ニュートリノから余次元理論超対称性理論、大統一理論へ

三者若手2003年夏の学校
(東京, 8/18-8/23, 2003)

波場直之（徳島大）

Plan of talk

0. Introduction
1. Standard Model
2. Beyond the SM
   2-1. extra dimensional theory, 2-2. SUSY
3. **ニュートリノ ν**
4. flavor&**質量階層** (**世代**) **構造**
   (quark,lepton**系の違いは何故？**)
5. **大統一理論** (GUT)
6. flavor&**質量階層** (**世代**) **構造　（その２）**
7. Big Questions
   7-1. **世代？**　7-2. 4**次元？**　7-3. **宇宙項？**
8. **素晴らしき未来へ**

0.Introduction

私事で恐縮ですが･･･

物理は既に宇宙始め１０$^{-43}$秒（M_{pl}）まで分かってしまったのです。それは、超対称性大統一理論、11次元超重力理論、10次元の超弦理論です。

(12, 3年前の科学雑誌)

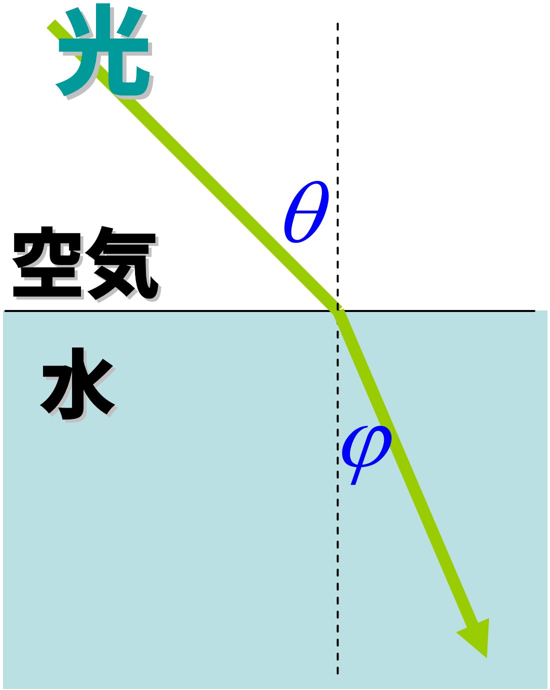
素粒子物理を目指すきっかけになった。。

→勉強してみたい。→･･･でも本当？？？

特に世代の謎について解明したい！！

素粒子物理：物質や力の粒子は何から出来ているのだろうか？ より基本的な物理法則の探求

例えば、光の屈折を考えてみませう。

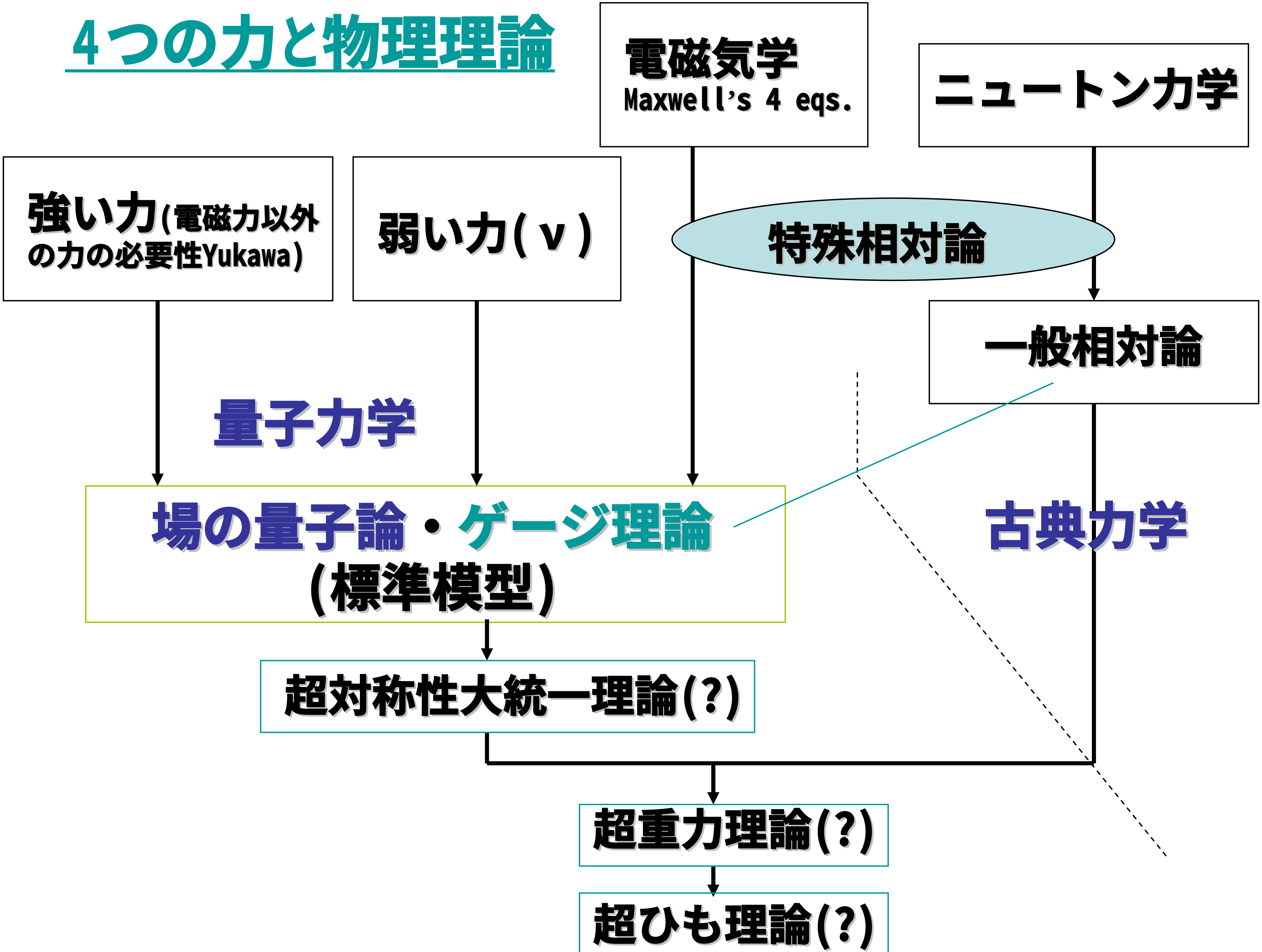


$$n = \frac{\sin\theta}{\sin\varphi} = \frac{c_{air}}{c_{water}}$$

フェルマーの定理
（光は最短時間経路を取る）
停留条件(1階微分)から簡単に導かれる

⇒量子電磁気学(QED)
U(1) gauge theory

4つの力と物理理論



電磁気学
Maxwell’s 4 eqs.

ニュートン力学

強い力(電磁力以外の力の必要性Yukawa)

弱い力(ν)

特殊相対論

一般相対論

量子力学

場の量子論・ゲージ理論
(標準模型)

古典力学

超対称性大統一理論(?)

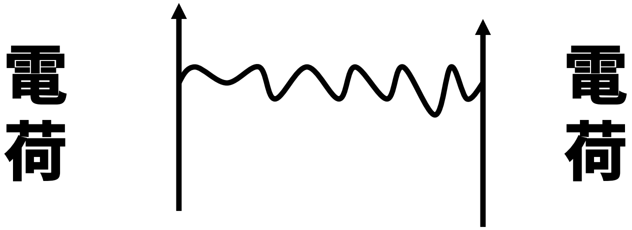
超重力理論(?)

超ひも理論(?)

ゲージ理論 力を伝えるgauge粒子⇒massless

光子 (γ)

massless 1つ

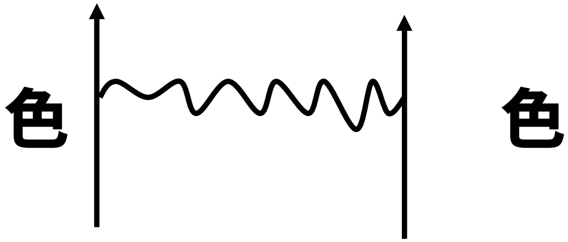


電磁相互作用

$U(1)_{em}$

グルーオン (G)

massless 8つ
confinement

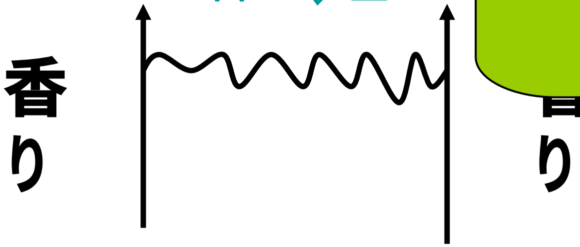


強い力

$SU(3)_C$

$W^{\pm}$、Z

massive 3つ
Higgs⇔対称性の
自発的破れ

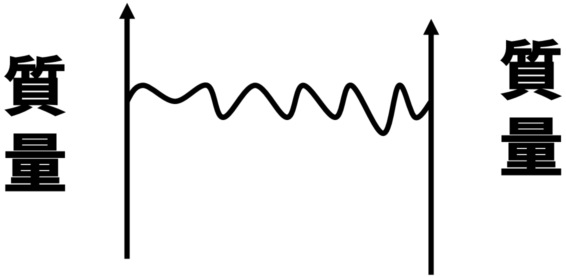


弱い力

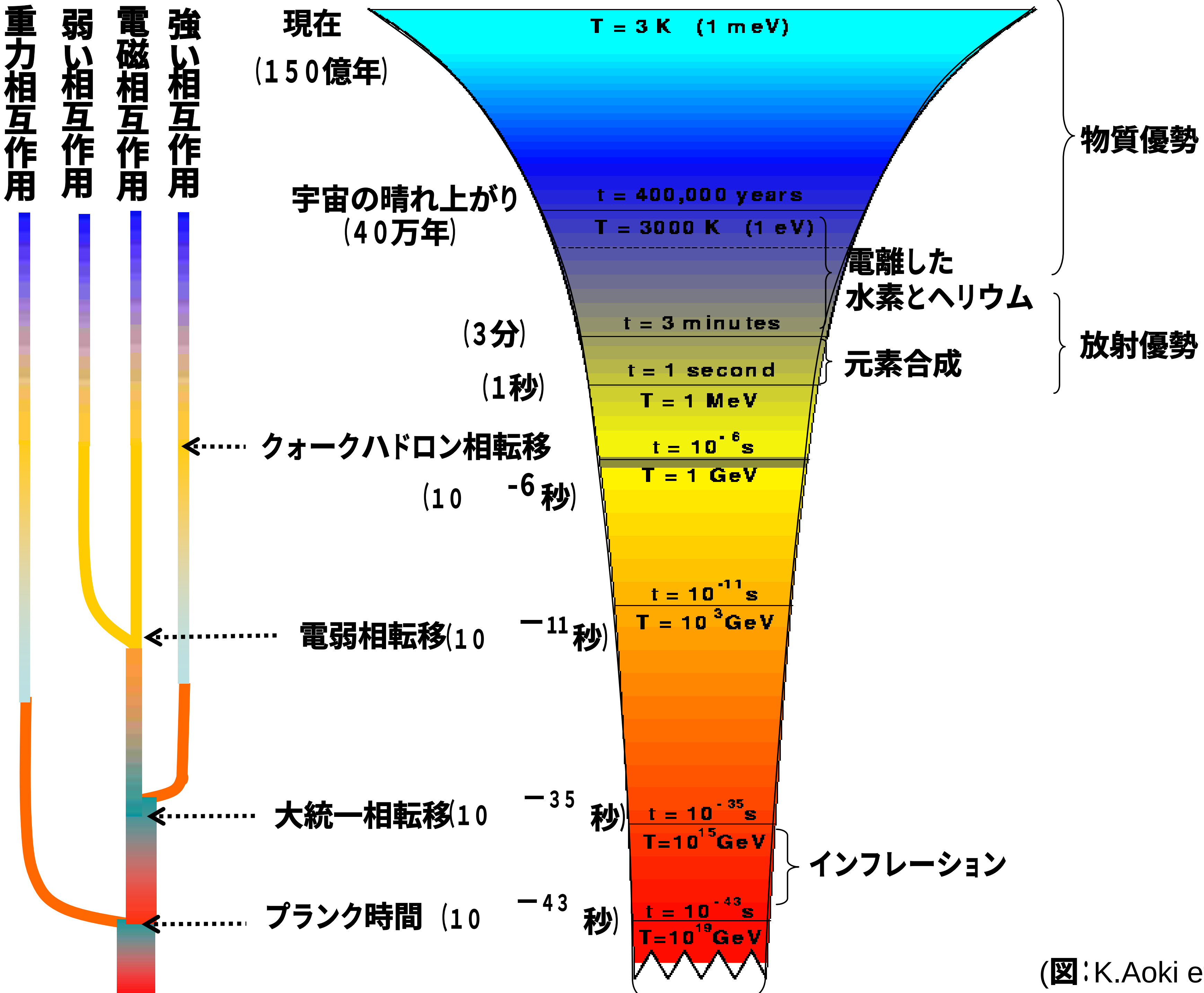
$SU(2)_L$

massless:
spin2

グラビトン (g)



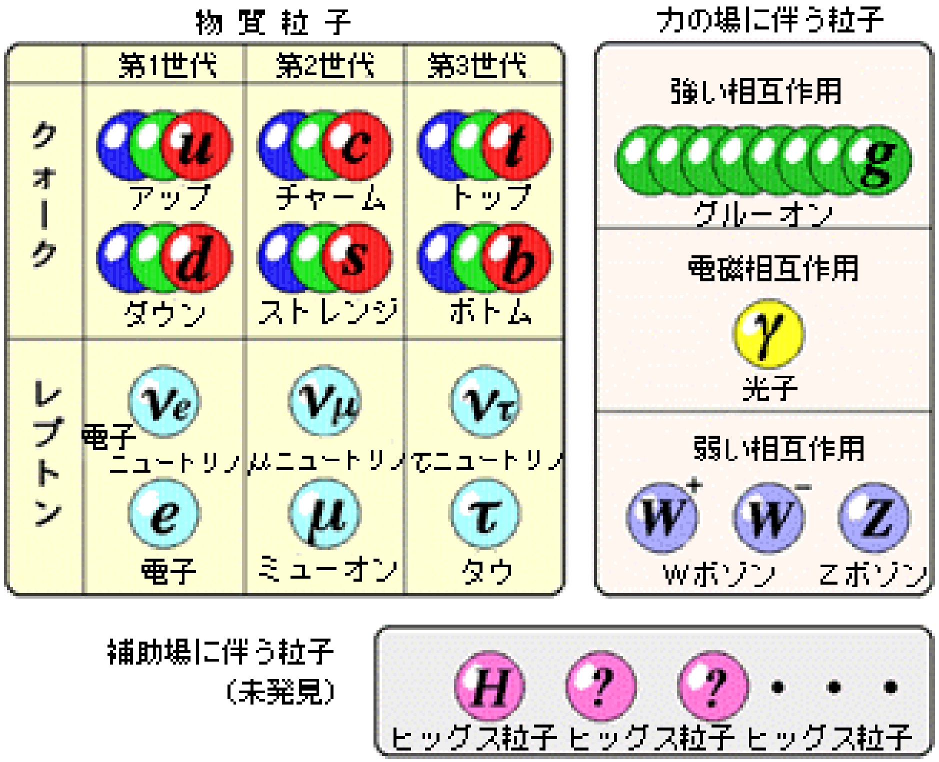
重力


重力相互作用
弱い相互作用
電磁相互作用
強い相互作用
現在
(150億年)
T = 3 K (1 meV)
物質優勢
宇宙の晴れ上がり
(40万年)
t = 400,000 years
T = 3000 K (1 eV)
電離した
水素とヘリウム
t = 3 minutes
(3分)
放射優勢
t = 1 second
元素合成
(1秒)
T = 1 MeV
クォークハドロン相転移
t = 10-6s
T = 1 GeV
(10-6秒)
t = 10-11s
T = 103GeV
電弱相転移(10−11秒)
大統一相転移(10−35秒)
t = 10-35s
T=1015GeV
インフレーション
プランク時間 (10−43秒)
t = 10-43s
T=1019GeV

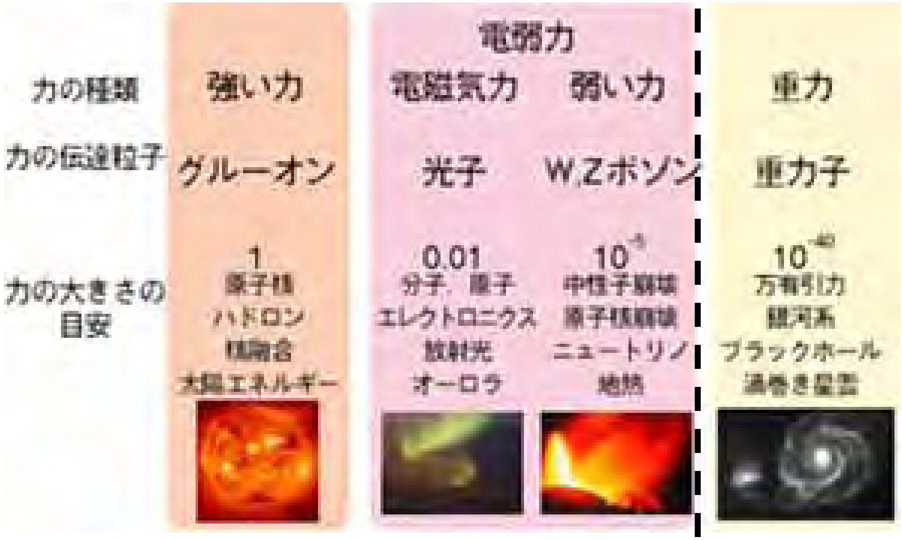
(図：K.Aoki et al)

1.Standard Model

O(100)GeV以下の素粒子物理をほぼ完璧に記述！



現在の素粒子像「標準模型」の世界



	電弱力			
力の種類	強い力	電磁気力	弱い力	重力
力の伝達粒子	グルーオン	光子	W,Zボソン	重力子
力の大きさの目安	1 原子核 ハドロン 核融合 太陽エネルギー	0.01 分子、原子 エレクトロニクス 放射光 オーロラ	10^{-5} 中性子崩壊 原子核崩壊 ニュートリノ 地熱	10^{-40} 万有引力 銀河系 ブラックホール 渦巻き星雲

重力？

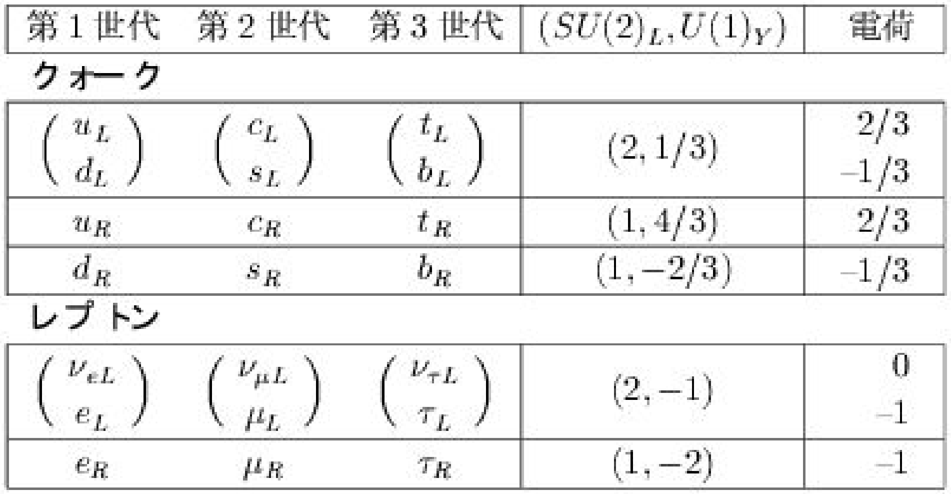
(KEK HP http://ccwww.kek.jp/public/greet/index.htmlから抜粋)

物質: spin1/2：quark & lepton

質量固有状態とweak int.
固有状態のミスマッチ
→quark_flavor **混合** (V_{CKM})

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$$

ν：massless (only left-handed)
→no lepton flavor**混合** (~~V_{MNS}~~)



第1世代	第2世代	第3世代	$(SU(2)_L, U(1)_Y)$	電荷
クォーク				
$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$\begin{pmatrix} c_L \\ s_L \end{pmatrix}$	$\begin{pmatrix} t_L \\ b_L \end{pmatrix}$	$(2, 1/3)$	$2/3$ $-1/3$
u_R	c_R	t_R	$(1, 4/3)$	$2/3$
d_R	s_R	b_R	$(1, -2/3)$	$-1/3$
レプトン				
$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$	$(2, -1)$	0 -1
e_R	μ_R	τ_R	$(1, -2)$	-1

(Kurimoto's lecture note)

Standard Model Lagrangian (renormalizable!)

$$L = L_{gauge} + L_{fermion} + L_{Higgs}$$

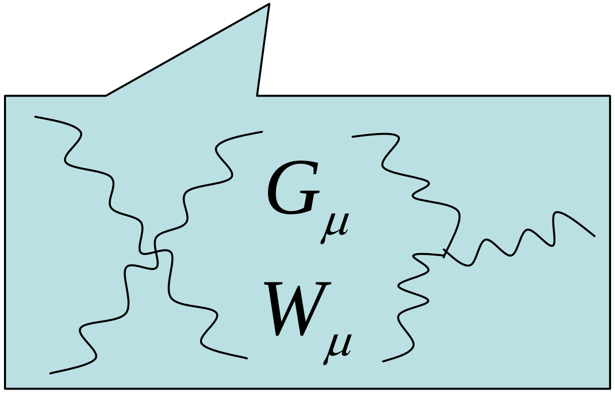
gauge sector:

$$L_{gauge} = -\frac{1}{2} Tr(G^{\mu\nu} G_{\mu\nu}) - \frac{1}{2} Tr(W^{\mu\nu} W_{\mu\nu}) - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}$$

“強い力” $SU(3)_C : \; G_{\mu\nu} = \partial_\mu G_\nu - \partial_\nu G_\mu \underline{- i g_3 [G_\mu, G_\nu]}$

“弱い力” $SU(2)_L : \; W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu \underline{- i g_2 [W_\mu, W_\nu]}$

“電磁力” $U(1)_Y : \; F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$



fermion sector: $L_{fermion} = L_{Kin.} + L_{Yukawa}$

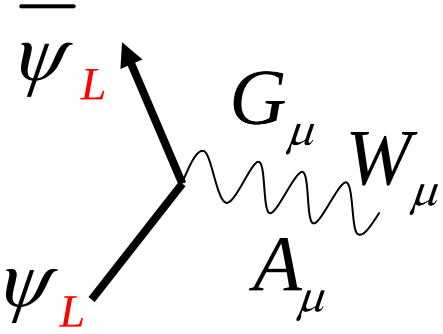
カイラル表示

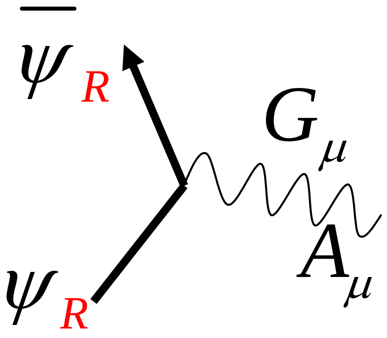
$$\psi_L = \frac{1-\gamma^5}{2}\psi, \quad \psi_R = \frac{1+\gamma^5}{2}\psi \qquad \gamma^5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \left(\frac{\text{Left}}{\text{Right}} \right) \gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$$

$$L_{Kin.} = i\overline{\psi}_L \gamma_\mu D^\mu \psi_L + i\overline{\psi}_R \gamma_\mu D^\mu \psi_R$$

$$D_\mu = \partial_\mu - ig_3 T_3^\alpha G_\mu^\alpha - ig_2 T_2^\alpha W_\mu^\alpha - ig_1 (Y/2) B_\mu$$

$$D_\mu = \partial_\mu - ig_3 T_{3*}^\alpha G_\mu^\alpha + ig_1 (Y/2) B_\mu$$





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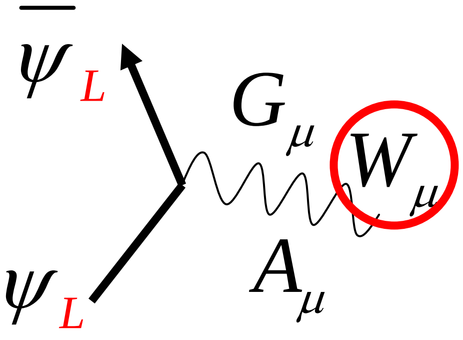
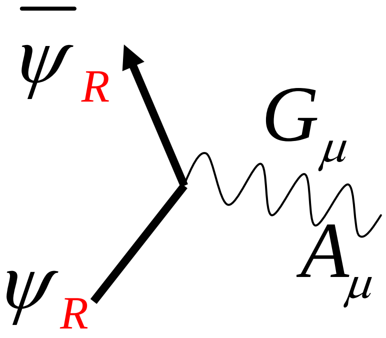
$$\begin{pmatrix} u \\ d \end{pmatrix}_L \begin{pmatrix} c \\ s \end{pmatrix}_L \begin{pmatrix} t \\ b \end{pmatrix}_L$$

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$$





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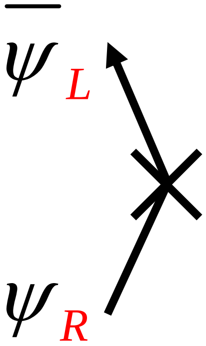
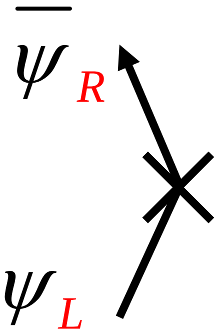
$$L_{mass} = -\underset{\searrow\ y\langle\phi\rangle}{\underline{m}}\ \bar{\psi}_L \psi_R - \underset{\searrow\ y^*\langle\phi^\dagger\rangle}{\underline{m^*}}\ \bar{\psi}_R \psi_L$$





Higgsの導入!

SU(2)$_L$のdoublet

$$\langle\phi\rangle = \begin{pmatrix} \langle\phi^0\rangle \\ \phi^- \end{pmatrix}$$

$$Q_Y(\phi) = -1$$

fermion sector: $L_{fermion} = L_{Kin.} + L_{Yukawa}$

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$$L_{mass} = -\underset{\substack{\searrow \\ y\langle\phi\rangle}}{\underline{m}}\ \bar{\psi}_L \psi_R - \underset{\substack{\searrow \\ y^*\langle\phi^\dagger\rangle}}{\underline{m^*}}\ \bar{\psi}_R \psi_L$$

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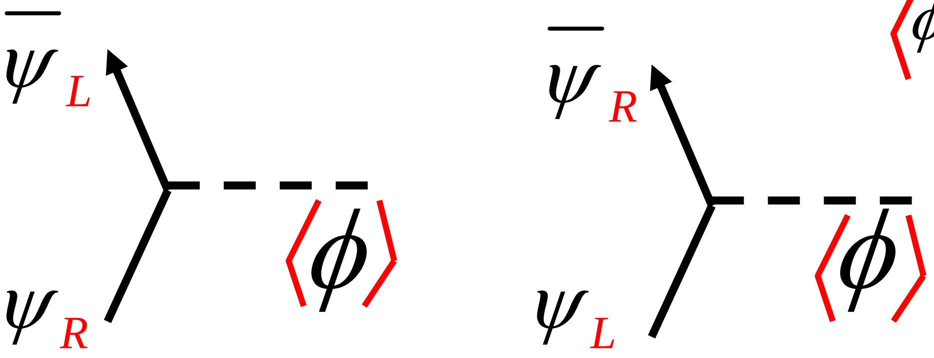

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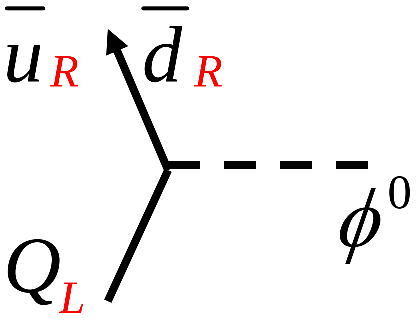
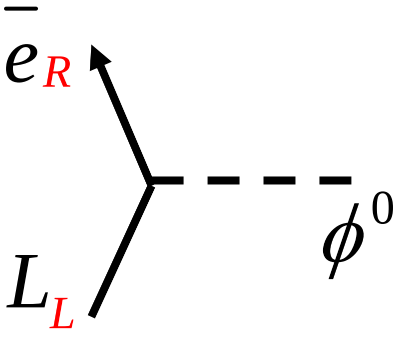
$$L_{Yukawa} = -\underline{y_u}\ \bar{u}_R\ \underline{\tilde{\phi}}\ Q_L - \underline{y_d}\ \bar{d}_R\ \underline{\phi}\ Q_L - \underline{y_e}\ \bar{e}_R \underline{\phi}\ L_L + h.c.$$

$$\tilde{\phi} = i\sigma^2\phi^*$$

$$\phi = \begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix}$$





fermion sector:

$$\langle\phi\rangle = \begin{pmatrix} \langle\phi^0\rangle \\ \phi^- \end{pmatrix}$$

$$L_{Yukawa} = -y_{u\,ij}\,\bar{u}_{Ri}\langle\tilde{\phi}\rangle Q_{Lj} - y_{d\,ij}\,\bar{d}_{Ri}\langle\phi\rangle Q_{Lj} - y_{e\,ij}\,\bar{e}_{Ri}\langle\phi\rangle L_{Lj} + h.c.$$

$$i, j = 1, 2, 3 \ (gen.\#)$$

$$U_L^{(u)\dagger}\, y_{uij}\langle\tilde{\phi}\rangle\, U_R^{(u)} \qquad U_L^{(d)\dagger}\, y_{dij}\langle\phi\rangle\, U_R^{(d)} \qquad U_L^{(e)\dagger}\, y_{eij}\langle\phi\rangle\, U_R^{(e)}$$

質量の固有状態のベースで書き直す。

quark

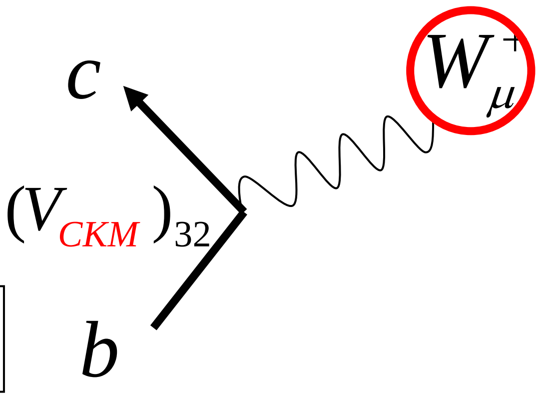
$$\begin{pmatrix} u \\ d \end{pmatrix}_L \begin{pmatrix} c \\ s \end{pmatrix}_L \begin{pmatrix} t \\ b \end{pmatrix}_L \Leftrightarrow \begin{pmatrix} u \\ d \end{pmatrix}_i \begin{pmatrix} c \\ s \end{pmatrix}_i \begin{pmatrix} t \\ b \end{pmatrix}_i$$

弱い相互作用の固有状態 **ミスマッチ！** **質量固有状態**

$$\begin{pmatrix} d \\ s \\ b \end{pmatrix}_L = \underline{\begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_i$$

$$V_{CKM} = U_L^{(u)\dagger} U_L^{(d)}$$

3: rotations
1: CP



(λ～12°カピボ角)

fermion sector:

$$\langle\phi\rangle = \begin{pmatrix} \langle\phi^0\rangle \\ \phi^- \end{pmatrix}$$

$$L_{Yukawa} = -y_{u\,ij}\,\bar{u}_{Ri}\langle\tilde{\phi}\rangle Q_{Lj} - y_{d\,ij}\,\bar{d}_{Ri}\langle\phi\rangle Q_{Lj} - y_{e\,ij}\,\bar{e}_{Ri}\langle\phi\rangle L_{Lj} + h.c.$$

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質量の固有状態のベースで書き直す。

lepton

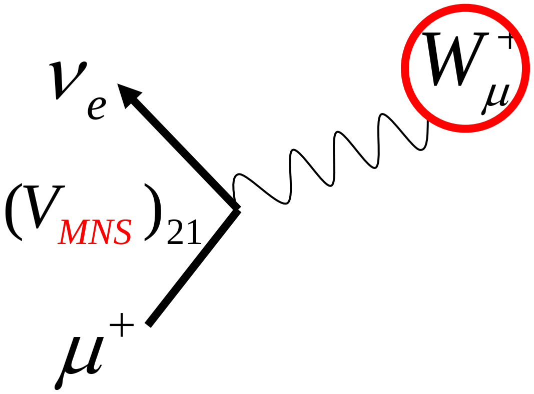
$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L \Leftrightarrow \begin{pmatrix} \nu_e \\ e \end{pmatrix}_i \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_i \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_i$$

ν：masslessなら同じに取れる ⇒ no LFV!! (例：$\mu \cancel{\to} e\gamma$)

if ν：masssive ⇒

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_L = \underline{\begin{pmatrix} 32.6^\circ & & \leq 9.2^\circ \\ & & \\ & & 45^\circ \end{pmatrix}} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_i$$

$$V_{MNS} = U_L^{(e)\dagger} U_l^{(\nu)}$$



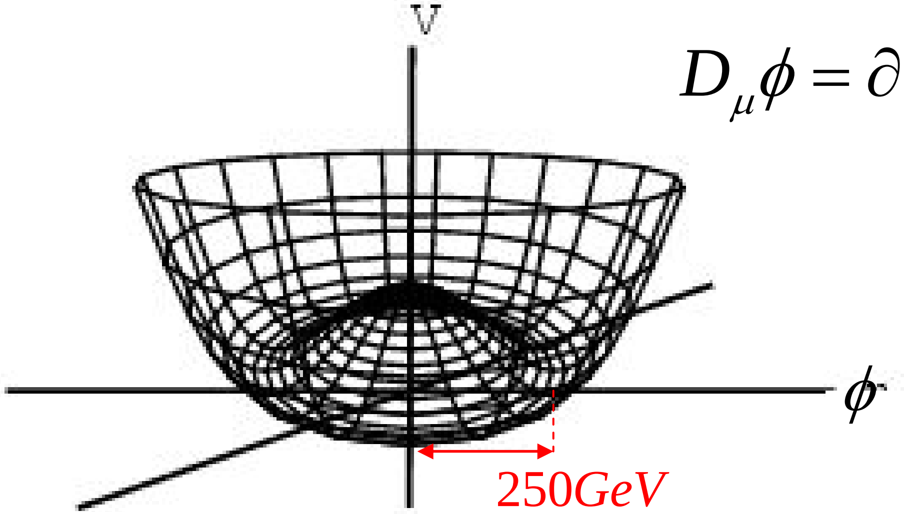
Higgs sector:

$$L_{Higgs} = |\underline{D_\mu \phi}|^2 - V(\phi)$$

$$\boxed{g^2\langle\phi\rangle^2 W^2, \ g^2\langle\phi\rangle^2 B^2}$$

$$V(\phi) = -m_\phi^2 |\phi|^2 + \lambda |\phi|^4$$

$$D_\mu \phi = \partial_\mu \phi - i g_2 T_2^\alpha \underline{W_\mu^\alpha} \phi - i g_1 (Y/2) \underline{B_\mu} \phi$$



$$\phi = \begin{pmatrix} \langle \phi^0 \rangle \\ \phi^- \end{pmatrix}$$

$$Q_Y(\phi) = -1$$

3つの自由度
→W±, Zに
吸収
(Higgs機構)

対称性の自発的破れ: $$\langle\phi\rangle = \frac{m_\phi}{\sqrt{2\lambda}} \sim 250GeV$$

⇒ **W±, Z gauge bosons 質量 (g⟨φ⟩)獲得**

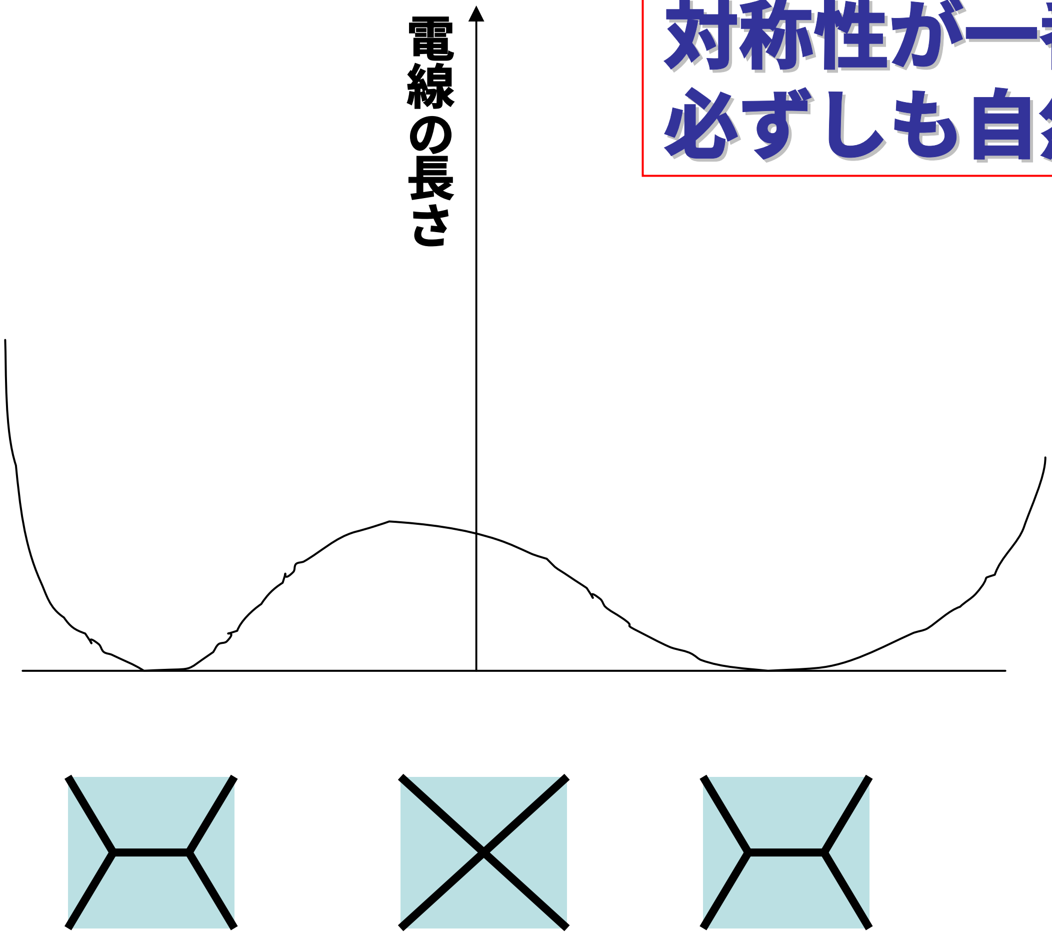
⇒ **SU(2)$_L$ × U(1)$_Y$ → U(1)$_{em}$**

W^{1,2,3}, B → (W±, Z) A(γ)

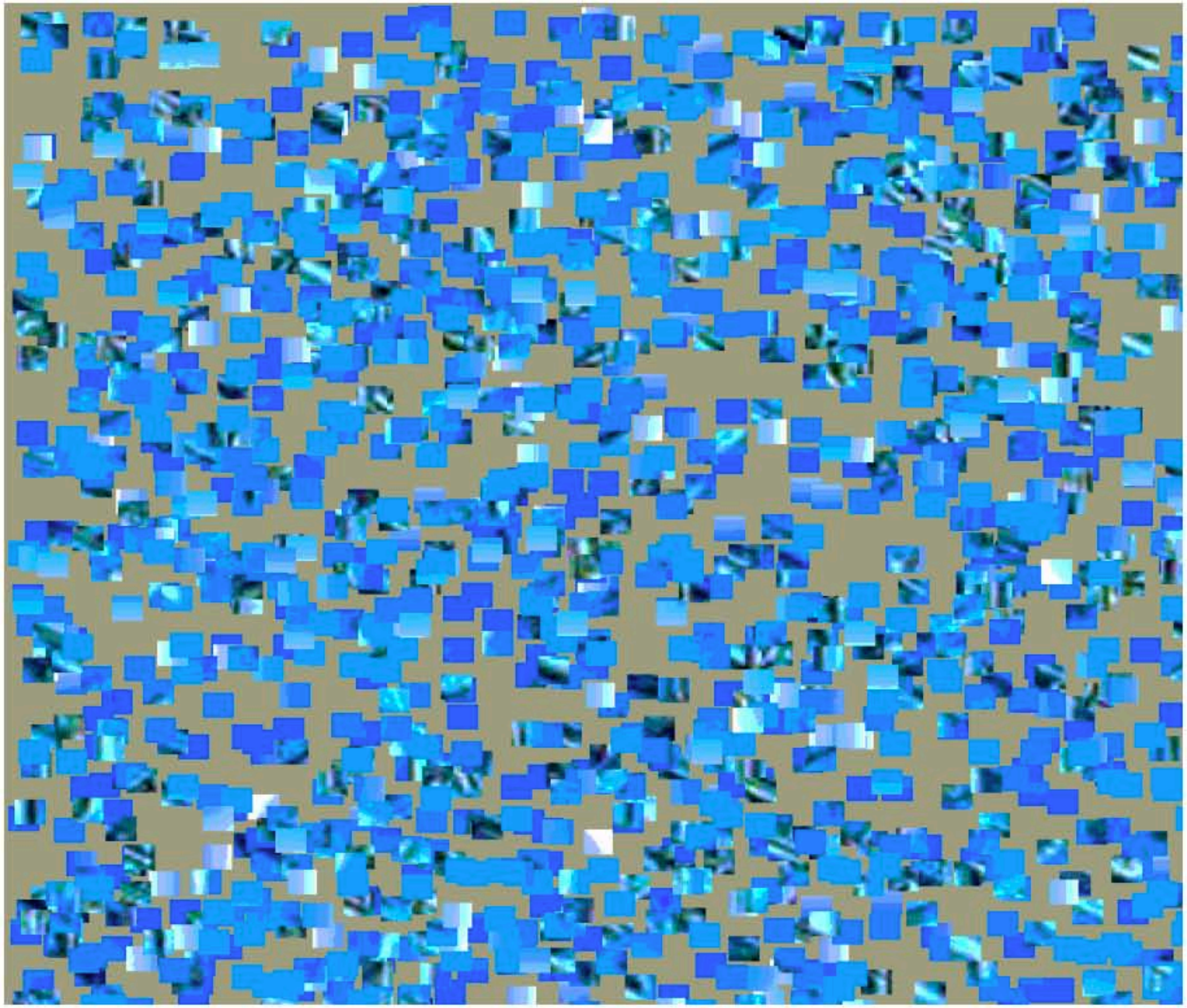
(Kurimoto's lecture note)

対称性の自発的破れ

対称性が一番高いものを
必ずしも自然は選ばない



対称性の自発的破れ

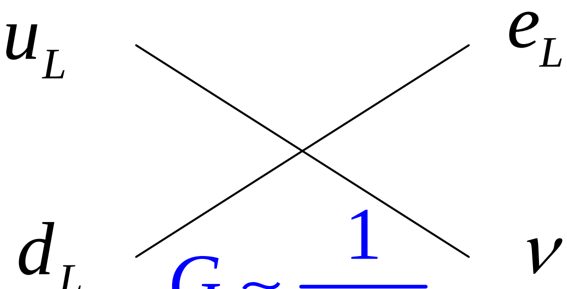


画像：金沢大学公開講座講義録CD-ROMより
制作：青木健一・伊藤祥一・石黒克也・木村剛・森祥寛

Higgsは、unitarity保存ため理論的に必要なんです

$\beta - decay:$ (弱い相互作用)

(例) $n \to p^+ e^- \nu$

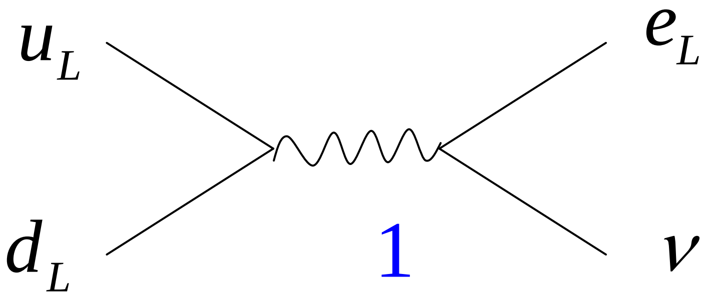


$$G \sim \frac{1}{M_W^2}$$

$$G(\overline{\psi}\psi\overline{\psi}\psi)$$

$$\sigma \sim \frac{E^2}{M_W^4}$$

$\Rightarrow$



$$\frac{1}{p^2 - M_W^2}$$

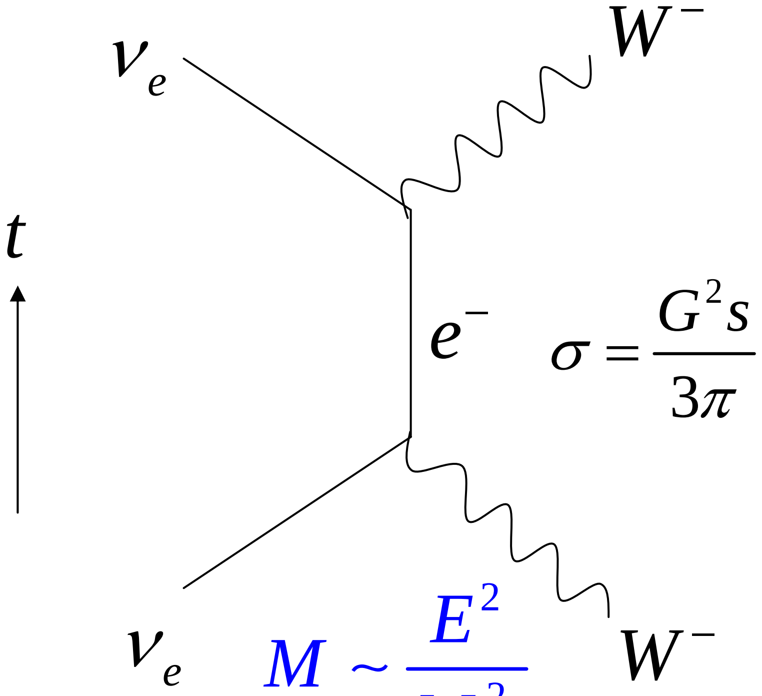
手で質量を与えた, 重たいゲージボソン

$$\sigma \to \frac{1}{E^2} \quad at \ E \to \infty$$

でOKだと期待される･･･が、しかし！

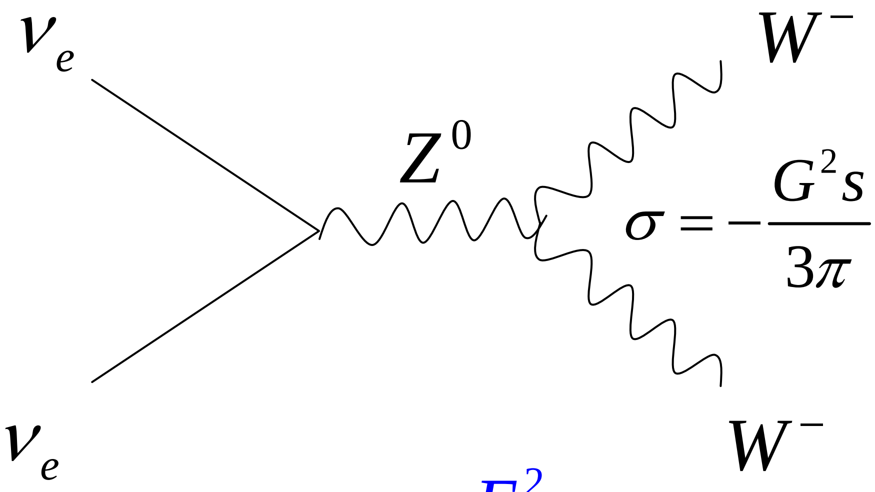
unitarity in the standard model

$$\boxed{\nu_e W^- \to \nu_e W^-}$$



$$\sigma = \frac{G^2 s}{3\pi}$$

$$M \sim \frac{E^2}{M_W^2}$$

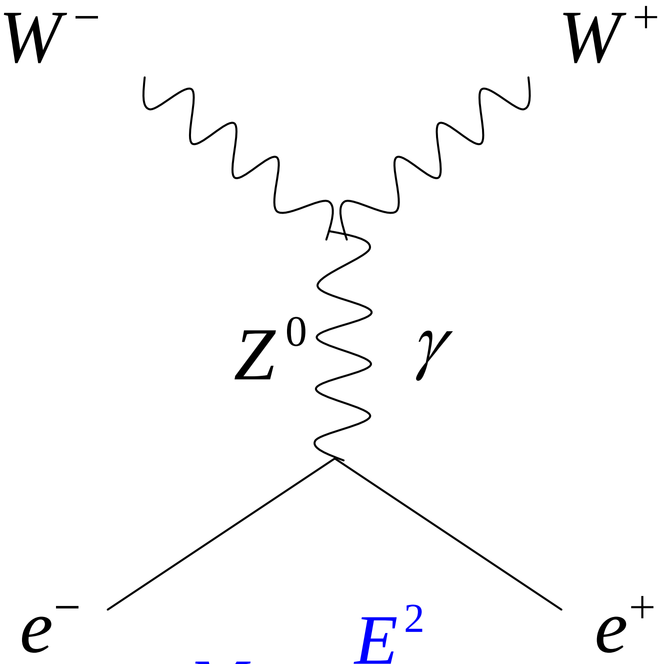


$$\sigma = -\frac{G^2 s}{3\pi}$$

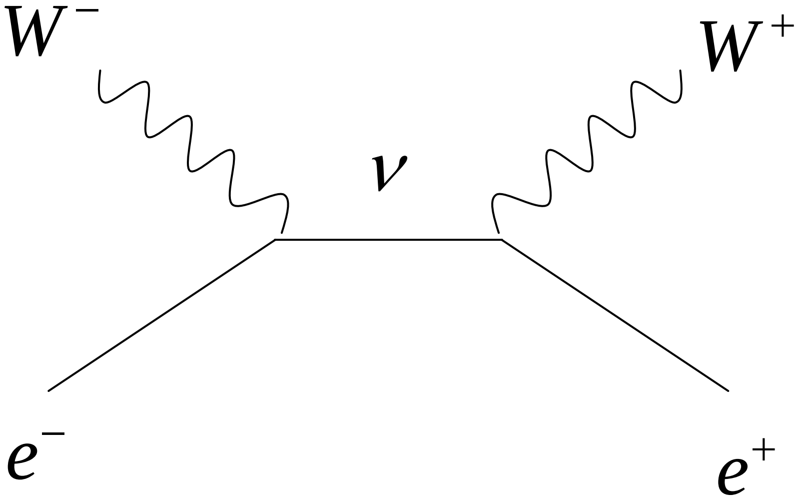
$$M \sim \frac{E^2}{M_W^2}$$

$$M \to O\ (\log E^2 / M_W^2)$$

$$\boxed{e^+e^- \to W^+W^-}$$



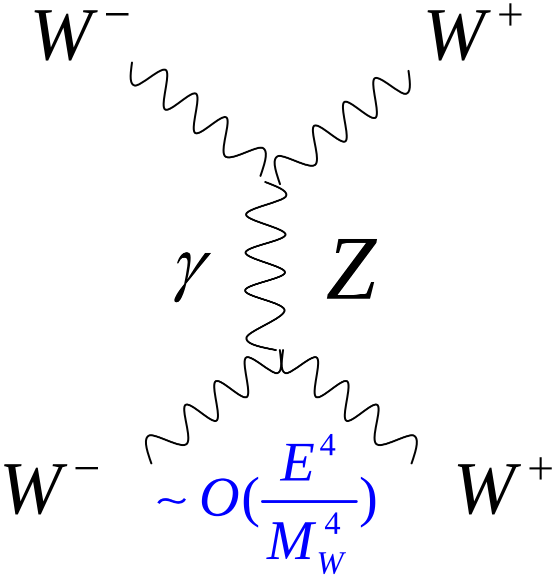
$$M \sim \frac{E^2}{M_W^2}$$

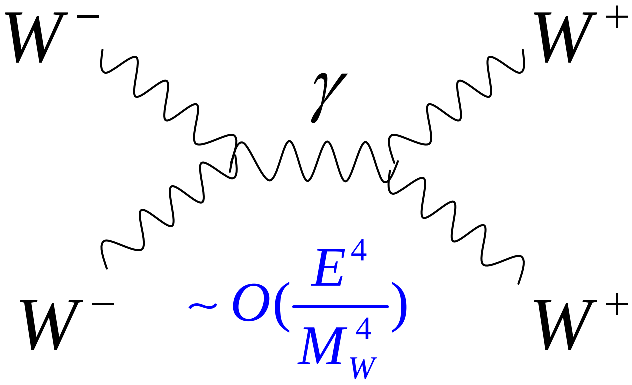


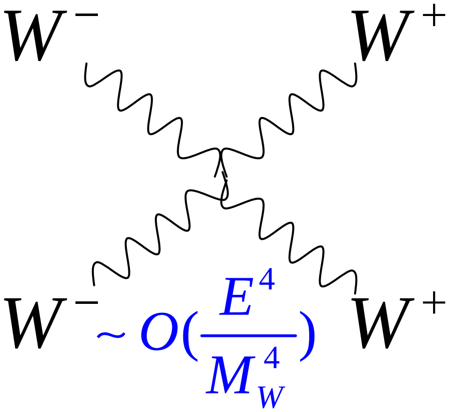
$$M \sim \frac{E^2}{M_W^2}$$

$$M \to O\ (\log E^2 / M_W^2)$$

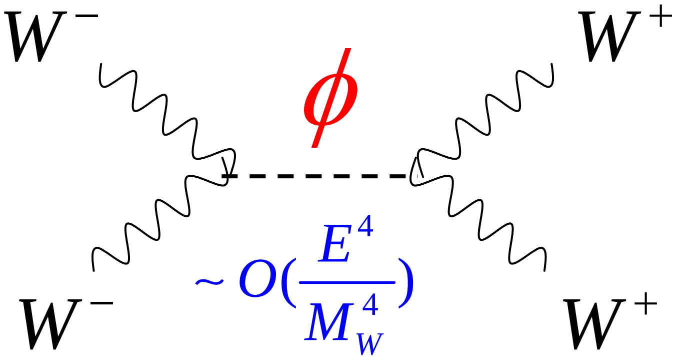
$$\boxed{W^+W^- \to W^+W^-}$$

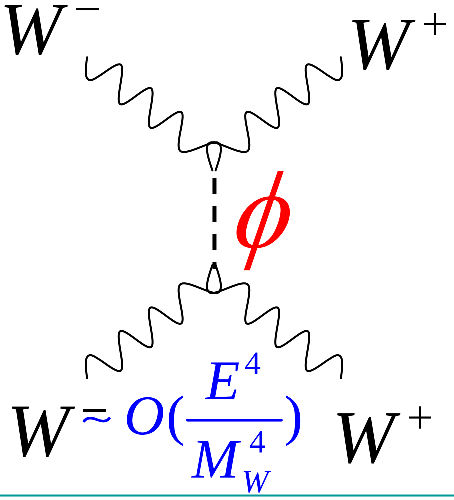






Higgs contributions !





