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Avalanche of Entanglement

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Motivation

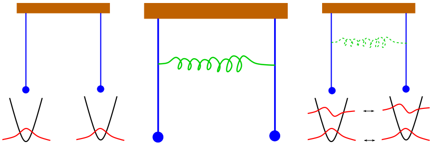
Linear fields: Gaussian (squeezed) states $\rightarrow$ pairs of particles

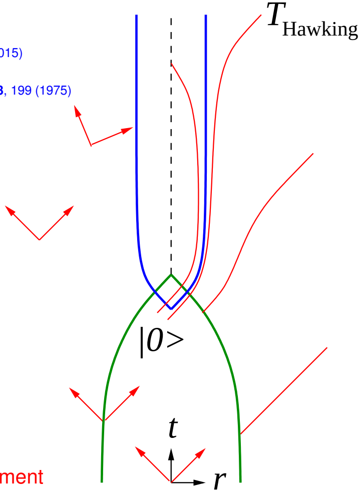
$\rightarrow$ bi-partite entanglement M. Hotta, R.S., W.G. Unruh, Phys. Rev. D **91**, 124060 (2015)

E.g., Hawking radiation S. W. Hawking, Nature **248**, 30 (1974); Comm. Math. Phys. **43**, 199 (1975)

$$T_{\mathrm{Hawking}} = \frac{1}{8\pi M} \frac{\hbar c^3}{G_{\mathrm{N}} k_{\mathrm{B}}}$$

Tearing apart of quantum vacuum fluctuations





But: trans-Plankian problem, information puzzle etc.

Non-linear interactions (“scrambling”) $\rightarrow$ multi-partite entanglement

Entanglement for Spins (QuBits)

Invariant under local unitary operations & non-increasing for local decoherence/dissipation etc.

- bi-partite entanglement → Bell states

$$|\mathrm{Bell}\rangle = \frac{|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle}{\sqrt{2}} \, ,$$

concurrence for arbitrary mixed states $\hat{\varrho}$ of two spins

$$C_2[\hat{\varrho}] = f\left(\mathrm{Eigenvalues}\left[\sqrt{\sqrt{\hat{\varrho}}\,\hat{R}\,\hat{\varrho}^*\,\hat{R}\,\sqrt{\hat{\varrho}}}\right]\right)$$

convex roof construction: minimization over all decompostitions into pure states!

- tri-partite entanglement → GHZ states

three-tangle τ_3 of three spins in pure state

$$|\mathrm{GHZ}\rangle_3 = \frac{|\uparrow\uparrow\uparrow\rangle + |\downarrow\downarrow\downarrow\rangle}{\sqrt{2}}$$

W-states $|\mathrm{W}\rangle_3 = (|\uparrow\downarrow\downarrow\rangle + |\downarrow\uparrow\downarrow\rangle + |\downarrow\downarrow\uparrow\rangle)/\sqrt{3}$???

- quadri-partite entanglement → GHZ_4 states

four-tangle(s) τ_4 of four spins in pure state

- entanglement entropy ↔ entanglement between two sub-systems of pure state $|\psi_{AB}\rangle$

$$S = -\mathrm{Tr}\left\{\hat{\varrho}_A \ln \hat{\varrho}_A\right\} \qquad \text{with} \qquad \hat{\varrho}_A = \mathrm{Tr}_B\left\{|\psi_{AB}\rangle\,\langle\psi_{AB}|\right\}$$

Quantum Ising Model in Transverse Field

Hamiltonian

$$\hat{H} = -J \sum_{i=1}^{N} \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z - \sum_{i=1}^{N} \hat{\sigma}_i^x$$

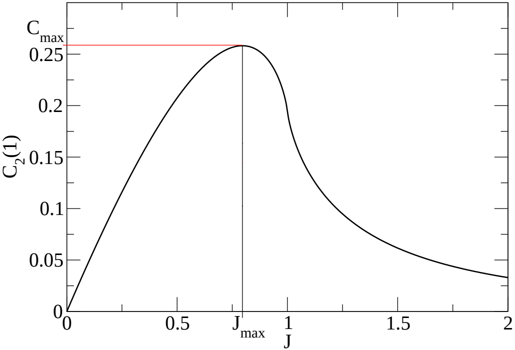
$J = 0$: paramagnetic state $|\rightarrow\rightarrow\rightarrow \dots\rangle$

$\rightarrow$ no entanglement

$J \rightarrow \infty$: ferromagnetic state

$$\frac{|\uparrow\uparrow\uparrow \dots\rangle + |\downarrow\downarrow\downarrow \dots\rangle}{\sqrt{2}}$$

$\rightarrow$ N-partite entanglement (GHZ type)



$J = 1$: critical point (phase transition)

$\rightarrow$ entanglement entropy $S \sim \ln N$ (violation of "area" law...)

cannot be explained by bi-partite entanglement C_2 alone $\rightarrow$ multi-partite entanglement!

