Doctoral Thesis

Nonperturbative Dynamics in Supersymmetric Gauge Theories

N. Maru*

Department of Physics, Nagoya University,

Nagoya 464-8602, Japan

* E-mail address maru@eken.phys.nagoya-u.ac.jp

Abstract

Recently, there has been a remarkable progress in the study of strong coupling dynamics of the gauge theories. Especially, in supersymmetric case, a certain sector can be determined *exactly*. In the former part of this thesis, we discuss the phase structure of $\mathcal{N} = 1$ supersymmetric gauge theories, especially focusing on the confining phase and the non Abelian Coulomb phase (duality). In the latter part, we discuss its phenomenological applications, especially focusing on the gauge mediated supersymmetry breaking.

Contents

Chapter 1

Introduction

The Standard Model (SM) describes the various experimental measurements very well, but none of particle physists believe the standard model as the fundamental theory. One of the problems in the standard model is the hierarchy problem, which appears in the Higgs sector, *i.e.* Higgs boson, which is introduced to trigger a spontaneous breaking of symmetry $SU(2)_L \times U(1)_Y \to U(1)_{em}$. Taking into account radiative corrections to the Higgs boson mass squared δm_H^2, one can easily see that δm_H^2 is quadratically divergent *i.e.* $\delta m_H^2 \sim \mathcal{O}(\alpha\Lambda^2)$, where α represents the gauge coupling factor, and Λ is a cut-off scale (nothing but a parameter in the standard model). The relation between the bare mass squared m_o^2 and the renormalized one m^2 is written as follows, $m^2 = m_o^2 + \delta m_H^2$. In order to give the electroweak scale, m^2 has to be $\mathcal{O}((100\text{GeV})^2)$. This needs a fine tuning of m_o^2 if the cut-off scale is assumed to be larger than the electroweak scale. For instance, if we assume the cut-off scale to be the Planck scale ($\sim 10^{19}$GeV), then the above Higgs mass relation implies $\mathcal{O}((100\text{GeV})^2) \simeq \mathcal{O}((10^{19}\text{GeV})^2) - \mathcal{O}((10^{19}\text{GeV})^2)$ for $\mathcal{O}(\alpha) \sim \mathcal{O}(1)$. This is quite an accidental result! Once assuming the cut-off scale to be much larger than the electroweak scale, we cannot help accepting a terrible fine tuning of the Higgs boson mass. In principle, though one cannot rule out this possibility, it appears to be implausible that the nature behaves like this. We have to explain how the small scale (the electroweak scale) arises from the fundamental scale of the theory (the Planck scale, for instance). This is called "the hierarchy problem".

One of the most attractive approaches to solve the hierarchy problem is "supersymmetry"(SUSY). In short, SUSY is a symmetry which transforms bosons into fermions and vice versa. The origin of the hierarchy problem is that there is no symmetry for scalars to guarantee the masslessness. Combining supersymmetry and chiral symmetry which guarantees the masslessness of the fermions, we can easily realize that scalars have to be massless from the requirement of symmetry. Indeed, the quadratic divergence of the Higgs scalar mass is milded by the cancellation between boson contributions and fermion ones.

In 1994, there was a remarkable breakthrough in supersymmetric gauge theories. First, Seiberg and Witten found the exact solutions in $\mathcal{N} = 2$ *four dimensional* SUSY gauge theories [1]. Second, Seiberg [2] conjectured the dual description of $\mathcal{N} = 1$ four dimensional SUSY gauge theories, which describes the same physics as in the original theory in the infrared limit. The key ingredient of these findings is "holomorphy". Thanks to this holomorphy, certain functions (the prepotential in $\mathcal{N} = 2$ case or the superpotential in $\mathcal{N} = 1$ case) are determined exactly. Therefore, we can investigate the strong coupling dynamics in more detail compared to non SUSY theories.

In this thesis, we focus on the issues on $\mathcal{N} = 1$ SUSY gauge theories. They are mainly separated into two parts. One is a theoretical aspect of $\mathcal{N} = 1$ SUSY gauge theories, the phase structure of the theory. Especially, the confining phase dynamics and the duality will be discussed. One will be able to understand through this discussion how powerful the holomorphy is. The other is a phenomenological aspect of $\mathcal{N}$=1 SUSY gauge theories, especially, the gauge mediated SUSY breaking. This issue is inevitable from the viewpoint of the phenomenological applications since SUSY must be broken at low energy. As the strong coupling dynamics of various theories have been revealed, a large number of SUSY breaking models have also been constructed. Therefore, it is worth while investigating the properties of each model applied to phenomenology.

This thesis is organized as follows. In chapter 2, the phase structure of SUSY $SU(N_c)$ gauge theory with N_f flavors is reviewed. It is instructive to review this theory because it includes the dynamical features which are common to those in other theories. Various phases appear as a function of N_f (No vacuum, confinement with chiral symmetry breaking, confinement without chiral symmetry breaking, a free magnetic phase, a non-Abelian Coulomb phase, a free electric phase.). Of these phases, the non-Abelian Coulomb phase is remarkable because Seiberg [2] found a dual pair which describe an equivalent physics in the infrared limit. Furthermore, a strong coupling dynamics of one theory can be described by a weak coupling dynamics of another theory and vice versa.

By using the duality, we can investigate the strong dynamics analytically. So, it is interesting to ask whether this duality is generic or not. Unfortunately, although many dualities have been discovered so far, we have no systematic approach to find a duality. In chapter 3 which is based on the work by myself and S. Kitakado, we found that the SO gauge group duality and the Sp gauge group duality are interrelated through the negative dimensional group. This negative dimensional group technique is useful and powerful in finding a new duality because if we know the $SO(Sp)$ group duality, we can easily obtain a new $Sp(SO)$ duality by a simple manipulation.

Another approach to search for a duality is to extract its informations from the confining phase dynamics. Since the duality must recover the result of the confining phase, the informations are quite valuable. The advantage of this approach is that the confining phase can be investigated systematically by symmetry and holomorphy. In chapter 4 which is based on the work by myself, I have studied the confining phase of SUSY $SO(12)$ gauge theory with one spinor and several vectors. The motivation to study this particular model is that I would like to clarify the structure of the duality in SUSY $SO(N_c)$ gauge theory with N_f vectors and N_Q spinors. The known dualities in this class have remarkable features which are not included in the duality Seiberg found. From this point, I think that we might be able to understand $\mathcal{N} = 1$ duality by clarifying this class of dualities. Along this line of thought, I constructed the exact superpotentials systematically, and conjectured the subgroup of the dual gauge group. Unfortunately, it turned out that we need more insight to find the above mentioned duality.

In the remainder of the thesis, we focus on the phenomenological aspects. In chapter 5, the concept of gauge mediated SUSY breaking is introduced. This is a scenario that SUSY breaking effects are communicated to the low energy by gauge interactions, which is an alternative to the supergravity (SUGRA) mediation scenario. Gauge mediated SUSY breaking has phenomenological advantages compared to the gravity mediation scenario, namely, the degeneracy of the sfermion masses is automatically achieved. Therefore, there is no flavor changing neutral currents (FCNC) problem. Next, the typical model by Dine, Nelson, Nir and Shirman (DNNS model) is reviewed. This model consists of three sectors, namely, the dynamical SUSY breaking (DSB) sector, the messenger sector and the observable sector. The messenger sector makes the mechanism to communicate SUSY breaking effects to the observable sector quite complicated. To make matters worse, in the true vacuum of the DNNS model, the color and the electroweak symmetries are broken at the scale which is higher than the weak scale. From these facts, the DNNS model became the driving force to simplifying, modifying the model or finding new models. In the next two sections, our new models are presented.

Chapter 6 is based on the work by N. Haba, T. Matsuoka and myself. We constructed the model in which the messenger sector is the effective theory of the DSB sector. More precisely, we identified the fields in the messenger sector with the massless composite fields in the DSB sector in contrast with the DNNS model in which the fields in the messenger sector are put by hand. This model simplifies the complicated structure of the DNNS model naturally. Furthermore, we have shown that in the true vacuum, the color and the electroweak symmetries are not broken at the scale higher than the weak scale. Then, we estimated various mass scales in the model, which are phenomenologically viable.

From the viewpoint of the simplicity of the model, it is natural to ask whether or not SUSY breaking effects can be communicated to the observable sector directly. This framework is referred to as "Direct Gauge Mediation (DGM)". In chapter 7 which is based on the work by myself, I constructed a new and a simple model of the DGM with an Affine quantum moduli space. The advantages of this model are as follows. First, the effective Kähler potential is calculable and simple because on Affine quantum moduli space, we can easily identify the massless composite fields by 't Hooft anomaly matching conditions. Second, SUSY breaking mechanism is simple. I have shown that this model avoids some problems (the SUGRA mediation dominance over the gauge mediation, the negative sfermion mass squared problem) in earlier DGM models. Furthermore, I have estimated various mass scales and it turned out that these are phenomenologically viable. One can find conclusion in chapter 8. In Appendix A, the superspace formalism, which is used throughout the thesis, is summarized.

Chapter 2

Phases of Supersymmetric QCD

As a warm up and to make our discussion clear, we review here the phases of $SU(N_c)$ supersymmetric QCD* .

2.1 Basics of SUSY QCD

The Lagrangian of SUSY $SU(N_c)$ QCD† is

$$\mathcal{L} = \int d^4\theta(Q^\dagger e^V Q + \bar{Q}e^{-V}\bar{Q}^\dagger) + \frac{1}{16\pi}\mathrm{Im}\int d^2\theta\tau W^{a\alpha}W^a_\alpha + h.c., \tag{2.1}$$

$$\tau \equiv \frac{\theta_{YM}}{2\pi} + \frac{4\pi i}{g^2}, \tag{2.2}$$

where Q and $\bar{Q}$ are chiral superfields, V is a vector superfield, τ denotes a complex gauge coupling (which is a useful combination in SUSY gauge theories), $W_{a\alpha}$ is a field strength tensor superfield (a is a gauge index and α is a spinor index.), θ in the Lagrangian represents Grassmannian coordinate, and θ_{YM} is the Yang-Mills theta parameter. The first term in (2.1) includes kinetic terms for quarks and squarks and the second term includes kinetic terms for gauge fields and gauginos.

This Lagrangian has the following classical symmetries:

$$SU(N_c) \times [SU(N_f)_L \times SU(N_f)_R \times U(1)_V \times U(1)_A \times U(1)_R] \tag{2.3}$$

* For further details, see [3, 4, 5].

† We consider here the massless case.

$SU(N_c)$ is a gauge symmetry, and symmetries in the bracket are flavor symmetries. Transformation properties of superfields are[‡]

$$Q^i_\alpha \qquad (\square\,;\square\,,\mathbf{1},1,1,R(Q)), \tag{2.4}$$

$$\bar{Q}^\alpha_i \qquad (\bar{\square}\,;\mathbf{1},\bar{\square}\,,-1,1,R(\bar{Q})), \tag{2.5}$$

$$W^\alpha \qquad (\mathbf{Adj};\mathbf{1},\mathbf{1},0,0,1), \tag{2.6}$$

where $\alpha = 1,\cdots,N_c$ and $i = 1,\cdots,N_f$. At the quantum level, $U(1)_A$ and $U(1)_R$ are broken by anomalies but we can always make an anomaly free R symmetry by taking a linear combination of $U(1)_A$ and $U(1)_R$. In other words, we can choose the charges so that only $U(1)_A$ is anomalous in the quantum theory. In fact, $U(1)_A, U(1)_R$ anomalies are

$$\partial_\mu j^\mu_A = 2N_f \frac{g^2}{32\pi^2} F\tilde{F}, \tag{2.7}$$

$$\partial_\mu j^\mu_R = \{(R(Q)+R(\bar{Q})-2)N_f + 2N_c\}\frac{g^2}{32\pi^2} F\tilde{F}, \tag{2.8}$$

where $\partial_\mu j^\mu_A, \partial_\mu j^\mu_R$ are the currents of $U(1)_A$ and $U(1)_R$, respectively.

If we define the current of anomaly free $U(1)_R$ as follows,

$$(j^\mu_R)_{AF} = j^\mu_R - \frac{(R(Q)+R(\bar{Q})-2)N_f + 2N_c}{2N_f} j^\mu_A, \tag{2.9}$$

where "AF" means anomaly free, then $\partial_\mu (j^\mu_R)_{AF} = 0$. So, quantum symmetries and the charges are[§]

$$SU(N_c) \quad \times \quad [SU(N_f)_L \times SU(N_f)_R \times U(1)_V \times U(1)^{AF}_R] \tag{2.10}$$

$$Q^i_\alpha \qquad (\square\,;\square\,,\mathbf{1},1,\frac{N_f-N_c}{N_f}), \tag{2.11}$$

$$\bar{Q}^\alpha_i \qquad (\bar{\square}\,;\mathbf{1},\bar{\square}\,,-1,\frac{N_f-N_c}{N_f}), \tag{2.12}$$

$$W^\alpha \qquad (\mathbf{Adj};\mathbf{1},\mathbf{1},0,1). \tag{2.13}$$

In order to study the vacuum structure in SUSY gauge theories, it is necessary to investigate "the flat direction", which means the field space where the scalar

‡ In general, $U(1)_R$ charges of Q and $\bar{Q}$ are arbitrary at the classical level.

