witt detector model
- 4. Origin of the quantum radiation
- 5. Conclusions

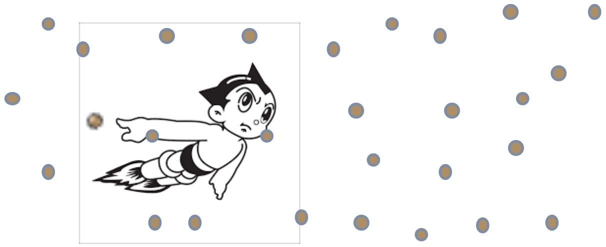
1. Introduction — Unruh effect

Unruh (1976)

- A observer in an uniformly accelerated motion sees the Minkowski vacuum as a thermally excited state.

a

acceleration



Unruh Temperature

$$T_U = \frac{a}{2\pi} = 4 \times 10^{-20} K \left(\frac{a}{9.8 m/s^2} \right)$$

☑ Analogy with the Hawking radiation by the equivalence principle

☑ The simplest system
relativity and quantum mechanics are important simultaneously

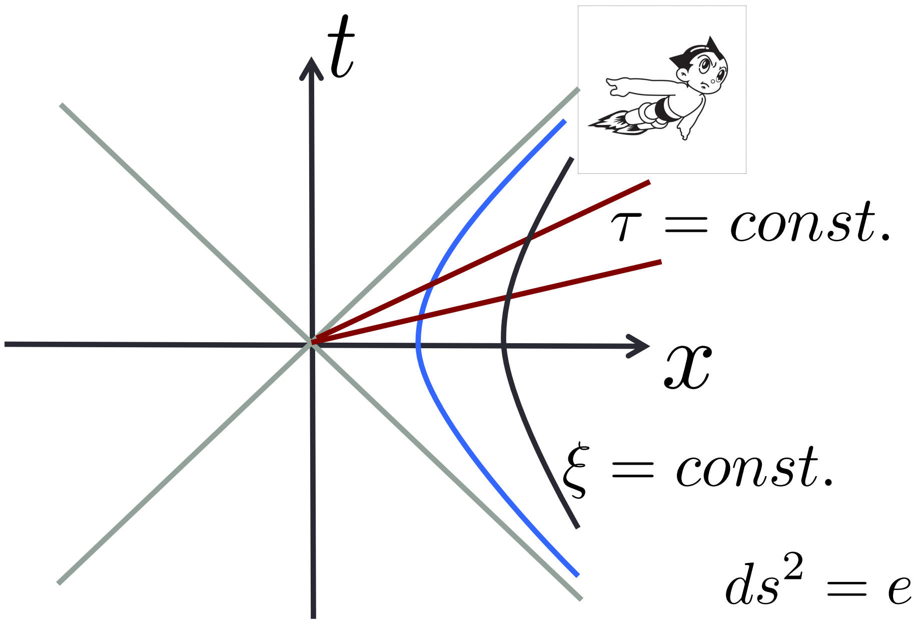
☑ Entanglement of quantum field

2. Unruh effect and entanglement of quantum field

Wald and Unruh (84)

Rindler coordinate :

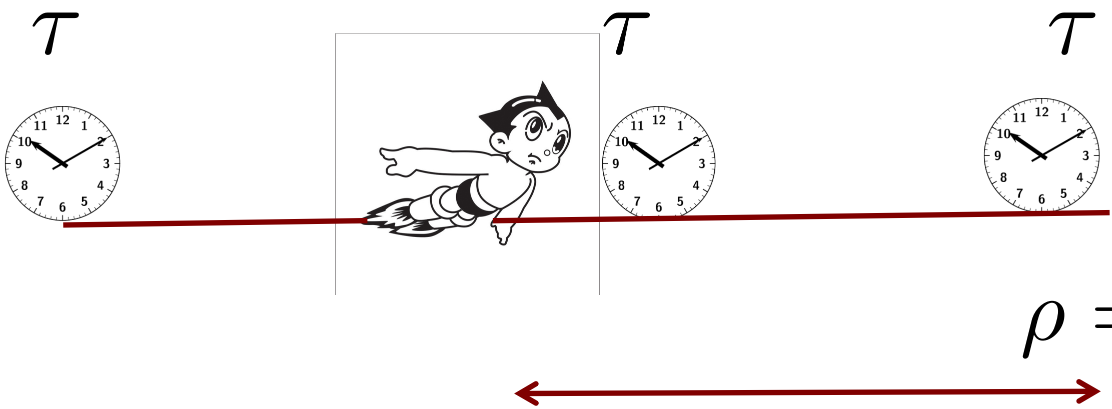
coordinate for a uniformly accelerating observer