Reduced density matrices $\hat{\varrho}_i = \mathrm{Tr}_{\mathrm{lattice}\setminus\{i\}}\{|\Psi\rangle\langle\Psi|\}$, $\hat{\varrho}_{ij} = \mathrm{Tr}_{\mathrm{lattice}\setminus\{ij\}}\{|\Psi\rangle\langle\Psi|\}$, etc.

Diagonalization (exact)

$\hat{\varrho}_{ij\dots} = \sum_\alpha p_\alpha \left|\Psi^\alpha_{ij\dots}\right\rangle\left\langle\Psi^\alpha_{ij\dots}\right|$

First two eigenvalues are dominant

$\hat{\varrho}_{ij\dots} \approx \sum_{\alpha=1}^{2} p_\alpha \left|\Psi^\alpha_{ij\dots}\right\rangle\left\langle\Psi^\alpha_{ij\dots}\right|$

→ rank-two matrices

→ calculation of tangles

first bi-partite C_2

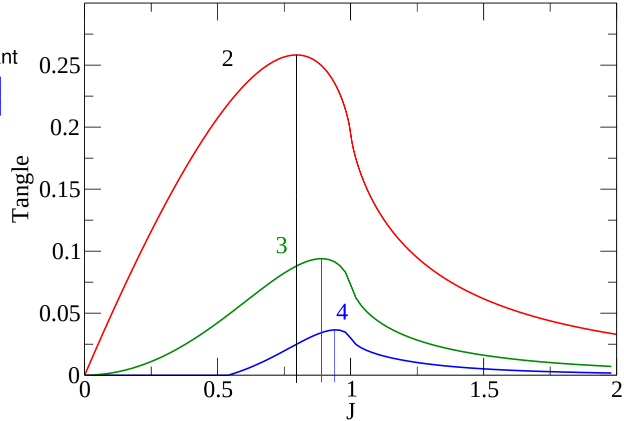
then tri-partite $\sqrt{\tau_3}$

later quadri-partite τ_4

…

finally N-partite

→ avalanche of entanglement



Correlations, e.g.,

$\langle \hat{\sigma}_i^z \hat{\sigma}_j^z \rangle^{\mathrm{corr}} = \langle \hat{\sigma}_i^z \hat{\sigma}_j^z \rangle - \langle \hat{\sigma}_i^z \rangle \langle \hat{\sigma}_j^z \rangle$

Correlated density matrices

$\hat{\varrho}_{ij}^{\mathrm{corr}} = \hat{\varrho}_{ij} - \hat{\varrho}_i \hat{\varrho}_j$ etc.

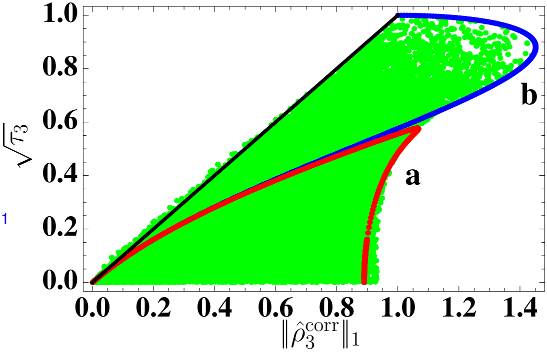
Upper bound (spectral norm)

$\langle \hat{\sigma}_i^z \hat{\sigma}_j^z \rangle^{\mathrm{corr}} = \mathrm{Tr}\{\hat{\sigma}_i^z \hat{\sigma}_j^z \hat{\varrho}_{ij}^{\mathrm{corr}}\} \leq ||\hat{\varrho}_{ij}^{\mathrm{corr}}||_1$

Concurrence for two spins

$C_2[\hat{\varrho}_{ij}] \leq ||\hat{\varrho}_{ij}^{\mathrm{corr}}||_1$

→ entanglement ↔ correlations



Question: three (or more) spins?

→ three-tangle $\sqrt{\tau_3[\hat{\varrho}_{ijk}]}$ as approximate lower bound for maximum correlation $||\hat{\varrho}_{ijk}^{\mathrm{corr}}||_1$

Questions: outlier states? four spins? . . .

