Entanglement Growth and Probability Distribution

Naoki Watamura (Nagoya Univ.)

arXiv : [hep-th] 1703.06589, M. Nozaki, NW.

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Entanglement Growth and Probability Distribution

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OUTLINE

- Introduction
- Entanglement Growth of Locally Excited States
    - Late Time Algebra
- Probability Distribution and EE.
- Summary

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Introduction

One of the ways to measure the correlation of 2 regions.

- Entanglement Entropy

  Many Applications in

  - Condensed Matter Context
  - Quantum Information Context
  - Quantum Field Theory Context
    - Relation to Quantum Gravity (Ryu-Takayanagi,…)
    - CFT central charge and c-theorem

      etc…

Important to understand the basic properties

the growth of EE with Locally Excited States

Introduction

One of the ways to measure the correlation of 2 regions.

- Entanglement Entropy

  Many Applications in

  - Condensed Matter Context
  - Quantum Information Context
  - Quantum Field Theory Context

Important to understand the basic properties

the growth of EE with Locally Excited States

- On of the easiest (simplest) deformation
- Finite
- Gauge Invariant (in Maxwell theory)

If Gauge Dependence is only on the Entangling surface

Entanglement Growth of Locally Excited States

Entanglement Growth of Locally Excited States

- The Setup.

