Superposition and Grover algorithm in the presence of a closed timelike curve

Ki Hyuk Yee (U. of Seoul),
with
Jeongho Bang(Korea Institue of Advance of Science), Doyeol Ahn (U. of Seoul)

The Relativistic Quantum Information North 2017
July 4-7, 2017, Yukawa Institute for Theoretical Physics

No-Go Theorem

Oszmaniec, Grudka, Horodecki and Wojcik, PRL 116, 110403 (2016)

Theorem 1.—Let α, β be nonzero complex numbers satisfying $|\alpha|^2 + |\beta|^2 = 1$ and let $\dim \mathcal{H} \geq 2$. There exists no nonzero completely positive map $\Lambda \in \mathcal{CP}(\mathcal{H}^{\otimes 2}, \mathcal{H})$ such that for all pure states $\mathbb{P}_1$, $\mathbb{P}_2$

$$\Lambda(\mathbb{P}_1 \otimes \mathbb{P}_2) \propto |\Psi\rangle\langle\Psi|, \tag{3}$$

where

$$|\Psi\rangle = \alpha|\psi\rangle + \beta|\phi\rangle \tag{4}$$

and $|\psi\rangle\langle\psi| = \mathbb{P}_1$, $|\phi\rangle\langle\phi| = \mathbb{P}_2$ and the representatives $|\psi\rangle$, $|\phi\rangle$ may in general depend on both $\mathbb{P}_1$ and $\mathbb{P}_2$.

No-Go Theorem

Oszmaniec, Grudka, Horodecki and Wojcik, PRL 116, 110403 (2016)

Theorem 1.—Let α, β be nonzero complex numbers satisfying $|\alpha|^2 + |\beta|^2 = 1$ and let $\dim \mathcal{H} \geq 2$. There exists no nonzero completely positive map $\Lambda \in \mathcal{CP}(\mathcal{H}^{\otimes 2}, \mathcal{H})$ such that for all pure states $\mathbb{P}_1$, $\mathbb{P}_2$

$$\Lambda(\mathbb{P}_1 \otimes \mathbb{P}_2) \propto |\Psi\rangle\langle\Psi|, \tag{3}$$

where

$$|\Psi\rangle = \alpha|\psi\rangle + \beta|\phi\rangle \tag{4}$$

and $|\psi\rangle\langle\psi| = \mathbb{P}_1$, $|\phi\rangle\langle\phi| = \mathbb{P}_2$ and the representatives $|\psi\rangle$, $|\phi\rangle$ may in general depend on both $\mathbb{P}_1$ and $\mathbb{P}_2$.

Theorem 2.—Let $\mathbb{P}_\chi$ be a fixed pure state on Hilbert space $\mathcal{H}$. There exists a *CP* map $\Lambda_{\text{sup}} \in \mathcal{CP}(\mathbb{C}^2 \otimes \mathcal{H}^{\otimes 2}, \mathcal{H})$ such that for all pure states $\mathbb{P}_\psi$, $\mathbb{P}_\phi$ on $\mathcal{H}$ satisfying

$$\text{tr}(\mathbb{P}_\chi \mathbb{P}_\psi) = c_1, \qquad \text{tr}(\mathbb{P}_\chi \mathbb{P}_\phi) = c_2, \tag{9}$$

we have

$$\Lambda_{\text{sup}}(\mathbb{P}_\nu \otimes \mathbb{P}_\psi \otimes \mathbb{P}_\phi) \propto |\Psi\rangle\langle\Psi|, \tag{10}$$

where $\mathbb{P}_\nu$, $|\nu\rangle = \alpha|0\rangle + \beta|1\rangle$, is an unknown qubit state and the vector $|\Psi\rangle$ is given by $|\Psi\rangle = \alpha|\psi\rangle + \beta|\phi\rangle$

$$P_{\text{succ}} = \text{tr}[\Lambda_{\text{sup}}(\mathbb{P}_\nu \otimes \mathbb{P}_\psi \otimes \mathbb{P}_\phi)] = \frac{c_1 c_2}{c_1 + c_2} \mathcal{N}_\Psi^2$$

$\mathcal{N}_\Psi^2$: Normalization Constant

No-Go Theorem

Can we superpose two unknown states assisted by closed timelike curve?

Closed Timelike Curve

JULY 1, 1935 PHYSICAL REVIEW VOLUME 48

The Particle Problem in the General Theory of Relativity

A. EINSTEIN AND N. ROSEN, *Institute for Advanced Study, Princeton*

(Received May 8, 1935)

The writers investigate the possibility of an atomistic theory of matter and electricity which, while excluding singularities of the field, makes use of no other variables than the $g_{\mu\nu}$ of the general relativity theory and the φ_μ of the Maxwell theory. By the consideration of a simple example they are led to modify slightly the gravitational equations which then admit regular solutions for the static spherically symmetric case. These solutions involve the mathematical representation of physical space by a space of two identical sheets, a particle being represented by a "bridge" connecting these sheets. One is able to understand why no neutral particles of negative mass are to be found. The combined system of gravitational and electromagnetic equations are treated similarly and lead to a similar interpretation. The most natural elementary charged particle is found to be one of zero mass. The many-particle system is expected to be represented by a regular solution of the field equations corresponding to a space of two identical sheets joined by many bridges. In this case, because of the absence of singularities, the field equations determine both the field and the motion of the particles. The many-particle problem, which would decide the value of the theory, has not yet been treated.

VOLUME 61, NUMBER 13 PHYSICAL REVIEW LETTERS 26 SEPTEMBER 1988

Wormholes, Time Machines, and the Weak Energy Condition

Michael S. Morris, Kip S. Thorne, and Ulvi Yurtsever
Theoretical Astrophysics, California Institute of Technology, Pasadena, California 91125

(Received 21 June 1988)

It is argued that, if the laws of physics permit an advanced civilization to create and maintain a wormhole in space for interstellar travel, then that wormhole can be converted into a time machine with which causality might be violatable. Whether wormholes can be created and maintained entails deep, ill-understood issues about cosmic censorship, quantum gravity, and quantum field theory, including the question of whether field theory enforces an averaged version of the weak energy condition.

