CENTRE FOR QUANTUM COMPUTATION & COMMUNICATION TECHNOLOGY

AUSTRALIAN RESEARCH COUNCIL CENTRE OF EXCELLENCE

Decoherence of the radiation from an accelerated quantum source

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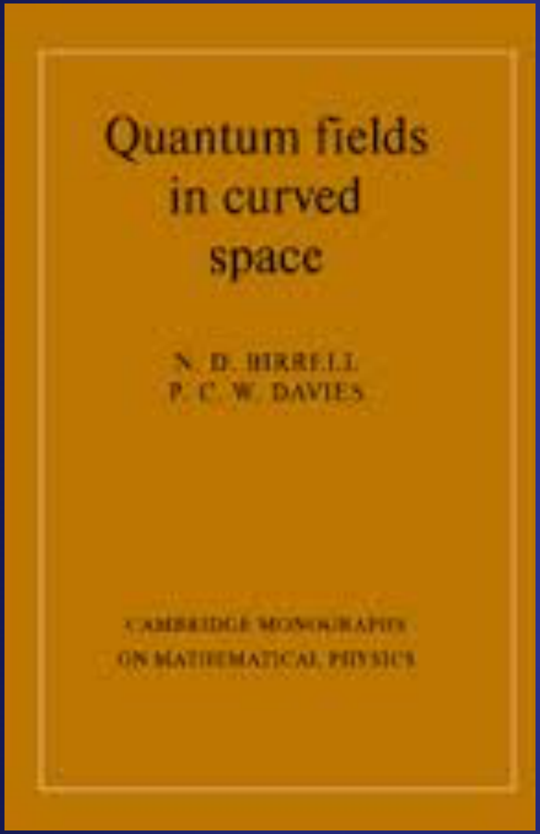
Quantum Optics Quantum Information Theory Group

Mathematical Motivation

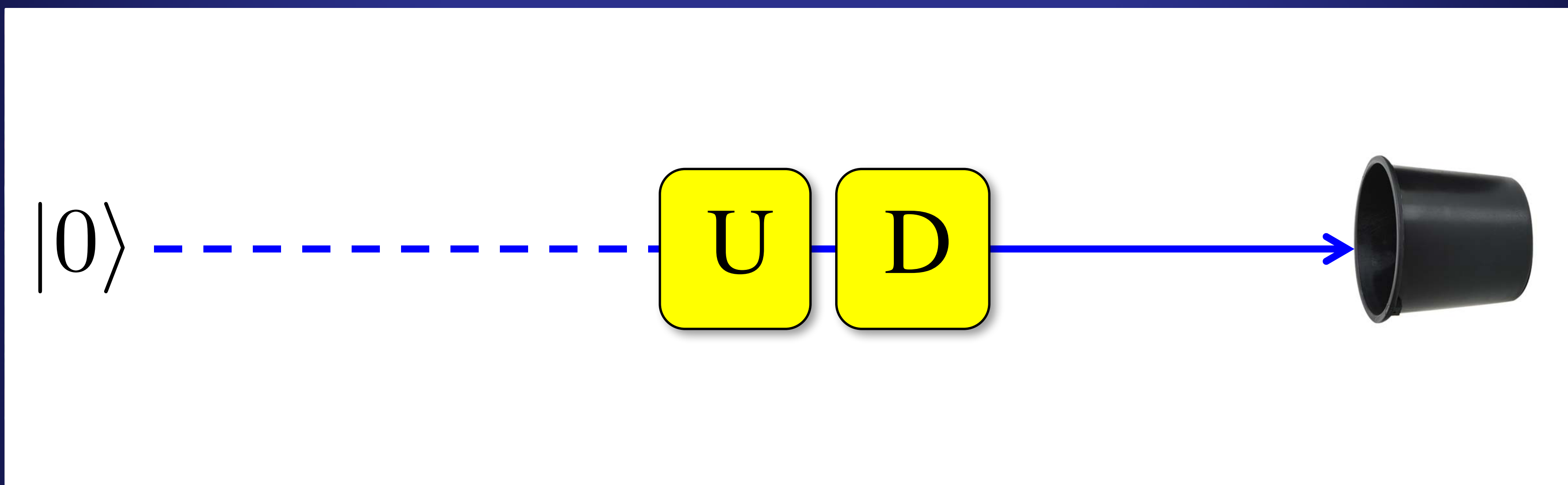


Radiation from accelerated objects has been studied for a long time, but…

... mostly solutions are: numerical; perturbative; and suffer from infra-red and ultra-violet divergences.

Mathematical Motivation

Problems arise from the **detector model** and **non-unitary interactions**



Start in vacuum

Single–mode unitary

Matched displacement

Broadband detector

Solve for expectation values in the **Heisenberg Picture**

Mathematical Motivation

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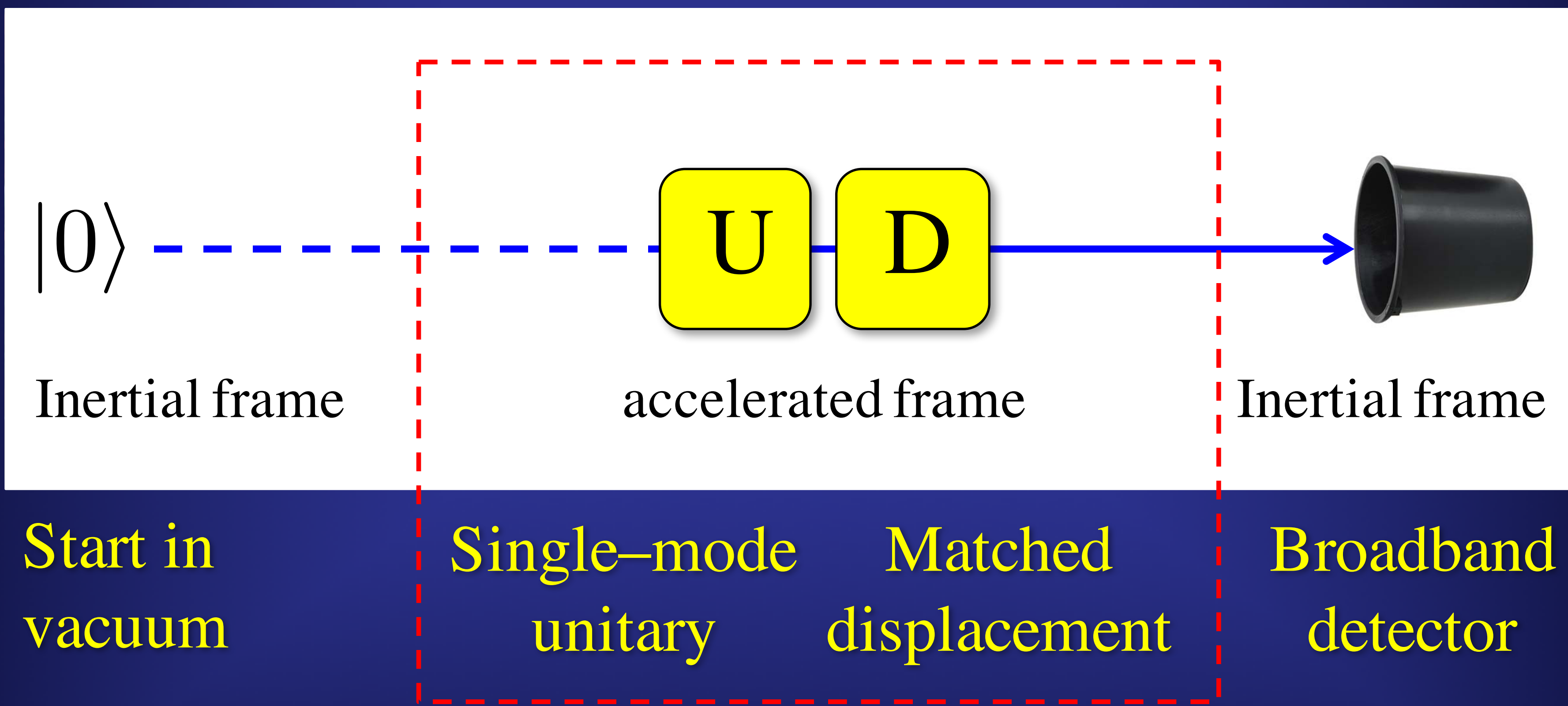
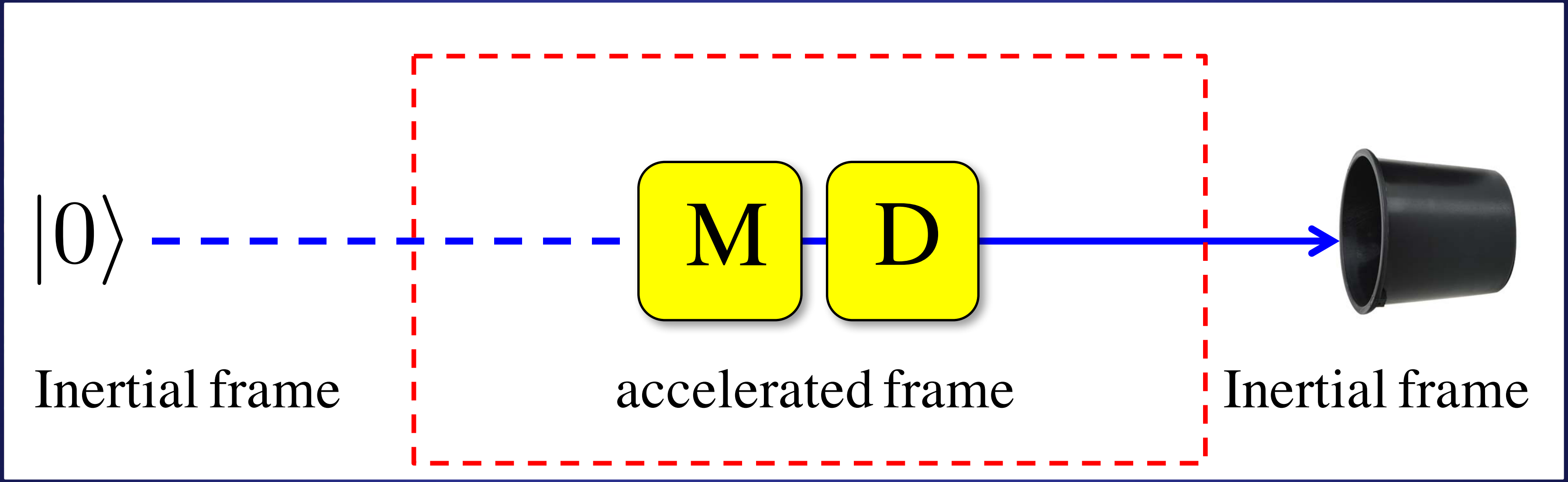


Solve for expectation values in the **Heisenberg Picture**

Mathematical Motivation

For example: **accelerated mirror**

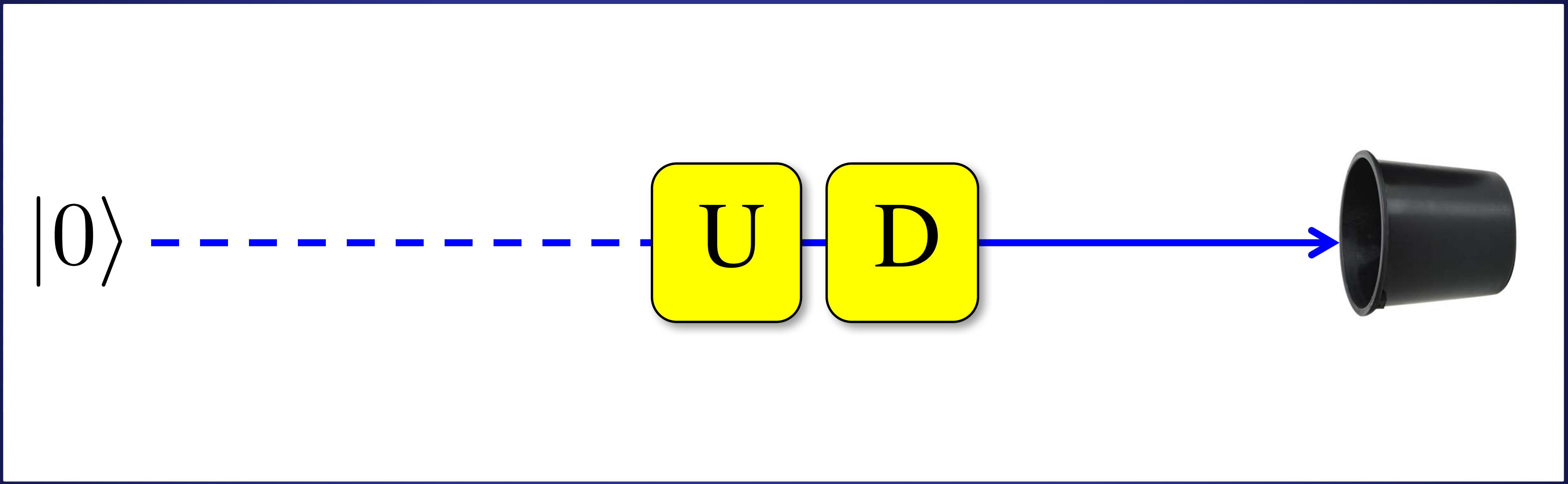


$$V = 1 + 8(1 - \cos\theta)\mathcal{I}_c\mathcal{I}_s$$

Solve for expectation values in the **Heisenberg Picture**

Physical Motivation

Pure state in, Unitary interaction, Pure state out!