$$t = \frac{e^{a\xi}}{a}\sinh a\tau$$

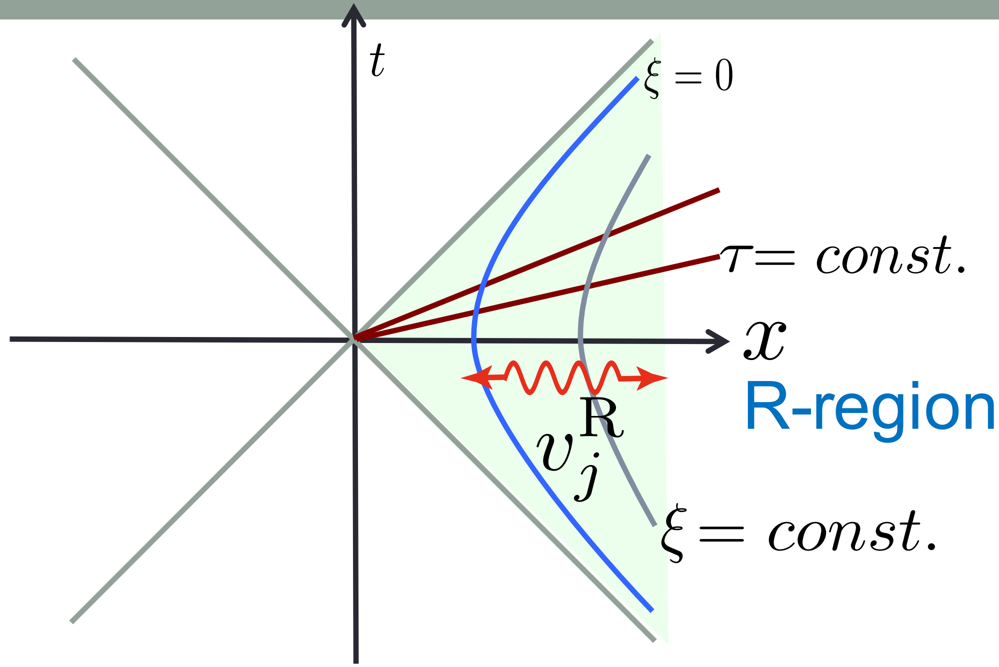
$$x = \frac{e^{a\xi}}{a}\cosh a\tau$$

$$ds^2 = e^{2a\xi}(d\tau^2 - d\xi^2) - d\mathbf{x}_\perp^2$$



$$\rho = \frac{1}{a}(e^{a\xi} - 1)$$

Rindler spacetime



$$ds^2 = e^{2a\xi}(d\tau^2 - d\xi^2) - d\mathbf{x}_\perp^2$$

massless scalar field

$$S = \frac{1}{2}\int d^4x\sqrt{-g}(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi)$$

quantized field

$$\hat{\phi}(x) = \int_0^\infty d\omega \int d^2k_\perp \left(\hat{a}^I_{\omega,\mathbf{k}_\perp} v^R_{\omega,\mathbf{k}_\perp}(x_R) + \text{h.c.}\right) = \sum_j \left(\hat{a}^I_j v^R_j(x) + \text{h.c.}\right),$$

$$v^R_j(x) = \sqrt{\frac{\sinh \pi\omega/a}{4\pi^4 a}}\, e^{-i\omega\tau} K_{i\omega/a}\left(\frac{\kappa e^{a\xi}}{a}\right) e^{i\mathbf{k}_\perp\cdot\mathbf{x}_\perp}, \qquad j = (\omega, \mathbf{k}_\perp)$$

right Rindler vacuum state : $\hat{a}^I_j|0,\mathrm{I}\rangle = 0$ *for any* $j = (\omega, \mathbf{k}_\perp)$

excited state $\quad |n_j, \mathrm{I}\rangle = \frac{1}{\sqrt{n_j!}}(\hat{a}^{I\dagger}_j)^{n_j}|0,\mathrm{I}\rangle$