O（E²/M²）もキャンセルする！

$$M \to O(\log E^2 / M_W^2)$$

Unitarity OK!

parameter # in Standard Model : 19(1)

gauge couplings: 3

quark mass: 6

lepton mass: 3

V_{CKM}: 4(1)

Θ: 1 $(G_{\mu\nu}\widetilde{G}^{\mu\nu})$

m_{ϕ}: 1

λ: 1

very successful ～O(100) GeV

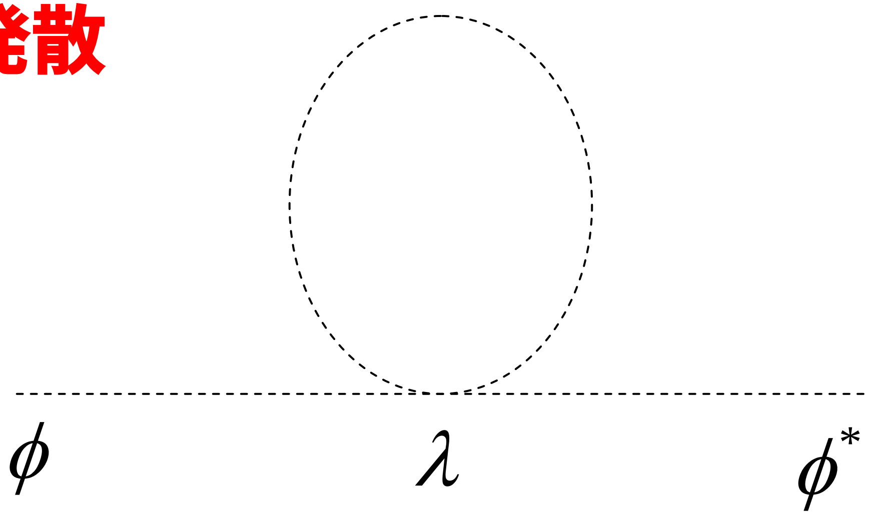
2.Beyond the Standard Model

Higgs sector of SM：

$$V(\phi) = -m_\phi^2 |\phi|^2 + \lambda |\phi|^4$$

$$\rightarrow \langle \phi \rangle = \frac{m_\phi}{\sqrt{2\lambda}} \sim 250\ GeV$$

2次発散



$$\simeq \lambda \Lambda^2$$

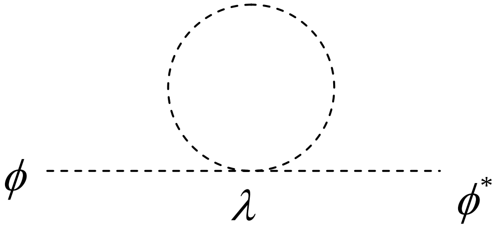
量子効果によりO（Λ^2）のmassを持ってしまう

m～O（100）GeVに留まる理由がない！

質量2次発散→SMはTeV以下の有効理論

☆摂動論的に低エネルギー⇒高エネルギーを見る (マクロからミクロ)

パワーの発散は繰り込みスキームに依存する



⇒ CutOffはTeVだ！

⇒ TeVから完全に新しい物理に (立場1)

Logの発散はスキームに“依存しない”

⇒ パワー発散無くして、

GUT, M_{Pl}まで視界を開こう！ (立場2)

☆Wilson流 (ミクロからマクロ)

ミクロ (基本理論) のスケールはrelevant OPにでるだろう。

$M_{GUT} \rightarrow M_W$で $M_W + \delta M_{GUT}$ずれる

立場1：TeVで完全に新しい物理に移行

① Higgsは複合粒子！

TC： $\langle \overline{F_T} F_T \rangle \neq 0$ ←QCD phase tr. $\langle \overline{\psi} \psi \rangle$ analogy (just scale up)

top mode condensation： $\langle \bar{t} t \rangle \neq 0$

……

② Large Extra Dimension (ADD) →§2-1

TeVで量子重力の世界が！

……

立場2：対称性でHiggsは軽いんだ！

③ 超対称性 (supersymmetry, SUSY) →§2-2

$m_\gamma=0$ ← gauge inv.
$m_{grav.}=0$ ← general cov.
$m_{fermion}\sim 0$ ← chiral sym.

chiral sym: $\psi_L \to e^{i\alpha}\psi_L,\ \psi_R \to e^{-i\alpha}\psi_R$

$$\xcancel{L_{mass} = -m\,\bar{\psi}_L\psi_R + h.c.}$$

t'Hooft naturalness cond.

$$m_{boson} \underset{\underset{\text{SUSY}}{\uparrow}}{=} m_{fermion} \ll M_{GUT,Pl}$$

④ HiggsはNG-boson!

$SU(6)_{global}$ ←$SU(5)_{gauge}$ (K. Inoue et al)

little Higgs

・・・・・

2-1.extra dimensional theory

TeVで余次元が！？

まず余次元理論を考える動機について

1、**重力の弱さを説明したい** → large extraD (ADD)

2、KK idear&more: 5D gravity = 4D gravity + 4D gauge + 4D scalar

3、field localization (brane, BG-vev, fixed points)
→ volume suppression,
→ geometrical understanding of particle physics

4、(24) Higgs**の起源** → cf. Hosotani mech.

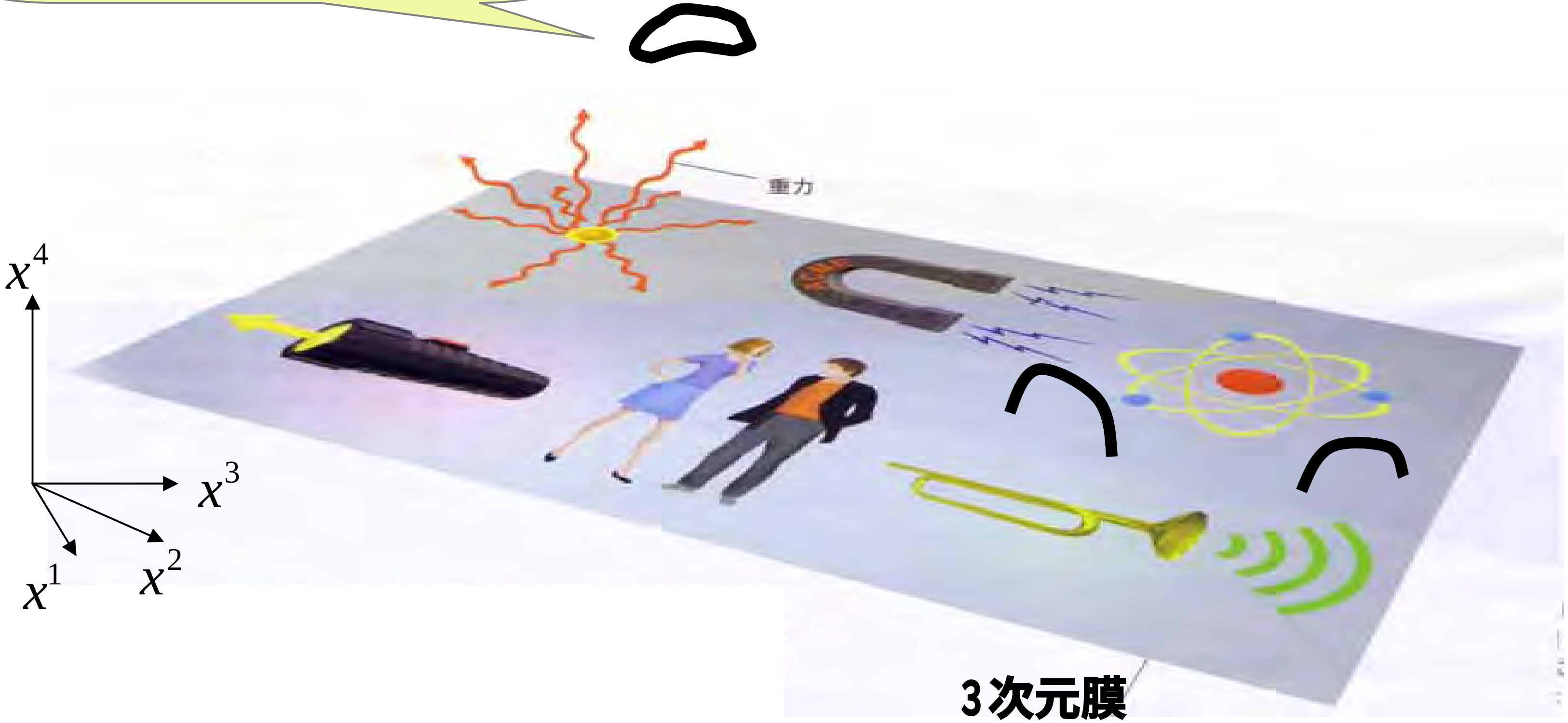
5、GUT**の問題点を回避したい** → extraD GUT

6、**重力の局在** (RS)、blane world、 string**との競合性、その他素粒子物理の新しい理解、それに、あったらそれだけで面白い！**

重力が何故他の3つの力に比べて弱いのか！？

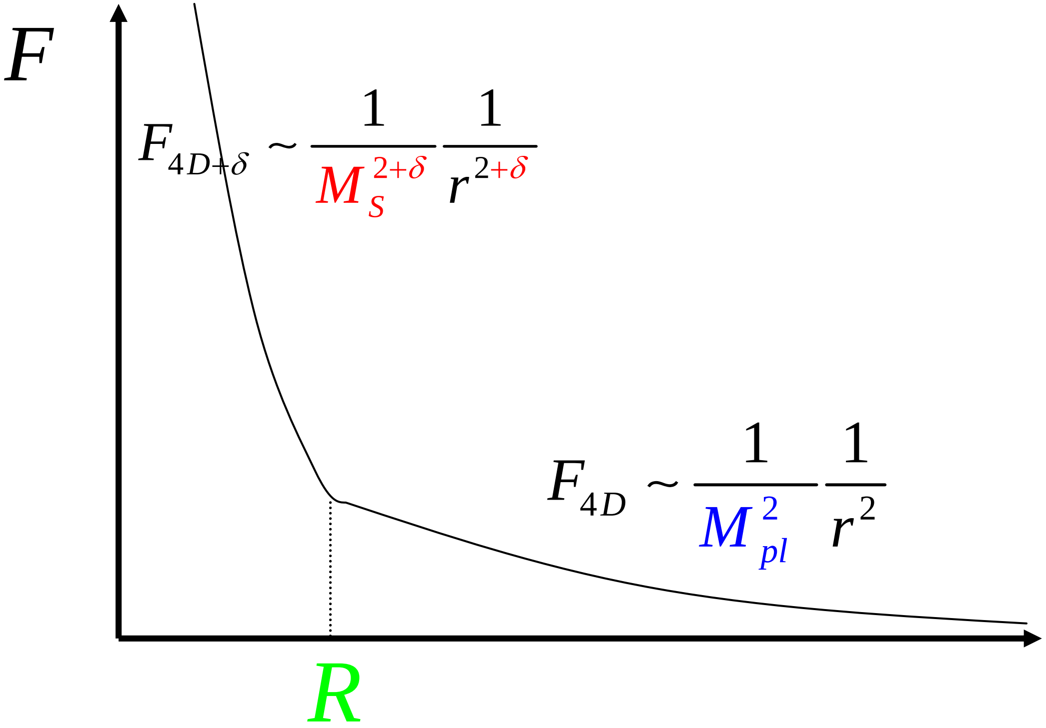
電磁気力：重力=1:0.00000000000000000000000000000000001

グラビトン
（重力を媒介する素粒子）
4次元から飛び出す！



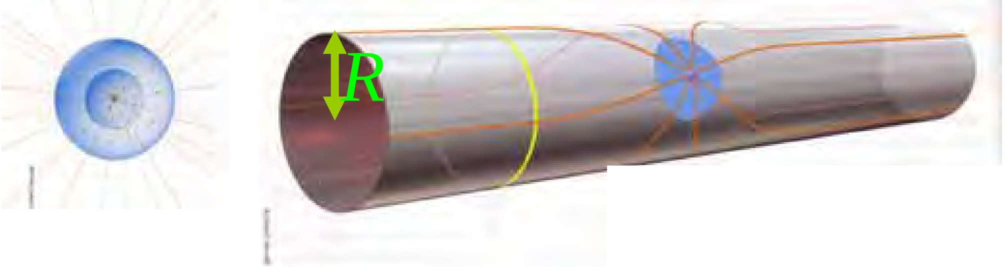
（図：サイエンスより）

large extraD (ADD)



$$F_{4D+\delta} \sim \frac{1}{M_S^{2+\delta}} \frac{1}{r^{2+\delta}}$$

$$F_{4D} \sim \frac{1}{M_{pl}^2} \frac{1}{r^2}$$



$$\boxed{M_{pl}^2 = M_S^{2+\delta} R^\delta}$$

M_{pl}: $10^{18}\ GeV$

M_S: $1\ TeV$

$\delta = 1 \rightarrow R \sim 10^{11} m$

$\delta = 2 \rightarrow R \sim 10^{-4} m \quad (R^{-1} \sim 10^{-3} eV)$

$\delta = 3 \rightarrow R \sim 10^{-9} m \quad (R^{-1} \sim 100\ eV$

⋮

重力が弱いのはRが大きいため！

cf. extraD GUT: small extraD $R^{-1} \sim 10^{16} GeV \Leftrightarrow R \sim 10^{-32} m \quad (\hbar c \approx 10^{-16} GeV \cdot m)$

（図：サイエンスより）

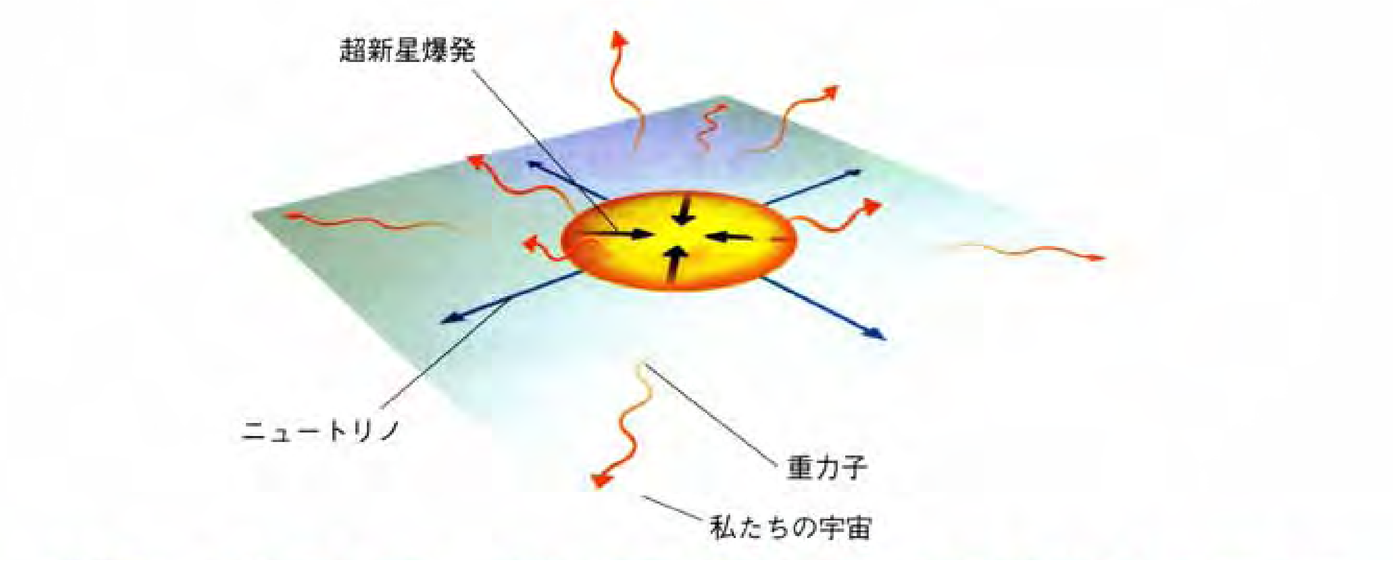
$$\boxed{M_{pl}^2 = M_S^{2+\delta} R^\delta}$$

$$S_{4+\delta} = -\frac{1}{M_S^{2+\delta}}\int d^{4+\delta}x\sqrt{-g}\; R^{(4+\delta)}$$

$$\Downarrow$$

$$S_4 = -\underbrace{\frac{1}{M_S^{2+\delta}}\underbrace{\int d^\delta x}_{R^\delta}}_{\frac{1}{M_{pl}^2}}\int d^4x\sqrt{-g_4}\; R^{(4)} + \cdots$$

現在の実験・観測からの制限



もっとも厳しい制限は超新星爆発からの制限
ほとんどのenergy**はニュートリノが持ち出す。**
重力子によるenergy**損失は余次元＃に依存。**

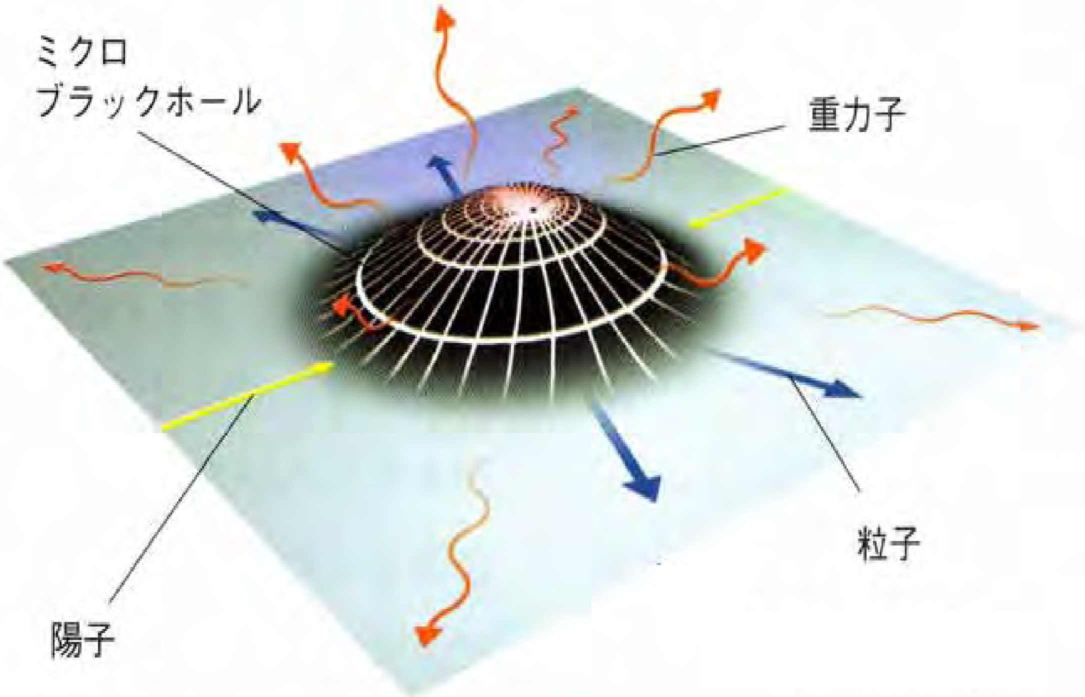
例：

(図：サイエンスより)

今後の実験

☆加速器実験（ＬＨＣ：大型ハドロンコライダー）

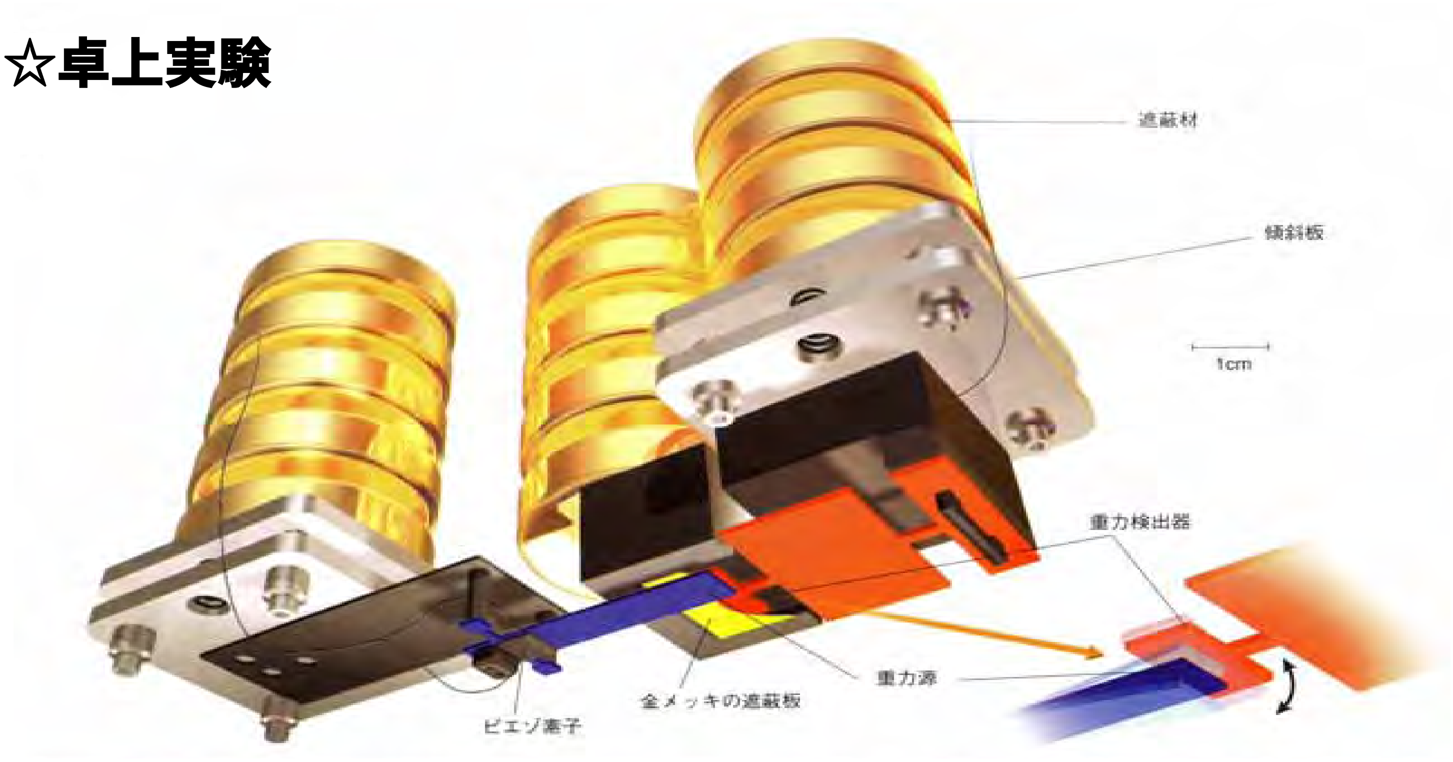
ミニブラックホール生成！ →ホーキング輻射で重力子・ＳＭ粒子等に崩壊



（図：サイエンスより）

今後の実験

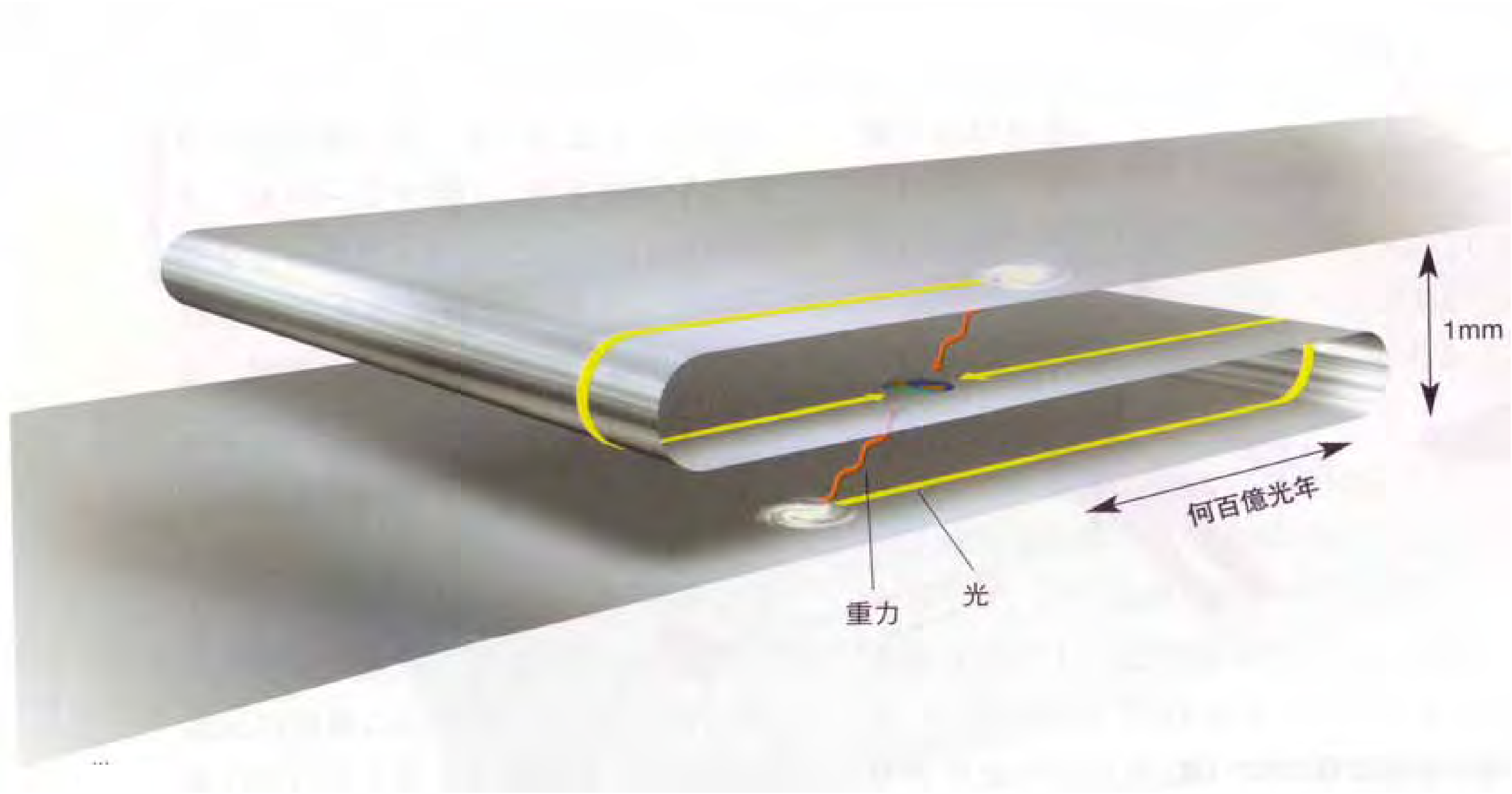
☆卓上実験



ねじれ振動はかり（コロラド大）：０．０５～１．０mmの重力
タングステン（青）の重力源→タングステン（赤）の検出器（電気信号）
各装置は別々に吊り下げられていて、電磁シールドが施されている。
低温（４K）での実験が計画されている。

（図：サイエンスより）

宇宙論：Dark matterも実は余次元の効果かも！？



（図：サイエンスより）

時空の概念？

M$_{\mathrm{Pl}}$スケール以上では、時空自体が定義出来ない？

不確定性関係式

$$\Delta x \geq \frac{\hbar}{\Delta p}$$

時空の概念？

M_{Pl}スケール以上では、時空自体が定義出来ない？

不確定性関係式

$$\Delta x \geq \frac{\hbar}{\Delta p} + \underline{\frac{G}{c^3}\Delta p}$$

quantum gravity

$$\rightarrow \Delta x \geq l^* \equiv \sqrt{\frac{G\hbar}{c^3}} \sim 1.6\times 10^{-33}\,cm$$

Wilson流： 高エネルギー（ミクロ）→低エネルギー（マクロ）の有効ラグランジャンを作ったとき対称性で許されるirelevant OP（dim4以上の繰り込み不可能な相互作用）は無限個現れる！

SMが繰り込み可能 ⇒ $\Lambda \gg M_W$!!

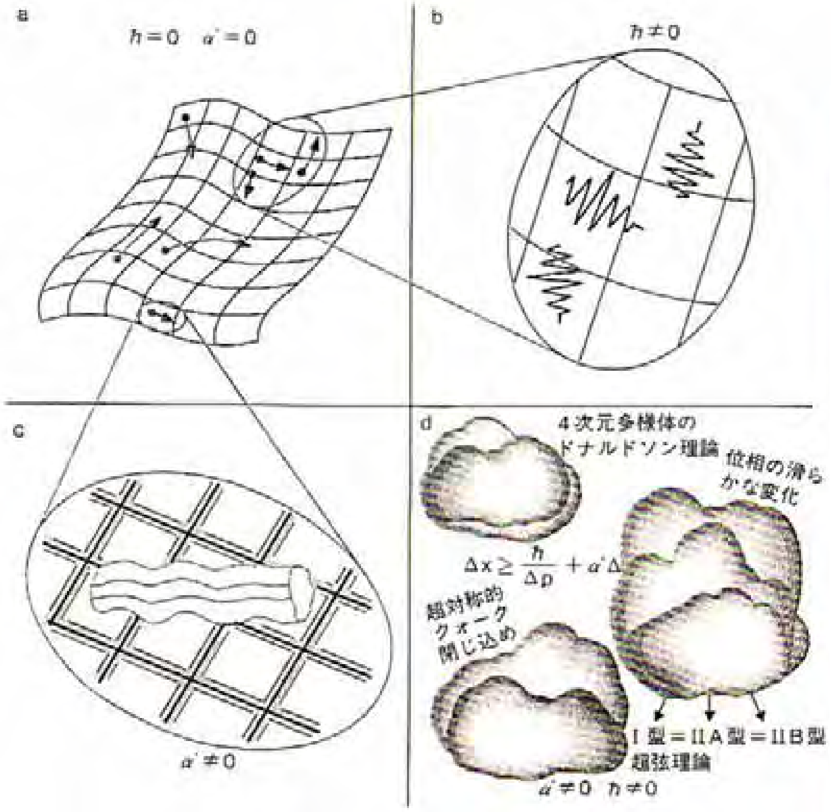
$$L_{eff} \supset L_{SM} + \underline{\frac{1}{\Lambda^2} QQQL} + \cdots$$

→陽子崩壊

Large extraD: **危険な項は対称性で禁止**or string inspired？

Minimum legth**の存在はl*よりミクロのスケールでは時空自体が定義出来ないということ。ということは、l*よりミクロの物理理論が存在するとしたら時空の概念なしに定式化されるだろう → M(atrix)理論？？**

stringの描像



$$\Delta x \geq \frac{\hbar}{\Delta p} + \underline{\alpha'\frac{\Delta p}{\hbar}}$$

第一項は、ハイゼンベルグの顕微鏡。

string**においては、**string scale**よりさらに加速しようとするとひも自身が膨らみ短距離を調査するどころか大きく延びたひもを観測だけになってしまう。**

この第二項のせいで、

α$'^{1/2}$ ~10^{-32}cm**の距離に関する不確定性が残る。**

(図: サイエンス)

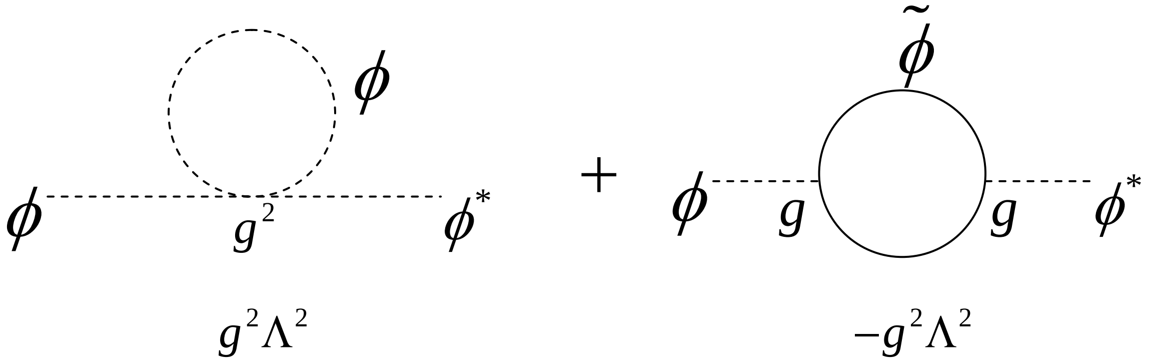
2-2. 超対称性理論(SUSY)

Higgsが軽い理由

$$m_{boson} = m_{fermion}$$

$m_{fermion}$ 《 M_{GUT} ← chiral symmetry

2次発散はキャンセル



$$\sim \frac{\alpha^2}{4\pi} m^2 \log(\frac{\Lambda}{m})$$

高エネルギー (MGUT) まで理論を適応してもいい。

→ しかも gauge coupling unification!

標準模型 (SM)

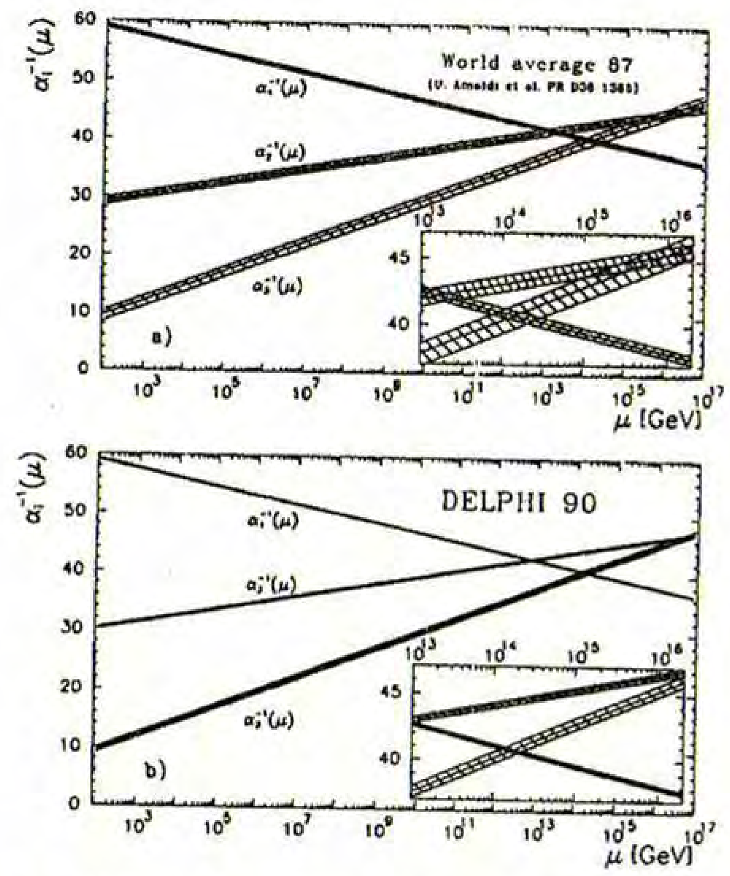


Fig. 1. (a) First order evolution of the three coupling constants in the minimal standard model (world average values in 1987 from ref. [1]). The small figure is a blow-up of the crossing area). (b) As above but using M_Z and $\alpha_s(M_Z)$ from DELPHI data. The three coupling constants disagree with a single unification point by more than 7 standard deviations.

最小超対称性標準模型 (MSSM)

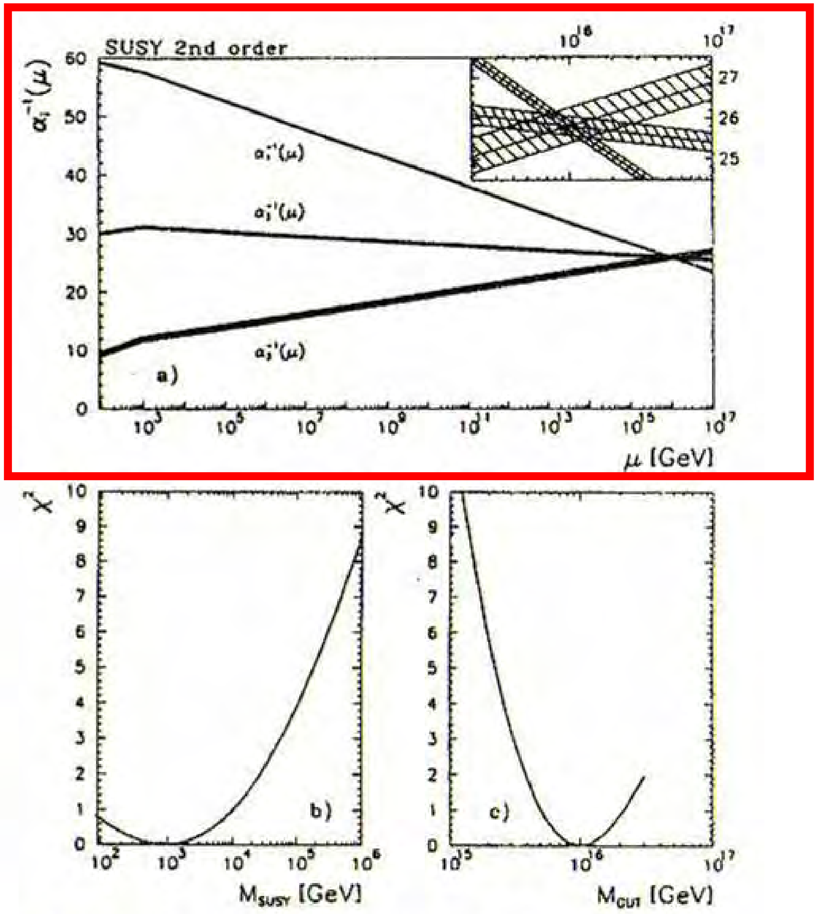
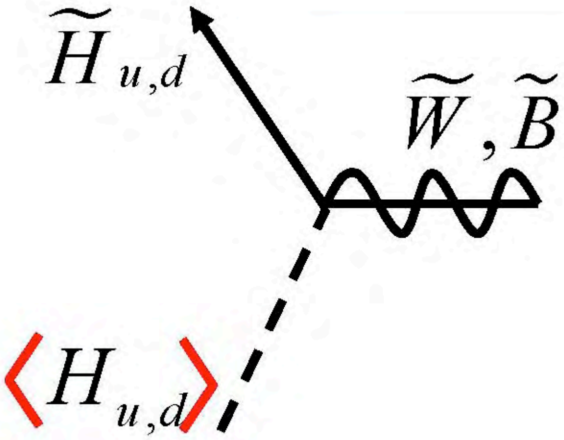


Fig. 2. (a) Second order evolution of the three coupling constants in the minimal SUSY model. M_{SUSY} has been fitted by requiring crossing of the couplings in a single point. The two lower plots show the χ^2 distribution for the SUSY scale M_{SUSY} (b) and for the unification scale M_{GUT} (c) taking into account their correlation.

(Amaldi, PLB260(1991)447)

SUSY粒子

quark (1/2)	squark (0)	$\tilde{q}$
lepton (1/2)	slepton (0)	$\tilde{l}$
gauge boson (1)	gaugino (1/2)	$\tilde{W}^{\pm}, \tilde{W}^{0}, \tilde{B}$ ..
Higgs (0)	higgsino (1/2)	$\tilde{H}_u, \tilde{H}_d$
graviton (2)	gravitino (3/2)	$\tilde{g}_{3/2}$



chargino:

$$(\tilde{W}^{+}, \tilde{H}_{u}^{+}, \tilde{W}^{-}, \tilde{H}_{d}^{-}) \longrightarrow \tilde{\chi}_1^{\pm}, \tilde{\chi}_2^{\pm}$$

neutraino:

$$(\tilde{B}, \tilde{W}^{0}, \tilde{H}_{d}^{0}, \tilde{H}_{u}^{0}) \longrightarrow \tilde{\chi}_1^{0}, \tilde{\chi}_2^{0}, \tilde{\chi}_3^{0}, \tilde{\chi}_4^{0}$$

SUSY algebra (graded Lie algebra)

Poincare algebra P_μ, $M_{\mu\nu}$ に反可換交換関係を入れて拡張

☆Coleman-Mandula, no-go theoremを回避

(Lorentz space-time sym.と内部対称性は常に可換)

super-charge: Q $(Q|F\rangle = |B\rangle,\ Q|B\rangle = |F\rangle)$

$[Q,M] \sim Q$, $[Q,P]=0$, $\{Q^\dagger, Q\} = P$ ($\Rightarrow$ **H**) cf.gauge→SUGRA

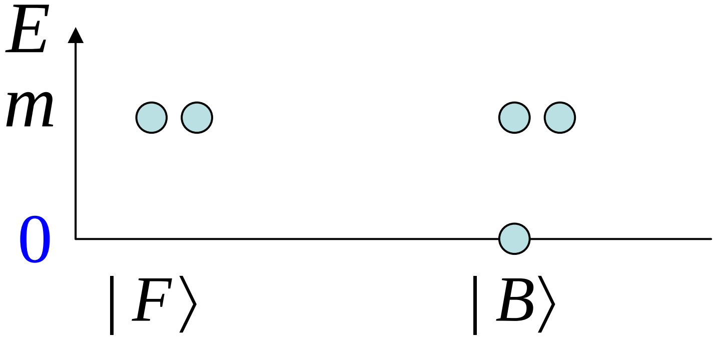
$\rightarrow [H,Q]=0$, $\langle 0|H|0\rangle \geqq 0$

$H|B\rangle = E|B\rangle \Rightarrow \langle F|F\rangle = \langle B|\, Q^\dagger Q\, |B\rangle = E\,\langle B|B\rangle = E \neq 0$

☆$E>0$：**必ずFとBは同じエネルギーでペア。**

☆$E=0$：$H|B\rangle = 0 \rightarrow Q|B\rangle = |F\rangle = 0$

→B**は**F**の相棒を持たない。**



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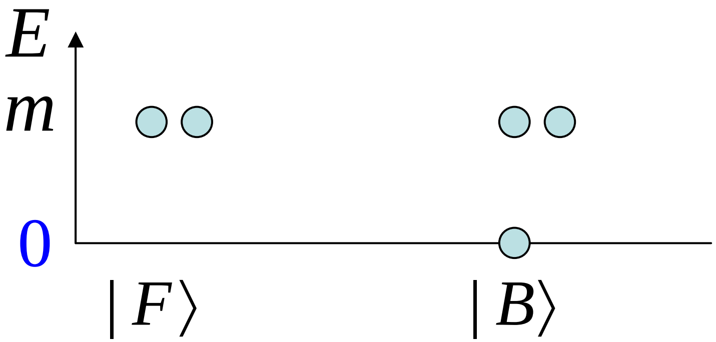


$H \sim Q^{\dagger}Q$

$\langle 0|H|0\rangle \sim \langle 0|Q^{\dagger}Q|0\rangle \geqq 0$

$Q|0\rangle \neq |0\rangle \Rightarrow$ SUSY breaking!