§ We take $R(Q) = R(\bar{Q})$, for simplicity.

potential vanishes. (the flat direction is also called "moduli space". We will also use this terminology in this thesis.)

Since the scalar potential in the present case is

$$V = \frac{1}{2} g^2 \sum_a (D^a)^2 = \frac{1}{2} g^2 \sum_a (Q^\dagger T^a Q - \bar{Q} T^a \bar{Q}^\dagger)^2, \tag{2.14}$$

the vanishing scalar potential condition is

$$0 = Q^\dagger T^a Q - \bar{Q} T^a \bar{Q}^\dagger = \mathrm{tr}[(T^a)^\beta_\alpha ((Q^\dagger Q)^\alpha_\beta - (\bar{Q}\bar{Q}^\dagger)^\alpha_\beta)] \tag{2.15}$$

$$\Rightarrow (Q^\dagger Q)^\alpha_\beta - (\bar{Q}\bar{Q}^\dagger)^\alpha_\beta \propto \delta^\alpha_\beta, \quad (\alpha, \beta = 1, \cdots, N_c), \tag{2.16}$$

where δ^α_β is N_c by N_c matrix. The classical moduli space is parametrized by the vacuum expectation value (VEV) of Q, $\bar{Q}$ modulo gauge and flavor rotations. For $N_f < N_c$,

$$\langle Q \rangle = \langle \bar{Q} \rangle = \begin{pmatrix} q_1 & & & \\ & q_2 & & \\ & & \ddots & \\ & & & q_{N_f} \\ & & & \end{pmatrix}. \tag{2.17}$$

And for $N_f \geq N_c$,

$$\langle Q \rangle = \begin{pmatrix} q_1 & & \\ & \ddots & \\ & & q_{N_c} \\ & & \end{pmatrix}, \quad \langle \bar{Q} \rangle = \begin{pmatrix} \bar{q}_1 & & \\ & \ddots & \\ & & \bar{q}_{N_c} \\ & & \end{pmatrix}, \tag{2.18}$$

where the quantity $|q_i|^2 - |\bar{q}_i|^2$ is independent of i. For $N_f < N_c$, the gauge symmetry $SU(N_c)$ is broken to $SU(N_c - N_f)$, so we can describe the classical moduli space in terms of VEVs of "mesons" $M^i_j = Q^i \bar{Q}_j$ (which match massless degrees of freedom correctly). For $N_f \geq N_c$, the gauge symmetry is completely broken, the classical moduli space is described by VEVs of mesons and "baryons" which are defined as

$$B \equiv \epsilon^{\alpha_1 \cdots \alpha_{N_c}} \epsilon_{i_1 \cdots i_{N_f}} Q^{i_1}_{\alpha_1} \cdots Q^{i_{N_c}}_{\alpha_{N_c}}, \tag{2.19}$$

$$\bar{B} \equiv \epsilon_{\alpha_1 \cdots \alpha_{N_c}} \epsilon^{i_1 \cdots i_{N_f}} \bar{Q}^{\alpha_1}_{i_1} \cdots \bar{Q}^{\alpha_{N_c}}_{i_{N_c}}. \tag{2.20}$$

Note that mesons and baryons are, in general, subject to the constraints as seen later.

2.2 $N_f < N_c$

As mentioned in the previous section, massless gauge invariant composites are mesons $M^i_j = Q^i\bar{Q}_j$ $(i, j = 1, \cdots, N_f)$. Baryons vanish identically due to the Bose statistics of superfields. Since the effective theory should be described by gauge invariant operators, the effective superpotential is a function of M^i_j. Taking into account holomorphy, symmetries, dimensional analysis, we can fix the form of the effective superpotential [7] (referred to as "ADS superpotential") except for the coefficient as

$$W_{\text{eff}} = C_{N_c,N_f}\left(\frac{\Lambda^{3N_c-N_f}_{N_c,N_f}}{\text{det}Q\bar{Q}}\right)^{\frac{1}{N_c-N_f}}. \tag{2.21}$$

Here Λ_{N_c,N_f} is the strong coupling scale of SUSY QCD and C_{N_c,N_f} is a constant depending on N_c, N_f. A coefficient in (2.21) is determined as follows. If we consider the limit $q_{N_f} \to \infty$, $SU(N_c)$ gauge group will be broken to $SU(N_c - 1)$ by Higgs mechanism, and the number of flavors is also reduced to $N_f - 1$. Matching the gauge coupling constant between the ultraviolet theory and the infrared theory at the scale q_{N_f} through the 1-loop RGE, we obtain

$$\Lambda^{3(N_c-1)-(N_f-1)}_{N_c-1,N_f-1} q^2_{N_f} = \Lambda^{3N_c-N_f}_{N_c,N_f}. \tag{2.22}$$

Using this relation, the superpotential (2.21) can be rewritten as follows,

$$W_{\text{eff}} = C_{N_c,N_f}\left(\frac{\Lambda^{3(N_c-1)-(N_f-1)}_{N_c-1,N_f-1}}{\text{det}'Q\bar{Q}}\right)^{\frac{1}{(N_c-1)-(N_f-1)}}, \tag{2.23}$$

where det' is the determinant of $(N_f - 1)$ dimensional flavor space. This corresponds to the superpotential for $SU(N_c - 1)$ gauge theory with $N_f - 1$ flavors, so $C_{N_c,N_f} = C_{N_c-1,N_f-1}$. Iterating the same arguments for $q_1 \cdots q_{N_f-1}$, we obtain

$$C_{N_c,N_f} = C_{N_c-N_f,0}. \tag{2.24}$$

Next, if we add a mass term $\delta W = m\bar{Q}_{N_f}Q^{N_f}$ to (2.21) and integrate out the massive mode, the superpotential for $SU(N_c)$ gauge theory with $N_f - 1$ flavors is obtained. In fact, if we add the mass term

$$W_{\text{eff}} = C_{N_c,N_f}\left(\frac{\Lambda^{3N_c-N_f}_{N_c,N_f}}{\text{det}Q\bar{Q}}\right)^{\frac{1}{N_c-N_f}} + m\bar{Q}_{N_f}Q^{N_f}, \tag{2.25}$$

by the equation of motion for $\bar{Q}_{N_f}Q^{N_f}$, we find

$$(\bar{Q}_{N_f}Q^{N_f})^{\frac{1}{N_c-N_f}} = \frac{C_{N_c,N_f}}{m(N_c-N_f)}\left(\frac{\Lambda_{N_c,N_f}^{3N_c-N_f}}{\det\bar{Q}Q}\right)^{\frac{1}{N_c-N_f}}. \tag{2.26}$$

Substituting this for $\bar{Q}Q$ in (2.25), we obtain

$$W_{\text{eff}} = (N_c - N_f + 1)\left(\frac{C_{N_c,N_f}}{N_c-N_f}\right)^{\frac{N_c-N_f}{N_c-N_f+1}}\left(\frac{\Lambda_{N_c,N_f-1}^{3N_c-N_f+1}}{\det\bar{Q}Q}\right)^{\frac{1}{N_c-N_f+1}}. \tag{2.27}$$

Here one-loop matching relation for the gauge coupling constant $m\Lambda^{3N_c-N_f} = \Lambda^{3N_c-N_f+1}$ is used. Since Eq. (2.27) corresponds to the superpotential for $SU(N_c)$ gauge theory with $N_f - 1$ flavors,

$$(N_c - N_f + 1)\left(\frac{C_{N_c,N_f}}{N_c-N_f}\right)^{\frac{N_c-N_f}{N_c-N_f+1}} = C_{N_c,N_f-1}. \tag{2.28}$$

Combining this and Eq. (2.24), one can derive

$$C_{N_c,N_f} = (N_c - N_f)C^{\frac{1}{N_c-N_f}}, \tag{2.29}$$

where C is a numerical constant. Direct instanton calculation [7] shows that this numerical constant is equal to one. Therefore, the final expression of the effective superpotential is

$$W_{\text{eff}} = (N_c - N_f)\left(\frac{\Lambda_{N_c,N_f}^{3N_c-N_f}}{\det Q\bar{Q}}\right)^{\frac{1}{N_c-N_f}}. \tag{2.30}$$

Note that the superpotential (2.30) is nonperturbative because non renormalization theorem [6] forbids to induce the superpotential perturbatively. Note that this type of superpotential cannot appear for $N_f \geq N_c$ because for $N_f = N_c$, the power is singular and for $N_f > N_c$, it is also singular in the weak coupling limit $\Lambda \to 0, Q\bar{Q} \to \infty$.

While W_{eff} is generated by one instanton for $N_f = N_c - 1$, for $N_f < N_c - 1$, it is generated by gaugino condensation in the unbroken $SU(N_c - N_f)$ gauge group [7].

As for the vacuum, the superpotential (2.30) pushes the minimum away to infinity, so the vacuum is not well-defined.

2.3 $N_f = N_c$ (quantum deformation)

The superpotential (2.30) is singular at $N_f = N_c$. Furthermore, since R-charges of the matter fields $Q, \bar{Q}$ vanish, it is impossible to construct any superpotentials. This leads to the fact that the classical flat direction cannot be lifted quantum mechanically. Recalling here that the gauge symmetry is completely broken, massless degrees of freedom $(N_f^2 + 1)$ and the degrees of freedom of the gauge invariant composites $(N_f^2 + 2)$ differ by one. This implies that $M, B, \bar{B}$ are not independent and one classical constraint

$$\det M - B\bar{B} = 0 \tag{2.31}$$

should exist. This constraint follows from the Bose statistics of superfields $Q, \bar{Q}$.

Seiberg argued [8] that this constraint is modified quantum mechanically as

$$\det M - B\bar{B} = \Lambda^{2N_c}. \tag{2.32}$$

This modification is due to one instanton effect because the right hand side of (2.32) is proportional to one instanton action. One may write this constraint using Lagrange multiplier superfield X as

$$W = X(\det M - B\bar{B} - \Lambda^{2N_c}). \tag{2.33}$$

The consistency of (2.33) is verified as follows. Adding the mass term for the last flavor $W = mM_{N_f}^{N_f}$ to (2.33) and integrating out massive mode using F-flatness conditions,

$$\begin{aligned}
0 &= \frac{\partial W}{\partial M_{N_f}^{N_f}} = X(\mathrm{cof}\,M)_{N_f}^{N_f} + m, && (2.34)\\
0 &= \frac{\partial W}{\partial X} = \det M - B\bar{B} - \Lambda^{2N_c}, && (2.35)\\
0 &= \frac{\partial W}{\partial B} = X\bar{B}, && (2.36)\\
0 &= \frac{\partial W}{\partial \bar{B}} = XB, && (2.37)\\
0 &= \frac{\partial W}{\partial M_i^{N_f}} = X(\mathrm{cof}\,M)_i^{N_f} \quad (i = 1, \cdots, N_f - 1), && (2.38)
\end{aligned}$$

where $(\mathrm{cof}M)^i_j$ is a cofactor of a $N_f \times N_f$ matrix defined as

$$(\mathrm{cof}M)^i_j = \frac{1}{(N_f-1)!}\epsilon^{ii_2\cdots i_{N_f}}\epsilon_{jj_2\cdots j_{N_f}}M^{j_2}_{i_2}\cdots M^{j_{N_f}}_{i_{N_f}} = \frac{\partial \mathrm{det}M}{\partial M^j_i}. \tag{2.39}$$

The third and fourth equations lead to $B = \bar{B} = 0$. The last equation can restrict the form of M as follows,

$$\begin{pmatrix} \tilde{M} & \\ & M^{N_f}_{N_f} \end{pmatrix}, \tag{2.40}$$

where $\tilde{M}$ denotes $(N_f-1)\times(N_f-1)$ matrix. Using (2.32) and the above results, one obtains

$$W_{\mathrm{eff}} = \frac{m\Lambda^{2N_c}}{\mathrm{det}\tilde{M}} = \frac{\tilde{\Lambda}^{2N_c+1}}{\mathrm{det}\tilde{M}}. \tag{2.41}$$

In the second equality, we used 1-loop matching relation of the gauge coupling constant $m\Lambda^{2N_c} = \tilde{\Lambda}^{2N_c+1}$, where $\tilde{\Lambda}$ is the strong coupling scale of $SU(N_c)$ gauge theory with N_c-1 flavors. This is nothing but ADS superpotential for $N_f = N_c-1$, which is a consistent result.

The origin of the moduli space $\langle M\rangle = \langle B\rangle = \langle\bar{B}\rangle = 0$ is not on the quantum moduli space (2.32). Therefore, chiral symmetries $SU(N_f)_L \times SU(N_f)_R \times U(1)_B \times U(1)_R$ are necessarily broken. It is straightforward to check that fundamental massless fermions and composite fermions satisfy 't Hooft anomaly matching conditions for unbroken global symmetries [8].