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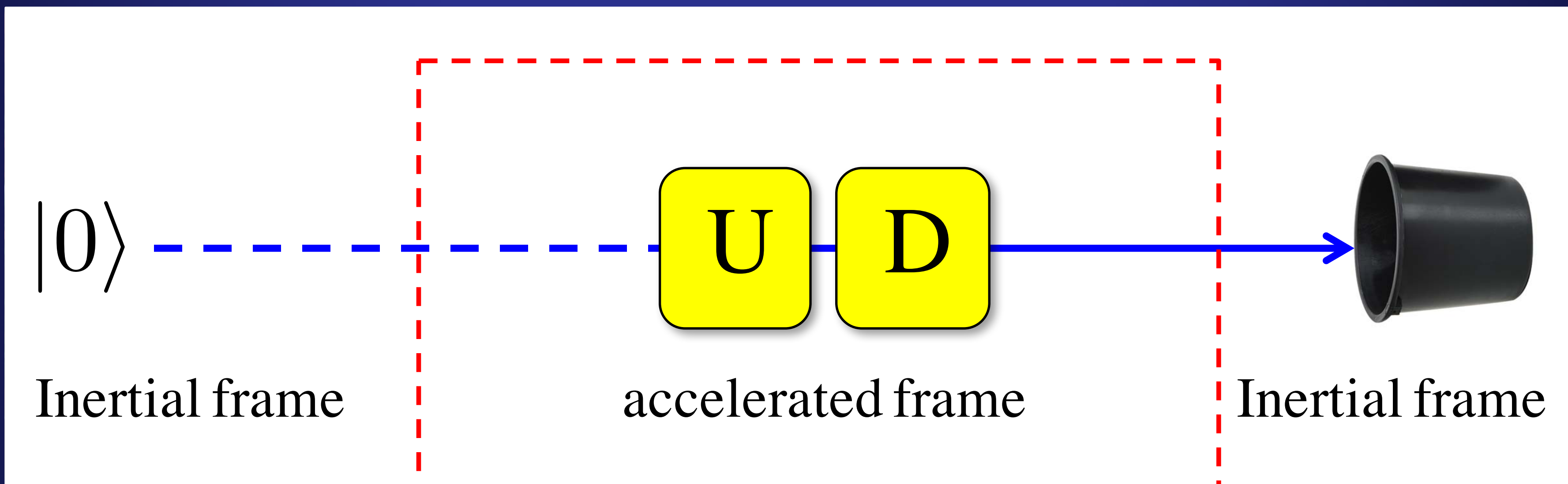
Physical Motivation

Pure state in, Unitary interaction, Pure state out...?



Start in vacuum

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Broadband detector

Solve for expectation values in the **Heisenberg Picture**

“Quantum circuit model for non-inertial objects: a uniformly accelerated mirror”
Daiqin Su, C. T. Marco Ho, Robert Mann, Timothy C. Ralph
New Journal of Physics **19**, 063017 (2017)

“Decoherence of the radiation from an accelerated quantum source”
Daiqin Su, T.C.Ralph, arXiv:1705.07432

Overview

* An accelerating quantum source
* Calculating the quantum statistics
* Decoherence
    - squeezed source
* Relationship to Black-Hole
    information paradox?

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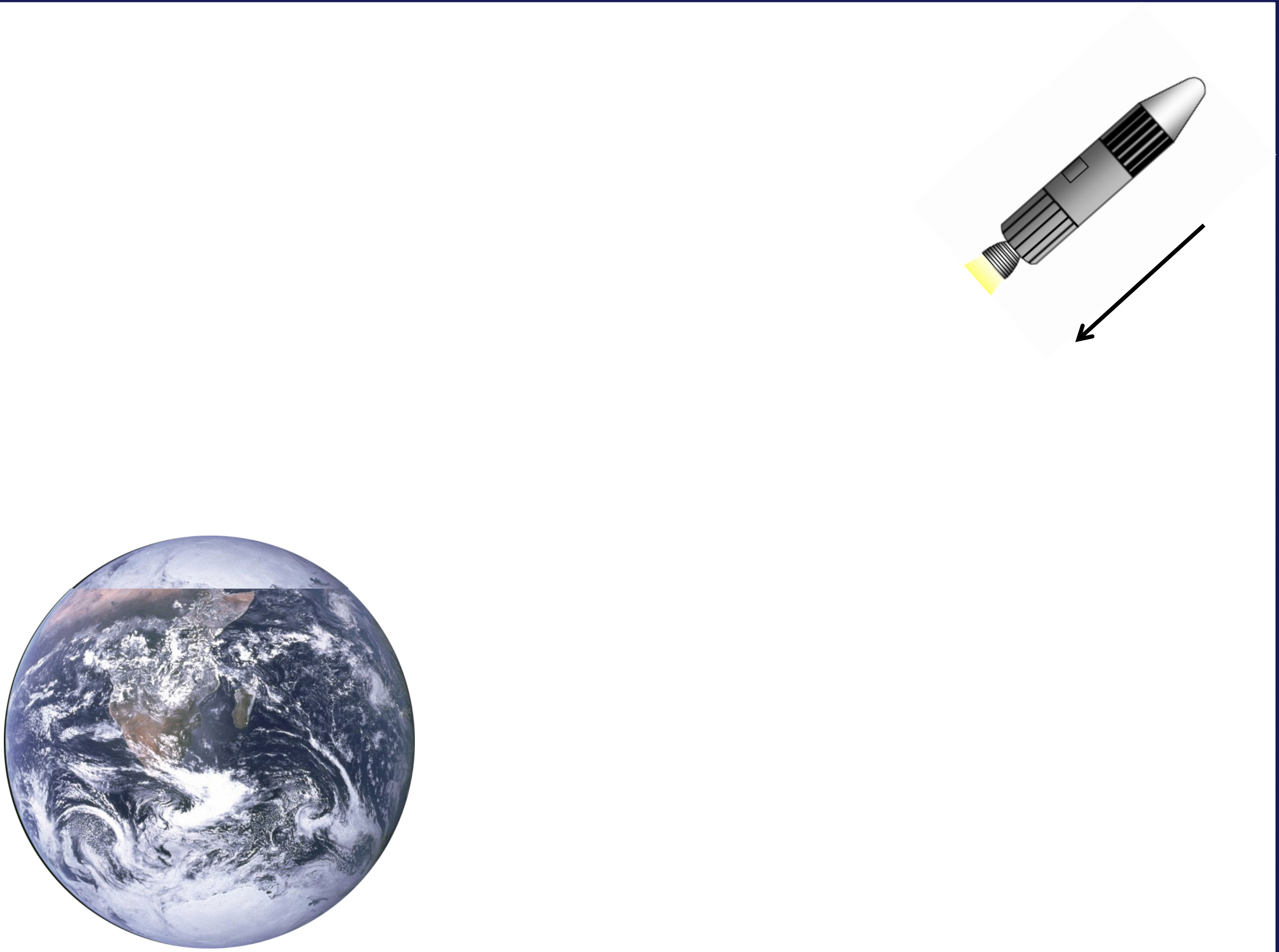
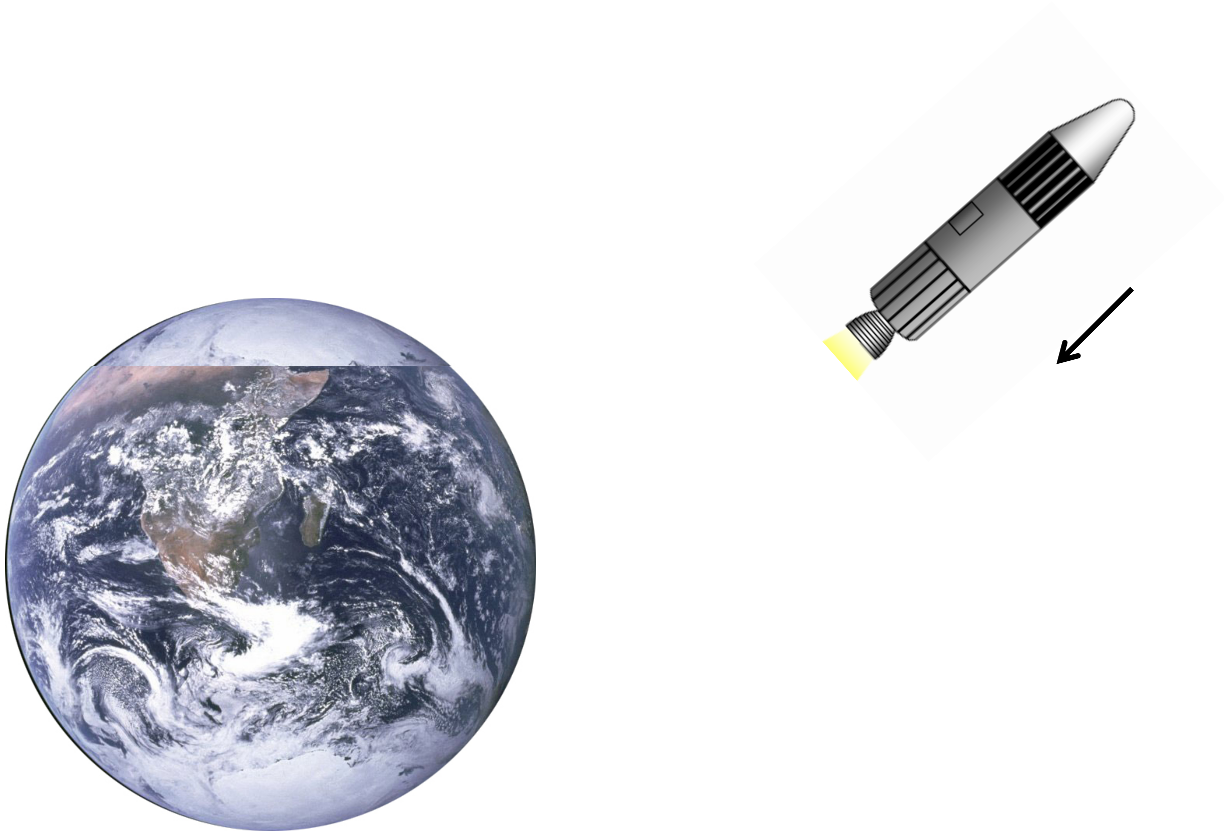
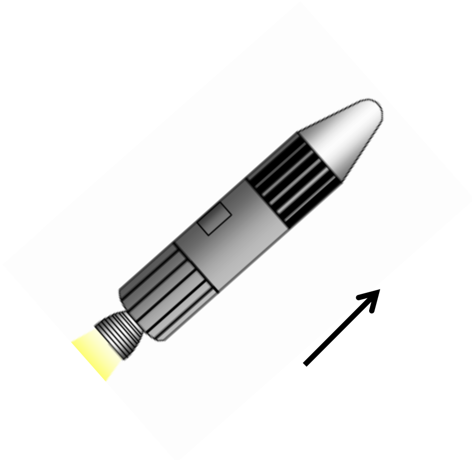
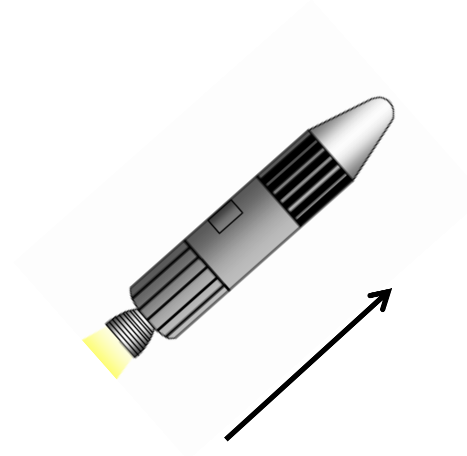
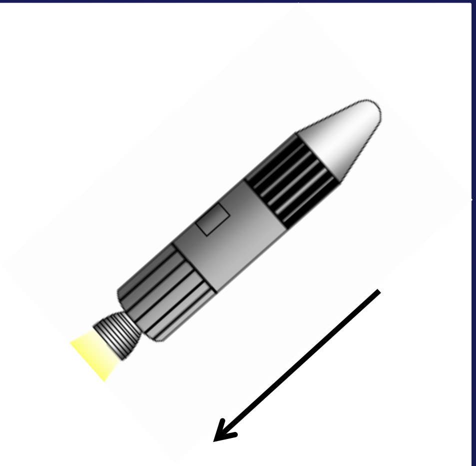