PACS numbers: 04.60.+n, 03.70.+k, 04.20.Cv

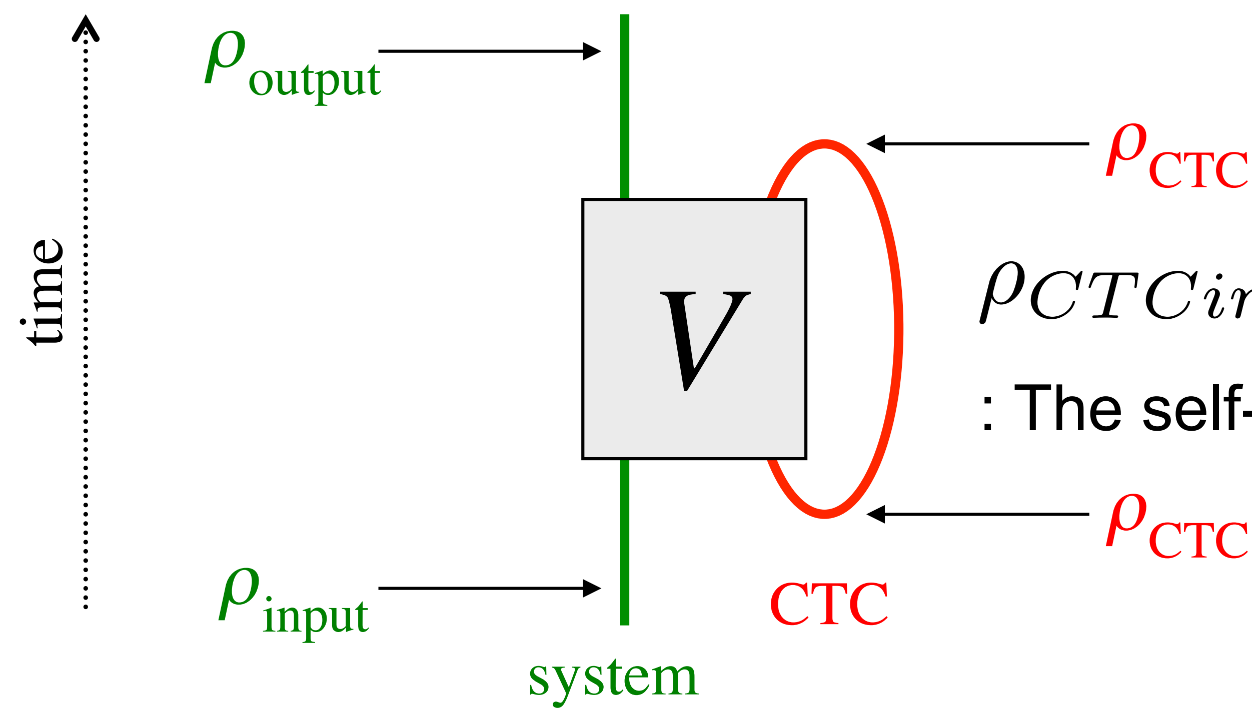
- Closed timelike curves (CTCs) : space time objects allowed by general relativity theory
- Recent works have shown CTCs enhance tasks
  - Solving NP-complete problems, the problem SAT

    Bacon, PRA 70, 032309 (2004).
  - Distinguishing arbitrary states Brun, Harrington and Wilde, PRL 102, 210402 (2009).
  - Unknown state cloning Ahn, Myers, Ralph and Mann, PRA 88, 022332 (2013).
  - Photonic simulation of the self-consistency condition

    Ringbauger, Broome, Myers, White and Ralph, Nat.Commun, 5, 2145 (2014).

Deutsch's Closed Timelike Curve (D-CTC)

D. Deutsch, PRD 44, 3197 (1991)



$$\rho_{CTCinput} = \rho_{CTCoutput}$$

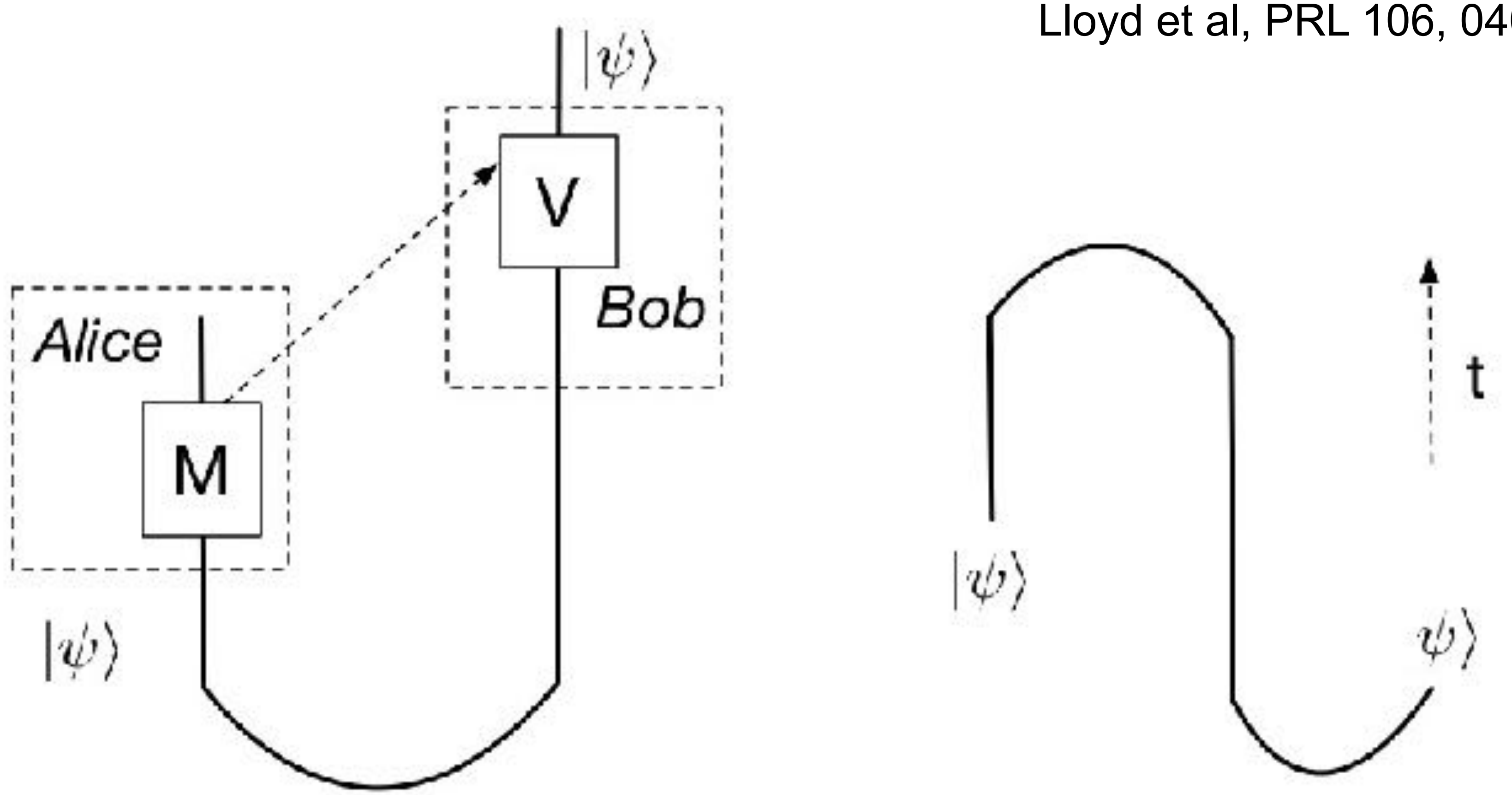
: The self-consistency condition

$$\rho_{CTCoutput} = \mathrm{Tr}_{system}[V(\rho_{input} \otimes \rho_{CTCinput})V^\dagger]$$

$$\rho_{output} = \mathrm{Tr}_{CTC}[V(\rho_{input} \otimes \rho_{CTC})V^\dagger]$$