Witten index = tr $(-1)^F$

=($|B\rangle$ # of E=0)ー($|F\rangle$ # of E=0)

≠0ならＳＵＳＹ破れない！

SUSY algebra (graded Lie algebra)

Poincare algebra P_{μ}, $M_{\mu\nu}$ **に反可換交換関係を入れて拡張**

☆Coleman-Mandula, no-go theorem**を回避**

(Lorentz space-time sym.**と内部対称性は常に可換**)

super-charge: Q $(Q|F\rangle = |B\rangle$, $Q|B\rangle = |F\rangle)$

$[Q,M]\sim Q$, $[Q, P] = 0$, $\{Q^{\dagger},Q\} = P$ $(\Rightarrow$ **H** $)$

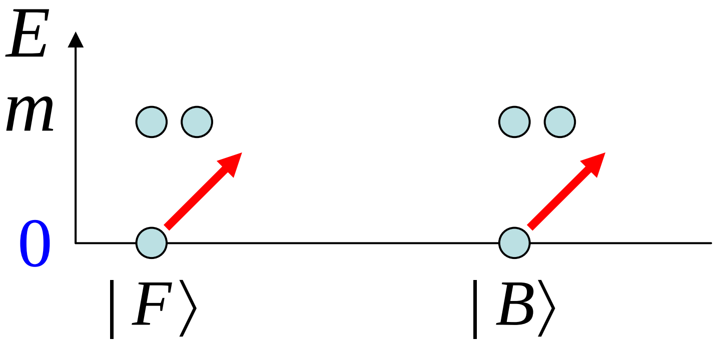
$\rightarrow [H, Q] = 0$, $\langle 0|H|0\rangle \geqq 0$

$H|B\rangle = E|B\rangle \Rightarrow \langle F|F\rangle = \langle B|Q^{\dagger}Q|B\rangle = E\langle B|B\rangle = E \neq 0$

☆E>0：**必ずFとBは同じエネルギーでペア。**

☆E=0：$H|B\rangle = 0 \rightarrow Q|B\rangle = |F\rangle = 0$

→B**は**F**の相棒を持たない。**



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$Q|0\rangle \neq |0\rangle \Rightarrow$ SUSY breaking!

Witten index = tr $(-1)^{F}$

=(|B⟩ # of E=0)ー(|F⟩ # of E=0)

＝0ならSUSY破れ得る！

SUSY algebra (graded Lie algebra)

Poincare algebra P_{μ}, $M_{\mu\nu}$ **に反可換交換関係を入れて拡張**

☆Coleman-Mandula, no-go theorem**を回避**

(Lorentz space-time sym.**と内部対称性は常に可換**)

super-charge: Q ($Q|F\rangle = |B\rangle$, $Q|B\rangle = |F\rangle$)

$[Q,M] \sim Q$, $[Q, P] = 0$, $\{Q^{\dagger}, Q\} = P$ (⇒ **H**)

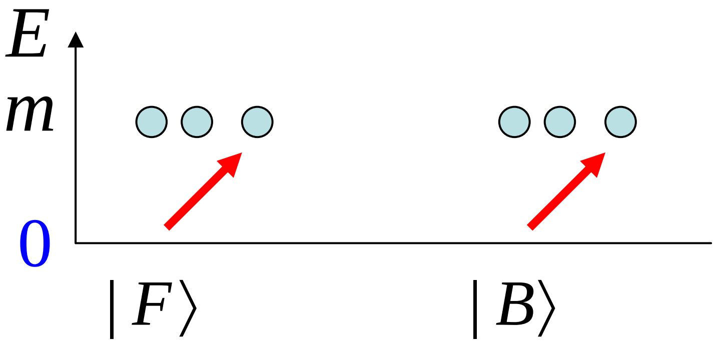
→ $[H, Q] = 0$, $\langle 0|H|0\rangle \geqq 0$

$H|B\rangle = E|B\rangle \Rightarrow \langle F|F\rangle = \langle B|Q^{\dagger}Q|B\rangle = E\langle B|B\rangle = E \neq 0$

☆E>0：**必ずＦとＢは同じエネルギーでペア。**

☆E=0：$H|B\rangle = 0 \rightarrow Q|B\rangle = |F\rangle = 0$

→B**は**F**の相棒を持たない。**



$H \sim Q^{\dagger}Q$

$\langle 0|H|0\rangle \sim \langle 0|Q^{\dagger}Q|0\rangle \geqq 0$

$Q|0\rangle \neq |0\rangle \Rightarrow$ SUSY breaking!

Witten index = tr $(-1)^F$

$= (|B\rangle \text{ \# of } E=0) - (|F\rangle \text{ \# of } E=0)$

＝0ならＳＵＳＹ破れ得る！

SUSYではEnergyの原点が決まる！

場の理論で出てきた始めの無限大

boson: $[a,a^\dagger]=1,\ [a,a]=[a^\dagger,a^\dagger]=0,$

fermion: $\{b,b^\dagger\}=1,\ \{b,b\}=\{b^\dagger,b^\dagger\}=0,$

$$H=1/2\,\boldsymbol{\omega}_B\{a^\dagger,a\}+1/2\,\boldsymbol{\omega}_F\{b^\dagger,b\}$$

$$=\boldsymbol{\omega}_B(n_B\underline{+\ 1/2})+\boldsymbol{\omega}_F(n_F\underline{-\ 1/2})$$

cancellation

$n_B=a^\dagger a,\quad n_F=b^\dagger b$ (number OP)

SUSY→$\boldsymbol{\omega}_B=\boldsymbol{\omega}_F=\boldsymbol{\omega}$

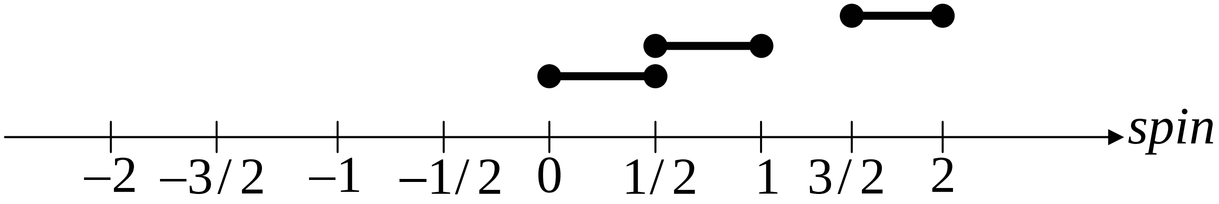
$$=\boldsymbol{\omega}(n_B+n_F)$$

N = 2, 4, 8 SUSY (super-chargeの数が増えた理論)

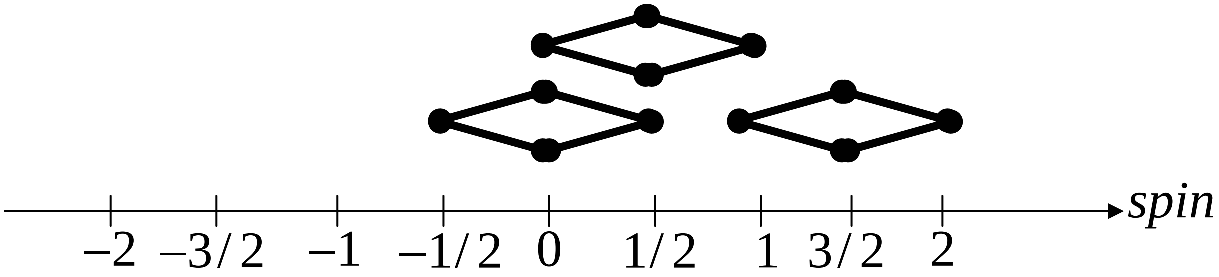
$$\{Q^{\dagger}, Q\}=P \quad \rightarrow \quad \{Q_n^{\dagger}, Q_m\}=P\,\delta_{nm}+C_{nm}$$

C:central charge→BPS ststes (Seiberg-Witten, Duality)

☆N=1:



☆N=2：1-loop exact



☆N=4：finite (1,1/2(4),0(6),-1/2(4),-1)

☆N=8：(2,3/2(8),1(28),1/2(56),0(70),-1/2(56),-1(28),-3/2(8),-2)

massless particle > spin2**はフリー理論。**spin1**以上の粒子は繰り込み不可能。**

Minimal Supersymmetric Standard Model (MSSM)

		SU(3)C	SU(2)L	U(1)Y
gauge		$g \leftrightarrow \tilde{g}$	$W^{\pm,0} \leftrightarrow \tilde{w}^{\pm,0}$	$B \leftrightarrow \tilde{b}$
			$W^{\pm}, Z, \gamma \leftrightarrow \tilde{w}^{\pm}, \tilde{z}, \tilde{\gamma} \; (\rightarrow \chi^{\pm}, \chi^{0})$	
matter	Q	($\underline{\square}$	$\square$	1/3)
	U	($\underline{\square}$	1	-4/3)
	D	($\square$	1	2/3)
	L	(1	$\square$	-1)
	E	(1	1	2)
Higgs	H_u	(1	$\square$	1)
	H_d	(1	$\square$	-1)

matterとHiggs区別せなあかん。。

R-parity

particle:+ sperticle:-

2つのHiggsが必要！

higgsino → anomaly

MSSM action

Kinetic term:

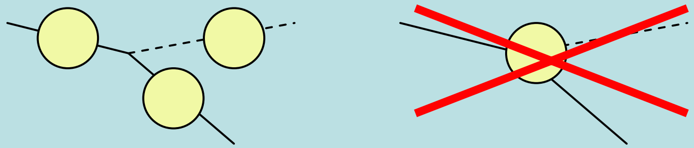
$$L = \underline{\int d^2\theta d^2\bar{\theta}\ Q^\dagger e^{g_3 G + g_2 W + \frac{g_Y}{3}} Q} + \cdots + \underline{\frac{1}{4g_3^2}\int d^2\theta\ G^\alpha G_\alpha} + \cdots + h.c.$$

$$|D\tilde{Q}|^2 + iQ\gamma^\mu D_\mu Q + i\sqrt{2}\,\tilde{Q}\tilde{g}\bar{Q} + \cdots \qquad \frac{1}{4g_3^2}G_{\mu\nu}^2 + i\tilde{g}\gamma^\mu D_\mu \bar{\tilde{g}} - \mathrm{Im}\left(\frac{1}{4g_3^2}\right)G_{\mu\nu}\widetilde{G}^{\mu\nu} + \cdots$$

Yukawa term:

$$L = \int d^2\theta\ \ W + h.c.$$

non-renomalization theorem



$$W = y_u\ QH_uU + y_d\ QH_dD + y_e\ LH_dE + \mu H_uH_d$$

$$+\lambda_d QLD + \lambda_e LLE + \lambda_u UDD \longleftarrow$$ **R－parityで禁止**

$$U(1)_R:\ \theta \to e^{i\alpha}\theta,\ \bar{\theta} \to e^{-i\alpha}\bar{\theta}$$

α＝πがR－parityに対応 (cf. chiral sym.)

Q，L，．．→1、 H→0、 μ項禁止（U（1）$_R$）、R－parityOK

parameter # of SM

SM･･･19 (1) $\left[\overset{3}{g_i},\ \overset{6}{M_{q_i}},\ \overset{3}{M_{l_i}},\ \overset{4(1)}{V_{KM}},\ \overset{1}{\theta},\ \overset{1}{m_H},\ \overset{1}{\lambda}\right]$

parameter # of MSSM

SM・・・19 (1) $\left[\begin{array}{ccccccc} 3 & 6 & 3 & 4(1) & 1 & 1 & 1 \\ g_i, & M_{qi}, & M_{li}, & V_{KM}, & \theta, & m_H, & \lambda \end{array}\right]$

MSSM・・・19 (1)

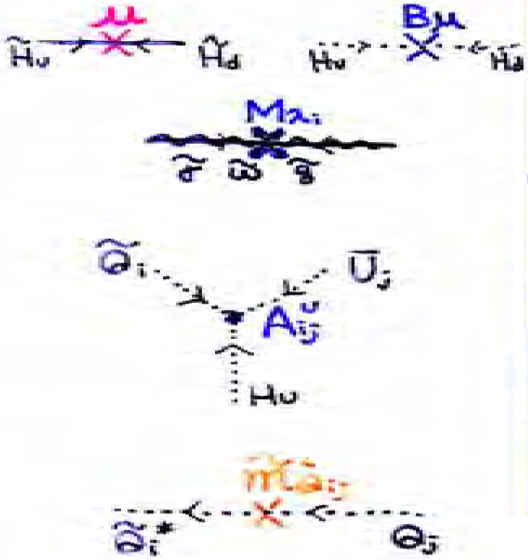
$$\left[\begin{array}{ccccc} 3 & 6 & 3 & 4(1) & 1 \\ g_i, & M_{qi}, & M_{li}, & V_{KM}, & \theta, \\ 1 & & & 1 & \\ \xi, & & & \mu & \end{array}\right.$$

parameter # of MSSM

SM・・・19 (1)　$\left[\overset{3}{g_i}, \overset{6}{M_{u_i}}, \overset{3}{M_{l_i}}, \overset{4(1)}{V_{KM}}, \overset{1}{\theta}, \overset{1}{m_\phi}, \overset{1}{\lambda} \right]$

MSSM・・・19 (1) +

$$\left[\begin{array}{l}
\overset{3}{g_i}, \overset{6}{M_{u_i}}, \overset{3}{M_{l_i}}, \overset{4(1)}{V_{KM}}, \overset{1}{\theta}, \qquad\qquad 19(1) \\
\overset{1}{\xi}, \overset{1}{m^2_{H_u}}, \overset{1}{m^2_{H_d}} \quad \overset{2(1)}{\mu} \quad \overset{2(1)}{B} \quad \overset{6(2\times3)\ (3)}{M_{\lambda_i}} \qquad\qquad 13 \\
\overset{18(=2\times9)\ (9)}{A^U_{ij}} = \begin{pmatrix} A_{uu} & A_{uc} & A_{ut} \\ A_{cu} & A_{cc} & A_{ct} \\ A_{tu} & A_{tc} & A_{tt} \end{pmatrix} \quad \overset{18\ (9)}{A^D_{ij}} \quad \overset{18\ (9)}{A^E_{ij}} \qquad\qquad 54(27) \\
\overset{9\ (3)}{\tilde{m}^2_{Q_{ij}}} = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix} \quad \overset{9(3)}{\tilde{m}^2_{U_{ij}}} \quad \overset{9(3)}{\tilde{m}^2_{D_{ij}}} \quad \overset{9(3)}{\tilde{m}^2_{L_{ij}}} \quad \overset{9(3)}{\tilde{m}^2_{E_{ij}}} \qquad\qquad 45(15)
\end{array}\right]$$

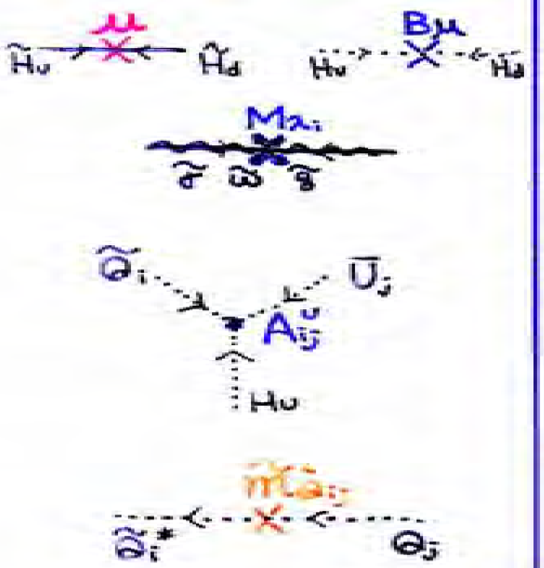


soft SUSY breaking
=2次発散が出ない
SUSY breaking

parameter # of MSSM

SM・・・19 (1) $\left[\ \overset{3}{g_i},\ \overset{6}{M_{Q_i}},\ \overset{3}{M_{L_i}},\ \overset{4(1)}{V_{KM}},\ \overset{1}{\theta},\ \overset{1}{m_\phi},\ \overset{1}{\lambda}\ \right]$

MSSM・・・125 (44)



$\overset{3}{g_i},\ \overset{6}{M_{Q_i}},\ \overset{3}{M_{L_i}},\ \overset{4(1)}{V_{KM}},\ \overset{1}{\theta},$ 19(1)

$\overset{1}{\xi},\ \overset{1}{m^2_{H_u}},\ \overset{1}{m^2_{H_d}}\quad \overset{2(1)}{\mu}\quad \overset{2(1)}{B}\quad \overset{6(=2\times3)\ (3)}{M_{\lambda_i}}$ 13

$\overset{18(=2\times9)\ (9)}{A^U_{ij}} = \begin{pmatrix} A_{uu} & A_{uc} & A_{ut} \\ A_{cu} & A_{cc} & A_{ct} \\ A_{tu} & A_{tc} & A_{tt} \end{pmatrix} \quad \overset{18(9)}{A^D_{ij}} \quad \overset{18(9)}{A^E_{ij}}$ 54(27)

$\overset{9(3)}{\tilde{m}^2_{Q_{ij}}} = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix} \quad \overset{9(3)}{\tilde{m}^2_{U_{ij}}} \quad \overset{9(3)}{\tilde{m}^2_{D_{ij}}} \quad \overset{9(3)}{\tilde{m}^2_{L_{ij}}} \quad \overset{9(3)}{\tilde{m}^2_{E_{ij}}}$ 45(15)

Rephasing

without μ, $\cancel{\text{SUSY}}$: $U(1)_B,\ U(1)_L^3,\ U(1)_{PQ},\ U(1)_R$

↓ 4 phase !

introduce μ, $\cancel{\text{SUSY}}$: $U(1)_B,\ U(1)_L$

cf. $A^E_{ij} = A\delta_{ij}$, $\tilde{m}^2_{L_{ij}} = \tilde{m}^2_{E_{ij}} = \tilde{m}^2\delta_{ij}$

$U(1)_L^3 \rightarrow U(1)_L^3$: 2 phase

soft SUSY breaking
=2次発散が出ない
SUSY breaking

too many parameters in SUSY (106(43))!!
SUSY breakingをきちんと考えねば！

Prediction & Phenomenology

Prediction & Phenomenology:

☆ light Higgs ＜ 160 GeV

☆ rich flavor & CP physics

lepton flavor violation (LFV) $\mu \to e\gamma$、・・・・・

$B \to K\phi$、・・・・・

EDM, g-2, ・・・・・

R-parity violation

・・・・・・・・

☆ baryogenesis, leptogenesis (AD etc)

☆ GUT, SUGRA, string, M-theory,・・・

☆ non-perturbative effects (SUSY breaking, composite model,…..)

Parameters (125(44)):

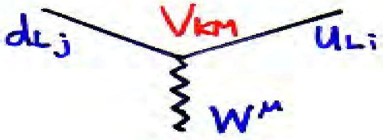
☆ naively too large FCNC, EDM → SUSY breaking (106(43))

☆ μ ?、θ ?、ξ ?

・・・・・

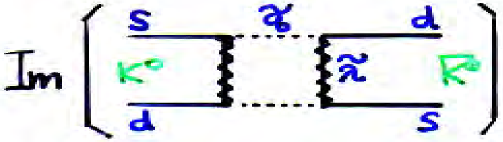
Phenomenological constraint

▷ FCNC ('82 Ellis & Nanopoulos ⋮)

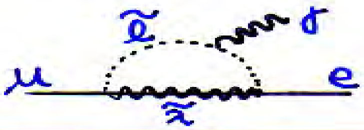


KM

SKM

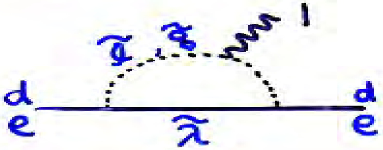


: $\frac{\Delta \tilde{m}_q^2}{\tilde{m}_q^2} \lesssim O(10^{-3})$ (D^0–$\bar{D}^0$, $K_L^0 \to \mu\mu$, $\to \mu e$, $\mu N \to e N$)



: $\frac{\Delta \tilde{m}_l^2}{\tilde{m}_l^2} \lesssim O(10^{-3})$ ($\mu \to e e \bar{e}$)

▷ EDM ($d_e \leq 10^{-27}$ e·cm , $d_N \leq 10^{-25}$ ecm)



:
- $\varphi_{SUSY} \lesssim O(10^{-2})$
- $\tilde{m} \gtrsim O(1)$ TeV

▷ $\mu \simeq O(100)$ GeV ≪ M_{GUT}, M_{Pl}

We need the underlyning theory for SUSY breaking!

SUSY flavor problem（degenerate解以外の解の可能性）

Sfermion masses of the first and second generations are severely constrained by $K^0-\bar{K}^0, \mu\rightarrow e\gamma$ etc.

$$\sin^2\theta_{\tilde{d}}\left(\frac{\Delta m^2_{\tilde{d}}}{\bar{m}^2_{\tilde{d}}}\right)^2\left(\frac{10TeV}{\bar{m}_{\tilde{d}}}\right)^2 \ll 1$$

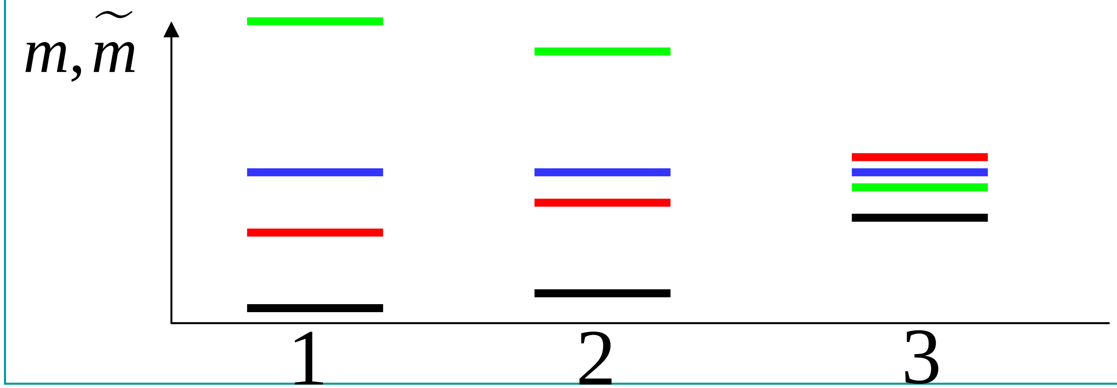
This expression is written in the basis where down sector quarks are diagonalized

① Alignment ($\sin^2\theta_{\tilde{d}} \simeq 0$)
② Degeneracy ($\Delta m^2_{\tilde{d}} \simeq 0$)
③ Decoupling ($\bar{m}_{\tilde{d}} \geq 10TeV$)

Nir,Seiberg (93)

Gabbiani,Gabrielli,Masiero,Silvestrini (96)

Dine,Kagan,Samuel (90), Dimopoulos,Giudice (95), Pomarol,Tommasini (95), Cohen,Kaplan,Nelson (96)



SUSY breaking mechanism scenario

☆ minimal gravity mediation (Polony etc) ←重力
☆ anomaly mediation ←重力
☆ gauge mediation
☆ anomalous U(1) mediation
☆ gaugino mediation -以下高次元のシナリオ-
☆ radion mediation
☆ KK mediation
☆ dilaton dominated scenario ←string inspired model
☆ moduli dominated scenario （ＫＫ） ←string inspired model
☆ SS breaking
・・・・・・・

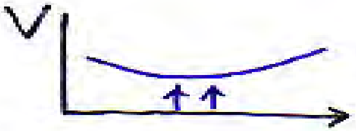
(spectra of KK=spectra of moduli)
(radion = SS in gauged $SU(2)_R$, $U(1)_R$ SUGRA)

SUSY algebra $\{Q, \bar{Q}\} \simeq P$

$\rightarrow H \simeq Q^{+}Q$

$\langle | H | \rangle \simeq \langle | Q^{+}Q | \rangle \geq 0$

$Q| \rangle \neq | \rangle$ ~~SUSY~~



$V = |F|^2 + D^2 \quad > 0$

☆ $D \neq 0$ (Fayet model)

(例) U(1) $\mathcal{L} = \xi \int d^4\theta\, V + \int d^4\theta\, \Phi^{+}\Phi$

$\rightarrow V \simeq (\xi + |\phi|^2)^2$

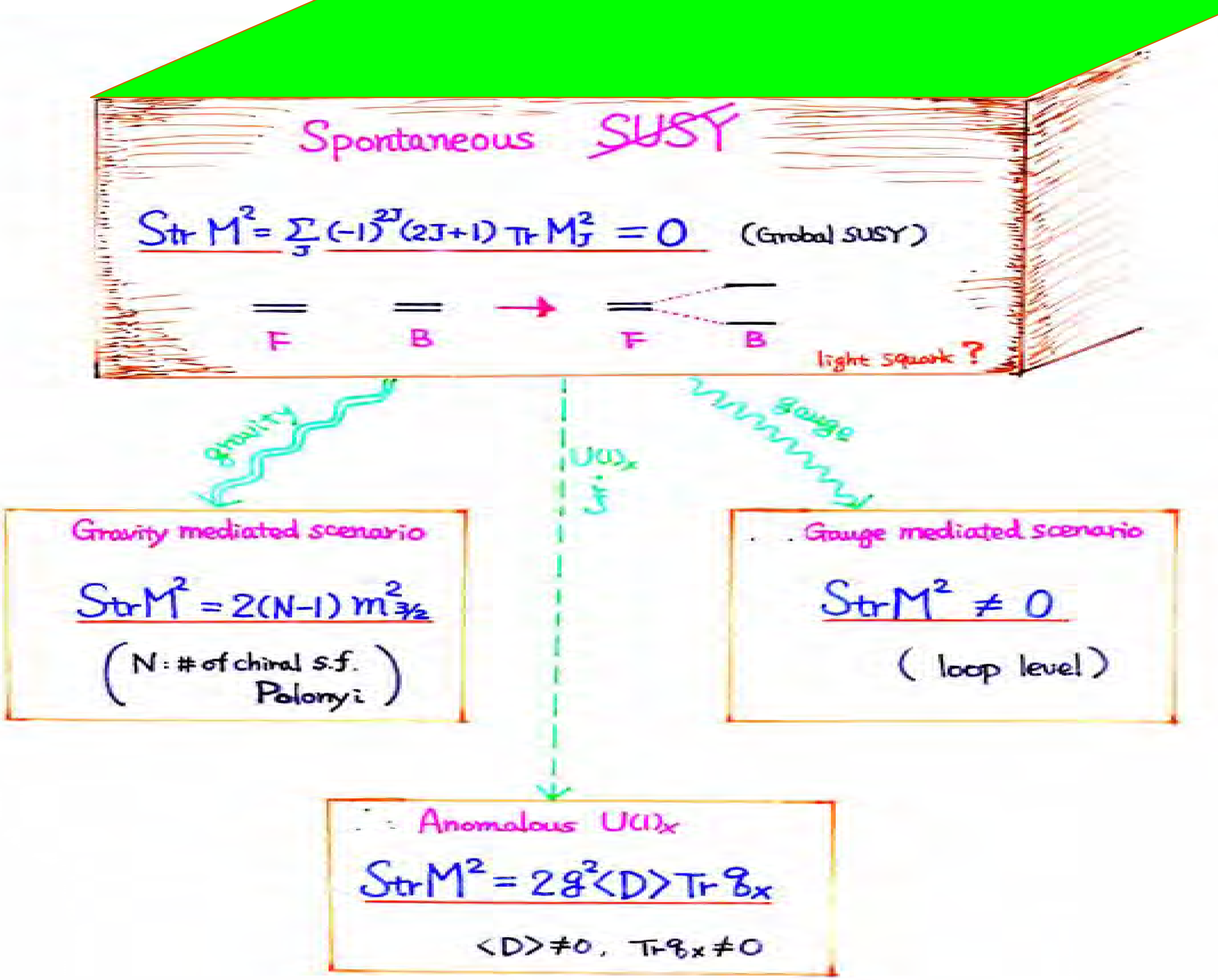
☆ $F \neq 0$ (O'Raifeartaigh model)

(例) ・ $W = \sigma^2 u \quad \rightarrow F_u = \sigma^2$

(例) ・ $W = X(Q^2 - \mu^2) + Q^2 Y$

$\rightarrow F_X = Q^2 - \mu^2,\ F_Y = Q^2,\ F_Q = \underbrace{2(X+Y)Q}_{=0}$

(はじめから ~~SUSY~~ の model。ξ, σ, μ は input parameter。)



Spontaneous ~~SUSY~~

$$\mathrm{Str}\,M^2=\sum_J(-1)^{2J}(2J+1)\,\mathrm{Tr}\,M_J^2=0$$ (Grobal SUSY)

∴ Gravity mediated scenario

$$\mathrm{Str}\,M^2=2(N-1)\,m_{3/2}^2$$

(N : # of chiral s.f.
Polonyi)

∴ Gauge mediated scenario

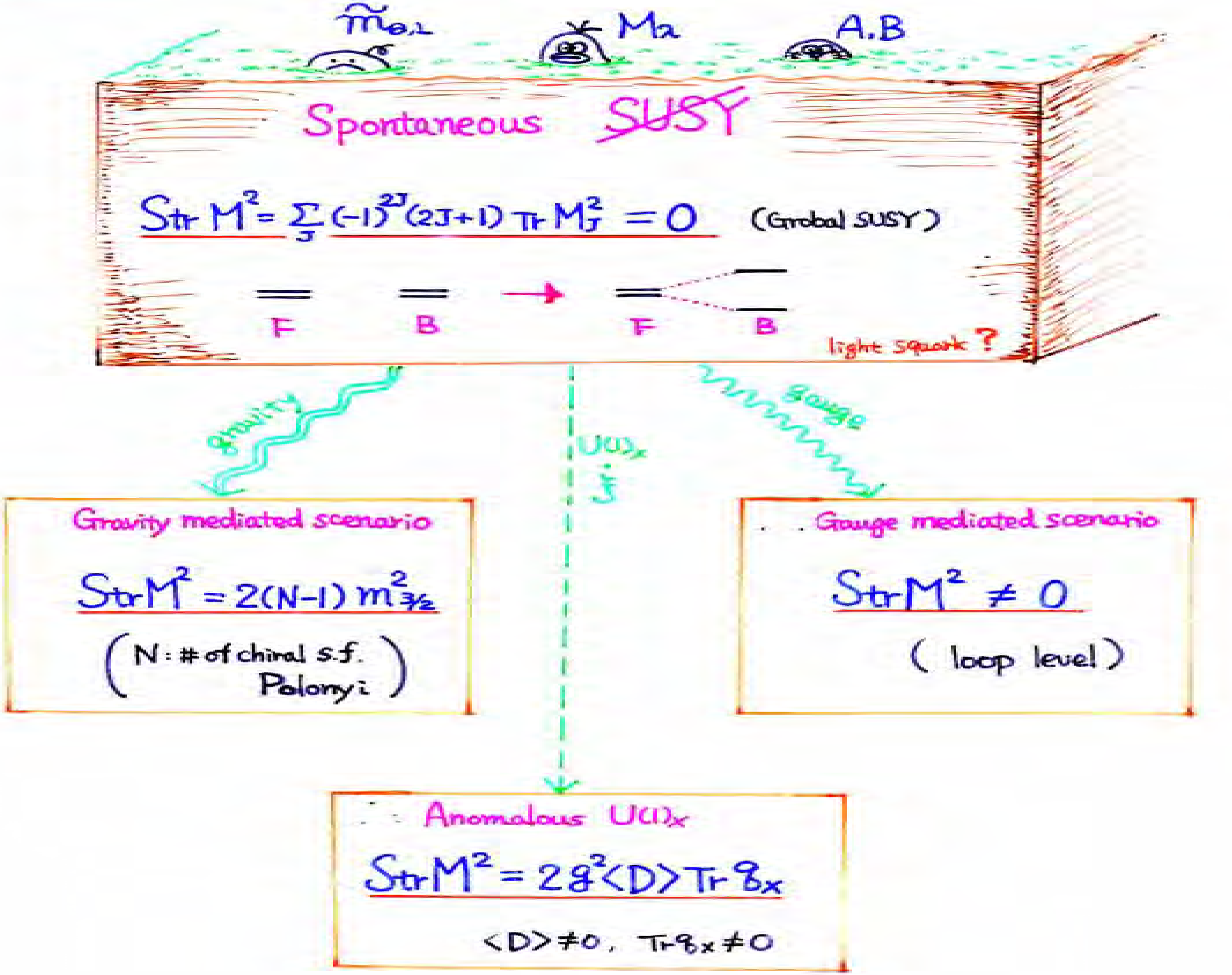
$$\mathrm{Str}\,M^2\neq 0$$

(loop level)

∴ Anomalous U(1)$_X$

$$\mathrm{Str}\,M^2=2g^2\langle D\rangle\,\mathrm{Tr}\,q_X$$

$\langle D\rangle\neq 0$, $\mathrm{Tr}\,q_X\neq 0$



Spontaneous ~~SUSY~~

$$\mathrm{Str}\,M^2 = \sum_J (-1)^{2J}(2J+1)\,\mathrm{Tr}\,M_J^2 = 0$$ (Global SUSY)

Gravity mediated scenario

$$\mathrm{Str}\,M^2 = 2(N-1)\,m_{3/2}^2$$

(N : # of chiral s.f.
Polonyi)

Gauge mediated scenario

$$\mathrm{Str}\,M^2 \neq 0$$

(loop level)

Anomalous $U(1)_X$

$$\mathrm{Str}\,M^2 = 2g^2\langle D\rangle\,\mathrm{Tr}\,q_X$$

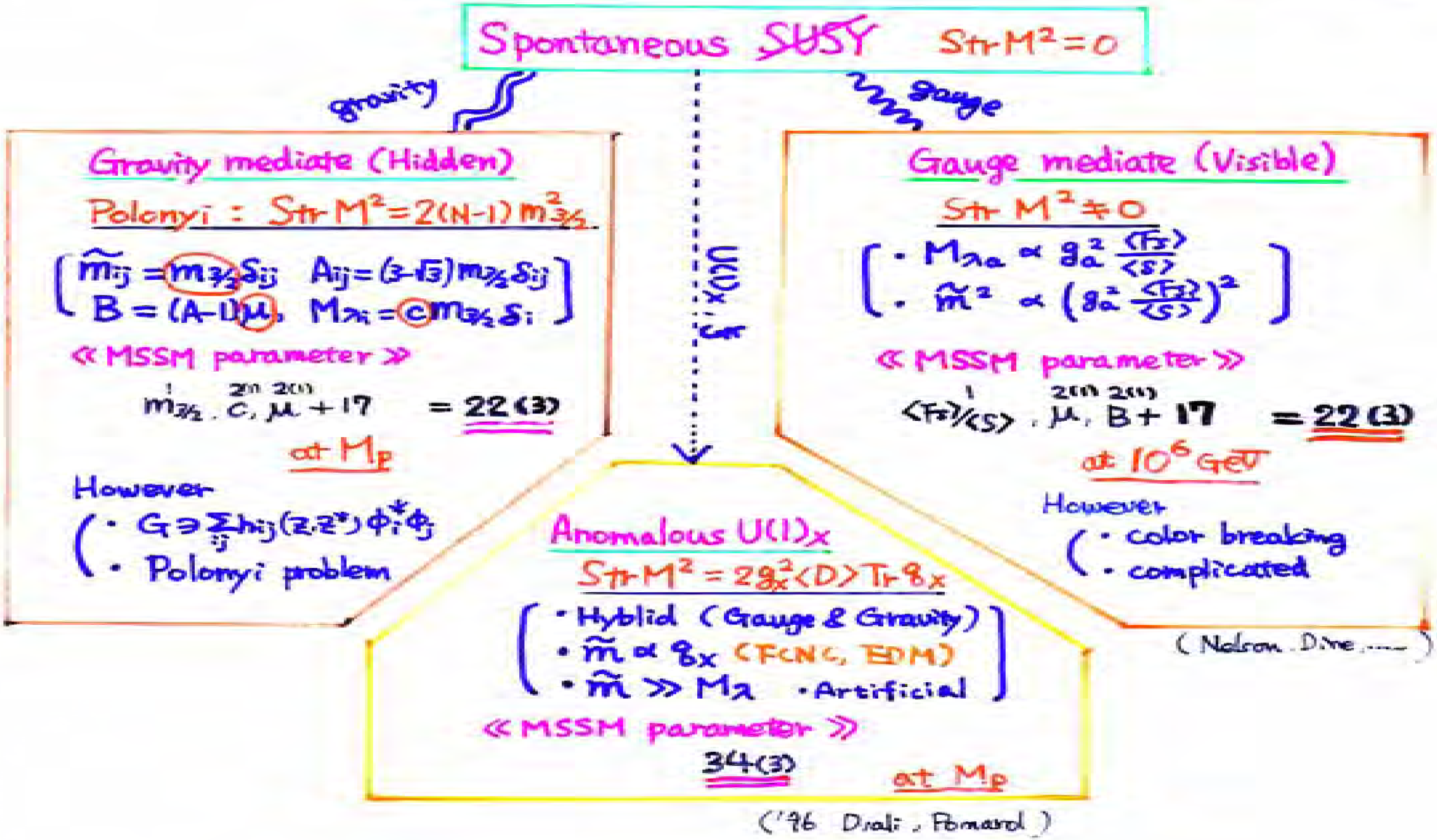
$\langle D\rangle \neq 0$, $\mathrm{Tr}\,q_X \neq 0$

SUSY parameters (125 (44) in MSSM)

- ~~SUSY~~ (106 (43))

[☆ FCNC $\frac{\Delta\tilde{m}^2}{m^2} \lesssim O(10^3)$

☆ EDM $\cdot\ \varphi \le O(10^{-2})$, $(\tilde{m} \ge O(1)\,\mathrm{TeV})$]



Spontaneous ~~SUSY~~ $\mathrm{Str}M^2 = 0$

Gravity mediate (Hidden)

Polonyi : $\mathrm{Str}M^2 = 2(N-1)m_{3/2}^2$

$$\left[\begin{array}{ll} \tilde{m}_{ij} = \boxed{m_{3/2}}\delta_{ij} & A_{ij} = (3-\sqrt{3})m_{3/2}\delta_{ij} \\ B = (A-1)\boxed{\mu}, & M_{\lambda_i} = \boxed{c}\,m_{3/2}\delta_i \end{array}\right]$$

« MSSM parameter »

$\overset{1}{m_{3/2}}\,.\ \overset{2(1)}{c},\ \overset{2(1)}{\mu} + 17 \quad = \underline{22\,(3)}$

at M_P

However

(· $G \ni \sum_{ij} h_{ij}(z,z^*)\phi_i^*\phi_j$

· Polonyi problem

Gauge mediate (Visible)

$\mathrm{Str}M^2 \neq 0$

$$\left[\begin{array}{l} \cdot\ M_{\lambda_a} \propto g_a^2 \frac{\langle F_S\rangle}{\langle S\rangle} \\ \cdot\ \tilde{m}^2 \propto \left(g_a^2 \frac{\langle F_S\rangle}{\langle S\rangle}\right)^2 \end{array}\right]$$

« MSSM parameter »

$\overset{1}{\langle F_S\rangle/\langle S\rangle}\,.\ \overset{2(1)}{\mu},\ \overset{2(1)}{B} + 17 \quad = \underline{22\,(3)}$

at 10^6 GeV

However

(· color breaking

· complicated

(Nelson . Dine ,----)

Anomalous $U(1)_X$

$\mathrm{Str}M^2 = 2g_X^2\langle D\rangle\,\mathrm{Tr}\,q_X$

[· Hyblid (Gauge & Gravity)

· $\tilde{m} \propto q_X$ (FCNC, EDM)

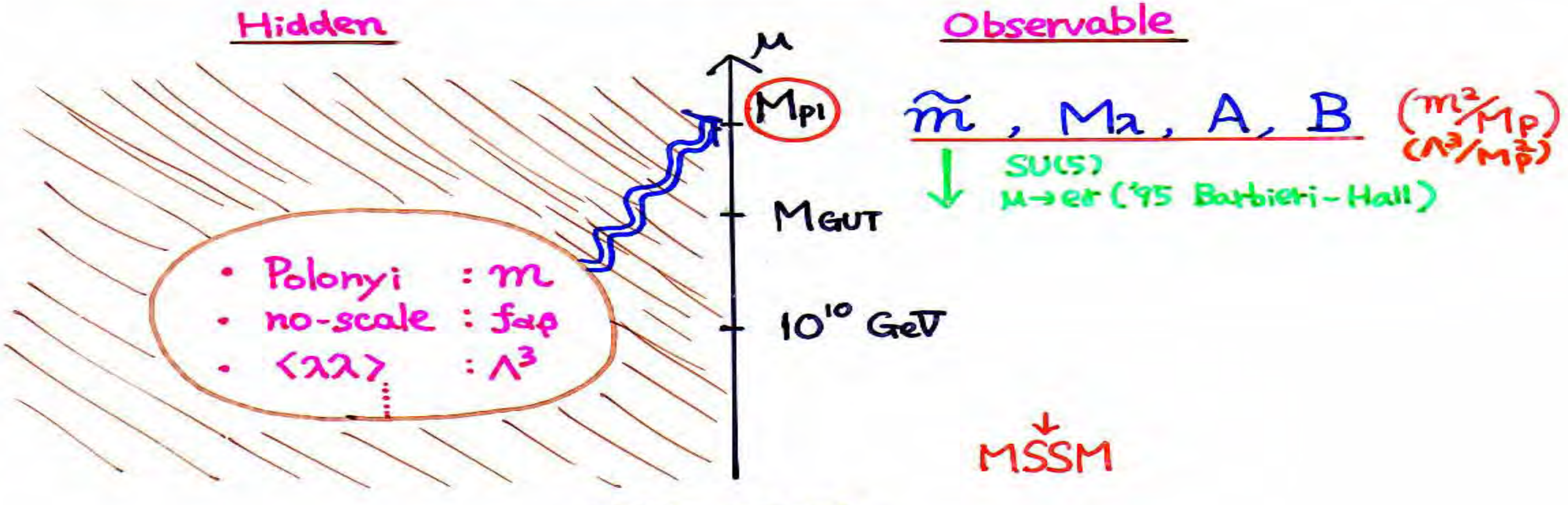
· $\tilde{m} \gg M_\lambda$ · Artificial]

« MSSM parameter »

$\underline{34(3)}$ at M_P

('96 Dvali , Pomarol)

☆ Gravity mediated scenario (Hidden sector)



Hidden

- Polonyi : m
- no-scale : $f_{\alpha\beta}$
- $\langle\lambda\lambda\rangle$: Λ^3

Observable

$\tilde{m}$, M_λ, A, B $\left(m^2/M_P\right)$ $\left(\Lambda^3/M_P^2\right)$

SU(5)

$\mu \to e\gamma$ ('95 Barbieri-Hall)

MSSM

SUGRA

Global	Local
$F_i = \frac{\partial W}{\partial \phi_i}$ $\longrightarrow$	$F_i = e^{G/2}(G^{-1})^j_i G_j - \frac{1}{4}\frac{\partial f_{\alpha\beta}}{\partial \phi_j}(G^{-1})^j_i \lambda^\alpha \lambda^\beta + \cdots\cdots$
$V = \lvert F_i \rvert^2 + \frac{1}{2} D_a^2$ $\longrightarrow$	$V = e^G\{G_k (G^{-1})^k_\ell G^\ell - 3\} + \frac{1}{2}\mathrm{Re}\, f^{-1}_{\alpha\beta} D_\alpha D_\beta$

(G: Kähler potential)

$$\mathcal{L}_{\tilde{\lambda}} = \int d^2\theta \left(f_{\alpha\beta} W^\alpha W^\beta + \text{h.c.} \right)$$

Polonyi

$$\boxed{G = \underline{z_p}^* \underline{z_p} + \phi^{i*}\phi_i + \ln \left| \underline{W_h(z_p)} + W_o(\phi_i) \right|^2}$$

$\underline{W_h(z_p)} = m_h^2 (z_p + \beta)$: $\dfrac{\partial W_h}{\partial z_p} = m_h^2$ (O'Raifeartaigh type)

$m_{3/2} \simeq \dfrac{m_h^2}{M_P} \left(= \dfrac{b m_h^2}{M_P} e^{\frac{a^2}{2}} \right)$ see-saw

$$\left[\begin{array}{ll} \langle z_p \rangle = a M_P & \langle W_h \rangle = b m_h^2 M_P \\ M_P \to \infty \text{ with fixing } m_{3/2} & \\ \langle V \rangle = 0 & (\text{cosmological const.} = 0) \end{array} \right]$$

$$\Downarrow$$

$$\underline{V_{ob} = \left| \frac{\partial \hat{W}_o}{\partial \phi_i} \right|^2 + \underline{m_{3/2}^2 |\phi_i|^2} + \underline{m_{3/2}} \left[\underline{\frac{\partial \hat{W}_o}{\partial \phi_i} \phi_i + (A-3) \hat{W}_o + h.c.} \right]}$$

$$\left(\hat{W}_o \equiv W_o e^{\frac{a^2}{2}},\ A \equiv a\left(a + \frac{1}{b}\right) \right)$$

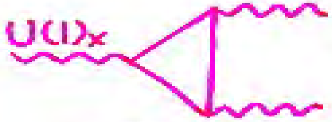
$$\begin{cases} \tilde{m}_{ij} = \underline{m_{3/2}}\, \delta_{ij} \\ A_{ij} = (3 - \sqrt{3})\, \underline{m_{3/2}}\, \delta_{ij} \\ B = (A - 1)\, \underline{m_{3/2}} \cdot \underline{\underline{\mu}} \end{cases} \quad \text{at } M_P$$

$$\underline{\mathrm{Str} M^2 = 2(N-1) m_{3/2}^2}$$

(N : # of chiral s.f.)

☆ Anomalous $U(1)_X$ ($\langle D\rangle \neq 0$, Fayet-Iliopoulos)

Anomalous $U(1)_X$ in SST ('84 Green-Schwarz)



$\mathrm{Tr}\, q_X \neq 0$

$$+ \quad \mathrm{Im}S \to \mathrm{Im}S + \Lambda \qquad = 0$$

$(f_{ab}^{i} = S\,\delta_{ab} k^{i})$

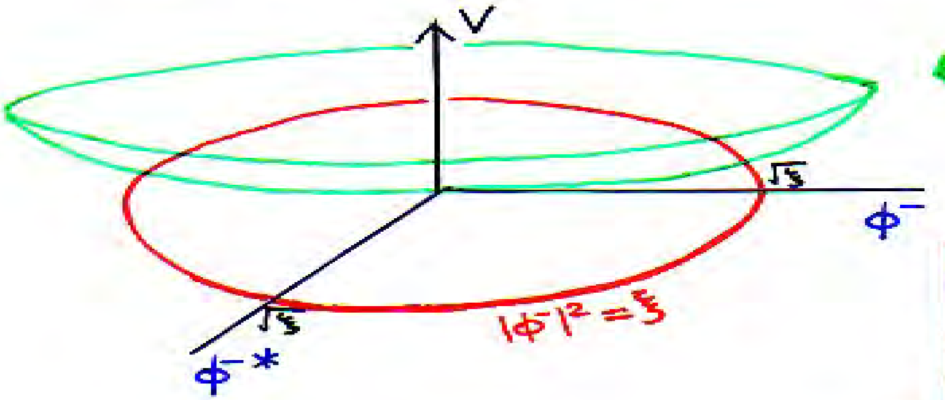
$$\Rightarrow \quad \xi = \frac{g_{ST}^2}{192\pi^2} \mathrm{Tr}\, q_X M_P^2 \equiv \varepsilon M_P^2 \quad (\varepsilon \simeq 10^{-2})$$

Model ('96 Dvali & Pomarol)

	ϕ^+	ϕ^-	Q_i
$U(1)_X$	$+1$	-1	$q_i > 0$

$\mathrm{tr}\, q_i > 0$

$$V_D = \frac{g_X^2}{2} D_X^2 = \frac{g_X^2}{2}\left(\underline{\underline{\xi - |\phi^-|^2}} + |\phi^+|^2 + \sum q_i |Q_i|^2\right)^2$$



$W = m\phi^+\phi^-$
$(V_F = m^2|\phi^-|^2 + m^2|\phi^+|^2)$

SUSY

$\langle\phi^+\rangle = \langle Q_i\rangle = 0, \quad \langle\phi^-\rangle = \sqrt{\xi - \frac{m^2}{g_X^2}} \equiv v$

$\langle D\rangle = \frac{m^2}{g_X^2}, \quad \langle F_{\phi^+}\rangle = mv \quad \langle F_{\phi^-}\rangle = 0$

$$V = \frac{g_x^2}{2}\left(\underbrace{\xi - |\phi^-|^2 + |\phi^+|^2 + \sum q_i|Q_i|^2}\right)^2 + m^2\left(|\phi^+|^2 + |\phi^-|^2\right)$$

$$\downarrow$$

$$\tilde{m}_i^2 = q_{xi} m^2$$

$m_{\phi^-}^2$: $0,\ 2g^2v^2$

$m_{\phi^+}^2$: $2m^2$

A_x^μ : $2g^2v^2$

$c\tilde{\lambda} + s\tilde{\Phi}$: $0 \dashrightarrow 3/2$!