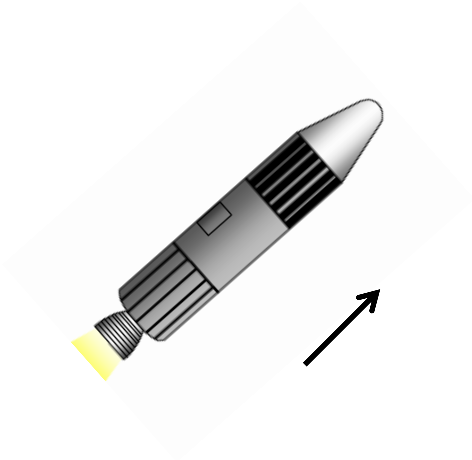


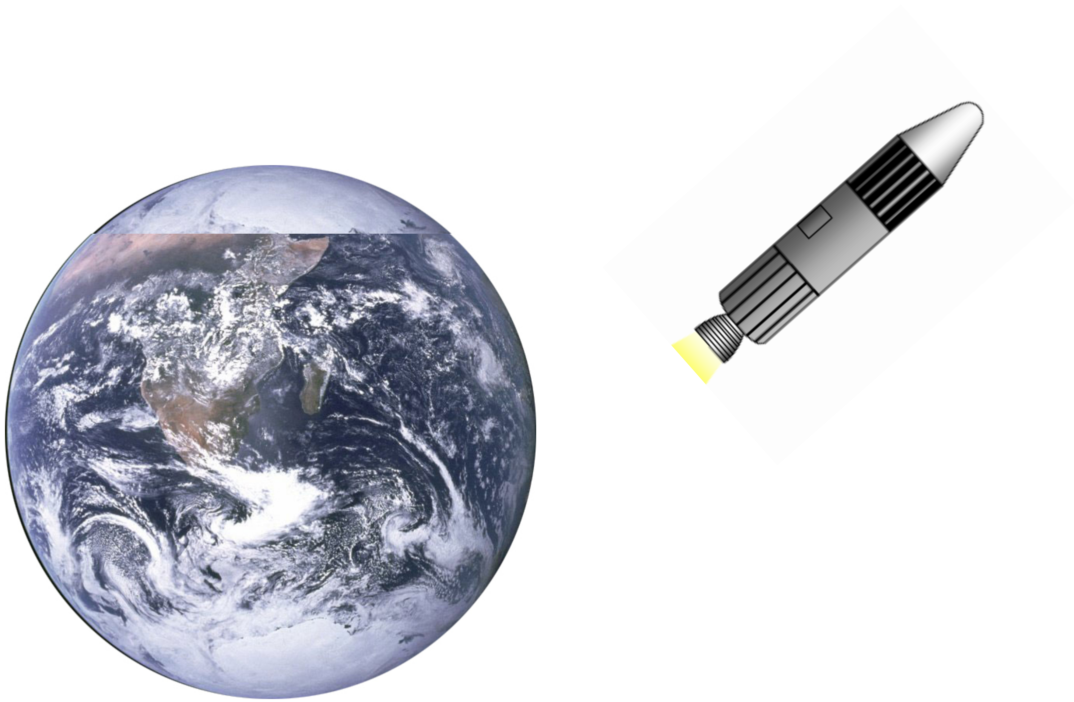


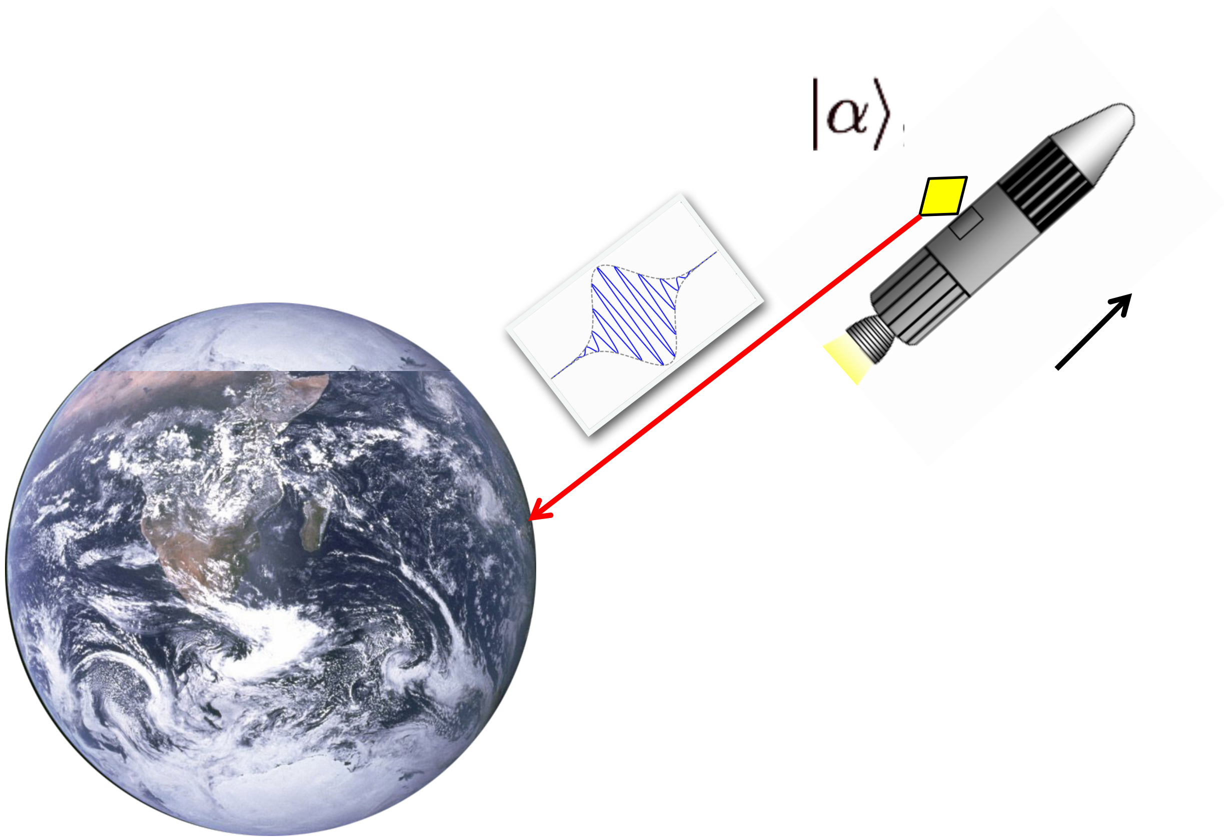


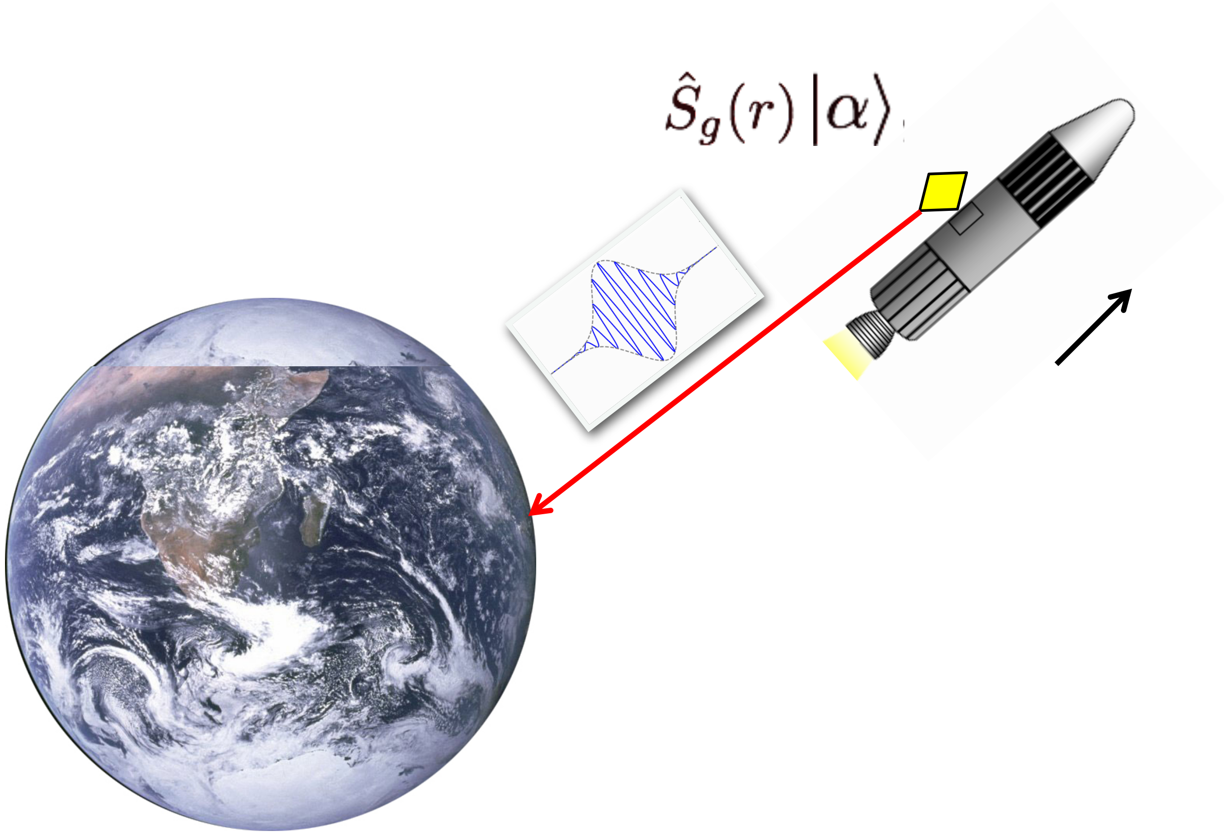


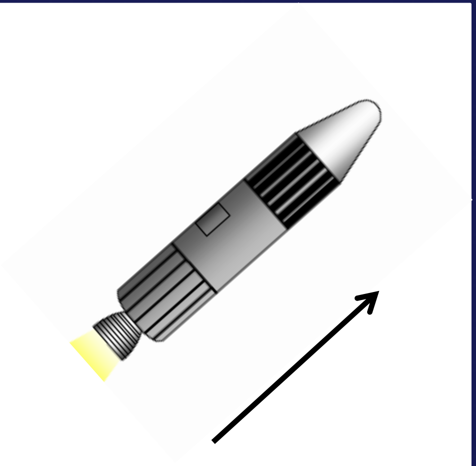





$|\alpha\rangle$


$\hat{S}_g(r)\,|\alpha\rangle$



Overview

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    - squeezed source
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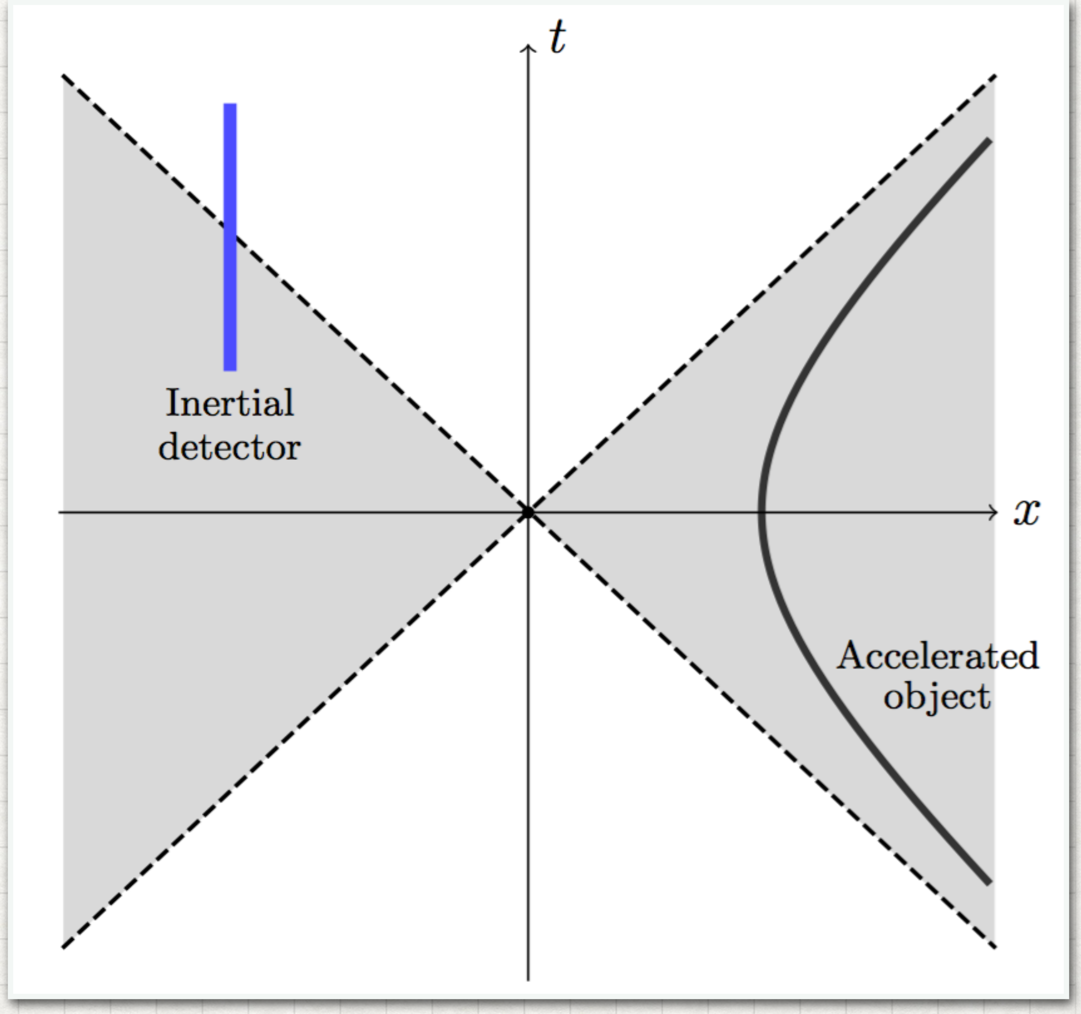