(3+1) D

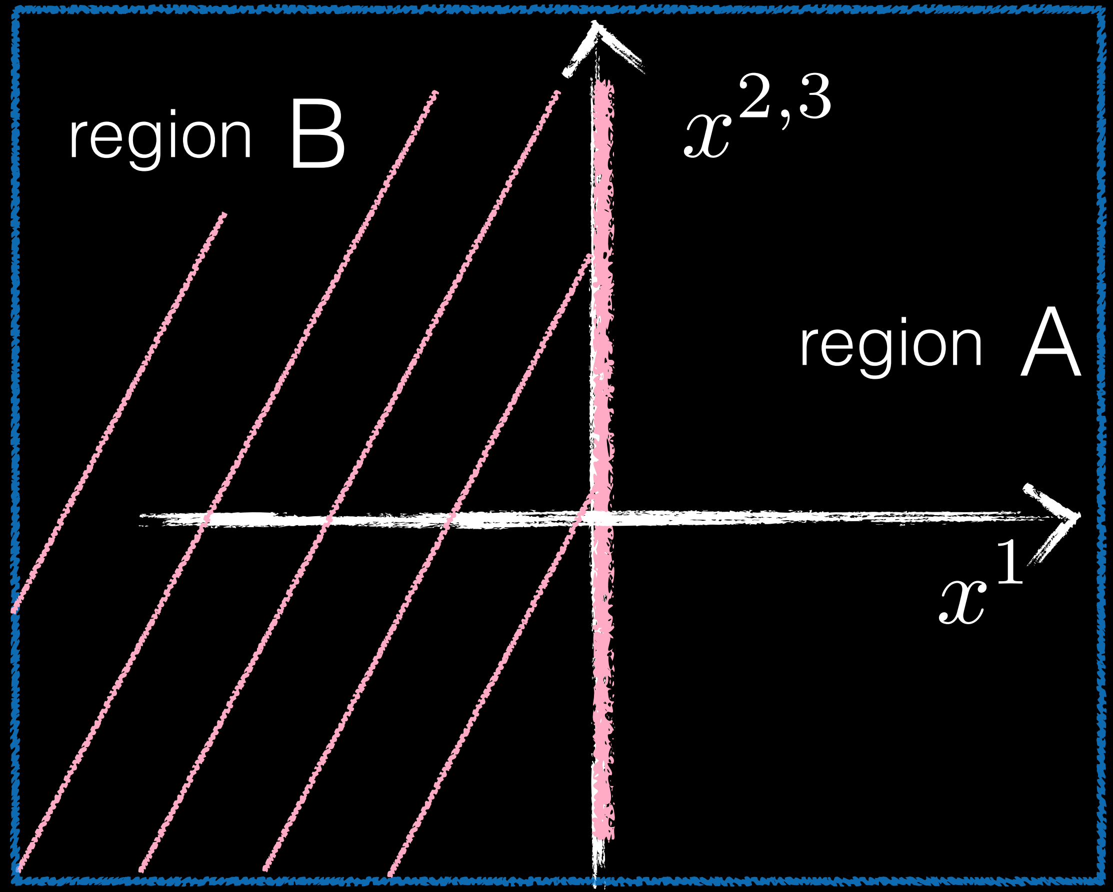
Time : $t = 0$

⬇

Euclid time : $\tau = 0$

region A : $x^1 > 0$

region B : $x^1 \leqq 0$



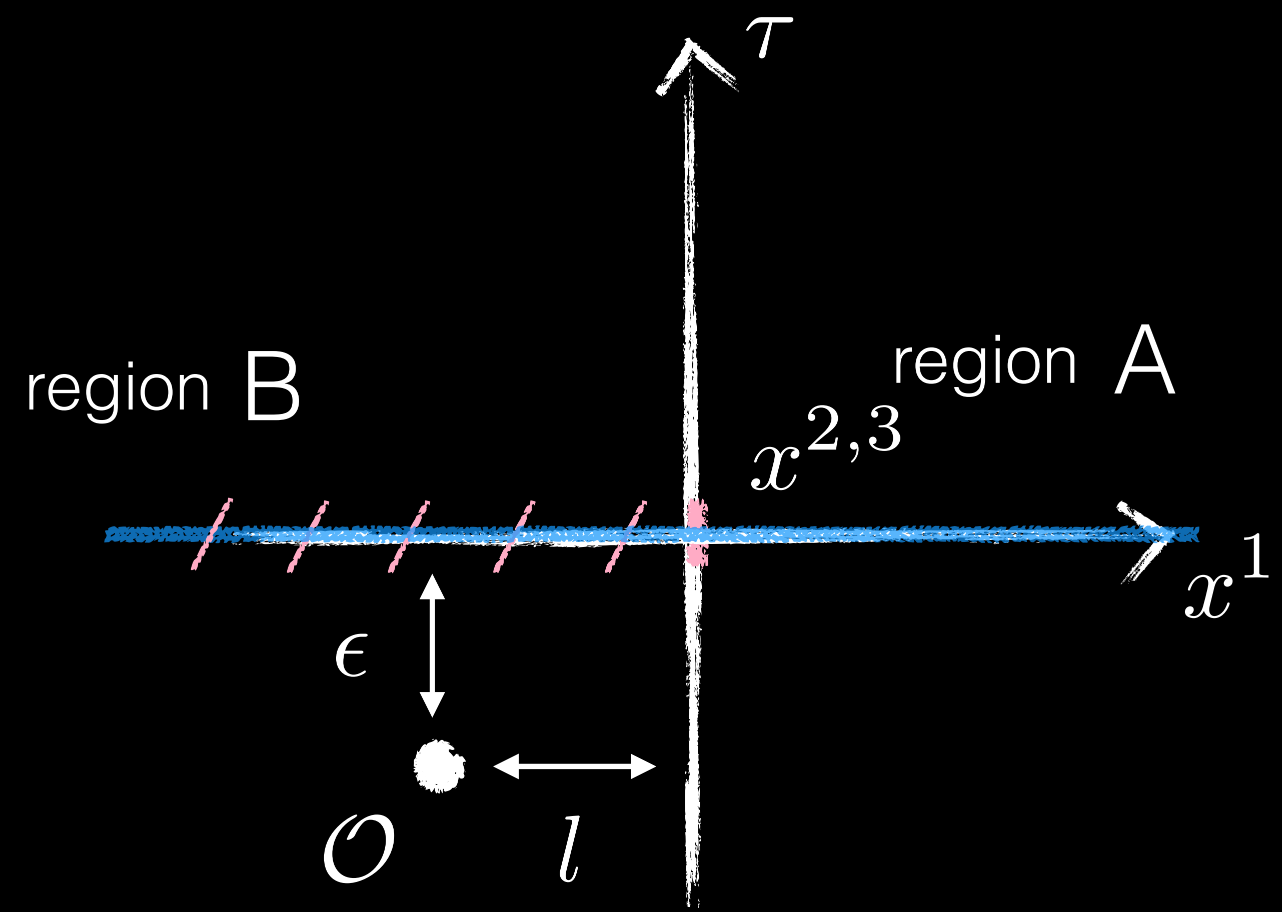
Entanglement Growth of Locally Excited States

- The Setup. (3+1) D

Euclid time : $\tau = 0$

region A : $x^1 > 0$

region B : $x^1 \leqq 0$



2 Rényi EE.

generated from

<u>Vacuum State</u>