The first case we consider is where $\langle B\rangle = \langle\bar{B}\rangle = 0$ and $\langle M^i_j\rangle = \delta^i_j$. In this case, $SU(N_f)_L \times SU(N_f)_R$ is broken to their diagonal $SU(N_f)_D$, and $U(1)_B, U(1)_R$ are unbroken. The explicit calculations of global anomalies are

$$\begin{array}{rcl} U(1)_R SU(N_f)^2 & E & (-1)\times N_c \times \mu(\square) + (-1)\times N_c\times\mu(\bar{\square}) = -2N_c\mu(\square), \\ & C & (-1)\times\mu(Adj) = -2N_f\mu(\square), \\ U(1)_R & E & (-1)\times N_cN_f + (-1)\times N_cN_f + 1\times(N_c^2-1) \\ & & = -2N_cN_f + N_c^2 - 1, \\ & C & (-1)\times(N_f^2-1) + (-1)\times 1 + (-1)\times 1 = -N_f^2 - 1, \\ U(1)_R^3 & E & (-1)^3\times N_cN_f + (-1)^3\times N_cN_f + 1\times(N_c^2-1) \\ & & = -2N_cN_f + N_c^2 - 1, \end{array}$$

$$
\begin{array}{rll}
 & C & (-1)^3 \times (N_f^2 - 1) + (-1)^3 \times 1 + (-1)^3 \times 1 = -N_f^2 - 1, \\
U(1)_R U(1)_B & E & (-1) \times 1 \times N_f N_c + (-1) \times 1 \times N_f N_c = -2N_c^2, \\
 & C & (-1) \times N_c^2 + (-1) \times N_c^2 = -2N_c^2,
\end{array} \tag{2.42}
$$

where E, C means "elementary" ,"composite", respectively and $\mu(R)$ is a Dynkin index for the representation R.

The second case is where $\langle B \rangle = -\langle \bar{B} \rangle = \Lambda^{N_f}, \langle M \rangle = 0$. In this case, $SU(N_f)_L \times SU(N_f)_R$ is unbroken, and $U(1)_B$ is broken. The explicit global anomalies are

$$
\begin{array}{rll}
SU(N_f)^3_{L,R} & E & N_c A(\square) \\
 & C & N_f A(\square), \\
U(1)_R SU(N_f)^2_{L,R} & E & (-1) \times N_c \times \mu(\square), \\
 & C & (-1) \times N_f \times \mu(\square), \\
U(1)_R & E & (-1) \times N_c N_f + (-1) \times N_c N_f + 1 \times (N_c^2 - 1) \\
 & & = -2N_c N_f + N_c^2 - 1, \\
 & C & (-1) \times (N_f^2 - 1) + (-1) \times 1 + (-1) \times 1 = -N_f^2 - 1, \\
U(1)^3_R & E & (-1)^3 \times N_c N_f + (-1)^3 \times N_c N_f + 1 \times (N_c^2 - 1) \\
 & & = -2N_c N_f + N_c^2 - 1, \\
 & C & (-1)^3 \times (N_f^2 - 1) + (-1)^3 \times 1 + (-1)^3 \times 1 \\
 & & = -N_f^2 - 1,
\end{array} \tag{2.43}
$$

where $A(R)$ denotes anomaly coefficient for the representation R.

2.4 $N_f = N_c + 1$ (s-confinement)

In this case, symmetry properties of $M, B, \bar{B}$ are following.

	$SU(N_f)$	$SU(N_f)$	$U(1)_B$	$U(1)_R$
M	$\square$	$\bar{\square}$	0	$2/N_f$
B	$\bar{\square}$	$\mathbf{1}$	$N_f - 1$	N_c/N_f
$\bar{B}$	$\mathbf{1}$	$\square$	$-N_f + 1$	N_c/N_f

We can easily derive the effective superpotential from symmetry and holomorphy [8]:

$$W_{\text{eff}} = \frac{1}{\Lambda^{2N_f-3}}(BM\bar{B} - \det M). \tag{2.44}$$

It is important that the relative sign is opposite because the equations of motion derived from (2.44) reproduce the classical constraints

$$\frac{\partial W_{\text{eff}}}{\partial M^i_j} = 0 \quad \Rightarrow \quad B_i \bar{B}^j - (\text{cof}M)^j_i = 0, \tag{2.45}$$

$$\frac{\partial W_{\text{eff}}}{\partial B_i} = 0 \quad \Rightarrow \quad M^i_j \bar{B}^j = 0, \tag{2.46}$$

$$\frac{\partial W_{\text{eff}}}{\partial \bar{B}^i} = 0 \quad \Rightarrow \quad B_j M^j_i = 0. \tag{2.47}$$

This is consistent with the fact that the classical moduli space cannot be lifted at the quantum level for $N_f \geq N_c$ as mentioned earlier.

We can also check that by adding the mass term $W = mM^{N_f}_{N_f}$ to (2.44), and integrating out the massive mode, the quantum constraint (2.32) is reproduced. Indeed, if we add the mass term explicitly

$$W = \frac{1}{\Lambda^{2N_f-3}}(BM\bar{B} - \det M) + mM^{N_f}_{N_f}, \tag{2.48}$$

and integrating out the massive modes, the quantum constraint (2.32) is obtained

$$0 \quad = \quad \frac{\partial W}{\partial M^{N_f}_{N_f}} = \frac{1}{\Lambda^{2N_f-3}}(B^{N_f}\bar{B}_{N_f} - (M^{-1})^{N_f}_{N_f}\det M) + m \tag{2.49}$$

$$\Leftrightarrow B^{N_f}\bar{B}_{N_f} - \det M + \Lambda^{2N_f-2} = 0. \tag{2.50}$$

Note that $\det M$ in the last equation is a determinant of $(N_f - 1)$ dimensional flavor space and the scale is also written by the scale of $N_f - 1$ flavors. This last equation is the constraint of the quantum deformation.

In contrast to $N_f = N_c$ case, the origin $\langle M \rangle = \langle B \rangle = \langle \bar{B} \rangle = 0$ is on the quantum moduli space (2.44). Therefore, 't Hooft matching conditions between the fundamental massless fermions and the composite massless fermions are satisfied everywhere on the moduli space. Explicit calculations go as follows,

$SU(N_f)^3_{L,R} \quad E \quad N_c A(\square\,)$

$$
\begin{array}{rcl}
 & C & N_f A(\square) + A(\overline{\square}) = (N_f - 1)A(\square), \\
U(1)_R SU(N_f)^2_{L,R} & E & \left(\frac{1}{N_f} - 1\right) N_c \times \mu(\square) = -\frac{N_c^2}{N_f}\mu(\square), \\
 & C & (\frac{2}{N_f} - 1)N_f \times \mu(\square) + \left(\frac{N_c}{N_f} - 1\right) \times \mu(\overline{\square}) \\
 & & = -\frac{(N_f-1)^2}{N_f}\mu(\square), \\
U(1)_B SU(N_f)^2_{L,R} & E & 1 \times N_c \times \mu(\square), \\
 & C & (N_f - 1) \times \mu(\overline{\square}), \\
U(1)_R & E & \left(\frac{1}{N_f} - 1\right) \times N_c N_f + \left(\frac{1}{N_f} - 1\right) \times N_c N_f + 1 \times (N_c^2 - 1) \\
 & & = -N_c^2 - 1, \\
 & C & \left(\frac{2}{N_f} - 1\right) \times N_f^2 + \left(\frac{N_c}{N_f} - 1\right) \times N_f + \left(\frac{N_c}{N_f} - 1\right) \times N_f \\
 & & = -N_c^2 - 1, \\
U(1)_R U(1)_B^2 & E & 1 \times \left(\frac{1}{N_f} - 1\right) \times N_f N_c + 1 \times \left(\frac{1}{N_f} - 1\right) \times N_f N_c = -2N_c^2, \\
 & C & \left(\frac{N_c}{N_f} - 1\right) \times (N_f - 1)^2 \times 2N_f = 2(N_c - N_f)N_c^2, \\
U(1)_R^2 U(1)_B & E & 1 \times \left(\frac{1}{N_f} - 1\right)^2 \times N_f N_c - 1 \times \left(\frac{1}{N_f} - 1\right)^2 \times N_f N_c = 0, \\
 & C & \left(\frac{N_c}{N_f} - 1\right)^2 \times (N_f - 1) \times N_f + \\
 & & \left(\frac{N_c}{N_f} - 1\right)^2 \times (-N_f + 1) \times N_f = 0, \\
U(1)_R^3 & E & \left(\frac{1}{N_f} - 1\right)^3 \times 2N_c N_f + 1 \times (N_c^2 - 1) \\
 & & = -N_f^2 + 6N_f - 12 + \frac{8}{N_f} - \frac{2}{N_f^2}, \\
 & C & \left(\frac{2}{N_f} - 1\right)^3 \times N_f^2 + \left(\frac{N_c}{N_f} - 1\right)^3 \times 2N_f \\
 & & = -N_f^2 + 6N_f - 12 + \frac{8}{N_f} - \frac{2}{N_f^2}.
\end{array}
\tag{2.51}
$$

Finally, we comment that the models with above features are referred to as "s-confinement". The s-confinement models based on simple groups are completely classified by C. Csáki, M. Schmaltz and W. Skiba [9].

2.5 $N_f \geq N_c + 2$

As in the previous sections, we can infer the form of the effective superpotential as

$$W_{\text{eff}} \sim B_{ij\cdots k} M^i_l M^j_m M^k_n \bar{B}^{lmn} - \det M. \tag{2.52}$$

However, this observation is not correct in the following respects. First, R-charge of (2.52) is not two. Second, $M, B, \bar{B}$ do not satisfy 't Hooft anomaly matching conditions without violating the previous results. This implies a signal for the termination of the confining phase.

Seiberg found a solution to this problem [2]. In the range $\frac{3}{2}N_c \leq N_f \leq 3N_c$, there are two equivalent descriptions which flow to the same infrared fixed point. At this fixed point, the gauge coupling is finite.

The exact beta function¶ in SUSY QCD is [10, 11]

$$\begin{aligned} \beta(g) &= -\frac{g^3}{16\pi^2}\frac{3N_c - N_f + \gamma(g^2)}{1 - \frac{g^2}{8\pi^2}N_c}, && (2.53) \\ \gamma(g^2) &= -\frac{g^2}{8\pi^2}\frac{N_c^2 - 1}{N_c} + \mathcal{O}(g^4), && (2.54) \end{aligned}$$

where $\gamma(g^2)$ is the anomalous dimension.

If we take N_c, N_f to infinity holding $N_c g^2$ and $\frac{N_f}{3N_c} = 1 - \epsilon (0 < \epsilon \ll 1)$ fixed, we can read

$$\begin{aligned} \beta_0 &= -3N_c + N_f = -3N_c\epsilon < 0 \Leftrightarrow N_f < 3N_c, && (2.55) \\ \beta_1 &= -N_c(3N_c - N_f) + \frac{N_f(N_c^2 - 1)}{N_c} \\ &\simeq N_c(-3N_c + 2N_f) > 0 \Leftrightarrow \frac{3}{2}N_c < N_f. && (2.56) \end{aligned}$$

There is a range where 1-loop beta function β_0 is negative, but 2-loop beta function β_1 is positive. This might be a non-trivial fixed point [13]. The coupling constant at the fixed point is determined from $\beta(g_*) = 0$, that is, $N_c g_*^2 = 8\pi^2\epsilon + \mathcal{O}(\epsilon^2)$.

¶ The gauge coupling constant appearing in the beta function is the "canonical" gauge coupling constant not the "holomorphic" gauge coupling constant. Related issues are discussed in Ref. [12].

The infrared theory is a non-trivial four dimensional superconformal field theory. The elementary quarks and gluons appear as interacting massless particles. Therefore, this phase is referred to as "the non-Abelian Coulomb phase".

Given that such a fixed point exists, we can use the superconformal algebra to derive some results about the theory. This algebra includes an R symmetry. The requirement that a representation of the superconformal algebra be unitary puts restriction on the scaling dimension D and the R-charge. In particular,

$$D \geq \frac{3}{2}|R| \tag{2.57}$$

and this inequality is saturated for chiral $D = \frac{3}{2}R$ or antichiral $D = -\frac{3}{2}R$ superfields. For the gauge invariant operators $Q\bar{Q}$,

$$D(Q\bar{Q}) = \frac{3}{2}R = \frac{3(N_f - N_c)}{N_f}. \tag{2.58}$$

This also follows from (2.53), $D = \gamma_* + 2 = \frac{3(N_f - N_c)}{N_f}$ because of $\gamma_* = -3\frac{N_c}{N_f} + 1$.

The bound $\frac{3}{2}N_c < N_f$ is also consistent with the unitarity bound $D \geq 1$ for the scalar fields. $D(Q\bar{Q}) \geq 1$ is rewritten as $\frac{3}{2}N_c < N_f$. Note that the bound is saturated for free fields.

Let us discuss these equivalent theories in more detail. We list below the matter content and its charge of symmetries and their superpotentials.