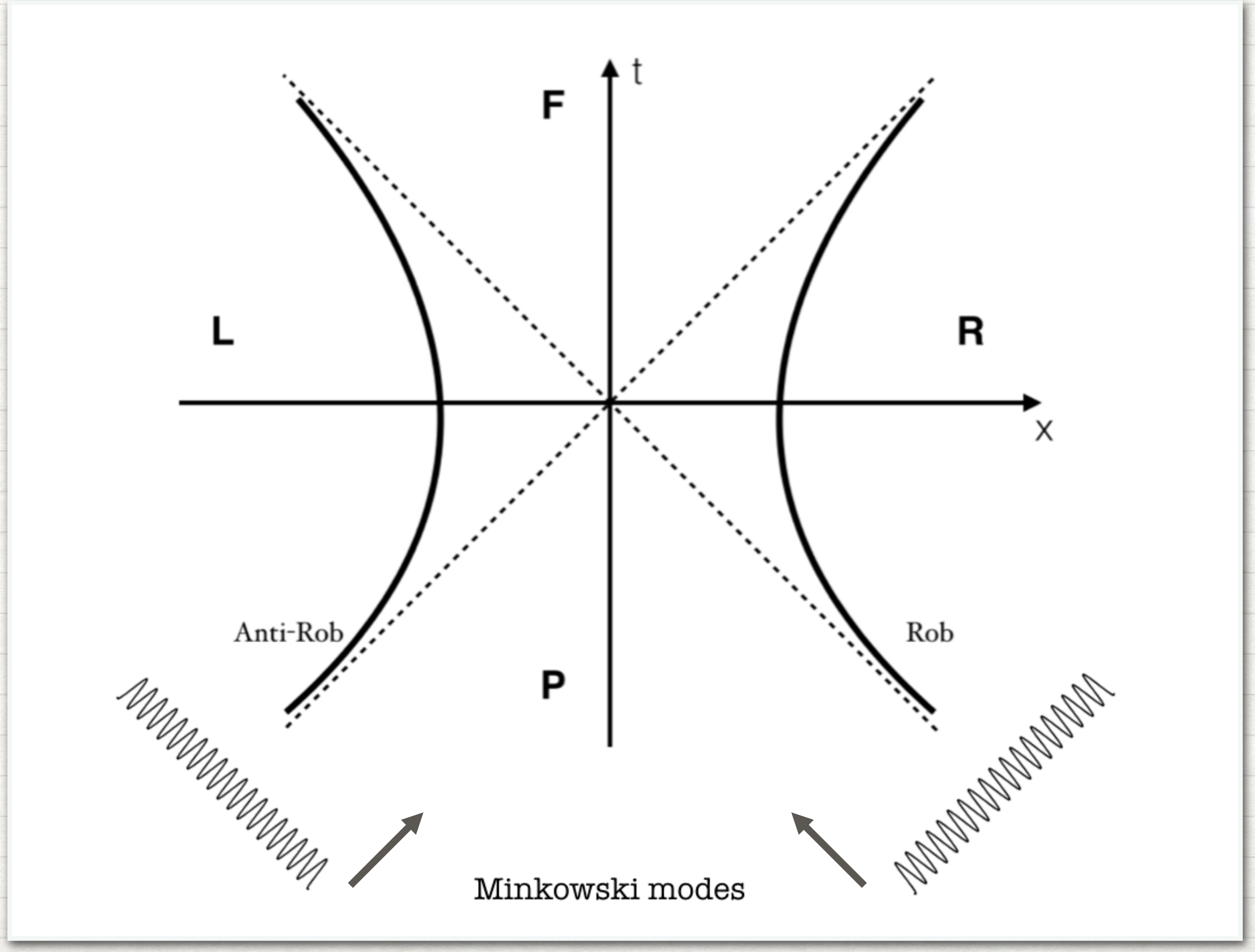
Radiation from accelerated objects

- Particle radiated by the accelerated object, detected by inertial observers

- Standard method: perturbation theory Feynman diagrams, renormalisation, etc.



Minkowski modes and Rindler modes



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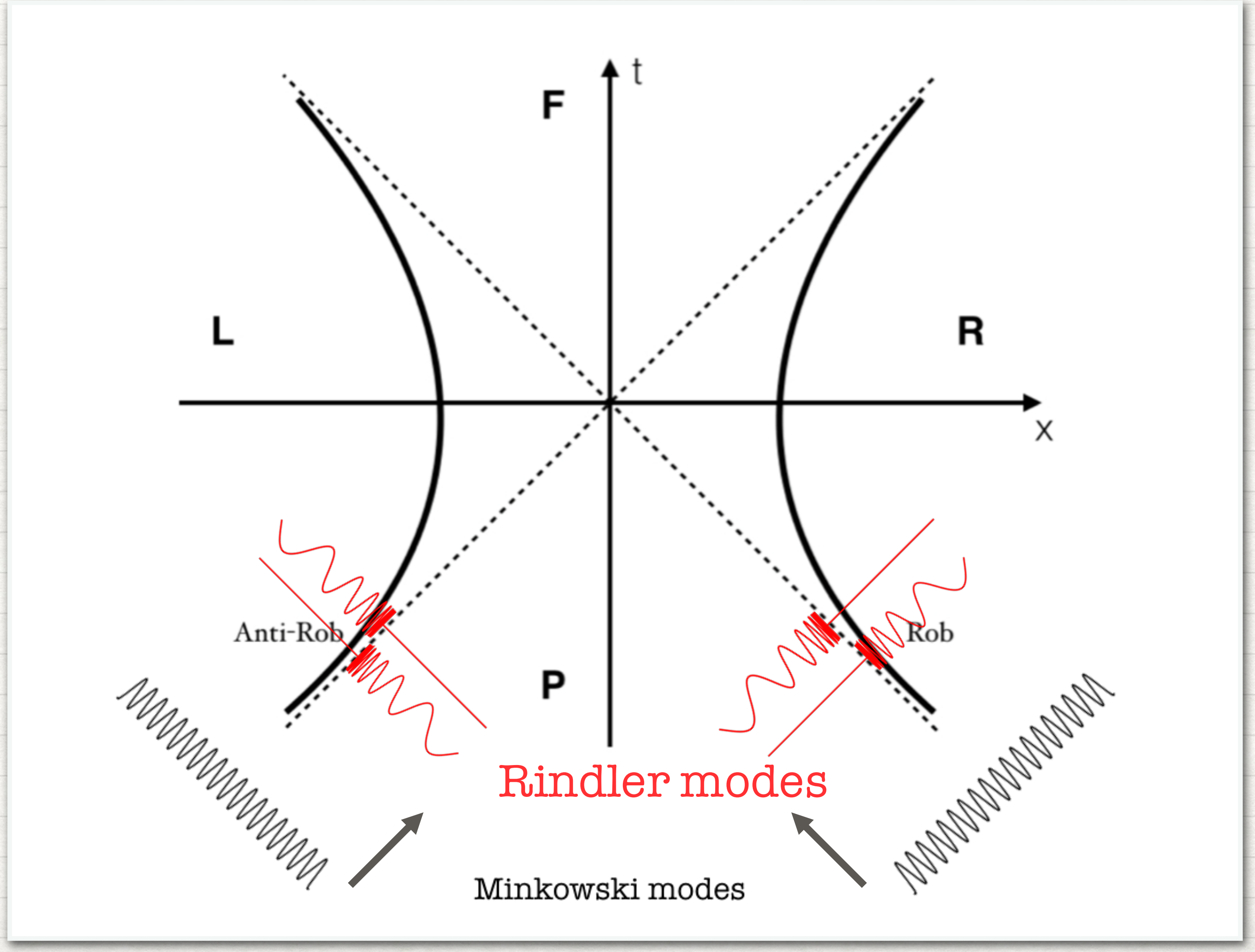
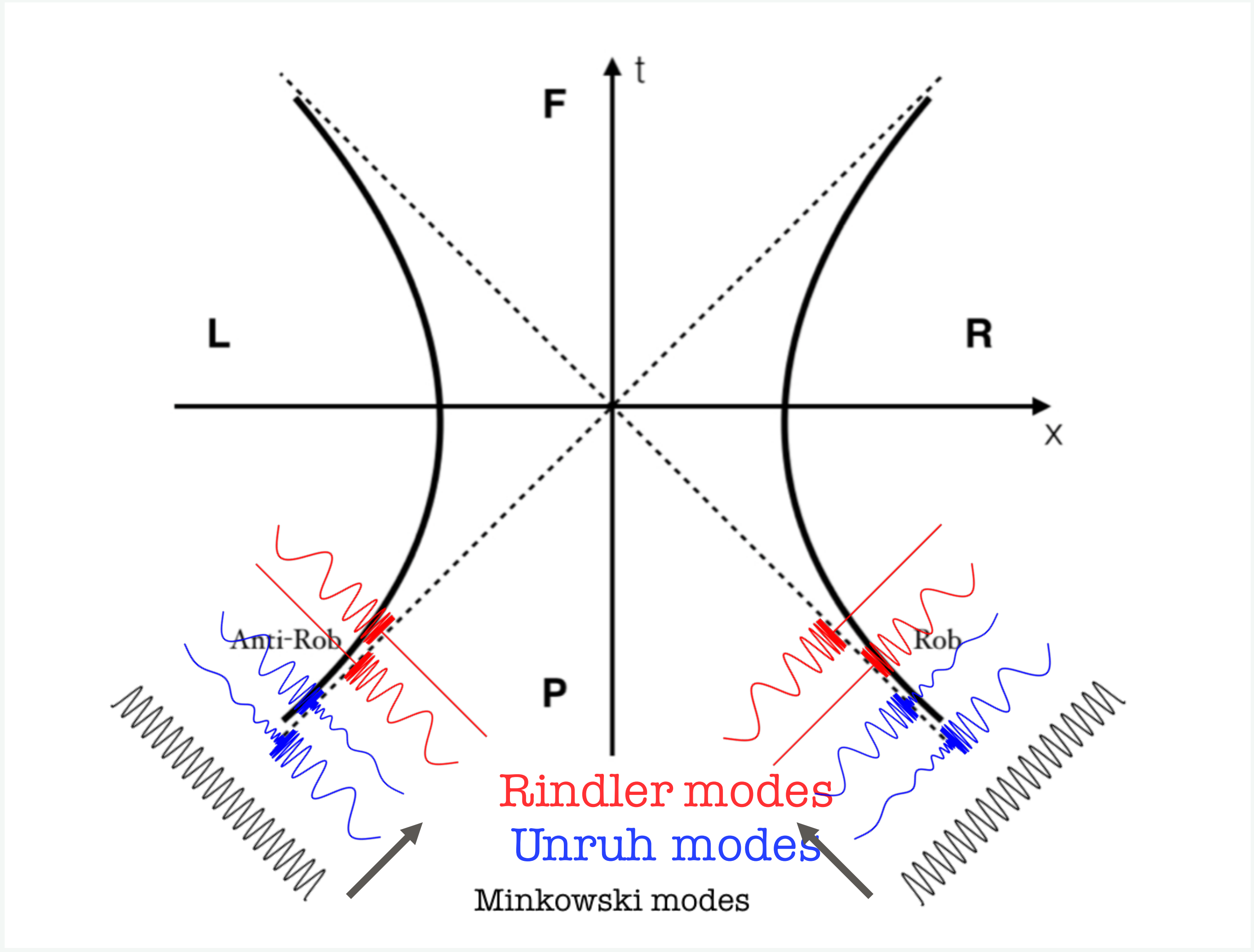