$$\rho^{\text{vac}} = |0\rangle\langle 0| \longrightarrow S_A^{(n),\text{G}}$$

<u>Excited State</u>

$$\rho^{\text{ex}} = \mathcal{O}(-t,-l)|0\rangle\langle 0|\mathcal{O}^\dagger(-t,-l) \longrightarrow S_A^{(n),\text{EX}}$$

The difference :

$$\Delta S_A^{(n)} = S_A^{(n),\text{EX}} - S_A^{(n),\text{G}}$$

Our Target

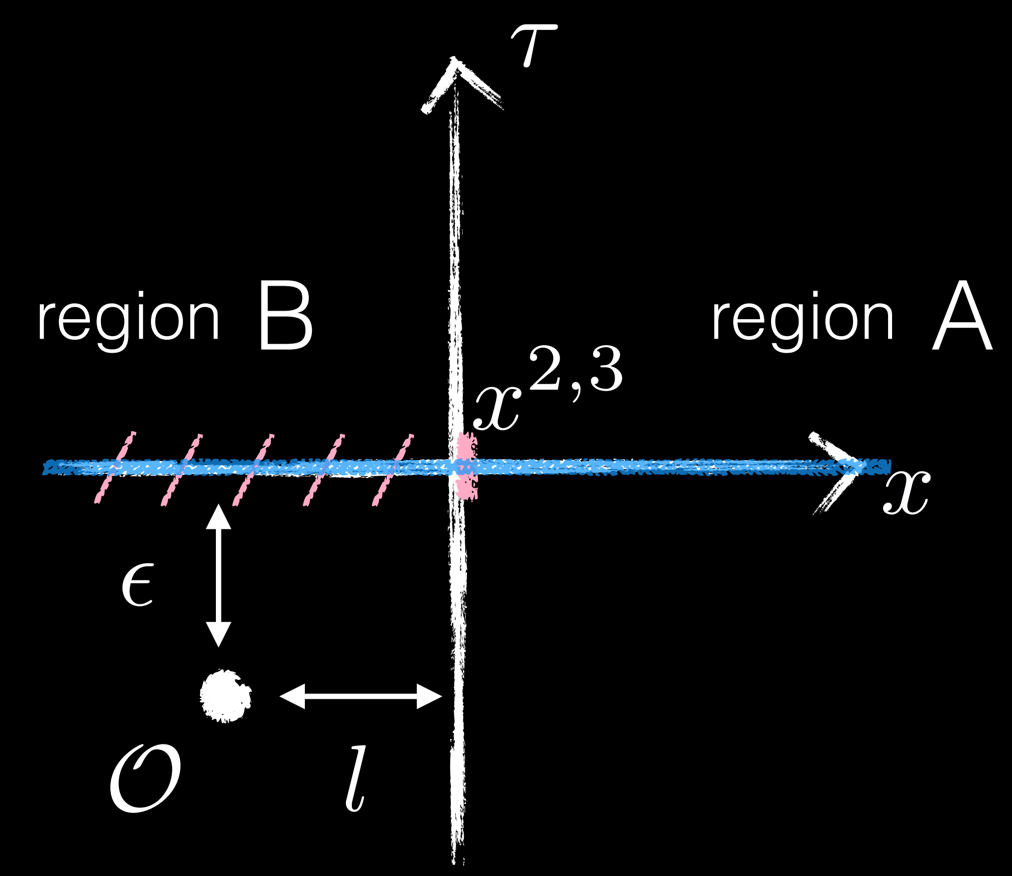
Entanglement Growth of Locally Excited States