Original theory

	$SU(N_c)$	$SU(N_f)_L$	$SU(N_f)_R$	$U(1)_B$	$U(1)_R$
Q	$\square$	$\square$	$\mathbf{1}$	1	$(N_f - N_c)/N_f$
$\bar{Q}$	$\bar{\square}$	$\mathbf{1}$	$\bar{\square}$	-1	$(N_f - N_c)/N_f$

$$W = 0$$

Dual theory

	$SU(N_c)$	$SU(N_f)_L$	$SU(N_f)_R$	$U(1)_B$	$U(1)_R$
q	$\square$	$\bar{\square}$	$\mathbf{1}$	$N_c/(N_f - N_c)$	N_c/N_f
$\bar{q}$	$\bar{\square}$	$\mathbf{1}$	$\square$	$N_c/(N_f - N_c)$	N_c/N_f
M	$\mathbf{1}$	$\square$	$\bar{\square}$	0	$2(N_f - N_c)/N_f$

$$W = \frac{1}{\mu} q M \bar{q}$$

Original theory is nothing but SUSY QCD discussed so far. The "dual" theory is equivalent to SUSY QCD in the infrared limit. (The meaning of "dual" will be explained.) The remarkable thing between two theories is that the gauge symmetry is not the same but the global symmetries are same. This is not so surprising because searching for dual description heavily depends on 't Hooft anomaly matching conditions. $U(1)_{B,R}$ charges of dual quarks $q, \bar{q}$ are determined through the mapping of baryons

$$\begin{aligned} B = Q^{N_c} \quad &\leftrightarrow \quad b = \sqrt{-(-\mu)^{N_c-N_f}\Lambda^{3N_c-N_f}}\,\epsilon^{i_1\cdots i_{N_f}} q^{N_f-N_c}, \\ \bar{B} = \bar{Q}^{N_c} \quad &\leftrightarrow \quad \bar{b} = \sqrt{-(-\mu)^{N_c-N_f}\Lambda^{3N_c-N_f}}\,\epsilon_{i_1\cdots i_{N_f}} \bar{q}^{N_f-N_c}. \end{aligned} \tag{2.59}$$

The normalization constant is determined by symmetry and dimensional analysis. Let us suppose that the normalization constant is C. The mass dimension of C is $2N_c - N_f$ since the mass dimension of B, b is N_c, $N_f - N_c$ respectively. Mass scales in the theory are $\Lambda^{3N_c-N_f}$ and μ.[‖] Therefore, we can write $C = (\Lambda^{3N_c-N_f})^a \mu^b$ without loss of generality, here a, b are constants. It leads to $(3N_c - N_f)a + b = 2N_c - N_f$ from the argument of mass dimension. As will be discussed later, the scale $\Lambda^{3N_c-N_f}$ and μ have charges $2N_f$ and $\frac{2N_f}{N_f-N_c}$ under anomalous $U(1)$. Taking into account that both baryons B and b have charge N_c, one finds that the normalization constant should be neutral under anomalous $U(1)$, namely, $2N_f a + \frac{2N_f}{N_f-N_c} b = 0$. These two considerations fix the normalization constant except for the sign. It is easy to see that the sign dependence is correct from $N_f = N_c + 2$ argument below.

We have to emphasize the following. First, M in the dual theory is an *elementary* superfield though the charges of M corresponds to those of meson $Q\bar{Q}$ in the original theory. Note that $Q\bar{Q} \leftrightarrow M$ identification is valid only in the infrared limit. Second, the superpotential in the dual theory is not only consistent with symmetries but also indispensable for the dual theory to possess the same $U(1)$ symmetries. If this superpotential does not exist, extra $U(1)$ symmetries appear. Mass scale μ in the

[‖] There is also a scale of the theory $\tilde{\Lambda}$, but this scale is related to $\Lambda^{3N_c-N_f}$ and μ through (2.60). (which will be discussed later).

dual superpotential is needed to compensate for the difference of mass dimension of $Q\bar{Q}$ and M at the ultraviolet.

Here, we comment on the meaning of "dual". Using anomalous $U(1)$ symmetries, we can derive the relation between the strong coupling scale of the electric theory Λ and the dual's $\tilde{\Lambda}$ as

$$\Lambda^{3N_c-N_f}\tilde{\Lambda}^{2N_f-3N_c} = (-1)^{N_f-N_c}\mu^{N_f}. \tag{2.60}$$

This is derived as follows. Let us note that $Q, \bar{Q}$ have charge $+1$ and $q, \bar{q}$ have charge $\frac{N_c}{N_f-N_c}$ (through the operator mapping $Q^{N_c} \leftrightarrow q^{N_f-N_c}$) under anomalous $U(1)$. Moreover, the strong coupling scale of the original theory and the dual theory have charge $2N_f$ and $\frac{2N_fN_c}{N_f-N_c}$, and μ has $\frac{2N_f}{N_f-N_c}$. We can fix the relation except for coefficient as

$$\Lambda^{3N_c-N_f}\tilde{\Lambda}^{2N_f-3N_c} = C(N_c, N_f)\mu^{N_f}, \tag{2.61}$$

where $C(N_c, N_f)$ is a dimensionless constant. N_c, N_f dependence is determined by the flat direction deformation and the mass term deformation as we fixed the coefficient of ADS superpotential before.

If we consider the weak coupling limit of the original theory $(\Lambda \to 0)^{**}$, then $\tilde{\Lambda} \to \infty$, that is, the dual is in the strong coupling regime and vice versa. This is the generalization of Dirac quantization condition. The relation of two theories under consideration is analogous to the electro-magnetic duality. Therefore, we use the terminology "dual" or we also use "electric" for the original theory and "magnetic" for the dual theory.

Next we comment on the scale relation (2.60). If we perform another duality transformation ($N_c \to \tilde{N}_c, \Lambda \to \tilde{\Lambda}$, $\mu \to \tilde{\mu}$), Eq. (2.60) becomes $\tilde{\Lambda}^{3\tilde{N}_c-N_f}\Lambda^{3N_c-N_f} = (-1)^{N_c}\tilde{\mu}^{N_f}$, therefore $\tilde{\mu} = -\mu$. This minus sign is crucial when we dualize again. The dual of the dual theory is $SU(N_c)$ gauge theory with N_f flavors of quarks $d^i, \bar{d}_{\bar{j}}$ and singlet mesons $M^i_{\bar{j}}$ and $N^{\bar{j}}_i = q_i q^{\bar{j}}$, and the superpotential

$$W = \frac{1}{\tilde{\mu}}N^{\bar{j}}_i d^i \bar{d}_{\bar{j}} + \frac{1}{\mu}M^i_{\bar{j}}N^{\bar{j}}_i$$

** In supersymmetric gauge theory, the gauge coupling constant and the strong coupling scale Λ are related through 1-loop RGE, $\exp\left(-\frac{8\pi}{g^2}+i\theta\right) = \left(\frac{\Lambda}{\mu}\right)^{b_0}$, where θ is theta angle, b_0 is 1-loop beta function coefficient, μ is renormalization scale.

$$= \frac{1}{\mu} N_i^{\bar{j}} (-d^i \bar{d}_{\bar{j}} + M_{\bar{j}}^i). \quad (2.62)$$

M and N are massive and can be integrated out using equations of motion,

$$0 = \frac{\partial W}{\partial M_{\bar{j}}^i} = N_i^{\bar{j}}, \quad (2.63)$$

$$0 = \frac{\partial W}{\partial N_{\bar{j}}^i} = -d^i \bar{d}_{\bar{j}} + M_{\bar{j}}^i. \quad (2.64)$$

The second equation $M = d\bar{d}$ indicates that the quarks $d, \bar{d}$ can be identified with the original quarks $Q, \bar{Q}$! Thus, the dual of the dual reproduces the original theory.

Although this Seiberg duality is not proved but only conjectured, there are many evidences which support this duality. First of all, as mentioned earlier, 't Hooft anomaly matching conditions are satisfied.

$$\begin{array}{rcl}
SU(N_f)^3_{L,R} & E & N_c A(\square) \\
& D & N_f A(\square) + (N_f - N_c) A(\bar{\square}) = N_c A(\square), \\
U(1)_R SU(N_f)^2_{L,R} & E & \left(-\frac{N_c}{N_f}\right) N_c \times \mu(\square) = -\frac{N_c^2}{N_f}\mu(\square), \\
& D & (\frac{N_c}{N_f} - 1)(N_f - N_c) \times \mu(\bar{\square}) + \frac{N_f - 2N_c}{N_f} \times N_f \mu(\square) \\
& & = -\frac{N_c^2}{N_f}\mu(\square), \\
U(1)_B SU(N_f)^2_{L,R} & E & 1 \times N_c \times \mu(\square), \\
& D & \frac{N_c}{N_f - N_c} \times (N_f - N_c)\mu(\bar{\square}), \\
U(1)_R & E & \left(-\frac{N_c}{N_f}\right) \times 2N_c N_f + 1 \times (N_c^2 - 1) = -N_c^2 - 1, \\
& D & \left(\frac{2(N_f - N_c)}{N_f} - 1\right) \times N_f^2 + \left(\frac{N_c}{N_f} - 1\right) \times 2N_f(N_f - N_c) \\
& & +(N_f - N_c)^2 - 1 = -N_c^2 - 1, \\
U(1)_R U(1)_B^2 & E & 1 \times \left(-\frac{N_c}{N_f}\right) \times 2N_f N_c = -2N_c^2, \\
& D & \left(\frac{N_c}{N_f} - 1\right) \times \left(\frac{N_c}{N_f - N_c}\right)^2 \times 2N_f(N_f - N_c) = -2N_c^2, \\
U(1)_R^3 & E & \left(-\frac{N_c}{N_f}\right)^3 \times 2N_c N_f + 1 \times (N_c^2 - 1) = \frac{-2N_c^4}{N_f^2} + N_c^2 - 1,
\end{array}$$

$$\begin{aligned} D \quad & \left(\frac{2(N_f-N_c)}{N_f}-1\right)^3 \times N_f^2 + \left(\frac{N_c}{N_f}-1\right)^3 \times 2N_f(N_f-N_c) \\ & +(N_f-N_c)^2-1 = \frac{-2N_c^4}{N_f^2}+N_c^2-1. \end{aligned} \tag{2.65}$$

Here D denotes "dual". Second, all gauge invariant operators of the electric theory are mapped to those of the magnetic theory.

$$Q\bar{Q} \quad \leftrightarrow \quad M, \tag{2.66}$$

$$B = Q^{N_c} \quad \leftrightarrow \quad b = q^{N_f-N_c}, \tag{2.67}$$

$$\bar{B} = \bar{Q}^{N_c} \quad \leftrightarrow \quad \bar{b} = \bar{q}^{N_f-N_c}. \tag{2.68}$$

This mapping is compatible with global symmetries. Note that the meson $q\bar{q}$ in the dual is redundant because of $0 = \partial W/\partial M = q\bar{q}$.

Third, the duality is preserved under various deformations of the theory. For example, adding the mass term for N_f-th quark $\delta W = mQ^{N_f}\bar{Q}_{N_f}$ and integrating out, we obtain the effective theory of $SU(N_c)$ gauge theory with (N_f-1) flavors. In the dual theory, this mass term corresponds to $\delta W = mM^{N_f}_{N_f}$. Solving the equations of motion for $M^{N_f}_{N_f}, M^{N_f}_i, M^i_{N_f} (i=1,\cdots,N_f-1)$, we find $\langle q^{N_f}\bar{q}_{N_f}\rangle = -m\mu$ and $\langle q_i\rangle = \langle\bar{q}^i\rangle = 0$. This breaks the gauge symmetry $SU(N_f-N_c)$ to $SU(N_f-N_c-1)$. Then, the effective theory is $SU(N_f-N_c-1)$ gauge theory with (N_f-1) flavors, gauge singlet meson M and the same form of the superpotential. The resulting pair of theories is nothing but theories which N_f-1 is substituted for N_f. This is a consistent result. The above discussion is incomplete for $N_f = N_c+2$, where the mass terms trigger complete magnetic gauge symmetry breaking. Then, the low energy should include instanton effects in the broken group. In this case, there are fermion zero modes, one mode for each $\psi_q, \psi_{\bar{q}}$. The superpotential coupling $\phi_M\psi_q\psi_{\bar{q}}$ lifts zero modes, then the instanton generates the superpotential

$$\begin{aligned} W_{\text{inst}} &= \tilde{\Lambda}^{-N_c+2}\det(\mu^{-1}\hat{M}) = \frac{\tilde{\Lambda}^{4-N_f}}{\langle q^{N_f+2}\bar{q}_{N_f+2}\rangle}\det(\mu^{-1}\hat{M}) \\ &= -\frac{\tilde{\Lambda}^{4-N_f}}{m\mu^{N_c+2}}\det\hat{M} = -\frac{\det\hat{M}}{m\Lambda^{2N_c-2}} = -\frac{\det\hat{M}}{\Lambda_L^{2N_c-1}}. \end{aligned} \tag{2.69}$$