Postselected Closed Timelike Curve (P-CTC)

Lloyd et al, PRL 106, 040403 (2011)

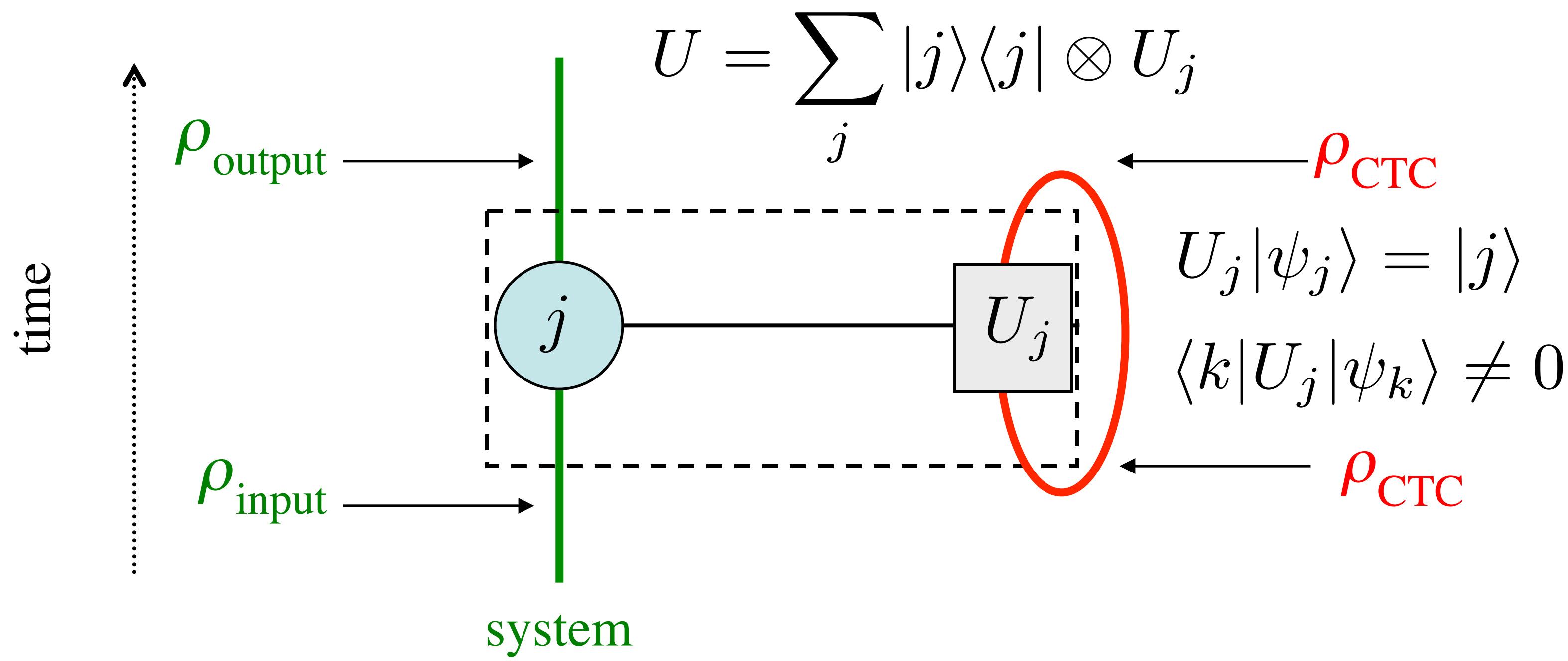


- A different approach to describing QM with CTCs invented by Bennett and Schumacher (never published)
- This approach based on *teleportation*.

- If guaranteed to postselect with certainty the outcomes of a measurement, one could teleport a copy of a state into the past.

Distinguishing nonorthogonal states

Brun, Harrington and Wilde, PRL 102, 210402 (2009).



Required to satisfy self-consistency condition

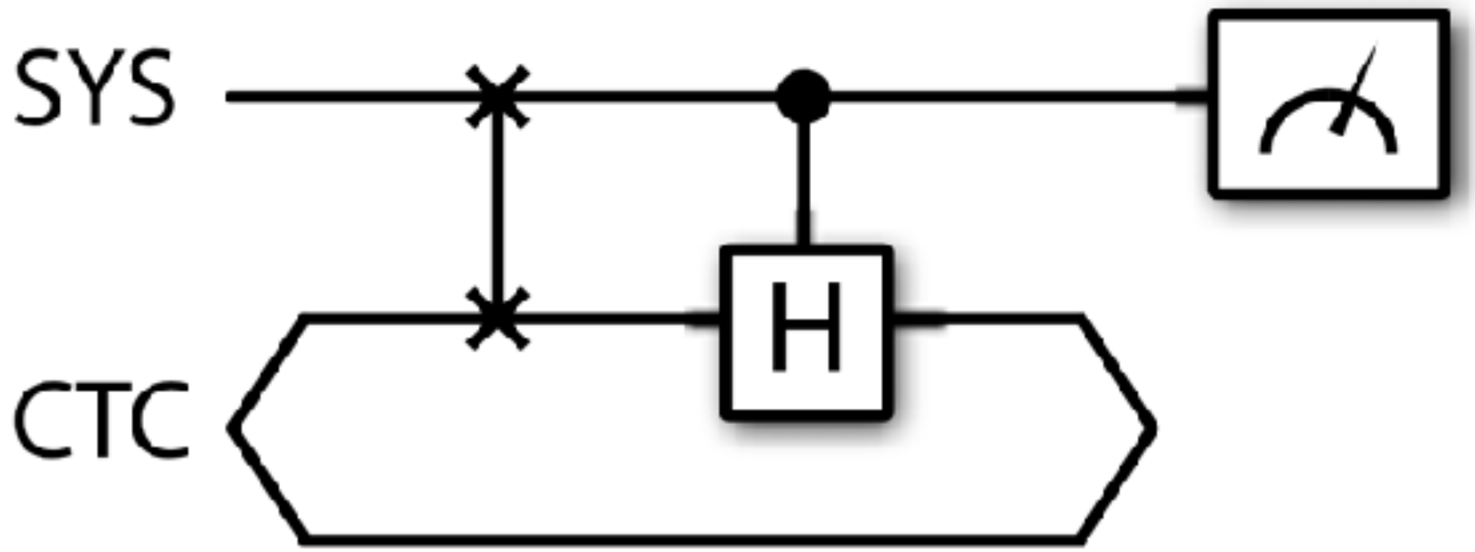
We can implement the following map

$$\forall j\ |\psi_j\rangle \rightarrow |j\rangle$$

$$|\psi_j\rangle\langle\psi_j| \otimes |j\rangle\langle j| (= \rho_{CTC}) \rightarrow SWAP \rightarrow |j\rangle\langle j| \otimes |\psi_j\rangle\langle\psi_j| \rightarrow U \rightarrow |j\rangle\langle j| \otimes |j\rangle\langle j|$$