Unruh modes



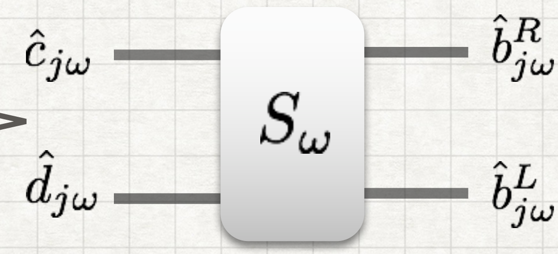
t
F
L
R
x
P
Anti-Rob
Rob
Rindler modes
Unruh modes
Minkowski modes

Relations between three sets of modes

Minkowski operators	Unruh operators	Rindler operators
$\hat{a}_{1k},\ \hat{a}_{2k}$	$\hat{c}_{1\omega}, \hat{d}_{1\omega}, \hat{c}_{2\omega}, \hat{d}_{2\omega}$	$\hat{b}^R_{1\omega}, \hat{b}^L_{1\omega}, \hat{b}^R_{2\omega}, \hat{b}^L_{2\omega}$

$$\hat{b}^R_{j\omega} = \cosh(r_\omega)\hat{c}_{j\omega} + \sinh(r_\omega)\hat{d}^\dagger_{j\omega}$$

$$\hat{b}^L_{j\omega} = \cosh(r_\omega)\hat{d}_{j\omega} + \sinh(r_\omega)\hat{c}^\dagger_{j\omega}$$



two mode squeezer

$j = 1, 2$ left-moving and right-moving modes

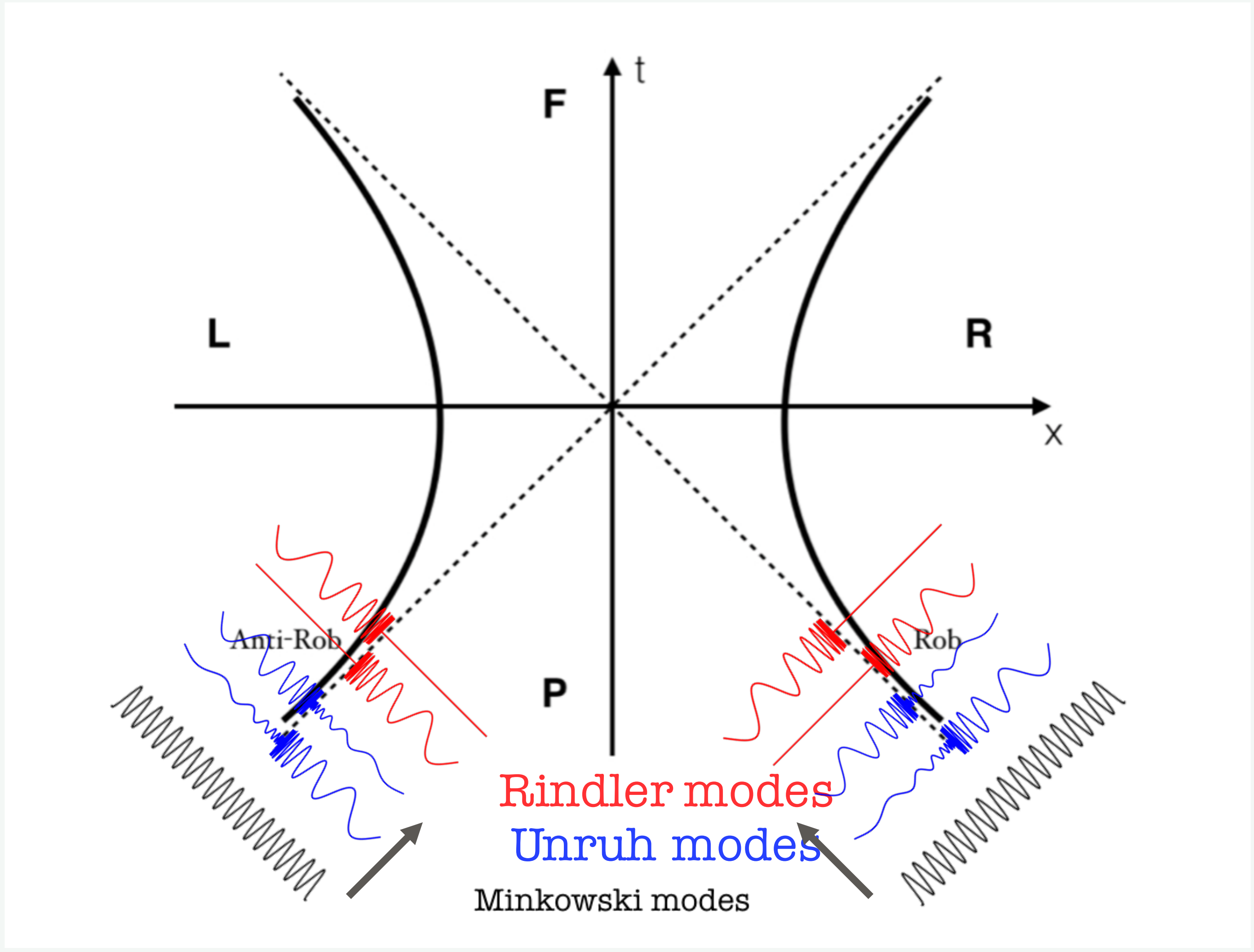
$\tanh(r_\omega) = e^{-\pi\omega}$

$$\hat{a}_{jk} = \int d\omega (A_{k\omega}\hat{c}_{j\omega} + B_{k\omega}\hat{d}_{j\omega})$$