- The Setup. (3+1) D

Euclid time : $\tau = 0$

region A : $x^1 > 0$

region B : $x^1 \leqq 0$



$$\rho^{\text{vac}} = |0\rangle\langle 0|$$
$$\rho^{\text{ex}} = \mathcal{O}(-t,-l)|0\rangle\langle 0|\mathcal{O}^\dagger(-t,-l)$$

The difference : $\Delta S_A^{(n)} = S_A^{(n),\text{EX}} - S_A^{(n),\text{G}}$

$$= -\frac{1}{n-1}\log \frac{\langle 0|\mathcal{O}\mathcal{O}^\dagger \cdots \mathcal{O}\mathcal{O}^\dagger|0\rangle_{\Sigma_n}}{\langle 0|0\rangle_{\Sigma_n}} \frac{\left(\langle 0|0\rangle_{\Sigma_1}\right)^n}{\left(\langle 0|\mathcal{O}\mathcal{O}^\dagger|0\rangle_{\Sigma_1}\right)^n}$$

Exact calculation in free theory

2n-point function on Σ_n

2-point function on Σ_1

Entanglement Growth of Locally Excited States

All Results are for Free theories

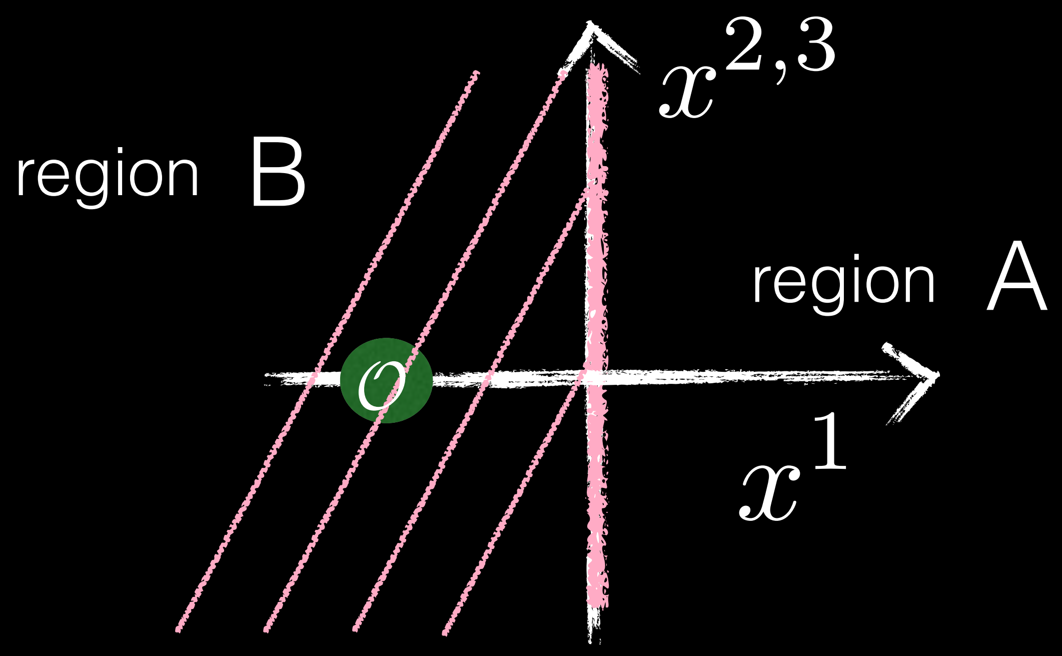
4D Massless Free Scalar Field

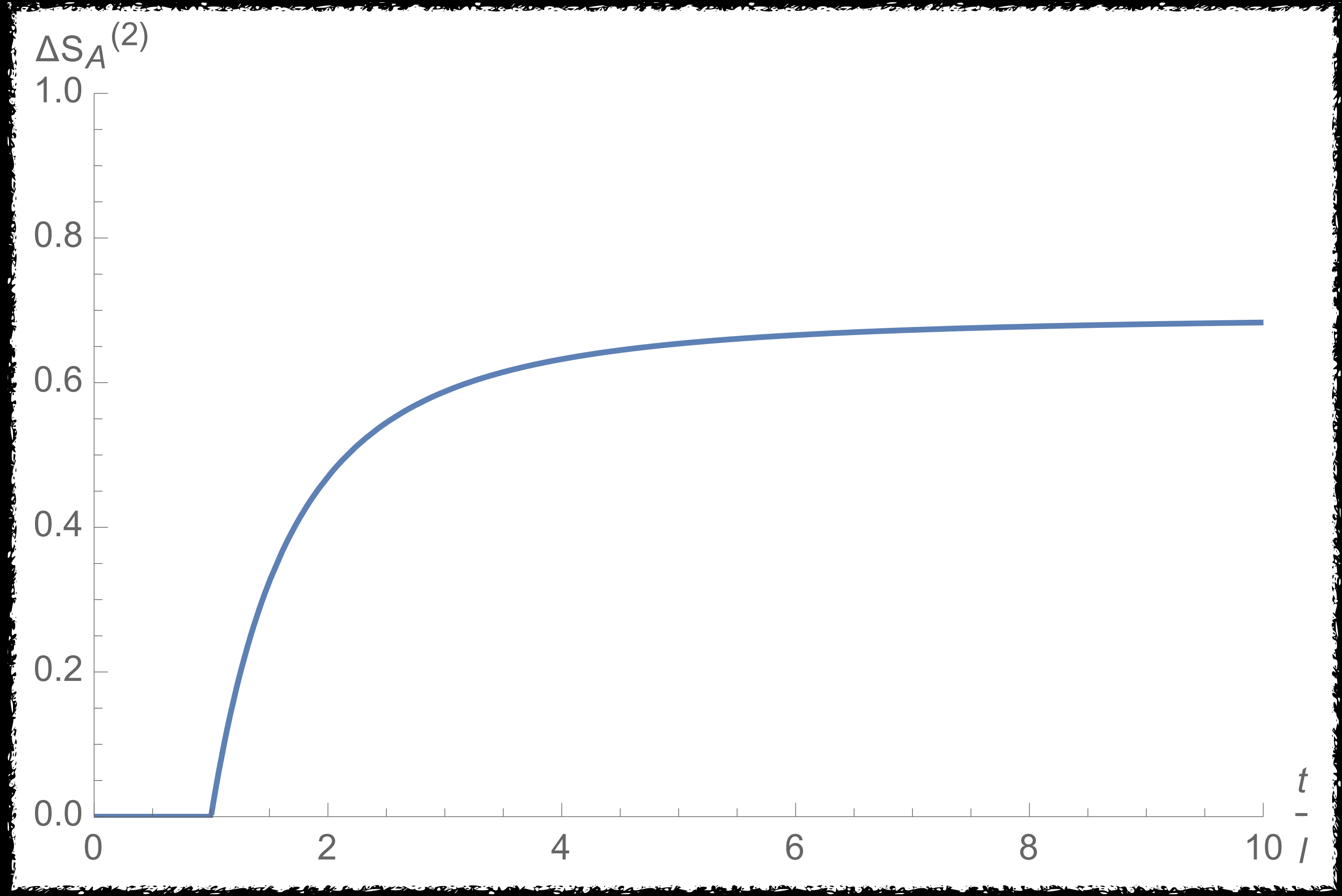
$$\mathcal{O} = \phi$$

At the late time limit:

$$t \to \infty$$

$$\Delta S_A^{(2)} \to \log 2$$





[M. Nozaki, T. Numasawa, T. Takayanagi], [M. Nozaki]

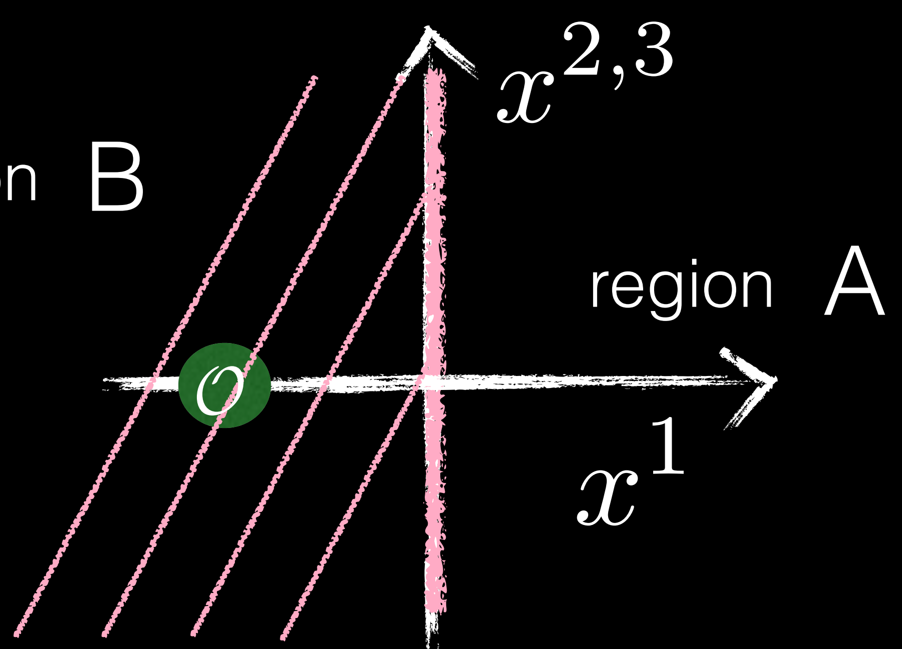
Entanglement Growth of Locally Excited States