Distinguishing nonorthogonal states

Brun and Wilde, Found Phys 42, 341 (2012)



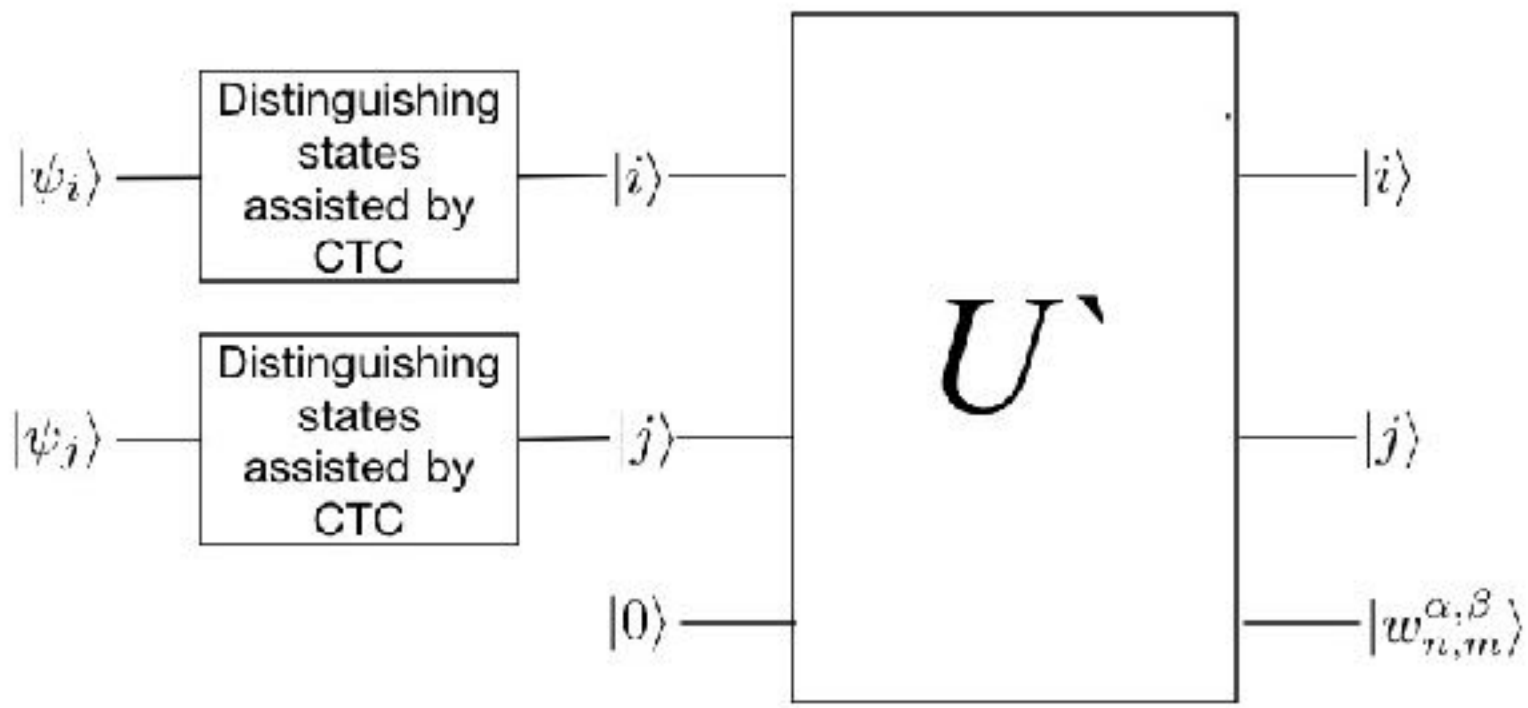
A P-CTC-assisted circuit that can distinguish.

Image credit : Brun and Wilde, Found Phys 42, 341 (2012)

- P-TCT also allows us to distinguish nonorthogonal states (The same circuit works as with DCTCs)

- However, P-CTC can only distinguish sets of *linearly independent* states.