Left Rindler coordinate

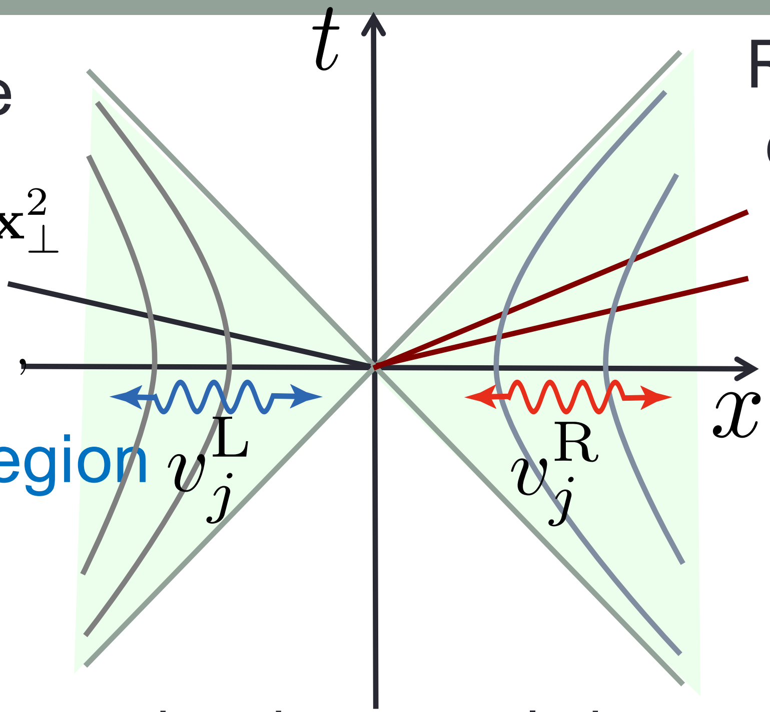
$$ds^2 = e^{2a\tilde{\xi}}(d\tilde{\tau}^2 - d\tilde{\xi}^2) - d\mathbf{x}_\perp^2$$

$$\hat{\phi}(x) = \sum_j \left(\hat{a}_j^{\mathrm{II}} v_j^L(x) + \mathrm{h.c.}\right),$$

$$|n_j, \mathrm{II}\rangle$$

$$j = (\omega, \mathbf{k}_\perp)$$

Right Rindler coordinate



$$|n_j, \mathrm{I}\rangle$$

Minkowski vacuum is expressed as the entangled state of the right–Rindler states and the left–Rindler states.

Wald, Unruh (1984)

$$|0, \mathrm{M}\rangle = \prod_j \left[N_j \sum_{n_j=0}^{\infty} e^{-\pi n_j \omega_j / a} |n_j, \mathrm{I}\rangle \otimes |n_j, \mathrm{II}\rangle \right] \quad N_j = \sqrt{1 - e^{-2\pi\omega_j/a}}$$

Partial trace with respect to the states of the L region

→ the thermal state with the Unruh temperature

$$\hat{\rho}_R = Tr_L(|0, M\rangle\langle 0, M|) = \prod_j N_j^2 [\sum_{n_j=0}^{\infty} e^{-2\pi n \omega_j / a} |n_j, I\rangle\langle n_j, I|]$$

Entanglements is an aspect of the Unruh effect

Possibilities of detecting the Unruh effect ?