$-s\tilde{\lambda} + c\tilde{\Phi}$: $g\sqrt{\xi - m^2/2g_x^2}$

$$\mathrm{Str}M^2 = 2g_x^2\langle D\rangle \mathrm{tr}\, q_x$$

gravity induced

- $\tilde{m}_{Q_i}^2 \simeq \frac{\langle F_{\phi^+}\rangle^2}{M_P^2} \simeq \varepsilon m^2 \ll q_i m^2$
- $M_{\tilde{\lambda}} \simeq c\frac{\langle F_{\phi^+}\phi^-\rangle}{M_P^2} \simeq c\varepsilon m$ $\left(\leftarrow \int c\frac{\phi^+\phi^-}{M_P^2}W^\alpha W_\alpha d^2\theta\right)$

$$\boxed{\underset{1}{\tilde{m}_{Q_i}} > \underset{\sqrt{\varepsilon}}{\tilde{m}_{Q_i}(\mathrm{grav})} > \underset{\varepsilon}{M_\lambda}}$$

$\left(\frac{\Delta\tilde{m}_Q^2}{\tilde{m}^2} \simeq \varepsilon \quad \text{FCNC}\right)$

(ex. $m \simeq 5\text{TeV}$ — 5TeV $\tilde{m}_{Q_{1,2}}$ naturalness — 500GeV $\tilde{m}_{Q_3}$ — 50 GeV M_λ) (EDM)

≪MSSM parameter≫

$A,\ m,\ \varepsilon,\ c,\ \mu,\ B,\ q_{xi}(Q_i, \bar{U}_i, \bar{D}_i)$ $g : M_a \cdots$

$2(1)\ 1\ \ 1\ \ 2(1)\ 2(1)\ 2(1)\ \ 9\ (= 3\times 3)\ \ +17(1) - 2\ = \underline{34(3)}$

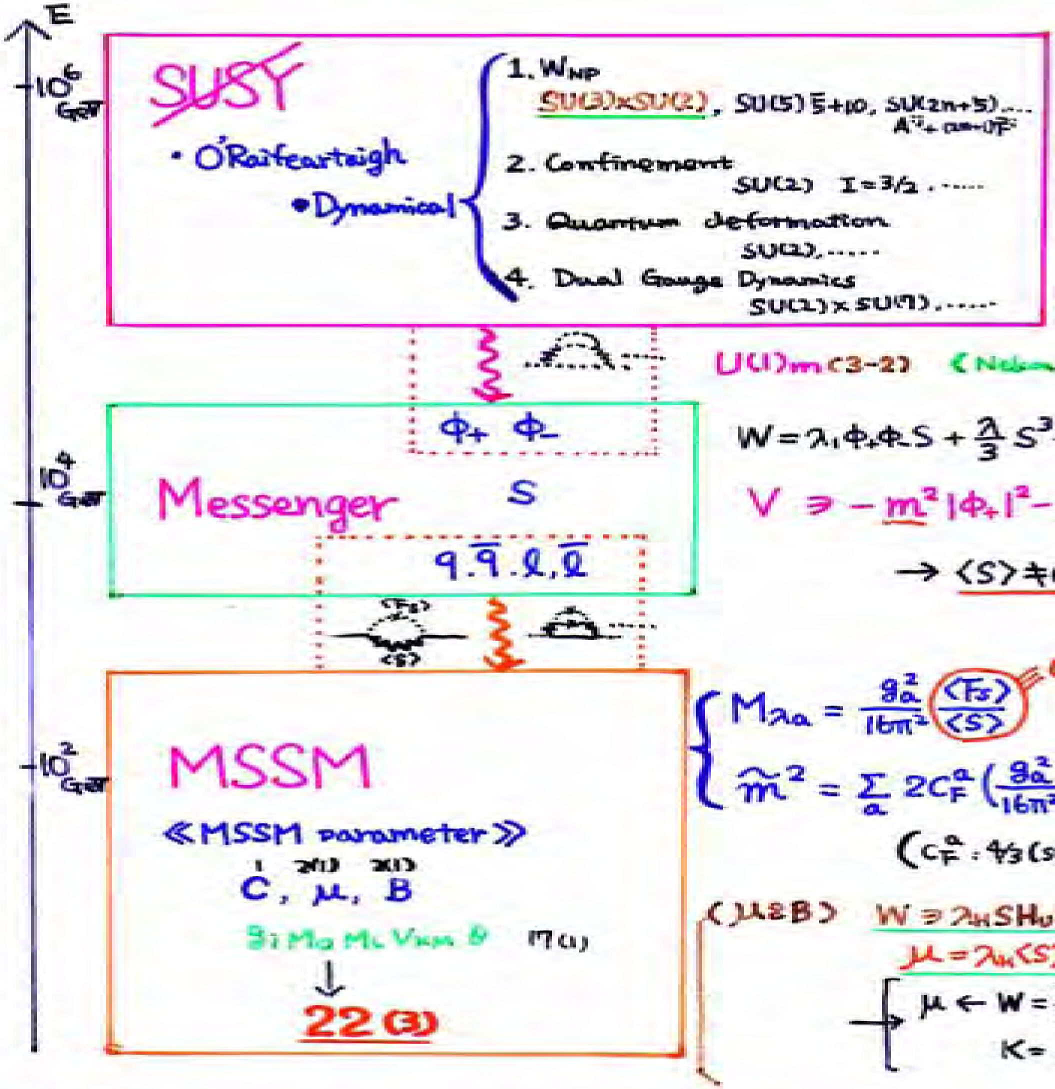
☆ Characteristic features

- Hybrid ($U(1)_X$ & gravity)
- $\tilde{m} \propto q_x$ ⟨FCNC, EDM⟩
- $\tilde{m} \gg M_\lambda$
- no polonyi problem

(▷ m : $SU(2)$ SQCD $N_f = N_c - 1$

$W = \lambda\frac{M}{M_P}\phi^+\phi^- + \frac{\Lambda^5}{M}$ $\quad M \equiv \bar{Q}Q$)

☆ Gauge mediated scenario (Visible sector)



3-2 model SU(3)×SU(2)

$Q, \bar{U}, \bar{D}, L$

$X_1 \equiv Q\bar{D}L$, $X_2 \equiv Q\bar{U}L$, $X_3 = \overset{\det}{(Q\bar{Q})}$

$$W_{3\text{-}2} = \underline{\lambda X_1} + \underline{\frac{\Lambda_3^7}{X_3}}$$

$v \simeq \Lambda/\lambda^{1/7}$

U(1)m (3-2) 《Nelson, Dine...》

$$W = \lambda_1 \phi_+ \phi_- S + \frac{\lambda}{3} S^3 + k S q \bar{q} + k' S \ell \bar{\ell}$$

$$V \ni -\underline{m}^2 |\phi_+|^2 - \underline{m}^2 |\phi_-|^2$$

$$\rightarrow \langle S\rangle \neq 0, \langle F_S\rangle \neq 0, \langle q, \ell\rangle = 0$$

$$\begin{cases} M_{\lambda_a} = \dfrac{g_a^2}{16\pi^2} \boxed{\dfrac{\langle F_S\rangle}{\langle S\rangle}} \equiv C \\ \tilde{m}^2 = \sum_a 2C_F^a \left(\dfrac{g_a^2}{16\pi^2}\right)^2 \boxed{\left(\dfrac{\langle F_S\rangle}{\langle S\rangle}\right)^2} \end{cases}$$

(C_F^a : 4/3 (SU(3)$_c$), 3/4 (SU(2)$_L$), $\frac{3}{5}Y^2$ (U(1)$_Y$))

(μ & B) $W \ni \lambda_H S H_u H_d$

$\mu = \lambda_H \langle S\rangle \ll B\mu = \lambda_H \langle F_S\rangle$

$$\rightarrow \left[\begin{array}{l} \mu \leftarrow W = \dfrac{XS^{2+p}}{M_P^p} + \dfrac{S^{m+3}}{M_P^m} + \dfrac{S^{n+1}}{M_P^n} H_u H_d \\ K = \dfrac{1}{M_P^2} X^\dagger X S^\dagger S \end{array} \right]$$

('95 Dine et al.)

★ Advantage
- predictive & calculable
- $\tilde{m}$ degeneracy (flavor blind) 〈FCNC〉
- no SUSY CP at low energy 〈EDM〉
- no Polonyi problem

★ characteristic feature $m_{3/2} \simeq \langle F\rangle / M_P \simeq 100$ keV (LSP) ($q\bar{q} \to \tilde{z} \to \tilde{G}\gamma\gamma$. $\cancel{E}_T$)

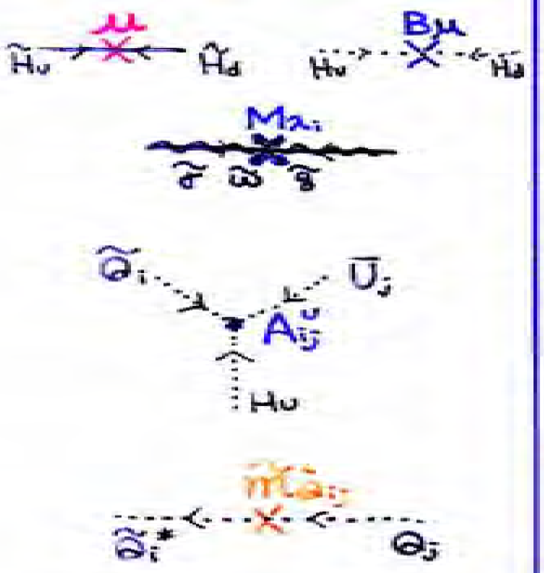
parameter # of MSSM

SM···19 (1)

$[\ \overset{3}{g_i},\ \overset{6}{M_{q_i}},\ \overset{3}{M_{L_i}},\ \overset{4(1)}{V_{KM}},\ \overset{1}{\theta},\ \overset{1}{m_\phi},\ \overset{1}{\lambda}\ ]$

$\tilde{m}_L \sim \tilde{m}_R$, *or parity violation in QCD*

MSSM···125 (44)



$\overset{3}{g_i},\ \overset{6}{M_{q_i}},\ \overset{3}{M_{L_i}},\ \overset{4(1)}{V_{KM}},\ \overset{1}{\theta},$ 17(1)

$\overset{1}{\xi},\ \overset{1}{m^2_{H_u}},\ \overset{1}{m^2_{H_d}}\quad \overset{2(1)}{\mu}\quad \overset{2(1)}{B}\quad \overset{6(2\times 3)\ (3)}{M_{\lambda_i}}$ 13

18 (=2×9) (9) $\quad$ 18 (9) $\quad$ 18 (9) $\quad$ 54(27)

$$A^U_{ij} = \begin{pmatrix} A_{UU} & A_{UC} & A_{UT} \\ A_{CU} & A_{CC} & A_{CT} \\ A_{TU} & A_{TC} & A_{TT} \end{pmatrix} \qquad A^D_{ij} \qquad A^E_{ij}$$

9 (3) $\quad$ 9(3) $\quad$ 9(3) $\quad$ 9(3) $\quad$ 9(3) $\quad$ 45(15)

$$\tilde m^2_{Q_{ij}} = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix} \quad \tilde m^2_{U_{ij}} \quad \tilde m^2_{D_{ij}} \quad \tilde m^2_{L_{ij}} \quad \tilde m^2_{E_{ij}}$$

Rephasing

without μ, ~~SUSY~~ : $U(1)_B,\ U(1)_L^3,\ U(1)_{PQ},\ U(1)_R$

↓ $\qquad$ 4 phase !

introduce μ, ~~SUSY~~ : $U(1)_B,\ U(1)_L$

cf. $A^E_{ij} = A\delta_{ij}$, $\tilde m^2_{L_{ij}} = \tilde m^2_{E_{ij}} = \tilde m^2 \delta_{ij}$

$U(1)_L^3 \rightarrow U(1)_L^3$: 2 phase

soft SUSY breaking
=2次発散が出ない
SUSY breaking

too many parameters in SUSY (106(43))!!

SUSY breakingをきちんと考えねば！

MSSM action

Kinetic term:

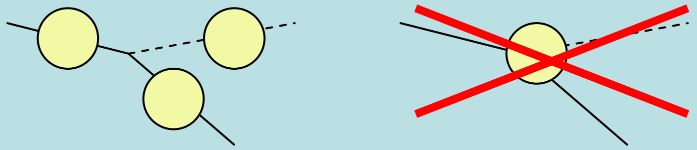
$$L=\underline{\int d^2\theta d^2\bar{\theta}\ Q^\dagger e^{g_3 G+g_2 W+\frac{g_Y}{3}}Q}+\cdots+\underline{\frac{1}{4g_3^2}\int d^2\theta\ G^\alpha G_\alpha}+\cdots+h.c.$$

$$|D\tilde{Q}|^2+iQ\gamma^\mu D_\mu Q+i\sqrt{2}\,\tilde{Q}\tilde{g}\bar{Q}+\cdots \qquad \frac{1}{4g_3^2}G_{\mu\nu}^2+i\tilde{g}\gamma^\mu D_\mu\bar{\tilde{g}}-\mathrm{Im}\left(\frac{1}{4g_3^2}\right)G_{\mu\nu}\widetilde{G}^{\mu\nu}+\cdots$$

Yukawa term:

$$L=\int d^2\theta\ \ W\ +h.c.$$

non-renomalization theorem



$$W=y_u\ QH_uU+y_d\ QH_dD+y_e\ LH_dE+\mu H_uH_d$$

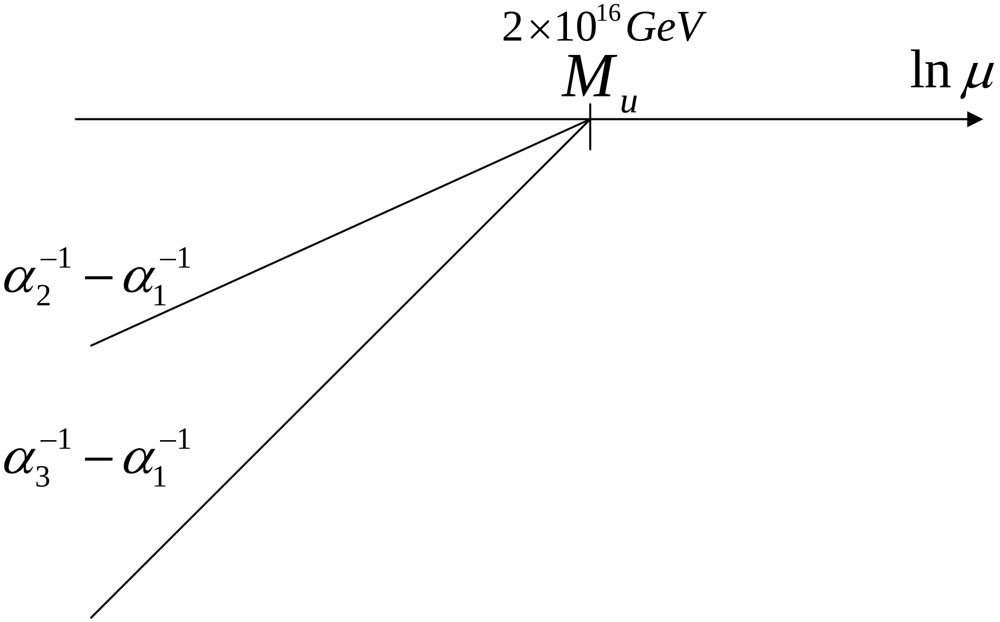
$$+\lambda_d QLD+\lambda_e LLE+\lambda_u UDD \longleftarrow$$ **R－parityで禁止**

$$U(1)_R:\ \theta\to e^{i\alpha}\theta,\ \bar{\theta}\to e^{-i\alpha}\bar{\theta}$$

α＝πがR－parityに対応　(cf. chiral sym.)

Q, L, . . →1、 H→0、 μ項禁止（$U(1)_R$）、R－parityOK

4D usual SU(5) SUSY GUT



$$\alpha_s(M_Z) = 0.1305 + \delta\alpha_s\,|_u + \delta\alpha_s\,|_{SUSY}$$

$$> \alpha_s^{\exp} = 0.117 \pm 0.002$$

$$for\ \ gauge\ \ coupling\ \ unification \Rightarrow M_T < M_u$$

$$for\ \ avoiding\ \ rapid\ \ p-decay \Rightarrow M_T > M_u$$

~~minimal 4D SU(5) SUSY GUT~~

→ How about 5D SU(5) GUT on S_1/Z_2?

5D SU(5) GUT on S_1/Z_2

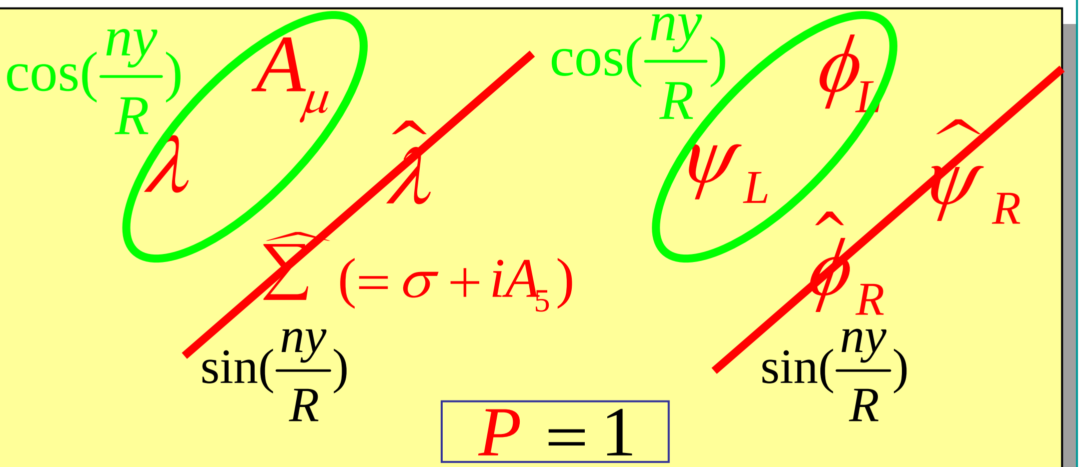
$$P:\ \ \phi(x^\mu,-y)=P\phi(x^\mu,y)$$

$$A_\mu(x^\mu,-y)=PA_\mu(x^\mu,y)P$$

$$A_5(x^\mu,-y)=-PA_5(x^\mu,y)P$$

$$\psi_L(x^\mu,-y)=P\psi_L(x^\mu,y)$$

$$\psi_R(x^\mu,-y)=-P\psi_R(x^\mu,y)$$



$$\boxed{P=1}$$

$$\boxed{T:\ \ diag.(-1,-1,-1,\ 1,\ 1)}$$

$$\left(\begin{array}{c} \cancel{H}_{T,\bar{T}} \\ \hline H_{u,d} \end{array}\right),\quad \left(\begin{array}{c|c} g\quad (\gamma) & \cancel{X,Y} \\ \hline \cancel{X,Y} & W,Z \end{array}\right)$$

$$\phi_{++}\sim\cos(\frac{ny}{R})\qquad \phi_{+-}\sim\cos(\frac{n+1/2}{R}y)$$

$$\phi_{-+}\sim\sin(\frac{ny}{R})\qquad \phi_{--}\sim\sin(\frac{n+1/2}{R}y)$$

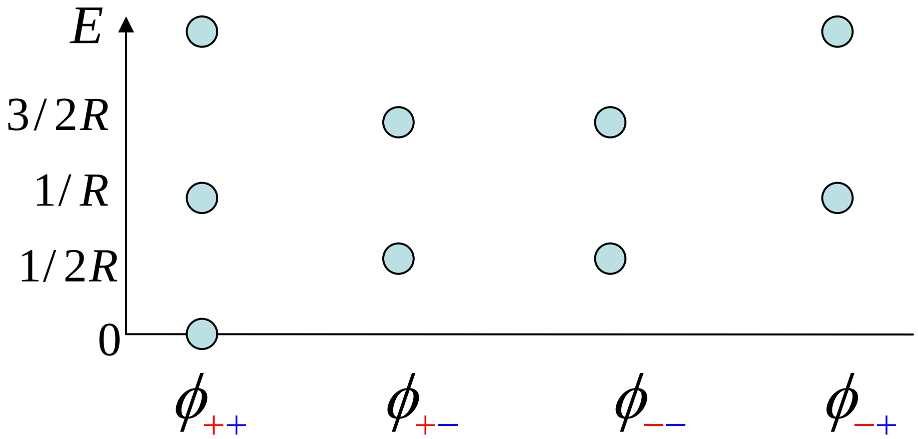
$$P'=TP$$

Parity at y=πR

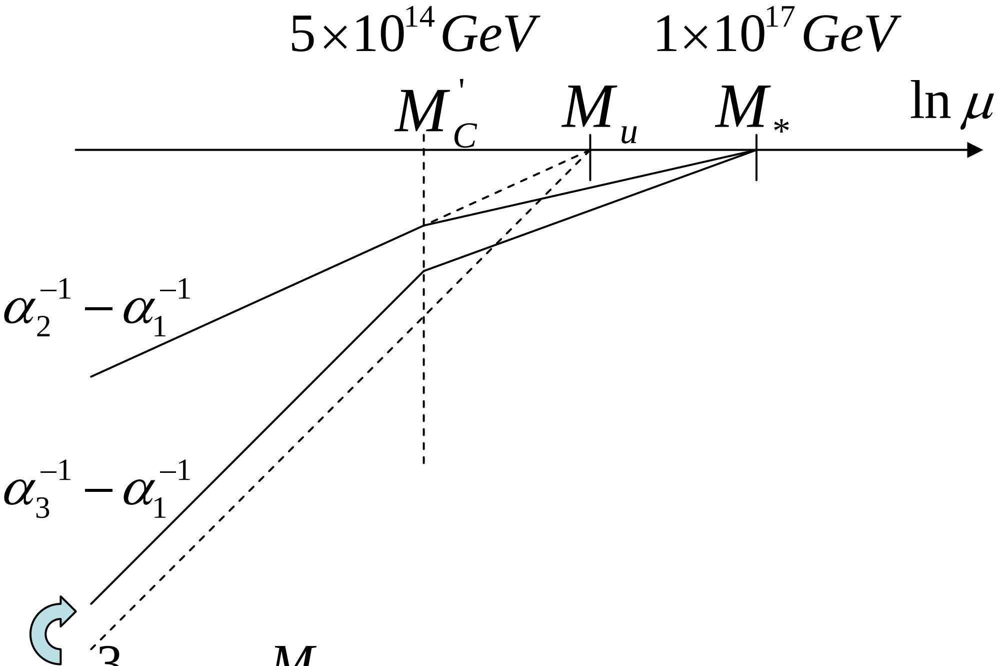
$$\phi(\pi R+y)=T\ \phi(-\pi R+y)=T\ P\ \phi(\pi R-y)$$

$$P':\ \ parity\ at\ \ y'\to -y'$$

$$(y'=\pi R+y)$$



5D SU(5) SUSY GUT (closing to each other by Log correction)

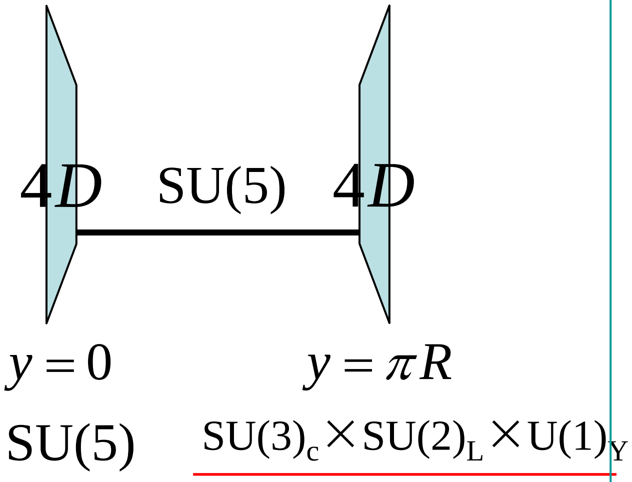


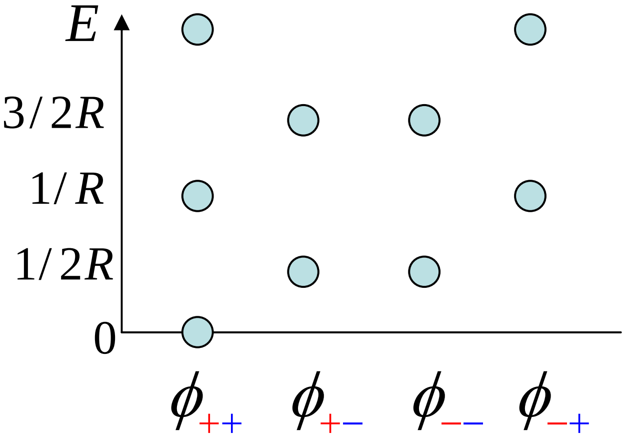
$$\delta\alpha_s \simeq -\frac{3}{7\pi}\alpha_s^2 \ln\frac{M_s}{M_C'}$$

$$\simeq -0.009 \sim -0.015$$

$$M_C' = M_C/\pi$$

$$\frac{M_*}{M_C'} \simeq 200$$





Plan of talk

0. Introduction
1. Standard Model
2. Beyond the SM
    2-1. extra dimensional theory, 2-2. SUSY
3. **ニュートリノ ν**
4. flavor&**質量階層** (**世代**) **構造**
    (quark,lepton**系の違いは何故？**)
5. **大統一理論** (GUT)
6. flavor&**質量階層** (**世代**) **構造　（その２）**
7. Big Questions
    7-1. **世代？**　7-2. 4**次元？**　7-3. **宇宙項？**
8. **素晴らしき未来へ**

3.ニュートリノν

(A).νに着目する理由

SM: m_ν=0 ← no reason!

cf. m_γ=0 ← gauge inv.
$m_{grav.}$=0 ← general cov.

one possibility:
NG-fermion of spontaneous ~~SUSY~~

$$\langle\delta\psi\rangle=\langle\{Q, \psi\}\rangle= F$$

$$J_{\mu\alpha}= F\partial_\mu\psi_\alpha \rightarrow \text{NO !}$$

↔

$$SU(2)_{FL}\times SU(2)_{FR}$$

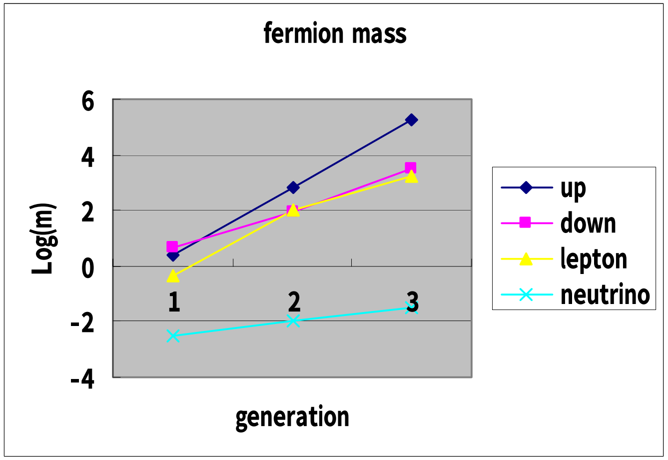
$$\langle\bar{\psi}\psi\rangle = f_\pi$$

$$j_\mu = f_\pi\partial_\mu\phi+\cdots$$

Π is NG-boson!

$m_\nu \neq 0 \Rightarrow$ beyond the SM!!

(B).何故、m_ν<<$m_{Q/L}$なのだろうか？



$$\mathrm{L} \sim \gamma \frac{\nu_L \nu_L \langle \phi \rangle \langle \phi \rangle}{M}$$

lepton数は２破れている

SMの繰り込み可能性

$$\Updownarrow$$

$M \gg M_Z$ and/or $\gamma \ll 1$

☆どうやってdim5 OPを出すか？

(i) see-saw mechanism

M≫M_W：Lepton＃br.
(cf) chiral sym.

$$L_{fund.} \sim \nu_L \phi N + MNN \; + h.c$$

$$\frac{\partial L_{fund.}}{\partial N} = 0$$

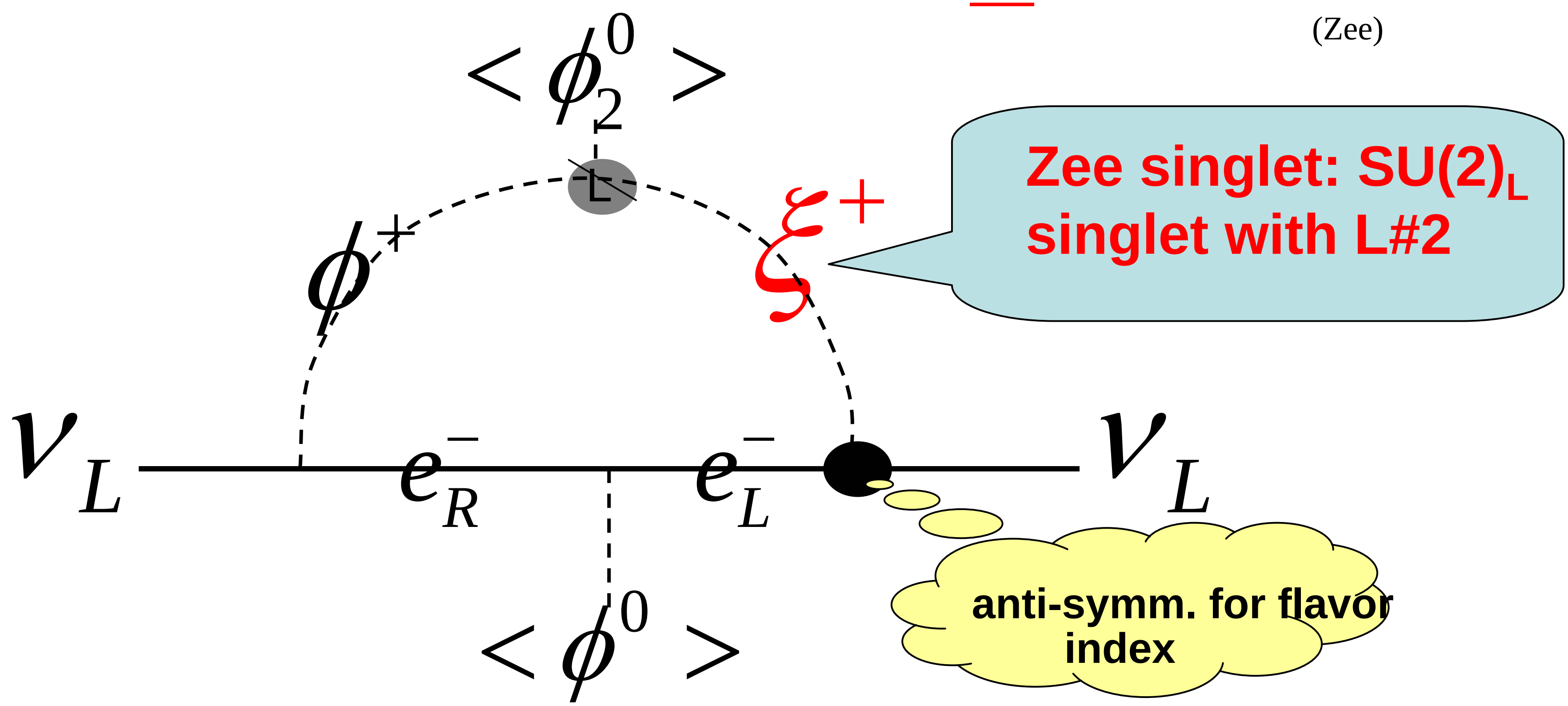
Integrate out heavy N

$$\begin{array}{cc} L & R \\ \end{array}$$

$$\begin{pmatrix} 0 & \langle\phi\rangle \\ \langle\phi\rangle & M \end{pmatrix} \xrightarrow[<\phi><<M]{} \begin{pmatrix} \frac{\langle\phi\rangle^2}{M} & 0 \\ 0 & M \end{pmatrix}$$

(ii) radiative induced m_ν

(Zee)



Zee singlet: SU(2)$_L$ singlet with L#2

anti-symm. for flavor index

$$\gamma \sim \frac{1}{4\pi^2}\,, \qquad M \sim m_{\xi^+}$$

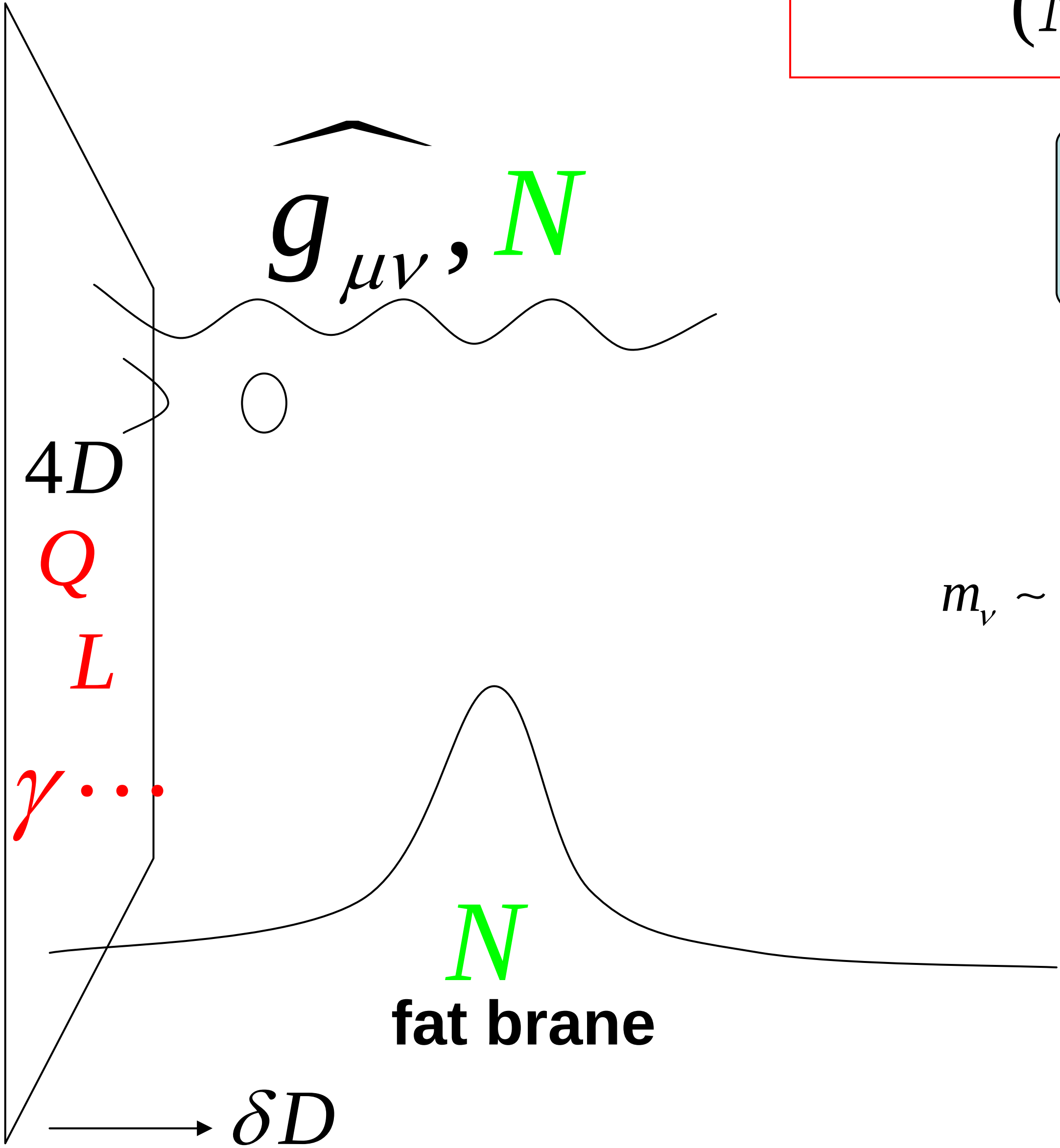
$$m_\nu \sim \begin{pmatrix} 0 & m_{e\mu} & m_{e\tau} \\ m_{\mu e} & 0 & m_{\mu\tau} \\ m_{\tau e} & m_{\tau\mu} & 0 \end{pmatrix}$$

(iii) extra dim.

volume suppression

$$\boxed{m_\nu^D \sim \frac{1}{(M_s R)^{\delta/2}} Y_\nu < \phi >}$$

δ :# of extraD ,
Ms:fund. scale of 4+ δ



$$m_\nu \sim \begin{array}{cc} \begin{array}{cccc} \nu_{N_R}^{(0)} & \nu_{N_R}^{(1)} & \nu_{N_R}^{(2)} & \end{array} & \\ \begin{pmatrix} m_D & m_D & m_D & \cdots \\ 0 & 1/R & 0 & \\ 0 & 0 & 2/R & \\ \vdots & & & \ddots \end{pmatrix} & \begin{array}{c} \nu_L^{(0)} \\ \nu_{N_L}^{(1)} \\ \nu_{N_L}^{(2)} \\ \end{array} \end{array}$$

distant suppression

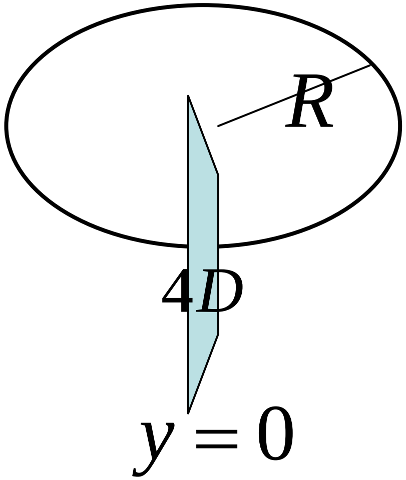
$$\boxed{m_\nu^D \propto e^{-(y-y_0)^2}}$$

☆volume suppression

$(1):\ M^4 \otimes S^1$

$$T:\ \phi(x^\mu, y + 2\pi R) = T\phi(x^\mu, y)$$

$[T \in U(N)]$

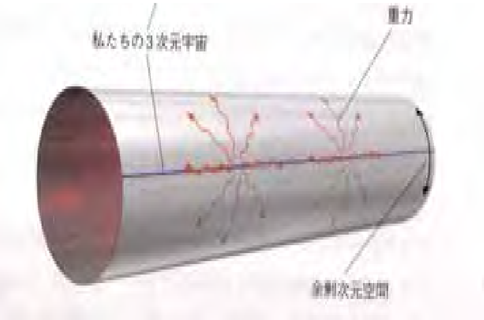


$$\phi(x^\mu, y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=-\infty}^{\infty} \phi^{(n)}(x^\mu)\ e^{i\frac{n}{R}y}$$

$$\int_{y=0}^{2\pi R} dy \int dx^\mu \,|(\partial^\mu + \partial^y + ig_5 A^\mu + ig_5 A^5)\phi(x^\mu, y)|^2$$

$$(g_4 = \frac{g_5}{\sqrt{2\pi R}})$$

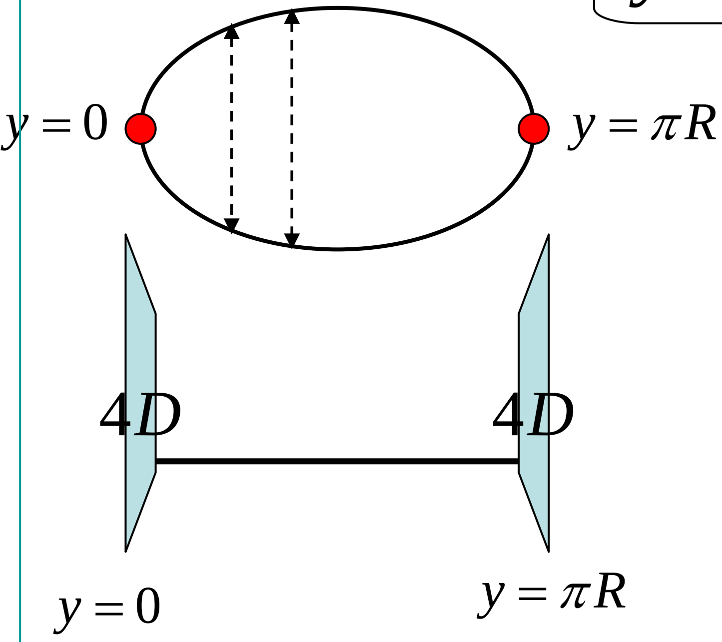
$$\Rightarrow \int dx^\mu \,|(\partial^\mu + ig_4 A^\mu + \underset{\substack{\Uparrow \\ \text{adjoint scalar}}}{ig_4 A^5})\phi^{(n)}(x^\mu)|^2 + \underset{\substack{\Uparrow \\ \text{KK mass}}}{(\frac{n}{R})^2} |\phi^{(n)}(x^\mu)|^2$$



(図：サイエンスより)

(2): $M^4 \otimes S^1 / Z^2$

$y = -y$



$$P:\ \phi(x^\mu, -y) = P\phi(x^\mu, y)$$

$[P^2 = 1 \ \because \phi(y) = P\phi(-y) = P^2\phi(y)]$

$$\phi_+(x^\mu, y) = \frac{1}{\sqrt{2^{\delta_{n,0}}\pi R}} \sum_{n=0}^{\infty} \phi_+^{(n)}(x^\mu)\ \cos(\frac{ny}{R})$$

$$\phi_-(x^\mu, y) = \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \phi_-^{(n)}(x^\mu)\ \sin(\frac{ny}{R})$$

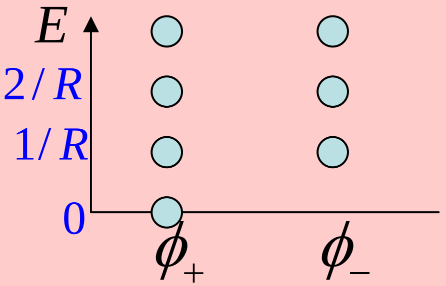
$$\int_{y=0}^{2\pi R} dy \int dx^\mu \,|(\partial^M + ig_5 A^M)\phi(x^\mu, y)|^2 \quad (g_4 = \frac{g_5}{\sqrt{2\pi R}})$$

$$\int_{y=0}^{2\pi R} dy\ \cos(\frac{ny}{R})\cos(\frac{my}{R}) = \int_{y=0}^{2\pi R} dy\ \frac{1}{2}[\cos(\frac{(n+m)y}{R}) + \cos(\frac{(n-m)y}{R})] = \frac{1}{2}(\delta_{n,m} + \delta_{n,-m})$$

$$\int_{y=0}^{2\pi R} dy\ \cos(\frac{ny}{R})\cos(\frac{my}{R})\cos(\frac{ky}{R}) = \frac{1}{4}(\delta_{n,m,k} + \delta_{n,-m,k} + \delta_{n,m,-k} + \delta_{n,-m,-k})$$

$$\phi^{(n)} - \phi^{(n)} - A^{(0)}:\ g_4$$

$$\phi^{(n)} - \phi^{(n)} - A^{(n)}:\ \frac{g_4}{\sqrt{2}}$$



$$A_\mu(x^\mu, -y) = PA_\mu(x^\mu, y)P$$

$$A_5(x^\mu, -y) = -PA_5(x^\mu, y)P$$

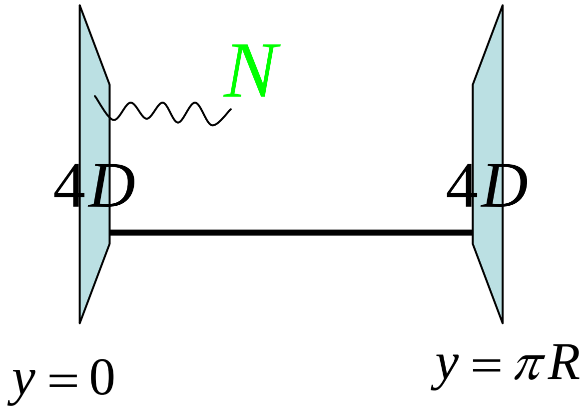
$$\psi_L(x^\mu, -y) = P\psi_L(x^\mu, y)$$

$$\psi_R(x^\mu, -y) = -P\psi_R(x^\mu, y)$$

$[\psi(x^\mu, -y) = Pi\gamma^y\psi(x^\mu, y)]$

5D: $\gamma^M = (\gamma^\mu, i\gamma^5)$

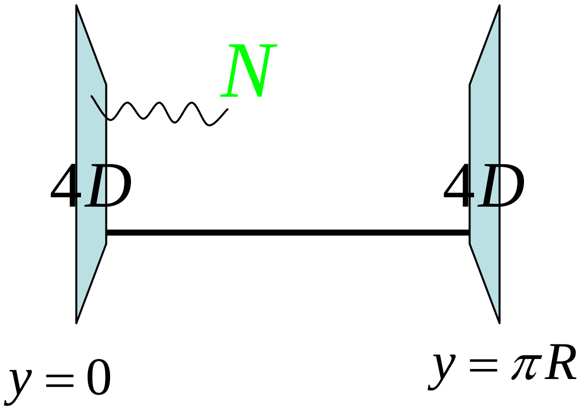
How about Yukawa?



$$W_Y = \frac{1}{\sqrt{M_*}} \int_{y=0}^{2\pi R} dy\, \delta(y)\, \overline{\psi_L}(x^\mu)\, \phi_+(x^\mu)\, \underline{\psi_R^{(n)}(x^\mu, y)}$$

How about Yukawa?

volume suppression!



$$W_Y = \frac{1}{\sqrt{M_*}} \int_{y=0}^{2\pi R} dy\, \delta(y)\, \overline{\psi_L}(x \quad \phi_+(x^\mu)\, \frac{1}{\sqrt{2^{\delta_{n,0}}\pi R}}\, \psi_R^{(n)}(x^\mu)\, \cos(\frac{ny}{R})$$

$$\Rightarrow\ W_Y^{4D} = \frac{1}{\sqrt{2\pi R M_*}}\, \overline{\psi_L}(x^\mu)\, \phi_+(x^\mu)\, \psi_R^{(0)}(x^\mu)$$

$$(\ \Rightarrow\ W_Y^{4D} = (\frac{1}{\sqrt{\pi R M_*}})^3\, \overline{\psi_L}^{(n)}(x^\mu)\, \phi_+^{(n)}(x^\mu)\, \psi_R^{(n)}(x^\mu))$$

large extraD

$$M_{pl}^2 = M_*^{2+\delta} R^\delta$$

$$\Rightarrow (\frac{1}{\sqrt{M_* R}})^\delta = \frac{M_*}{M_{pl}} (\sim 10^{-15})$$

$$\rightarrow m_\nu \sim 10^{-4} eV$$

$$m_\nu \sim \begin{array}{cccccc} \nu_{N_R}^{(0)} & \nu_{N_R}^{(1)} & \nu_{N_R}^{(2)} & & & \\ m_D & m_D & m_D & \cdots & & \nu_L^{(0)} \\ 0 & 1/R & 0 & & & \nu_{N_L}^{(1)} \\ 0 & 0 & 2/R & & & \nu_{N_L}^{(2)} \\ \vdots & & & \ddots & & \end{array}$$

∞個のsterile ν ($\Theta_{\alpha s}$〜1/R)

small mixing MSW for $\nu_e - \nu_s$

☆distant suppression

$5D:\ \gamma^M = (\gamma^\mu, i\gamma^5)$

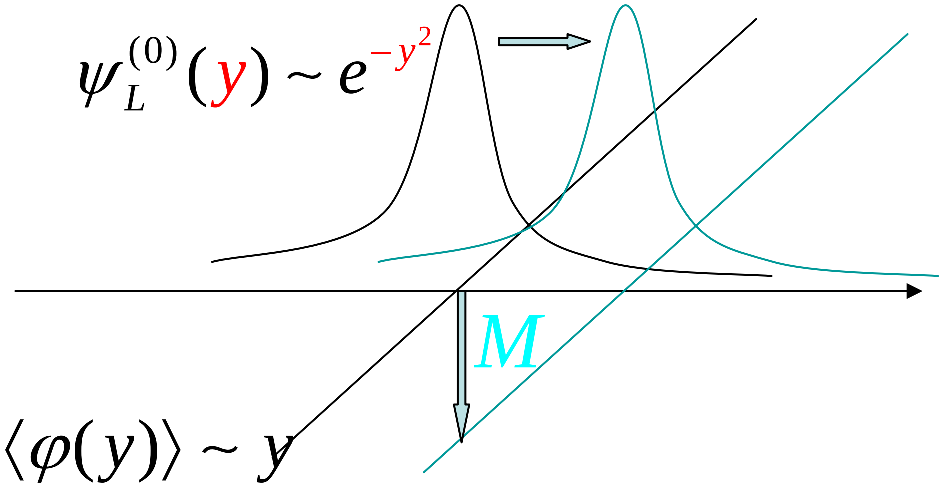
$$i\gamma^M\partial_M = \begin{pmatrix}\partial_y I & \sigma^\mu\partial_\mu \\ \overline{\sigma}^\mu\partial_\mu & -\partial_y I\end{pmatrix} \quad \begin{pmatrix}L \\ R\end{pmatrix} \quad \gamma^5 = \begin{pmatrix}-1 & \\ & 1\end{pmatrix} \quad \begin{aligned}\sigma^\mu &= (1,\sigma^i), \\ \overline{\sigma}^\mu &= (1,-\sigma^i)\end{aligned}$$

$$(i\gamma^\mu\partial_\mu - \gamma^5\partial_5 - \langle\varphi(y)\rangle)\ \psi(x^\mu, y) = 0$$

$$\Rightarrow \quad \psi_L(x^\mu, y) \sim e^{-\int\langle\varphi(y)\rangle dy}\psi_L^{(0)}(x^\mu)$$

chiral projection!