Superposing two unknown states



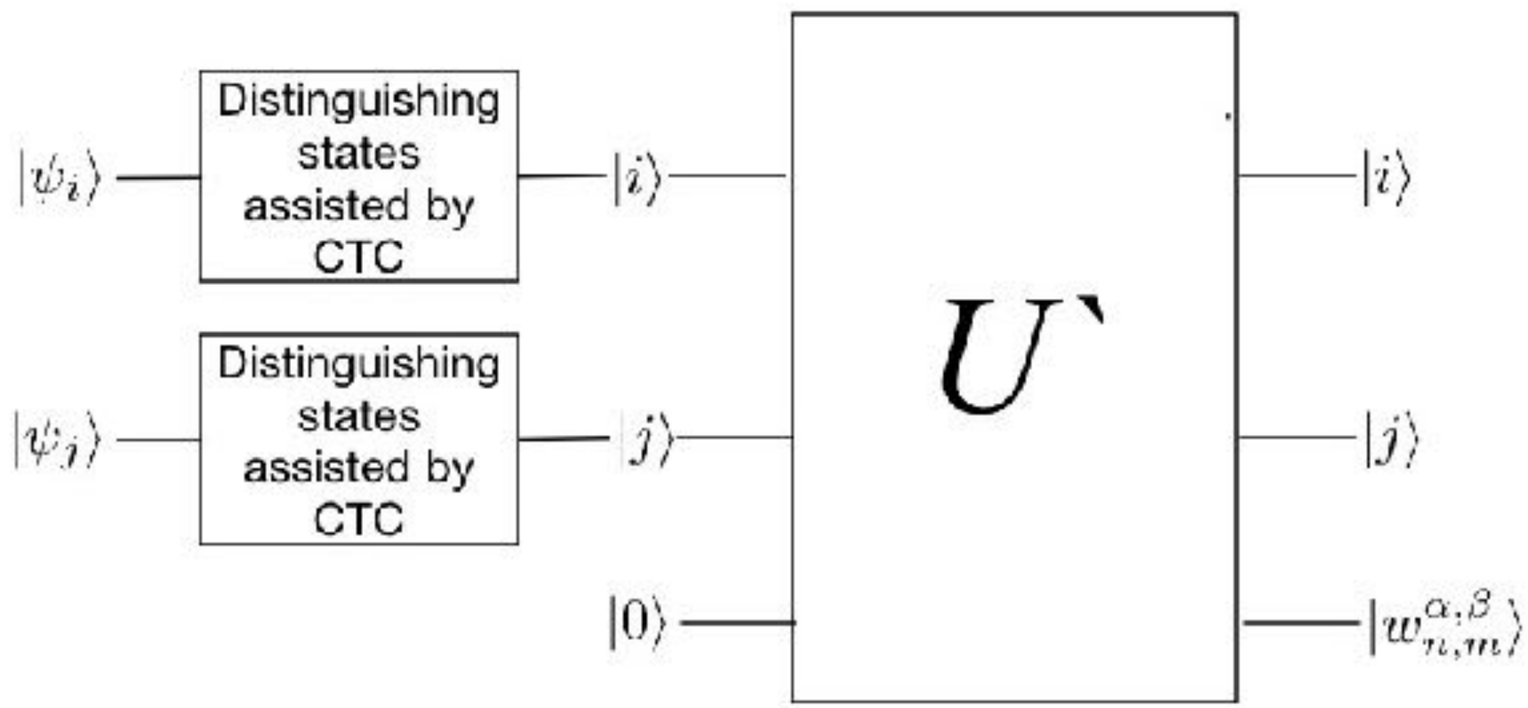
$$U' = \sum_{n,m=0}^{N-1} |n\rangle\langle n| \otimes |m\rangle\langle m| \otimes U_{\alpha,\beta}^{n,m}$$

$$U_{\alpha,\beta}^{n,m}|0\rangle = |w_{\alpha,\beta}^{n,m}\rangle = \alpha|\psi_n\rangle + \beta|\psi_n\rangle$$

$U_{\alpha,\beta}^{n,m}$ can be constructed by Gram Schmidt process on the set $S = |w_{\alpha,\beta}^{n,m}\rangle \cup \{|\psi_n\rangle\}_{n=0}^{N-1}$

- Using D-CTCs, superposing two unknown states is possible

Superposing two unknown states



$$U' = \sum_{n,m=0}^{N-1} |n\rangle\langle n| \otimes |m\rangle\langle m| \otimes U_{\alpha,\beta}^{n,m}$$

$$U_{\alpha,\beta}^{n,m}|0\rangle = |w_{\alpha,\beta}^{n,m}\rangle = \alpha|\psi_n\rangle + \beta|\psi_m\rangle$$

$U_{\alpha,\beta}^{n,m}$ can be constructed by Gram Schmidt process on the set $S = |w_{\alpha,\beta}^{n,m}\rangle \cup \{|\psi_n\rangle\}_{n=0}^{N-1}$

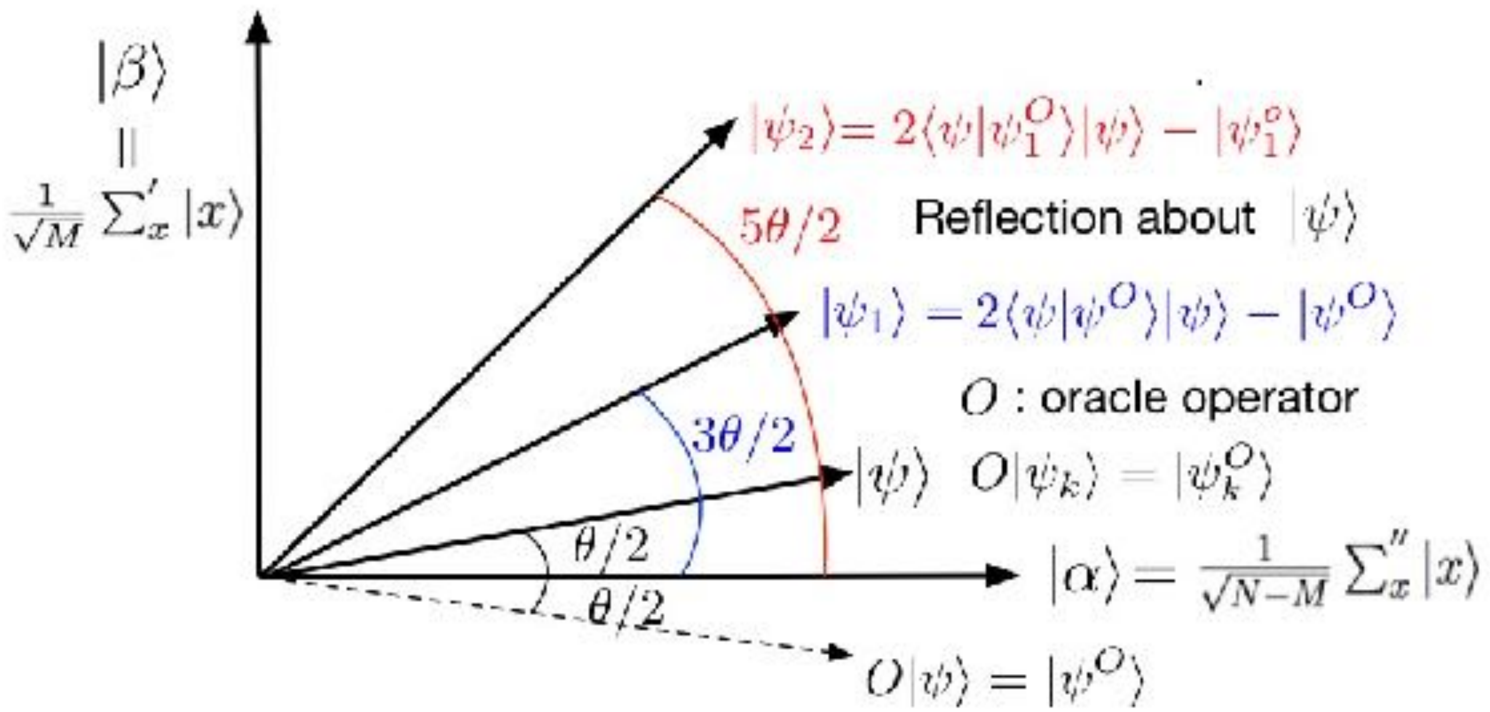
- Using D-CTCs, superposing two unknown states is possible
- Using P-CTC, superposing two unknown states in the set of *linearly independent states* is possible.