electron acceleration with an intense laser

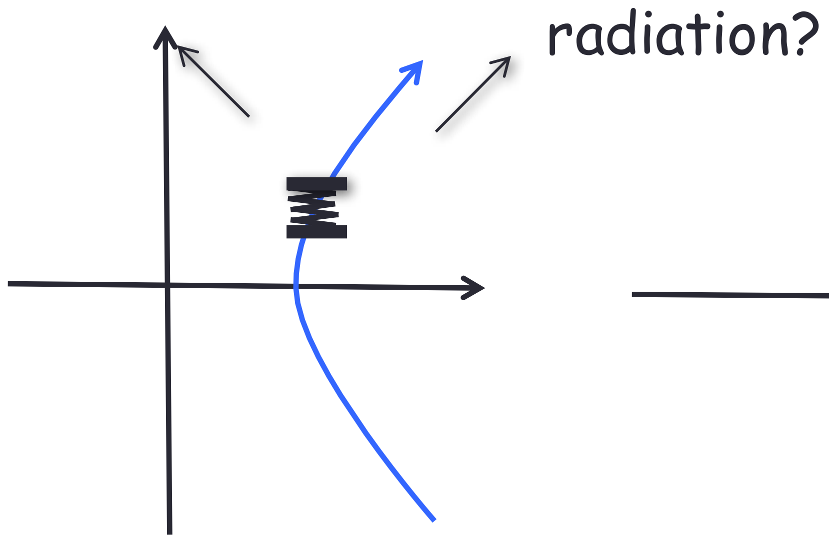
Chen, Tajima (99)

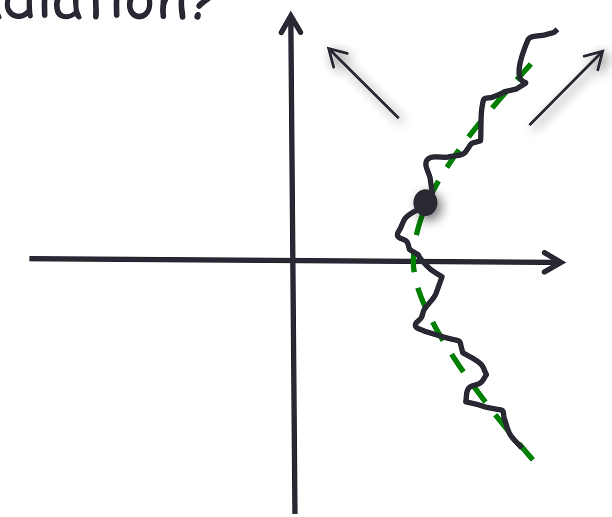
Schutzhold, et al (06)

c.f. Bell, Leinaas (83)

Cozzella, et al (17)

Radiation from Unruh effect ?





Is there radiation from thermal equilibrium system?

→ Importance of an interference effect

→ But, there is non-vanishing quantum radiation

→ What is the origin of the quantum radiation?

Lin and Hu (06)

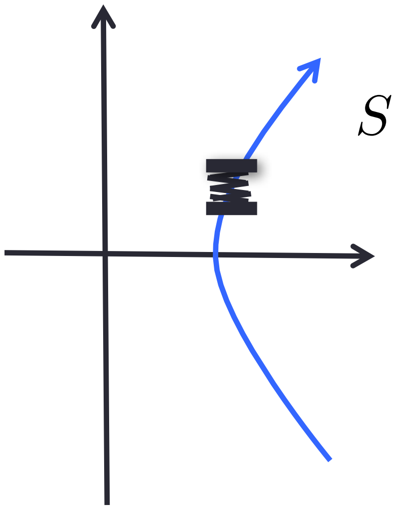
Iso, et al (11)

Oshita, KY, Zhang (16

Iso, et al. (17)

3. Quantum radiation in Unruh de Witt detector model

Lin and Hu (06)



$$S[Q,\phi] = \frac{m}{2}\int d\tau\left((\dot{Q}(\tau))^2 - \Omega_0^2 Q^2(\tau)\right) + \frac{1}{2}\int d^4x\partial^\mu\phi(x)\partial_\mu\phi(x)$$

$$+S_{\rm int}[Q,\phi]$$

$$S_{\rm int}[Q,\phi] = \lambda\int d^4x d\tau Q(\tau)\phi(x)\delta^4(x - z(\tau))$$