Here $\hat{M}$ is $(N_f-1)\times(N_f-1)$ matrix. We used the scale matching relation in the second and the last equalities, the equation of motion for $M^{N_c+2}_{N_c+2}$ in the third equality. The scale relation (2.60) is used in the fourth equality. One can see that this superpotential is generated by 1-instanton effect in completely broken magnetic gauge group. Following Ref. [14], the 't Hooft vertex for 1-instanton in $SU(2)$ gauge theory with $N_f(=N_c+2)$ flavors is given by

$$\psi_q^{2N_f}\tilde{\lambda}^4\tilde{\Lambda}^{-N_c-4}, \tag{2.70}$$

where $\tilde{\lambda}$ is the gaugino in the magnetic theory, and ψ_q is obviously the fermionic components of q. First, inserting gaugino-fermion-scalar vertex $\psi_q\tilde{\lambda}\phi_q^*$ four times,we convert the 't Hooft vertex to

$$\psi_q^{2(N_f-2)}(\phi_q^*)^4\tilde{\Lambda}^{-N_c-4}. \tag{2.71}$$

Next, we use the superpotential coupling $\frac{1}{\mu}\psi_M\phi_q\psi_{\bar{q}}$ twice to obtain

$$\frac{1}{\mu^2}\psi_q^{2(N_f-3)}(\phi_q^*)^4\psi_M^2\phi_q^2\tilde{\Lambda}^{-N_c-4}. \tag{2.72}$$

Finally, we use the superpotential coupling $\frac{1}{\mu}\phi_M\psi_q\psi_q$ to obtain the term

$$\frac{1}{\mu^{N_f-1}}\phi_M^{N_f-3}\psi_M^2(\phi_q^*)^4\phi_q^2\tilde{\Lambda}^{-N_c-4}. \tag{2.73}$$

Substituting VEV for ϕ_q, we obtain

$$\frac{1}{\mu^{N_f-1}}\phi_M^{N_f-3}\psi_M^2\langle(\phi_q^*)\rangle^4\langle\phi_q\rangle^2\tilde{\Lambda}^{-N_c-4}. \tag{2.74}$$

The integration over the instanton size will result in the additional factor of $\langle\phi_q^*\rangle^{-4}$ because the dependence on $\langle\phi_q^*\rangle$ has to be cancelled in order for the superpotential to be holomorphic. Thus, 1-instanton in the completely broken $SU(2)$ group gives a contribution to the superpotential of the form

$$\frac{\det(\frac{1}{\mu}M)}{\langle\phi_q\rangle^2}\tilde{\Lambda}^{-N_c+4}. \tag{2.75}$$

Adding this to the tree level superpotential, we obtain the following superpotential

$$\begin{aligned} W &= \frac{1}{\mu}Mq\bar{q} - \frac{1}{\Lambda_L^{2N_c-1}}\det M \\ &= \frac{1}{\Lambda^{2N_c-1}}(MB\bar{B} - \det M), \end{aligned} \tag{2.76}$$

where we used $q = \sqrt{\mu/\Lambda_L^{2N_c-1}}B$, $\bar{q} = \sqrt{\mu/\Lambda_L^{2N_c-1}}\bar{B}$. This is the same superpotential as that of s-confinement phase (2.44). In the electric description, Eq.(2.44) is due to the strong coupling effects. In the magnetic description, it is rederived in a *weakly* coupled framework.

One more example of the deformation is "flat direction deformation". If VEV of N_f-th quark is non-zero and large compared to the dynamical scale, the effective theory will become $SU(N_c-1)$ gauge theory with (N_f-1) flavors. In the dual theory, this deformation corresponds to $\langle M_{N_f}^{N_f}\rangle \neq 0$. Since M is a gauge singlet, the gauge symmetry is unbroken. The effective theory is $SU(N_f-N_c)$ gauge theory with (N_f-1) flavors. The resulting pairs of theories are nothing but theories where N_c-1 and N_f-1 are substituted for N_c and N_f.

Classically, the electric and magnetic theories have different moduli space. It is only after taking non-perturbative effects into account that they are seen to be identical. For example, in the electric theory, there is a classical constraint $\text{rank}\langle M\rangle \leq N_c$. In the dual theory, M is an independent field whose VEV is unconstrained to all orders of perturbation theory. The constraint arises in the dual theory by quantum effects. The equation of motion for M sets $\langle N\rangle = 0$. However, the magnetic theory has $N_f - \text{rank}\langle M\rangle$ massless flavors, so if $N_f - \text{rank}\langle M\rangle < N_f - N_c$, the superpotential, which is analogous to ADS superpotential, is generated dynamically and there is no vacuum with $\langle N\rangle = 0$. Therefore, the vacua of the dual theory also satisfies $\text{rank}\langle M\rangle \leq N_c$ as a result of quantum effects rather than as a classical constraint.

Similarly, for $\text{rank}\langle M\rangle = N_c$, the magnetic theory has $N_f - N_c = \tilde{N}_c$ massless flavors and a constraint similar to (2.32)

$$\det N - b\bar{b} = \tilde{\Lambda}_L^{2\tilde{N}_c} \tag{2.77}$$

appears, where $\tilde{\Lambda}_L^{2\tilde{N}_c} = \det'\langle\mu^{-1}M\rangle\tilde{\Lambda}^{3\tilde{N}_c-N_f}$, $\det'\langle\mu^{-1}M\rangle$ means the product of the N_c non zero eigenvalues of $\langle\mu^{-1}M\rangle$. Using the equation of motion for M, *i,e,* $\langle N\rangle = 0$ and the mapping (2.59) and the scale relation (2.60), the constraint (2.77) becomes

$$\langle B\bar{B}\rangle = \det'\langle M\rangle. \tag{2.78}$$

This is a constraint which is classical in the electric theory but quantum in the dual theory. It is quite interesting that classical relations arise via quantum effects in the dual theory.

Before closing this chapter, we comment on phases of other ranges of N_f which are not touched upon. For $N_f \geq 3N_c$, the original theory is not asymptotically free. The low energy spectrum is elementary quarks and gluons. The phase in this range is referred to as "non-Abelian free electric phase". For $N_c + 2 \leq N_f \leq \frac{3}{2}N_c$, the magnetic theory is not asymptotically free. The low energy spectrum consists of $SU(N_f - N_c)$ gauge fields and $M, q, \bar{q}$. The phase in this range is referred to as "non-Abelian free magnetic phase". The phases of SUSY QCD is summarized in Fig. 2.1.

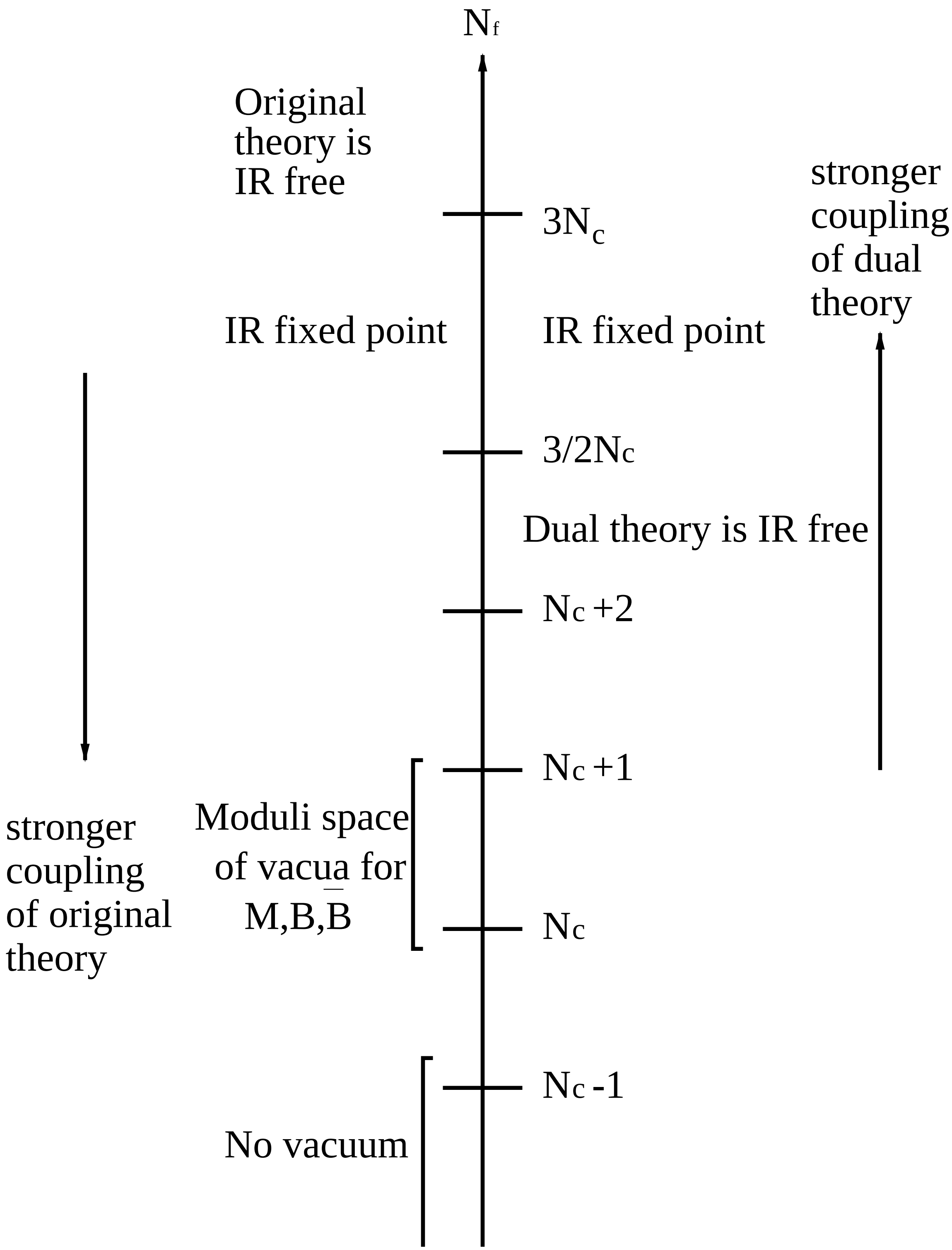

Figure 2.1: Phases of SUSY QCD

Chapter 3

Negative Dimensional Group and Duality

Duality proposed by Seiberg [2] gives a clue to analyze the strong coupling dynamics. Many authors have proposed dualities which have various gauge groups and various matter contents [15]-[45].

In this chapter we point out that the similarities of dualities in $\mathcal{N} = 1$ supersymmetric SO, SP gauge theories can be explained by using the trick of negative dimensional groups. In constructing the duality in $\mathcal{N} = 1$ supersymmetric gauge theories one of the key ideas is anomaly matching [48]. Anomaly matching conditions heavily depend on the representations and the charges of matter fields in the model. Therefore we have to find the matter content of the model by trial and error. However if we know one $SO(SP)$ duality we can easily obtain another $SP(SO)$ duality automatically satisfying anomaly matching conditions by using the trick of negative dimensional groups.

The idea of negative dimensional groups is not new and goes back to Penrose [49], who has constructed the $SU(2)$ $(= SP(2))$ representations in terms of $SO(-2)$. Since then many relations have been observed among the expressions for the $SU(N)$, $SO(N)$ and $SP(N)$ group invariants under the substitution $N \to -N$ [50]. On the other hand, Parisi and Sourlas [51] have observed that a Grassmann space of dimension N can be interpreted as an ordinary space of dimension $-N$. In supersymmetric theories, the $N \to -N$ relations are, in a sense, built in and we are going to utilize this property. We also give a comment on the problem which one encounters in applying the trick of negative dimensional group to *Spin* group duality.

This chapter is based on the work in collaboration with S. Kitakado [47].

3.1 Similarities in SO and SP duality

In order to show the advantages of our trick of negative dimensional groups we focus here on dualities with SO and SP gauge groups. * It is well known that these models strongly resemble each other in appearance. As an example, we shall take the model proposed by Intriligator [27]. In Ref. [27], duality in supersymmetric SO and SP gauge theories are discussed. The electric theory of SO dual model is a $\mathcal{N} = 1$ supersymmetric $SO(2N_c)^\dagger$ gauge theory with $2N_f$ fields Q^i in the fundamental representation and a symmetric traceless tensor X. The anomaly free global symmetries are $SU(2N_f) \times U(1)_R$ with the fields transforming as

$$
\begin{array}{ll}
Q^i & \left(\square \,, 1 - \dfrac{2(N_c - k)}{(k+1)N_f} \right), \\
X & \left(1, \dfrac{2}{k+1} \right).
\end{array}
\tag{3.1}
$$

The superpotential is

$$W = g_k \mathrm{Tr} X^{k+1}. \tag{3.2}$$

The magnetic theory is $\mathcal{N} = 1$ supersymmetric $SO(2\tilde{N}_c)$ gauge theory, where $\tilde{N}_c \equiv k(N_f + 2) - N_c$, with $2N_f$ fields q^i in the fundamental representation, a symmetric traceless tensor Y and singlets $M_j (j = 1, \cdots, k)$. The anomaly free global symmetries are $SU(2N_f) \times U(1)_R$ with the fields transforming as

$$
q^i \quad \left(\bar{\square} \,, 1 - \frac{2(\tilde{N}_c - k)}{(k+1)N_f} \right),
$$

* As will be explained later, $SU(N)$ group is self-dual under $N \to -N$. Therefore we don't discuss SU group here.