Superposing two unknown states

- What can we do if superposing two unknown states is possible?

No Superposition Theorem and Grover Algorithm

- Standard Grover Algorithm



- After k iterations

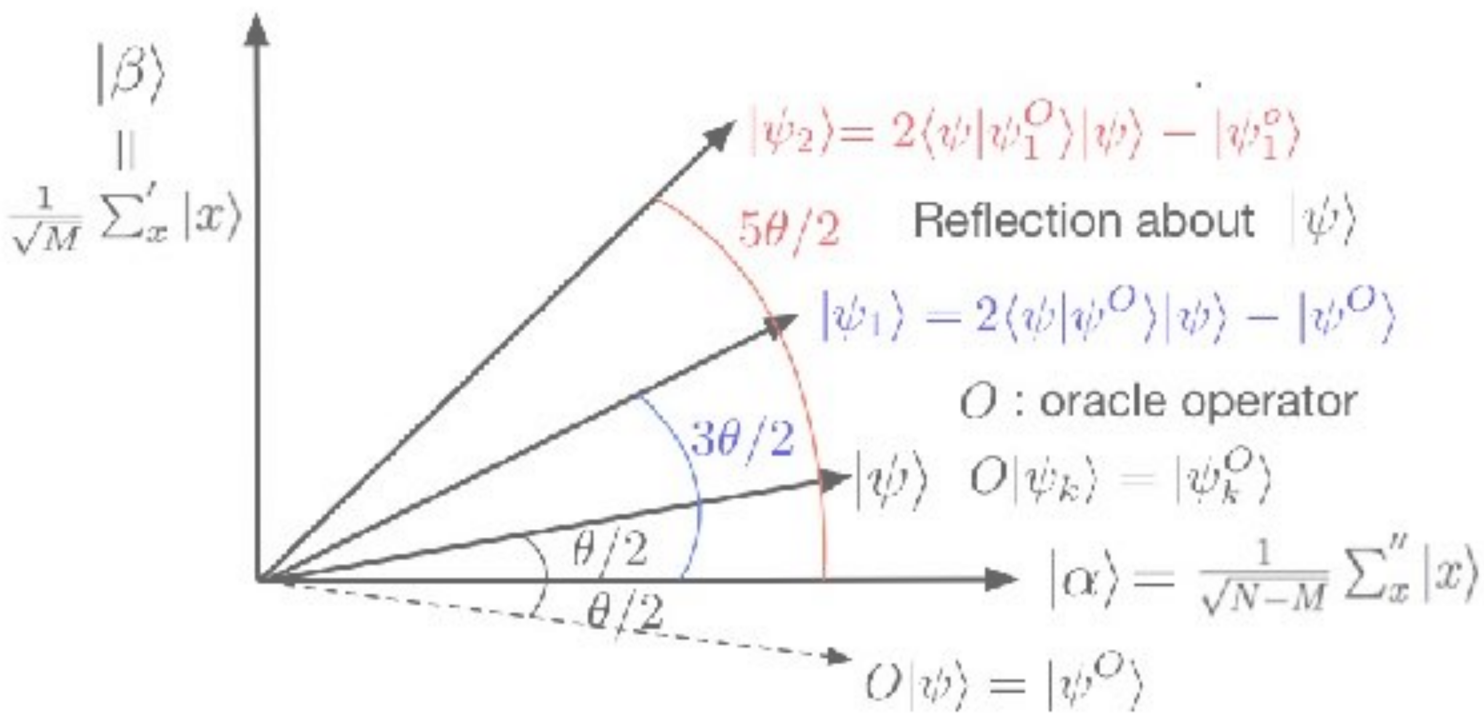
$$|\psi_k\rangle = 2\langle\psi|\psi_{k-1}^O\rangle|\psi\rangle - |\psi_{k-1}^O\rangle = \cos\frac{(2k+1)\theta}{2}|\alpha\rangle + \sin\frac{(2k+1)\theta}{2}|\beta\rangle$$

- Total number of Iteration

$R = \mathcal{O}(\sqrt{\frac{N}{M}})$ N : # of elements in data base
M : # of solutions of the search problem

No Superposition Theorem and Grover Algorithm

- Standard Grover Algorithm



- After k iterations

$$|\psi_k\rangle = 2\langle\psi|\psi_{k-1}^O\rangle|\psi\rangle - |\psi_{k-1}^O\rangle = \cos\frac{(2k+1)\theta}{2}|\alpha\rangle + \sin\frac{(2k+1)\theta}{2}|\beta\rangle$$

- Total number of Iteration

$R = \mathcal{O}(\sqrt{\frac{N}{M}})$ N : # of elements in data base
M : # of solutions of the search problem

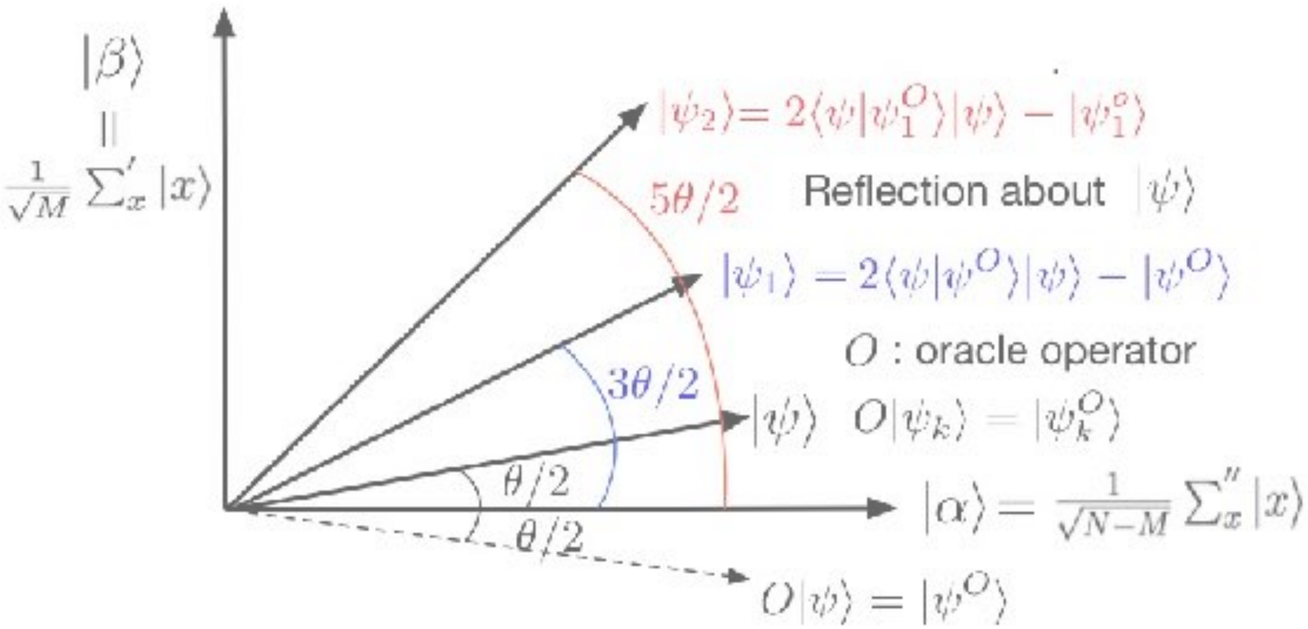
- Can we do better?