Initial time : $\tau_0 \to -\infty$

☑ $$\partial^\mu\partial_\mu\phi(x) = \lambda\int d\tau Q(\tau)\delta_D^{(4)}(x - z(\tau))$$

$$\longrightarrow \phi(x) = \phi_{\rm h}(x) + \phi_{\rm inh}(x)$$

$\phi_{\rm h}(x)$: vacuum fluctuations

$$\phi_{\rm inh}(x) = \lambda\int d\tau Q(\tau)G_R(x - z(\tau))$$

☑ $$\left(\frac{d^2}{d\tau^2} + 2\gamma\frac{d}{d\tau} + \Omega^2\right)Q(\tau) = \frac{\lambda}{m}\phi_h(z(\tau))$$

random force from vacuum fluctuations

$$\langle E\rangle = \frac{m}{2}\left(\langle\dot{Q}^2(\tau)\rangle + \Omega^2\langle Q^2(\tau)\rangle\right)$$

$$\simeq \frac{a}{2\pi} = T_U$$

law of qui-partition of energy

Two point function of the field

$$\phi(x) = \phi_{\rm h}(x) + \phi_{\rm inh}(x)$$

$$\langle\phi(x)\phi(y)\rangle = \langle\phi_{\rm h}(x)\phi_{\rm h}(y)\rangle + \underbrace{\langle\phi_{\rm h}(x)\phi_{\rm inh}(y)\rangle + \langle\phi_{\rm inh}(x)\phi_{\rm h}(y)\rangle} + \langle\phi_{\rm inh}(x)\phi_{\rm inh}(y)\rangle$$

vacuum fluctuation + interference term + naïve radiation term

cancelation!

A + B – B

$$\langle\phi_{\rm h}(x)\phi_{\rm inh}(y)\rangle + \langle\phi_{\rm inh}(x)\phi_{\rm h}(y)\rangle + \langle\phi_{\rm inh}(x)\phi_{\rm inh}(y)\rangle = A$$

Energy flux can be derived from the two point function

$$T_{0i} = \lim_{y\to x}\left\langle \frac{\partial\phi(x)}{\partial x^0}\frac{\partial\phi(y)}{\partial y^i}\right\rangle = \lim_{y\to x}\frac{\partial}{\partial x^0}\frac{\partial}{\partial y^i}\left\langle \phi(x)\phi(y)\right\rangle$$

$$\frac{dE}{dt} \sim \frac{a\lambda}{4\pi m}\frac{a^2}{2\pi\Omega^2}$$

non-vanishing quantum radiation flux consistent with Lin and Hu (06)

4. Origin of the quantum radiation?

Flux is estimated in F-region

$$\langle \phi_{\rm h}(x)\phi_{\rm inh}(y)\rangle \qquad x, y \in \text{F region}$$

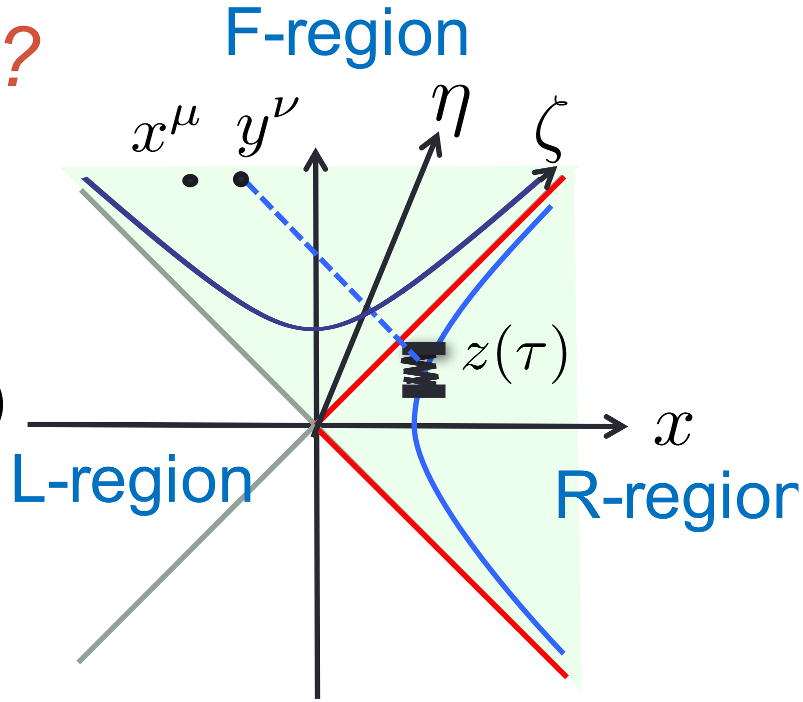
$$\phi_{\rm inh}(y) = \lambda \int d\tau Q(\tau) G_R(y - z(\tau))$$

