Mon & Wed is for theory,
Tues & Thur is for experiments,
Fri is for “drinkin` and thinkin
- Close enough!

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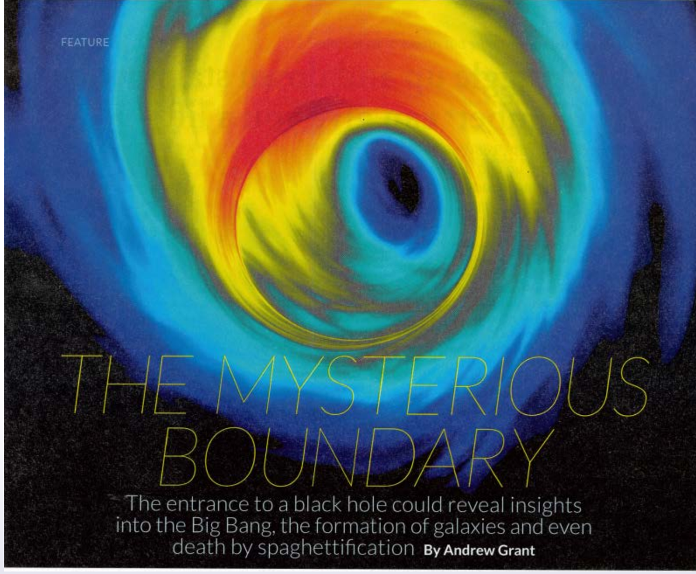
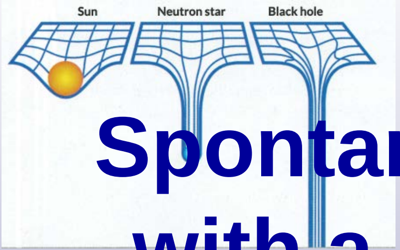


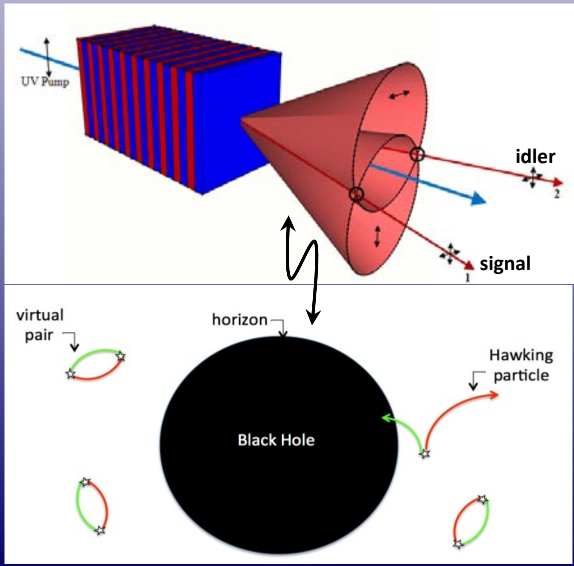




# Spontaneous parametric down conversion with a depleted pump as an analogue for gravitational particle production

RQI 2017

Kyoto, Japan

4—7July2017





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Collaborator: Perry Rice,

Univ. of Miami, Oxford, OH



$$ - AFOSR LRIR: “Relativistic Quantum Information”

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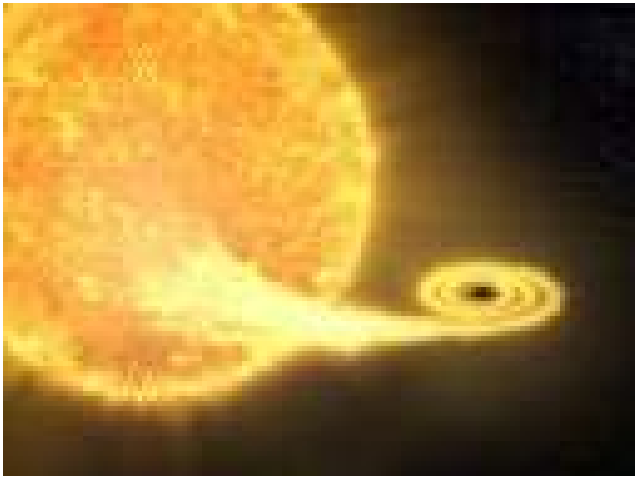
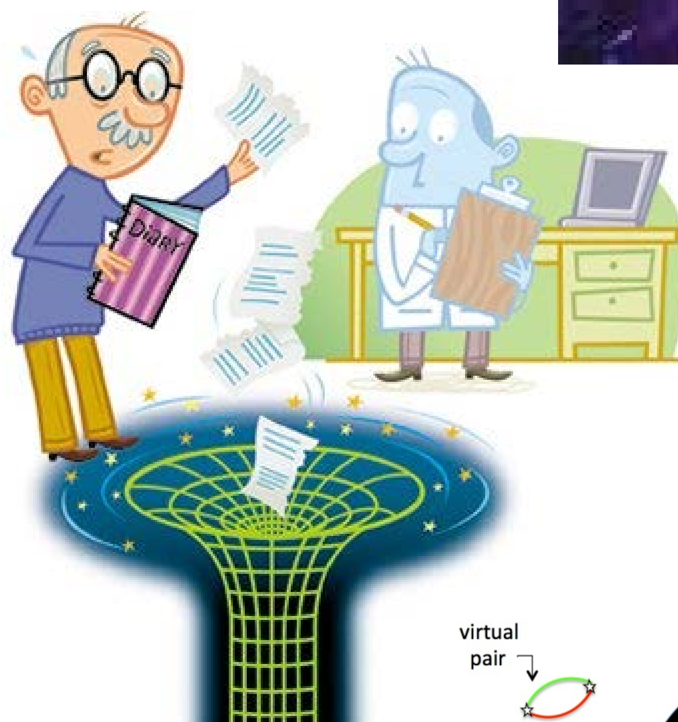
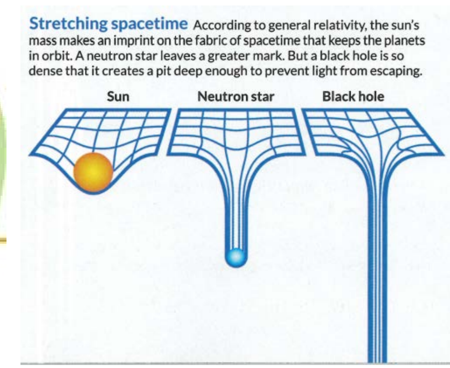
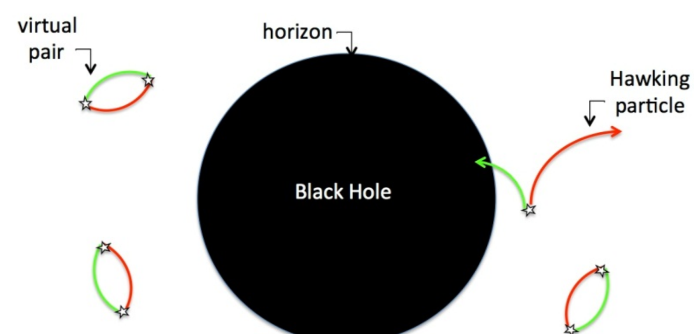
PM: Dr. Tatjana Curcic

# Black Hole Information Problem



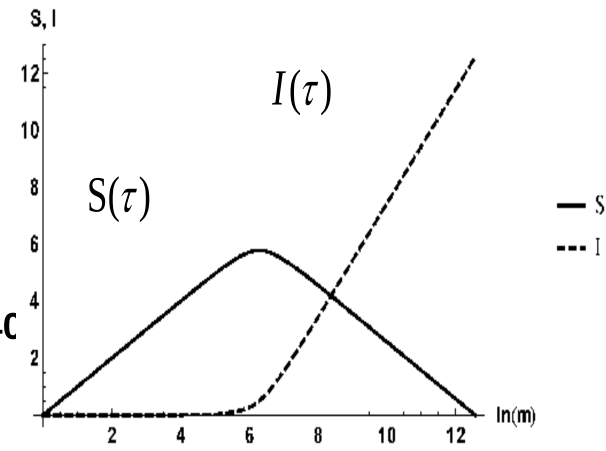


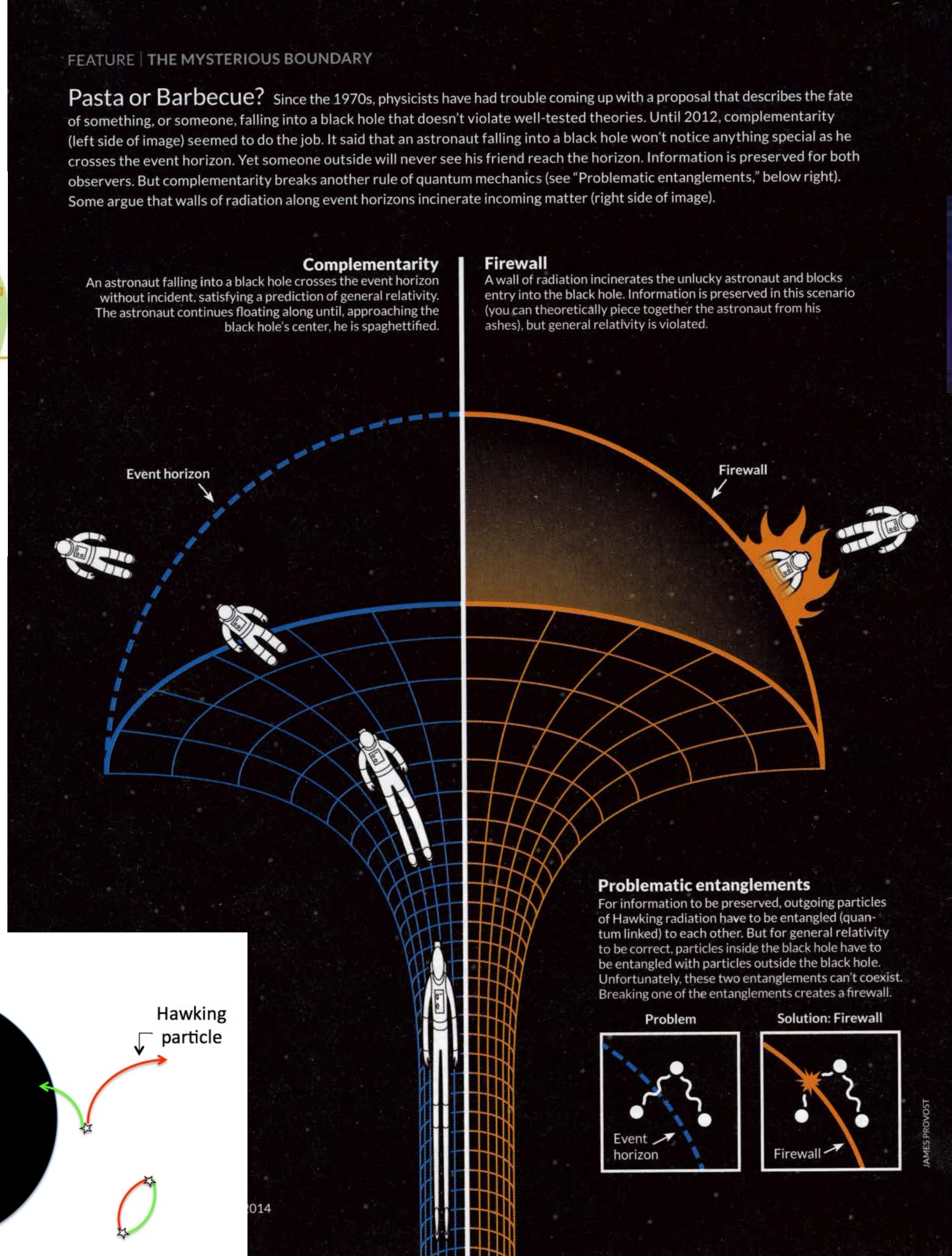



# Outline

- **Classical information transmission capacity of quantum black holes;**
  **Adami & Ver Steeg,** Class. Q. Grav. **31** (2014) 075015; arXiv:gr-qc/0407090v8
  - Classical information is not lost in black hole dynamics; re-emitted in stimulated emission
  - Hawking radiation is spontaneous emission
- **Analogy to SPDC (spontaneous parametric down conversion)**
  - Hawking radiation is a two-mode squeezed state; observed state is thermal
- **Depleted BH `pump' model (PDC)** **(Alsing: CQG 32, 075010, (2015); arXiv:1408.4491)**
  - Quantized the BH `pump' source
  - Short time behavior, Long time behavior
  - Page Information Curves




- **One Shot Decoupling Model (Bradler & Adami: arXiv:1505.0284**
  **Alsing & Fanto: CQG 33, 015005 (2016), arXiv:1507.00429)**
  - Suggested by Alsing: CQG:2015 Future Work; closer analogy to SPDC
  - Page Information Curves redux
- **Summary and Conclusion**



FEATURE | THE MYSTERIOUS BOUNDARY

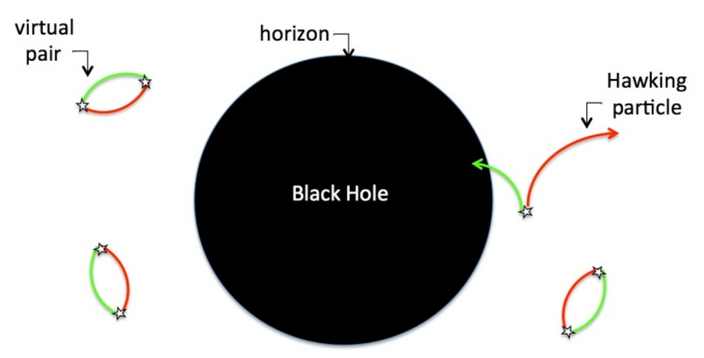
**Pasta or Barbecue?** Since the 1970s, physicists have had trouble coming up with a proposal that describes the fate of something, or someone, falling into a black hole that doesn't violate well-tested theories. Until 2012, complementarity (left side of image) seemed to do the job. It said that an astronaut falling into a black hole won't notice anything special as he crosses the event horizon. Yet someone outside will never see his friend reach the horizon. Information is preserved for both observers. But complementarity breaks another rule of quantum mechanics (see "Problematic entanglements," below right). Some argue that walls of radiation along event horizons incinerate incoming matter (right side of image).

**Complementarity**
An astronaut falling into a black hole crosses the event horizon without incident, satisfying a prediction of general relativity. The astronaut continues floating along until, approaching the black hole's center, he is spaghettified.

**Firewall**
A wall of radiation incinerates the unlucky astronaut and blocks entry into the black hole. Information is preserved in this scenario (you can theoretically piece together the astronaut from his ashes), but general relativity is violated.

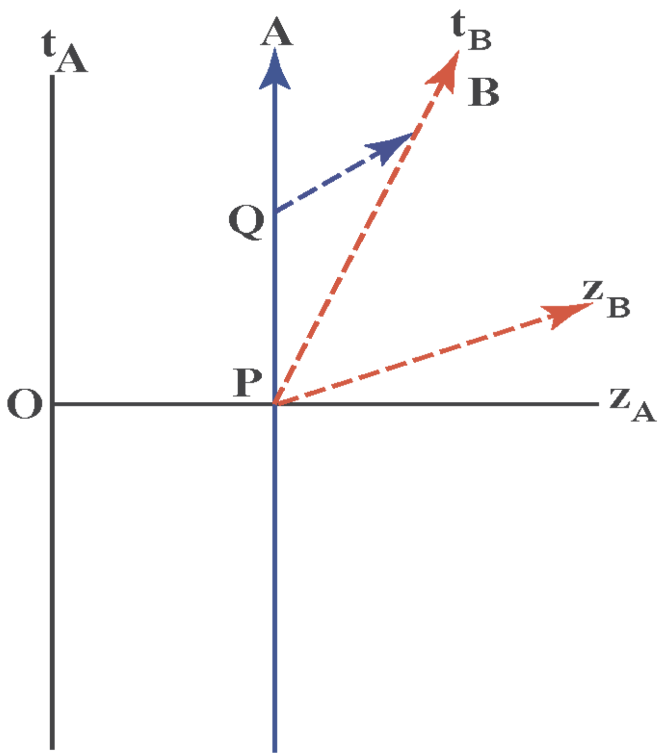
**Problematic entanglements**
For information to be preserved, outgoing particles of Hawking radiation have to be entangled (quantum linked) to each other. But for general relativity to be correct, particles inside the black hole have to be entangled with particles outside the black hole. Unfortunately, these two entanglements can't coexist. Breaking one of the entanglements creates a firewall.