$N=2 \rightarrow N=1$

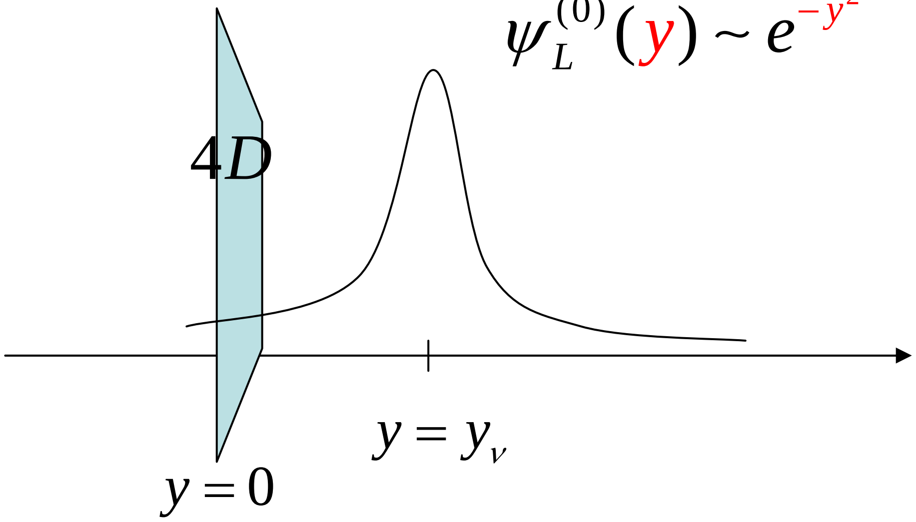


for examples,

$\langle\varphi(y)\rangle \sim y \ \rightarrow\ \psi_L \sim e^{-y^2}$

$\langle\varphi(y)\rangle \sim \varepsilon(y) \ \rightarrow\ \psi_L \sim e^{-|y|}$

$\cdots$



$$\psi_L^{(0)}(y) \sim e^{-y^2}$$

$$m_\nu \sim e^{-y_\nu{}^2 M_*^2} \langle H_u \rangle$$

distant suppression!

$$\to e^{-y_\nu{}^2 M_*^2} \sim 10^{-(12\sim14)}$$

$$\to y_\nu \sim (6\sim7)\ \ M_*^{-1}$$

hierarchy = distance of extraD

(C). ν 振動

lepton	quark
$\begin{pmatrix}\nu_e\\\nu_\mu\\\nu_\tau\end{pmatrix}_{\underset{\text{int.}}{\alpha}} = \left(\quad ? \quad \right)\begin{pmatrix}\nu_e\\\nu_\mu\\\nu_\tau\end{pmatrix}_{\underset{\text{mass}}{i}}$ 	$\begin{pmatrix}d\\s\\b\end{pmatrix}_\alpha = \begin{pmatrix}1 & \lambda & \lambda^3\\ \lambda & 1 & \lambda^2\\ \lambda^3 & \lambda^2 & 1\end{pmatrix}\begin{pmatrix}d\\s\\b\end{pmatrix}_i$ 

$$|\nu_\alpha(t)\rangle = \sum_i V^{MNS}_{\alpha i}\, e^{-iE_i t}\,|\nu_i(0)\rangle$$

$$P_{\alpha\to\beta} = |\langle \nu_\beta(t)|\nu_\alpha(0)\rangle|^2 = \left|\sum_i V_{\alpha i}(V^\dagger)_{i\beta}\, e^{-iE_i t}\right|^2$$

ex.) $V_{MNS} = \left(\begin{array}{cc} \nu_e & \nu_\mu \\ \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{array}\right)$

$$P_{\nu_e\to\nu_e} = 1 - \underline{\sin^2 2\theta}\ \underline{\sin^2\left(\frac{\Delta m^2}{4E}t\right)} \qquad \Delta m^2 \equiv m_2^2 - m_1^2$$

$$\rightarrow \underline{1-\frac{1}{2}\sin^2 2\theta \qquad \text{(time average)}}$$

$\boxed{\text{Q.}}$ Why we can not observe quark oscillation???

(C). ν 振動

lepton	quark
$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}_{\alpha} = \begin{pmatrix} & ? & \end{pmatrix}\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}_{i}$ int. mass	$\begin{pmatrix}d\\ s\\ b\end{pmatrix}_{\alpha} = \begin{pmatrix}1 & \lambda & \lambda^3\\ \lambda & 1 & \lambda^2\\ \lambda^3 & \lambda^2 & 1\end{pmatrix}\begin{pmatrix}d\\ s\\ b\end{pmatrix}_{i}$
	

$$|\nu_\alpha(t)\rangle = \sum_i V_{\alpha i}^{MNS}\, e^{-iE_i t}\, |\nu_i(0)\rangle$$

$$P_{\alpha\to\beta} = |\langle\nu_\beta(t)|\nu_\alpha(0)\rangle|^2 = \left|\sum_i V_{\alpha i}(V^\dagger)_{i\beta}\, e^{-iE_i t}\right|^2$$

ex.)
$$V_{MNS} = \begin{array}{cc} \nu_e & \nu_\mu \\ \end{array}\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}$$

$$P_{\nu_e\to\nu_e} = 1 - \underline{\sin^2 2\theta}\, \sin^2\underline{\left(\frac{\Delta m^2}{4E}t\right)} \qquad \Delta m^2 \equiv m_2^2 - m_1^2$$

$$\to \underline{1 - \frac{1}{2}\sin^2 2\theta} \qquad \text{(time average)}$$

$\boxed{Q.}$ Why we can not observe quark oscillation ???

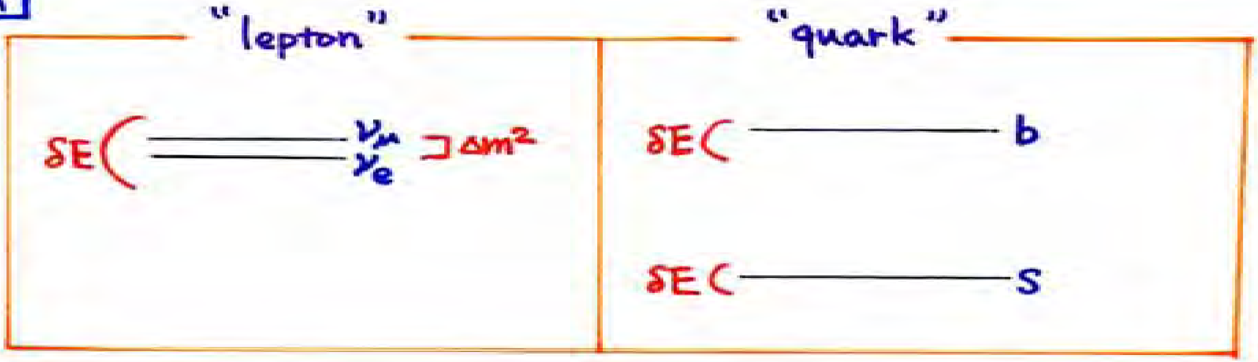
$\boxed{A'}$ decay to the light Q/L ??

typical life time $\tau \sim \alpha_s^{-2}, G_F^{-2}$ $(\tau_{\pi^\pm} \sim 10^{-8}(s), \tau_{K^\pm} \sim 10^{-8}(s), \tau_{B^\pm} \sim 10^{-12}(s))$

$$\frac{\Delta m^2}{4E} \sim \frac{1\,\mathrm{GeV}^2}{100\,\mathrm{GeV}} \sim 10^{-2}\,\mathrm{GeV} \sim \frac{10^{-2}\,\mathrm{GeV}}{2\times10^{-16}\,\mathrm{GeV}\cdot s} \sim 10^{14}\,(1/s)$$

$(\hbar c \sim 2\times10^{-16}\,\mathrm{GeV}\cdot s)$

A



$$|\nu_\alpha(t)\rangle = \sum_i V^{NMS}_{\alpha i}\, \underline{e^{-iE_it}}\, |\nu_i(0)\rangle$$

$$\boxed{\delta E\cdot\delta t \gtrsim \hbar}$$

uncertainty principle of QM.

* If we can measure $\delta E\left(\text{———}\,\nu_\mu\right.$, $\delta E\left(\text{———}\,\nu_e\right.$ $\}$ do not interfere! ν oscillation disappear!

$$\left(\begin{matrix}\delta E \to 0\\ \delta t \to \infty\end{matrix}\right.$$

: time information is lost!
(position)

$$P_{\nu_e\to\nu_e} = |\langle\nu_e|1|\nu_e\rangle|^2 + |\langle\nu_e|2|\nu_e\rangle|^2 = \cos^4\theta + \sin^4\theta$$

$$= \underline{1 - \frac{1}{2}\sin^2 2\theta}$$

added incoherently

4. flavor&質量階層 (世代) 構造(quark,lepton系の違いは何故？)

世代構造の実験date

CKM (quark**系**) **情報**：

加速器実験

MNS (lepton**系**) **情報**：

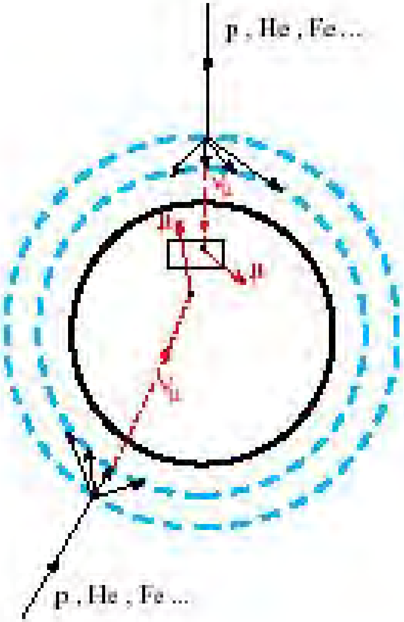
ν振動実験

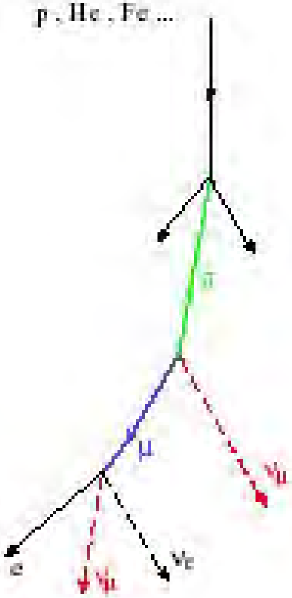
atmospheric → 2-3**世代**

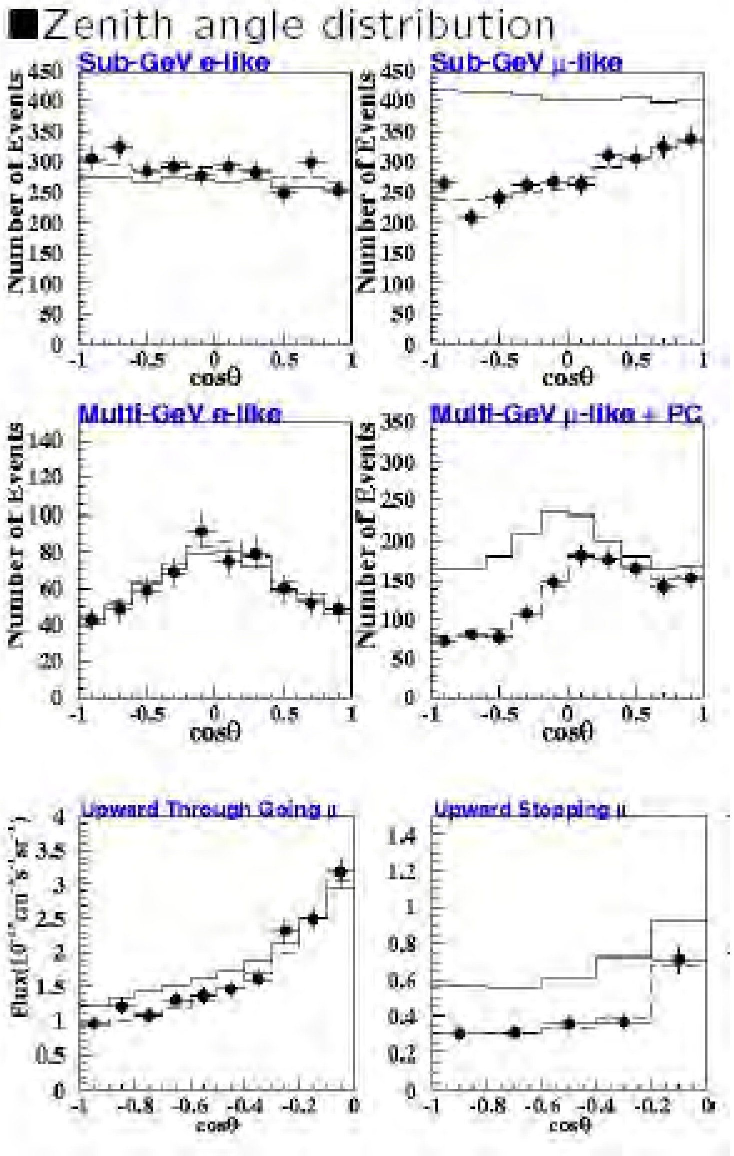
solar → 1-2**世代**

原子炉(CHOOZ) → 1-3**世代**

atmospheric → 2-3世代



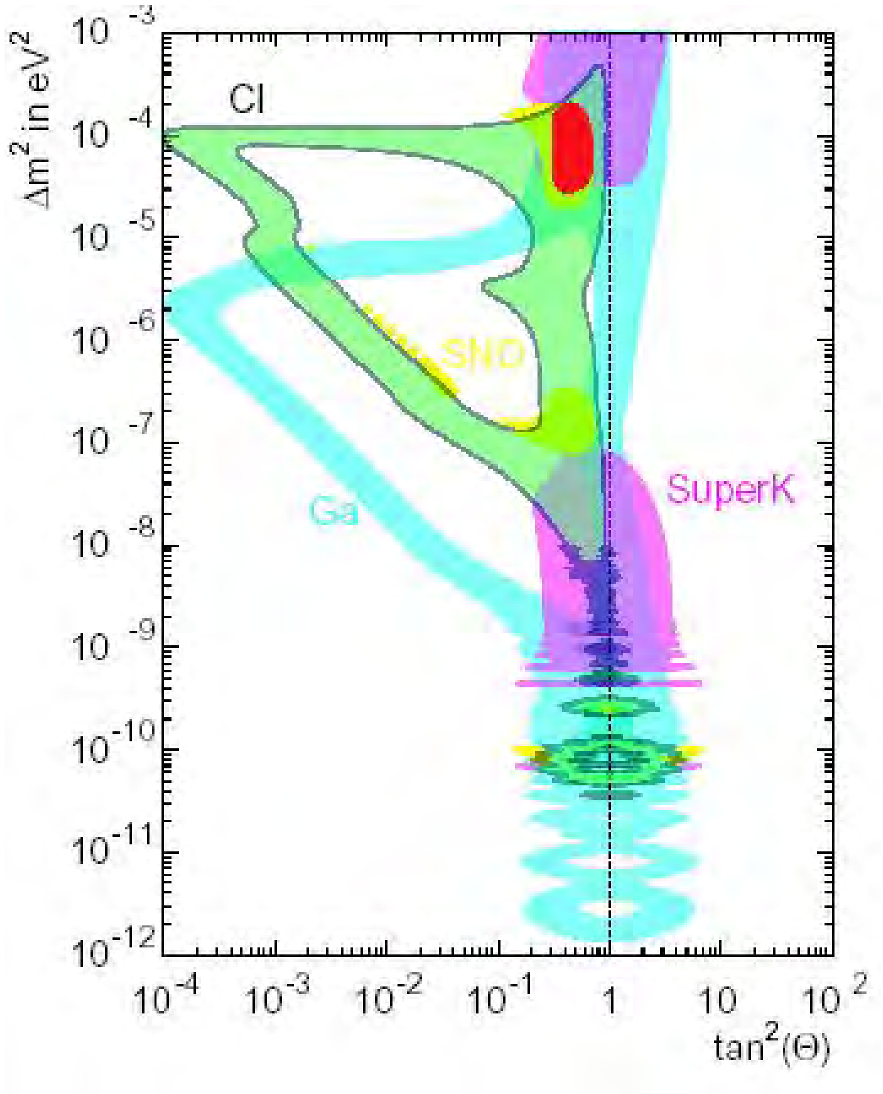


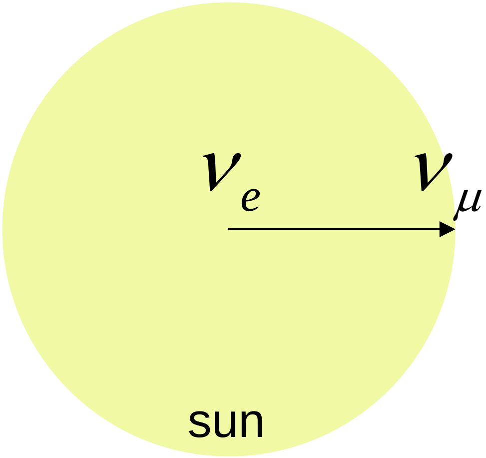


理論的：$\nu_\mu : \nu_e = 2 : 1$

(T.Ohta's OHP)

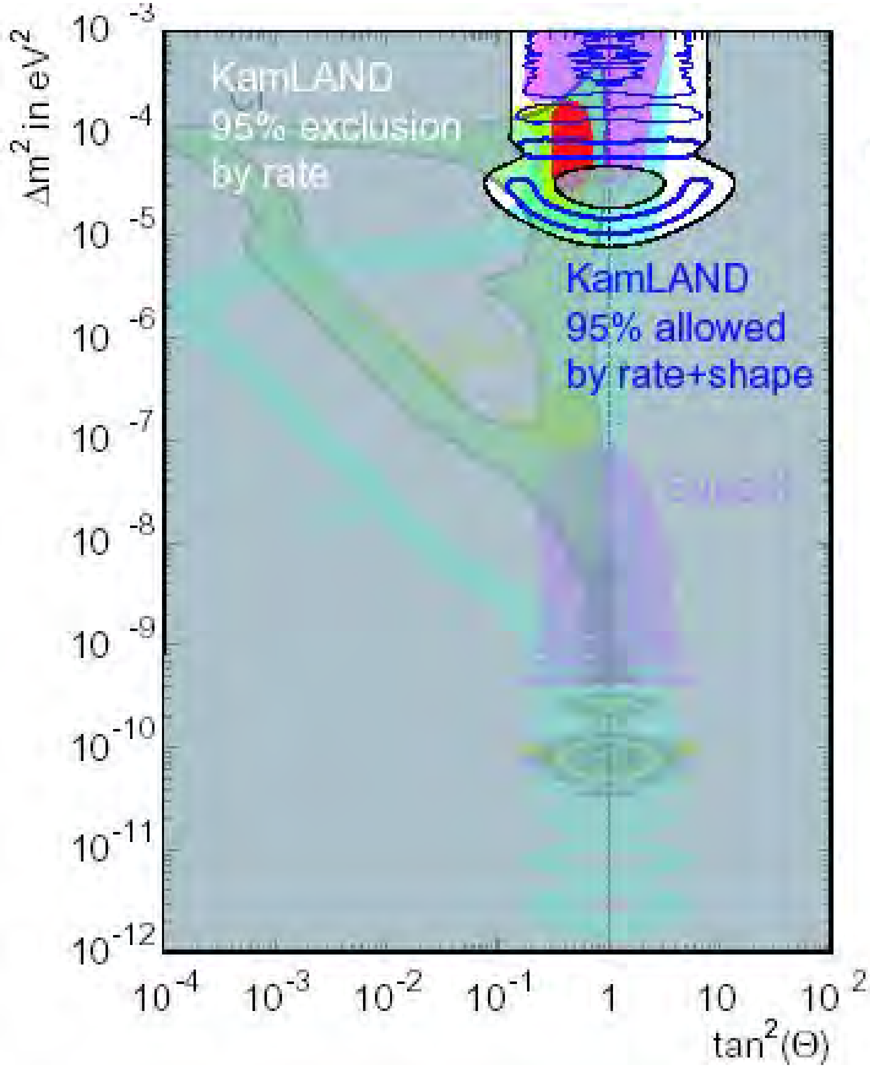
solar → 1-2世代





(T.Ohta's OHP)

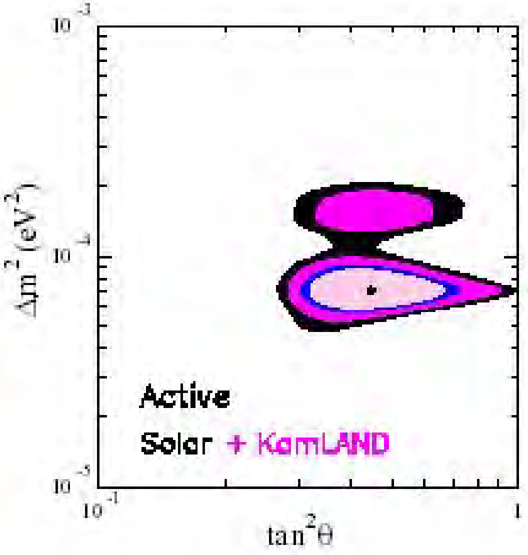
solar → 1-2世代



after KamLAND

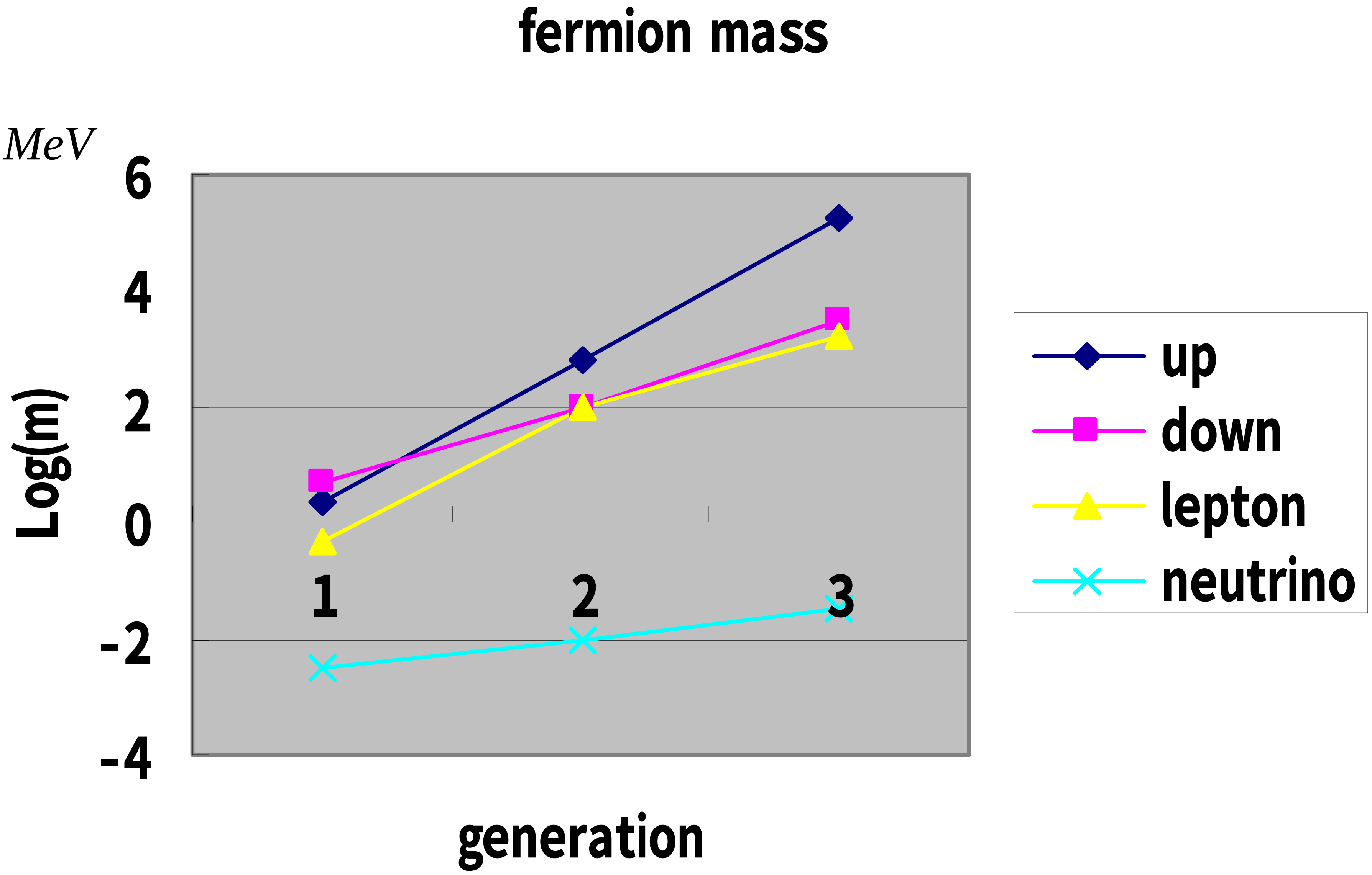
■After KamLAND

▶LMA is the only remaining solution.



(T.Ohta's OHP)

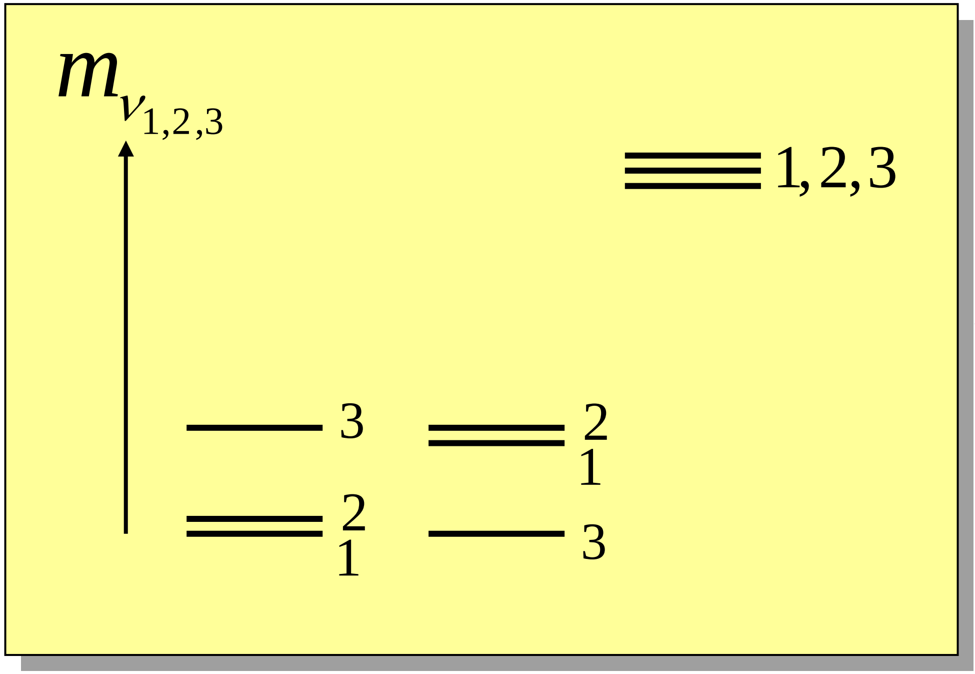
質量階層構造:


fermion mass
MeV
6
4
2
0
-2
-4
Log(m)
1
2
3
generation
up
down
lepton
neutrino

質量階層構造:

$$m_u : m_c : m_t \sim \lambda^{7\sim8} : \lambda^4 : 1$$

$$m_d : m_s : m_b \sim m_e : m_\mu : m_\tau \sim \lambda^{4\sim5} : \lambda^2 : 1$$



solar

$$\delta m_{12}^2 \simeq (0.4 \sim 2.8)\times 10^{-4}\, eV^2$$

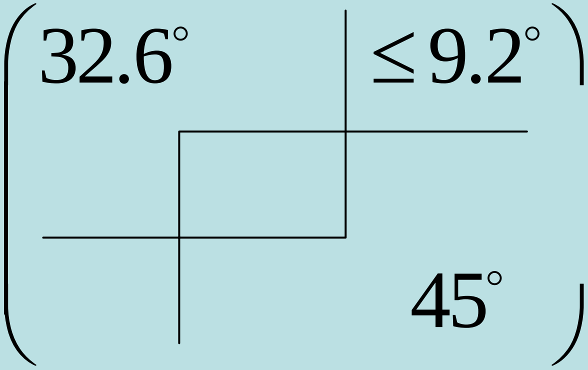
atm

$$\delta m_{23}^2 \simeq (1.2 \sim 5)\times 10^{-3}\, eV^2$$

世代構造:

small mixing in quark　　　　large mixing in lepton

$$V_{CKM} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \Longleftrightarrow V_{MNS} \sim$$



$$\begin{pmatrix} 32.6^\circ & & \le 9.2^\circ \\ & & \\ & & 45^\circ \end{pmatrix}$$

$$\theta_{12} \simeq 25.6^\circ \sim 42.0^\circ \qquad \delta m_{12}^2 \simeq (0.4 - 2.8) \times 10^{-4} eV^2$$

$$\theta_{23} \simeq 33.2^\circ \sim 45.0^\circ \qquad \delta m_{23}^2 \simeq (1.2 \sim 5) \times 10^{-3} eV^2$$

$$\theta_{13} \le 9.2^\circ$$

$$\delta \sim ?$$

世代構造の差はν特有の効果だろうか？

(i) see-saw enhance

see-saw ←ν特有: Majorana mass を持てるのはν_Rのみ

all flavor mixings in *m* →small

$$m_\nu^D \simeq \begin{pmatrix} \lambda^6 & \lambda^6 & 0 \\ 0 & \lambda^4 & \lambda^4 \\ 0 & 0 & 1 \end{pmatrix} \qquad M_R \simeq \begin{pmatrix} \lambda^{12} & 0 & 0 \\ 0 & \lambda^{10} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

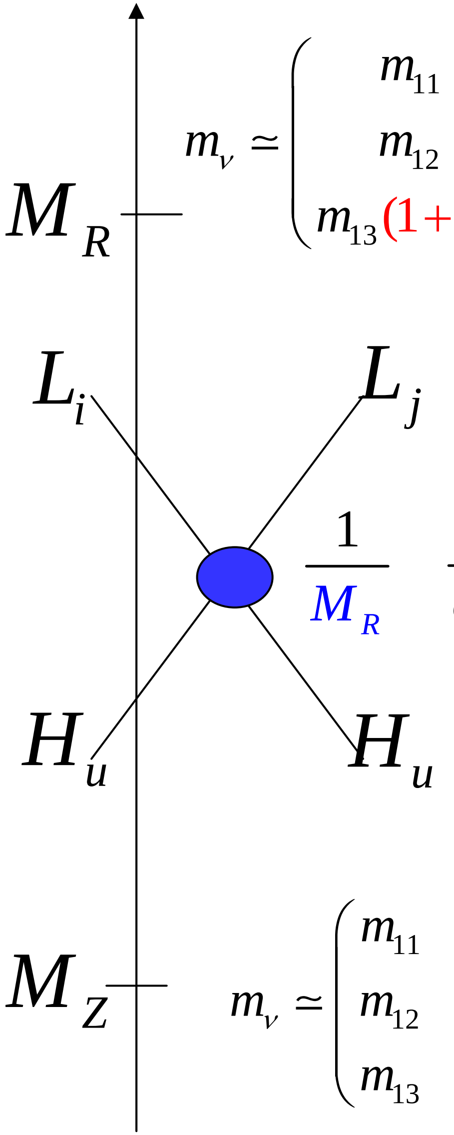
(Altarelli,…)

see-saw mechanism

$$\frac{m_\nu^{DT} m_\nu^D}{M_R} \Longrightarrow m_\nu^l = \begin{pmatrix} \lambda^2 & \lambda^2 & 0 \\ \lambda^2 & 1+\lambda^2 & 1 \\ 0 & 1 & 1+\lambda^2 \end{pmatrix} \frac{<\phi>^2}{\lambda^2 M_R}$$

(ii) RGE effect

縮退した mass を持てるのはνのみ



$$m_\nu \simeq \begin{pmatrix} m_{11} & m_{12} & m_{13}(1+\xi) \\ m_{12} & m_{22} & m_{23}(1+\xi) \\ m_{13}(1+\xi) & m_{23}(1+\xi) & m_{33}(1+2\xi) \end{pmatrix}$$

1 *parameter*

$$\xi = 0.01 \sim 0.1$$

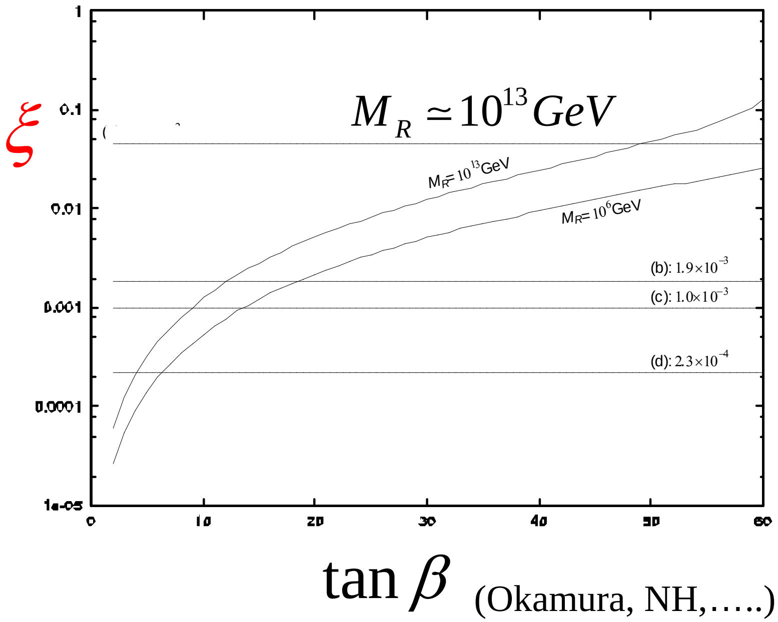
$$(\tan\beta \simeq 30 \sim 50, M_R \simeq 10^{13}GeV)$$

$$\frac{1}{M_R} \qquad \frac{d}{dt}\kappa_{ij} = (\gamma_i + \gamma_j + 2\gamma_H)\kappa_{ij}$$

- *CP*位相は不変！

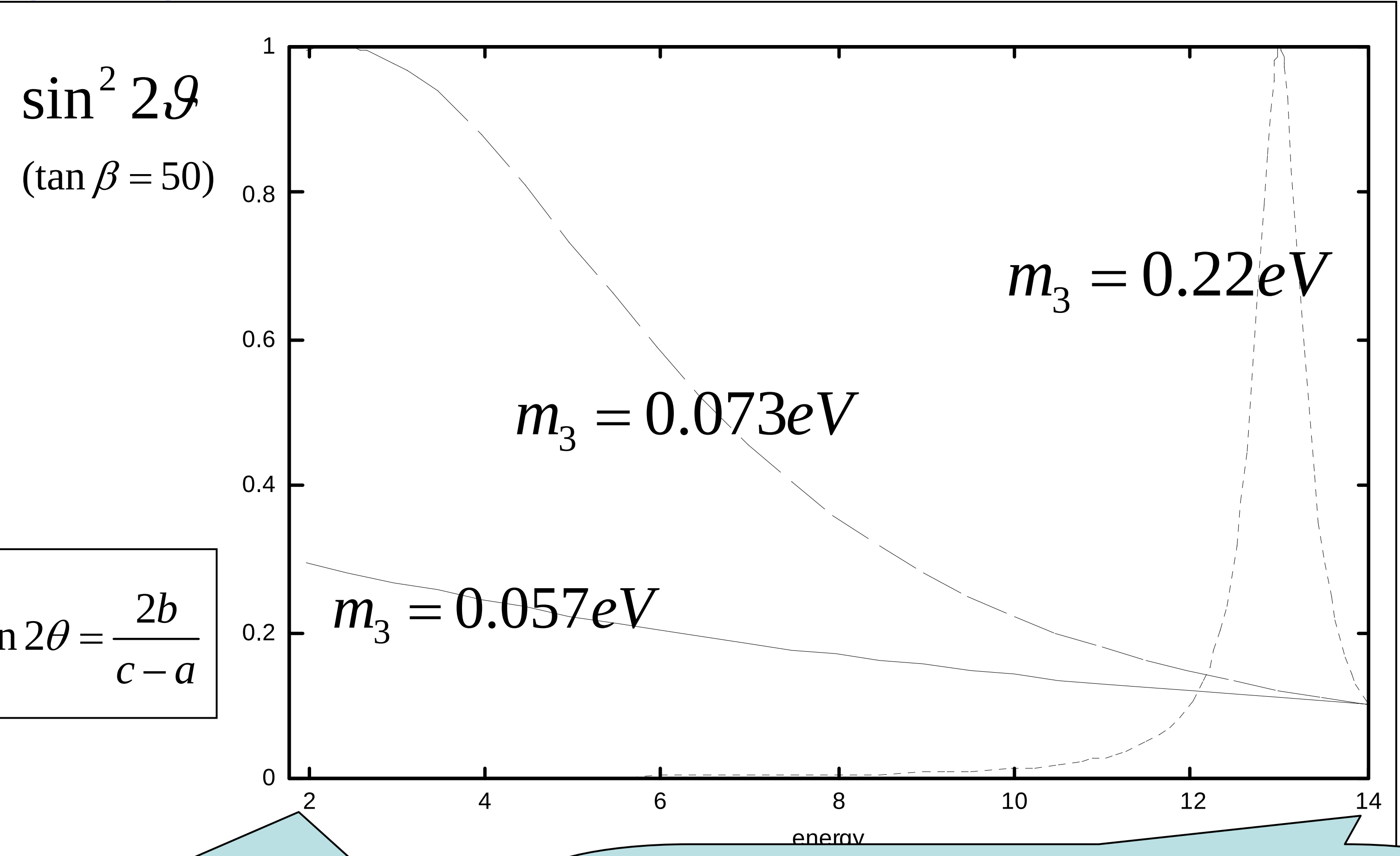
$$\frac{d}{dt}\ln\kappa_{ij} = \frac{d}{dt}\ln|\kappa_{ij}| + i\frac{d}{dt}\varphi_{ij}$$
$$= (\gamma_i + \gamma_j + 2\gamma_H)$$

$$m_\nu \simeq \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{12} & m_{22} & m_{23} \\ m_{13} & m_{23} & m_{33} \end{pmatrix}$$



(Okamura, NH,…..)

mixing angleは繰り込み効果でどう変わるか？



$$\begin{pmatrix} a & b \\ b & c \end{pmatrix} \to \tan 2\theta = \frac{2b}{c-a}$$

$$\begin{pmatrix} 1 & * \\ * & 1 \end{pmatrix} \to \tan 2\theta = \frac{2(\delta m/m)}{1-1}$$

$$\begin{pmatrix} 1 & *(1+\xi) \\ *(1+\xi) & 1(1+2\xi) \end{pmatrix} \to \tan 2\theta = \frac{2(\delta m/m)}{2\xi}$$

$(\delta m/m) \sim 0.01 \sim 0.1eV \ [m \sim 0.1 \sim 1eV]$

同符号で縮退の場合は、高エネルギーで小混合！

(iii) psudo-Dirac

→ 2 mass squared difs.

$\Rightarrow$
- If LSND is correct, one more δm_L^2 is needed (~1eV^2)
- ν_L # is 3 (LEP) !

$\Rightarrow$ Introduction of ν_S ($SU(2)_L$ singlet $\times U(1)_Y$)

// ν_R is just sterile!

▷ pseudo-Dirac

(Wolfenstein ('81)
Kobayashi-Lim-Nojiri (96))

$$\begin{pmatrix} 0 & m \\ m & M \end{pmatrix} \xrightarrow[\substack{m \gg M \\ \text{(cf. see-saw)}}]{} \begin{pmatrix} m+\frac{M}{2} & 0 \\ 0 & -m+\frac{M}{2} \end{pmatrix}$$

- $\Delta m^2 = 2mM$
- $\theta \sim 45^\circ$ $(\nu_L - \nu_R)$!

Singular see-saw

(Chun, Kim, Lee '98)

M_R : rank 2

$$m_\nu \simeq \begin{array}{c} \begin{array}{cccccc} \nu_e & \nu_\mu & \nu_\tau & S_e & S_\mu & S_\tau \end{array} \\ \left(\begin{array}{ccc|c:c:c} & & & m & m & m \\ & 0 & & m & m & m \\ & & & m & m & m \\ \hline m & m & m & M & o & M \\ \hdashline m & m & m & o & o & o \\ \hdashline m & m & m & M & o & M \end{array}\right) \end{array} \xrightarrow[M\,(\gg m)]{\text{integrate out}} \left\{ \begin{array}{c} \overbrace{\begin{array}{cc} \nu_\mu & S_\mu \end{array}}^{\text{p-Dirac}} \\ \begin{pmatrix} m^2/M & m \\ m & o \end{pmatrix} \\ \\ \begin{array}{cc} \nu_e & \nu_\tau \end{array} \\ \begin{pmatrix} m^2/M & m^2/M \\ m^2/M & m^2/M \end{pmatrix} \end{array} \right)$$

$\theta_a \sim 45^\circ$ ($\nu_\mu + S_\mu$ ═══] $\delta m_a^2 \sim m\frac{m^2}{M}$ $(\sim \epsilon^3 M^2)$

$\nu_\mu - S_\mu$ ═══

$\delta m_L^2 \sim m^2$ $(\sim \epsilon^2 M^2)$

ν_τ ═══ $\delta m_{\odot}^2 \sim \left(\frac{m^2}{M}\right)^2$ $(\sim \epsilon^4 M^2)$

ν_e ═══

$\left(m \sim \epsilon M \rightarrow \begin{cases} M \sim 1\,\mathrm{keV} \\ \epsilon \sim 10^{-3} \end{cases} \right)$

In order to fit all data,

(NH, Chikira, Mimura, EPJC16(00)701)

we must introduce hierarchy in m_ν^D !

simplest ex.)

$$m_\nu^D \sim \begin{pmatrix} \epsilon & \epsilon & \epsilon \\ 1 & 1 & \epsilon \\ \epsilon & \epsilon & \epsilon \end{pmatrix} m \rightarrow \begin{pmatrix} \cdot\,\text{atm} & \cdot\,\text{LSND} \\ \cdot\,\text{Solar} & \text{VO}\ (\nu_e - \nu_\tau) \end{pmatrix}$$

pseudo-Dirac is interesting!

However!

$\nu_{\text{active}} - \nu_{\text{sterile}}$ maximal mix is O.K. ?? (X)

(X BBN $N_\nu \leq 3.6$ ([illegible]))

Super－K

$\nu_\mu - \nu_s$

は実験で否定された。

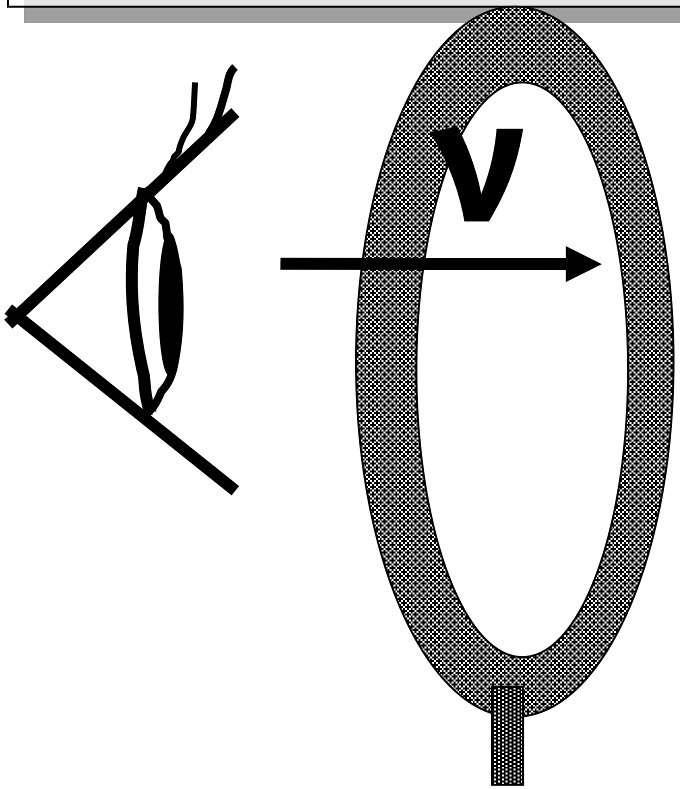
5.大統一理論 (GUT)

$-q(e^-)=q(p^+)$　21桁も！

$|q(e^-)+q(p^+)|/e<10^{-21}$ from neutrality of matter experiment (assumed $q(n)=q(p^+)+q(e^-)$)

quark ⇔leptonの対称性!