$Q(\tau)$: detector

$\phi_{\rm h}(z(\tau))$ vacuum fluctuation

$$\left(\frac{d^2}{d\tau^2} + 2\gamma\frac{d}{d\tau} + \Omega^2\right) Q(\tau) = \frac{\lambda}{m}\phi_h(z(\tau))$$

$$\langle \phi_h(x)\phi_h(z(\tau))\rangle$$



Correlation of vacuum fluctuations of the points, one is in F-region and the other is on the detector trajectory (R-region) is the key

Similarity

$$\langle \phi_{\rm h}(x)\phi_{\rm inh}(y)\rangle \longleftrightarrow \langle \phi_h(x)\phi_h(z(\tau))\rangle$$

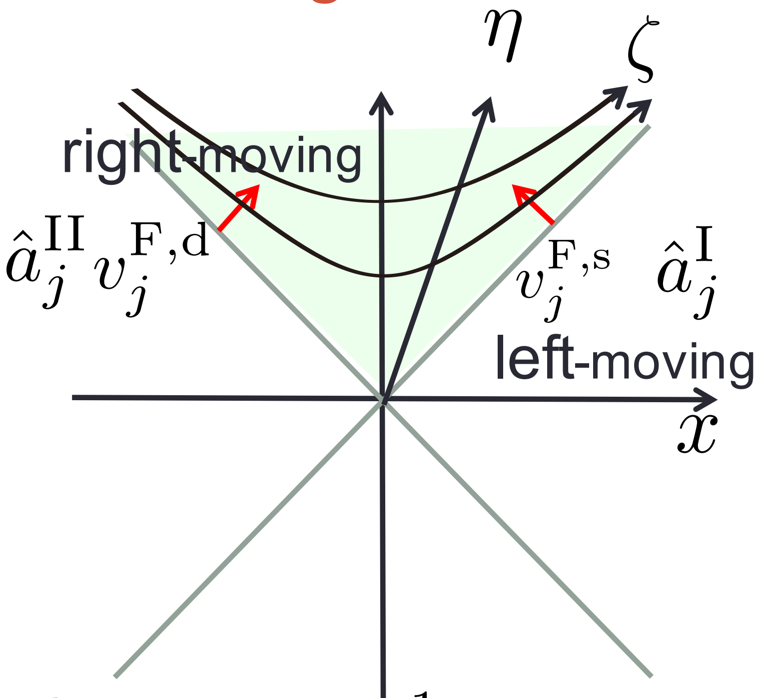
Kasner degenerate expanding universe in F-region

$$ds^2 = e^{2a\eta}(d\eta^2 - d\zeta^2) - d\mathbf{x}_\perp^2$$

Quantization of the massless scalar field

left moving wave + right-moving wave

$$\hat{\phi}(x) = \hat{\phi}^{F,s} + \hat{\phi}^{F,d}$$
$$= \sum_j \left(\hat{a}_j^{\rm I} v_j^{F,s}(x) + \hat{a}_j^{\rm II} v_j^{F,d}(x) + {\rm h.c.} \right),$$