Answer is negative

- C. H. Bennett, E. Bernstein, G. Brassard, and U. Vazirani, SIAM J. Comput. 26, 15101524 (1997)

No Superposition Theorem and Grover Algorithm

- Standard Grover Algorithm



- After k iterations

$$|\psi_k\rangle = 2\langle\psi|\psi_{k-1}^O\rangle|\psi\rangle - |\psi_{k-1}^O\rangle = \cos\frac{(2k+1)\theta}{2}|\alpha\rangle + \sin\frac{(2k+1)\theta}{2}|\beta\rangle$$

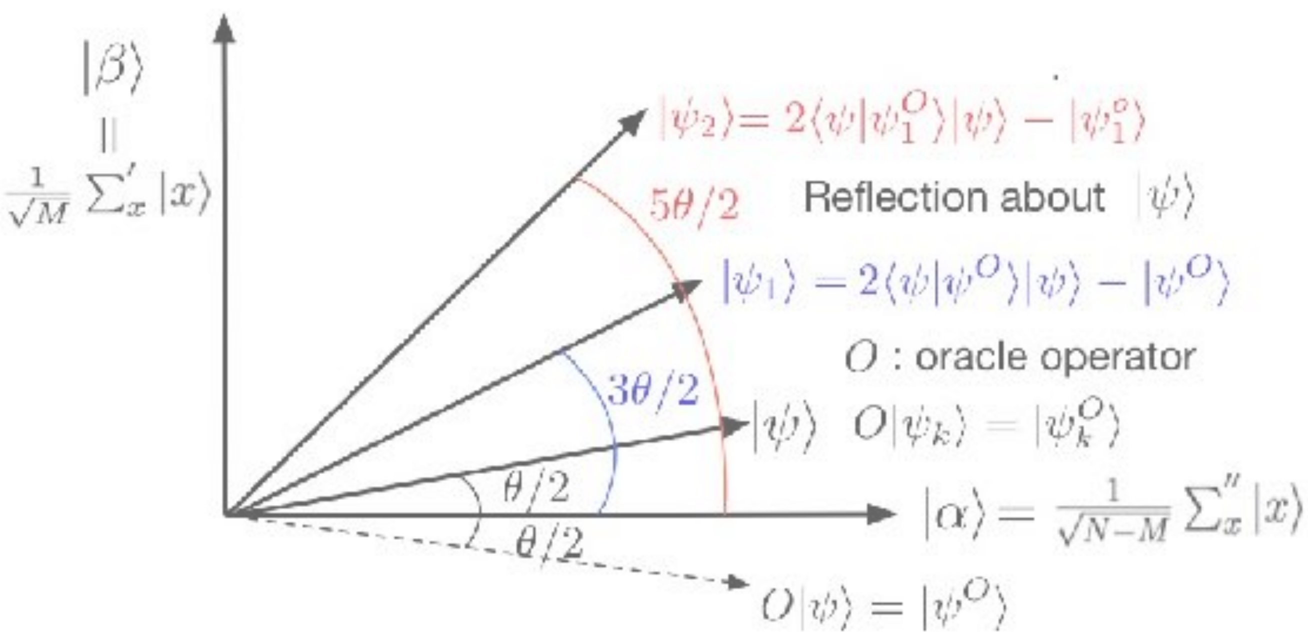
- Total number of Iteration

$R = \mathcal{O}(\sqrt{\frac{N}{M}})$ N : # of elements in data base
M : # of solutions of the search problem

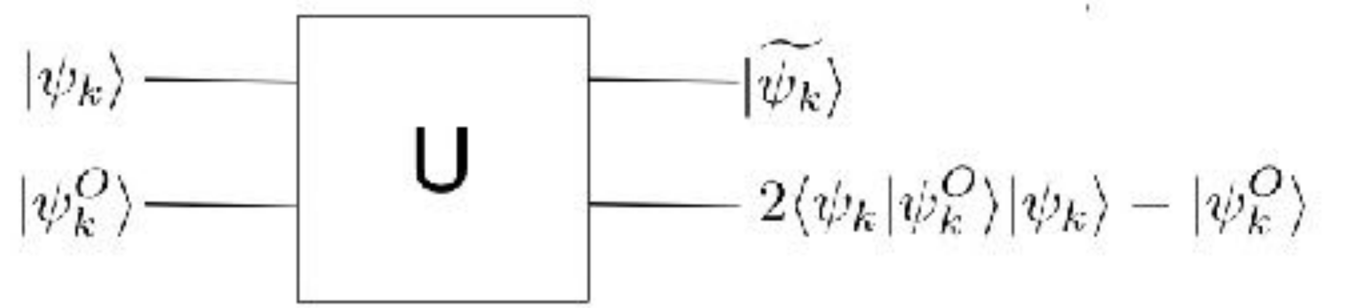
- What if superposition state $2\langle\psi_k|\psi_k^O\rangle|\psi_k\rangle - |\psi_k^O\rangle$ created from two unknown states $|\psi_k\rangle$ and $|\psi_k^O\rangle$ assisted by CTC is possible?

No Superposition Theorem and Grover Algorithm

- Standard Grover Algorithm



- What if superposition state $2\langle\psi_k|\psi_k^O\rangle|\psi_k\rangle - |\psi_k^O\rangle$ created from two unknown states $|\psi_k\rangle$ and $|\psi_k^O\rangle$ assisted by CTC is possible?



Exponential speed up possible!