大統一理論



質量階層性と世代構造を通して大統一理論の世界を研究しよう！

ν がkey word

$$SU(3)_C \times SU(2)_L \times U(1)_Y \subset SU(5),\ SO(10),\ E_6,\ E_8 \cdots$$

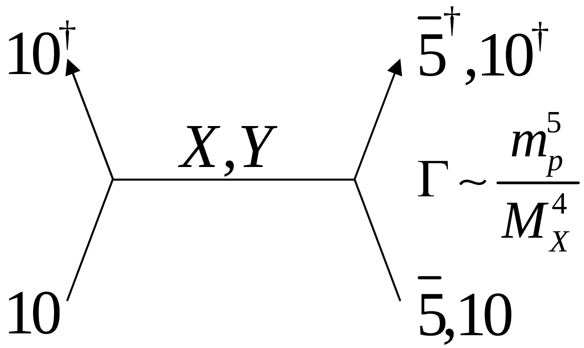
charge の量子化が群論で保障される

quark ⟷ lepton : **同じ表現に！**

predictions

1. proton-decay

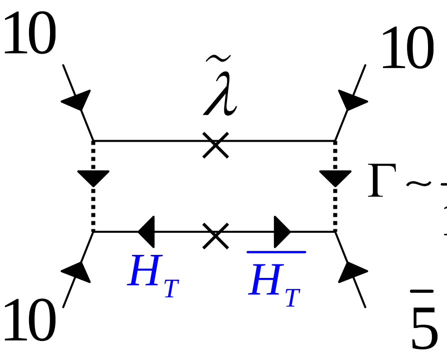
$p^+ \to e^+ \pi^0$ (non-SUSY)



$$\Gamma \sim \frac{m_p^5}{M_X^4}$$

$Q^\dagger \bar{U} L^\dagger \bar{D},\ Q^\dagger \bar{U} Q^\dagger \bar{E}$

$p^+ \to K^+ \nu$ (SUSY)



$$\Gamma \sim \frac{\alpha^2}{16\pi} \frac{m_p^5}{M_X^2 m_{susy}^2}$$

$QQQL,\ \bar{U}\bar{U}\bar{D}\bar{E}$

2. gauge coupling unification $\to \frac{M_{\lambda_3}}{\alpha_3} = \frac{M_{\lambda_2}}{\alpha_2} = \frac{M_{\lambda_1}}{\alpha_1}$

3. $m_b = m_\tau \cdots$

$SU(5)$ $\quad 10=(Q,\overline{U},\overline{E})$ $\quad \bar{5}=(\overline{D},L)$ $\quad 1=(\overline{N})$

$H_5,\ \overline{H}_5$

$SO(10)$ $\quad 16=10+\bar{5}+1$

$10_H\,(+10'_H)$

E_6 $\quad 27=16+10+1$

湯川相互作用

$$W=\underbrace{10\cdot 10\cdot H_5}_{m_u}+\underbrace{10\cdot\bar{5}\cdot\overline{H}_5}_{m_d,\,m_e}+\underbrace{\bar{5}\cdot 1\cdot H_5}_{m_\nu^D}+\underbrace{M\cdot 1\cdot 1}_{M_R}$$

TD-splitting

GUTのHiggs系に存在する大問題！

$$W_H \simeq \lambda_\Sigma tr\Sigma^3 + \mu_\Sigma tr\Sigma^2 + \underline{f_h\ H\langle\Sigma\rangle\overline{H} + \mu_h H\overline{H}}$$

$$H[\,f_h V_{GUT}\begin{pmatrix}2&&&&\\&2&&&\\&&2&&\\&&&-3&\\&&&&-3\end{pmatrix} + \mu_h\begin{pmatrix}1&&&&\\&1&&&\\&&1&&\\&&&1&\\&&&&1\end{pmatrix}\overline{H}\,]\qquad \begin{pmatrix}H_T\\ H_D\end{pmatrix}$$

$$\begin{cases} 2f_h\ V_{GUT} + \mu_h > 10^{16}GeV \\ -3f_h\ V_{GUT} + \mu_h \sim 10^2 GeV \end{cases}$$

10^{14} $fine-tune!$

解決案:

☆ missing partner

$\langle 75_H\rangle,\quad 50_H + \overline{50}_H\,(\sim(3,1)_{-2} + (\bar{3},1)_2)$

☆ pNG boson

$SU(6)_{gl} \supset SU(5)$

☆ $SU(5)_{fl}\times U(1)$

☆ DW mechanism

$\langle 45_H\rangle = diag.(\sigma,\sigma,\sigma,0,0)$

…………

6. flavor&質量階層 (世代) 構造（その２）

１０表現 in SU(5)

(Babu,Barr)

$$10=(Q,\overline{U},\overline{E})\quad \overline{5}=(\overline{D},L)\quad 1=(\overline{N})$$

$$W=y^{u}10\cdot 10\cdot H_5+y^{d/e}10\cdot\overline{5}\cdot\overline{H_5}+y^{\nu}\overline{5}\cdot 1\cdot H_5+M_R\cdot 1\cdot 1$$

$$10_3\to 10_3,\ 10_2\to\lambda^2 10_2,\ 10_1\to\lambda^4 10_1$$

$$W = y^u 10\cdot 10\cdot H_5 + y^{d/e} 10\cdot \bar{5}\cdot \overline{H_5} + y^{\nu} \bar{5}\cdot 1\cdot H_5 + M_R\cdot 1\cdot 1$$

$$10_i = (Q,\overline{U},\overline{E})_i \quad [10_3 \to 10_3,\ 10_2 \to \lambda^2 10_2,\ 10_1 \to \lambda^4 10_1]$$

$$m_u \simeq \begin{array}{c} L \qquad\qquad\qquad R \\ \begin{pmatrix} \lambda^8 & \lambda^6 & \lambda^4 \\ \lambda^6 & \lambda^4 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} \end{array} \langle H_u \rangle \Longrightarrow \begin{array}{c} L \\ \begin{pmatrix} 1 & \lambda^2 & \lambda^4 \\ \lambda^2 & 1 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} \end{array} \begin{pmatrix} \lambda^8 & & \\ & \lambda^4 & \\ & & 1 \end{pmatrix} \begin{array}{c} R \\ \begin{pmatrix} 1 & \lambda^2 & \lambda^4 \\ \lambda^2 & 1 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} \end{array} \langle H_u \rangle$$

$$m_d \simeq \begin{pmatrix} \lambda^4 & \lambda^4 & \lambda^4 \\ \lambda^2 & \lambda^2 & \lambda^2 \\ 1 & 1 & 1 \end{pmatrix} \langle H_d \rangle \Longrightarrow \begin{pmatrix} 1 & \lambda^2 & \lambda^4 \\ \lambda^2 & 1 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} \begin{pmatrix} \lambda^4 & & \\ & \lambda^2 & \\ & & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \langle H_d \rangle$$

$$m_l \simeq \begin{pmatrix} \lambda^4 & \lambda^2 & 1 \\ \lambda^4 & \lambda^2 & 1 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} \langle H_d \rangle \Longrightarrow \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \lambda^4 & & \\ & \lambda^2 & \\ & & 1 \end{pmatrix} \begin{pmatrix} 1 & \lambda^2 & \lambda^4 \\ \lambda^2 & 1 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} \langle H_d \rangle$$

$$m_\nu \simeq \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \frac{\langle H_u \rangle^2}{M_R} \Longrightarrow \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \frac{\langle H_u \rangle^2}{M_R}$$

$$W = y^u 10\cdot 10\cdot H_5 + y^{d/e} 10\cdot \bar{5}\cdot \overline{H_5} + y^\nu \bar{5}\cdot 1\cdot H_5 + M_R \cdot 1\cdot 1$$

$$10_i = (Q,\overline{U},\overline{E})_i \qquad [10_3 \to 10_3,\ 10_2 \to \lambda^2 10$$

$$\begin{array}{cc} & L \qquad\qquad R \\ m_u \simeq & \begin{pmatrix} \lambda^8 & \lambda^6 & \lambda^4 \\ \lambda^6 & \lambda^4 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} \langle H_u \rangle \end{array} \Rightarrow L \begin{pmatrix} 1 & \lambda^2 & \lambda^4 \\ \lambda^2 & 1 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix}^\dagger \begin{pmatrix} \lambda^8 & & \\ & & \\ & & \end{pmatrix} \cdots \langle H_u \rangle$$

$$m_d \simeq \begin{pmatrix} \lambda^4 & \lambda^4 & \lambda^4 \\ \lambda^2 & \lambda^2 & \lambda^2 \\ 1 & 1 & 1 \end{pmatrix} \langle H_d \rangle \Rightarrow \begin{pmatrix} 1 & \lambda^2 & \lambda^4 \\ \lambda^2 & 1 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} \begin{pmatrix} \lambda^4 & & \\ & \lambda^2 & \\ & & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \langle H_d \rangle$$

$$m_l \simeq \begin{pmatrix} \lambda^4 & \lambda^2 & 1 \\ \lambda^4 & \lambda^2 & 1 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} \langle H_d \rangle \Rightarrow \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}^\dagger \begin{pmatrix} \lambda^4 & & \\ & & \\ & & \end{pmatrix} \begin{pmatrix} 1 & \lambda^2 & \lambda^4 \\ & & \\ & & \end{pmatrix} \cdots$$

$$m_\nu \simeq \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \frac{\langle H_u \rangle^2}{M_R} \Rightarrow \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & & \\ & & \\ & & \end{pmatrix} \cdots$$

$$V_{CKM} \simeq \begin{pmatrix} 1 & \lambda^2 & \lambda^4 \\ \lambda^2 & 1 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix}$$

$$V_{MNS} \simeq \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

どうやって１０に階層性を持たすか？

(i) FN mechanism

(ii) composite model

(iii) extra dim. [fat brane]

(iv) extra dim. [orbifold]

…

(i) FN mechanism

(Murayama,NH,
Hisano,Kurosawa,Nomura….)

U(1) flavor symmetry, anomalous U(1),...

$$Q_f(10_3)=0,\ \ Q_f(10_2)=1,\ \ Q_f(10_1)=2$$

$$Q_f(\varphi_f)=-1\quad \frac{\langle\varphi_f\rangle}{M_*}\simeq\lambda^2$$

(for example)

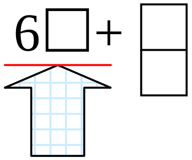
$$L\supset\frac{\langle\varphi_f\rangle^2}{M_*{}^2}10_1\frac{\langle\varphi_f\rangle}{M_*}10_2H_u\rightarrow m_{u12}\simeq\lambda^6\langle H_u\rangle$$

$$\frac{\langle\varphi_f\rangle^2}{M_*{}^2}10_1\overline{5}_iH_d\rightarrow m_{d1i}\simeq\lambda^4\langle H_d\rangle$$

(ii) composite model

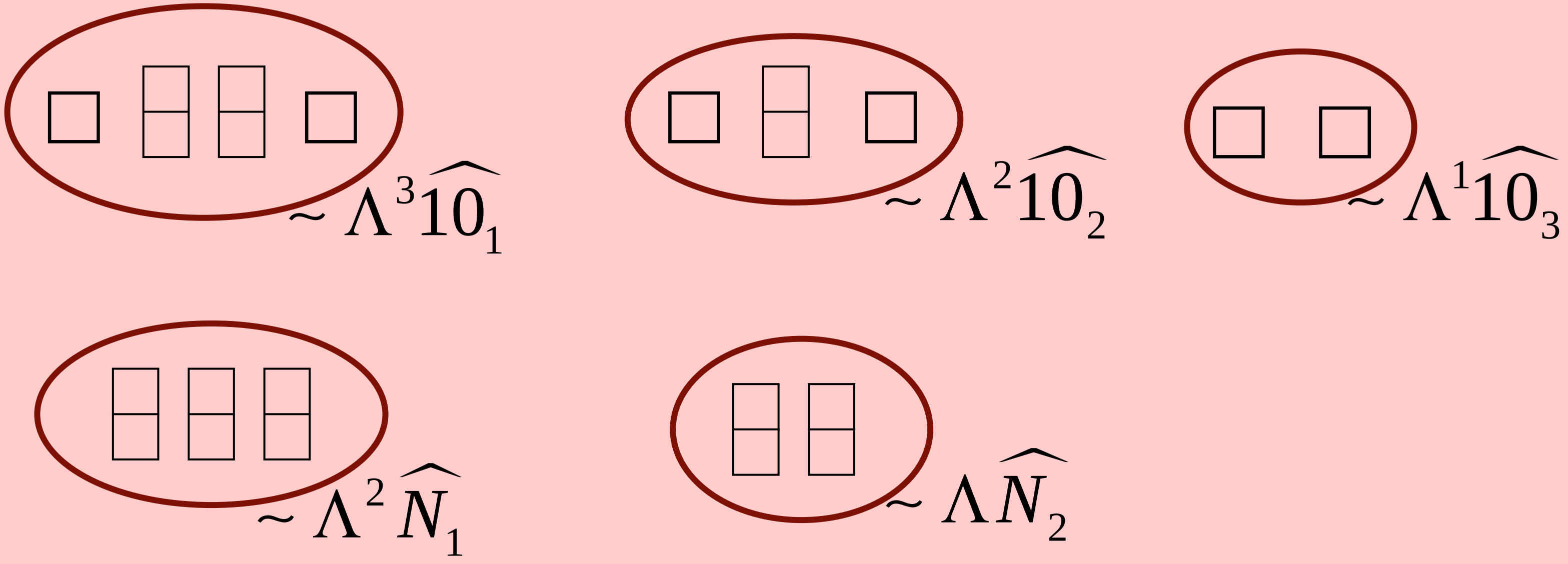
(Nelson,Strasller
NH)

N=1 SQCD –confinement- Sp(2N) with 6□+



1+5

Sp(6)×SU(5) $M_{GUT} < \Lambda < M_{Pl}$



$\sim \Lambda^3 \widehat{10}_1$ $\sim \Lambda^2 \widehat{10}_2$ $\sim \Lambda^1 \widehat{10}_3$

$\sim \Lambda^2 \widehat{N}_1$ $\sim \Lambda \widehat{N}_2$

They are massless composite, others are elementary particles.

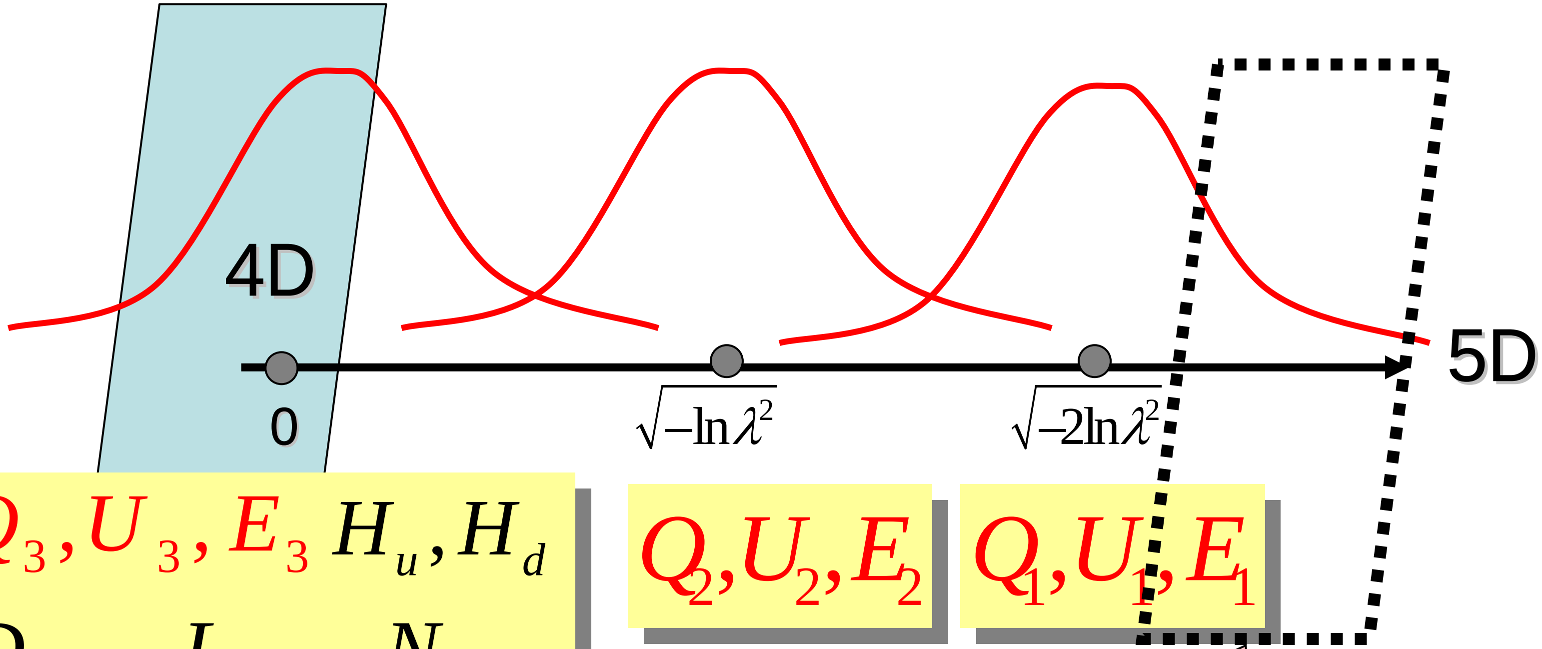
$\frac{\Lambda}{M_{Pl}} \sim \lambda^2$ → suitable mass hierarchies & flavor mixings

(iii) extra dim. [fat brane]

(Schmaltz,…
Kaplan,
Blanco, ..)

(Maru, NH)

Yukawa Hierarchy ⟷ Geography in ExtraD



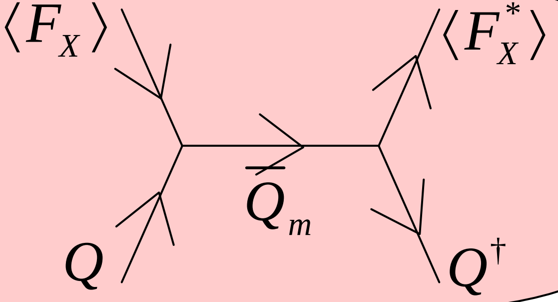
$Q_3, U_3, E_3\ H_u, H_d$
$D_{1,2,3}, L_{1,2,3}, N_{1,2,3}$

Q_2, U_2, E_2

Q_1, U_1, E_1

decoupling ~~SUSY~~

$X = F\vartheta^2$ millar fields



(iv) extra dim. [orbifold]

準備＆練習しましょう

余次元理論を考える動機について

1．**重力の弱さを説明したい** → large extraD (ADD)

2、KK idear&more: 5D gravity = 4D gravity + 4D gauge + 4D scalar

3、field localization (brane, BG-vev, fixed points)

→ volume suppression,

→ geometrical understanding of particle physics

4、(24) Higgs**の起源** → cf. Hosotani mech.

5、GUT**の問題点を回避したい** → extraD GUT

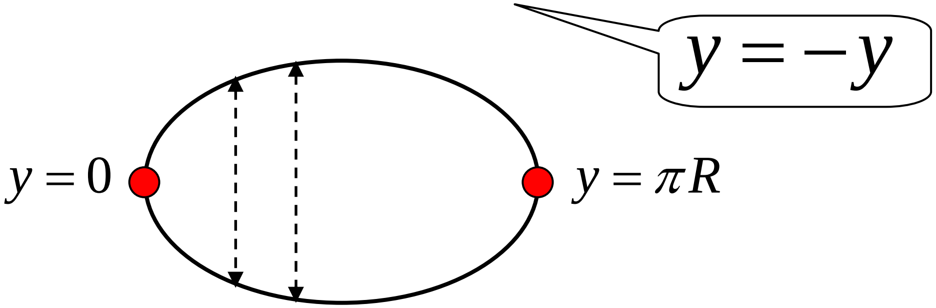
6、**重力の局在** (RS)、blane world、 string**との競合性、その他素粒子物理の新しい理解、それに、あったらそれだけで面白い！**

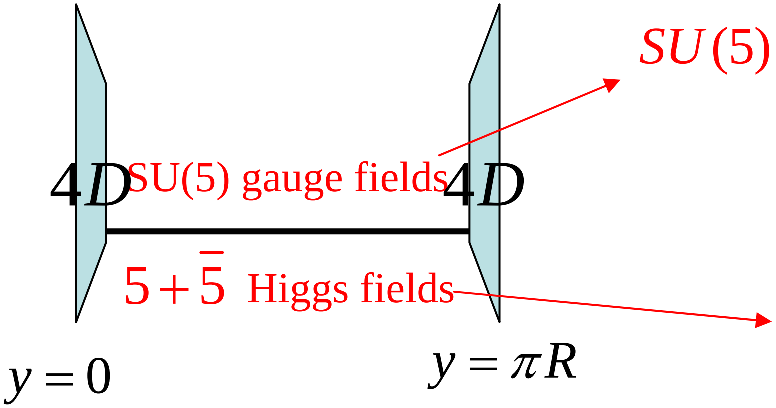
new idea for the solution of TD-splitting

(Kawamura, Hall,Nomura)

5D theory with no Σ_{24}

$$M^4 \otimes S^1/Z^2$$

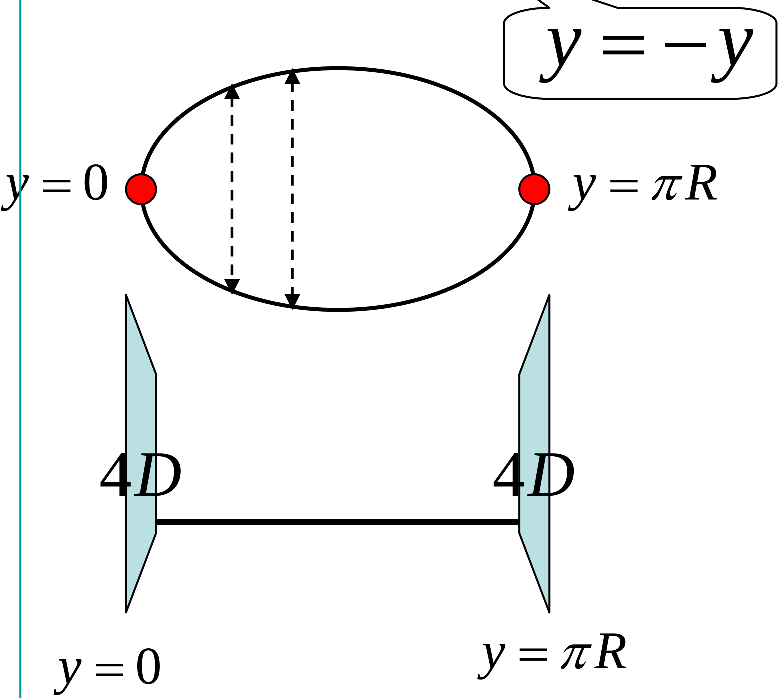




$SU(5) \to SU(3)_C \times SU(2)_L \times U(1)_Y$

$TD-splitting$

$$M^4 \otimes S^1 / Z^2$$



$$\boxed{P: \ \phi(x^\mu, -y) = P\phi(x^\mu, y)}$$

$$[P^2 = 1 \ \because \phi(y) = P\phi(-y) = P^2\phi(y)]$$

$$A_\mu(x^\mu, -y) = PA_\mu(x^\mu, y)P$$

$$A_5(x^\mu, -y) = -PA_5(x^\mu, y)P$$

$$\psi_L(x^\mu, -y) = P\psi_L(x^\mu, y)$$

$$\psi_R(x^\mu, -y) = -P\psi_R(x^\mu, y)$$

$$[\psi(x^\mu, -y) = Pi\gamma^y\psi(x^\mu, y)]$$

$$5D: \ \gamma^M = (\gamma^\mu, i\gamma^5)$$

$$\boxed{T: \ \phi(x^\mu, y + 2\pi R) = T\phi(x^\mu, y)}$$

$$\phi_+(x^\mu, y) = \frac{1}{\sqrt{2^{\delta_{n,0}}\pi R}} \sum_{n=0}^{\infty} \phi_+^{(n)}(x^\mu) \ \cos\left(\frac{ny}{R}\right)$$

$$\phi_{++}(x^\mu, y) = \frac{1}{\sqrt{2^{\delta_{n,0}}\pi R}} \sum_{n=0}^{\infty} \phi_{++}^{(n)}(x^\mu) \ \cos\left(\frac{ny}{R}\right)$$

$$\phi_{+-}(x^\mu, y) = \frac{1}{\sqrt{\pi R}} \sum_{n=0}^{\infty} \phi_{+-}^{(n)}(x^\mu) \ \cos\left(\frac{n+1/2}{R}y\right)$$

$$\phi_-(x^\mu, y) = \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \phi_-^{(n)}(x^\mu) \ \sin\left(\frac{ny}{R}\right)$$

$$\phi_{--}(x^\mu, y) = \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \phi_{--}^{(n)}(x^\mu) \ \sin\left(\frac{n+1/2}{R}y\right)$$

$$\phi_{-+}(x^\mu, y) = \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \phi_{-+}^{(n)}(x^\mu) \ \sin\left(\frac{n}{R}y\right)$$

5D SU(5) GUT on S_1/Z_2

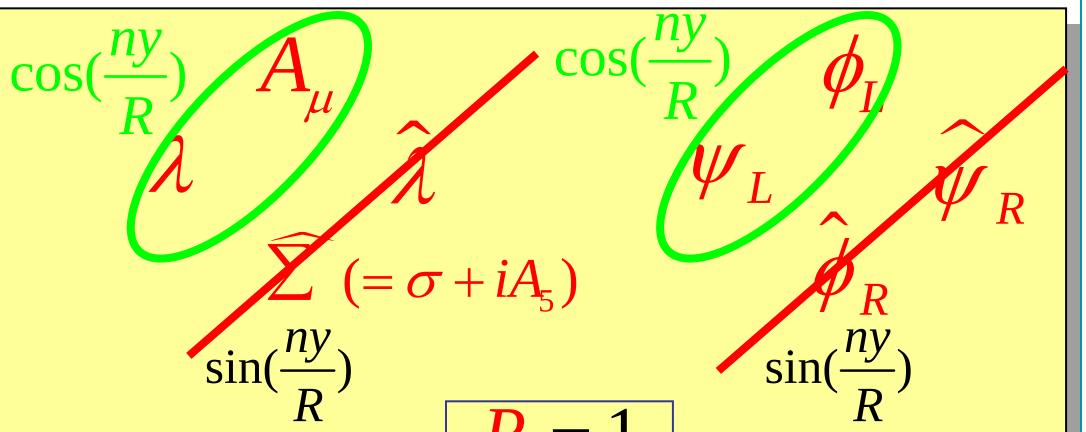
$$P: \quad \phi(x^\mu,-y) = P\phi(x^\mu,y)$$

$$A_\mu(x^\mu,-y) = PA_\mu(x^\mu,y)P$$

$$A_5(x^\mu,-y) = -PA_5(x^\mu,y)P$$

$$\psi_L(x^\mu,-y) = P\psi_L(x^\mu,y)$$

$$\psi_R(x^\mu,-y) = -P\psi_R(x^\mu,y)$$



$$\boxed{P = 1}$$

$$\boxed{T: \quad diag.(-1,-1,-1,\ 1,\ 1)}$$

$$\left(\begin{array}{c} \cancel{H}_{T,\bar T} \\ \hdashline H_{u,d} \end{array}\right), \quad \left(\begin{array}{c:c} g \quad (\gamma) & \cancel{X,Y} \\ \hdashline \cancel{X,Y} & W,Z \end{array}\right)$$

$$\phi_{++} \sim \cos(\frac{ny}{R}) \qquad \phi_{+-} \sim \cos(\frac{n+1/2}{R}y)$$

$$\phi_{-+} \sim \sin(\frac{ny}{R}) \qquad \phi_{--} \sim \sin(\frac{n+1/2}{R}y)$$

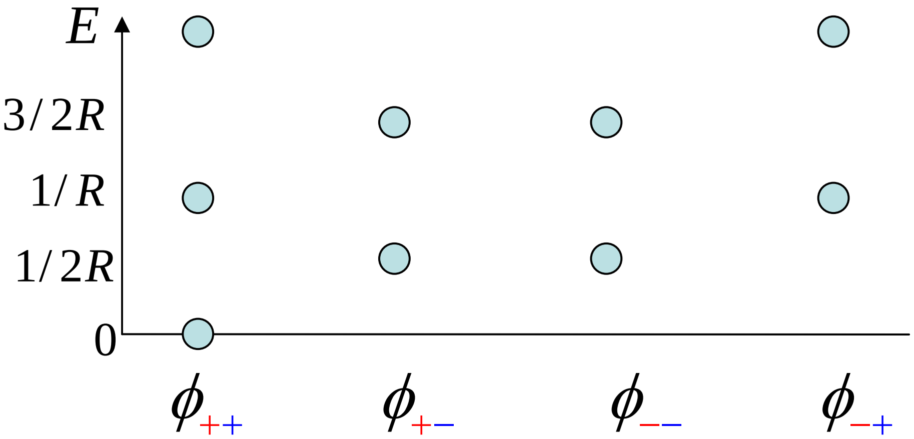
$$P' = TP$$

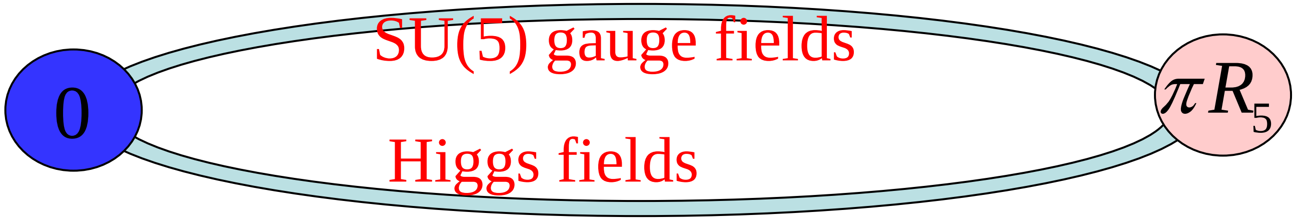
Parity at y=πR

$$\phi(\pi R + y) = T\ \phi(-\pi R + y) = T\ P\ \phi(\pi R - y)$$

$$P':\quad parity\ at\ y' \to -y'$$

$$(y' = \pi R + y)$$





$$\phi(x^\mu, y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=-\infty}^{\infty} \phi^{(n)}(x^\mu)\; e^{i\frac{ny}{R}} \;\Rightarrow H_{u,d},\;\; g, W, \gamma$$

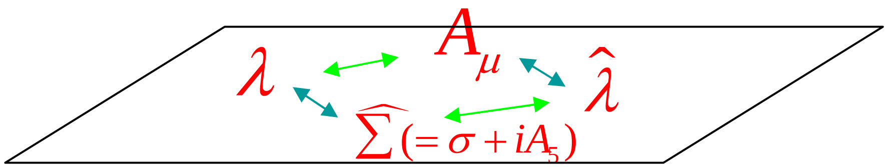
$$H_{T,\bar{T}},\;\; X, Y$$

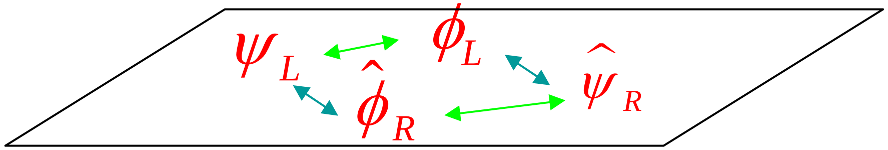
$$\widehat{H}_{T,\bar{T}},\;\; \widehat{\Sigma}_{X,Y}$$

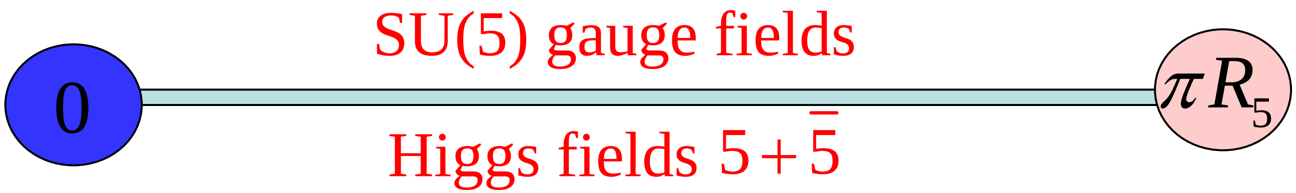
$$\widehat{H}_{u,d}\;\; \widehat{\Sigma}_{g,W,\gamma}$$

chiral super-fields

$N = 2$







SU(5) gauge fields

Higgs fields $5+\bar{5}$

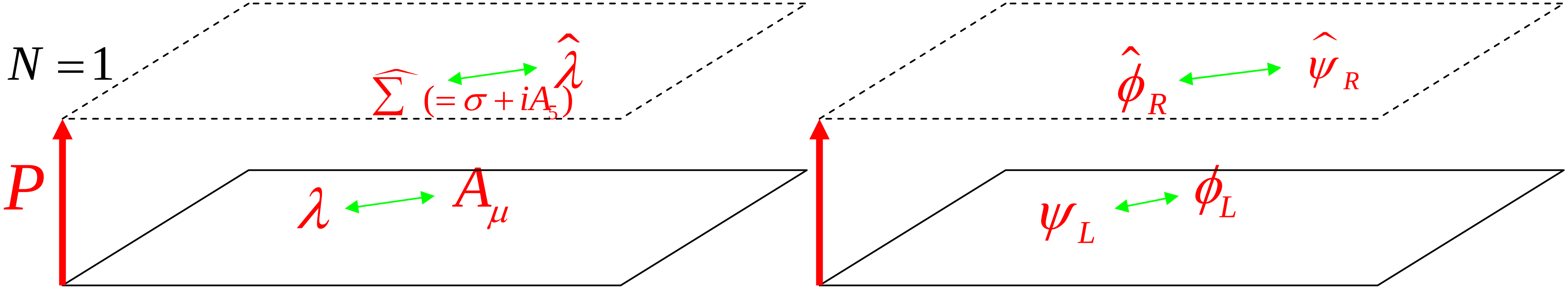
$$\phi_+(x^\mu, y) = \frac{1}{\sqrt{2^{\delta_{n,0}}\pi R}} \sum_{n=0}^{\infty} \phi_+^{(n)}(x^\mu)\ \cos(\frac{ny}{R}) \Rightarrow H_{u,d},\ g, W, \gamma$$

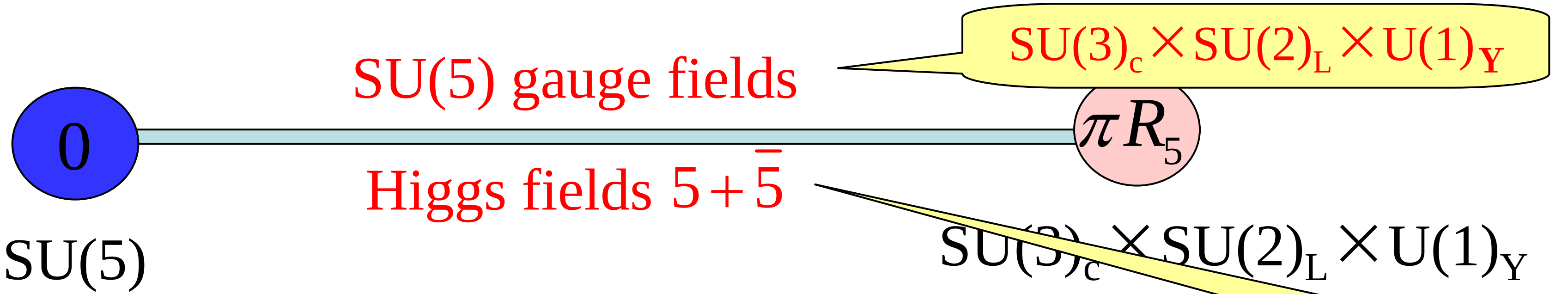
$$H_{T,\bar{T}},\ X, Y$$

$$\phi_-(x^\mu, y) = \frac{1}{\sqrt{\pi R}} \sum_{n=0}^{\infty} \phi_-^{(n)}(x^\mu)\ \sin(\frac{ny}{R}) \quad \Rightarrow \widehat{H}_{T,\bar{T}},\ \widehat{\Sigma}_{X,Y}$$

$$\widehat{H}_{u,d} \qquad \widehat{\Sigma}_{g,W,\gamma}$$

$$\boxed{T = diag.(-1,-1,-1,\ 1,\ 1)}$$





TD splitting

$$\phi_{++}(x^\mu,y)=\frac{1}{\sqrt{2^{\delta_{n,0}}\pi R}}\sum_{n=0}^{\infty}\cos(\frac{ny}{R}) \Rightarrow H_{u,d},\quad g,W,\gamma$$

$$\phi_{+-}(x^\mu,y)=\frac{1}{\sqrt{\pi R}}\sum_{n=0}^{\infty}\cos(\frac{n+1/2}{R}y) \Rightarrow H_{T,\bar{T}},\quad X,Y$$

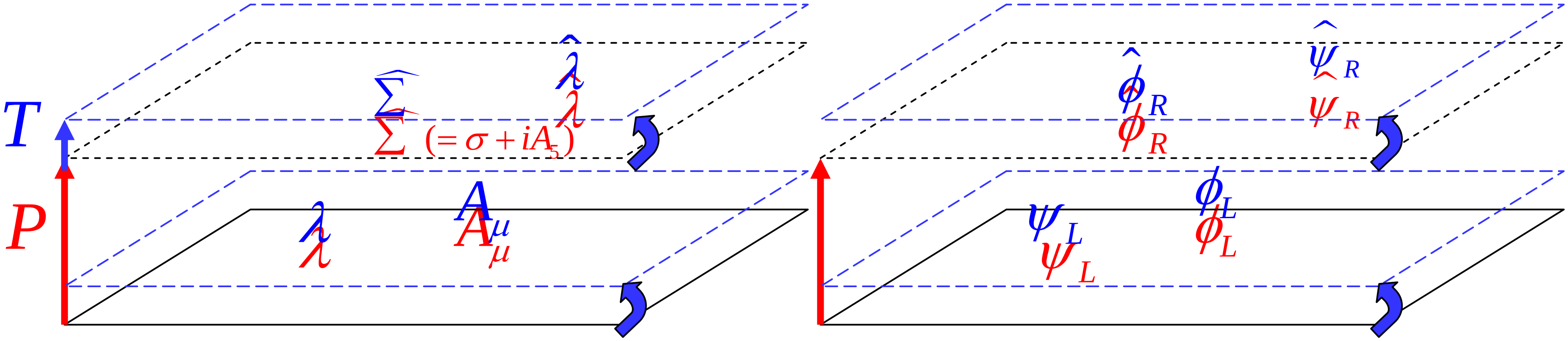
(y=0 brane)

$$\phi_{--}(x^\mu,y)=\frac{1}{\sqrt{\pi R}}\sum_{n=0}^{\infty}\sin(\frac{n+1/2}{R}y) \Rightarrow \hat{H}_{T,\bar{T}},\quad \hat{\Sigma}_{X,Y}$$

(y=πR brane)

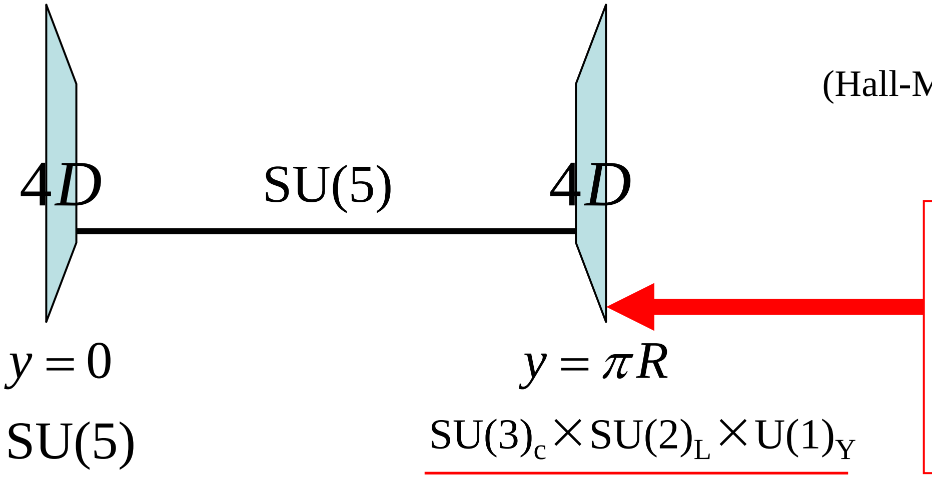
$$\phi_{-+}(x^\mu,y)=\frac{1}{\sqrt{\pi R}}\sum_{n=0}^{\infty}\sin(\frac{n+1}{R}y) \Rightarrow \hat{H}_{u,d}\quad \hat{\Sigma}_{g,W,\gamma}$$

$$\boxed{T=diag.(-1,-1,-1,\ 1,\ 1)}$$



What going unitarity in orbifold gauge theory?

(Hall-Murayama-Nomura)



$4D$　SU(5)　$4D$

$y = 0$

SU(5)

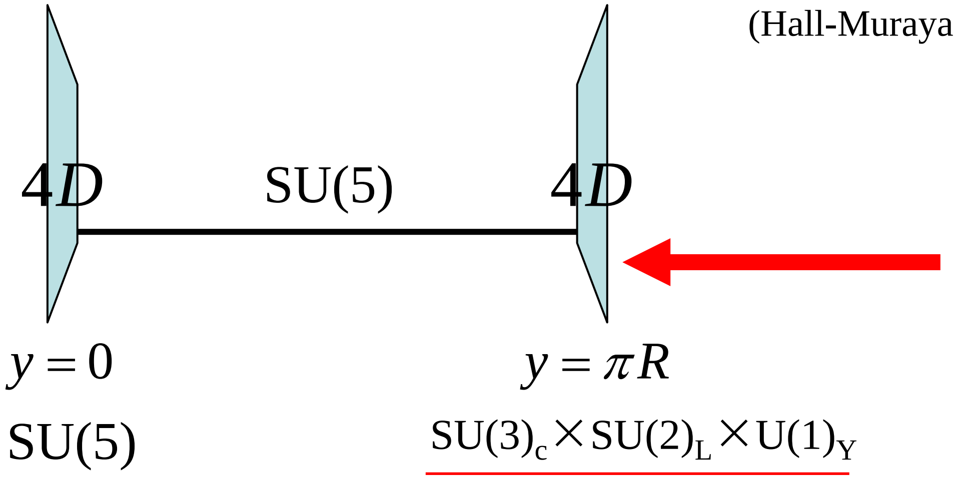
$y = \pi R$

$SU(3)_c \times SU(2)_L \times U(1)_Y$

主張：SU(5)ではなく、
$SU(3)_c \times SU(2)_L \times U(1)_Y$
のmultipletを入れてよい！！

重いgauge boson → unitarity**は、大丈夫か？**
(**この模型では、**SU(5)**は「手で」破っている・・・**)

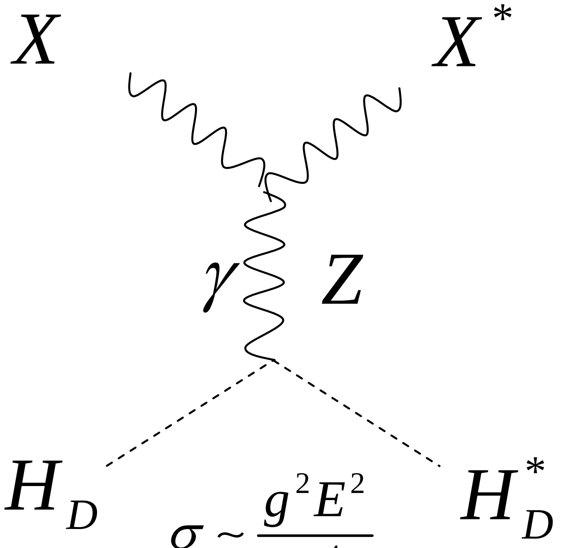
(Hall-Murayama-Nomura)



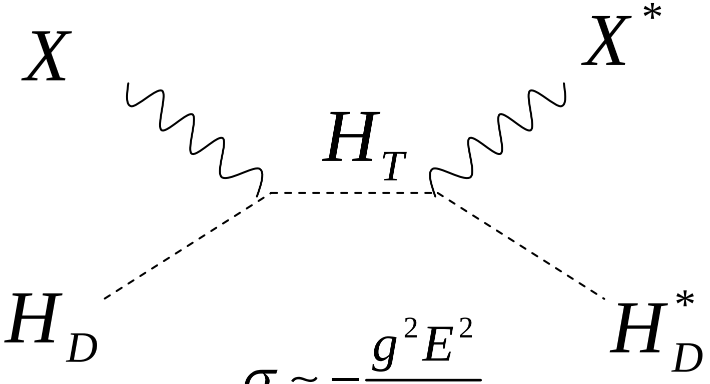
H_D のみ導入してやる

$$\boxed{H_D H_D^* \to X \ \ X^*}$$

（４Ｄ理論）



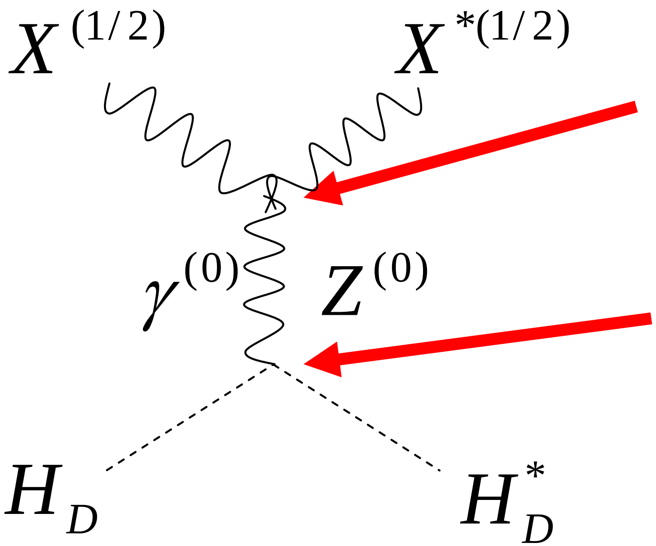
$$\sigma \sim \frac{g^2 E^2}{m^4}$$



$$\sigma \sim -\frac{g^2 E^2}{m^4}$$

$$\sigma \to g^2 \log(\frac{E^2}{m^2})$$

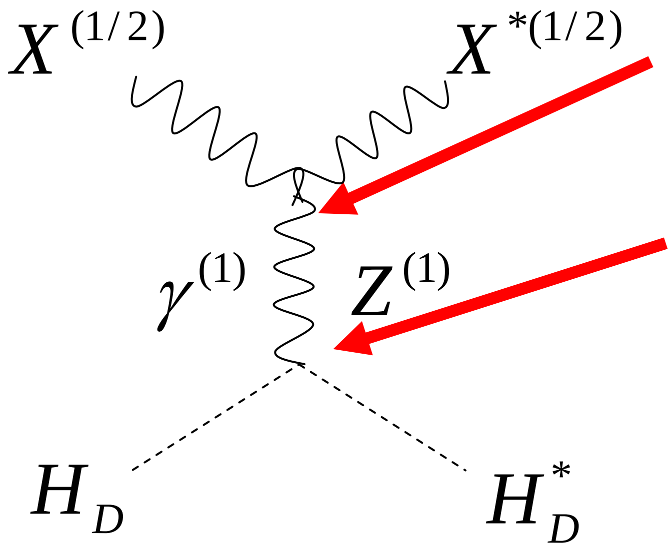
$\boxed{H_D H_D^* \to X^{(1/2)} X^{(1/2)*}}$ （orbifold **5 D理論**） *no* $H_T! \to$ *unitarity*?



$$g_5\int_0^{2\pi R} dy\left(\frac{1}{\sqrt{\pi R}}\cos\frac{y}{2R}\right)^2 \frac{1}{\sqrt{2\pi R}} = \frac{g_5}{\sqrt{2\pi R}} = g_4$$

$$g_5\int_0^{2\pi R} dy\frac{1}{\sqrt{2\pi R}}\delta(y-\pi R) = \frac{g_5}{\sqrt{2\pi R}} = g_4$$

$$\Rightarrow \sigma \sim \frac{g_4^2E^2}{m^4}$$



$$g_5\int_0^{2\pi R} dy\left(\frac{1}{\sqrt{\pi R}}\cos\frac{y}{2R}\right)^2 \frac{1}{\sqrt{\pi R}}\cos\frac{y}{R} = \frac{g_5}{\sqrt{4\pi R}} = \frac{g_4}{\sqrt{2}}$$