$$v_j^{F,s}(x) = \frac{-i}{4\pi\sqrt{a\sinh(\pi\omega/a)}} e^{-i\omega\zeta} J_{-i\omega/a}\left(\frac{\kappa e^{a\eta}}{a}\right) e^{-i\mathbf{k}_\perp\cdot\mathbf{x}_\perp},$$

$$t = \frac{1}{a} e^{a\eta} \cosh a\zeta$$

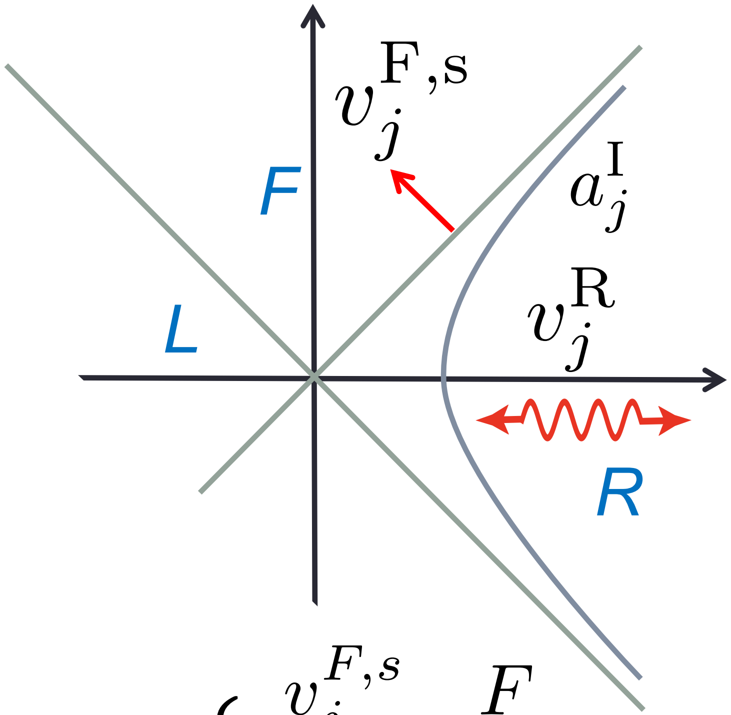
Left-moving wave Kasner mode $j = (\omega, \mathbf{k}_\perp)$

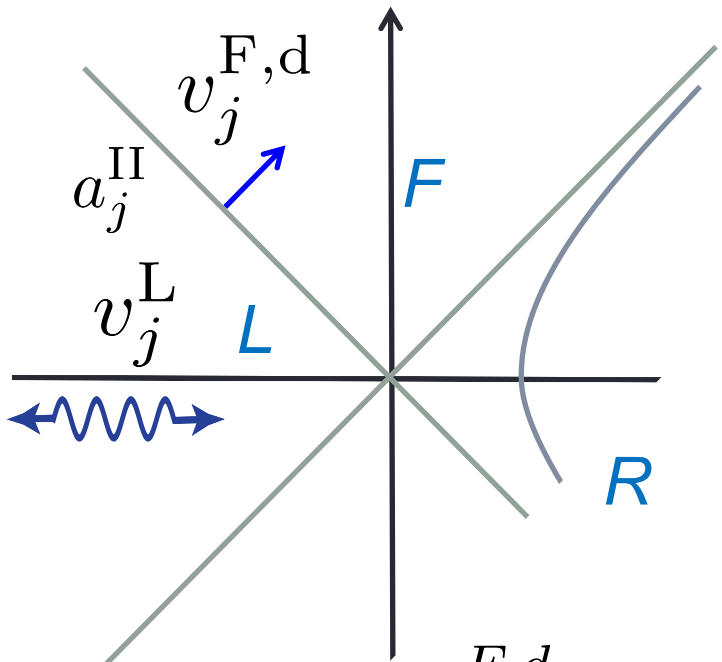
$$x = \frac{1}{a} e^{a\eta} \sinh a\zeta$$

$$v_j^{F,d}(x) = \frac{-i}{4\pi\sqrt{a\sinh(\pi\omega/a)}} e^{i\omega\zeta} J_{-i\omega/a}\left(\frac{\kappa e^{a\eta}}{a}\right) e^{i\mathbf{k}_\perp\cdot\mathbf{x}_\perp},$$