† In order to see the relation with the SP groups, we are restricting our discussion to the even dimensional $SO(2N_c)$ groups leaving aside the $SO(2N_c + 1)$ groups.

$$
\begin{array}{ll}
Y & \left(1, \dfrac{2}{k+1}\right), \\
M_j & \left(\square\!\square\,, \dfrac{2(j+k)}{(k+1)} - \dfrac{4(N_c-k)}{(k+1)N_f}\right).
\end{array}
\tag{3.3}
$$

The superpotential is

$$
W = \mathrm{Tr}Y^{k+1} + \sum_{j=1}^{k} M_j q Y^{k-j} q. \tag{3.4}
$$

On the other hand, the electric theory of SP dual model is a $\mathcal{N}=1$ supersymmetric $SP(2N_c)$ gauge theory with $2N_f$ fields Q^i in the fundamental representation and an antisymmetric traceless tensor X ‡ . The global symmetries are $SU(2N_f) \times U(1)_R$ with fields transforming as

$$
\begin{array}{ll}
Q^i & \left(\square\,, 1 - \dfrac{2(N_c+k)}{(k+1)N_f}\right), \\
X & \left(1, \dfrac{2}{k+1}\right).
\end{array}
\tag{3.5}
$$

The superpotential is

$$
W = g_k \mathrm{Tr}X^{k+1}. \tag{3.6}
$$

The magnetic theory is $\mathcal{N}=1$ supersymmetric $SP(2\tilde{N}_c)$ gauge theory, where $\tilde{N}_c \equiv k(N_f-2) - N_c$, with $2N_f$ fields q^i in the fundamental representation, an antisymmetric traceless tensor Y and singlets $M_j (j = 1, \cdots, k)$. The global symmetries are $SU(2N_f) \times U(1)_R$ with the fields transforming as

$$
\begin{array}{ll}
q^i & \left(\overline{\square}\,, 1 - \dfrac{2(\tilde{N}_c+k)}{(k+1)N_f}\right), \\
Y & \left(1, \dfrac{2}{k+1}\right), \\
M_j & \left(\begin{array}{c}\square\\[-6pt]\square\end{array}, \dfrac{2(j+k)}{(k+1)} - \dfrac{4(N_c+k)}{(k+1)N_f}\right).
\end{array}
\tag{3.7}
$$

The superpotential is

$$
W = \mathrm{Tr}Y^{k+1} + \sum_{j=1}^{k} M_j q Y^{k-j} q. \tag{3.8}
$$

It is easy to recognize that the representations and the charges of fields are quite similar. Furthermore, it can be seen that we can obtain $SP(SO)$ duality from the

‡ In this thesis, we denote the symplectic group as $SP(2N_c)$ whose fundamental representation is $2N_c$ dimensional.

$SO(SP)$ duality by changing the signs of N_c, N_f into $-N_c$, $-N_f$, and exchanging a symmetric (an antisymmetric) tensor for an antisymmetric (a symmetric) tensor. This feature is not specific to this model and is applicable to SO, SP dual models discovered so far [17, 26, 22].

3.2 Negative dimensional group

Group theoretically, these can be anticipated by considering the negative dimensional groups first proposed by Penrose [49]. This is a technique to calculate the algebraic invariants. Using this technique, we can find the peculiar relations for dimensions of the irreducible representations of the classical groups $SU(N), SO(N), SP(N)$ [50]. If λ_s is a Young tableau with s boxes and if the dimensions of the corresponding irreducible representations of $SU(N), SO(N)$ and $SP(N)$ are denoted by $D\{\lambda_s; N\}$, $D[\lambda_s; N]$ and $D\langle\lambda_s; N\rangle$, respectively, it was noticed by King [50] that

$$\begin{aligned} D\{\lambda_s; N\} &= (-1)^s D\{\tilde{\lambda}_s; -N\}, \\ D[\lambda_s; N] &= (-1)^s D\langle\tilde{\lambda}_s; -N\rangle. \end{aligned} \tag{3.9}$$

Here $\tilde{\lambda}$ stands for the "transposed" (rows and columns interchanged) Young tableau. The simplest example of the relations (3.9) is a two index tensor,

$$\square\!\square \quad \text{for} \quad SU(N), SP(2N)(SO(2N)) \leftrightarrow \begin{smallmatrix}\square\\[-2pt]\square\end{smallmatrix} \quad \text{for} \quad SU(-N), SO(-2N)(SP(-2N)). \tag{3.10}$$

Moreover, it is useful to give the relations among the generalized Casimirs of the classical groups in totally symmetric and totally antisymmetric representations.

$$\begin{aligned} C_p^{SU(N)}(1,1,\cdots,1) &= (-1)^{p-1} C_p^{SU(-N)}(r,0,\cdots,0), \\ C_p^{SO(2N)}(1,1,\cdots,1) &= (-1)^{p-1} C_p^{SP(-2N)}(r,0,\cdots,0), \\ C_p^{SP(2N)}(1,1,\cdots,1) &= (-1)^{p-1} C_p^{SO(-2N)}(r,0,\cdots,0), \end{aligned} \tag{3.11}$$

where $C_p(1,1,\cdots,1)$ and $C_p(r,0,\cdots,0)$ mean the p-th order generalized Casimir in totally antisymmetric and totally symmetric rank-r tensor representations respectively. These relations Eqs. (3.9) and (3.11) are necessary to take anomaly matching conditions into account.

We can express these results symbolically as follows [50],

$$\begin{aligned} SU(-N) &\cong \overline{SU(N)}, \\ SO(-2N) &\cong \overline{SP(2N)}, \\ SP(-2N) &\cong \overline{SO(2N)}, \end{aligned} \tag{3.12}$$

where the overbar means symmetrization and antisymmetrization are interchanged.

Since supersymmetric theories are "invariant" under this interchange we can use this technique as a useful method of obtaining a new dual theory from a known dual theory through the extrapolation to the negative dimensional groups. The procedures are :

- Change the sign of the group dimension, $N_c \leftrightarrow -N_c$, $N_f \leftrightarrow -N_f$.

- Interchange the symmetrization and antisymmetrization of the representations.

We can actually convince ourselves that with these procedures Eq. (3.7) follows from Eq. (3.3). This negative dimensional group technique is very powerful since the anomaly matching is automatically satisfied.

3.3 Discussion

In this chapter, we have pointed out that the similarities of dualities in $\mathcal{N} = 1$ supersymmetric SO, SP gauge theories can be explained by using the trick of negative dimensional groups. If we know the duality in supersymmetric $SO(SP)$ gauge theory, we can easily obtain another duality in supersymmetric $SP(SO)$ gauge theory automatically satisfying the anomaly matching conditions by extrapolating the groups to the negative dimensions. By this trick we can also know the representations and the charges of the fields. On the other hand, when there is no known duality in $SO(SP)$ gauge theory this trick is powerless.

Explicit application of this trick in finding new dualities is left for future studies. It is interesting to see whether this trick is applicable to other groups. If we try to apply the trick of negative dimensional groups to the *Spin* group dualities [32]-[37], we immediately get into trouble because the spinors in negative dimensions (called "spinsters" in [52]) have an infinite dimensional representation. This can be seen as follows. Let us recall how Dirac constructed the spinor representation. He considered the orthogonal norm as the squared of the linear form,

$$(x^1)^2 + (x^2)^2 + \cdots + (x^n)^2 = (\gamma^1 x^1 + \gamma^2 x^2 + \cdots + \gamma^n x^n)^2, \tag{3.13}$$

where x^i means components of n-dimensional vector. This requires the γ^i to satisfy the anticommutation relations which define a Clifford algebra C_n

$$\{\gamma^i, \gamma^j\} = 2\delta^{ij}. \tag{3.14}$$

Note that there exists a $2^{[n/2]} \times 2^{[n/2]}$ matrix representation of C_n. Now consider a matrix in the defining representation of the orthogonal group $O(n)$ acting on the Euclidean vector $\boldsymbol{x}$ as

$$x^i \to x'^i = O^i_j x^j. \tag{3.15}$$

In order for Eq. (3.13) to be invariant, γ^i is transformed as follows,

$$\gamma^i \to \gamma'^i = O^i_j \gamma^j. \tag{3.16}$$

Since the γ^i also satisfy the Clifford algebra (3.14) and the matrix representation is unique, we must have

$$\gamma'^i = S(O)\gamma S(O)^{-1}, \tag{3.17}$$

where S is a $2^{[n/2]} \times 2^{[n/2]}$ matrix depending on the orthogonal matrix O. In fact, the S forms a double-valued unitary representation of $O(n)$, which we call the spinor representation.

Next, we consider the same procedure on a Grassmann tensor space since the negative dimensional group is defined in terms of their action on a Grassmann space in a sense. In this case, it is natural to consider the symplectic form $\mathbf{\Theta^2} \equiv \mathbf{\Theta^i J_{ij} \Theta^j}$, where J_{ij} is an invariant metric of $Sp(n)$. Then, we look for algebraic quantities β^i such that

$$\begin{aligned} \mathbf{\Theta^2} &\equiv \Theta^i J_{ij} \Theta^j = 2(\Theta^1\Theta^2 + \Theta^3\Theta^4 + \cdots + \Theta^{n-1}\Theta^n) \\ &= (\beta^1\Theta^2 - \beta^2\Theta^1 + \cdots + \beta^{n-1}\Theta^n - \beta^n\Theta^{n-1})^2 = (\beta^i J_{ij}\Theta^j)^2. \end{aligned} \tag{3.18}$$

This leads to the commutation relation

$$[\beta^i, \beta^j] = 2J^{ij}. \tag{3.19}$$

We can represent the β^i by a unitary irreducible infinite dimensional representation. The remaining argument is same except that O is a matrix in the defining representation of $Sp(n)$ in this case, and the spinster is represented by S, which is an infinite dimensional matrix depending on the symplectic matrix O.

It seems to be impossible to match the global anomalies from the above argument. One may think that the above problem can be avoided in the case of $Spin(8)$ gauge theory [33] because vector, spinor and conjugate spinor representations are equivalent in this case. However, it can be shown that duality does not work in the theory extrapolated from $Spin(8)$ duality. For the present, we have no idea how spinsters should be treated. It is also interesting to consider the case of exceptional groups. The relations among dimensions of the irreducible representations and Casimirs are also known for exceptional groups [53]. We have described the substitution $N \to -N$ just as a useful trick for studying the duality structures of supersymmetric theories leaving aside the direct significance it might have in such theories. Thus it might be interesting to study the symmetries under $N \to -N$ directly in the supersymmetric theories where duality is realized explicitly. We hope to report elsewhere on these together with the related problems.

Chapter 4

Confining Phase in SUSY SO(12) Gauge Theory

Our understanding of non-perturbative nature in $\mathcal{N}=1$ supersymmetric gauge theories has much advanced since the pioneering works of Seiberg and his collaborators [8, 17]. Especially, the physics of confining phase in $\mathcal{N}=1$ supersymmetric gauge theories was enriched (quantum deformed moduli space, "s-confinement"). These works have been also extended to the theories with various types of gauge groups and matter contents [9, 56, 57, 58]. Furthermore, these theories have recently been applied to construction of models with dynamical supersymmetry breaking or SUSY composite models.

In this chapter, we present the confining phase in $\mathcal{N}=1$ supersymmetric $SO(12)$ gauge theory with $N_f \leq 7$ vectors and one spinor. There are two motivations we are interested in this particular model. First, from the theoretical point of view, it will provide useful informations for finding the dual to $SO(N_c)(N_c > 10)$ with an arbitrary number of vectors and spinors. Although the duality of this class of models has only been generalized to $SO(10)$ [37], the known dualities have the following remarkable properties which are not contained in Seiberg's duality [2]:

(i). Chiral-Nonchiral duality [32, 33, 36].

(ii). Each gauge group belongs to different Cartan families [32]-[37].

(iii). Reducibility to the exceptional group (G_2) duality [32, 38, 39].

(iv). Simple and Semi-simple group duality* [37].

(v). Identification of massive spinors and Z_2 monopoles under duality [59].

It is interesting to ask whether these properties persist in the duality for $SO(N_c)(N_c > 10)$ theory. However, from the result of Ref. [37] looking for this class of duals seems to be highly non-trivial. Cho [56] has already investigated in detail the confining phase of $SO(11)$ gauge theory with $N_f \le 6$ vectors and a spinor and extracted some clues in search for duals. It is interesting to pursue further following the line of his argument in order to clarify the dual to $SO(N_c)(N_c > 10)$ theory. Second, as mentioned in the above paragraph, the theory under consideration may provide phenomenologically viable models with dynamical supersymmetry breaking or SUSY composite models.