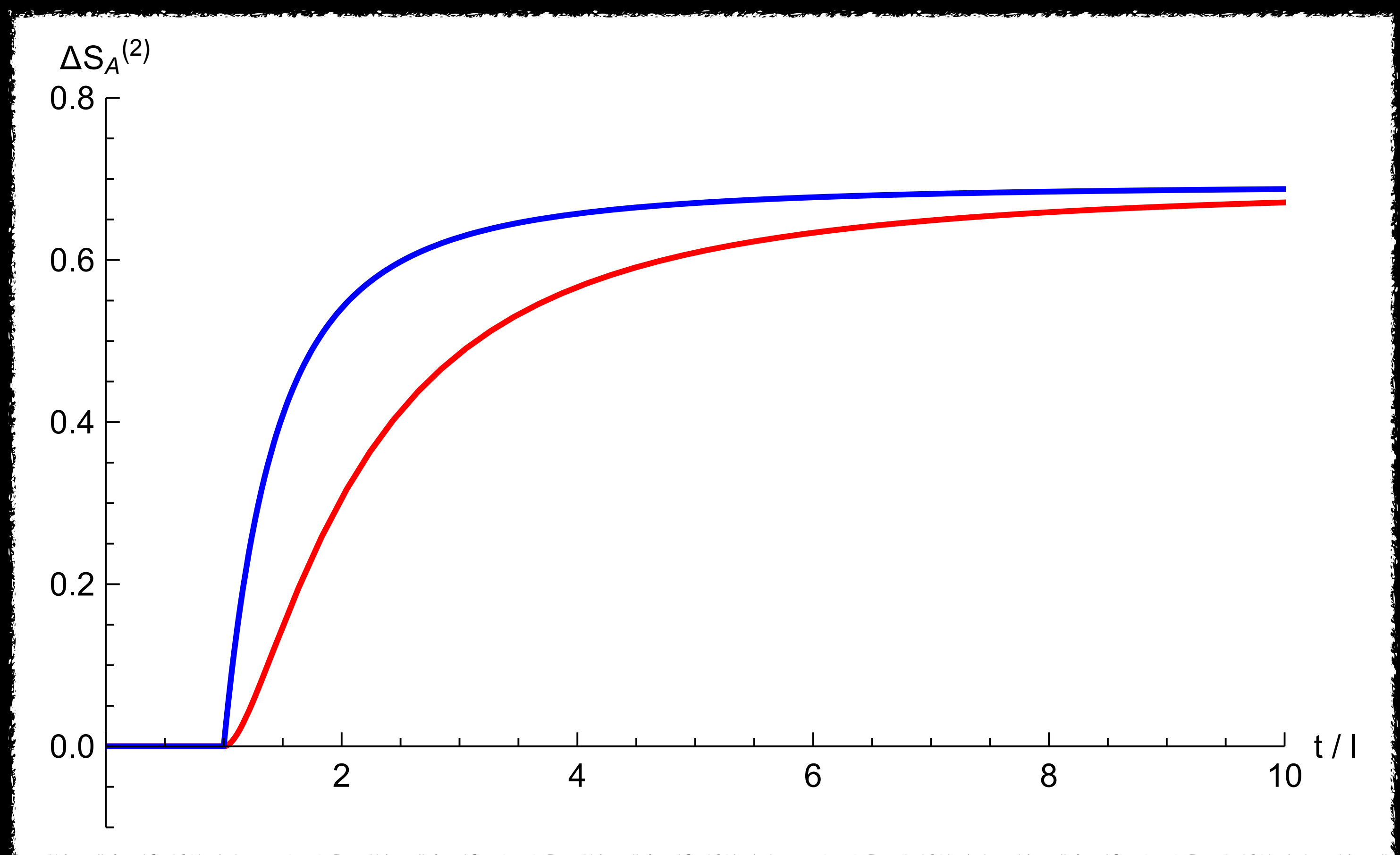
All Results are for Free theories

4D Free Maxwell

Red: $E_1(B_1)$ $t \to \infty$

Blue: $E_{2,3}(B_{2,3})$ $\Delta S_A^{(n)} \to \log 2$



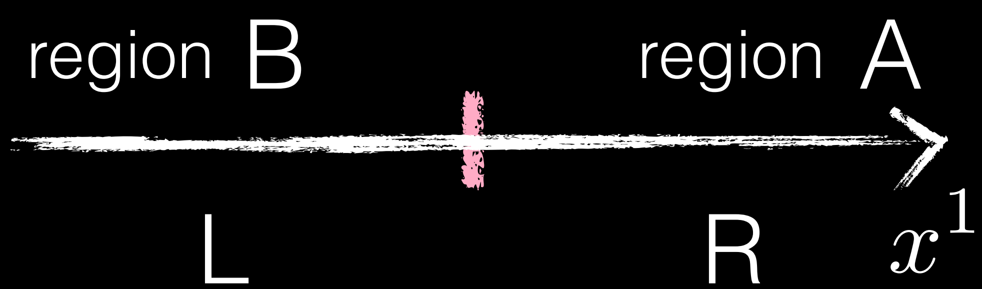


[M. Nozaki, NW]

The Late Time Algebra

The late time behavior of $\Delta S_A^{(n)}$ can be understood from the following algebra obtained from the QFT propagator

- Left / Right movers



Left / Right is along the x^1 direction

$$\hat{\phi} = \hat{\phi}_L + \hat{\phi}_R + \hat{\phi}_L^\dagger + \hat{\phi}_R^\dagger$$

Commutation Relations

$$\left[\hat{\phi}_L, \hat{\phi}_L^\dagger\right] = G^{(n)}(\Delta\theta)$$

$$\left[\hat{\phi}_R, \hat{\phi}_R^\dagger\right] = G^{(n)}(2\pi - \Delta\theta)$$

The others are zero

Propagators on n-sheeted Riemann surface after taking the limit $t \to \infty$

$$G^{(n)}(\theta - \theta') = \langle \phi(\theta)\phi(\theta') \rangle_{\Sigma_n}$$

The Late Time Algebra

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$$\hat{\phi} = \hat{\phi}_L + \hat{\phi}_R + \hat{\phi}_L^\dagger + \hat{\phi}_R^\dagger$$

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The others are zero

$$\mathcal{H}_{\rm tot} = \mathcal{H}_A \otimes \mathcal{H}_B$$

$$\mathcal{H}_A = \mathrm{Span}\{|0\rangle, \hat{\phi}_R^\dagger|0\rangle, \cdots\}$$

$$\mathcal{H}_B = \mathrm{Span}\{|0\rangle, \hat{\phi}_L^\dagger|0\rangle, \cdots\}$$

Reduced density matrix

$$\hat{\rho}_A = \mathrm{tr}_{\mathcal{H}_B}\hat{\rho}$$

REE

$$\Delta S_A^{(n)} = \frac{1}{1-n}\log\left[\mathrm{tr}_{\mathcal{H}_A}\left(\hat{\rho}_A\right)^n\right]$$

Extending the LTA to finite time agrees with the QFT result, also in higher dimensions, both Scalar and Maxwell.

Example : $\hat{\rho} = \dfrac{1}{\mathcal{N}^2}\hat{\phi}|0\rangle\langle 0|\hat{\phi}^\dagger$