$\hat{c}_{j\omega}|0_M\rangle = \hat{d}_{j\omega}|0_M\rangle = 0$

Unruh modes share the same vacuum with Minkowski modes

Unruh modes



t
F
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x
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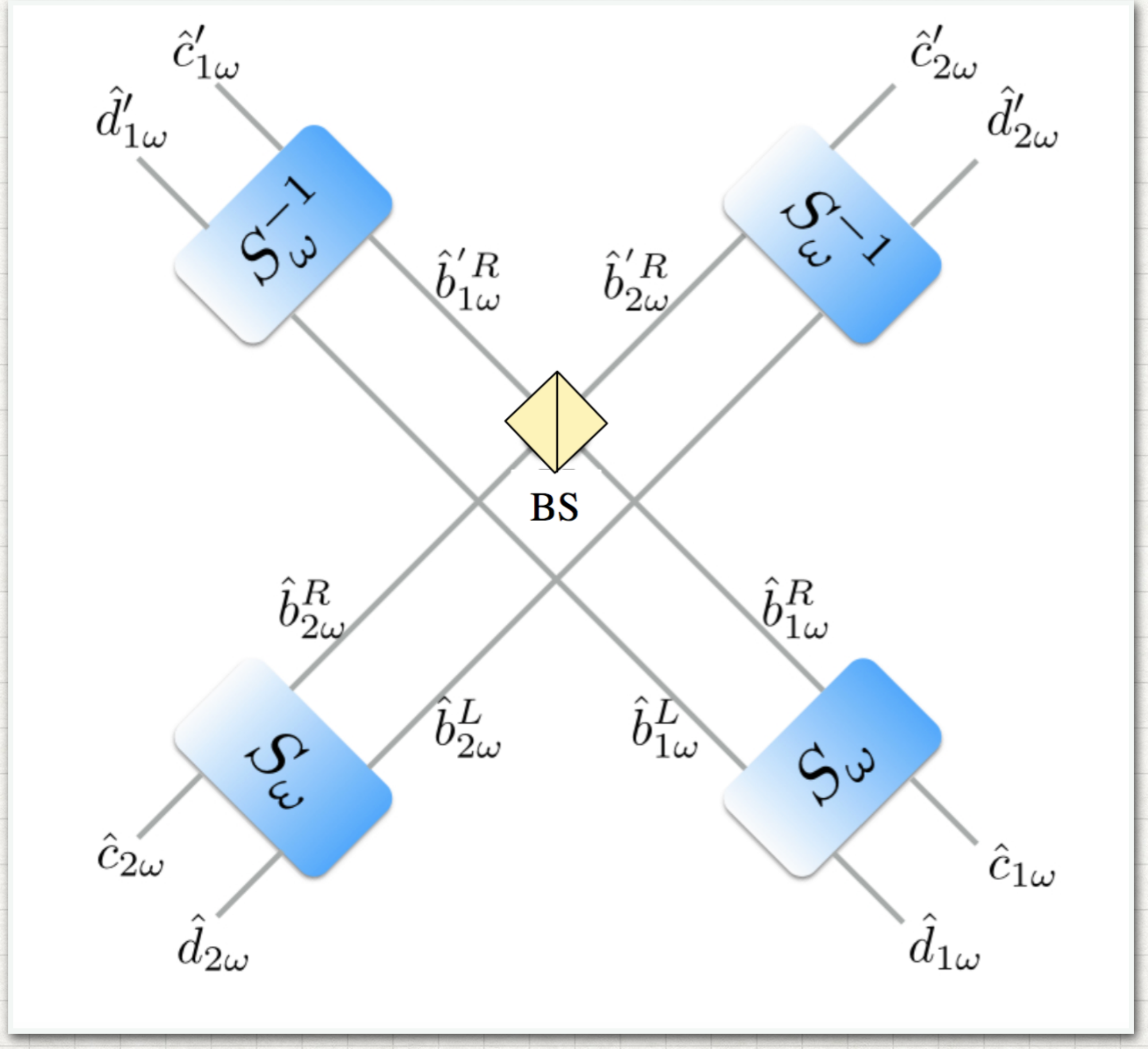
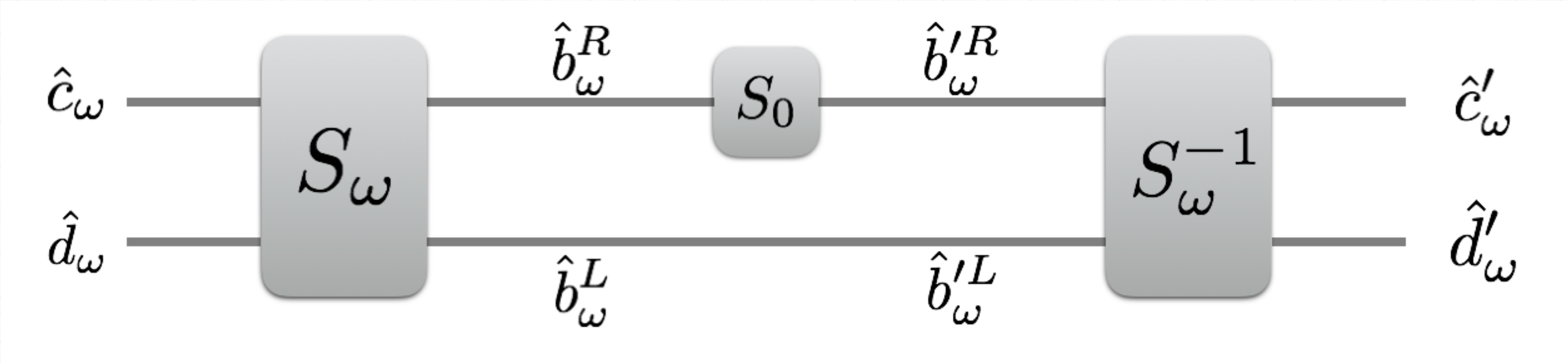
Anti-Rob
Rob
Rindler modes
Unruh modes
Minkowski modes

Quantum circuit model: accelerated mirror



Daiqin Su, et al, New Journal of Physics **19**, 063017 (2017)

Quantum circuit model: accelerated time independent interaction



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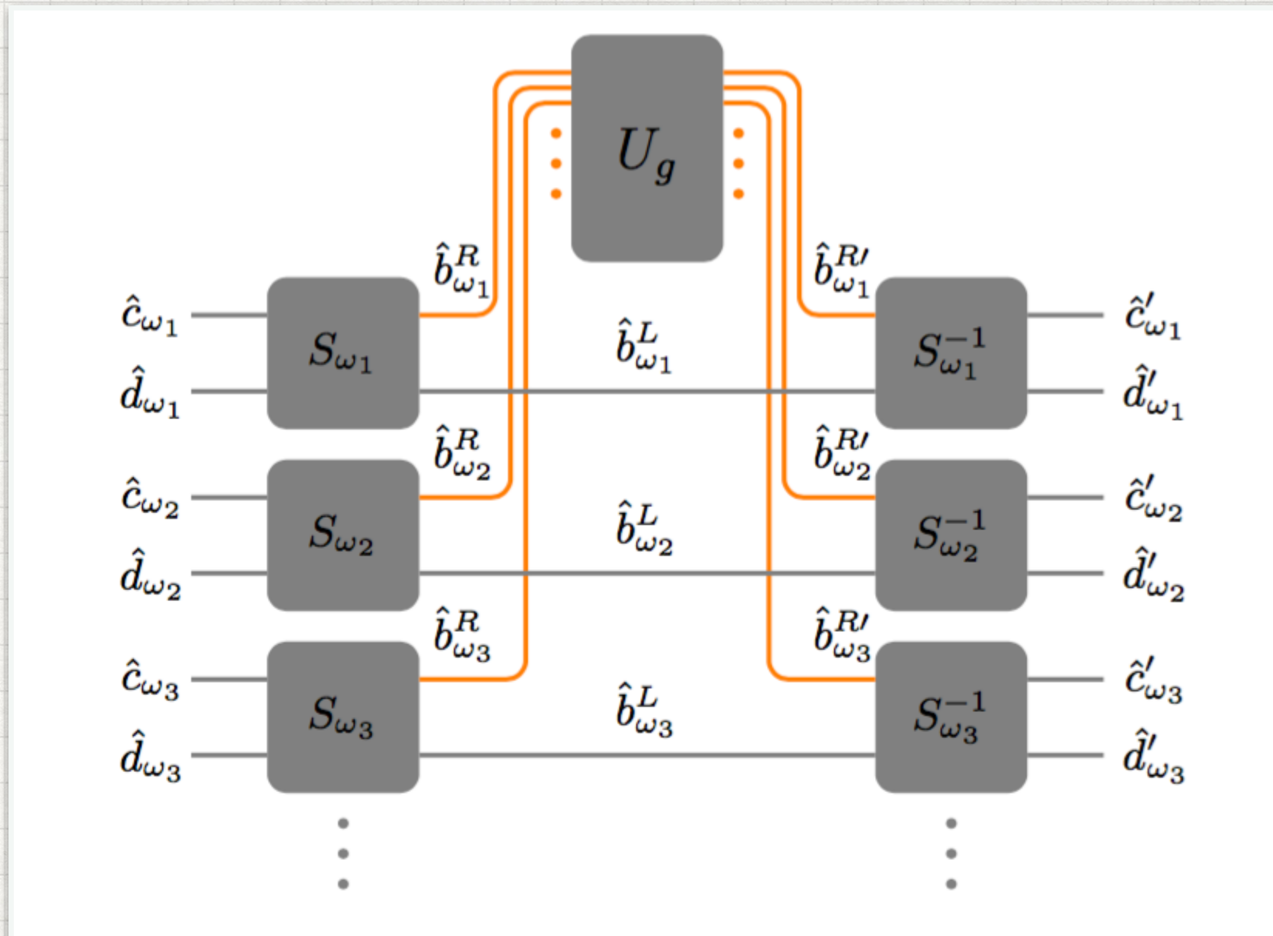
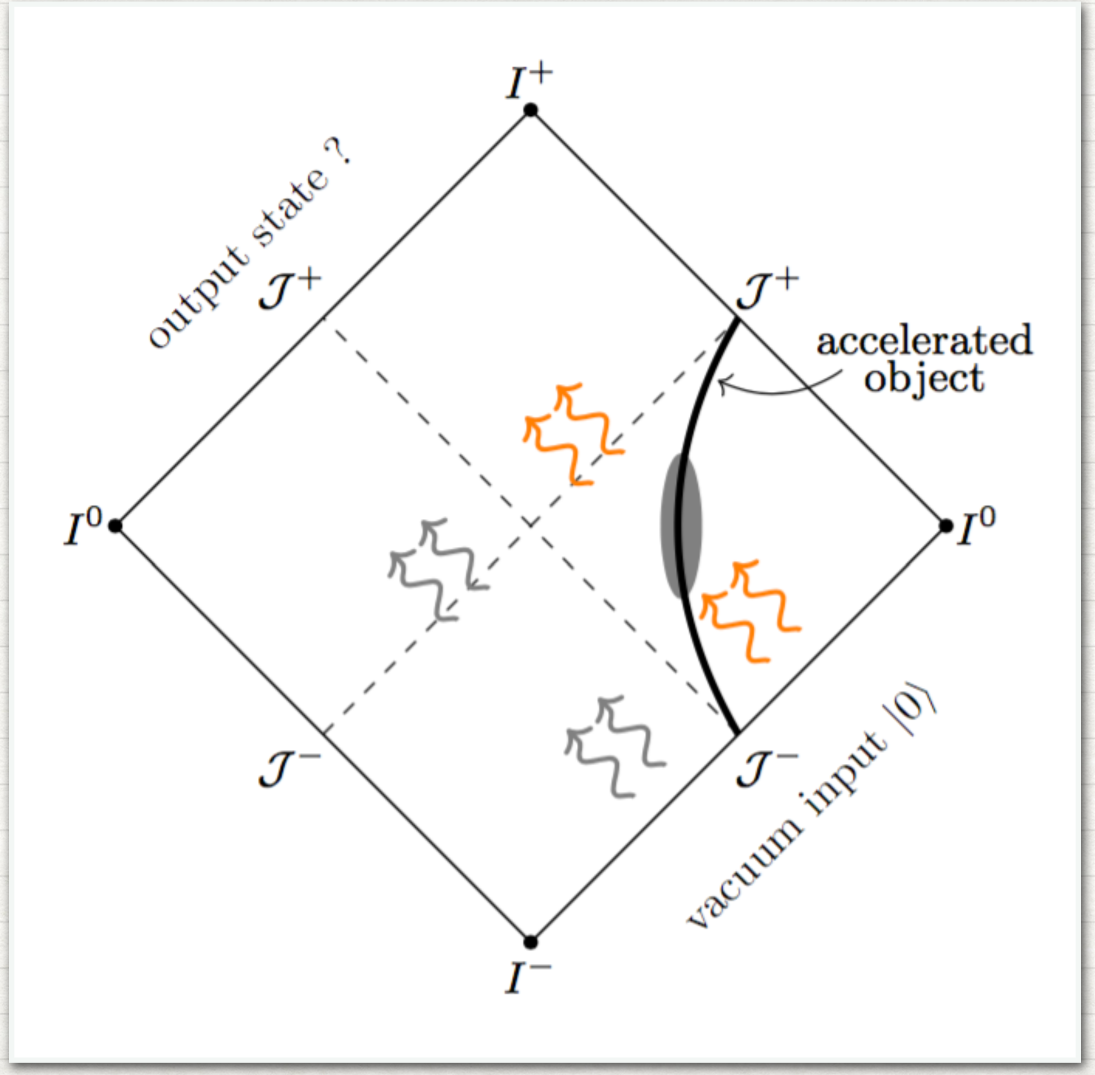
Circuit for time dependent interactions



Penrose diagram of the problem



Self-Homodyne detection

Inertial detector

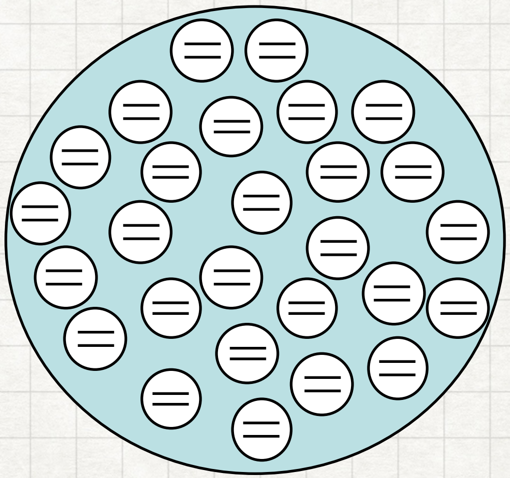
$$\hat{N} = \int \mathrm{d}k \ \hat{a}_k^\dagger \hat{a}_k$$