This chapter is based on the work [54].

4.1 A Model

The model we consider has the following symmetry groups

$$G = SO(12)_{gauge} \times [SU(N_f)_V \times U(1)_V \times U(1)_Q \times U(1)_R]_{global} \tag{4.1}$$

under which the superfields transform as†

$$V^i_\mu \sim (\mathbf{12}, \Box\ , 1, 0, 0), \tag{4.2}$$

$$Q_\alpha \sim (\mathbf{32}, \mathbf{1}, 0, 1, 0) \tag{4.3}$$

and no tree level superpotential. Note that since each of the $U(1)$ symmetries in Eq. (4.1) are anomalous, the action is transformed as

$$S \to S - C\alpha \int d^4x \frac{g^2}{32\pi^2} F\tilde{F}, \tag{4.4}$$

* We do not consider here the duality derived by using deconfinement technique [23].

† We implicitly regard the 32 dimensional $SO(12)$ spinor as a projection $Q = P_- Q_{64}$ where Q_{64} means the 64 dimensional spinor of $SO(13)$ and $P_- = \frac{1}{2}(1 - \Gamma_{13})$.

where C denotes the anomaly coefficient of the corresponding $U(1)_V SO(12)^2$, $U(1)_Q SO(12)^2$ or $U(1)_R SO(12)^2$ anomalies and α is a transformation parameter. If the theta parameter in the Lagrangian is shifted under these anomalous $U(1)$'s as $\theta \to \theta + C\alpha$, then anomalies can be cancelled. Recalling the relation

$$\left(\frac{\Lambda}{\mu}\right)^{b_0} = \exp\left(-\frac{8\pi^2}{g^2(\mu)} + i\theta\right), \tag{4.5}$$

where b_0 represents 1-loop beta function coefficient ‡

$$b_0 = \frac{1}{2}[3\mu(\mathrm{Adj}) - \sum_{\mathrm{matter}} \mu(R)] = 26 - N_f, \tag{4.6}$$

and Λ is the strong coupling scale of the theory, the spurion superfield Λ^{b_0} is transformed as

$$\Lambda^{b_0} \sim (1, 1, 2N_f, 8, 12 - 2N_f). \tag{4.7}$$

Using these symmetries and holomorphy, we can easily fix the form of the dynamically generated superpotential W_{dyn} for the small value of N_f so that $U(1)_R$ charge of W_{dyn} be 2 and $U(1)_V, U(1)_Q$ charges vanish. The results are summarized in Table 4.1. $N_f = 6$ case is special because R charge of Λ^{b_0} vanishes. This means that we cannot construct the dynamically generated superpotential. However, the classical constraint among matter superfields is modified by non-perturbative effects and this quantum constraint can be included in the superpotential by using the Lagrange multiplier superfield X [8]. Thus, $N_f = 6$ case is analogous to $N_f = N_c$ SUSY QCD (quantum deformation of moduli space). Furthermore, we will immediately find that $N_f = 7$ case is analogous to $N_f = N_c + 1$ SUSY QCD ("s-confinement") [8], and $N_f \leq 5$ case is analogous to $N_f \leq N_c - 1$ SUSY QCD (runaway superpotential) [7].

In order to describe the low energy effective theory, we need to find gauge invariant operators which behave as the moduli space coordinate § . It is in general

‡ μ denotes quadratic Dynkin index defined as Tr $T^a(R)T^b(R) = \mu(R)\delta^{ab}$ (T^a: the generators of the group, R: representation which the superfield belongs to). We use the following values : $\mu(\mathbf{12}) = 2$, $\mu(\mathbf{32}) = 8$, $\mu(\mathbf{66}) = 20$.

§ Vacuum expectation values (VEVs) of these gauge invariant operators are in one to one correspondence to the solutions of D-flatness conditions [61].

N_f	$R(\Lambda^{b_0})$	W_{dyn}
0	12	$(\Lambda^{26}/Q^8)^{1/6}$
1	10	$(\Lambda^{25}/Q^8V^2)^{1/5}$
2	8	$(\Lambda^{24}/Q^8V^4)^{1/4}$
3	6	$(\Lambda^{23}/Q^8V^6)^{1/3}$
4	4	$(\Lambda^{22}/Q^8V^8)^{1/2}$
5	2	Λ^{21}/Q^8V^{10}
6	0	$X(Q^8V^{12} - \Lambda^{20})$
7	-2	Q^8V^{14}/Λ^{19}

Table 4.1: The form of dynamically generated superpotentials

troublesome to do this task. However, if the gauge symmetry breaking pattern is known at generic points on the moduli space, one can easily identify these gauge invariant operators. We illustrate below how it works in the present model. The gauge symmetry breaking pattern we utilize is [62]

$$\begin{aligned} SO(12) &\xrightarrow{<32>} SU(6) \xrightarrow{<12>} SU(5) \xrightarrow{<12>} SU(4) \\ &\xrightarrow{<12>} SU(3) \xrightarrow{<12>} SU(2) \xrightarrow{<12>} 1. \end{aligned} \tag{4.8}$$

With this information in hand, counting degrees of freedom of gauge invariant operators is nothing but a group theoretical exercise. We display in Table 4.2 parton degrees of freedom, unbroken subgroups, eaten degrees of freedom by Higgs mechanism and hadron degrees of freedom. We are now in a position to construct gauge invariant operators explicitly. Before doing this, we need to notice that $SO(12)$ spinor product decomposes into the following irreducible representations

$$32 \times 32 = [0]_A + [2]_S + [4]_A + [\tilde{6}]_S, \tag{4.9}$$

where [n] represents rank-n antisymmetric tensor, subscripts "A" and "S" mean antisymmetry and symmetry under spinor exchange, and the tilde of the last term implies that the rank-6 tensor is self-dual. Since our model has only one spinor, gauge invariant operators can include $[2]_S$ and $[\tilde{6}]_S$ in Eq. (4.9).

N_f	Parton DOF	Unbroken Subgroup	Eaten DOF	Hadron DOF
0	32	SU(6)	$66-35=31$	1
1	44	SU(5)	$66-24=42$	2
2	56	SU(4)	$66-15=51$	5
3	68	SU(3)	$66-8=58$	10
4	80	SU(2)	$66-3=63$	17
5	92	1	66	26
6	104	1	66	38
7	116	1	66	50

Table 4.2: Degrees of freedom of independent gauge invariants

Taking this into account, we can construct gauge invariant composites as follows[¶]

$$
\begin{aligned}
L &= \frac{1}{2!2!}(Q^T\Gamma^{[\mu}\Gamma^{\nu]}CQ)(Q^T\Gamma_{[\mu}\Gamma_{\nu]}CQ) \sim (\mathbf{1};\mathbf{1};4N_f;4R),\\
M^{(ij)} &= (V^{i\mu})^T V^j_\mu \sim (\mathbf{1};\ \begin{array}{|c|c|}\hline \ & \ \\ \hline\end{array}\ ;-8;2R),\\
N^{[ij]} &= \frac{1}{2}Q^T \not{V}^i \not{V}^j CQ \sim (\mathbf{1};\ \begin{array}{|c|}\hline \ \\ \hline \ \\ \hline\end{array}\ ;2N_f-8;4R),\\
P^{[ijklmn]} &= \frac{1}{6!}Q^T \not{V}^{[i}\not{V}^j\not{V}^k\not{V}^l\not{V}^m\not{V}^{n]}CQ \sim (\mathbf{1};\ \begin{array}{|c|}\hline \ \\ \hline \ \\ \hline \ \\ \hline \ \\ \hline \ \\ \hline \ \\ \hline\end{array}\ ;2N_f-24;8R),\\
R^{[ijklmn]} &= \frac{1}{6!}\epsilon^{\mu_1\cdots\mu_{12}}(Q^T\Gamma_{\mu_1}\Gamma^\nu CQ)(Q^T\Gamma_\nu\Gamma_{\mu_2}\cdots\Gamma_{\mu_6}CQ)V^i_{\mu_7}V^j_{\mu_8}V^k_{\mu_9}V^l_{\mu_{10}}V^m_{\mu_{11}}V^n_{\mu_{12}}\\
&\sim (\mathbf{1};\ \begin{array}{|c|}\hline \ \\ \hline \ \\ \hline \ \\ \hline \ \\ \hline \ \\ \hline \ \\ \hline\end{array}\ ;4N_f-24;10R), \qquad (4.10)
\end{aligned}
$$

where the square bracket means the antisymmetrization of the corresponding indices, $\not{V}^i \equiv V^i_\mu\Gamma^\mu$ and we use here the following SO(12) Gamma matrices,

¶ The representations and the charge in Eq. (4.10) are those under Eq. (4.11).

$$\Gamma_1 = \sigma_2 \otimes \sigma_3 \otimes \sigma_3 \otimes \sigma_3 \otimes \sigma_3 \otimes \sigma_3, \qquad \Gamma_2 = -\sigma_1 \otimes \sigma_3 \otimes \sigma_3 \otimes \sigma_3 \otimes \sigma_3 \otimes \sigma_3,$$
$$\Gamma_3 = 1 \otimes \sigma_2 \otimes \sigma_3 \otimes \sigma_3 \otimes \sigma_3 \otimes \sigma_3, \qquad \Gamma_4 = -1 \otimes \sigma_1 \otimes \sigma_3 \otimes \sigma_3 \otimes \sigma_3 \otimes \sigma_3,$$
$$\Gamma_5 = 1 \otimes 1 \otimes \sigma_2 \otimes \sigma_3 \otimes \sigma_3 \otimes \sigma_3, \qquad \Gamma_6 = -1 \otimes 1 \otimes \sigma_1 \otimes \sigma_3 \otimes \sigma_3 \otimes \sigma_3,$$
$$\Gamma_7 = 1 \otimes 1 \otimes 1 \otimes \sigma_2 \otimes \sigma_3 \otimes \sigma_3, \qquad \Gamma_8 = -1 \otimes 1 \otimes 1 \otimes \sigma_1 \otimes \sigma_3 \otimes \sigma_3,$$
$$\Gamma_9 = 1 \otimes 1 \otimes 1 \otimes 1 \otimes \sigma_2 \otimes \sigma_3, \qquad \Gamma_{10} = -1 \otimes 1 \otimes 1 \otimes 1 \otimes \sigma_1 \otimes \sigma_3,$$
$$\Gamma_{11} = 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes \sigma_2, \qquad \Gamma_{12} = -1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes \sigma_1,$$
$$\Gamma_{13} = \sigma_3 \otimes \sigma_3 \otimes \sigma_3 \otimes \sigma_3 \otimes \sigma_3 \otimes \sigma_3,$$

and C is a charge conjugation matrix.

In order to see that these gauge invariant operators are in fact the coordinates of the moduli space, one has to check whether the total degrees of freedom of these gauge invariant operators in Eq. (4.10) coincide with hadronic degrees of freedom. In Table 4.3, these degrees of freedom are listed as a function of N_f. For $N_f \leq 5$, hadronic degrees of freedom and that of L, M, N, P and R agree with each other. For $N_f = 6$, degrees of freedom of L, M, N, P and R are larger than those of hadrons by one. This implies that L, M, N, P and R are not independent and a single constraint among them exists. This statement is also consistent with the previous argument for dynamically generated superpotentials. For $N_f = 7$, 14 constraints are expected to come from the equations of motion as $N_f = N_c + 1$ SUSY QCD.

N_f	Hadron DOF	L	M	N	P	R	Constraints
0	1	1	0	0	0	0	
1	2	1	1	0	0	0	
2	5	1	3	1	0	0	
3	10	1	6	3	0	0	
4	17	1	10	6	0	0	
5	26	1	15	10	0	0	
6	38	1	21	15	1	1	−1
7	50	1	28	21	7	7	−14

Table 4.3: Hadron degrees of freedom count

We can obtain more non-trivial support which convinces us that gauge invariant operators L, M, N, P and R are the moduli. One of the powerful methods to study the low energy spectrum is 't Hooft anomaly matching [48]. To see that anomalies between elementary fields and composite ones match, we take anomaly free symmetry group instead of Eq. (4.1)

$$G_{AF} = SO(12)_{gauge} \times [SU(N_f) \times U(1) \times U(1)_R]_{global}, \tag{4.11}$$

where new $U(1)$ and $U(1)_R$ are linear combinations of the original $U(1)$'s in Eq.(4.1). Matter superfields transform under these symmetries as

$$V^i_\mu \sim (\mathbf{12}, \square, -4, R), \tag{4.12}$$

$$Q_\alpha \sim (\mathbf{32}, \mathbf{1}, N_f, R), \tag{4.13}$$

where $R = \frac{N_f - 6}{N_f + 4}$. We can calculate anomalies and see that anomalies match for $N_f = 7$; $SU(7)^3 : 12A(\square), SU(7)^2U(1)_R : -\frac{120}{11}\mu(\square), SU(7)^2U(1) : -48\mu(\square), U(1)_R : -\frac{434}{11}, U(1) : -112, U(1)_R^2U(1) : \frac{2128}{121}, U(1)_RU(1)^2 : -\frac{29120}{11}, U(1)_R^3 : -\frac{28154}{1331}, U(1)^3 : 5600$, where $A(\square)$ and $\mu(\square)$ are cubic and quadratic Dynkin indices for fundamental representation of $SU(N_f)$. Recalling that anomalies are saturated in $N_f = N_c + 1$ SUSY QCD, we are convinced that this coincidence for $N_f = 7$ is very natural and gives a strong support that the theory in this case is in "s-confinement" phase.