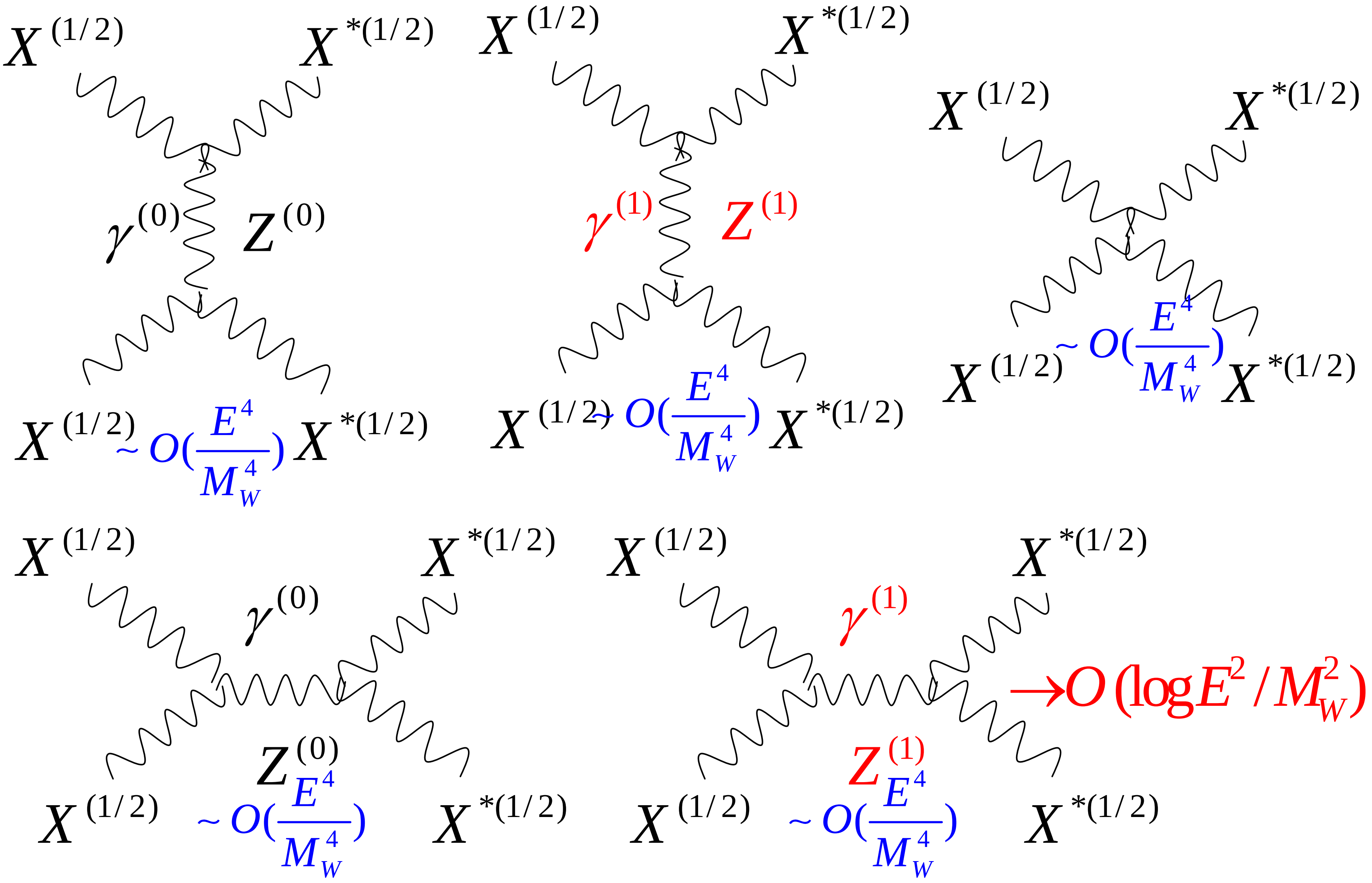
$$g_5\int_0^{2\pi R} dy\frac{1}{\sqrt{\pi R}}\cos\frac{y}{R}\delta(y-\pi R) = -\frac{g_5}{\sqrt{\pi R}} = -\sqrt{2}g_4$$

$$\Rightarrow \sigma \sim -\frac{g_4^2E^2}{m^4}$$

$$\to O(\log E^2/M_W^2)$$

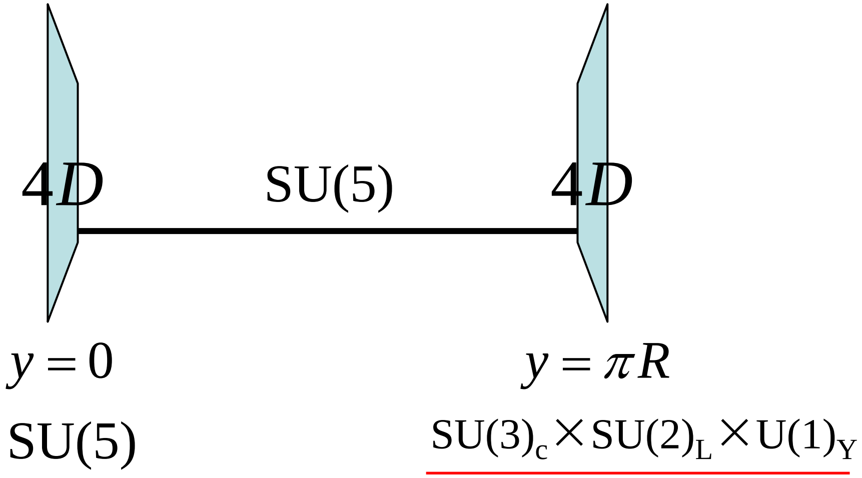
$$X^{(1/2)}X^{(1/2)*}\to X^{(1/2)}X^{(1/2)*}$$

(Abe,Higashide,Kobayashi,Matsunaga,N.H.)



proton decay in extraD GUT (S_1/Z_2)

gauge coupling unification (Hall-Nomura)



$4D$ SU(5) $4D$

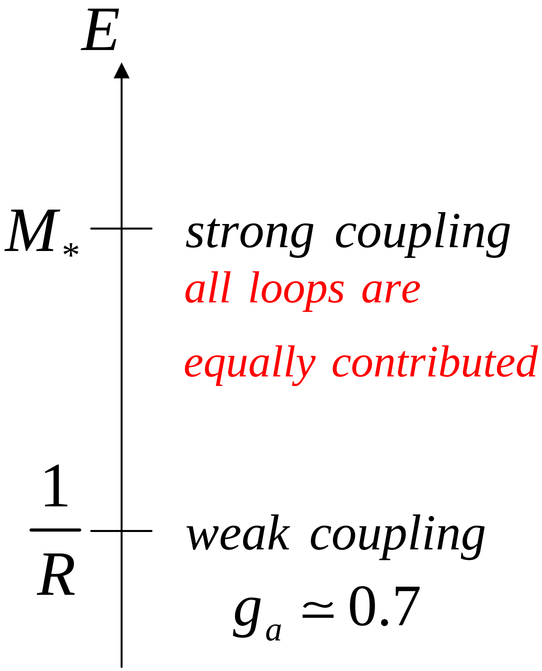
$y=0$ $y=\pi R$

SU(5) $\mathrm{SU(3)_c \times SU(2)_L \times U(1)_Y}$

$$S=\int d^4xdy\left[\frac{1}{g_5^2}F^2+\delta(y)\frac{1}{\tilde{g}^2}F^2+\delta(y-\pi R)\frac{1}{\tilde{g}_a^2}F_a^2\right]$$

$$\frac{1}{g_5^2}\sim\frac{M_*}{24\pi^3} \qquad \frac{1}{\tilde{g}_a^2}\sim\frac{1}{16\pi^2}$$

$$\frac{2\pi R}{g_5^2}=\frac{1}{g_{5(4D)}^2}$$



E

M_* *strong coupling*

all loops are

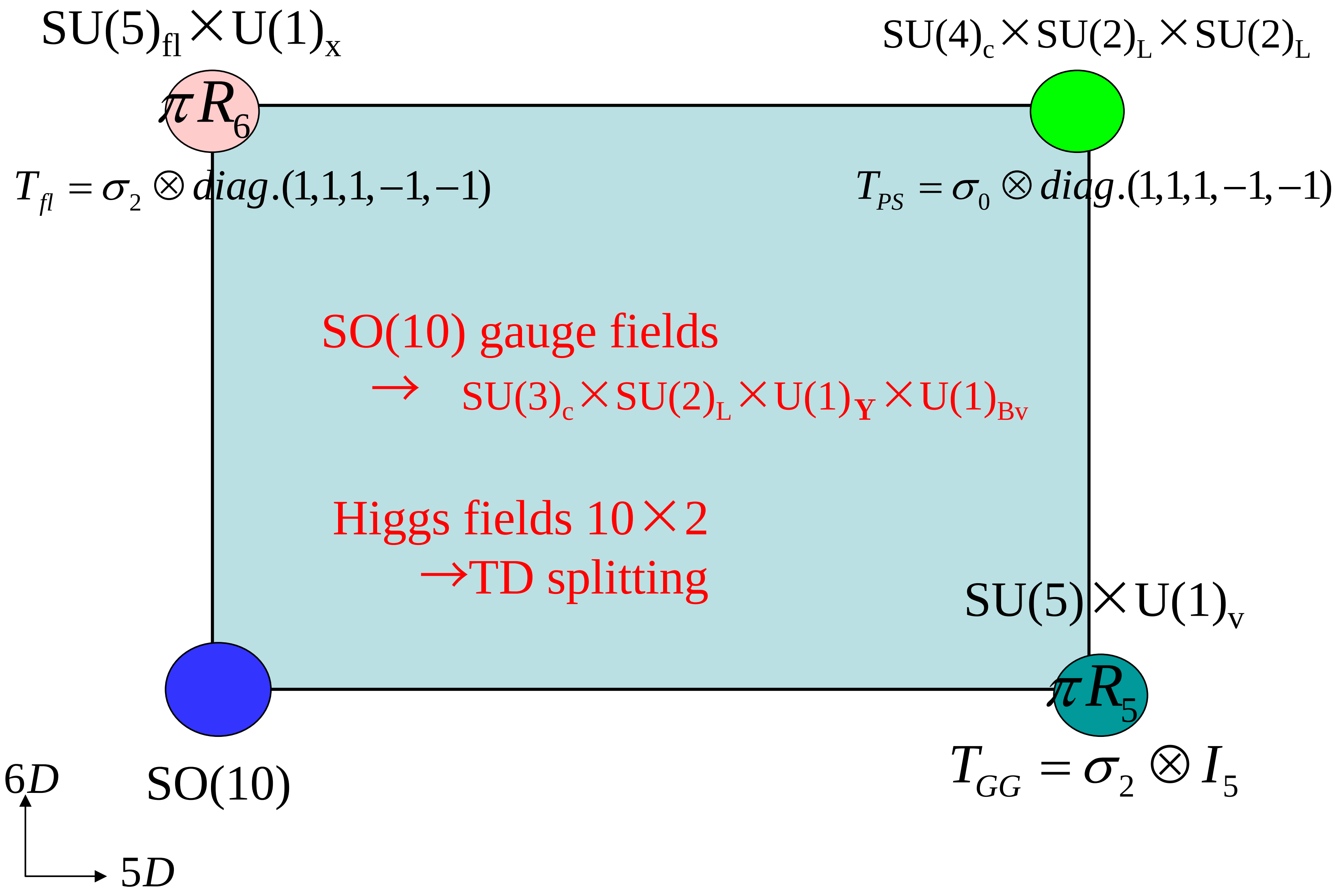
equally contributed

$\frac{1}{R}$ *weak coupling*

$g_a\simeq 0.7$

(Asaka,..Hall,Nomura,.Shimizu,Kondo,NH)

6D SO(10) GUT on T_2/Z_2



$SU(5)_{fl} \times U(1)_x$

$SU(4)_c \times SU(2)_L \times SU(2)_L$

πR_6

$T_{fl} = \sigma_2 \otimes diag.(1,1,1,-1,-1)$

$T_{PS} = \sigma_0 \otimes diag.(1,1,1,-1,-1)$

SO(10) gauge fields

$\rightarrow$ $SU(3)_c \times SU(2)_L \times U(1)_{\mathbf{Y}} \times U(1)_{Bv}$

Higgs fields 10×2

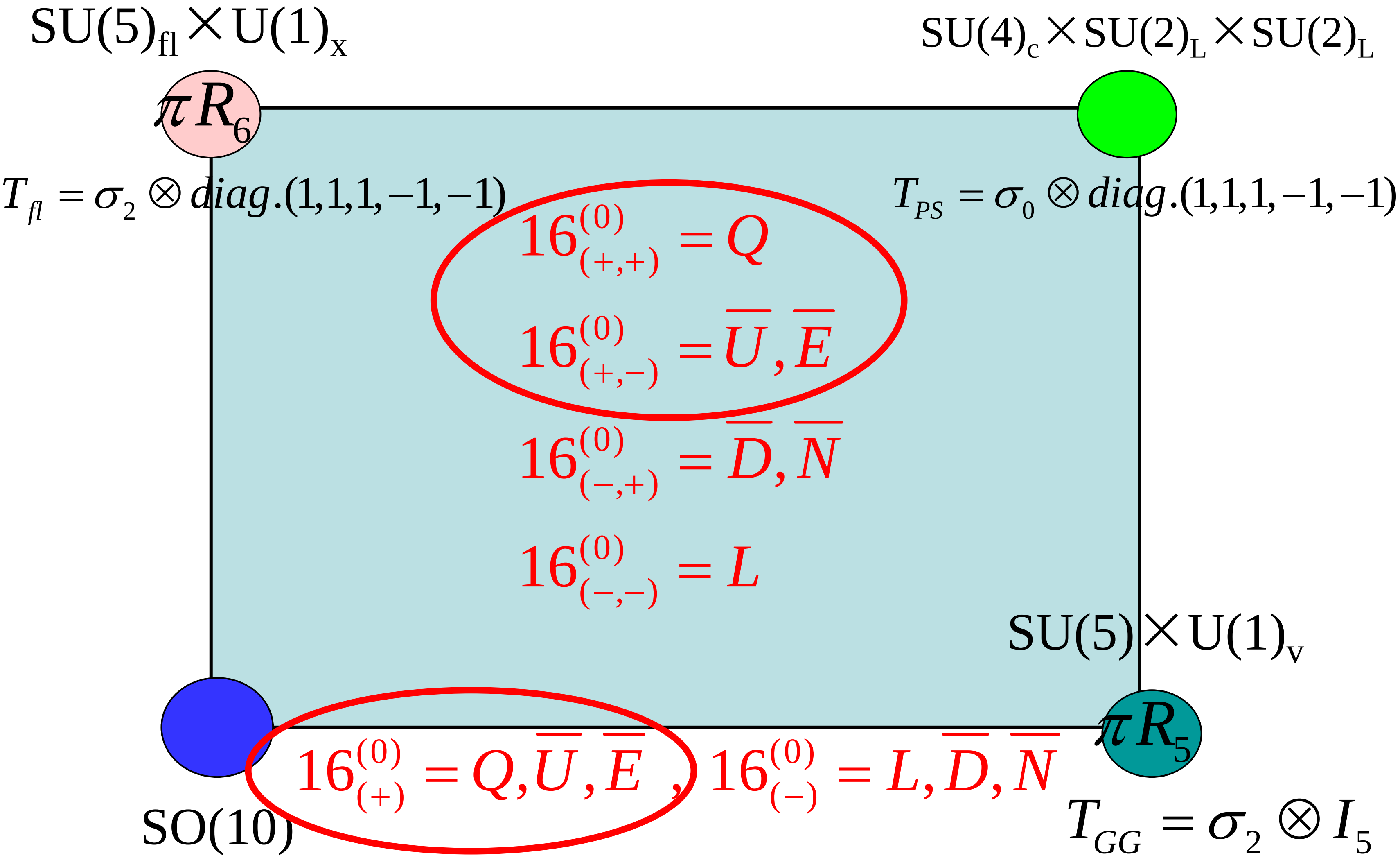
→TD splitting

$SU(5) \times U(1)_v$

πR_5

$T_{GG} = \sigma_2 \otimes I_5$

SO(10)



$\mathrm{SU}(5)_{\mathrm{fl}} \times \mathrm{U}(1)_{\mathrm{x}}$

πR_6

$T_{fl} = \sigma_2 \otimes diag.(1,1,1,-1,-1)$

$\mathrm{SU}(4)_{\mathrm{c}} \times \mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{SU}(2)_{\mathrm{L}}$

$T_{PS} = \sigma_0 \otimes diag.(1,1,1,-1,-1)$

$16^{(0)}_{(+,+)} = Q$

$16^{(0)}_{(+,-)} = \overline{U}, \overline{E}$

$16^{(0)}_{(-,+)} = \overline{D}, \overline{N}$

$16^{(0)}_{(-,-)} = L$

$\mathrm{SU}(5) \times \mathrm{U}(1)_{\mathrm{v}}$

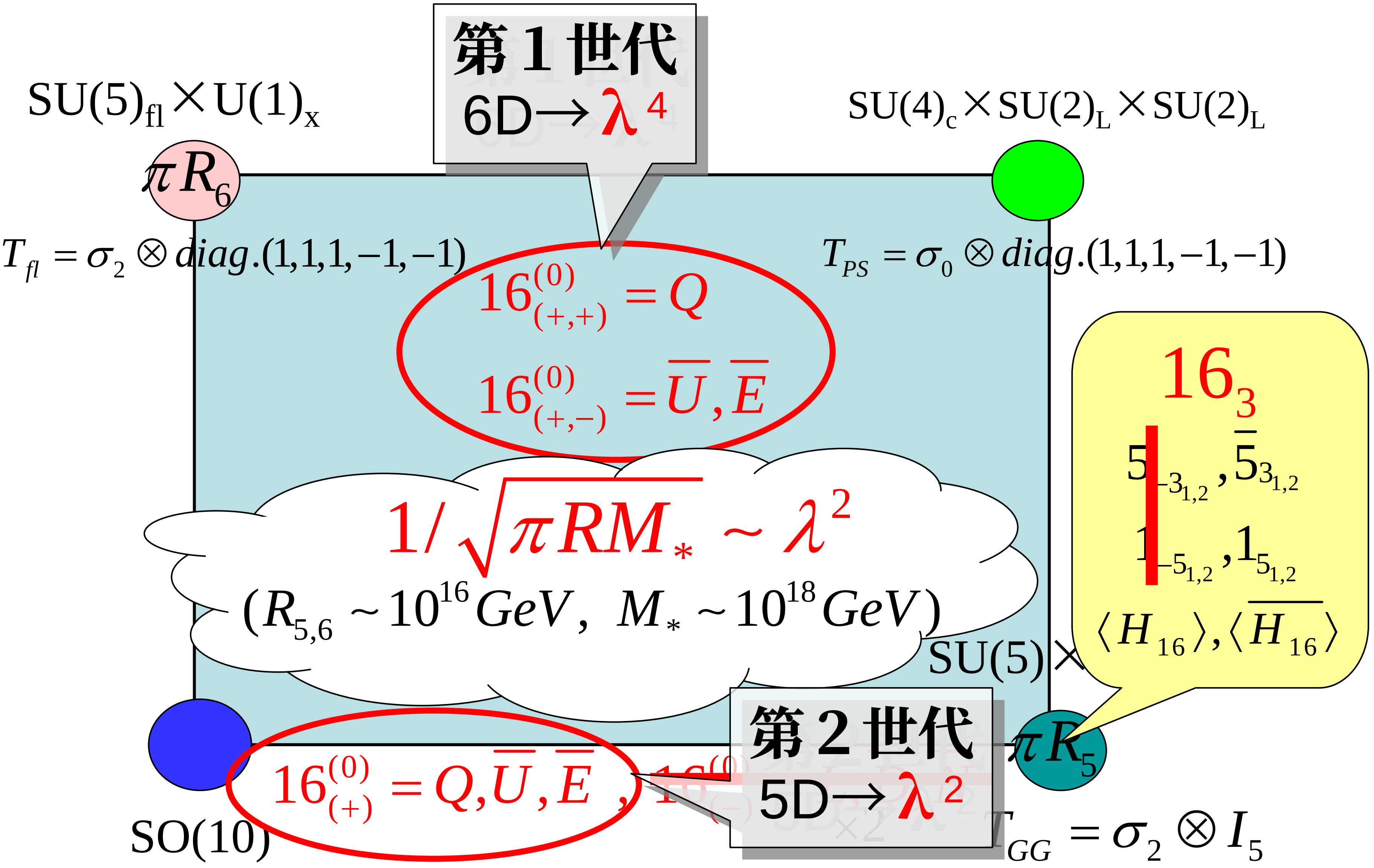
πR_5

$16^{(0)}_{(+)} = Q, \overline{U}, \overline{E}$, $16^{(0)}_{(-)} = L, \overline{D}, \overline{N}$

SO(10)

$T_{GG} = \sigma_2 \otimes I_5$

(Shimizu,N.H.)



$\mathrm{SU(5)_{fl} \times U(1)_x}$

πR_6

$T_{fl} = \sigma_2 \otimes diag.(1,1,1,-1,-1)$

第 1 世代

6D→λ^4

$\mathrm{SU(4)_c \times SU(2)_L \times SU(2)_L}$

$T_{PS} = \sigma_0 \otimes diag.(1,1,1,-1,-1)$

$16^{(0)}_{(+,+)} = Q$

$16^{(0)}_{(+,-)} = \overline{U}, \overline{E}$

$1/\sqrt{\pi R M_*} \sim \lambda^2$

$(R_{5,6} \sim 10^{16} GeV,\ M_* \sim 10^{18} GeV)$

16_3

$5_{-3_{1,2}}, \bar{5}_{3_{1,2}}$

$1_{-5_{1,2}}, 1_{5_{1,2}}$

$\langle H_{16} \rangle, \langle \overline{H_{16}} \rangle$

SU(5)×

$16^{(0)}_{(+)} = Q, \overline{U}, \overline{E}$, $16^{(0)}_{(-)}$

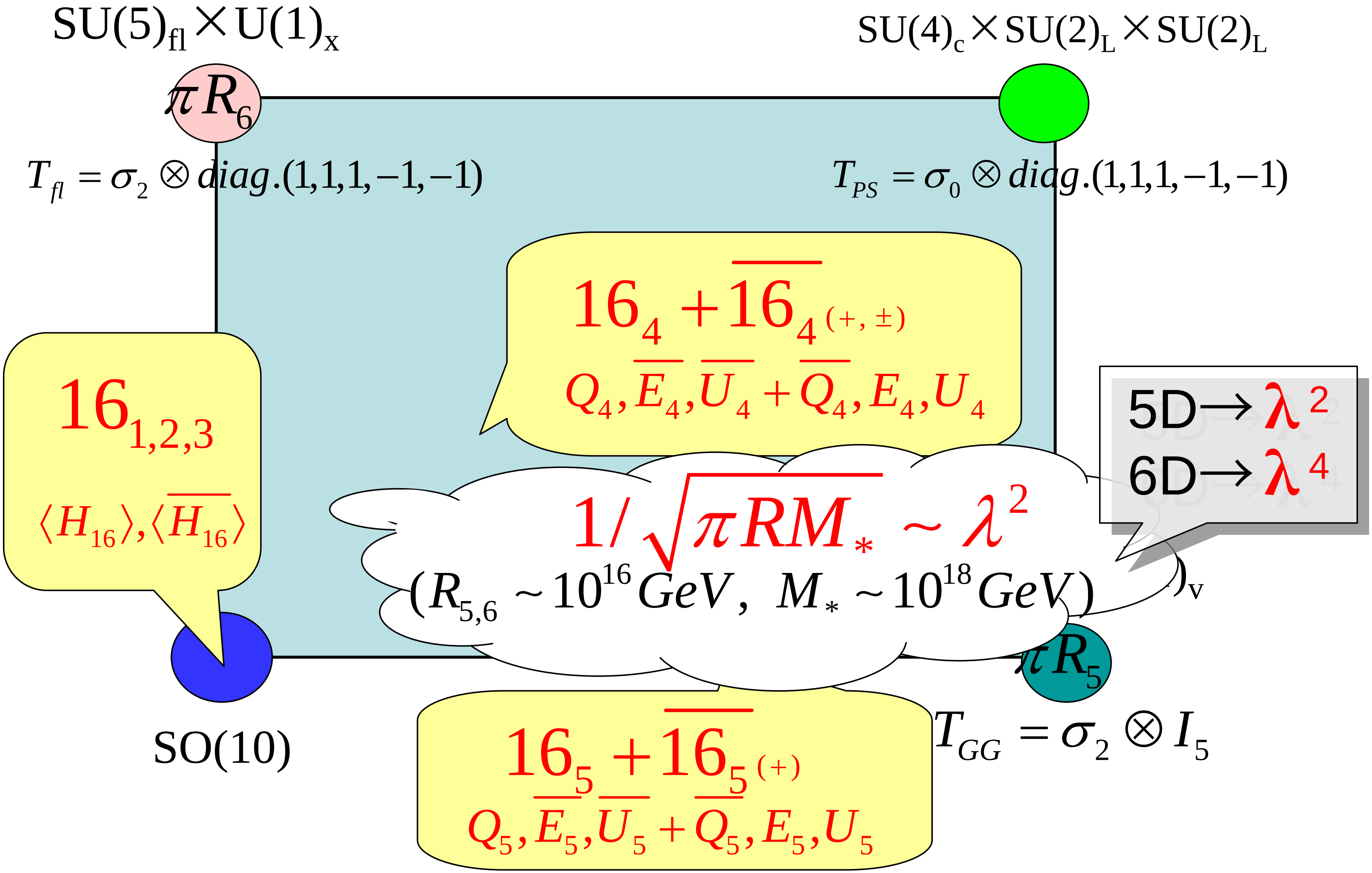
第 2 世代

5D→λ^2

πR_5

SO(10)

$T_{GG} = \sigma_2 \otimes I_5$



$SU(5)_{\rm fl}\times U(1)_{\rm x}$

πR_6

$T_{fl}=\sigma_2\otimes diag.(1,1,1,-1,-1)$

$SU(4)_{\rm c}\times SU(2)_{\rm L}\times SU(2)_{\rm L}$

$T_{PS}=\sigma_0\otimes diag.(1,1,1,-1,-1)$

$16_4+\overline{16}_4{}^{(+,\,\pm)}$

$Q_4,\overline{E}_4,\overline{U}_4+\overline{Q}_4,E_4,U_4$

$16_{1,2,3}$

$\langle H_{16}\rangle,\langle\overline{H_{16}}\rangle$

5D→ λ^2

6D→ λ^4

$1/\sqrt{\pi RM_*}\sim\lambda^2$

$(R_{5,6}\sim 10^{16}GeV,\ M_*\sim 10^{18}GeV)$

$)_{\rm V}$

πR_5

SO(10)

$T_{GG}=\sigma_2\otimes I_5$

$16_5+\overline{16}_5{}^{(+)}$

$Q_5,\overline{E}_5,\overline{U}_5+\overline{Q}_5,E_5,U_5$

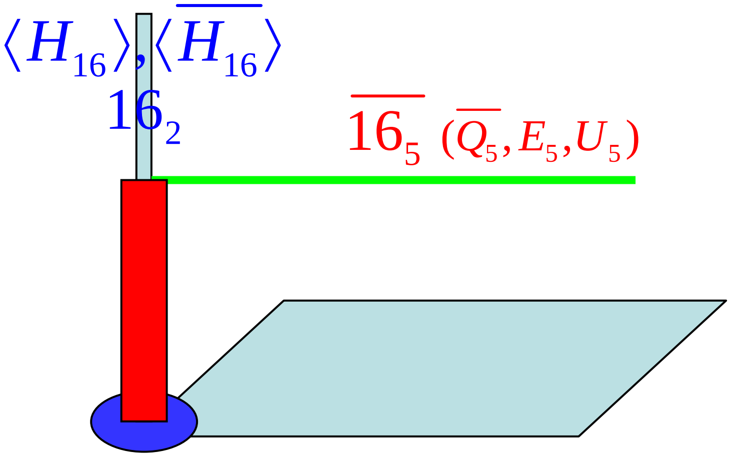
$$10=(Q,\overline{U},\overline{E})\quad \overline{5}=(\overline{D},L)\quad 1=(\overline{N})$$

$$W=y^{u}10\cdot 10\cdot H_5+y^{d/e}10\cdot\overline{5}\cdot\overline{H_5}+y^{\nu}\overline{5}\cdot 1\cdot H_5+M_R\cdot 1\cdot 1$$

$$10_3\to 10_3,\ \underline{10_2\to\lambda^2 10_2},\ \underline{10_1\to\lambda^4 10_1}$$

5D

6D



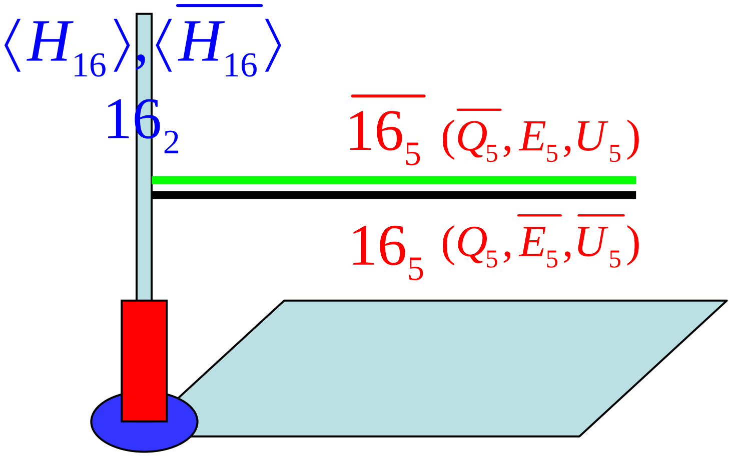
$\langle H_{16}\rangle,\langle\overline{H_{16}}\rangle$

16_2

$\overline{16}_5\ (\overline{Q_5},E_5,U_5)$

2-5 mixing mass

$$\lambda^2\,\frac{\langle H_{16}\rangle\langle\overline{H_{16}}\rangle}{M_*}\gg$$



$\langle H_{16}\rangle,\langle\overline{H_{16}}\rangle$

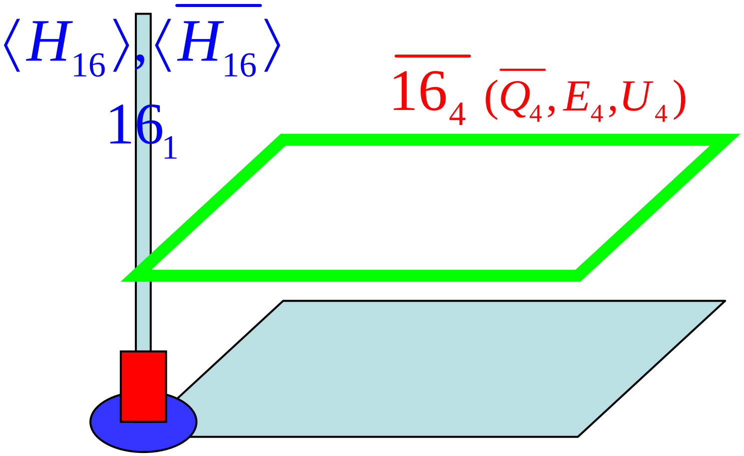
16_2

$\overline{16}_5\ (\overline{Q_5},E_5,U_5)$

$16_5\ (Q_5,\overline{E_5},\overline{U_5})$

5-5 mixing mass

$$\lambda^4\,\frac{\langle H_{16}\rangle\langle\overline{H_{16}}\rangle}{M_*}$$



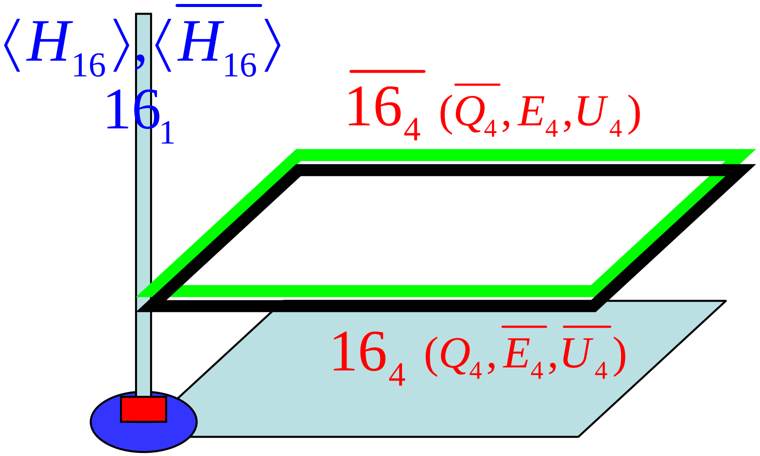
$\langle H_{16}\rangle,\langle\overline{H_{16}}\rangle$

16_1

$\overline{16}_4\ (\overline{Q_4},E_4,U_4)$

1-4 mixing mass

$$\lambda^4\,\frac{\langle H_{16}\rangle\langle\overline{H_{16}}\rangle}{M_*}\gg$$



$\langle H_{16}\rangle,\langle\overline{H_{16}}\rangle$

16_1

$\overline{16}_4\ (\overline{Q_4},E_4,U_4)$

$16_4\ (Q_4,\overline{E_4},\overline{U_4})$

4-4 mixing mass

$$\lambda^8\,\frac{\langle H_{16}\rangle\langle\overline{H_{16}}\rangle}{M_*}$$

$$W=\frac{\langle H_{16}\rangle\langle\overline{H_{16}}\rangle}{M_*}[(\lambda^4 16_1\overline{16_4}+\lambda^8 16_4\overline{16_4})+(\lambda^2 16_2\overline{16_5}+\lambda^4 16_5\overline{16_5})]$$

$$\circ 16_i\mapsto(Q_i,\overline{E_i},\overline{U_i})$$

$$\begin{cases}16_l^{(1)}\simeq\lambda^4 16_1-16_4\\ 16_H^{(1)}\simeq 16_1+\lambda^4 16_4\end{cases}\qquad\qquad\begin{cases}16_l^{(2)}\simeq\lambda^2 16_2-16_5\\ 16_H^{(2)}\simeq 16_2+\lambda^2 16_5\end{cases}$$

$$\Downarrow\qquad\qquad\qquad\qquad\Downarrow$$

$$\begin{cases}16_1\simeq\lambda^4 16_l^{(1)}+16_H^{(1)}\\ 16_4\simeq 16_l^{(1)}-\lambda^4 16_H^{(1)}\end{cases}\qquad\qquad\begin{cases}16_2\simeq\lambda^2 16_l^{(2)}+16_H^{(2)}\\ 16_5\simeq 16_l^{(2)}-\lambda^2 16_H^{(2)}\end{cases}$$

$$W_Y=16_i 16_i H$$

$$16_1\to\lambda^4 16_l^{(1)}\qquad(Q_1,\overline{E_1},\overline{U_1})\to\lambda^4(Q_1,\overline{E_1},\overline{U_1})$$

$$16_2\to\lambda^2 16_l^{(2)}\qquad(Q_2,\overline{E_2},\overline{U_2})\to\lambda^2(Q_2,\overline{E_2},\overline{U_2})$$

$$10=(Q,\overline{U},\overline{E})\quad \overline{5}=(\overline{D},L)\quad 1=(\overline{N})$$

$$W=y^{u}10\cdot 10\cdot H_{5}+y^{d/e}10\cdot\overline{5}\cdot\overline{H_{5}}+y^{\nu}\overline{5}\cdot 1\cdot H_{5}+M_{R}\cdot 1\cdot 1$$

$$10_{3}\rightarrow 10_{3},\ \underline{10_{2}\rightarrow\lambda^{2}10_{2}},\ \underline{10_{1}\rightarrow\lambda^{4}10_{1}}$$

$$W = y^u\, 10 \cdot 10 \cdot H_5 + y^{d/e}\, 10 \cdot \bar{5} \cdot \overline{H_5} + y^{\nu}\, \bar{5} \cdot 1 \cdot H_5 + M_R \cdot 1 \cdot 1$$

$$10_i = (Q, \overline{U}, \overline{E})_i \quad [10_3 \to 10_3,\ 10_2 \to \lambda^2 10_2,\ 10_1 \to \lambda^4 10_1]$$

$$\begin{array}{cccc} & L \qquad\qquad R & & L \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad R \end{array}$$

$$m_u \simeq \begin{pmatrix} \lambda^8 & \lambda^6 & \lambda^4 \\ \lambda^6 & \lambda^4 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} \langle H_u \rangle \Longrightarrow \begin{pmatrix} 1 & \lambda^2 & \lambda^4 \\ \lambda^2 & 1 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} \begin{pmatrix} \lambda^8 & & \\ & \lambda^4 & \\ & & 1 \end{pmatrix} \begin{pmatrix} 1 & \lambda^2 & \lambda^4 \\ \lambda^2 & 1 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} \langle H_u \rangle$$

$$m_d \simeq \begin{pmatrix} \lambda^4 & \lambda^4 & \lambda^4 \\ \lambda^2 & \lambda^2 & \lambda^2 \\ 1 & 1 & 1 \end{pmatrix} \langle H_d \rangle \Longrightarrow \begin{pmatrix} 1 & \lambda^2 & \lambda^4 \\ \lambda^2 & 1 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} \begin{pmatrix} \lambda^4 & & \\ & \lambda^2 & \\ & & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \langle H_d \rangle$$

$$m_l \simeq \begin{pmatrix} \lambda^4 & \lambda^2 & 1 \\ \lambda^4 & \lambda^2 & 1 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} \langle H_d \rangle \Longrightarrow \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \lambda^4 & & \\ & \lambda^2 & \\ & & 1 \end{pmatrix} \begin{pmatrix} 1 & \lambda^2 & \lambda^4 \\ \lambda^2 & 1 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} \langle H_d \rangle$$

$$m_\nu \simeq \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \frac{\langle H_u \rangle^2}{M_R} \Longrightarrow \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \frac{\langle H_u \rangle^2}{M_R}$$

$$W = y^u 10\cdot 10\cdot H_5 + y^{d/e} 10\cdot \bar{5}\cdot \overline{H_5} + y^\nu \bar{5}\cdot 1\cdot H_5 + M_R\cdot 1\cdot 1$$

$$10_i = (Q,\overline{U},\overline{E})_i \quad [10_3 \to 10_3,\ 10_2 \to \lambda^2 10_2,\ 10_1 \to \lambda^4 10_1]$$

L R L

$$m_u \simeq \begin{pmatrix}\lambda^8 & \lambda^6 & \lambda^4\\ \lambda^6 & \lambda^4 & \lambda^2\\ \lambda^4 & \lambda^2 & 1\end{pmatrix}\langle H_u\rangle \Rightarrow \begin{pmatrix}1 & \lambda^2 & \lambda^4\\ \lambda^2 & 1 & \lambda^2\\ \lambda^4 & \lambda^2 & 1\end{pmatrix}^\dagger \begin{pmatrix}\lambda^8 & & \\ & & \\ & & \end{pmatrix} \cdots \rangle$$

$$m_d \simeq \begin{pmatrix}\lambda^4 & \lambda^4 & \lambda^4\\ \lambda^2 & \lambda^2 & \lambda^2\\ 1 & 1 & 1\end{pmatrix}\langle H_d\rangle \Rightarrow \begin{pmatrix}1 & \lambda^2 & \lambda^4\\ \lambda^2 & 1 & \lambda^2\\ \lambda^4 & \lambda^2 & 1\end{pmatrix} \begin{pmatrix}\lambda^4 & & \\ \lambda^2 & & \\ & & 1\end{pmatrix} \begin{pmatrix} & & \\ 1 & 1 & 1\\ 1 & 1 & 1\end{pmatrix}\langle H_d\rangle$$

$$m_l \simeq \begin{pmatrix}\lambda^4 & \lambda^2 & 1\\ \lambda^4 & \lambda^2 & 1\\ \lambda^4 & \lambda^2 & 1\end{pmatrix}\langle H_d\rangle \Rightarrow \begin{pmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{pmatrix}^\dagger \begin{pmatrix}\lambda^4 & & \\ \lambda & & \\ & & \end{pmatrix} \begin{pmatrix}1 & \lambda^2 & \lambda^4\\ & & \\ & & \end{pmatrix} \cdots \rangle$$

$$m_\nu \simeq \begin{pmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{pmatrix}\frac{\langle H_u\rangle^2}{M_R} \Rightarrow \begin{pmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{pmatrix} \begin{pmatrix}1 & & \\ & 1 & \\ & & \end{pmatrix} \cdots \frac{\ \ ^2}{\ }$$

$$V_{CKM} \simeq \begin{pmatrix}1 & \lambda^2 & \lambda^4\\ \lambda^2 & 1 & \lambda^2\\ \lambda^4 & \lambda^2 & 1\end{pmatrix}$$

$$V_{MNS} \simeq \begin{pmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{pmatrix}$$

When $10_i = (Q, \overline{U}, \overline{E})_i$ produce hierarchy,

Good Points:

$$m_u : m_c : m_t \sim \lambda^8 : \lambda^4 : 1$$

$$m_d : m_s : m_b \sim m_e : m_\mu : m_\tau \sim \lambda^4 : \lambda^2 : 1$$

$$m_{\nu_e} : m_{\nu_\mu} : m_{\nu_\tau} \sim 1 : 1 : 1$$

small flavor mixing in Quark ⇔ large flavor mixing in Lepton

Bad Points:

1,2**世代の質量** $m_d, m_e \cdots,$ $m_\mu \sim m_s$

too large U_{e3}, too small V_{us}

coefficients of O(1):

determination of O(1) coefficients

↓

high rep. Higgs & vector-like fields at high energy

[16 vector like, 45 Higgs ……(Babu-Barr)]

↓

integrating out heavy fields

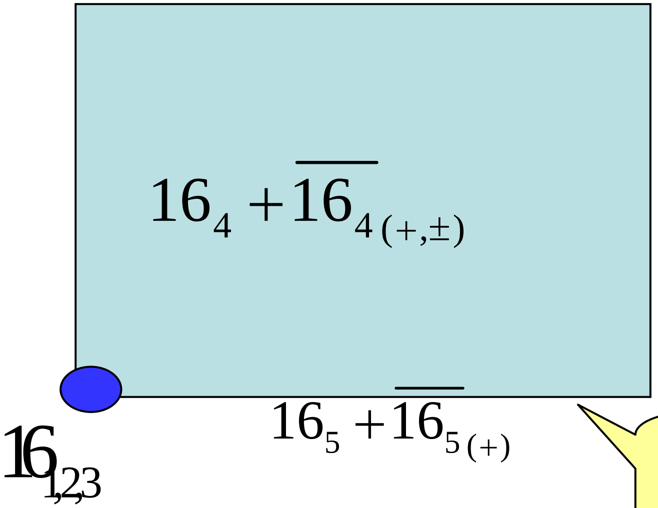
$$m_u \simeq \begin{pmatrix} 0 & -4d\lambda^6 & 0 \\ -4d\lambda^6 & c\lambda^4 & 0 \\ 0 & b\lambda^2 & 1 \end{pmatrix} \langle H_u \rangle, \quad m_d \simeq \begin{pmatrix} 4d\lambda^4 & d\lambda^4 & d\lambda^4 \\ d/5\lambda^2 & d\lambda^2 & d\lambda^2 \\ c/2 & b & 1 \end{pmatrix} \langle H_d \rangle,$$

$$m_l \simeq \begin{pmatrix} \lambda^4 & 0 & 0 \\ b\lambda^4 & -2c\lambda^2 & 1 \\ 0 & -b\lambda^2 & 5 \end{pmatrix} \langle H_d \rangle, \quad m_\nu \simeq \begin{pmatrix} e & e & 0 \\ 0 & c & 2.5 \\ 0 & 2.5 & 5 \end{pmatrix} \frac{\langle H_u \rangle^2}{M_R}$$

$$(b = 4,\ \ c = 3.6,\ \ d = 2,\ \ e = 1)$$

(Maru,Nakamura,NH)

modification



$$16_4 + \overline{16}_{4(+,\pm)}$$

$$16_{1,2,3}$$

$$16_5 + \overline{16}_{5(+)}$$

$$16_5 + \overline{16}_{5(-)}$$

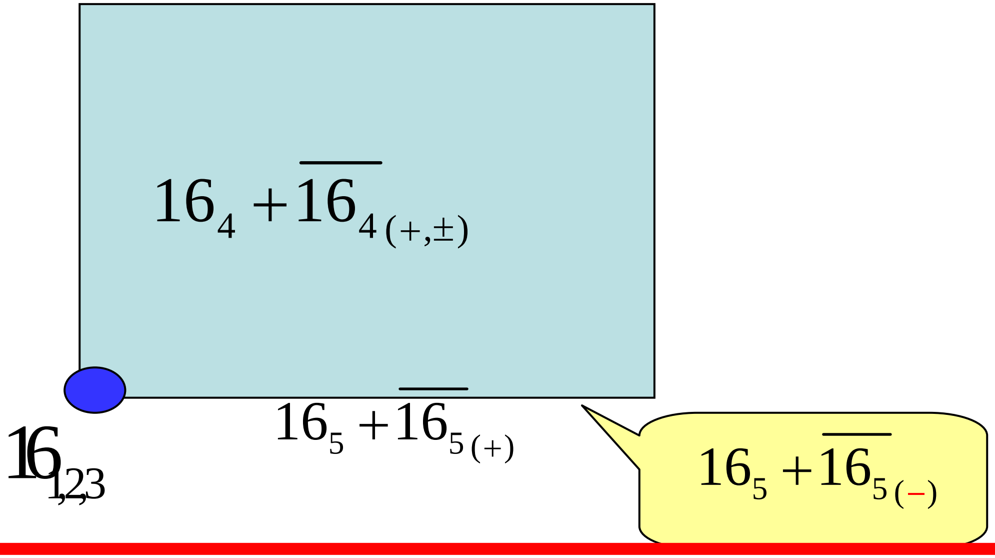
$$V_{CKM} \simeq \begin{pmatrix} 1 & \lambda^2 & \lambda^4 \\ \lambda^2 & 1 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix}$$

$$m_u \simeq \begin{pmatrix} \lambda^8 & \lambda^6 & \lambda^4 \\ \lambda^6 & \lambda^4 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} \langle H_u \rangle, m_d \simeq \begin{pmatrix} \lambda^4 & \lambda^4 & \lambda^4 \\ \lambda^2 & \lambda^2 & \lambda^2 \\ 1 & 1 & 1 \end{pmatrix} \langle H_d \rangle$$

$$m_l \simeq \begin{pmatrix} \lambda^4 & \lambda^2 & 1 \\ \lambda^4 & \lambda^2 & 1 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} \langle H_d \rangle, m_\nu \simeq \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \frac{\langle H_u \rangle^2}{M_R}$$

$$V_{MNS} \simeq \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

modification



$16_4 + \overline{16}_{4(+,\pm)}$

$16_{1,2,3}$

$16_5 + \overline{16}_{5(+)}$

$16_5 + \overline{16}_{5(-)}$

$$V_{CKM} \simeq \begin{pmatrix} 1 & \lambda^2 & \lambda^4 \\ \lambda^2 & 1 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix}$$

$$m_u \simeq \begin{pmatrix} \lambda^8 & \lambda^6 & \lambda^4 \\ \lambda^6 & \lambda^4 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} \langle H_u \rangle, m_d \simeq \begin{pmatrix} {\color{red}\lambda^6} & \lambda^4 & \lambda^4 \\ {\color{red}\lambda^4} & \lambda^2 & \lambda^2 \\ {\color{red}\lambda^2} & 1 & 1 \end{pmatrix} \langle H_d \rangle$$

$$m_l \simeq \begin{pmatrix} {\color{red}\lambda^6} & {\color{red}\lambda^4} & {\color{red}\lambda^2} \\ \lambda^4 & \lambda^2 & 1 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} \langle H_d \rangle, m_\nu \simeq \begin{pmatrix} {\color{red}\lambda^4} & {\color{red}\lambda^2} & {\color{red}\lambda^2} \\ {\color{red}\lambda^2} & 1 & 1 \\ {\color{red}\lambda^2} & 1 & 1 \end{pmatrix} \frac{\langle H_u \rangle^2}{M_R}$$

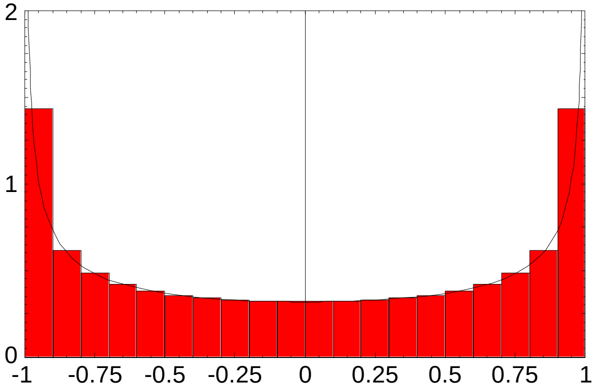
$$V_{MNS} \simeq \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & \lambda^2 \\ 1/2 & -1/2 & 1/\sqrt{2} \\ -1/2 & 1/2 & 1/\sqrt{2} \end{pmatrix}$$

☆ランダム係数

(Murayama,Hall,..
Murayama,NH)