# Simple Derivation of Unruh Effect: zero vs. constant acceleration

Constant Velocity

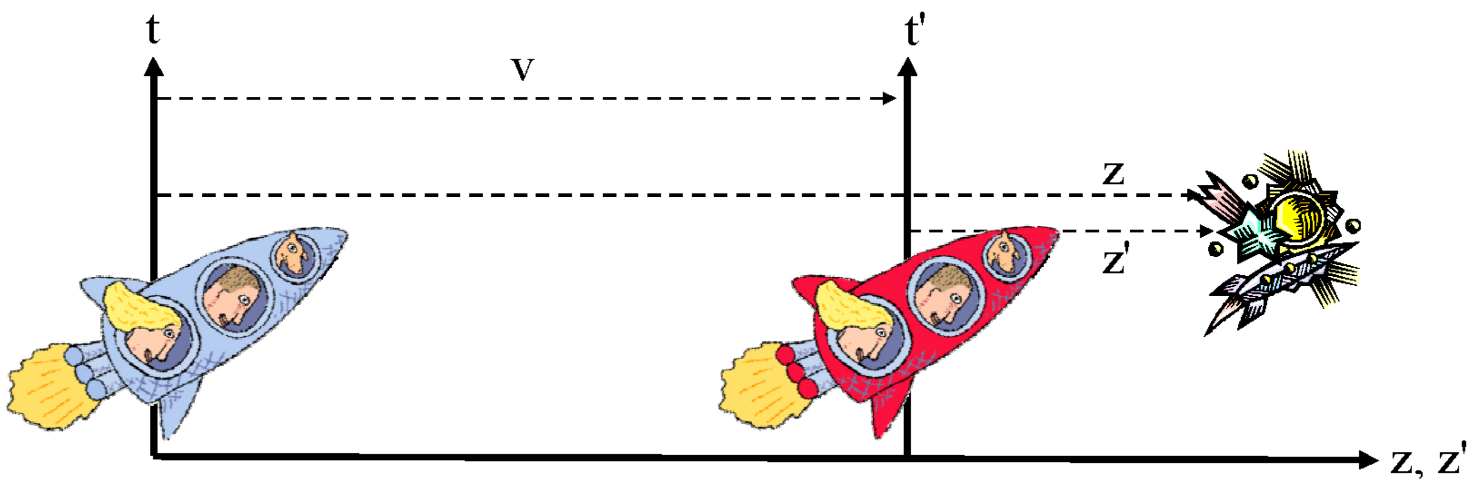



(a)

# Simple Derivation of Unruh Effect: Bosons

## Frequency Transformations in SR: a = 0 (constant velocity)

Alsing & Milonni, Am.J.Phys. **72** 1524 (2004);   T. Padmanabhan, "Gravitation: Foundations & Frontiers," Cambridge (2010).




**Lorentz Transformation**

$$ct = ct' \cosh r + z' \sinh r, \qquad z = ct' \sinh r + z' \cosh r,$$

**For** $k = \omega/c$, **the phase of a plane wave** $e^{i\phi(t,z)}$ **transforms as**

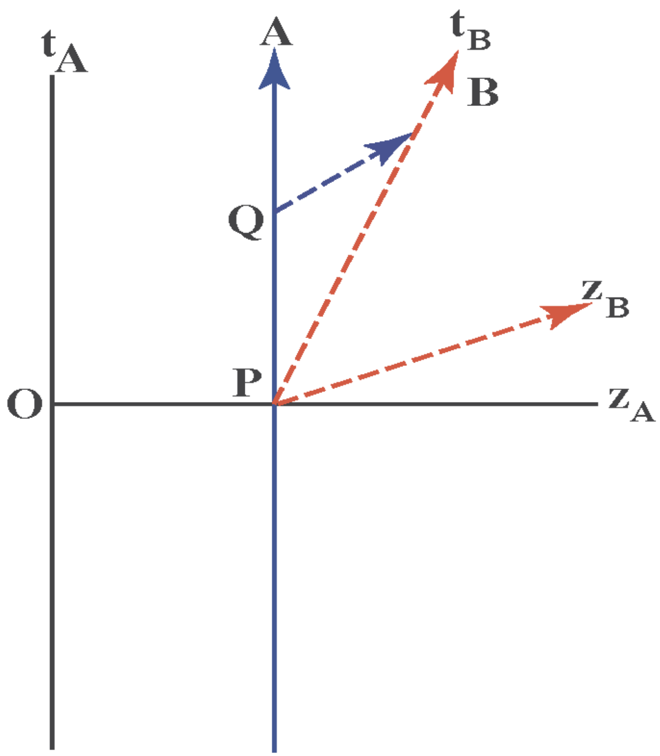
$$\phi = k \cdot z \equiv kz \pm \omega t = k(z \pm ct) = k e^{\pm r}(z' \pm ct') \equiv k' \,(z' \pm ct')$$

**Transformed frequency/phase**

$$\phi' = \phi, \qquad \omega' = \omega \, e^{\pm r}$$

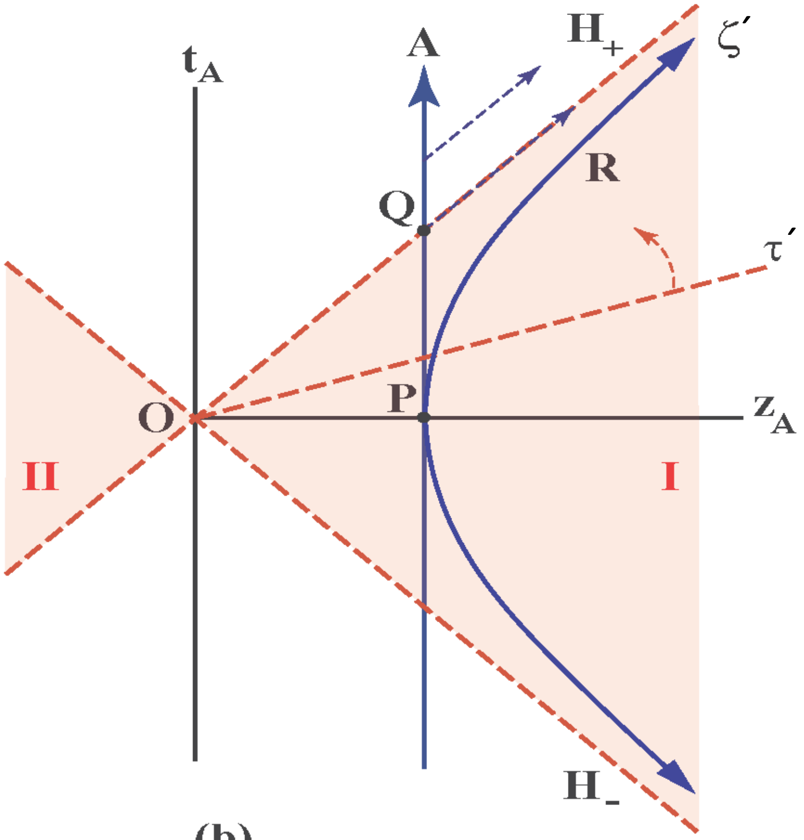
# Simple Derivation of Unruh Effect: zero vs. constant acceleration

Constant Velocity




(a)

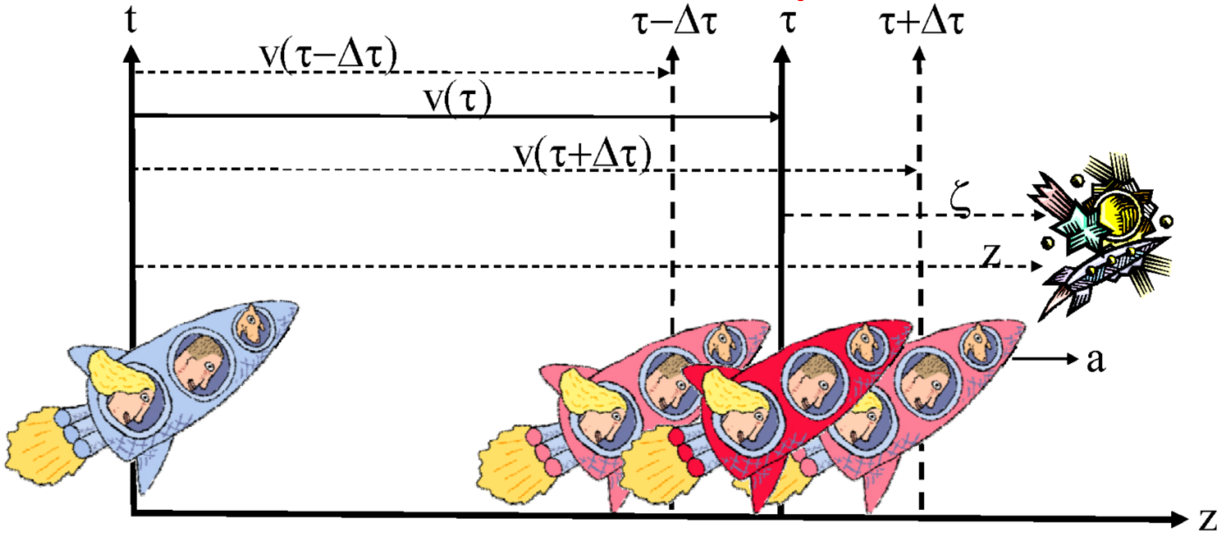
Constant Acceleration




(b)

# Simple Derivation of Unruh Effect: Bosons

**Frequency Transformations in SR: a = constant; (uniform acceleration)**




**Rindler Transformation**

$$ct = \zeta \sinh(a\tau/c) \qquad z = \zeta \cosh(a\tau/c),$$

On orbit of accelerated (Rindler) observer $\zeta = (c^2/a)$

$e^{i\phi(t,z)} \Rightarrow$

$$\phi \equiv kz \pm \omega t = k(z \pm ct) = k\zeta\, e^{\pm a\tau/c} = \omega(c/a)\, e^{\pm a\tau/c}$$

Time dependent Doppler shifted frequency/phase

$$\omega'(\tau) = \omega\, e^{\pm a\tau/c}, \qquad \phi(\tau) = \omega\,(c/a)\, e^{\pm a\tau/c}$$

# Simple Derivation of Unruh Effect: Bosons

**Noise Spectrum seen by (Rindler) accelerated observer: spin 0**

$$S(\Omega)=\left|\int_{-\infty}^{\infty} d\tau e^{i\Omega\tau} e^{i\phi(\tau)}\right|^2=\left|\int_{-\infty}^{\infty} d\tau e^{i\Omega\tau} e^{i(\omega c/a)e^{a\tau/c}}\right|^2.$$

**Changing variables to $y=e^{a\tau/c}$, we have**

$$\left[s=i\Omega c/a=i\Omega/(a/c) \quad b=-i\omega c/a,\ -i=e^{-i\pi/2}\right]$$

$$\left[\int_0^\infty dy\, y^{s-1}e^{-by}=e^{-s\ln b}\,\Gamma(s) \quad \mathrm{Re}\, b>0,\ \mathrm{Re}\, s>0\right]$$

$$\begin{aligned}\int_{-\infty}^{\infty} d\tau e^{i\Omega\tau} e^{i(\omega c/a)e^{a\tau/c}} &= \frac{c}{a}\int_0^\infty dy\, y^{(i\Omega c/a-1)} e^{i(\omega c/a)y} \\ &= \frac{c}{a}\Gamma\left(\frac{i\Omega c}{a}\right)\left(\frac{\omega c}{a}\right)^{-i\Omega c/a} e^{-\pi\Omega c/2a},\end{aligned}$$

**where $\Gamma$ is the gamma function. Then, since**

$$\left|\Gamma\left(\frac{i\Omega c}{a}\right)\right|^2=\frac{\pi}{(\Omega c/a)\sinh(\pi\Omega c/a)},$$

**we have**

$$S(\Omega)=\left|\int_{-\infty}^{\infty} d\tau e^{i\Omega\tau} e^{i(\omega c/a)e^{a\tau/c}}\right|^2=\frac{2\pi c}{\Omega a}\frac{1}{e^{2\pi\Omega c/a}-1}$$

**The Planck factor**

$$\frac{1}{e^{2\pi\Omega c/a}-1}\equiv\frac{1}{e^{\hbar\Omega/kT_{Unruh}}-1} \Rightarrow kT_{Unruh}=\frac{\hbar a/c}{2\pi}$$

**is indicative of a Bose Einstein (BE) distribution.**

Alsing & Milonni, Am.J.Phys. **72** 1524 (2004)

# Simple Derivation of Unruh Effect: Fermions

**Noise Spectrum seen by (Rindler) accelerated observer: spin 1/2**

A Dirac spin 1/2 particle is a spinor wave function

$$\psi_\alpha = u_\alpha\, e^{i(kz\pm\omega t)} = u_\alpha e^{i\phi(t,z)}$$

To boost into the instantaneous rest frame of the accelerated observer, we must transform not only the phase of the plane wave, but also the spinor

$$\begin{aligned}\psi_\alpha(t,z) \to \psi_\alpha(\tau) &= \left(e^{a\tau/2\,\boldsymbol{\gamma}^0\boldsymbol{\gamma}^3}\right)_{\alpha\beta} u_\beta\, e^{i\phi(\tau)} \equiv \hat{S}_{\alpha\beta}\, u_\beta\, e^{i\phi(\tau)} \\ &= \left[\cosh(a\tau/2)\,\boldsymbol{I} + \sinh(a\tau/2)\,(\boldsymbol{\gamma}^0\boldsymbol{\gamma}^3)\right]_{\alpha\beta} u_\beta\, e^{i\phi(\tau)}\end{aligned}$$

where

$$\boldsymbol{\gamma}^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad \boldsymbol{\gamma}^3 = \begin{pmatrix} 0 & \sigma_z \\ -\sigma_z & 0 \end{pmatrix},$$

For $u = [1,0,1,0]^T$ , we have $\hat{S}(\tau)\,u = e^{(a\tau/2c)}\,u$ (eigenstate)

$$S(\Omega) = \left|\int_{-\infty}^{\infty} d\tau e^{i\Omega\tau}\, e^{a\tau/2c}\, e^{i(\omega c/a)e^{a\tau/c}}\right|^2 = \frac{2\pi c}{\omega a}\frac{1}{e^{2\pi\Omega c/a}+1} \qquad \textbf{(Fermi-Dirac)}$$

Where we have used

$$i\Omega c/a \to i\Omega c/a + 1/2 \quad \Rightarrow \quad |\Gamma(i\Omega c/a + 1/2)|^2 = \frac{\pi}{\cosh(\pi\Omega c/a)}$$

Alsing & Milonni, Am.J.Phys. **72** 1524 (2004)

# Flat Minkowski Spacetime: Modes

**Metric and Wave Equation:**

$$ds^2 = dt^2 - dz^2, \qquad \Rightarrow \qquad (\partial_t^2 - \partial_z^2)\phi = 0.$$

**Solutions:**

$$\omega > 0, \qquad f_k = \frac{1}{\sqrt{4\pi\omega}} e^{i(kz-\omega t)}, \qquad k^2 = \omega^2$$

**Inner Product:**

$$(\phi_1, \phi_2) = -i \int_{\Sigma_t} (\phi_1 \, \partial_t \, \phi_2^* - \phi_2 \, \partial_t \, \phi_1^*) \, dz$$

$$(f_{k_1}, f_{k_2}) = \delta(k_1 - k_2), \qquad (f_{k_1}^*, f_{k_2}^*) = -\delta(k_1 - k_2), \qquad (f_{k_1}, f_{k_2}^*) = 0$$

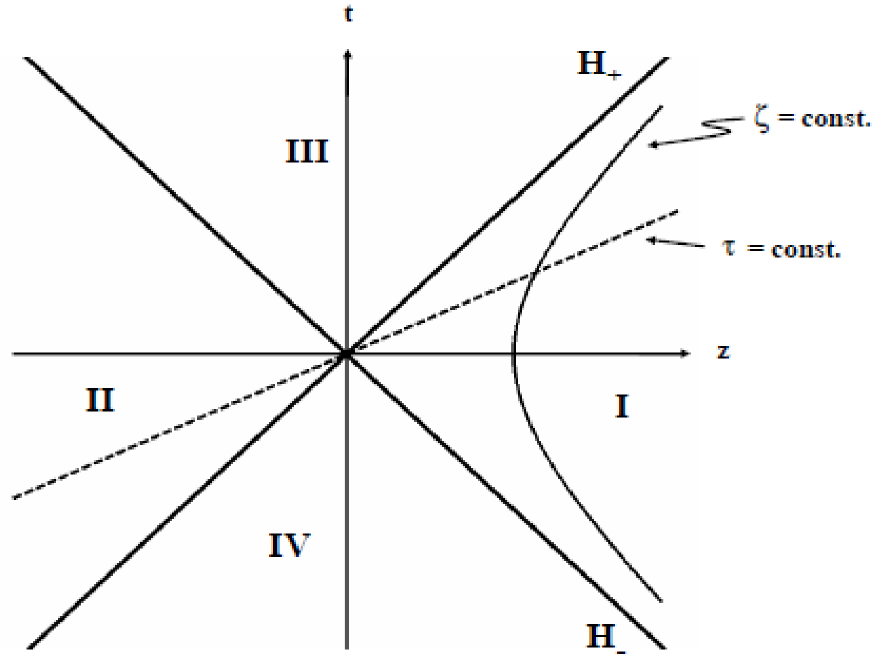
**Definition of Positive and Negative Frequency Modes:**

$$\text{Pos freq:} \qquad \partial_t \, f_k = -i \, \omega \, f_k, \qquad \text{Neg freq:} \qquad \partial_t \, f_k^* = i \, \omega \, f_k^*$$

**Field Decomposition in terms of Minkowski modes:**

$$\hat{\phi} = \int dk \, (\hat{a}_k \, f_k + \hat{a}_k^\dagger \, f_k^*), \qquad [\hat{a}_{k_1}, \hat{a}_{k_2}^\dagger] = \delta(k_1 - k_2), \qquad [\hat{a}_{k_1}, \hat{a}_{k_2}] = [\hat{a}_{k_1}^\dagger, \hat{a}_{k_2}^\dagger] = 0$$