Kumar and Paraoanu, EPL, 93, 20005, 2011

- After k iterations

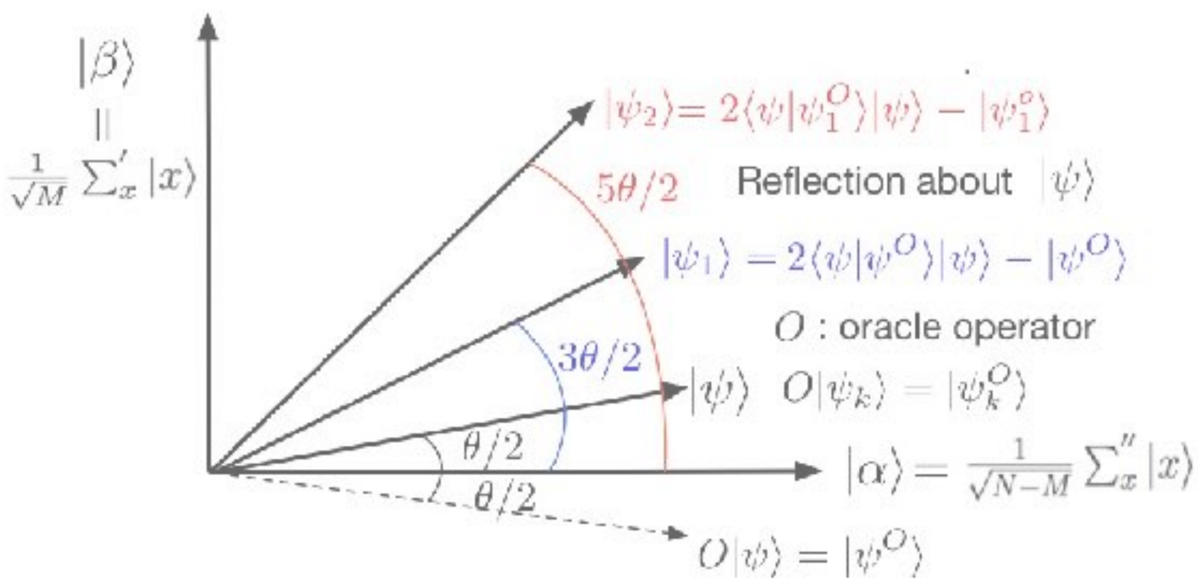
$$|\psi_k\rangle = 2\langle\psi|\psi_{k-1}^O\rangle|\psi\rangle - |\psi_{k-1}^O\rangle = \cos\frac{(2k+1)\theta}{2}|\alpha\rangle + \sin\frac{(2k+1)\theta}{2}|\beta\rangle$$

- Total number of Iteration

$R = \mathcal{O}(\sqrt{\frac{N}{M}})$ N : # of elements in data base
M : # of solutions of the search problem

No Superposition Theorem and Grover Algorithm

- Standard Grover Algorithm



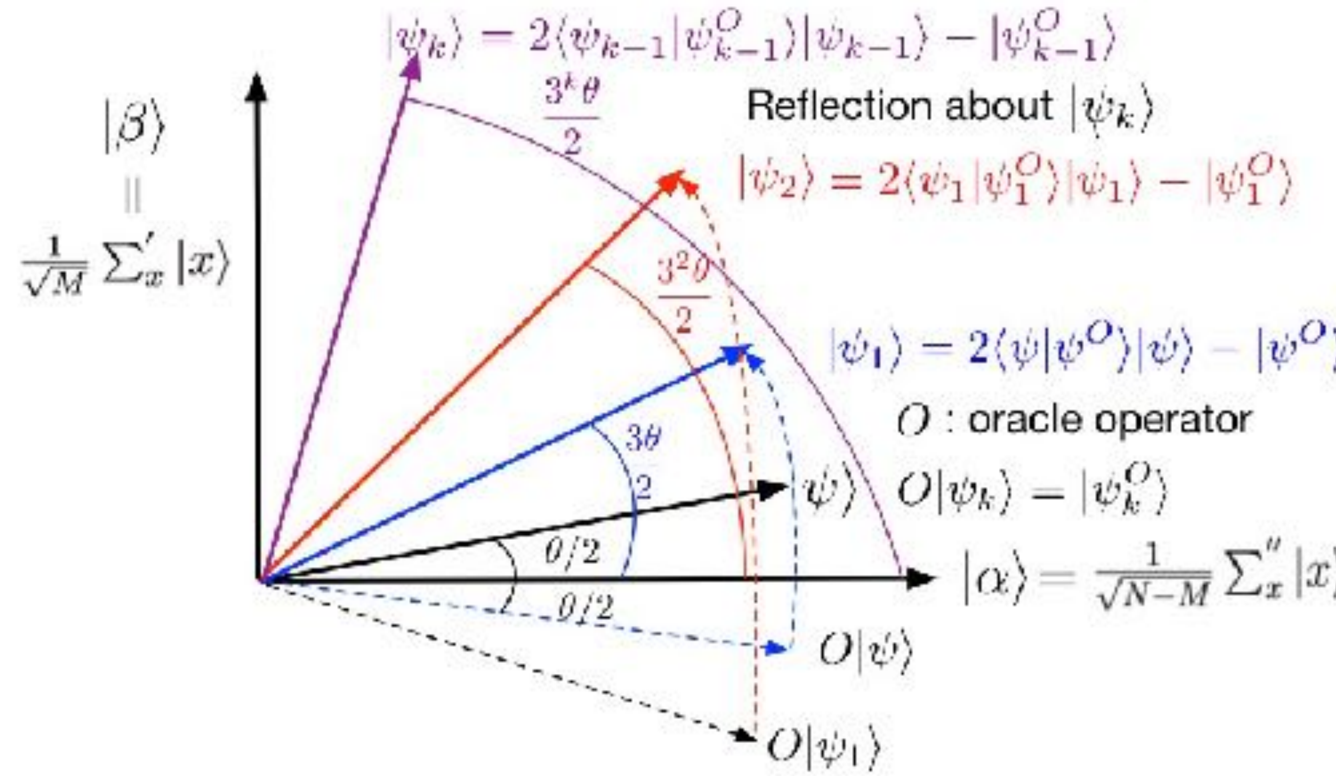
- After k iterations

$$|\psi_k\rangle = \cos\frac{(2k+1)\theta}{2}|\alpha\rangle + \sin\frac{(2k+1)\theta}{2}|\beta\rangle$$

- Total number of Iteration

$R = \mathcal{O}(\sqrt{\frac{N}{M}})$ N : # of elements in data base
M : # of solutions of the search problem

- Grover Algorithm if $2\langle\psi_k|\psi_k^O\rangle|\psi_k\rangle - |\psi_k^O\rangle$ can be created by superposing $|\psi_k\rangle$ and $|\psi_k^O\rangle$



- After k iterations

$$|\psi_k\rangle = \cos\frac{3^k\theta}{2}|\alpha\rangle + \sin\frac{3^k\theta}{2}|\beta\rangle$$

- Total number of Iteration

$$R_{\text{mod}} = \mathcal{O}\left(\log_3\sqrt{\frac{N}{M}}\right)$$

Exponential reduction in # of iteration!

Conclusion

- We can show that the superposition of two unknown states is possible assisted by CTC.
- If the superposition of two unknown states is possible assisted by CTC,

  the exponential speed up of Grover search algorithm could be possible

Thank you for your attention!