right-moving wave Kasner mode

Description of the Minkowski vacuum state with entanglement





$$v_j^{\mathrm{I}}(x) = \begin{cases} v_j^{F,s} & F \\ v_j^{R} & R \\ 0 & L \end{cases} \qquad v_j^{\mathrm{II}}(x) = \begin{cases} v_j^{F,d} & F \\ 0 & R \\ v_j^{L} & L \end{cases}$$

$$\hat{\phi}(x) = \sum_j \left(\hat{a}_j^{\mathrm{I}} v_j^{\mathrm{I}}(x) + \hat{a}_j^{\mathrm{II}} v_j^{\mathrm{II}}(x) + \mathrm{h.c.} \right),$$

$$|0, M\rangle = \prod_j \left[N_j \sum_{n_j=0}^{\infty} e^{-\pi n_j \omega / a} |n_j, \mathrm{I}\rangle \otimes |n_j, \mathrm{II}\rangle \right]$$

$$\langle 0,M|\phi_{\rm h}(x)\phi_{\rm h}(z(\tau))|0,M\rangle \qquad \phi(x)=\phi^{F,d}+\phi^{F,s}$$

$$=\langle 0,M|\phi^{F,d}(x)\phi(z(\tau))|0,M\rangle+\langle 0,M|\phi^{F,s}(x)\phi(z(\tau))|0,M\rangle$$

||

Right-moving Kasner mode

right Rindler mode

A

(non-vanishing flux)

Origin of radiation

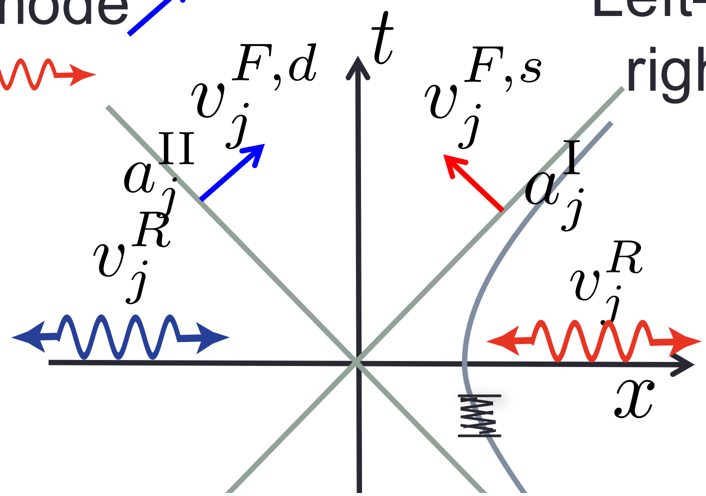
||

Left-moving Kasner mode

right Rindler mode

B

(cancel out)



$$=-\frac{i}{8\pi^2\rho_0(x)}\int_{-\infty}^{+\infty}d\omega e^{-i\omega\tau}\left(\frac{e^{\pi\omega/a}}{e^{2\pi\omega/a}-1}e^{i\omega\tau_+^x}-\frac{1}{e^{2\pi\omega/a}-1}e^{i\omega\tau_-^x}\right)$$

Interference term

A

B

$$\langle 0,M|\phi_{\rm h}(x)\phi_{\rm inh}(y)|0,M\rangle$$

$$=-\frac{\lambda^2}{(4\pi)^2\rho_0(x)\rho_0(y)}\int_{-\infty}^{+\infty}d\omega e^{-i\omega\tau_-^y}h(\omega)\left(\frac{e^{\pi\omega/a}}{e^{2\pi\omega/a}-1}e^{i\omega\tau_+^x}-\frac{1}{e^{2\pi\omega/a}-1}e^{i\omega\tau_-^x}\right)$$

5. Conclusions

- ✓ non-vanishing quantum radiation due to the Unruh effect
  - Unruh de Witt detector model in a uniformly accelerated motion

- ✓ Origin of the quantum radiation is
  the entanglement of the states of the field
  between the right-moving Kasner mode and the Rindler mode.