# The Unruh Effect: Modes




Rinder coordinates:

$$t = \frac{\pm 1}{a} e^{a\zeta} \sinh(a\tau), \quad z = \frac{\pm 1}{a} e^{a\zeta} \cosh(a\tau), \quad (I : z > 0,\ z > |t|, \quad II : z < 0,\ |z| > |t|),$$

Metric and Wave Equation:

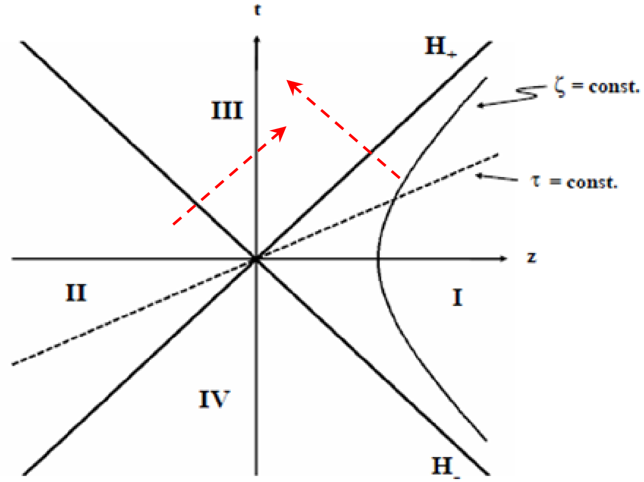
$$ds^2 = e^{2a\zeta}(d\tau^2 - d\zeta^2), \quad \Rightarrow \quad e^{-2a\zeta}(\partial_\tau^2 - \partial_\zeta^2)\phi = 0.$$

# The Unruh Effect: Modes (cont.)

**Solutions:**

$$g_k^{(1)} = \begin{cases} \frac{1}{\sqrt{4\pi\omega}} e^{ik\zeta - i\omega\tau} & \text{I} \\ 0 & \text{II} \end{cases}$$

$$g_k^{(2)} = \begin{cases} 0 & \text{I} \\ \frac{1}{\sqrt{4\pi\omega}} e^{ik\zeta + i\omega\tau} & \text{II} \end{cases}$$




**Inner Product:**

$$(g_{k_1}^{(1)}, g_{k_2}^{(1)}) = \delta(k_1 - k_2), \qquad (g_{k_1}^{(2)}, g_{k_2}^{(2)}) = \delta(k_1 - k_2), \qquad (g_{k_1}^{(1)}, g_{k_2}^{(2)}) = 0$$

**Positive and Negative Frequency Modes:**

2 Pos freq modes: $\partial_\tau \, g_k^{(1)} = -i\,\omega g_k^{(1)}, \qquad \partial_{(-\tau)} \, g_k^{(2)} = -i\,\omega g_k^{(2)},$

2 Neg freq modes: $\{g_k^{(1)*}, g_k^{(2)*}\}$

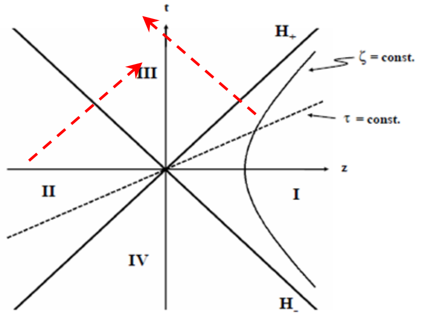
**Field Decomposition in terms of Rindler modes:**

$$\phi = \int dk \left( \hat{b}_k^{(1)} \, g_k^{(1)} + \hat{b}_k^{(1)\dagger} \, g_k^{(1)*} + \hat{b}_k^{(2)} \, g_k^{(2)} + \hat{b}_k^{(2)\dagger} \, g_k^{(2)*} \right)$$

$$[\hat{b}_{k_1}^{(i)}, \hat{b}_{k_2}^{(j)\dagger}] = \delta_{i,j} \, \delta(k_1 - k_2), \quad [\hat{b}_{k_1}^{(i)}, \hat{b}_{k_2}^{(j)}] = [\hat{b}_{k_1}^{(i)\dagger}, \hat{b}_{k_2}^{(j)\dagger}] = 0$$

Sean Carroll, *Spacetime and Geometry*, Chap 9, (2004)

# The Unruh Effect: The Minkowski Vacuum State in terms of Rindler Modes




Minkowski Vacuum

$$|0\rangle_M = \frac{1}{\cosh r}\sum_{n=0}^{\infty} \tanh r\, |n\rangle_I \otimes |n\rangle_{II}, \quad \tanh r = e^{-\pi\Omega}, \quad \Omega \equiv \frac{\omega}{a/c}$$

Density matrix for observer Rob in region I: $\rightarrow$ a Thermal State

$$\hat{\rho}^{(I)} = Tr_{II}[\,|0\rangle_M\langle 0|\,] = (1 - e^{-2\pi\Omega})\sum_{n=0}^{\infty} e^{-2\pi\Omega n}\, |n\rangle_I\langle n|$$

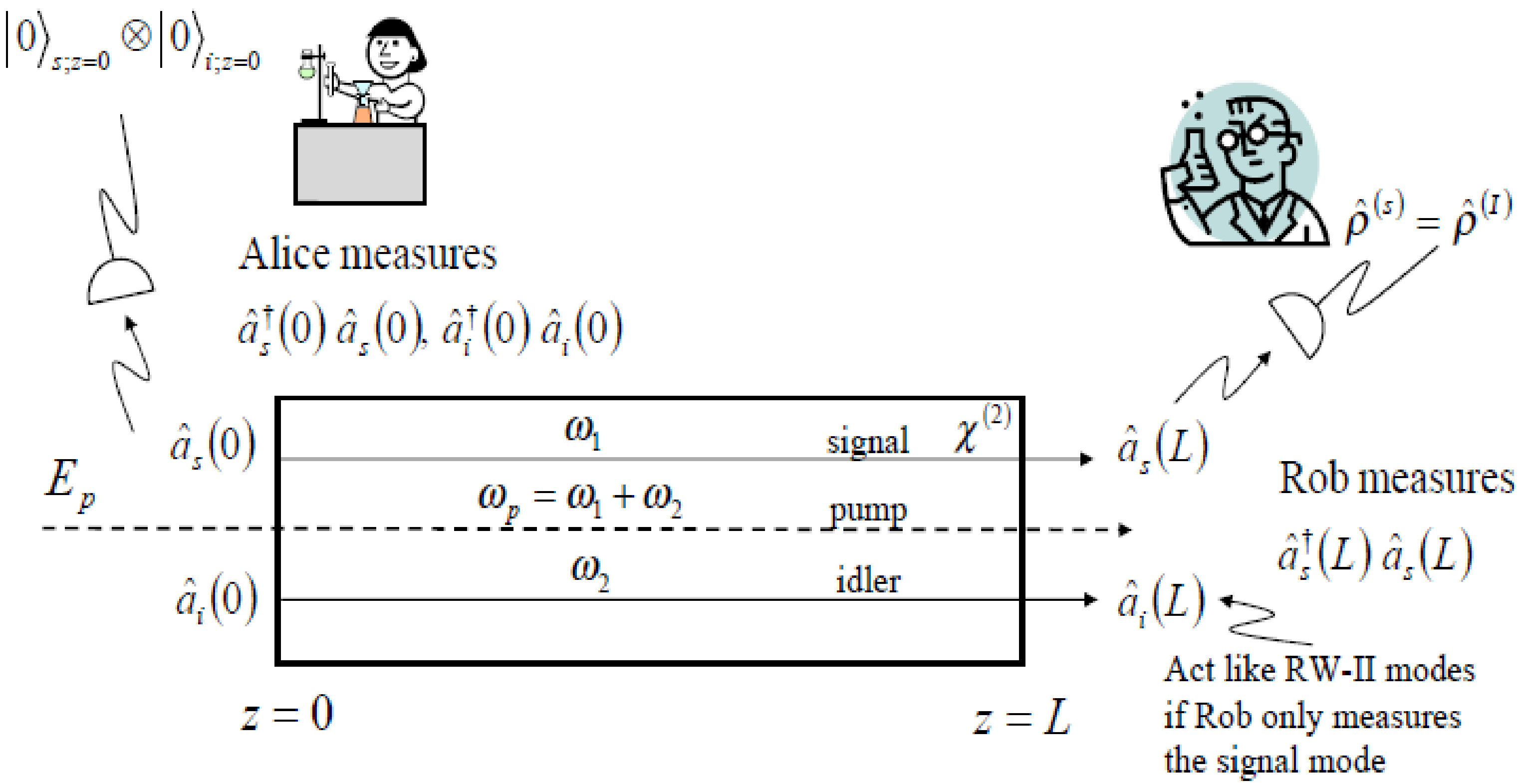
Mean number of particles measured in region I: a Bose-Einstein distribution

$$\langle \hat{n}_I \rangle = Tr_I[\,\hat{b}_I^\dagger \hat{b}_I\, \hat{\rho}^{(I)}\,] = \frac{1}{e^{2\pi\Omega} - 1} \equiv \frac{1}{e^{\hbar\omega/k_B T_U} - 1}$$

Unruh Temperature

$$T_U = \frac{\hbar(a/c)}{2\pi k_B}$$

# Analogy between non-degenerate parametric down conversion and the Unruh-Hawking effect




A detector at z=L measures $\hat{n}_s(L)$ and $\hat{n}_i(L)$.
If Rob measures only the signal modes, i.e. Rindler Wedge I (RW-I)modes, the idler modes act like Rindler Wedge II (RW-II) modes, to which he has no access, and must trace over. Here Rob *chooses* to ignore the idler modes and canalways recover them in principle. This is not the case in the Unruh-Hawking effectwhere Rob, constrained to RW-I, is *causally disconnected* from RW-II, and is forced to trace overthe latter, since he can never, even in principle, acquire information from that region.

## Again, Rob sees "out" particles in the "in" vacuum

$$ {}_{s;z=0}\langle 0| \otimes {}_{i;z=0}\langle 0|\ \hat{a}_s^\dagger(L,\omega_1)\hat{a}_s^\dagger(L,\omega_1)\ |0\rangle_{s;z=0} \otimes |0\rangle_{i;z=0} = \sinh^2 r $$

# A Brief Survey of the Hawking Effect

Schwarzschild Metric

$$ds^2 = \left(1 - \frac{2GM}{r}\right) dt^2 - \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 - r^2\, d\Omega^2$$

Linearize about horizon $r = r_s \equiv 2GM$

$$ds^2 \approx \kappa^2 \rho^2 dt^2 - d\rho^2 - \frac{1}{4\kappa^2} d\Omega^2,$$
$$\approx \kappa^2 e^{2\kappa z}(dt^2 - dz^2), \quad \rho(z) = e^{\kappa z}$$

$\kappa = GM/r_s^2 = 1/(4GM)$ = constant surface gravity of Black Hole

Radial Wave Equation $\phi = f(t,r)/r\, Y_{lm}(\theta, \phi)$

$$\frac{\partial^2 f}{\partial t^2} - \frac{\partial^2 f}{\partial r_*^2} + V(r_*) f = 0$$

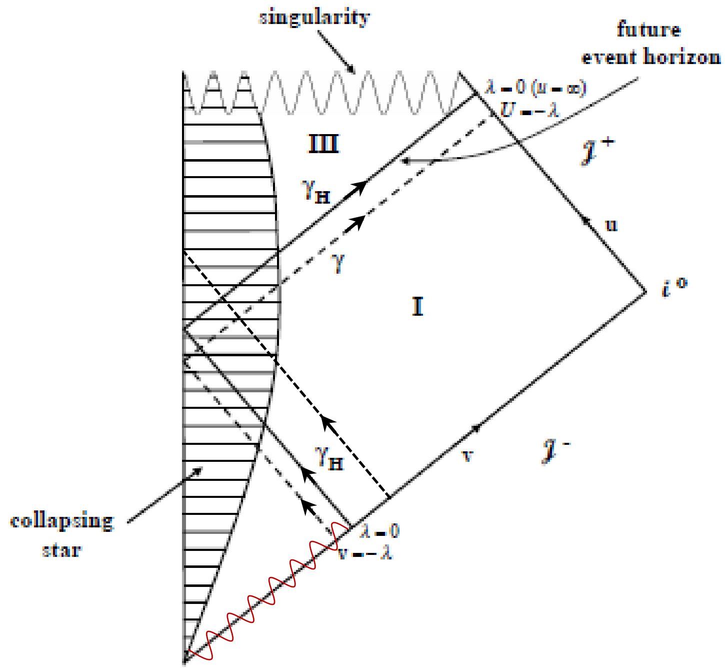
where $r_* = r + r_s \ln(r/r_s - 1)$, and $V(r_*) \to 0$ as $r_* \to -\infty$ $(r \to r_s)$.

$$T_U = \frac{\hbar(a/c)}{2\pi k_B} \Rightarrow \frac{\hbar(\kappa/c)}{2\pi k_B} = T_H$$

$$\kappa = \frac{GM}{r_s^2} = \frac{c^4}{4GM},$$
surface gravity

$$r_s = \frac{2GM}{c^2}$$
Schwarzschild radius

# The Hawking Effect: Modes




Lightcone coordinates:

$$u = t - r_*, \qquad v = t + r_*$$

Incoming and Outgoing Solutions

$$\begin{aligned} f_\omega(t, r_*) &= a\, e^{-i\omega t}\, e^{i\omega r_*} + b\, e^{-i\omega t}\, e^{-i\omega r_*} \\ &= a\, e^{-i\omega u} + b\, e^{-i\omega v} \\ &\equiv \quad f_\omega^{out} \quad + \quad f_\omega^{in} \end{aligned}$$

# Classical information transmission capacity of quantum black holes

Class. Quantum Grav. **31** (2014) 075015

C Adami and G Ver Steeg

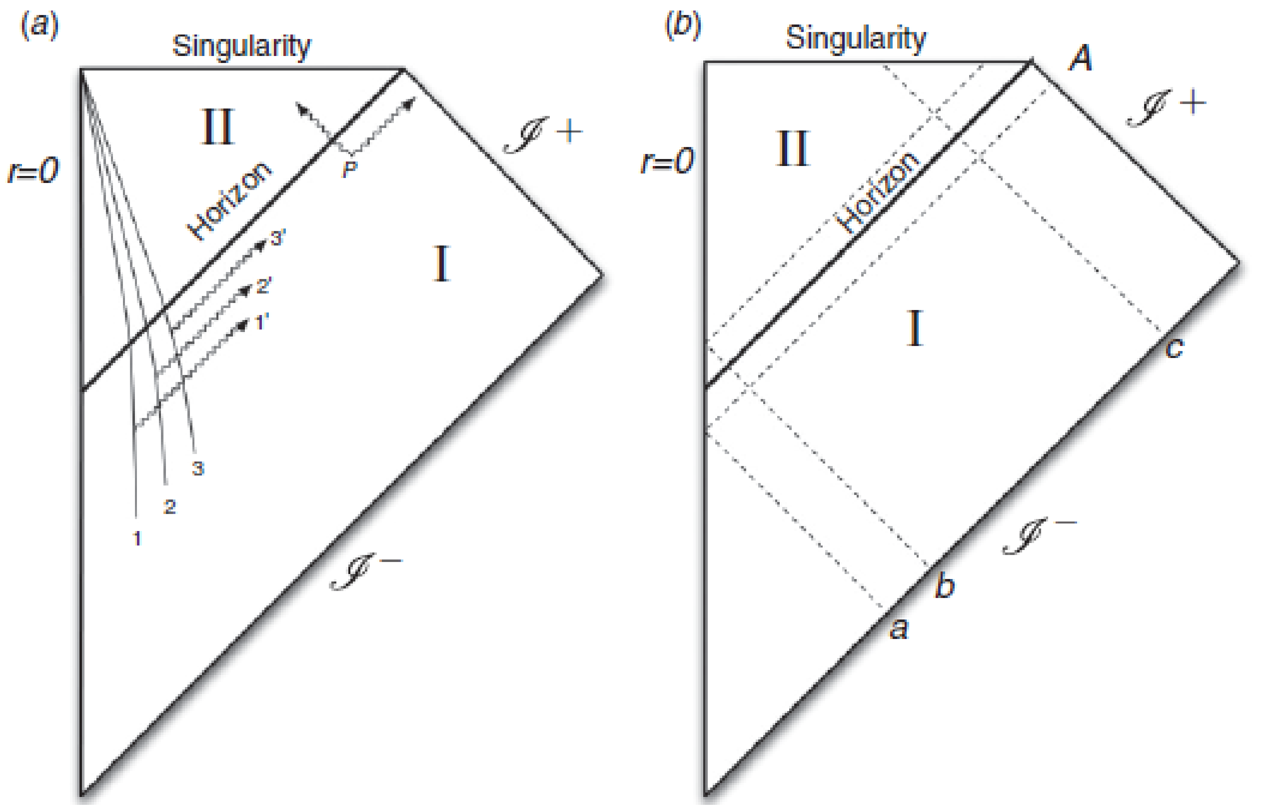
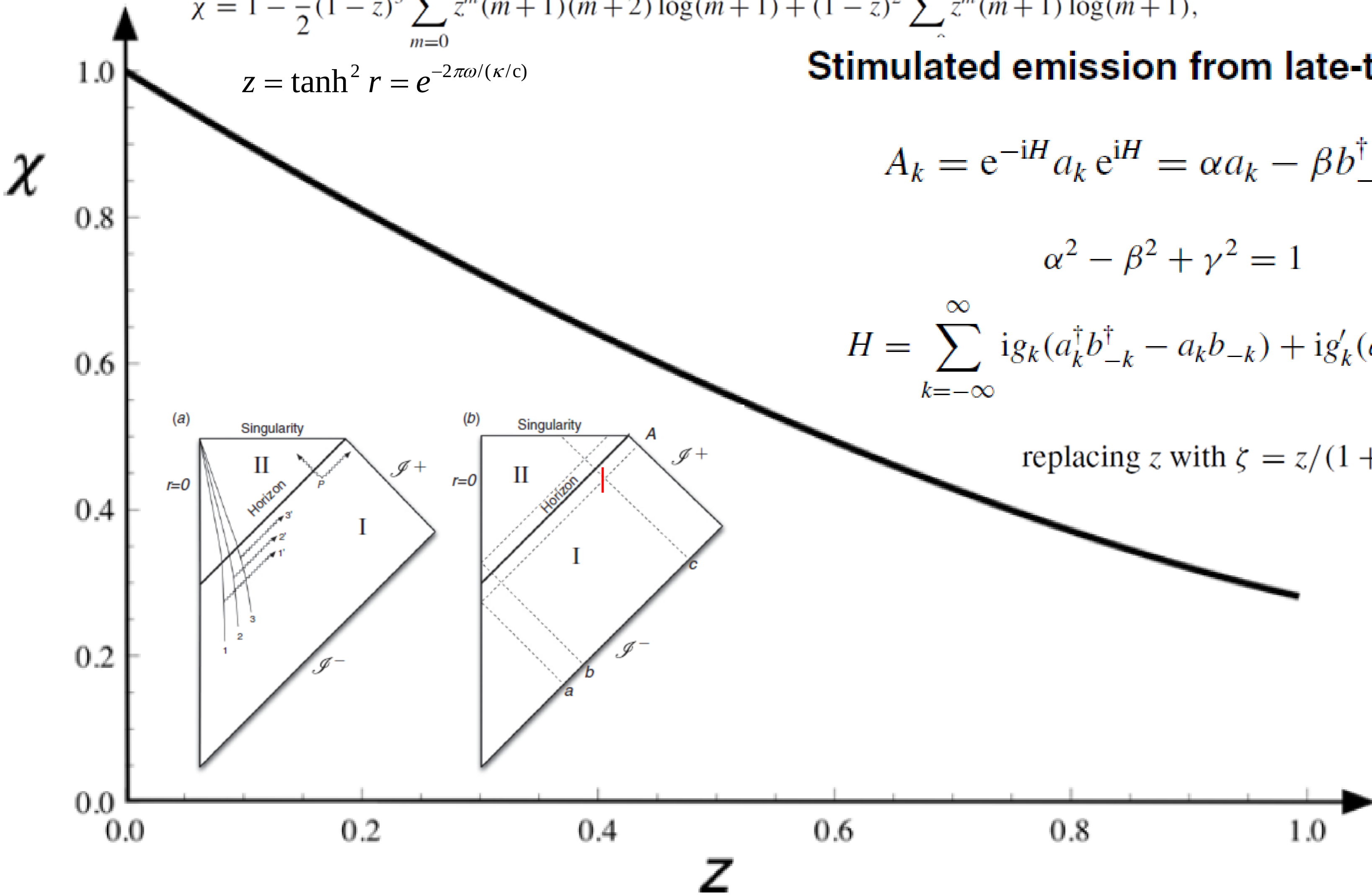