Self-Homodyne detection

Inertial detector

$$\hat{N} = \int \mathrm{d}k \; \hat{a}_k^\dagger \hat{a}_k$$

Many two-level atoms



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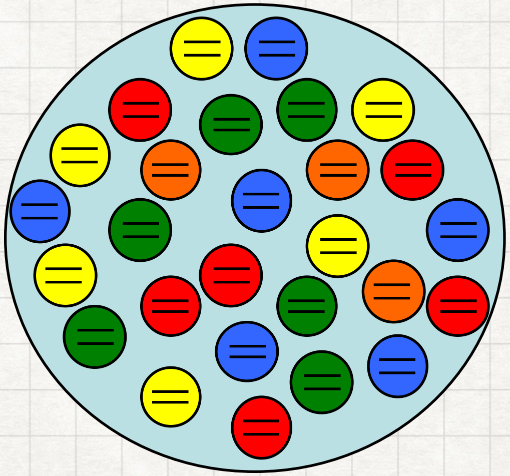

Inhomogeneously broadened

Self-Homodyne detection

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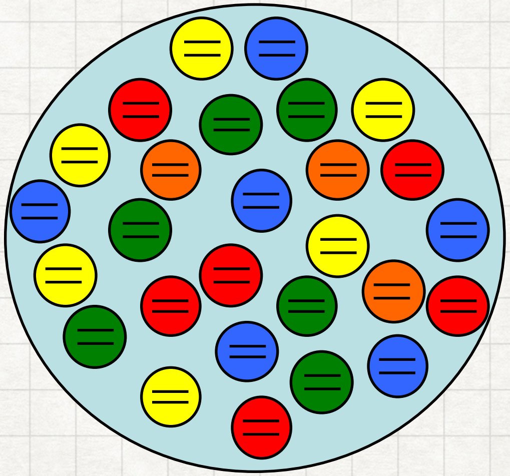
Many two-level atoms



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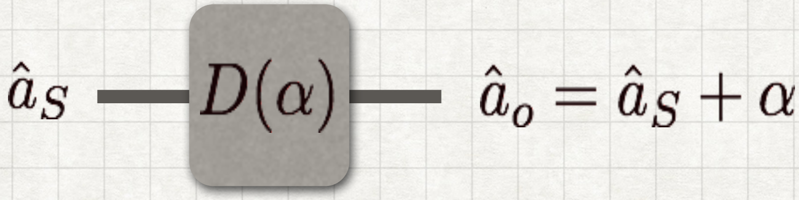


$$\hat{N} \approx |\alpha|^2 + |\alpha| \hat{X}(\phi)$$

$$\left(\Delta X(\phi)\right)^2 = \frac{\left(\Delta N\right)^2}{|\alpha|^2}$$

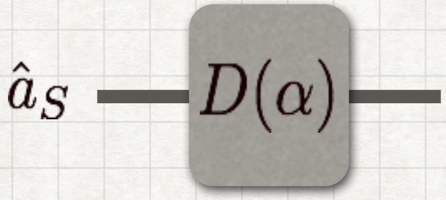
Self-Homodyne detection

Inertial detector

$$\hat{N} = \int \mathrm{d}k \; \hat{a}_k^\dagger \hat{a}_k$$

$$\hat{a}'_k \; = \; \int \mathrm{d}\omega \; \left(A_{k\omega}\hat{c}'_\omega + B_{k\omega}\hat{d}'_\omega\right)$$

$$\begin{aligned} \hat{N} \; &= \int \mathrm{d}k \int \mathrm{d}\omega_1 \int \mathrm{d}\omega_2 (A^*_{k\omega_1}\hat{c}'^\dagger_{\omega_1} + B^*_{k\omega_1}\hat{d}'^\dagger_{\omega_1})(A_{k\omega_2}\hat{c}'_{\omega_2} + B_{k\omega_2}\hat{d}'_{\omega_2}) \\ &= \int \mathrm{d}\omega \; (\hat{c}'^\dagger_\omega \hat{c}'_\omega + \hat{d}'^\dagger_\omega \hat{d}'_\omega), \end{aligned}$$



$$\hat{a}_o = \hat{a}_S + \alpha$$

$$\hat{N} \approx \; |\alpha|^2 + |\alpha|\hat{X}(\phi)$$

$$\left(\Delta X(\phi)\right)^2 = \frac{\left(\Delta N\right)^2}{|\alpha|^2}$$

Accelerated displacement

$$U_g = D_g$$

Displacement

$$\hat{D}_g(\alpha) = \exp\left(\alpha \hat{b}_g^{R\dagger} - \alpha^* \hat{b}_g^{R}\right)$$

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Unruh modes

$$\begin{cases} \hat{c}'_\omega = \hat{c}_\omega + \alpha g^*(\omega)\cosh(r_\omega) \\ \hat{d}'_\omega = \hat{d}_\omega - \alpha^* g(\omega)\sinh(r_\omega) \end{cases}$$

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Minkowski modes

Displacement amplitude

$$\hat{a}'_k = \hat{a}_k + \alpha \int \mathrm{d}\omega A_{k\omega} g^*(\omega)\cosh(r_\omega) - \alpha^* \int \mathrm{d}\omega B_{k\omega} g(\omega)\sinh(r_\omega)$$

Coherent state as observed by inertial observers

Overview II

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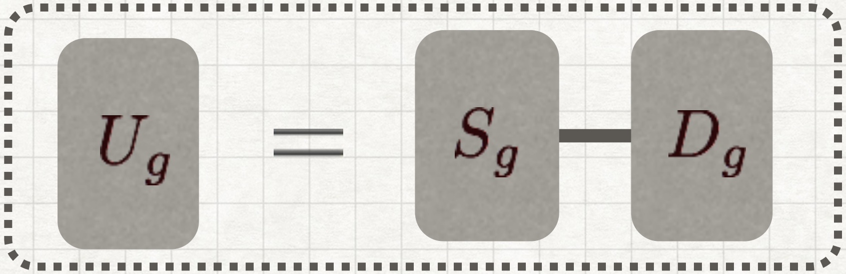
  - squeezed source
* Relationship to Black-Hole information paradox?

Accelerated single-mode squeezer



Single-mode squeezer

$$\hat{S}_g(r) = \exp\left\{\frac{r}{2}\left(\hat{b}_g^{R\dagger}\right)^2 - \frac{r}{2}\left(\hat{b}_g^{R}\right)^2\right\}$$

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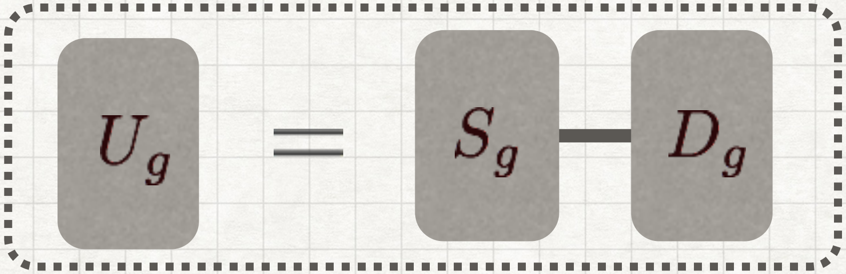

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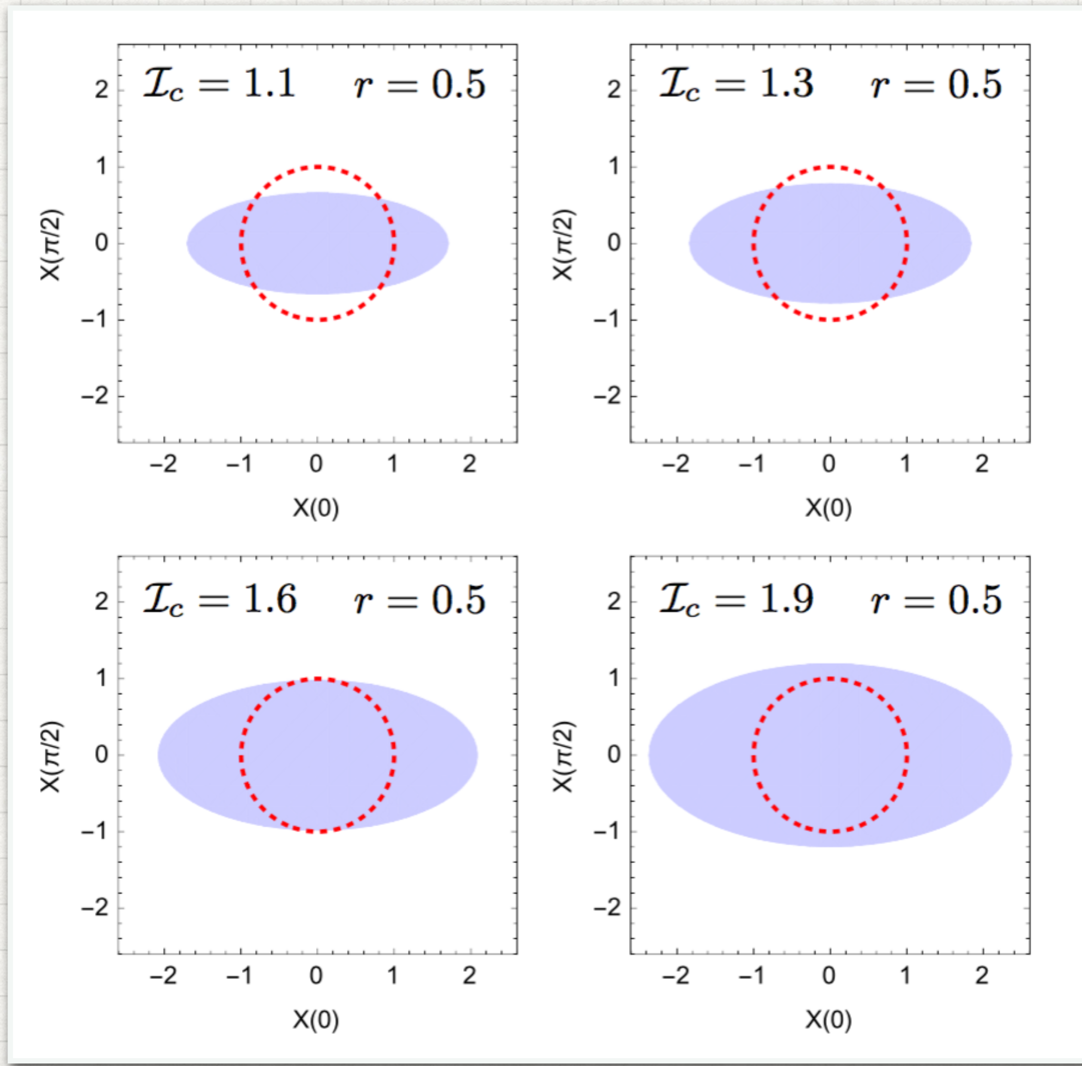
Maximum & minimum variance

$$\begin{aligned} V_{\max} &= e^{2r} + 4\mathcal{I}_c(\mathcal{I}_c - 1)(e^{r} - 1)^2, \\ V_{\min} &= e^{-2r} + 4\mathcal{I}_c(\mathcal{I}_c - 1)(e^{-r} - 1)^2. \end{aligned}$$

$$\mathcal{I}_c = \int_0^\infty \mathrm{d}\omega\ |g(\omega)|^2 \frac{e^{2\pi\omega/a}}{e^{2\pi\omega/a} - 1}$$

$$a \to 0, \quad \mathcal{I}_c \to 1$$

Accelerated single-mode squeezer



Red circle:
vacuum noise

Blue ellipse
variance of output state

Accelerated single-mode squeezer

non-unitary

Accelerated single-mode squeezer

non-unitary

$$\hat{a}'_k = \int \mathrm{d}\omega \, \left(A_{k\omega}\hat{c}'_\omega + B_{k\omega}\hat{d}'_\omega\right)$$