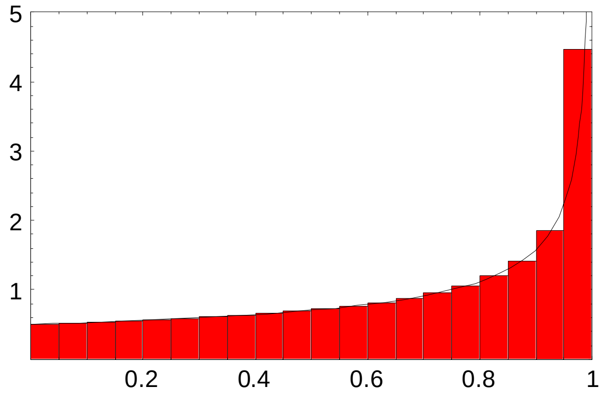
$$m_\nu \simeq \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \frac{\langle H_u \rangle^2}{M_R}$$

$$d\delta = \frac{1}{\cos\delta} d(\sin\delta)$$

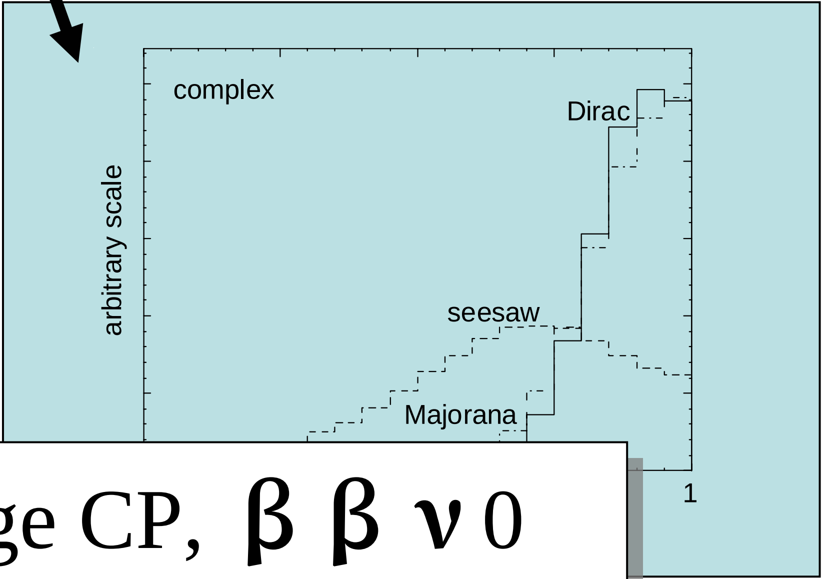


$\sin\delta$

$$d(\sin^2\theta) = \frac{1}{4\cos 2\theta} d(\sin^2 2\theta)$$



$\sin^2 2\theta$



予言：large U$_{e3}$, large CP, ββν0

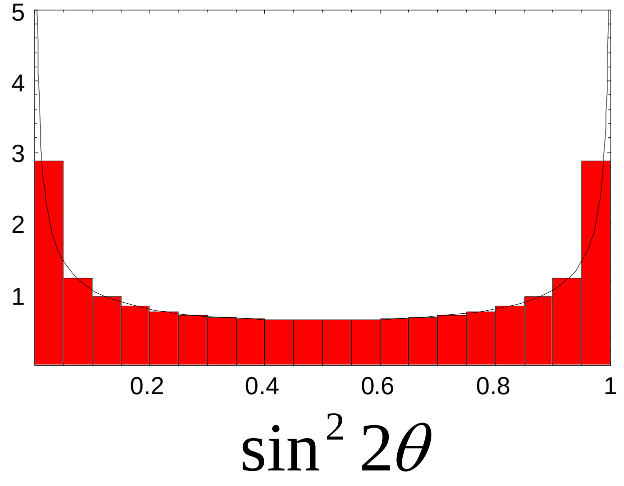
real Majorana (3×3)

$$dO = d\vartheta_{12}d(\sin\vartheta_{13})d\vartheta_{23}$$

$$dM_{11}\wedge dM_{12}\wedge dM_{22} = \Delta m\ d\overline{m}\wedge d(\Delta m)\wedge d\theta$$

$$[d(\theta) = \frac{\cos^4 2\theta}{16\cos^2\theta\sin^2\theta}d(\tan^2 2\theta)]$$

$$d(\theta) = \frac{1}{4\cos 2\theta\sin 2\theta}d(\sin^2 2\theta)$$



complex Majorana (3×3)

$$dU = d(\sin^2\vartheta_{12})d(\cos^4\vartheta_{13})d(\sin^2\vartheta_{23})$$

$$d^2M_{11}d^2M_{12}d^2M_{22} = (m_1^2 - m_2^2)dm_1^2dm_2^2dU$$

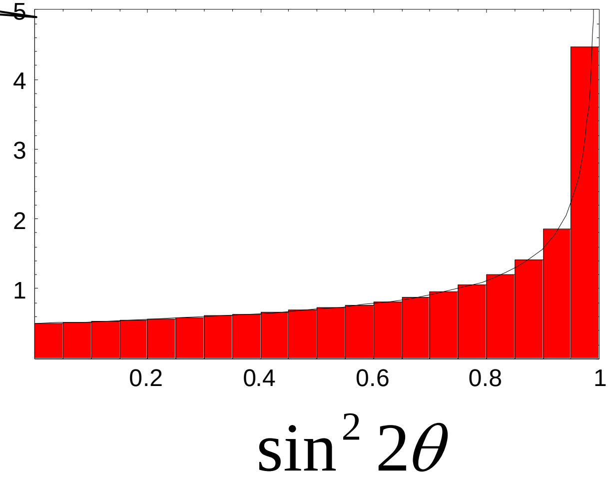
$$dU = d(\sin^2\theta)d\eta d\omega d\phi$$

distributions in angles ← Haar measure

$$dU = ds_{12}^2dc_{13}^2ds_{23}^2d\delta d\eta d\phi_1d\phi_2d\chi_1d\chi_2$$

$$dU = e^{i\eta}e^{i\phi_1\lambda_3+i\phi_2\lambda_8}\begin{pmatrix}1 & 0 & 0\\ 0 & c_{23} & s_{23}\\ 0 & -s_{23} & c_{23}\end{pmatrix}\begin{pmatrix}c_{13} & 0 & s_{13}e^{-i\delta}\\ 0 & 1 & 0\\ -s_{13}e^{i\delta} & 0 & c_{13}\end{pmatrix}\begin{pmatrix}c_{12} & s_{12} & 0\\ -s_{12} & c_{12} & 0\\ 0 & 0 & 1\end{pmatrix}e^{i\chi_1\lambda_3+i\chi_2\lambda_8}$$

$$d(\sin^2\theta) = \frac{1}{4\cos 2\theta}d(\sin^2 2\theta)$$



The MNS matrix: $U_{MNS}=U_l^{\dagger}U_\nu$

$$m_l\sim\begin{pmatrix}\lambda^4&\lambda^2&1\\\lambda^4&\lambda^2&1\\\lambda^4&\lambda^2&1\end{pmatrix},\rightarrow m_l\,m_l^{\dagger}\sim\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix}$$

(Q): U_lとU_νからのピーク同士がcancelしないか？

(A): Phaseの自由度のおかげでcancelしない。

$$U_l\sim e^{i\eta_l}\begin{pmatrix}e^{i\varpi_l}&\\&e^{-i\varpi_l}\end{pmatrix}\begin{pmatrix}\cos\theta_l&\sin\theta_l\\-\sin\theta_l&\cos\theta_l\end{pmatrix}\begin{pmatrix}e^{i\phi_l}&\\&e^{-i\phi_l}\end{pmatrix}$$

$$U_\nu\sim e^{i\eta_\nu}\begin{pmatrix}e^{i\varpi_\nu}&\\&e^{-i\varpi_\nu}\end{pmatrix}\begin{pmatrix}\cos\theta_\nu&\sin\theta_\nu\\-\sin\theta_\nu&\cos\theta_\nu\end{pmatrix}\begin{pmatrix}e^{i\phi_\nu}&\\&e^{-i\phi_\nu}\end{pmatrix}$$

The mixing angle θ in $U_{MNS}=U_l^{\dagger}U_\nu$ is given by

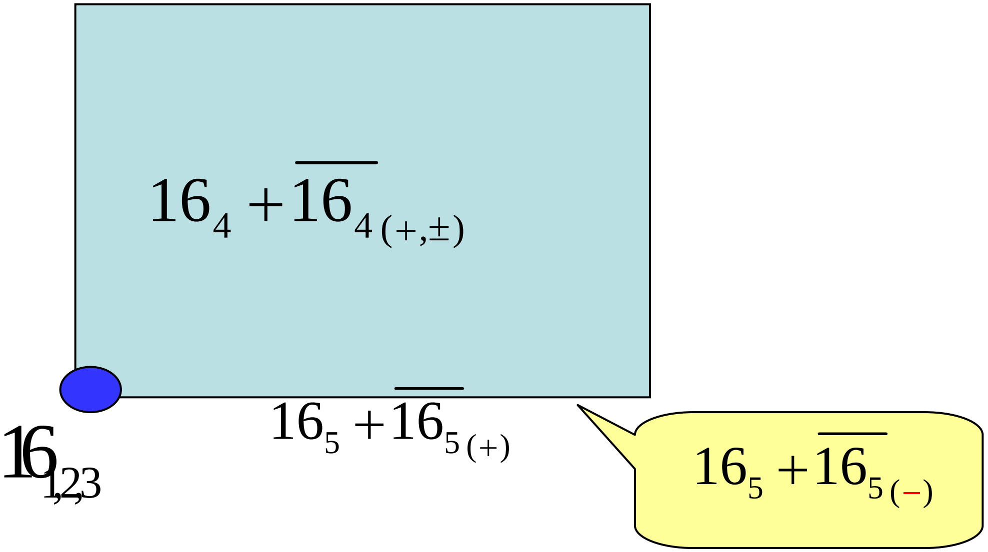
$$\sin^2\theta=\cos^2\theta_l\sin^2\theta_\nu+\sin^2\theta_l\cos^2\theta_\nu-2\cos\theta_l\sin\theta_\nu\sin\theta_l\cos\theta_\nu\cos2(\varpi_\nu-\varpi_l)$$

$peak\quad at\quad\theta\sim\frac{\pi}{4}$

$peak\quad at\quad\theta_l\sim\theta_\nu\sim\frac{\pi}{4}$

$flat$

modification



$16_4+\overline{16}_{4(+,\pm)}$

$16_{1,2,3}$

$16_5+\overline{16}_{5(+)}$

$16_5+\overline{16}_{5(-)}$

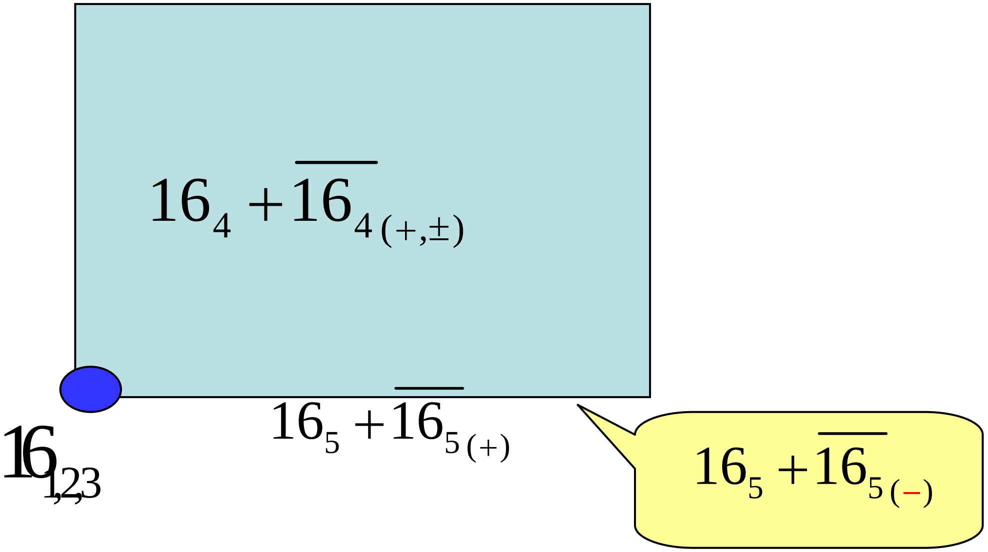
$$V_{CKM}\simeq\begin{pmatrix}1 & \lambda^2 & \lambda^4\\ \lambda^2 & 1 & \lambda^2\\ \lambda^4 & \lambda^2 & 1\end{pmatrix}$$

$$m_u\simeq\begin{pmatrix}\lambda^8 & \lambda^6 & \lambda^4\\ \lambda^6 & \lambda^4 & \lambda^2\\ \lambda^4 & \lambda^2 & 1\end{pmatrix}\langle H_u\rangle, m_d\simeq\begin{pmatrix}\lambda^4 & \lambda^4 & \lambda^4\\ \lambda^2 & \lambda^2 & \lambda^2\\ 1 & 1 & 1\end{pmatrix}\langle H_d\rangle$$

$$m_l\simeq\begin{pmatrix}\lambda^4 & \lambda^2 & 1\\ \lambda^4 & \lambda^2 & 1\\ \lambda^4 & \lambda^2 & 1\end{pmatrix}\langle H_d\rangle, m_\nu\simeq\begin{pmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{pmatrix}\frac{\langle H_u\rangle^2}{M_R}$$

$$V_{MNS}\simeq\begin{pmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{pmatrix}$$

modification



$16_4 + \overline{16}_{4(+,\pm)}$

$16_5 + \overline{16}_{5(+)}$

$16_{1,2,3}$

$16_5 + \overline{16}_{5(-)}$

$$V_{CKM} \simeq \begin{pmatrix} 1 & \lambda^2 & \lambda^4 \\ \lambda^2 & 1 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix}$$

$$m_u \simeq \begin{pmatrix} \lambda^8 & \lambda^6 & \lambda^4 \\ \lambda^6 & \lambda^4 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} \langle H_u \rangle, m_d \simeq \begin{pmatrix} \lambda^6 & \lambda^4 & \lambda^4 \\ \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^2 & 1 & 1 \end{pmatrix} \langle H_d \rangle$$

$$m_l \simeq \begin{pmatrix} \lambda^6 & \lambda^4 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} \langle H_d \rangle, m_\nu \simeq \begin{pmatrix} \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^2 & 1 & 1 \\ \lambda^2 & 1 & 1 \end{pmatrix} \frac{\langle H_u \rangle^2}{M_R}$$

$$V_{MNS} \simeq \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & \lambda^2 \\ 1/2 & -1/2 & 1/\sqrt{2} \\ -1/2 & 1/2 & 1/\sqrt{2} \end{pmatrix}$$

- flavor&**質量階層 (世代) 構造の解明に向けての探求**-

質量階層性　&　世代構造

（lepton sector: large flavor mix ⇔quark sector: small flavor mix）

★§4　**ν特有の効果**
- see-saw enhancement
- RGE

★§6　**SU(5) GUT 10表現→質量階層性 & flavor mix**

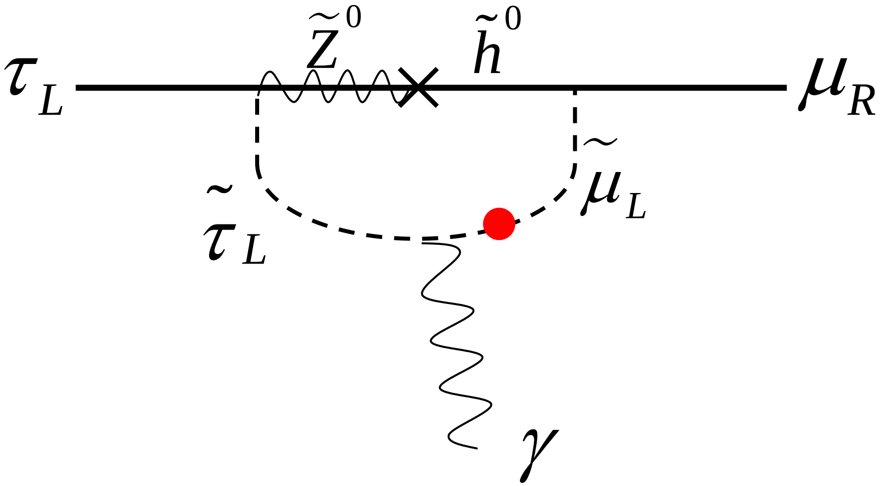
→ 0(1)**係数の評価**

★ LFV, B, ν textures, ……….

(例1）LFV

(MNS origin: SM+m$_\nu$→small, while SUSY→large contrivution in general)

L_{Li}の1-2 & 2-3 large混合 ⇒ μ→e γ, τ→μ γ,…

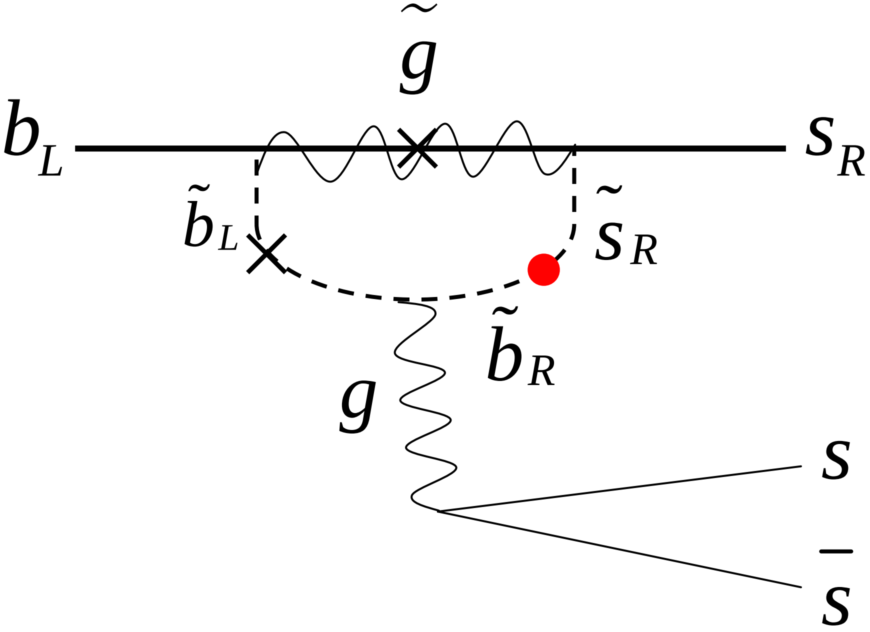


★ LFV, B, ν textures, ……….

(例2）B崩壊： **$5^*_i=(L_L, D_R)_i$** **(GUT base)**

$\underline{L_{Li}\text{の2-3 large混合} \Rightarrow D_{Ri}\text{の2-3 large混合}}$

$B \to K\phi$:



SMではRの混合は物理的ではない。
BUT, SUSYでは物理的！
（cf：neucleon parity violation）

7. Big Questions

まだまだ分かっていないことがた〜くさん！

独断と偏見で幾つか紹介しますね。

7-1: 世代って何だろう？何故三世代？

7-2: 何故四次元？

7-3: 宇宙項の謎

7-1. 世代って何だろう？何故三世代？

ＬＥＰ→$\frac{M_Z}{2}$より軽いνは３世代

でも、$\frac{M_Z}{2}$より重い世代はあっていい

様々な説明の試み

(A): 超弦理論(Calabi-Yau,orbifold)

(B): 3-3-1 model

(C): fixed points of extra dim.

(d): G_{flavor} (ETC,SU(1,1),...)

・・・・・・

(A):超弦理論(Calabi-Yau,orbifold)

KK modeは無限個(世代数無限)
→zero modeのみが低エネルギーの“世代”

6次元

世代数＝(オイラー数)/2

$$(\Box_4+\Box_6)\psi=0$$

$\Box_6\psi=0$ ←massless (zero mode)

↑6次元調和フォームである。
6次元多様体の**調和フォーム数**
＝Betti 数

$$\chi(M_6)=2(h_{2,1}-h_{1,1})$$

世代
(27,3)

反世代
(27*,3*)

$(E_6, SU(3))$

cf. 4世代模型：

$$CP^4:\sum_{i=1}^{5} z_i^{\,5}=0\rightarrow h_{2,1}=5,\ \ h_{1,1}=1$$

3世代模型：

$$CP^3\times CP^{3'}:f(z_i,z_i')=0\rightarrow h_{2,1}=9,h_{1,1}=6$$

CP^N: N+1 complex→z=λz (λnonzero)
for examples, $CP_1=S^2$

(B):3-3-1 model

(Frampton….., 1992)

$$SU(3)_c \times SU(3)_L \times U(1)_X \supset SU(3)_c \times SU(3)_L \times U(1)_X$$

gauge anomaly free ⇒ 世代数＝カラー数！

→new exsotic quark, bilepton gauge boson

$$\ell_{1,2,3} = \begin{pmatrix} e \\ \nu_e \\ e^c \end{pmatrix}, \begin{pmatrix} \mu \\ \nu_\mu \\ \mu^c \end{pmatrix}, \begin{pmatrix} \tau \\ \nu_\tau \\ \tau^c \end{pmatrix} : (1, 3^*, 0),$$

$$Q_{1,2} = \begin{pmatrix} u \\ d \\ D \end{pmatrix}, \begin{pmatrix} c \\ s \\ S \end{pmatrix} : (3, 3, -\frac{1}{3}),$$

$$Q_3 = \begin{pmatrix} t \\ b \\ T \end{pmatrix} : (3, 3^*, \frac{2}{3}),$$

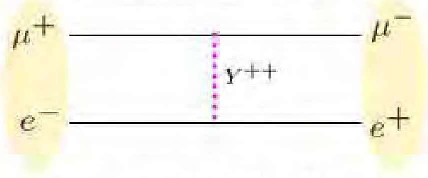
$$d^c, s^c, b^c : (3^*, 1, \frac{1}{3}),$$

$$u^c, c^c, t^c : (3^*, 1, -\frac{2}{3}),$$

$$D^c, S^c : (3^*, 1, \frac{4}{3}), \qquad T^c : (3^*, 1, -\frac{5}{3}),$$

D, S, T : Extra Quarks

- **Wrong muon decay**
- **Unitarity of CKM matrix**
- **Muoniumu-Antimuonium Conversion**



$M_{Y^{++}} > 800\text{GeV}$ **(Willmann *et al*, 1999)**

In the minimal set of Higgs,

$$\frac{M_{Z'}^2}{M_Y^2} \simeq \frac{4}{3} \frac{\cos^2\theta_W}{1 - 4\sin^2\theta_W}$$

⇨ $M_Y < 700\text{GeV}$

Minimal Model is excluded(?)

(Y.Mimura’s OHP)

(C):fixed points of extra dim.

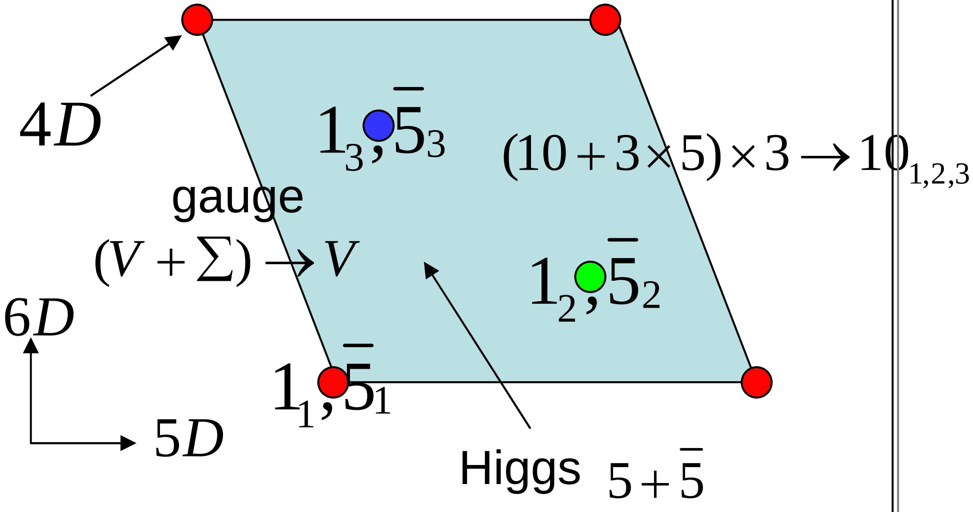
6次元空間 (5,6次元座標をorbifold compactification)

T_2/Z_3 orbifold

$z \to z + 2\pi R i$

$z \to z + 2\pi R e^{2\pi i/3}$

$z = z e^{2\pi i/3}$



3つのfixed point=3世代

(Yanagida-Watari (02))

Democratic mass matrix

$$m_{q/l} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, m_\nu = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$U = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ 0 & -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{pmatrix} \Rightarrow \begin{array}{l} \sin^2 2\theta_{12} = 1 \\ \sin^2 2\theta_{23} = \frac{8}{9} \\ \sin^2 2\theta_{13} = 0 \end{array}$$

$V_{CKM} = U_u{}^\dagger U_d \sim I$ →small mixing

$V_{MNS} = U_l{}^\dagger$ →large mixing

(D): G_{flavor} (ETC,SU(1,1),...)

ETC:	u	c	t	T_u	T_u	...
	d	s	b	T_d	T_d	...
	e	μ	τ	T_e	T_e	...

→ N個の世代

SU(1,1): $1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \ .....$

$\bar{4} \quad \bar{5} \quad \bar{6} \ .....$

(Inoue-Yamashita)

→ ∞個の世代

→F ?

7-2. 何故四次元？

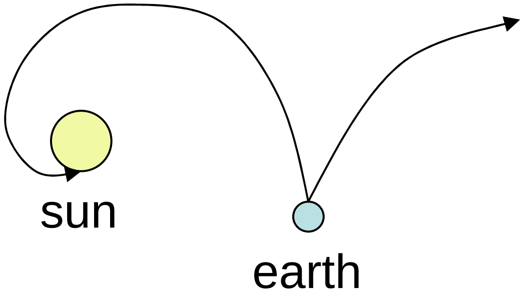
d+1次元時空：

(M.Sakamoto’s lecture note)

☆ 太陽系

万有引力 $\sim \dfrac{1}{r^{d-2}}$

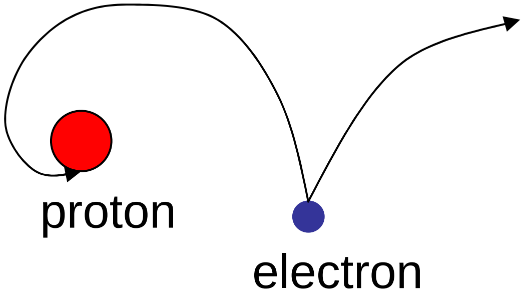
遠心力 $\sim \dfrac{1}{r^{2}}$



☆ 原子系

クーロン力 $\sim \dfrac{1}{r^{d-2}}$

不確定性関係からの“力” $\sim \dfrac{1}{r^{2}}$



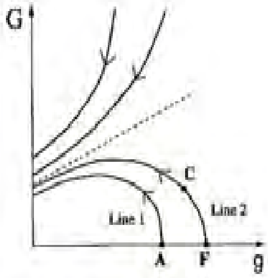
４Ｄは非常に特別な次元。

☆ duality (曲率: 2 form)

$$\tilde{F}^{\mu\nu} = \varepsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$$

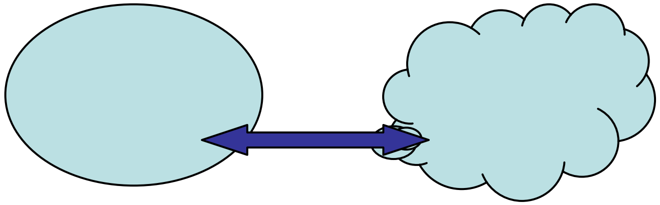
☆ gauge coupling → dimension-less (marginal)

gauge theoryの **繰り込み可能性**



cf. SM繰り込み可能→cutoff十分high energy

☆ R^{4}は∞個の微分構造を持つ



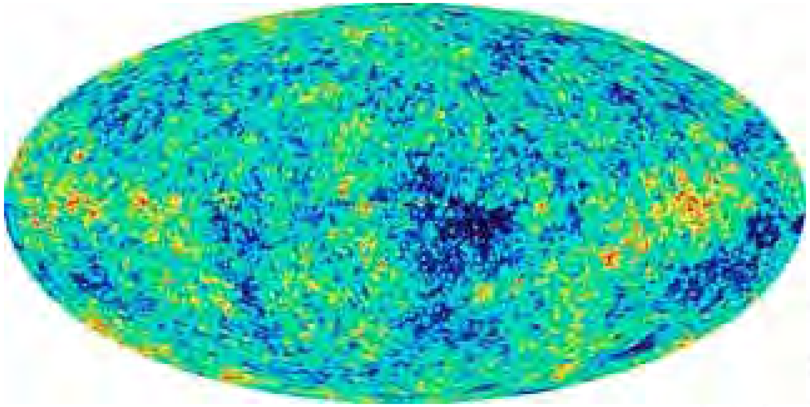
位相同形だが微分同相でない多様体

for examples, S^7 : 28

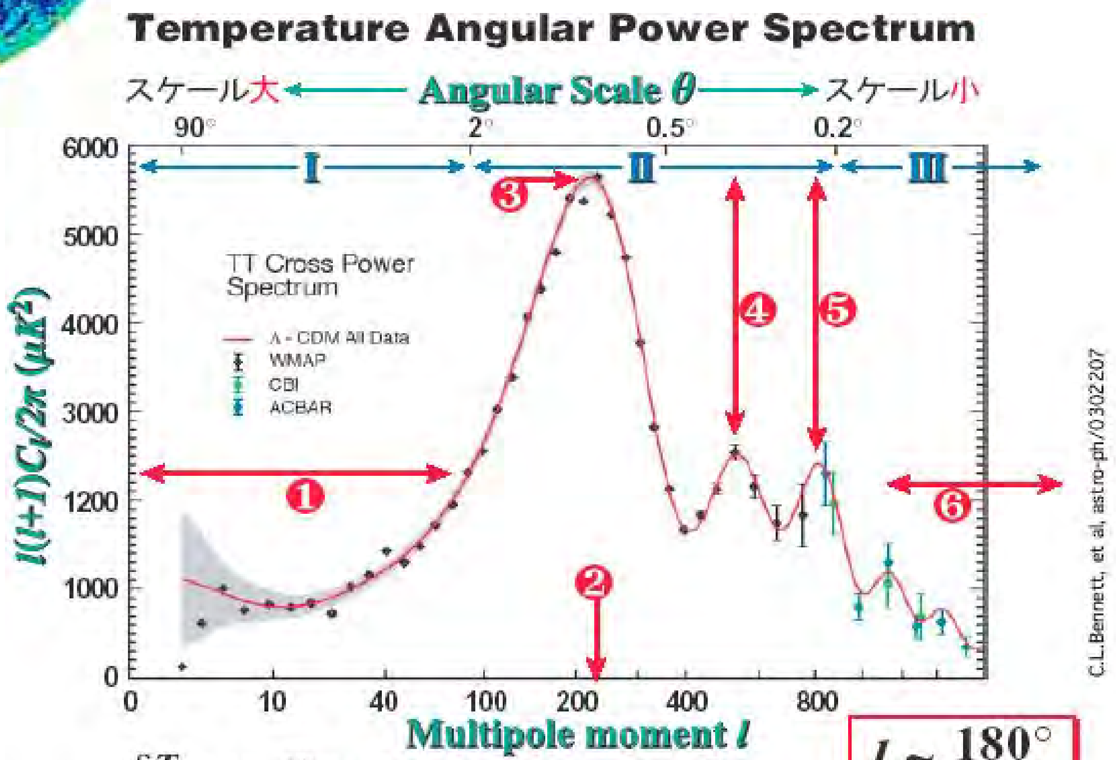
(graph: K.Aoki’s lecture note)

7-3. 宇宙項の謎

宇宙背景放射: 温度ゆらぎ　δT / T



WMAP



$$\frac{\delta T}{T}(\theta,\phi)=\sum_{lm} a_{lm} Y_{lm}(\theta,\phi),\quad C_l=<|a_{lm}|^2>$$

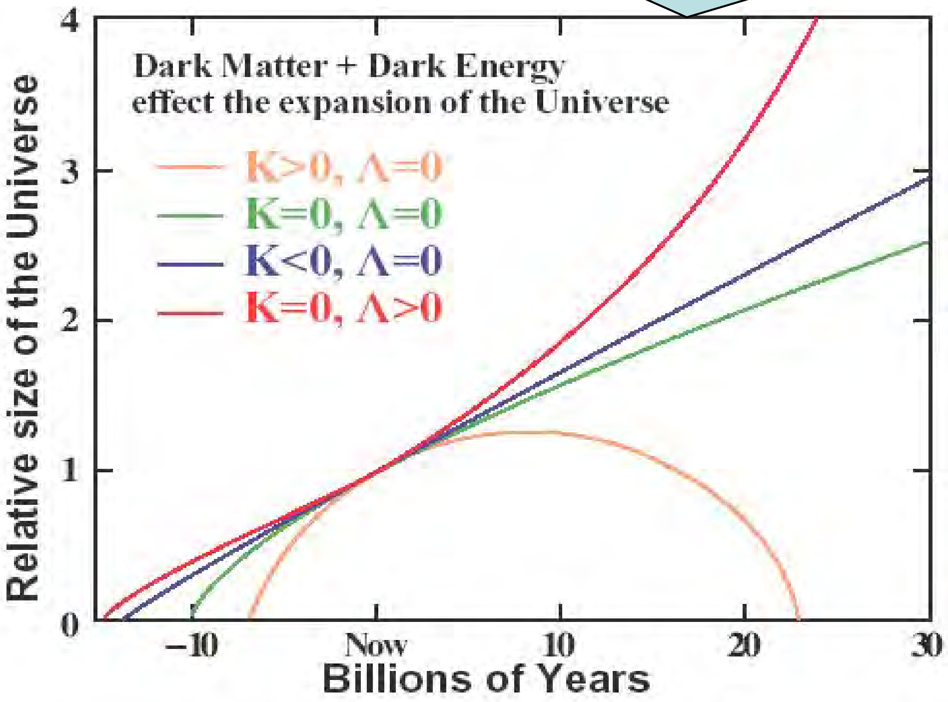
$$\boxed{l \sim \frac{180^\circ}{\theta}}$$

(M.Sakamoto’s OHP, WMAP HP)

わかったこと

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\rho - \frac{Kc^2}{a^2} + \frac{\Lambda c^2}{3}$$

1．宇宙は現在第二のインフレーション時期
2．宇宙年齢問題（$t_0 = 2/3H_0$～92億年≪
古い球状星団110～160億年）クリアー
3．再電離時期　z～20



WMAP ⊕ CBI, ACBAR
2dFGRS, Lyman α

$\Omega_\nu h^2 < 0.0076$ (95% CL)

$$\Omega_\nu h^2 = \frac{\sum_i m_i}{93.5\,eV}$$

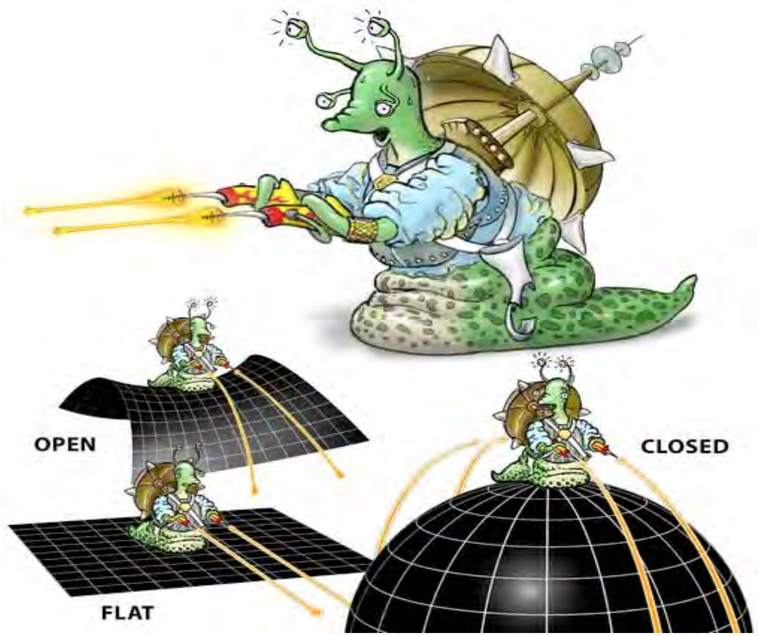
（晴れ上がり の時に相対論的なニュートリノに対してのみ和を取る。）

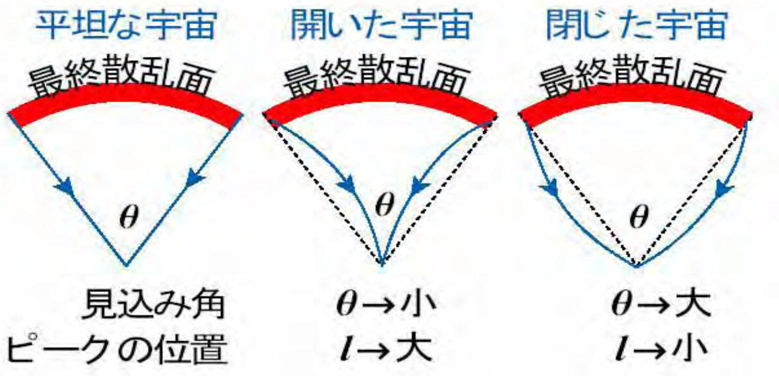
$m_{\nu_e} \sim m_{\nu_\mu} \sim m_{\nu_\tau}$を仮定

$m_\nu < 0.23\,eV$

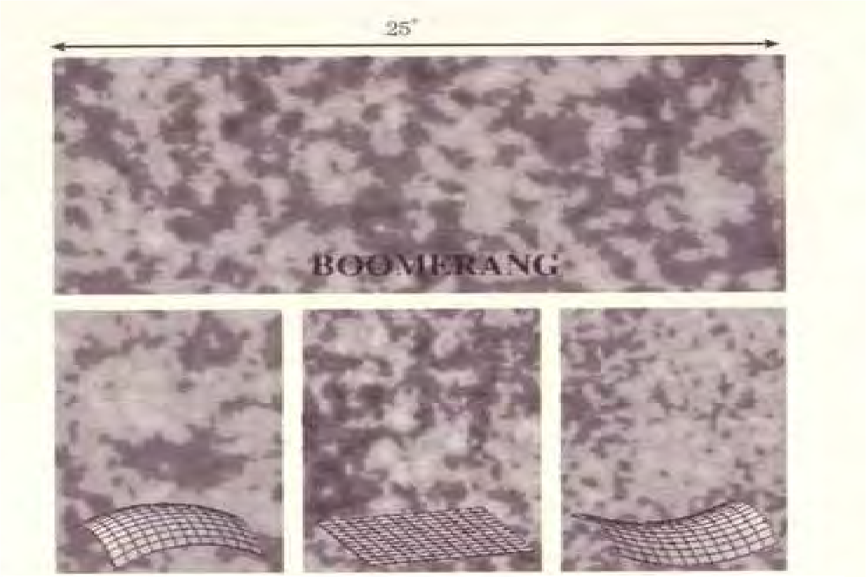
(M.Sakamoto’s OHP, WMAP HP)

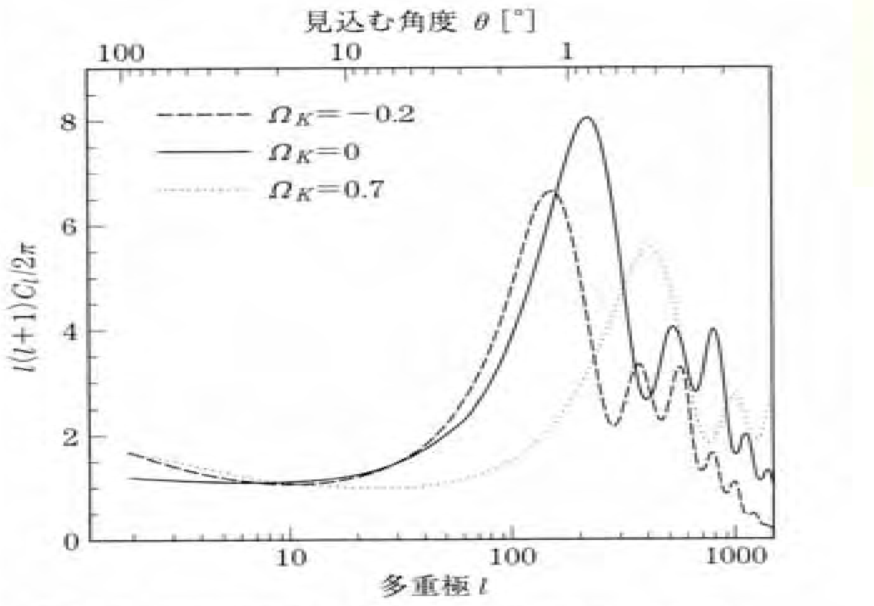
K (曲率) ＝0





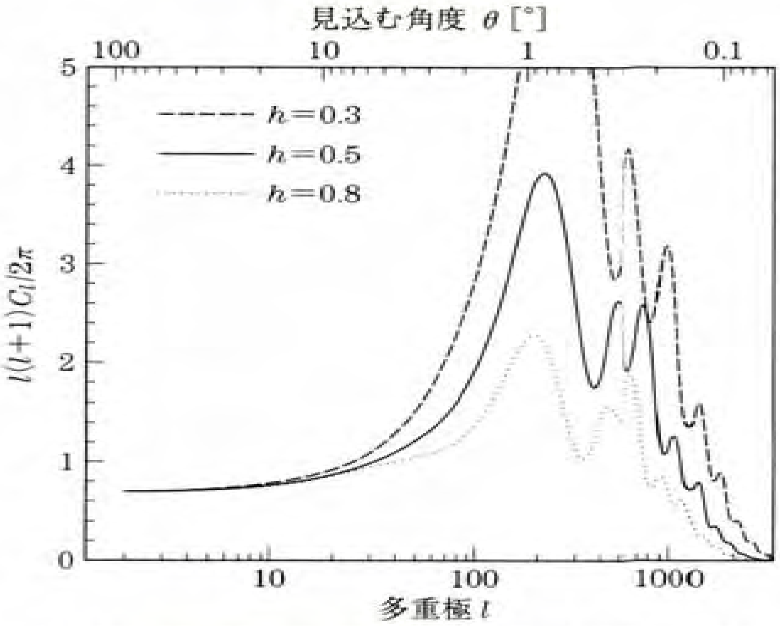
平坦な宇宙	開いた宇宙	閉じた宇宙
最終散乱面	最終散乱面	最終散乱面
見込み角 ピークの位置	θ→小 l→大	θ→大 l→小



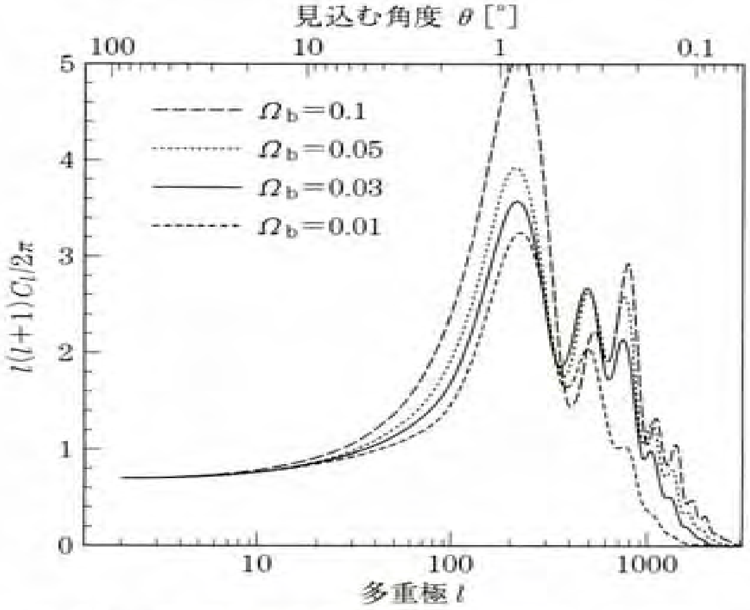


(M.Sakamoto's OHP, WMAP HP, N.Sugiyama岩波書店)

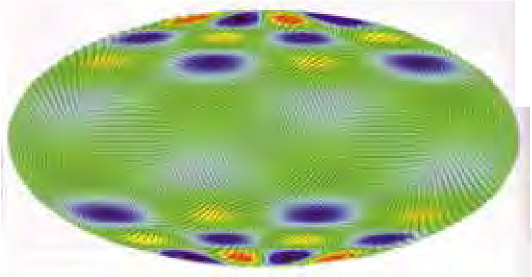
Ω_m依存性



Ω_b依存性



(N.Sugiyama岩波書店)



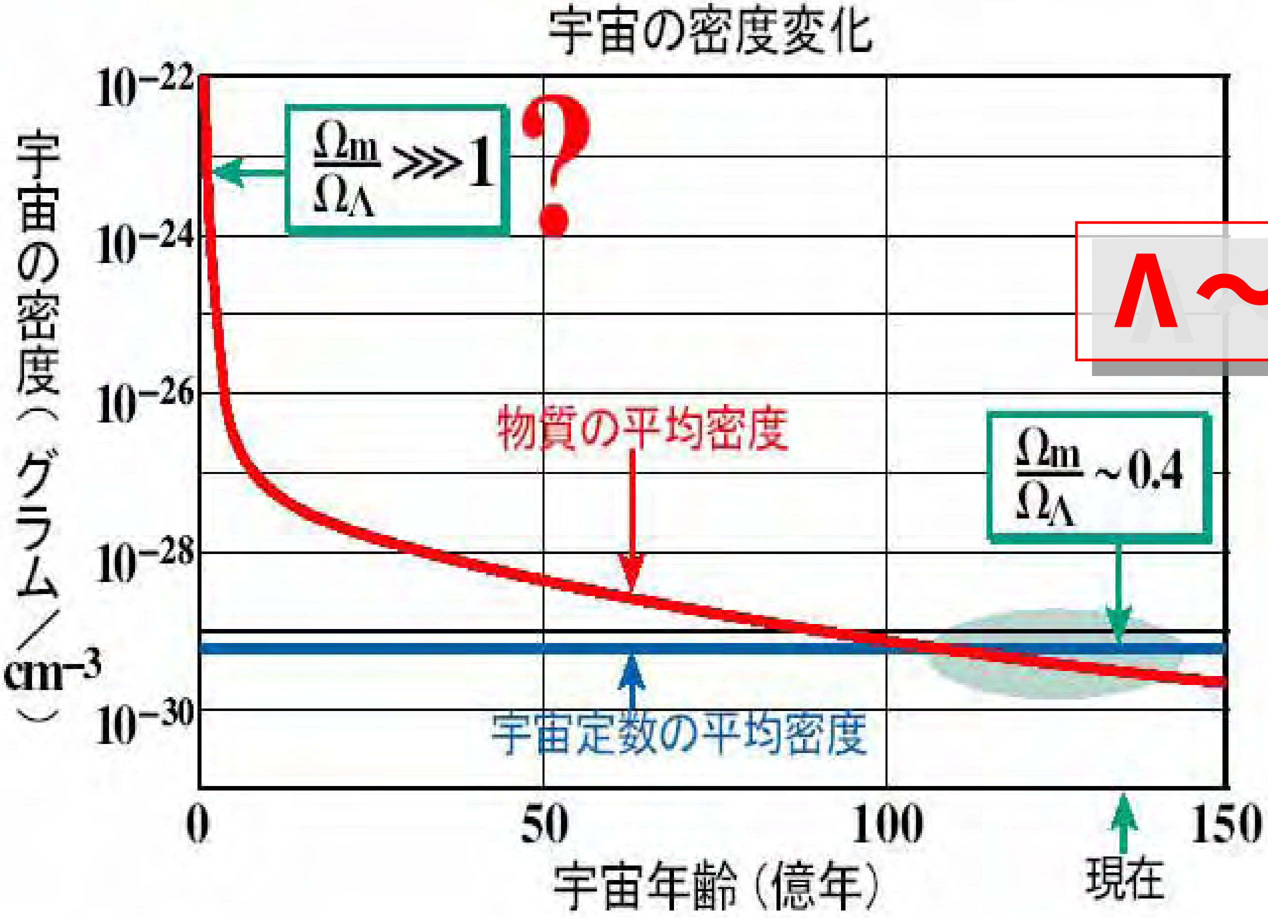
cf. **初期宇宙の重力波が観測**
出来るかも？

(サイエンス)

宇宙定数は量子重力と関係するはずだろう

$\rightarrow \Lambda \sim M_{Pl}$ $(\sim 10^{19}\mathrm{GeV})$

BUT、$\Lambda / M_{Pl} \sim 10^{-120}$！？



$\Lambda \sim m_\nu{}^4$!?!

(M.Sakamoto’s OHP)

8.素晴らしき未来へ

我々は素晴らしい時代に研究者として生まれてきた！
想像もしなかったこと（ν関係 large **MNS，Λ等など）沢山分かってきた。**

LHC, TEV: Higgs, SUSY, extraD
Mini-Boom: strile ν
Hyper-K: p-decay, precision measur. of ν
LBL (MINOS, OPERA, J-PARC): ν matter effect, CP, ν_τ appearance
PLANCK: Λ

…………

数学の美しさより自然（物理）の方が美しいかもしれない。険しい道かも
しれないけれど、物理を好きでいること愛していることが一番大事。
真理への探求に向かって一緒に頑張りましょう！