Hierarchy of Correlations

Small J: hierarchy of correlations

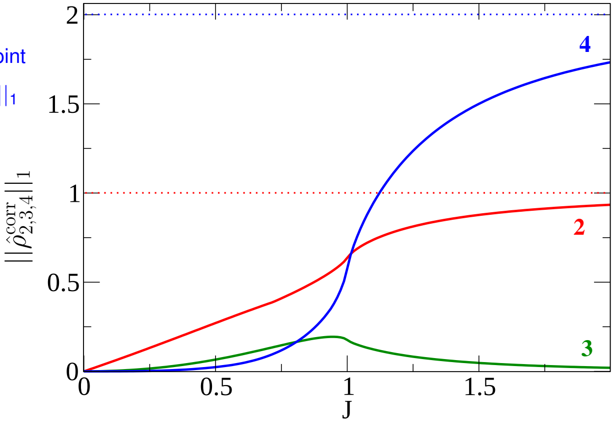
two-point $\gg$ three-point $\gg$ four-point

$||\hat{\varrho}_{ij}^{\mathrm{corr}}||_1 \gg ||\hat{\varrho}_{ijk}^{\mathrm{corr}}||_1 \gg ||\hat{\varrho}_{ijkl}^{\mathrm{corr}}||_1$

Violation of hierarchy at $J \approx 0.7$

$\rightarrow$ approximation schemes

$\langle\sigma_i^x\sigma_j^x\sigma_k^x\sigma_l^x\rangle =$
$\langle\sigma_i^x\rangle\langle\sigma_j^x\rangle\langle\sigma_k^x\rangle\langle\sigma_l^x\rangle +$
$\langle\sigma_i^x\sigma_j^x\rangle^{\mathrm{corr}}\langle\sigma_k^x\rangle\langle\sigma_l^x\rangle + \cdots +$
$\langle\sigma_i^x\sigma_j^x\sigma_k^x\rangle^{\mathrm{corr}}\langle\sigma_l^x\rangle + \cdots +$
$\langle\sigma_i^x\sigma_j^x\sigma_k^x\sigma_l^x\rangle^{\mathrm{corr}}$



Bose-Hubbard Model

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Hamiltonian

$$\hat{H} = -J \sum_{i=1}^{N} \left(\hat{b}_i^\dagger \hat{b}_{i+1} + \hat{b}_{i+1}^\dagger \hat{b}_i \right) + \frac{1}{2} \sum_{i=1}^{N} \hat{b}_i^\dagger \hat{b}_i^\dagger \hat{b}_i \hat{b}_i$$

Small J: paramagnetic → Mott insulator

Large J: ferromagnetic → superfluid

Small J: hierarchy of correlations

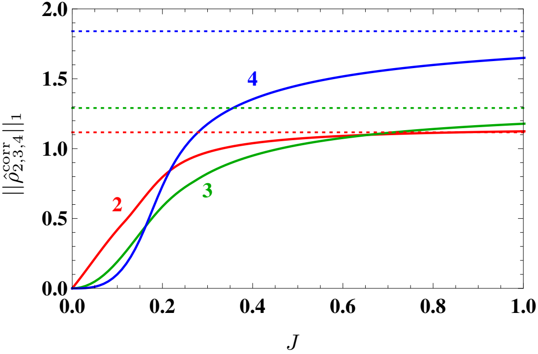
two-point ≫ three-point ≫ four-point

$||\hat{\varrho}_{ij}^{\mathrm{corr}}||_1 \gg ||\hat{\varrho}_{ijk}^{\mathrm{corr}}||_1 \gg ||\hat{\varrho}_{ijkl}^{\mathrm{corr}}||_1$

Violation of hierarchy at $J \approx 0.16$

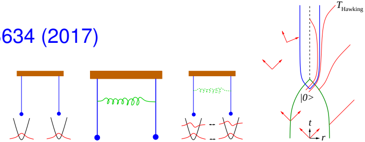
well before the critical point

(here $J_{\mathrm{crit}} \approx 0.3$)



Summary

K.V. Krutitsky, A. Osterloh, R.S., Nature Scientific Reports **7**, 3634 (2017)

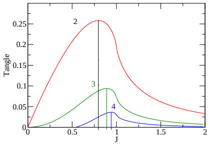


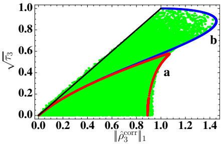
- Motivation: bi-partite entanglement ↔ pairs (partners)

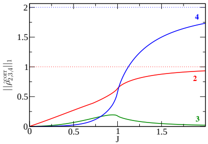
  multi-partite entanglement ↔ ???

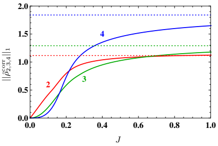
- Ising model: avalanche of entanglement → correlations

Hubbard model









Accuracy of Approximation

approximation 1)

$$\hat{\varrho}_{ij} \approx p_1 \left|\Psi_{ij}^1\right\rangle \left\langle\Psi_{ij}^1\right| + (1 - p_1) \left|\Psi_{ij}^2\right\rangle \left\langle\Psi_{ij}^2\right|$$

approximation 2)

$$\hat{\varrho}_{ij} \approx \frac{p_1 \left|\Psi_{ij}^1\right\rangle \left\langle\Psi_{ij}^1\right| + p_2 \left|\Psi_{ij}^2\right\rangle \left\langle\Psi_{ij}^2\right|}{p_1 + p_2}$$

note that $p_3 < 0.5\ \%$