**Figure 1.** (*a*) Penrose diagram of the spacetime of a black hole, with accretion of arbitrarily labeled particles 1,2,3, from past null infinity ($\mathscr{I}^-$). Once the particles cross the event horizon into region II, they are indistinguishable (leading to a loss of information) unless they leave a signature outside (in region I) via stimulated emission (labeled 1′, 2′, 3′). (*b*) Modes *a*, *b*, *c* and *A* are concentrated in a region of null infinity indicated by the letter (note that *a* and *b* actually overlap on $\mathscr{I}^-$).

# Channel (Holevo) Capacity

$$\chi = 1 - \frac{1}{2}(1-z)^3 \sum_{m=0}^{\infty} z^m (m+1)(m+2)\log(m+1) + (1-z)^2 \sum_{m=0}^{\infty} z^m (m+1)\log(m+1),$$

$$z = \tanh^2 r = e^{-2\pi\omega/(\kappa/c)}$$




**Stimulated emission from late-time modes**

$$A_k = \mathrm{e}^{-\mathrm{i}H} a_k\, \mathrm{e}^{\mathrm{i}H} = \alpha a_k - \beta b_{-k}^{\dagger} + \gamma c_k,$$

$$\alpha^2 - \beta^2 + \gamma^2 = 1$$

$$H = \sum_{k=-\infty}^{\infty} \mathrm{i}g_k (a_k^{\dagger} b_{-k}^{\dagger} - a_k b_{-k}) + \mathrm{i}g_k' (a_k^{\dagger} c_k - a_k c_k^{\dagger}).$$

replacing $z$ with $\zeta = z/(1+z)$

Class. Quantum Grav. **31** (2014) 075015 C Adami and G Ver Steeg

# Black Hole Information Problem

**Evolution of black hole theories**

Black holes have given physicists headaches since Stephen Hawking proposed his eponymous radiation. A time line of proposals to prevent black holes from destroying information:

| Year | Event |
| --- | --- |
| **1916** | Einstein's general theory of relativity lays a framework for existence of black holes, with massive gravity. Information stays safely locked inside. |
| **1974-1976** | Hawking shows that black holes evaporate over time. That means information inside disappears. Physicists are baffled. |
| **Late 1990s** | Complementarity, proposed by physicist Leonard Susskind, temporarily solves the problem of information loss. |
| **2004** | Hawking accepts Susskind and Juan Maldacena's assertion that black holes preserve information. General relativity and quantum mechanics are safe. |
| **2012** | Polchinski *et al* say complementarity violates rules of quantum entanglement. Implication: a wall of fire at the event horizon. |
| **2014** | Solutions put forth include fuzzy event horizons, a new take on complementarity and wormholes. |

duction to the recent paper by Lloyd and Preskill [7, 11] which we quote *The crux of the puzzle is this: if a pure quantum state collapses to form a BH, the geometry of the evaporating BH contains spacelike surfaces crossed by both the collapsing body inside the event horizon and nearly all of the emitted Hawking radiation outside the event horizon. If this process is unitary, then the quantum information encoded in the collapsing matter must also be encoded (perhaps in a highly scrambled form) in the outgoing radiation; hence the infalling quantum state is cloned in the radiation, violating the linearity of quantum mechanics.*

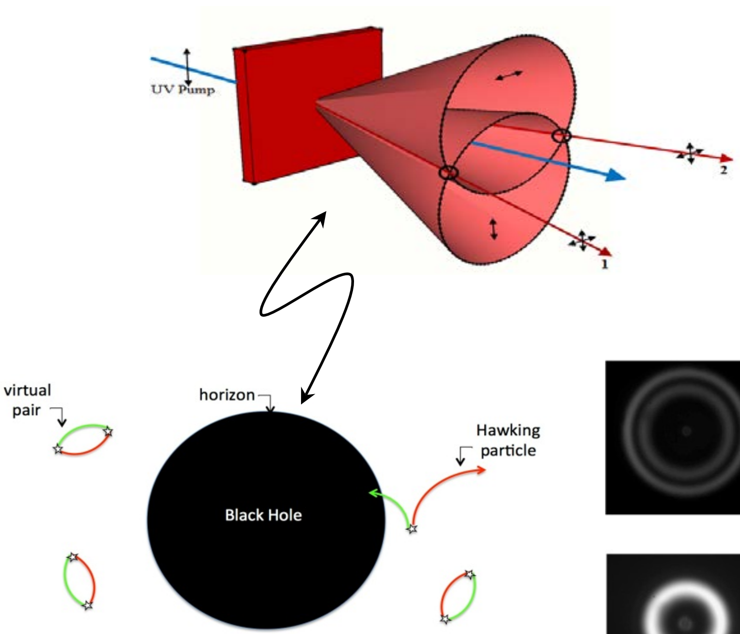
In the majority of these approaches the Hawking radiation is canonically taken to be of the form $\sum_n \sqrt{p_n}\ |n\rangle_{\text{int}}|n\rangle_{\text{ext}}$ where Hilbert space of the BH is taken to be of the tensor product form $\mathcal{H} = \mathcal{H}_{\text{ext}} \bigotimes \mathcal{H}_{\text{int}}$ for the interior (int) and exterior (ext) of the BH. The action of evaporation is to move some subsystem from the BH interior to the exterior [15] $\mathcal{H}_{\text{int}} \rightarrow \mathcal{H}_{bh} \bigotimes \mathcal{H}_r$ via $|n\rangle_{\text{int}} \rightarrow (U\ |n\rangle_{bh,r})$ where U denotes the unitary process that might be thought of as 'selecting' the subsystem to'eject.' Here $|n\rangle_{\text{int}}$ is the initial state of the BH

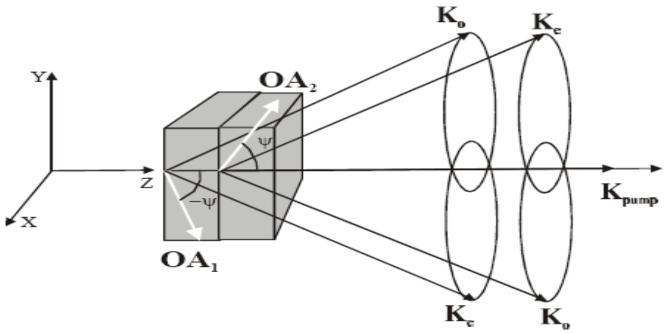
# BH as PDC with depleted pump

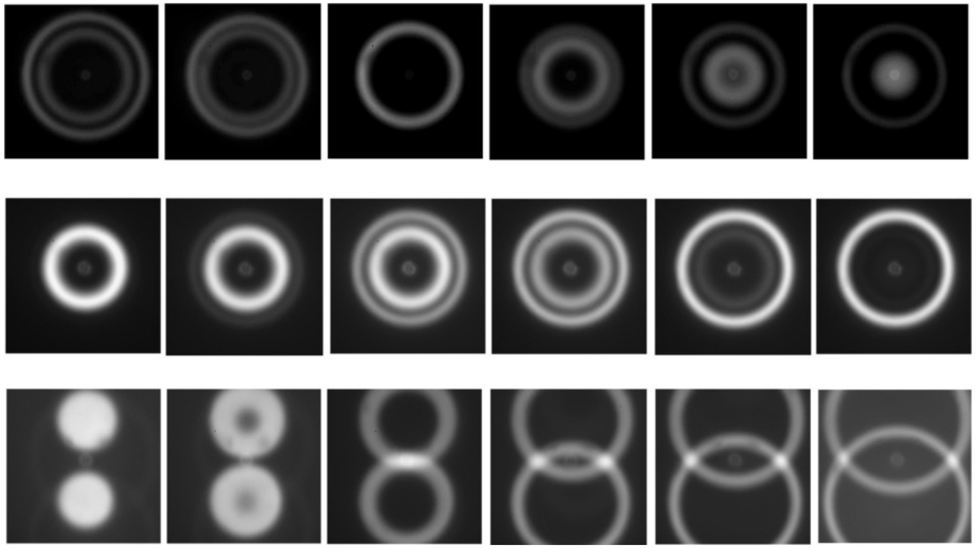
P.M. Alsing, Classical & Quant. Grav. **32**, 075010 (2015);   arXiv:1408.4491

The trilinear Hamiltonian $H_{p,s,\bar{i}}$

$$H_{p,s,\bar{i}} = r(a_p\, a_s^\dagger\, a_{\bar{i}}^\dagger + a_p^\dagger\, a_s\, a_{\bar{i}})$$



# Justification for Model

D.G. Boulware, *Hawking radiation and thin shells*, Phys. Rev. D. **13**, 2169 (1976)
U.H. Gerlach *The mechanism of blackbody radiation from an incipient black hole*, Phys. Rev. D, **14**, 1479 (1976)
A. Saini and D. Stojkovic, *Radiation from a collapsing object is manifestly covariant*, arxiv:1503.01487v3; T. Vachaspati, D. Stojkovic and L.M. Krauss, *Observation of incipient black holes and the information loss problem*, arxiv:gr-qc/0609024v3
G.L. Alberghi, R. Casadio, G.P. Vacca and G. Venturi, *Gravitational collapse of a radiating shell*, arxiv:gr-qc/0102014v2.

collapsing thin shell of matter
scalar boson field coupled to this classical gravitation field

$$H = 1/2\, r\, \xi(\tau)\, ({a^\dagger}^2 + a^2)$$

quantized harmonic oscillator with a (exponentially) time varying frequency

well known to generate single mode squeezed states

---

If two bosons were coupled to the field,
or a single, complex boson field

$$H = 1/2\, r\, \xi(\tau)\, ({a^\dagger}_s\, {a^\dagger}_{\bar{i}} + a_s\, a_{\bar{i}})$$

---

The next logical step, to incorporate the quantum statistics of the pump In quantum optics, such semi-classical models are familiar

$$H_{p,s,\bar{i}} = r\left(a_p a_s^\dagger a_{\bar{i}}^\dagger + a_p^\dagger a_s a_{\bar{i}}\right)$$

# BH as PDC with depleted pump

**The trilinear Hamiltonian $H_{p,s,\bar{i}}$**

$$H_{p,s,\bar{i}} = r(a_p\, a_s^\dagger\, a_{\bar{i}}^\dagger + a_p^\dagger\, a_s\, a_{\bar{i}})$$

$$|n\rangle_L \equiv |n_{p0} - n\rangle_p |n_{s0} + n\rangle_s |n\rangle_{\bar{i}}, \quad |\psi\rangle_{in} = |0\rangle_L = |n_{p0}\rangle_p |n_{s0}\rangle_s |0\rangle_{\bar{i}},$$

$$H_{p,s,\bar{i}} = r(J_+^{(p\bar{i})}\, a_s + J_-^{(p\bar{i})}\, a_s^\dagger) \qquad SU(2) \qquad J_+^{(p\bar{i})} = a_p^\dagger\, a_{\bar{i}}$$

$$H_{p,s,\bar{i}} = r(a_p^\dagger\, K_-^{(s\bar{i})} + a_p\, K_+^{(s\bar{i})}), \qquad SU(1,1) \qquad K_+^{(s\bar{i})} = a_s^\dagger\, a_{\bar{i}}^\dagger$$

$$c_n(t) = {}_L\langle n| e^{-iH_{p,s,\bar{i}}t} |\psi\rangle_{in} \qquad |\psi\rangle_{out} = \sum_{n=0}^{\infty} c_n |n\rangle_L$$

$$\begin{aligned} i\frac{dc_n(t)}{dt} &= r\sqrt{n_{p0} - n}\,\sqrt{(n+1)(2\kappa+n)}\, c_{n+1}(t) \\ &+ r\sqrt{n_{p0} - n + 1)}\,\sqrt{n(2\kappa+n-1)}\, c_{n-1}(t), \quad c_n(0) = \delta_{n,0}, \quad 2\kappa = n_{s0} + 1. \end{aligned}$$

see Heisenberg approach: P. Nation and M. Blencowe: New J. Phys. **12** 095013 (2010), arXiv: 1004.0522

# BH as PDC with depleted pump

**B. Early times: the non-depleted pump regime** $n_{p0}$ □ $n_{s0}, n$

For early times the condition $n_{p0} \gg n_{s0},\, n$ holds, and the simplest approximation is to approximate the terms $\sqrt{n_{p0}-n}$ and $\sqrt{n_{p0}-n+1}$ by $\sqrt{n_{p0}}$ which leads

$$i\,\frac{dc_n(t)}{d\tau} = \sqrt{n\,(n+n_{s0})}\,c_{n+1}(t) + \sqrt{(n+1)(n+1+n_{s0})}\,c_{n-1}(t).$$

with solution

$$|\psi_{t_<}(\tau)\rangle_{out} = \sum_{n=0}^{n_{p0}} c_n(\tau)\,|n\rangle_L = \sum_{n=0}^{n_{p0}} c_n(\tau)\,|n_{p0}-n\rangle_p |n_{s0}+n\rangle_s |n\rangle_{\bar{i}}$$

$$\approx |n_{p0}\rangle_p \otimes \sum_{n=0}^{\infty} c_n(\tau)\,|n_{s0}+n\rangle_s |n\rangle_{\bar{i}} \equiv |n_{p0}\rangle_p \otimes |\psi_{t_<}(\tau)\rangle_{s,\bar{i}},$$

$$c_n^<(\tau) = \frac{(-i\,\tanh\tau)^n}{(\cosh\tau)^{n_{s0}+1}}\sqrt{\binom{n_{s0}+n}{n}}, \qquad \tau = \sqrt{n_{p0}}\,r\,t,$$

# BH as PDC with depleted pump

$$p_<(n,\tau) = |c_n^<(\tau)|^2 = \frac{\tanh^{2n}\tau}{(\cosh^2\tau)^{n_{s0}+1}}\binom{n_{s0}+n}{n} \equiv (1-z)^{n_{s0}+1} z^n \binom{n_{s0}+n}{n}, \qquad z = \tanh^2\tau. \tag{39}$$

One has $\sum_{n=0}^{\infty} p_<(n,\tau) = 1$ upon noting the identity [9, 19] $\sum_{n=0}^{\infty} z^n \binom{n_{s0}+n}{n} = (1-z)^{-(n_{s0}+1)}$. The average number of particles in region $I$ is given by $n_{s0} + \bar{n}_<(\tau)$ where (taking $n_{p0} \to \infty$)

$$\bar{n}_<(\tau) = \sum_{n=0}^{\infty} n\, p(n,\tau) = (n_{s0}+1)\,\frac{z}{1-z} = (n_{s0}+1)\sinh^2\tau, \tag{40}$$

which allows one to write Eq.(39) as

$$p_<(n,\tau) = (n_{s0}+1)^{n_{s0}+1}\binom{n_{s0}+n}{n}\frac{\bar{n}_<^n(\tau)}{(\bar{n}_<(\tau)+n_{s0}+1)^{n+n_{s0}+1}}, \tag{41}$$

which reduces to the standard thermal probability distribution $p_{thermal}(n,\tau) = \bar{n}^n/(\bar{n}+1)^{n+1}$ with $\bar{n}_{thermal} = \sinh^2\tau$ wehn $n_{s0} = 0$.