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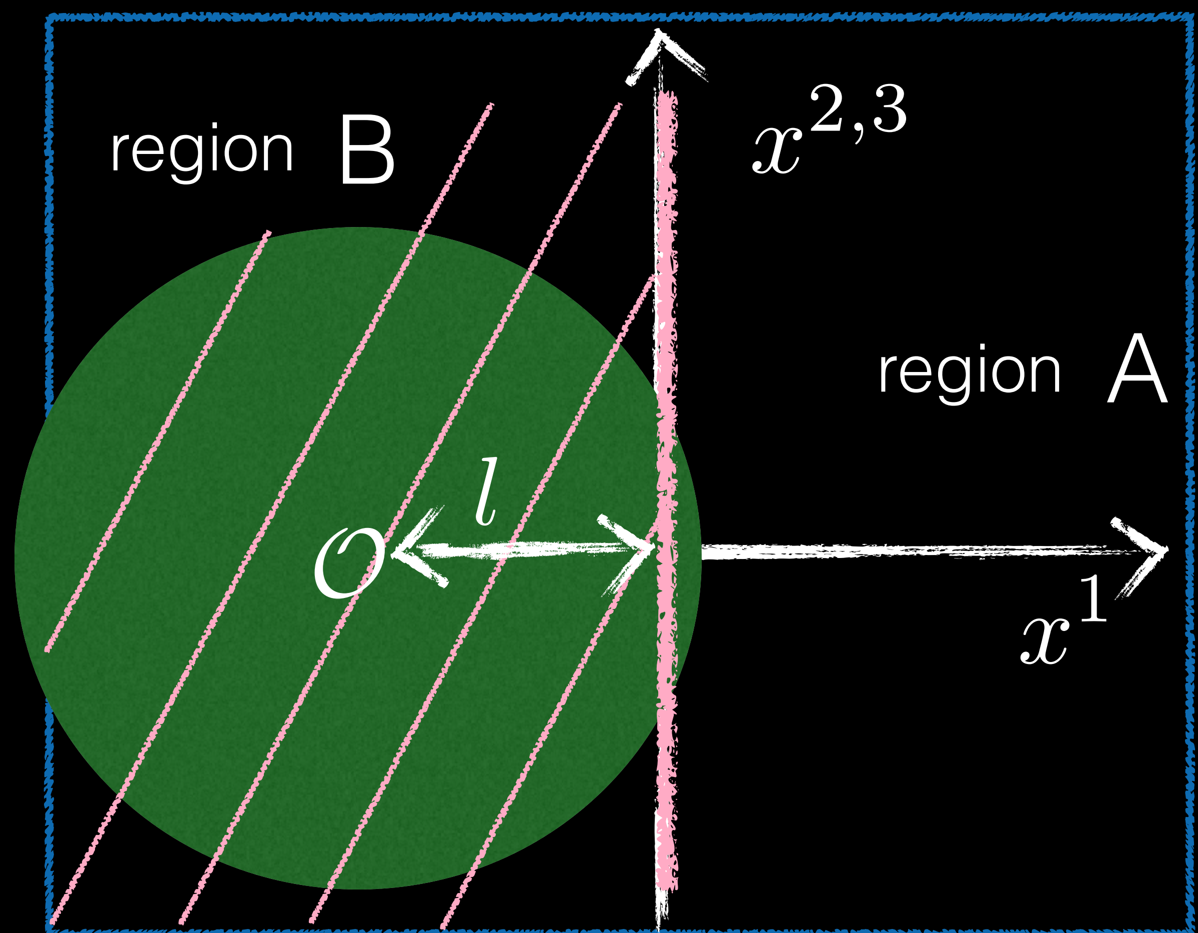
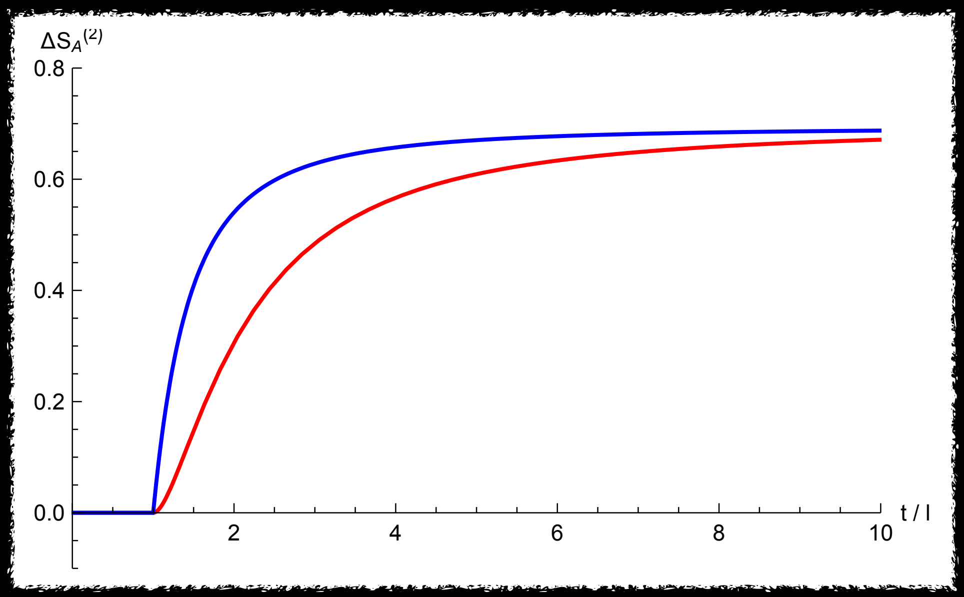
Probability Distribution and EE

It looks like a spherically propagating particle.

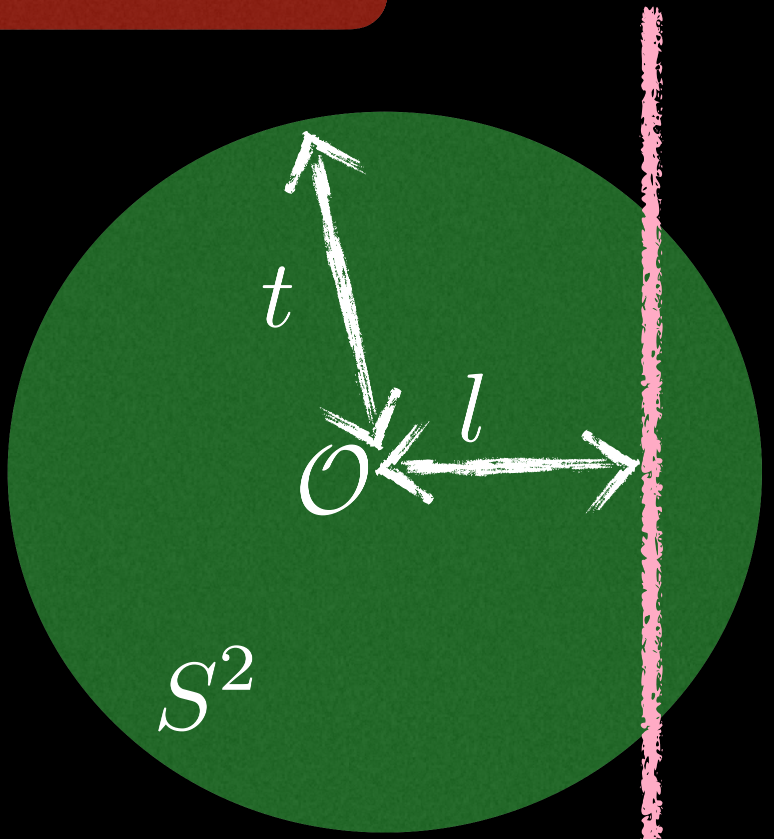




For 4D Free Massless Scalar Theory, this is the case!