two sets of Unruh modes → one set of Minkowski modes

Accelerated single-mode squeezer

non-unitary

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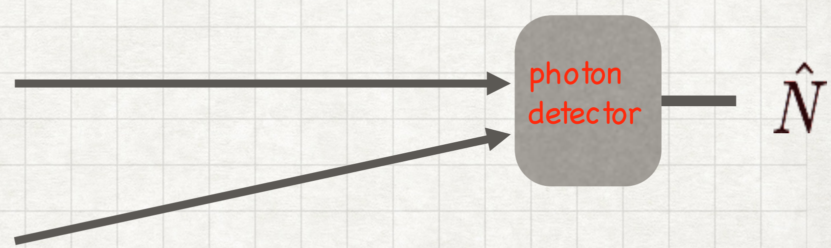

interference information lost

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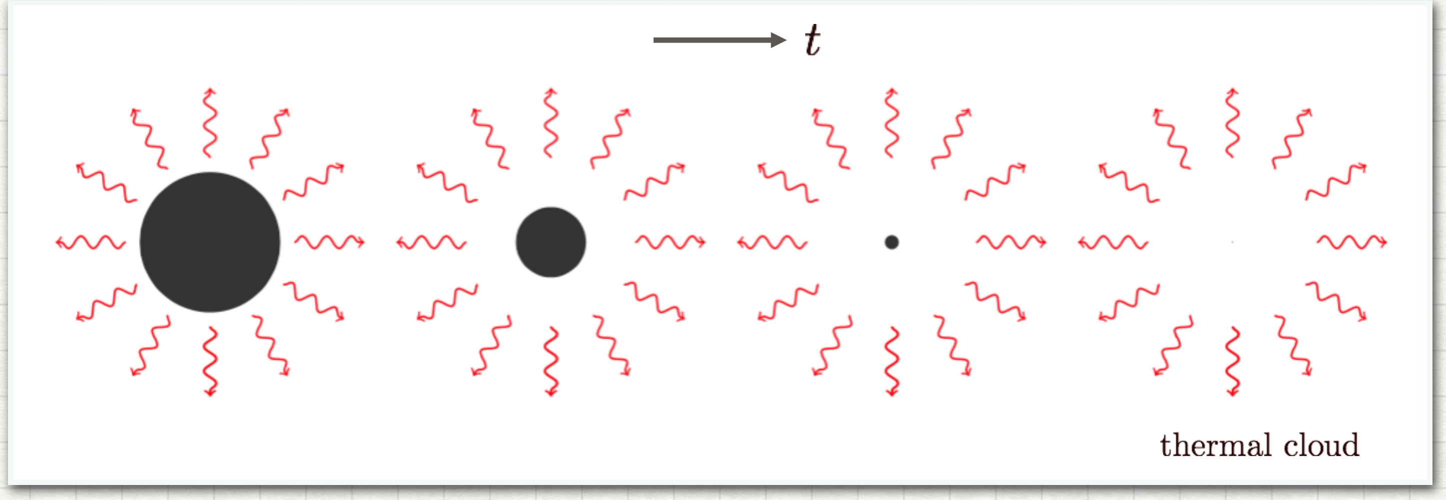
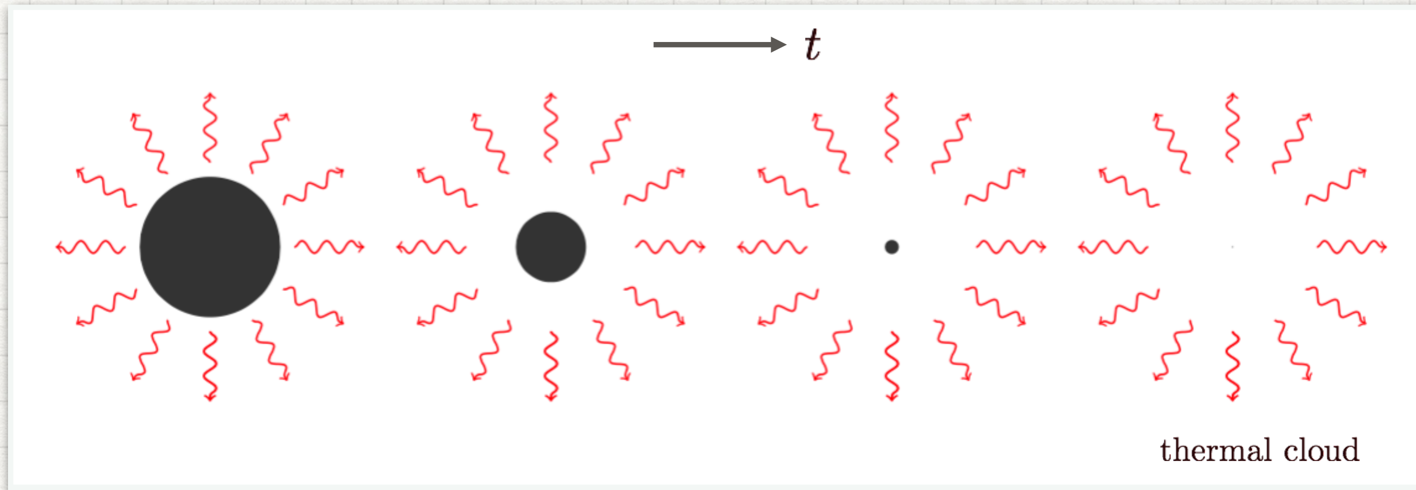
Black hole information paradox



Black hole information paradox



A pure initial state

black hole formation

black hole evaporation

A mixed final state

Unitary evolution is violated in the presence of gravity?

S. Hawking , Phys. Rev. D ***14****, 2460(1976)*

acknowledgements

Daiqin Su

Marco Ho

Rob Mann

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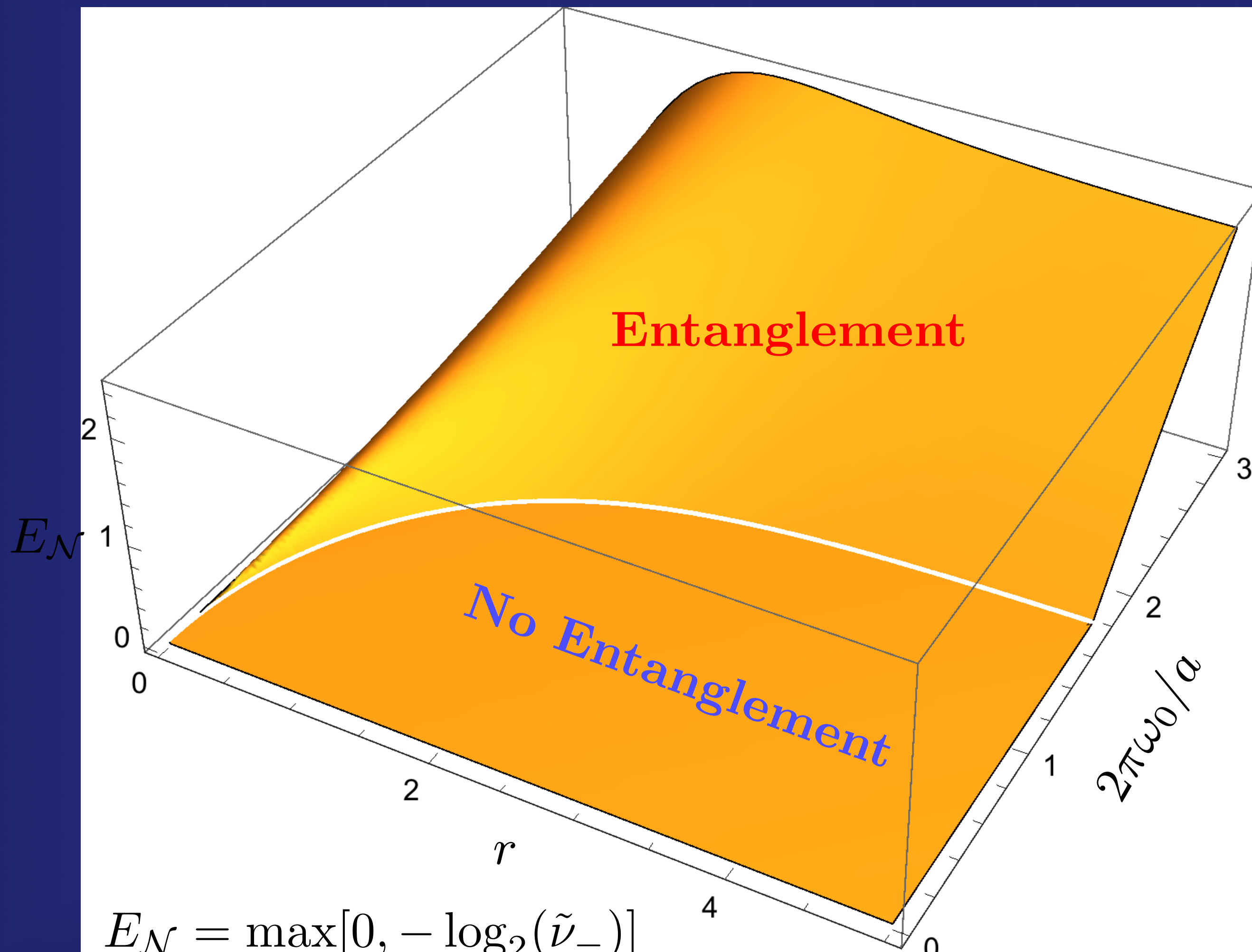
Decoherence of Entanglement



$$E_{\mathcal{N}} = \max[0, -\log_2(\tilde{\nu}_-)]$$

$$\tilde{\nu}_- = e^{-2r} + 4\mathcal{I}_c(\mathcal{I}_c - 1)(e^{-r} - 1)^2$$

Localised wave packet modes

- Localised wave packet modes

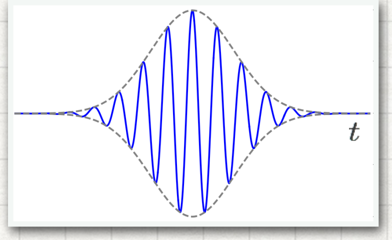
$$\boxed{\hat{b}_g^R = \int_0^\infty \mathrm{d}\omega \; g(\omega)\hat{b}_\omega^R}$$

$g(\omega)$ – localised wave packet

$$\int_0^\infty \mathrm{d}\omega \; |g(\omega)|^2 = 1$$

$$\Longrightarrow \quad [\hat{b}_g^R, \hat{b}_g^{R\dagger}] = 1$$

finite bandwidth localised in time



- Localised unitary operator $\hat{U}_g$

$$\boxed{\hat{b}_g^{R\prime} = \hat{U}_g^\dagger \hat{b}_g^R \hat{U}_g}$$

$g_{\perp i}(\omega)$ is orthogonal to $g(\omega)$

$$\int_0^\infty \mathrm{d}\omega \; g(\omega) g_{\perp i}^*(\omega) = 0$$

$$\boxed{\hat{b}_{g_{\perp i}}^R = \hat{U}_g^\dagger \hat{b}_{g_{\perp i}}^R \hat{U}_g}$$

$$\text{inverse relation}: \; \hat{b}_\omega^R = g^*(\omega)\hat{b}_g^R + \sum_i g_{\perp i}^*(\omega)\hat{b}_{g_{\perp i}}^R$$

- Transformation of single frequency modes

$$\boxed{\hat{b}_\omega^{R\prime} = \hat{b}_\omega^R + g^*(\omega)\left(\hat{U}_g^\dagger \hat{b}_g^R \hat{U}_g - \hat{b}_g^R\right)} \quad \longleftarrow$$

mixing of different frequency modes

Accelerated single-mode squeezer

Minimum variance

$$\frac{2\pi\omega_0}{a} = \ln\left(\frac{\sqrt{1+\coth(r/2)}+1}{\sqrt{1+\coth(r/2)}-1}\right)$$