# BH as PDC with depleted pump

**C. Late times: the depleted pump regime** $n_{p0} \approx n_{s0}, n$

$$n = n_{p0} \, \sin^2 \theta,$$

$$\theta_n = \sin^{-1}(n/n_{p0}),$$

$$\frac{d\theta}{dn} = \frac{1}{2\sqrt{n\,(n_{p0} - n)}}.$$

$$g(n) = \sqrt{2}\,\frac{\Gamma(1 + n/2)}{\Gamma(1/2 + n/2)}, \qquad g(n-1)\,g(n) = n,$$

$$G(n) = g(n_{p0} - n)\,g(n)\,g(2\kappa + n - 1)$$

$$\tilde{C}_n(t) = \sqrt{G(n)}\,\tilde{c}_n(t), \qquad \tilde{c}_n(t) = (-i)^n\,c_n(t).$$

$$\frac{d\tilde{C}_n(t')}{dt'} + G(n)\left(\tilde{C}_{n+1}(t') - \tilde{C}_{n-1}(t')\right) = 0, \qquad t' = r\,t.$$

PHYSICAL REVIEW A VOLUME 1, NUMBER 2 FEBRUARY 1970

**Quantum-Mechanical Amplification and Frequency Conversion with a Trilinear Hamiltonian**

Daniel F. Walls*†
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and
Richard Barakat
*Division of Engineering and Applied Physics, Harvard University, Cambridge, Massachusetts 02138*
(Received 23 June 1969)

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PHYSICAL REVIEW A VOLUME 2, NUMBER 2 AUGUST 1970

**Coherent Spontaneous Emission***

Rodolfo Bonifacio†
and
Giuliano Preparata‡

p336

# Channel (Holevo) Capacity $\chi_{s,\bar{s}}(z)$

$$\chi^{(s,\bar{s})}(z,\theta) = \max_p S\left[p\,\rho^{(s)}(0)\otimes\rho^{(\bar{s})}(0) + (1-p)\,\rho^{(s)}(0)\otimes\rho^{(\bar{s})}(0)\right] - p\left(S[\rho^{(s)}(0)] + S[\rho^{(\bar{s})}(0)]\right) - (1-p)\left(S[\rho^{(s)}(1)] + S[\rho^{(\bar{s})}(1)]\right),$$

$$\equiv H\left[p\,p_k^{(s)}(0)\,p_m^{(\bar{s})}(0) + (1-p)\;p_k^{(s)}(1)\,p_m^{(\bar{s})}(1)\right] - p\left(H[p_k^{(s)}(0)] + H[p_m^{(\bar{s})}(0)]\right) - (1-p)\left(H[p_k^{(s)}(0)] + H[p_m^{(\bar{s})}(0)]\right)$$

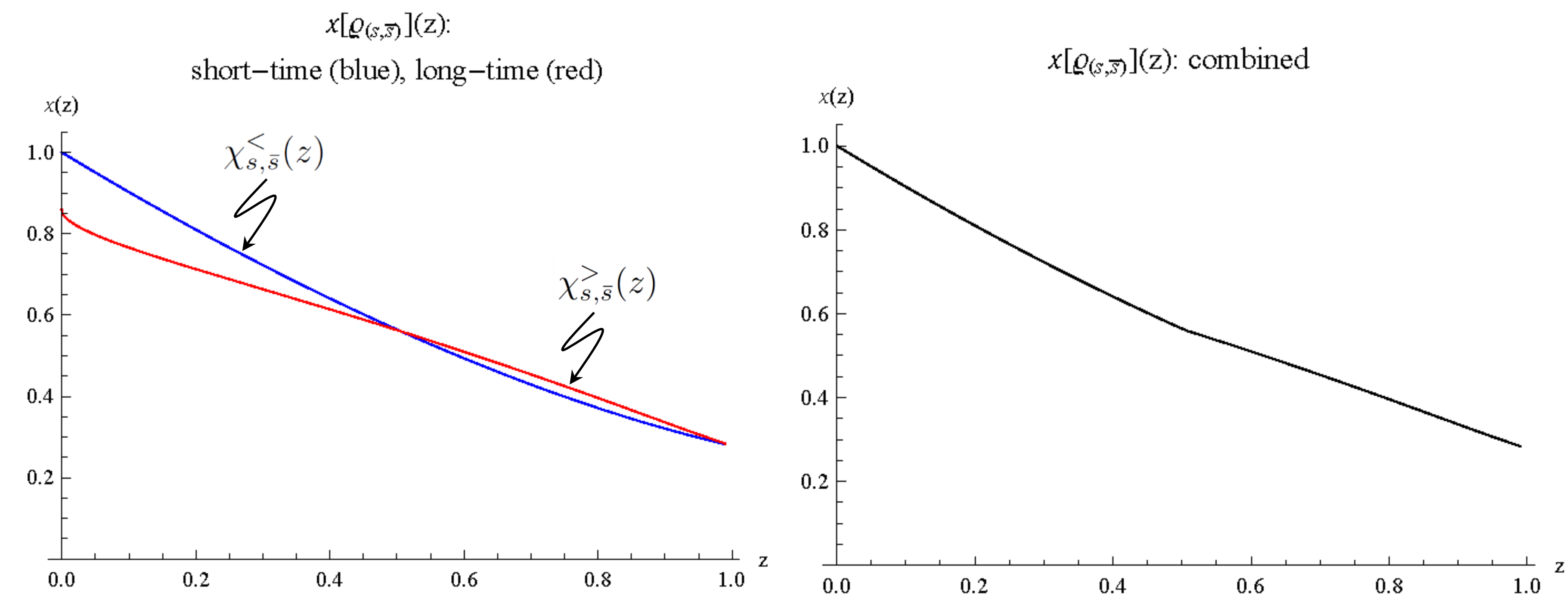
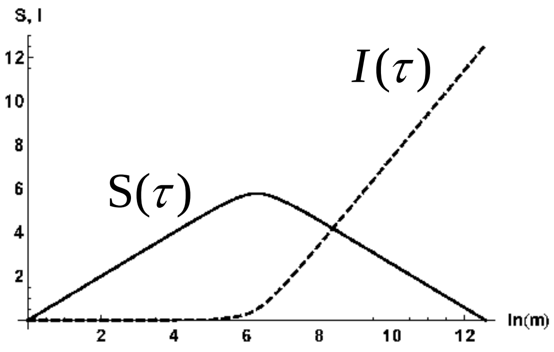



FIG. 7: (Left): $\chi^{<}_{s,\bar{s}}(z)$ short-times (blue curve -also Adami and ver Steeg Eq.(40) [1]) and $\chi^{<}_{s,\bar{s}}(z)$ long-times (red curve), plotted for $0 \leq z = \tanh^2\tau \lesssim 1$. (Right) $\chi_{s,\bar{s}}(z)$ combined formula with crossover time $z^* = 0.506407$ Eq.(63) in the limit $n_{p0} \to \infty$.

# Page Information Curves

$$I(\tau) = S_{thermal}(\tau) - S(\rho_s(\tau))$$

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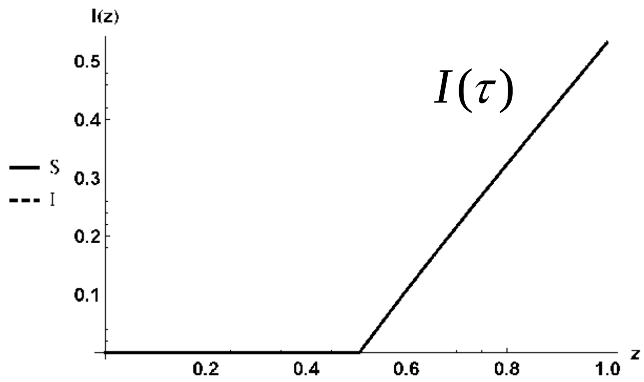
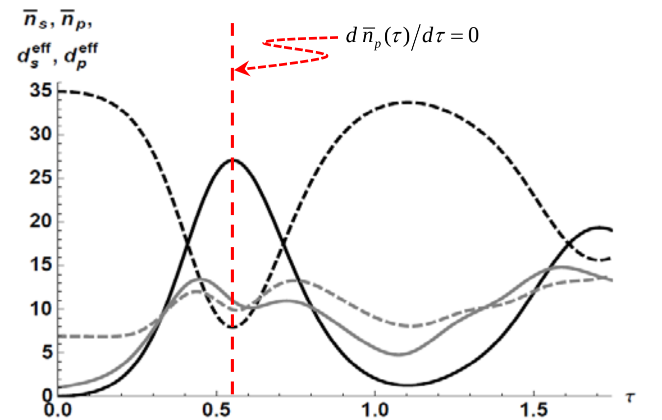
Page, PRL **71**, 3743 (1993); gr-qc/9306083




$$|\psi\rangle_{in} = |n_{p0}\rangle_p |n_{s0}\rangle_s |0\rangle_{\bar{i}}$$

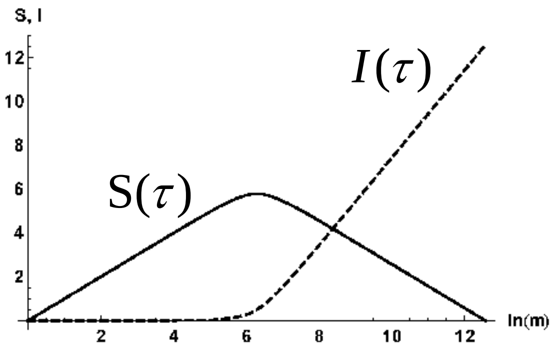



$$|\psi\rangle_{in} = |\alpha\rangle_p |0\rangle_s |0\rangle_{\bar{i}}$$

$$\bar{n}_p(0) = \alpha^2 = 35 \quad \Delta n_p(0) = \alpha = 5.92$$

# Page Information Curves

$$I(\tau) = S_{thermal}(\tau) - S(\rho_s(\tau))$$

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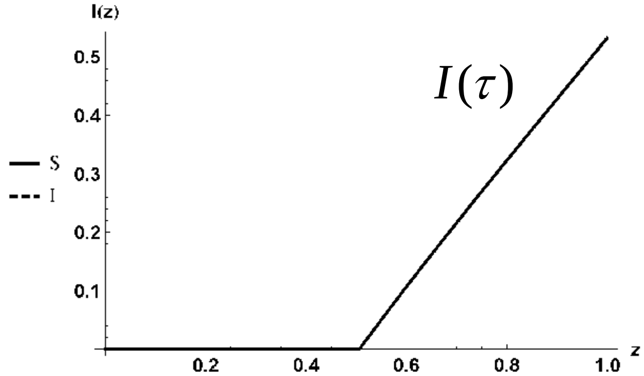
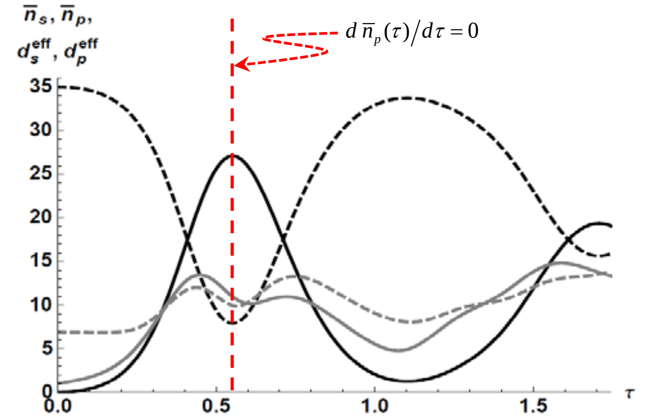
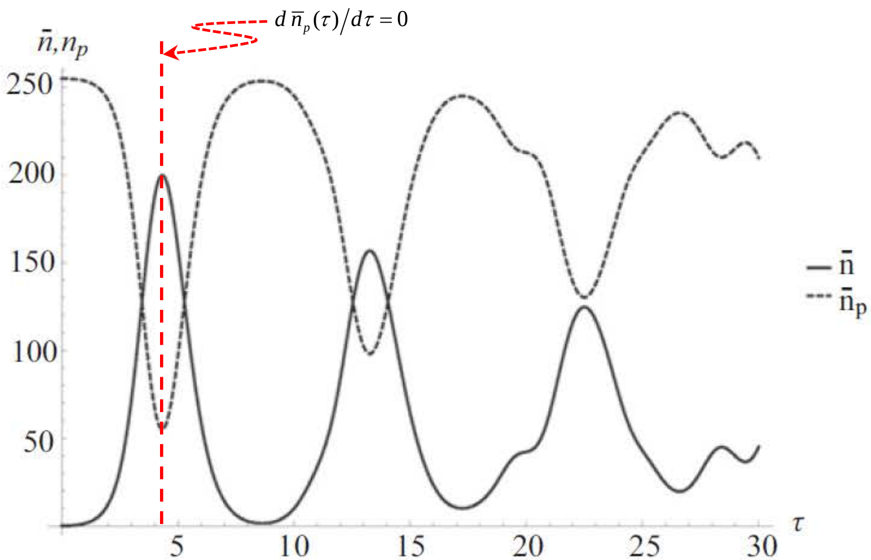


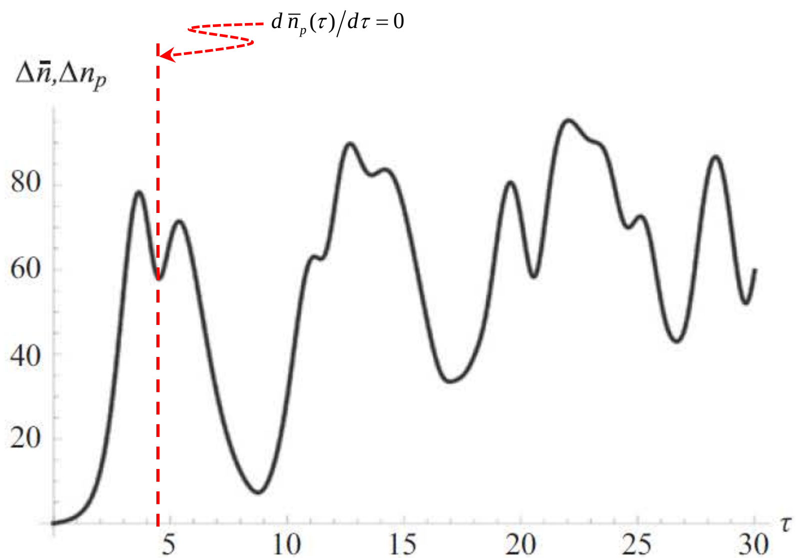

$$|\psi\rangle_{in} = |n_{p0}\rangle_p |n_{s0}\rangle_s |0\rangle_{\bar{i}}$$




$$|\psi\rangle_{in} = |\alpha\rangle_p |0\rangle_s |0\rangle_{\bar{i}}$$

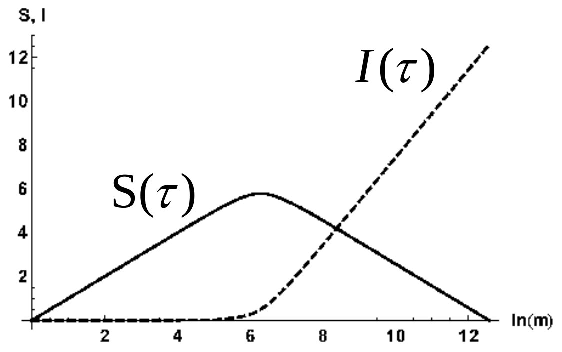
$$\bar{n}_p(0) = \alpha^2 = 35 \quad \Delta n_p(0) = \alpha = 5.92$$