Probability Distribution and EE

For free massless Scalar in 4d



$$c = 1$$
$$t > l$$

Area of the Left side

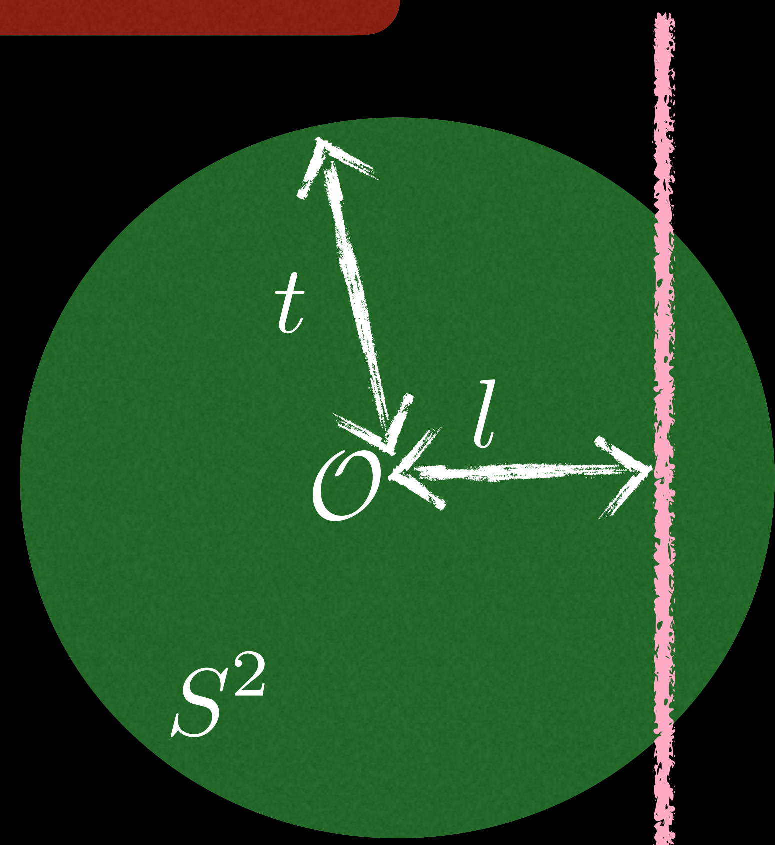
$$\frac{S_B(t)}{S_{all}(t)}$$

Area of the Right side

$$\frac{S_A(t)}{S_{all}(t)}$$

Probability Distribution and EE

For free massless Scalar in 4d



$$c = 1$$
$$t > l$$

Area of the Left side

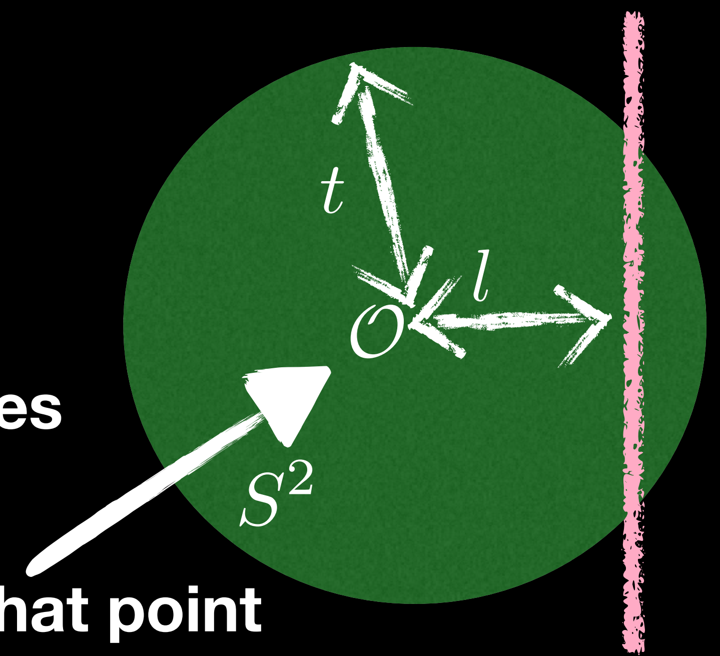
$$\frac{S_B(t)}{S_{all}(t)} = \frac{G^{(n)}(\Delta\theta)}{G^{(1)}(\Delta\theta)}$$

Area of the Right side

$$\frac{S_A(t)}{S_{all}(t)} = \frac{G^{(n)}(2\pi - \Delta\theta)}{G^{(1)}(\Delta\theta)}$$

Probability Distribution and EE

For free massless Scalar in 4d



$c = 1$

Let's consider a model of propagating quasi-particles

An inserted operator creates one quasi-particle at that point

$$\mathcal{O} = \phi$$

The Density Matrix

$$\rho = P_1|0,1\rangle\langle 0,1| + P_2|1,0\rangle\langle 1,0|$$

The particle is in A P_1

The particle is in B P_2

Area of the Right side

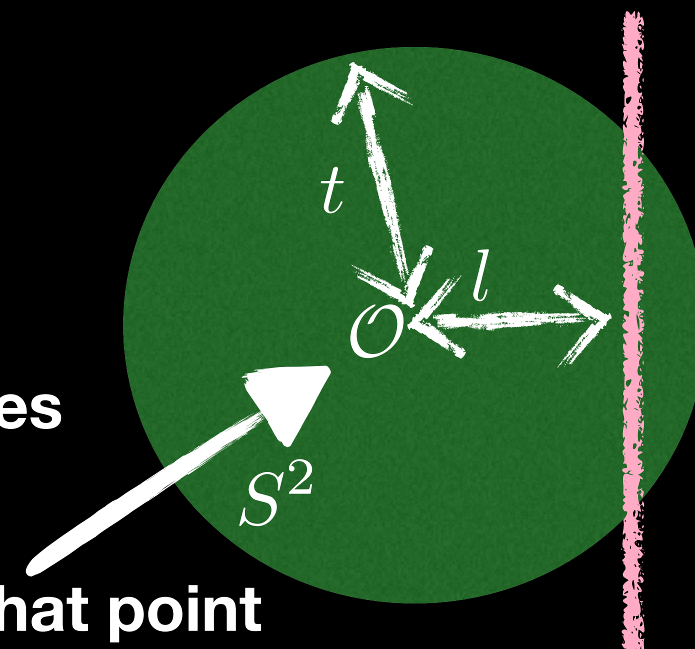
$$\frac{S_A(t)}{S_{all}(t)} = \frac{G^{(n)}(2\pi - \Delta\theta)}{G^{(1)}(\Delta\theta)}$$