# Page Information Curves

$$I(\tau) = S_{thermal}(\tau) - S(\rho_s(\tau))$$

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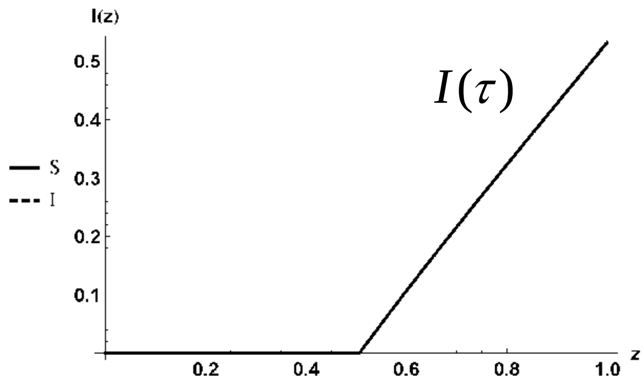
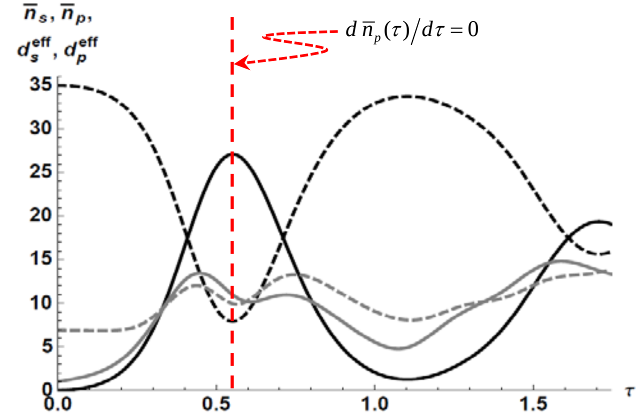
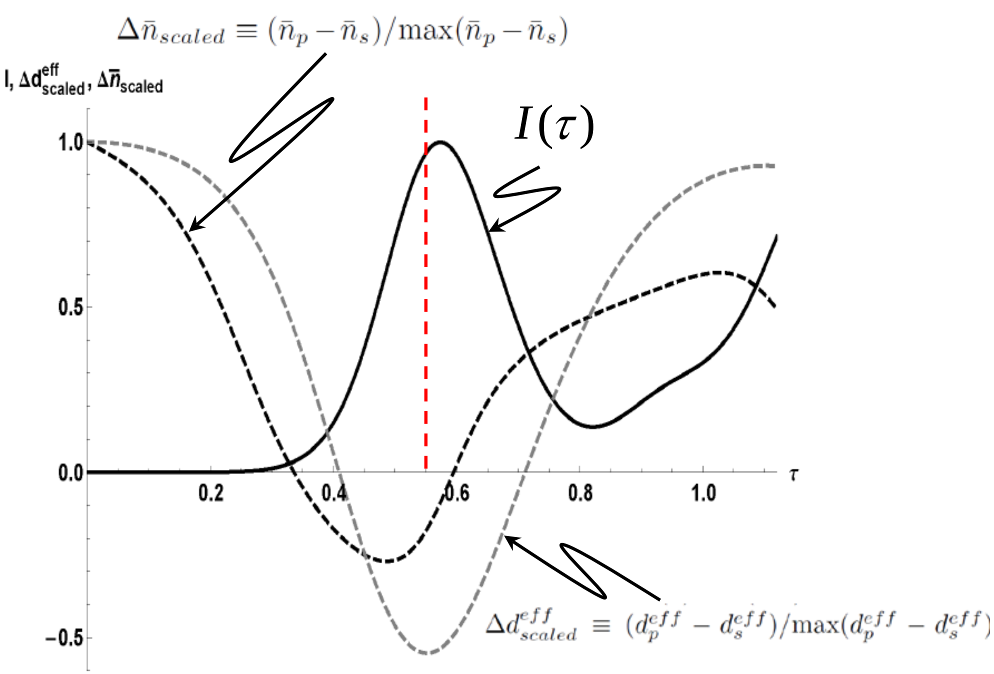


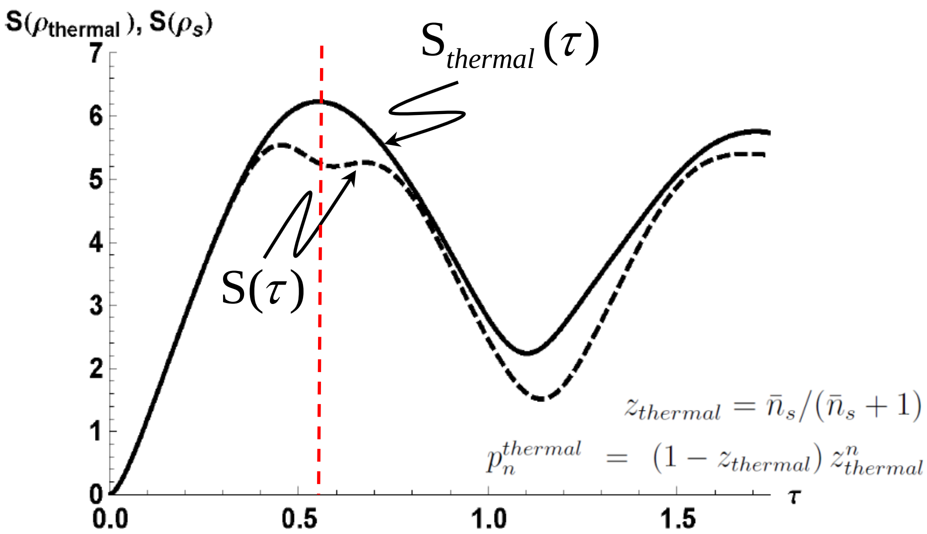

$$|\psi\rangle_{in} = |n_{p0}\rangle_p |n_{s0}\rangle_s |0\rangle_{\tilde{i}}$$



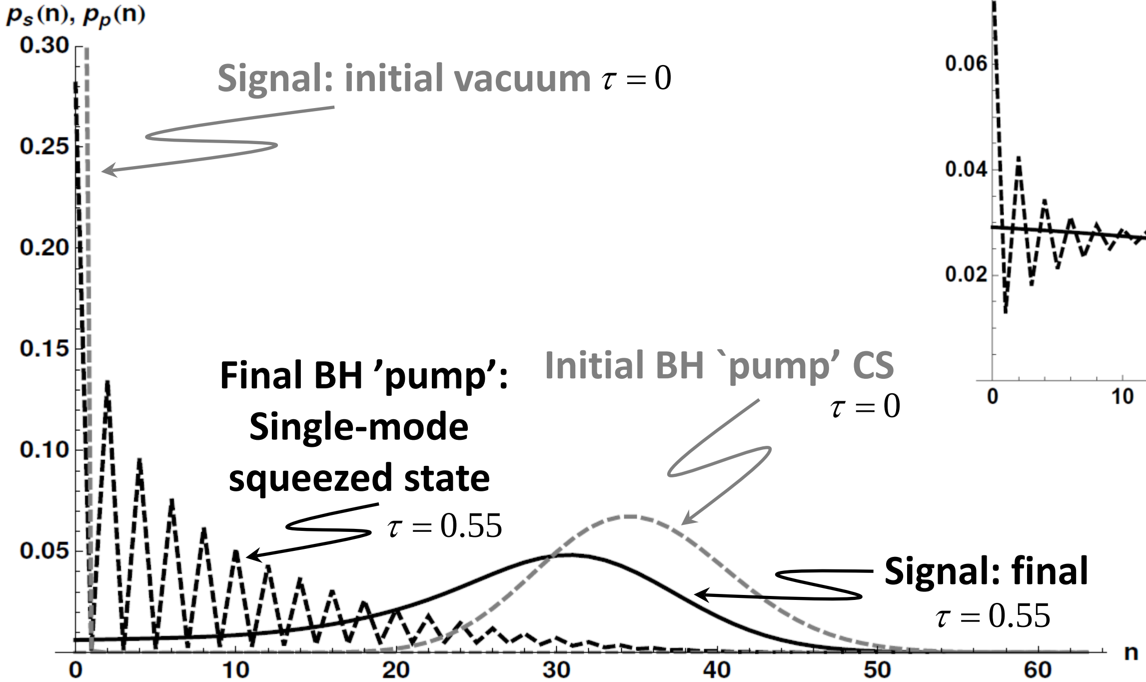

$$|\psi\rangle_{in} = |\alpha\rangle_p |0\rangle_s |0\rangle_{\tilde{i}}$$

$$\bar{n}_p(0) = \alpha^2 = 35 \quad \Delta n_p(0) = \alpha = 5.92$$



# Relative Entropy of BH 'pump' to emitted HawkRad signal



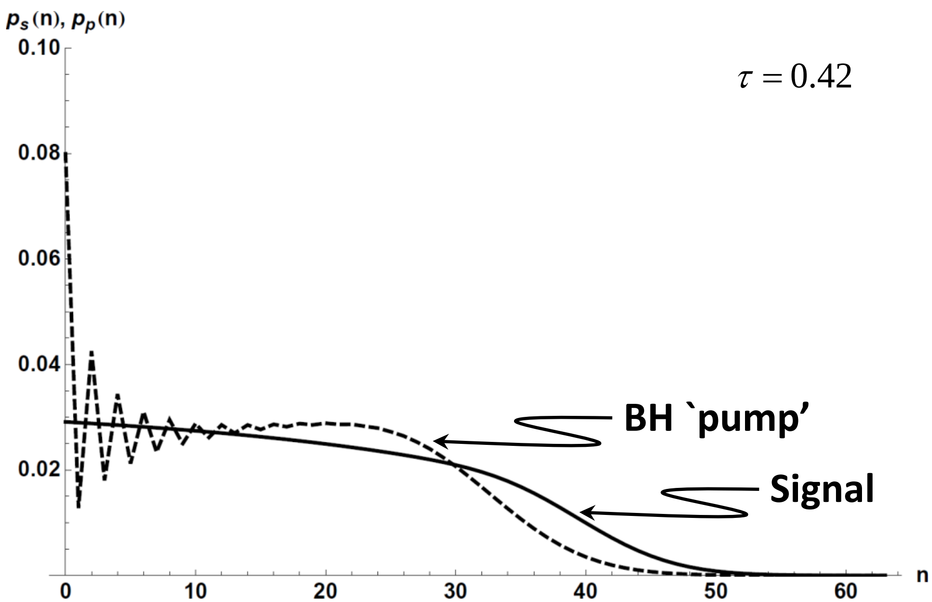




**Figure 17.** Probability distributions $p_s(n,\tau)$ and $p_p(n,\tau)$ in the computational basis (Fock states)from numerical integration of Eq.(124). Initial distributions: (gray-solid) $p_s(n,0)$, (gray-dashed) $p_p(n,0)$ with BH $\bar{n}_p(0) = |\alpha|^2 = 35$. Late time distributions $\tau \approx 0.55$ when $dn_p/d\tau = 0$: (black-solid) $p_s(n,\tau)$ and (black-dashed) $p_p(n,\tau)$.

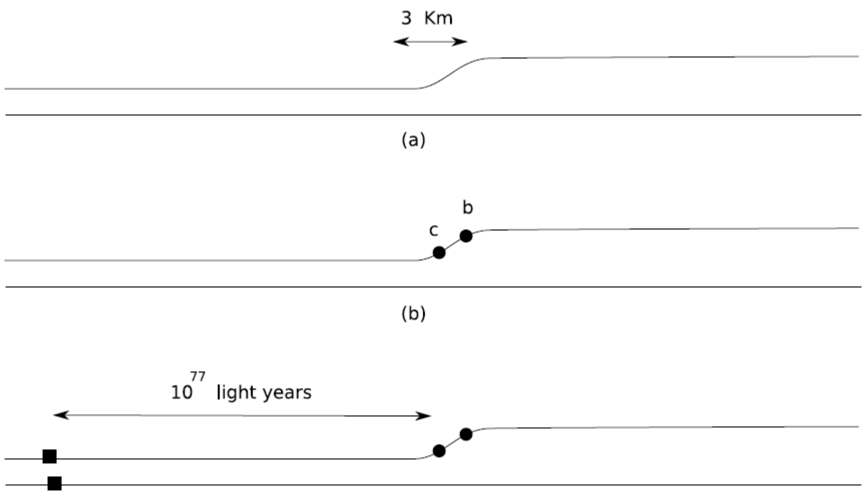
# Outline
# One Shot Decoupling Model

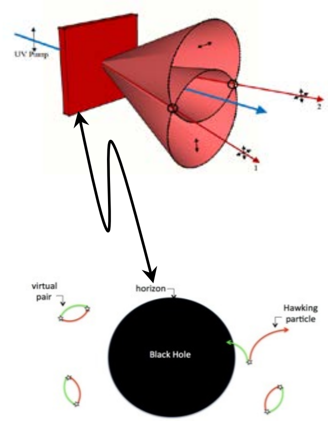
- **Justification for use of trilinear Hamiltonian for BH evaporation/particle production**
  - Semi-classical Hamiltonian for a collapsing spherical shell
- **One Shot Decoupling Model of Bradler and Adami, arXiv:1505.02840**
  - Simplified version of Master Equation suggested by **Alsing: CQG 32, 075010, (2015); arXiv:1408.4491**
- **Analytic formulation by Alsing and Fanto, CQG 33, 015005 (2016), arXiv:1507.00429**
  - Extension of models by Alsing and by Nation and Blencowe
  - Page Information Curves
- **Summary and Conclusion**

# Justification for Model

The information paradox: A pedagogical introduction

**Samir D. Mathur** arXiv:0909.1038v2 [hep-th] 25 Jan 2011



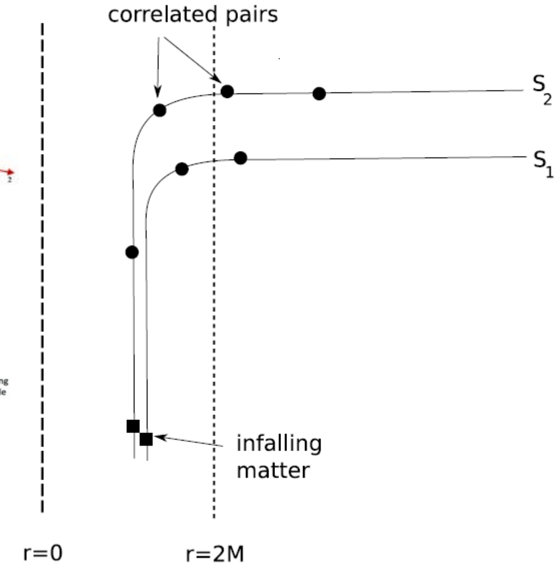




Figure 1: (a) Spacelike slices in an evolution; the intrinsic geometry of the slice distorts in the region between the right and left sides (b) Particle pairs are created in the region of distortion

Figure 2: A schematic set of coordinates for the Schwarzschild hole. Spacelike slices are $t = const$ outside the horizon and $r = const$ inside. Infalling matter is very far from the place where pairs are created ($\sim 10^{77}$ light years) when we measure distances along the slice. Curvature length scale is $\sim 3\ km$ all over the region of evolution covered by the slices $S_i$.

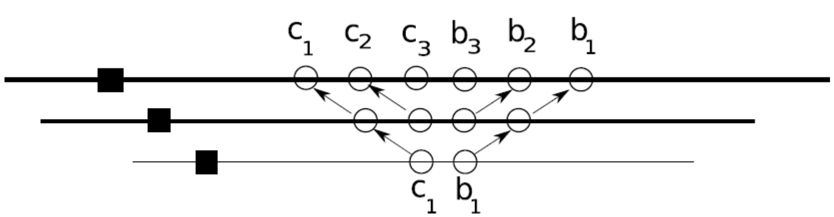



Figure 4: The creation of Hawking pairs. The new quanta $c_{n+1}, b_{n+1}$ are not created by interaction with either the matter $|\psi\rangle_M$ (represented by the black square) or with the earlier created pairs. Rather the creation is by a Schwinger process which moves $|\psi\rangle_M$ further away from the place of pair creation, and also moves the earlier created $c, b$ quanta away from the place of pair creation. The new pairs are created in a state which to leading order is entangled between the new $b, c$ quanta but not entangled with anything else. Small corrections to this leading order state does not change this entanglement significantly, so the entanglement keeps growing all through the radiation process, unlike the case of radiation from normal hot bodies.

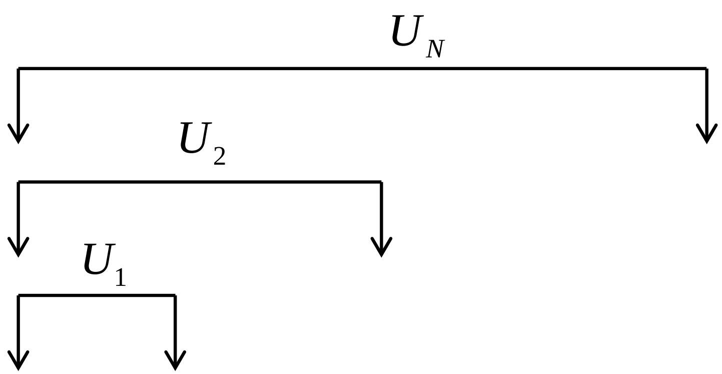
# Spontaneous parametric down conversion as an analogue for gravitational particle production

## One Shot Decoupling Model

Bradler and Adami, arXiv:1505.0284

**BH `pump' mode** **empty Hawking radiation modes**

$$|\psi(\tau)\rangle = U(\tau,0)|\Psi(0)\rangle = \mathcal{T} e^{-i\int d\tau' \, H_{p,s,\bar{i}}(\tau')}\,|\Psi(0)\rangle \approx \prod_{k=1}^{N} \underbrace{e^{-iH_{p,s_k,\bar{i}_k}\Delta\tau}}_{U_k} |n_{p0}\rangle_p \prod_{k'=1}^{N} |0\rangle_{s_{k'},\bar{i}_{k'}}$$




$$|\Psi(0)\rangle = |n_{p0}\rangle_p \prod_{k'=1}^{N} |0\rangle_{s_{k'},\bar{i}_{k'}} = |n_{p0}\rangle_p \otimes |0\rangle_{s_1,\bar{i}_1} \otimes |0\rangle_{s_2,\bar{i}_2} \otimes \ldots \otimes |0\rangle_{s_N,\bar{i}_N}$$

# Spontaneous parametric down conversion as an analogue for gravitational particle production

## One Shot Decoupling Model

$$
\begin{aligned}
|\Psi(1)\rangle &= \sum_{n_1=0}^{n_{p0}} \sqrt{p_{n_1}^{(n)}(z)}\; |n_{p0}-n_1\rangle_p |n_1\rangle_{s_1,\bar{i}_1} \otimes \prod_{k'=2}^{N} |0\rangle_{s_{k'},\bar{i}_{k'}}, \qquad z \ll z^*, \quad |0\rangle_s |0\rangle_{\bar{i}} \equiv |0\rangle_{s,\bar{i}} \\
&\equiv \sum_{n_1=0}^{n_{p0}} \sqrt{p_{n_1}^{(n_{p0})}(z)}\; |n_{p0}-n_1\rangle_p |n_1\rangle_1, \qquad p_{n_1}^{(n_{p0})}(z) = \frac{(1-z)}{(1-z^{n_{p0}+1})}\, z^{n_1}, \qquad \sum_{n_1=0}^{n_{p0}} p_{n_1}^{(n_{p0})} = 1, \\
&\approx |n_{p0}\rangle_p \otimes \sum_{n_1=0}^{n_{p0}} \sqrt{p_{n_1}^{(n_{p0})}(z)}\; |n_1\rangle_1, \qquad n_{p0} \gg n_1, \\
&\equiv |n_{p0}\rangle_p \otimes |\phi^{(sqzd)}\rangle_1, \qquad |\phi^{(sqzd)}\rangle_1 = (1-z) \sum_{n_1=0}^{n_{p0}\to\infty} z^{n_1} |n_1\rangle_1
\end{aligned}
$$

$$
\begin{aligned}
|\Psi(2)\rangle &= U_{p,2}\,|\Psi(1)\rangle = \sum_{n_1=0}^{n_{p0}} \sum_{n_2=0}^{n_{p0}-n_1} \sqrt{p_{n_1}^{(n_{p0})}\, p_{n_2}^{(n_{p0})-n_1}}\; |(n_{p0}-n_1)-n_2\rangle_p\, |n_1\rangle_1\, |n_2\rangle_2, \\
&\approx |n_{p0}\rangle_p \otimes \sum_{n_1=0}^{n_{p0}} \sqrt{p_{n_1}^{(n_{p0})}}\; |n_1\rangle_1 \otimes \sum_{n_2=0}^{n_{p0}-n_1} \sqrt{p_{n_2}^{(n_{p0}-n_1)}}\; |n_2\rangle_2, \qquad n_{p0} \gg n_1, n_2, \qquad p_{n_2}^{(n_{p0}-n_1)}(z) = \frac{(1-z)}{(1-z^{(n_{p0}-n_1)+1})}\, z^{n_2}, \\
&\approx |n_{p0}\rangle_p \otimes |\phi^{(sqzd)}\rangle_1 \otimes |\phi^{(sqzd)}\rangle_2, \qquad \sum_{n_2=0}^{n_{p0}-n_1} p_{n_2}^{(n_{p0}-n_1)} = 1.
\end{aligned}
$$

# Spontaneous parametric down conversion as an analogue for gravitational particle production

## Reduced Density Matrices

$$|\Psi(N)\rangle \approx \sum_{k=0}^{n_{p0}} \sqrt{P_k^{(N)}}\, |n_{p0}-k\rangle_p\, |\Phi_k'^{(N)}\rangle$$

$$\tilde{P}_k^{(N)} = (1-z)^N z^k \binom{k+N-1}{k}, \quad P_k^{(N)} = \frac{\tilde{P}_k^{(N)}}{\sum_{k'=0}^{n_{p0}} \tilde{P}_{k'}^{(N)}}$$

$$\begin{aligned} |\Phi'^{(N=4)}_{j_N=2}\rangle &= (|2,0,0,0\rangle + |0,2,0,0\rangle + |0,0,2,0\rangle + |0,0,0,2\rangle + |1,1,0,0\rangle \\ &+ |1,0,1,0\rangle + |1,0,0,1\rangle + |0,1,1,0\rangle + |0,1,0,1\rangle + |0,0,1,1\rangle)/\sqrt{10}, \quad \left.\binom{j_N+N-1}{j_N}\right|_{N=4,j_N=2} = 10, \end{aligned}$$

$$\left|\Phi'^{N}_{j_N}\right\rangle = \left[\sum_{j_1=0}^{j_N} \sum_{j_2=j_1}^{j_N} \sum_{j_3=j_2}^{j_N} \cdots \sum_{j_{N-1}=j_{N-2}}^{j_N} \prod_{i=1}^{N} \frac{1}{\sqrt{(1-z^{n_{p0}-j_{i-1}+1})}} |j_i - j_{i-1}\rangle_i, \right]$$

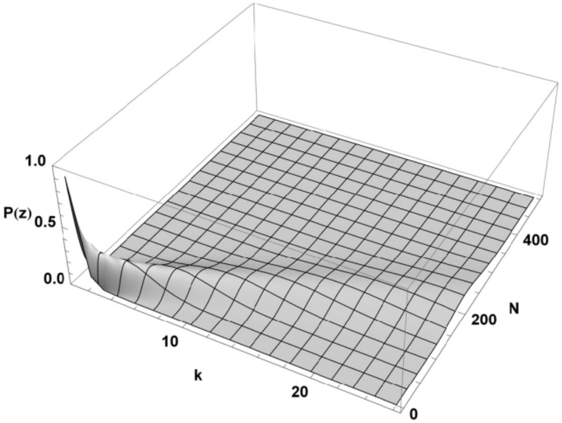
$$\rho_{s,\bar{i}}(N) = \sum_{k=0}^{n_{p0}} P_k^{(N)} |\Phi_k'^{(N)}\rangle_{s,\bar{i}} \langle \Phi_k'^{(N)}|$$

$$\rho_p(N) = \sum_{k=0}^{n_{p0}} P_k^{(N)} |n_{p0}-k\rangle_p \langle n_{p0}-k|,$$

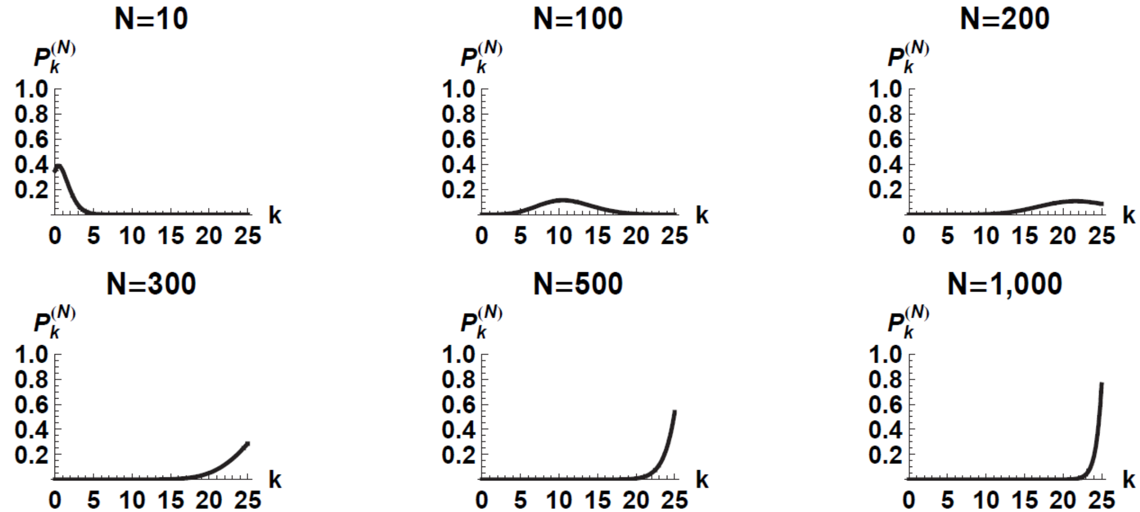
(notation: $j_N \equiv k$)

# Spontaneous parametric down conversion as an analogue for gravitational particle production

## Probabilities




Plot of $P_k^{(N)}(z)$, for $n_{p0} = 25$ and $z = 0.1$.



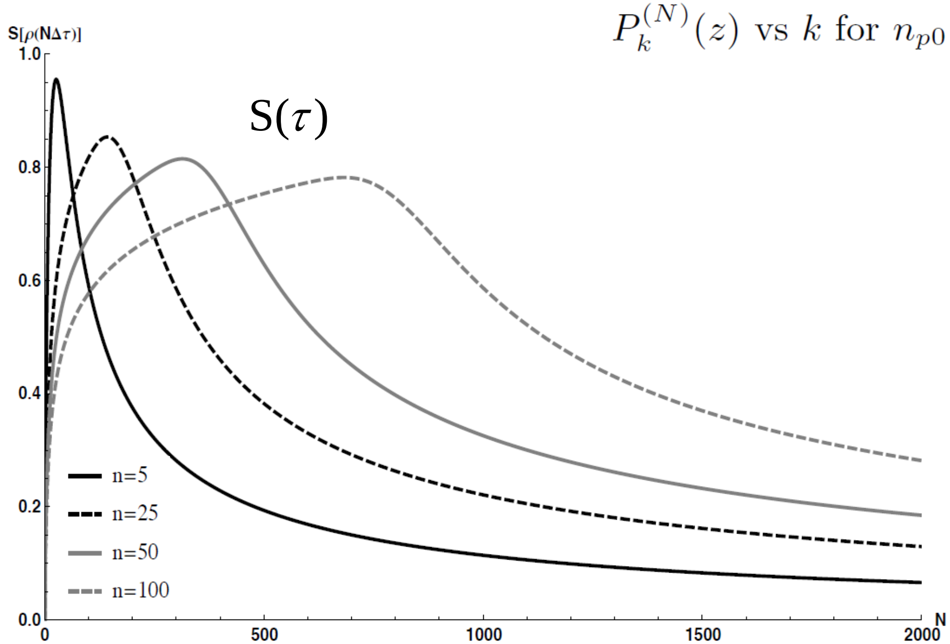

$P_k^{(N)}(z)$ vs $k$ for $n_{p0} = 25$, $z = 0.1$

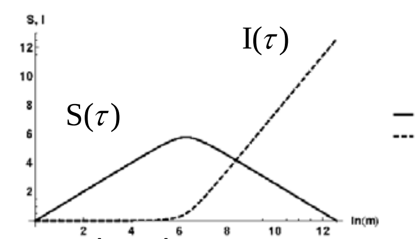
## Entropy

$S_p^{(N)}(z)$ vs $N$

$z = 0.1$

$n_{p0} = (5, 25, 50, 100)$

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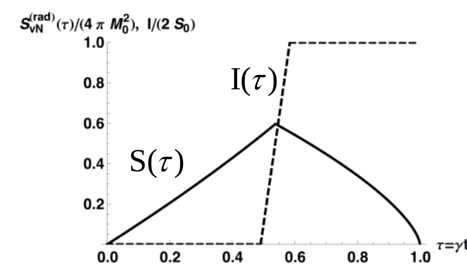


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# Spontaneous parametric down conversion as an analogue for gravitational particle production

## Original Probabilities

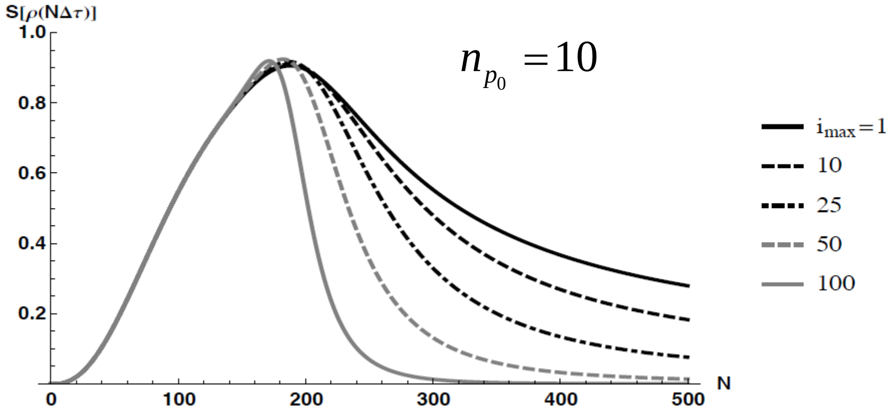
$$|\Psi(N)\rangle = (1-z)^{N/2} \sum_{j_1=0}^{n_{p0}} \sum_{j_2=j_1}^{n_{p0}} \sum_{j_3=j_2}^{n_{p0}} \cdots \sum_{j_N=0}^{n_{p0}} \sqrt{z^{j_N}} |n_{p0}-j_N\rangle_N \otimes \prod_{i=1}^{N} \frac{1}{\sqrt{(1-z^{n_{p0}-j_i}+1)}} |j_i - j_{i-1}\rangle_i, \qquad j_0 \equiv 0$$

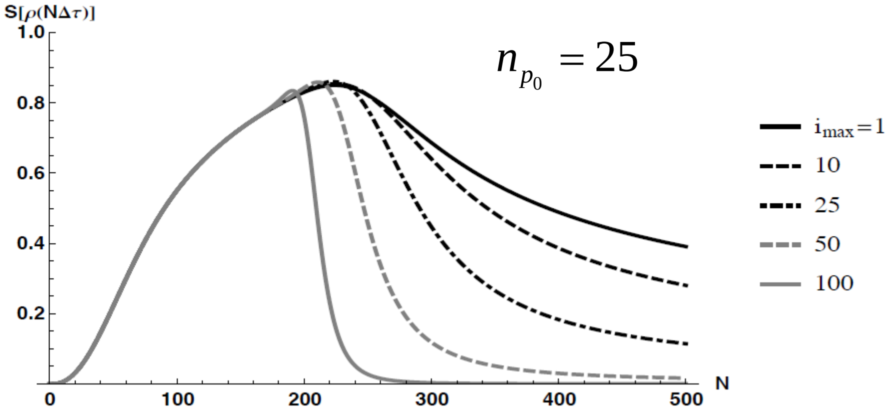
$$= (1-z)^{N/2} \sum_{j_N=0}^{n_{p0}} \sqrt{z^{j_N}} |n_{p0}-j_N\rangle_N \otimes \left[ \sum_{j_1=0}^{j_N} \sum_{j_2=j_1}^{j_N} \sum_{j_3=j_2}^{j_N} \cdots \sum_{j_{N-1}=j_{N-2}}^{j_N} \prod_{i=1}^{N} \frac{1}{\sqrt{(1-z^{n_{p0}-j_{i-1}}+1)}} |j_i - j_{i-1}\rangle_i, \right]$$

$$\equiv (1-z)^{N/2} \sum_{j_N=0}^{n_{p0}} \sqrt{z^{j_N}} |n_{p0}-j_N\rangle_N \otimes |\Phi^{(N)}_{j_N}\rangle, \qquad \tilde{P}^{(N)}_k = (1-z)^N z^k \begin{pmatrix} k+N-1 \\ k \end{pmatrix}, \qquad P^{(N)}_k = \frac{\tilde{P}^{(N)}_k}{\sum_{k'=0}^{n_{p0}} \tilde{P}^{(N)}_{k'}}$$

## Refinement of Probabilities (notation: $j_N \equiv k$)

$$\tilde{P}^{(N)}_k = (1-z)^N \frac{z^k}{(1-z^{n_{p0}-k}+1)^{N-1}} \begin{pmatrix} k+N-1 \\ k \end{pmatrix}, \qquad P'^{(N)}_k = \frac{\tilde{P}^{(N)}_k}{\sum_{k'=0}^{n_{p0}} \tilde{P}^{(N)}_{k'}}$$