Area of the Left side

$$\frac{S_B(t)}{S_{all}(t)} = \frac{G^{(n)}(\Delta\theta)}{G^{(1)}(\Delta\theta)}$$

Probability Distribution and EE

For free massless Scalar in 4d



$c = 1$

Let's consider a model of propagating quasi-particles

An inserted operator creates one quasi-particle at that point

$$\mathcal{O} = \phi$$

Area of the Right side

$$P_1 = \frac{S_A(t)}{S_{all}(t)}$$

The Density Matrix

$$\rho = P_1 |0,1\rangle\langle 0,1| + P_2 |1,0\rangle\langle 1,0|$$

Area of the Left side

$$P_2 = \frac{S_B(t)}{S_{all}(t)}$$

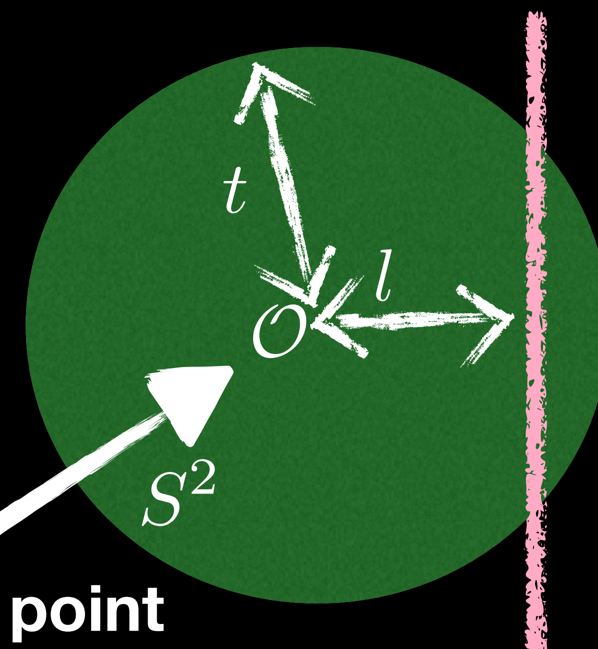
n-th Rényi EE

$$S^{(n)} = \frac{1}{1-n} \log \mathrm{Tr} \rho^n$$

Perfectly agrees with the QFT result
In finite t

Probability Distribution and EE

For free massless Scalar in 4d



$c = 1$

Let's consider a model of propagating quasi-particles

An inserted operator creates one quasi-particle at that point

$$\mathcal{O} = \phi$$

$$P_1 = \frac{S_A(t)}{S_{all}(t)}$$

$$P_2 = \frac{S_B(t)}{S_{all}(t)}$$

The Density Matrix

$$\rho = P_1|0,1\rangle\langle 0,1| + P_2|1,0\rangle\langle 1,0|$$

This is the same for assuming

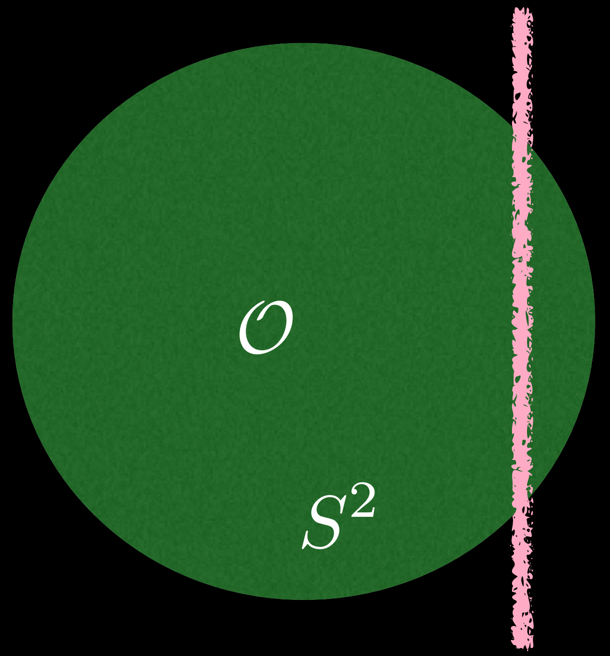
$$\left[\phi_L, \phi_L^\dagger\right] = G^{(n)}(\Delta\theta)$$

$$\left[\phi_R, \phi_R^\dagger\right] = G^{(n)}(2\pi - \Delta\theta)$$

is valid for finite t in LTA

Probability Distribution and EE

For free massless Scalar in 4d



For insertion of more than 1 operators at the same point

$$\mathcal{O} =: \phi^k :$$ **k quasi-particles**

$$\rho = \sum_{l=0}^{k} {}_kC_l (P_1(t))^{k-l} (P_2(t))^k |l, k-l\rangle\langle l, k-l|$$

l in B, k-l in A

n-th Rényi EE

$$S^{(n)} = \frac{1}{1-n} \log \mathrm{Tr} \rho^n$$

This kind of description for other fields and other dimensions are under investigation.

Perfectly agrees with the QFT result In finite t

Summary

- We investigate the property of EE of a state excited by acting with a local operator.
- The late time behavior can be obtained from the „Late Time Algebra“(LTA),
- The commutation relations in LTA are defined from the propagators of corresponding QFT
- In (3+1)D free massless scalar field theory, the QFT result can be described with an model of quasi-particle which is propagating spherically.

Thank you very much!