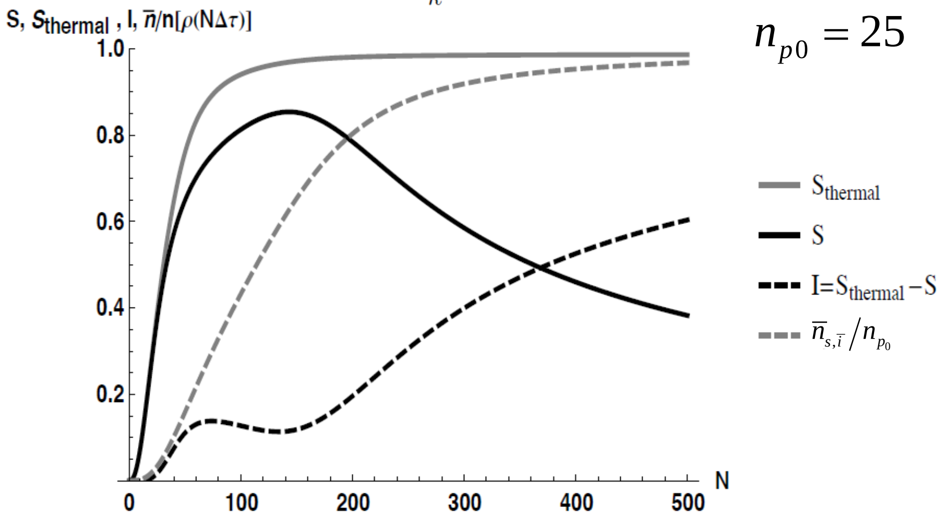



# Spontaneous parametric down conversion as an analogue for gravitational particle production

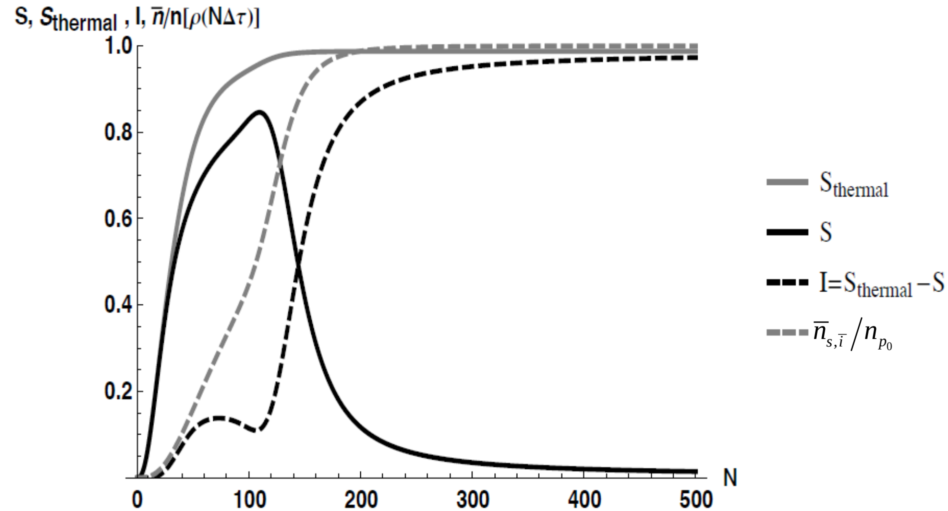
## Page Information Curves $I(\tau) = S_{thermal}(N) - S(\rho_{s,\bar{i}}(N))$

$P_k^{(N)}$

$n_{p0} = 25$




$P_k^{\prime(N)}$




$$\tilde{P}_k^{(N)} = (1-z)^N z^k \binom{k+N-1}{k}, \qquad P_k^{(N)} = \frac{\tilde{P}_k^{(N)}}{\sum_{k'=0}^{n_{p0}} \tilde{P}_{k'}^{(N)}}$$

$$\tilde{P}_k^{(N)} = (1-z)^N \frac{z^k}{(1-z^{n_{p0}-k+1})^{N-1}} \binom{k+N-1}{k},$$

$n_{p0} = 100$

$$P_k^{\prime(N)} = \frac{\tilde{P}_k^{(N)}}{\sum_{k'=0}^{n_{p0}} \tilde{P}_{k'}^{(N)}}$$

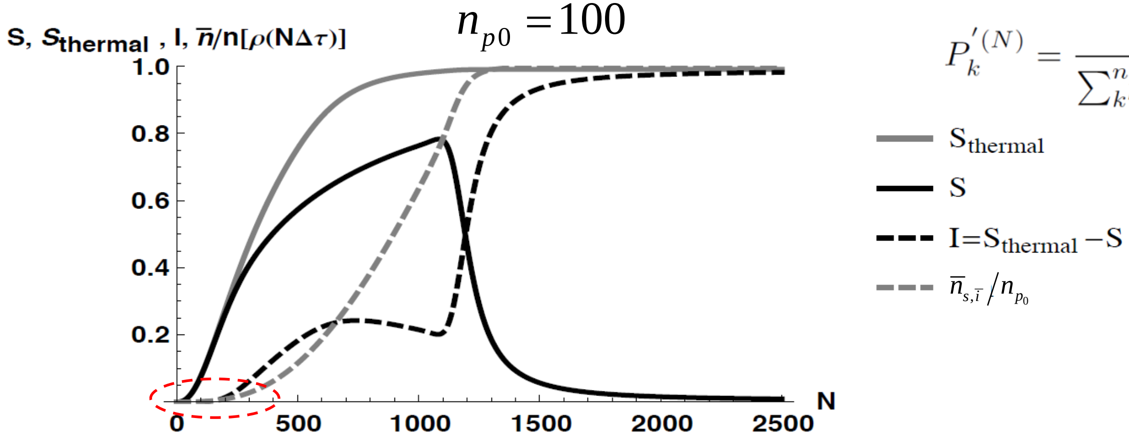
$\binom{k+N-1}{k}$ summands

estimate of only

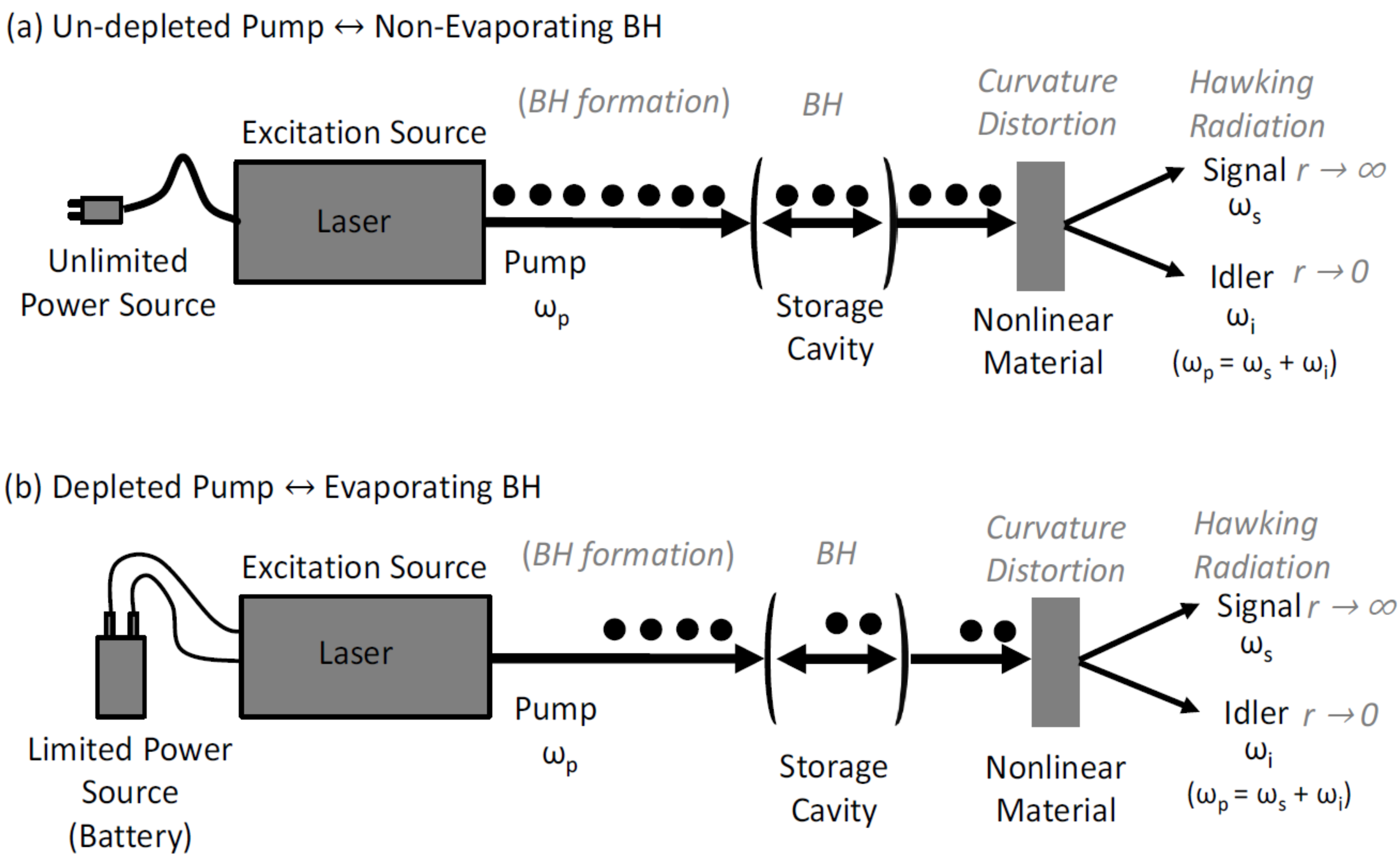
$k = 10$ nonzero terms per sum

$10^{27}$ total number of nonzero summand

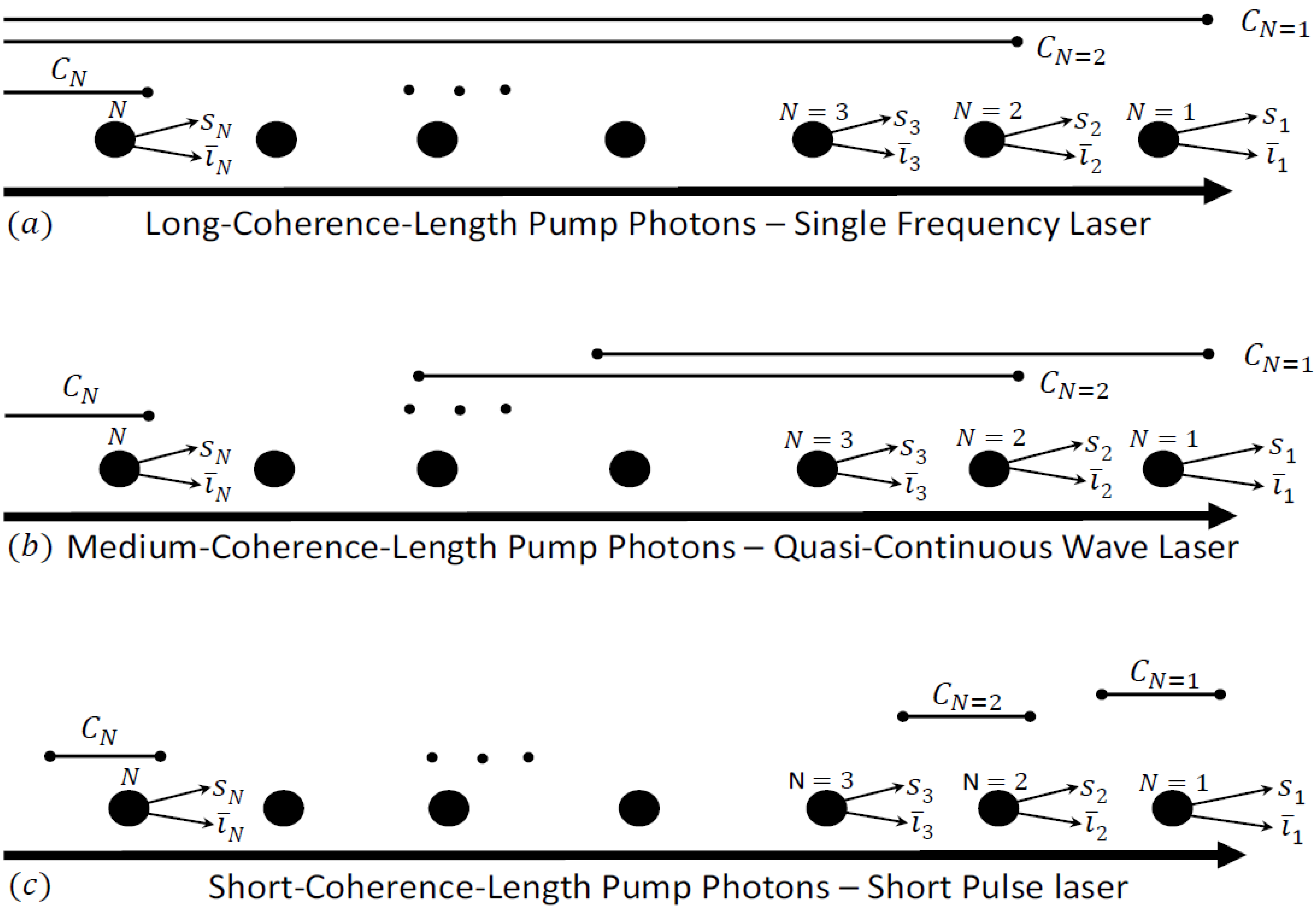
$k = n_{p0} = 100,\ N = 2500$



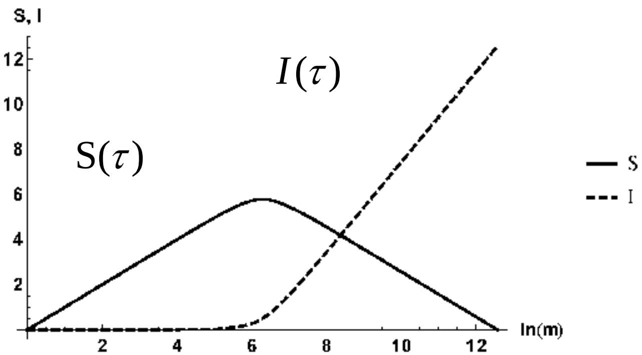
# Analogy of BH evaporation to SPDC process



# Consideration of coherence length of BH `pump' source particles



# Conclusion

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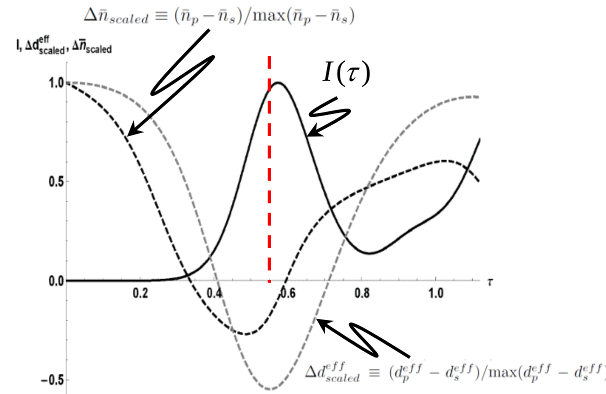
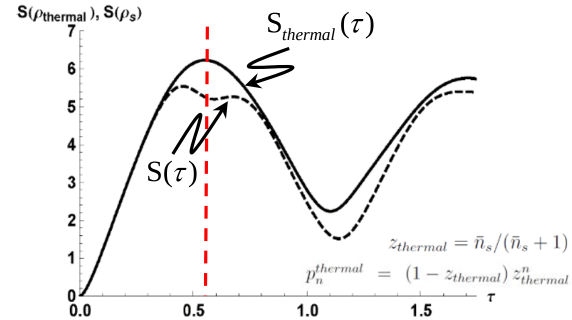
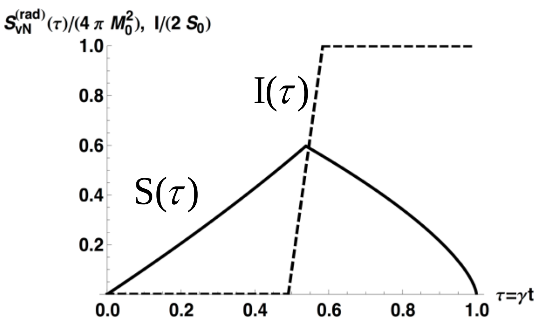



n=100




Alsing: CQG 32, 075010, (2015)

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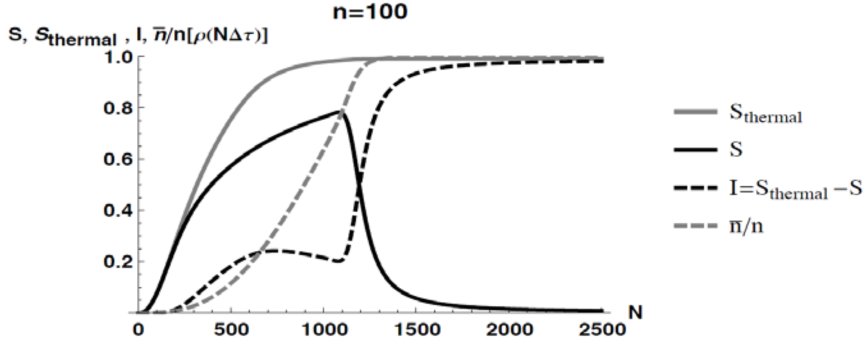


Alsing and Fanto, CQG 33, 015005 (2016)

$$|\Psi(N)\rangle \approx \sum_{k=0}^{n_{p0}} \sqrt{P^{(N)}_k}\, |n_{p0}-k\rangle_p\, |\Phi'^{(N)}_k\rangle \quad \tilde{P}^{(N)}_k = (1-z)^N z^k \binom{k+N-1}{k}, \quad P^{(N)}_k = \frac{\tilde{P}^{(N)}_k}{\sum_{k'=0}^{n_{p0}} \tilde{P}^{(N)}_{k'}}$$

$$|\psi\rangle_{out} = \sum_{n=0}^{\infty} c_n |n\rangle_L \quad |n\rangle_L \equiv |n_{p0}-n\rangle_p |n_{s0}+n\rangle_s |n\rangle_{\bar{i}}, \quad |\psi\rangle_{in} = |0\rangle_L = |n_{p0}\rangle_p |n_{s0}\rangle_s |0\rangle_{\bar{i}},$$

$$|c_k^{<,(n_{s0})}(z,\tau)|^2 = p_k^{(n_{p0}\to\infty)} = (1-z)^{n_{s0}+1} z^k \binom{k+n_{s0}}{k}, \qquad 0 \le z \le z^* \qquad \boxed{n_{s0} = N-1}$$