For $N_f \geq 8$, it is impossible to satisfy anomaly matching conditions without violating $N_f \leq 7$ result even if other gauge invariant operators are added. This implies that a confining phase of this model terminates at $N_f = 7$.

4.2 Low energy superpotentials

In this section, we determine explicitly the low energy superpotentials in terms of L, M, N, P and R. Since we know which of gauge invariant operators are moduli in the previous section, it is straightforward to work out which of operators should be in the superpotential. Following Ref. [56], we first determine the quantum deformed constraint in $N_f = 6$ theory. Dimensional analysis, symmetries and holomorphy restrict the superpotential as follows

$$
\begin{aligned}
W_{N_f=6} &= X(R^2 + P^2 L + 2P\mathrm{Pf}N + L^2 \mathrm{det} M \\
&+ \frac{1}{2!4!}\epsilon_{i_1 i_2 i_3 i_4 i_5 i_6}\epsilon_{j_1 j_2 j_3 j_4 j_5 j_6} L N^{i_1 j_1} N^{i_2 j_2} M^{i_3 j_3} M^{i_4 j_4} M^{i_5 j_5} M^{i_6 j_6} \\
&+ \frac{1}{4!2!}\epsilon_{i_1 i_2 i_3 i_4 i_5 i_6}\epsilon_{j_1 j_2 j_3 j_4 j_5 j_6} N^{i_1 j_1} N^{i_2 j_2} N^{i_3 j_3} N^{i_4 j_4} M^{i_5 j_5} M^{i_6 j_6} - \Lambda_6^{20}),
\end{aligned}
\tag{4.14}
$$

where Λ_6 is the strong coupling scale of $SO(12)$ gauge theory with 6 vector flavors and a spinor. Coefficients of each terms are determined so that it reproduce the superpotential in $SO(11)$ gauge theory with 5 flavors [56]. (If VEV $< V_{12}^6 > \neq 0$ is given, $SO(12)$ gauge theory with 6 flavors under consideration here reduces to $SO(11)$ gauge theory with 5 flavors)

One may also determine these coefficients by using the symmetry breaking along the spinor flat direction $SO(12) \xrightarrow{<32>} SU(6)$, explicitly $< \mathbf{32} >^T = (0, a, 0, \cdots, 0, a, 0)$. V_μ^i decomposes into $\mathbf{6} + \bar{\mathbf{6}}$ under $SU(6)$, which is explicitly

$$
V_\mu = \begin{pmatrix} q_1 + \bar{q}_1 \\ i(q_1 - \bar{q}_1) \\ q_2 + \bar{q}_2 \\ i(q_2 - \bar{q}_2) \\ q_3 + \bar{q}_3 \\ i(q_3 - \bar{q}_3) \\ q_4 + \bar{q}_4 \\ i(q_4 - \bar{q}_4) \\ q_5 + \bar{q}_5 \\ i(q_5 - \bar{q}_5) \\ q_6 + \bar{q}_6 \\ i(q_6 - \bar{q}_6) \end{pmatrix}, \tag{4.15}
$$

where $q_i, \bar{q}_i (i = 1, \cdots, 6)$ mean $SU(6)$ quarks, antiquarks, respectively. The reduced theory is $SU(6)$ gauge theory with $6(\square + \bar{\square})$, therefore it has a single quantum constraint [8].

According to this decomposition rule, $SO(12)$ gauge invariant operators are decomposed into the following $SU(6)$ meson m^{ij}, baryon b and anti-baryon $\bar{b}$;

$$
\begin{aligned}
L &\rightarrow 12a^4, \\
M^{ij} &\rightarrow 2(m^{ij}+m^{ji}), \\
N^{[ij]} &\rightarrow -4ia^2(m^{ij}-m^{ji}), \\
P &\rightarrow 64ia^2(b+\bar{b})+2ia^2\epsilon_{ijklmn}m^{ij}m^{kl}m^{mn}, \\
R &\rightarrow 64a^4(b-\bar{b}).
\end{aligned} \tag{4.16}
$$

Using this information, one can also determine coefficients so that $\det m - b\bar{b} = \Lambda^{12}$ be reproduced.

The superpotential of $N_f = 7$ case can be found in a similar way. In this case, the superpotential must have the following features. 1. It is smooth everywhere on the moduli space. 2. Equations of motion give classical constraints among vectors and a spinor. 3. Adding mass term for one vector flavor to this superpotential and integrating out this massive vector, the superpotential (4.14) must be reproduced. The result is$^{\|}$

$$
\begin{aligned}
W_{N_f=7} = & \frac{1}{\Lambda^{19}}(M^{ij}R_iR_j - 2iN^{ij}P_iR_j + LP_iP_jM^{ij} \\
& + L^2\det M + \frac{1}{3!2^2}\epsilon_{i_1\cdots i_7}P_jM^{ji_1}N^{i_2i_3}N^{i_4i_5}N^{i_6i_7} \\
& + \frac{1}{3!4!}\epsilon_{i_1i_2i_3i_4i_5i_6i_7}\epsilon_{j_1j_2j_3j_4j_5j_6j_7}LN^{i_1j_1}N^{i_2j_2}M^{i_3j_3}M^{i_4j_4}M^{i_5j_5}M^{i_6j_6}M^{i_7j_7} \\
& + \frac{1}{4!3!}\epsilon_{i_1i_2i_3i_4i_5i_6i_7}\epsilon_{j_1j_2j_3j_4j_5j_6j_7}N^{i_1j_1}N^{i_2j_2}N^{i_3j_3}N^{i_4j_4}M^{i_5j_5}M^{i_6j_6}M^{i_7j_7}).
\end{aligned} \tag{4.17}
$$

By adding the mass terms for vector fields $\delta W = m_{ij}M^{ij}$ to the superpotential (4.14) and integrating out each massive vectors successively, we can readily derive the superpotentials for $N_f \leq 5$ systematically. As a matter of fact, we obtain

$$
\begin{aligned}
W_{N_f=5} = & \Lambda_5^{21}/(L^2\det M + \frac{1}{2!3!}LN^{i_1j_1}N^{i_2j_2}M^{i_3j_3}M^{i_4j_4}M^{i_5j_5}\epsilon_{i_1i_2i_3i_4i_5}\epsilon_{j_1j_2j_3j_4j_5} \\
& + \frac{1}{4!}N^{i_1j_1}N^{i_2j_2}N^{i_3j_3}N^{i_4j_4}M^{i_5j_5}\epsilon_{i_1i_2i_3i_4i_5}\epsilon_{j_1j_2j_3j_4j_5}), \\
W_{N_f=4} = & 2\left(\frac{\Lambda_4^{22}}{L^2\det M + \frac{1}{2!2!}LN^{i_1j_1}N^{i_2j_2}M^{i_3j_3}M^{i_4j_4}\epsilon_{i_1i_2i_3i_4}\epsilon_{j_1j_2j_3j_4} + (\mathrm{Pf}N)^2}\right)^{1/2},
\end{aligned}
$$

$^{\|}$ Although this superpotential has already been derived in Ref. [9] by using the index argument, a PMN^3 term was missing. Without this term, the result of Ref. [56] cannot be correctly recovered.

$$
\begin{aligned}
W_{N_f=3} &= 3\left(\frac{\Lambda_3^{23}}{L^2\mathrm{det}M + \frac{1}{2!}LM^{i_1j_1}N^{i_2j_2}N^{i_3j_3}\epsilon_{i_1i_2i_3}\epsilon_{j_1j_2j_3}}\right)^{1/3}, \\
W_{N_f=2} &= 4\left(\frac{\Lambda_2^{24}}{L^2\mathrm{det}M + LN^2}\right)^{1/4}, \\
W_{N_f=1} &= 5\left(\frac{\Lambda_1^{25}}{L^2M}\right)^{1/5}, \\
W_{N_f=0} &= 6\left(\frac{\Lambda_0^{26}}{L^2}\right)^{1/6},
\end{aligned} \tag{4.18}
$$

where the strong coupling scales for each flavor are related to each other through one-loop matching of gauge coupling as follows

$$
\begin{aligned}
\Lambda_0^{26} &= m_{11}\Lambda_1^{25} = m_{11}m_{22}\Lambda_2^{24} = m_{11}m_{22}m_{33}\Lambda_3^{23} \\
&= m_{11}m_{22}m_{33}m_{44}\Lambda_4^{22} = m_{11}m_{22}m_{33}m_{44}m_{55}\Lambda_5^{21} \\
&= m_{11}m_{22}m_{33}m_{44}m_{55}m_{66}\Lambda_6^{20} = m_{11}m_{22}m_{33}m_{44}m_{55}m_{66}m_{77}\Lambda_7^{19}.
\end{aligned} \tag{4.19}
$$

It is worth to note that one can confirm the above superpotentials (4.18) to recover correctly the superpotentials for $N_f \leq 4$ in $SO(11)$ theory [56], which is obtained when one flavor vector field has non-vanishing VEV.

Before closing this section, we briefly discuss dynamical supersymmetry breaking. For $N_f = 6$, if we take the tree level superpotential as

$$
W_{tree} = \lambda_1 S_1 V^2 + \lambda_2 S_2 Q^2 V^6 + \lambda_3 S_3 Q^4 V^6, \tag{4.20}
$$

where $S_{1,2,3}$ are singlet superfields, then equations of motion with respect to $S_{1,2,3}$ and the quantum constraint (4.14) are incompatible. Therefore, supersymmetry is dynamically broken [63, 64]. For $N_f = 0$, since we cannot add terms which lift a classical flat direction (*i.e.*L) preserving $U(1)_R$ to the tree level superpotential, supersymmetry remains unbroken. The same argument seems to be applicable for $1 \leq N_f \leq 5$ ** .

** In Ref. [9], the authors construct a model with dynamical supersymmetry breaking for $N_f = 1$ by promoting a global $U(1)$ to a local $U(1)$ and adding singlets to cancel $U(1)$ gauge anomaly.

4.3 Discussion

In this chapter, we have studied the confining phase in $\mathcal{N}=1$ $SO(12)$ SUSY gauge theory with $N_f \leq 7$ vectors and a spinor. Utilizing the gauge symmetry breaking pattern at generic points on the moduli space, which plays a crucial role in our study, we have identified gauge invariant operators which behave as the moduli coordinate. Then we have derived explicitly low energy superpotentials for $N_f \leq 7$.

Some clues in search for duals are obtained from the results of this work. Let us suppose that the dual with the gauge group $\tilde{G}$ exists for $N_f \geq 8$. The original $SO(12)$ theory breaks down to $SU(6)+N_f(\square+\bar{\square})$ along the spinor flat direction. On the other hand, the gauge group of the dual theory is usually unbroken since in the dual theory the gauge invariant operators develop VEV, in the present case, it corresponds to L. Since $SU(6)+N_f(\square+\bar{\square})(N_f \geq 8)$ theory is dual to $SU(N_f-6)+N_f(\square+\bar{\square})$ [2], we can guess that at least $\tilde{G}$ must include $SU(N_f-6)$ as a subgroup to preserve the duality along this direction. Furthermore, the superpotential in the dual theory must recover $N_f=7$ superpotential. We also note that $N_f=8$ case in our model is known to be self-dual [44].

Although it seems to be quite difficult to find a dual which is compatible with the above requirements, we hope that this work will provide useful informations to search for the dual to $SO(N_c)(N_c \geq 11)$ theory.

Chapter 5

Gauge Mediated SUSY Breaking

5.1 Introduction

In the previous chapters, we have discussed phases of $\mathcal{N}=1$ supersymmetric gauge theories. This is a theoretical aspect in a sense. Once we turn to the phenomenological application of SUSY theories, we are necessarily faced with the issue of supersymmetry breaking. If the real world is supersymmetric, then the mass of bosons and fermions are degenerate. However, we have not yet observed, for instance, the scalar electron (selectron), which is the superpartner of the electron. Therefore, we cannot help concluding that supersymmetry is broken at low energy. Moreover, we emphasize that the mechanism of supersymmetry breaking plays an important role in the spectrum of the soft terms. This provides crucial informations on experimental methods of the superparticle searches.

There are two questions for supersymmetry breaking. How is SUSY broken? To answer this question, we have to recall first "the supertrace theorem"[66]. In a global supersymmetric theory with a gauge group free from gravitational anomalies [67], the sum of the tree level squared mass, weighted by the corresponding number of degrees of freedom, is equal in the bosonic and fermionic sectors, namely

$$\mathrm{Str}\mathcal{M}^2 = \sum_J (-1)^{2J}(2J+1)\mathcal{M}_J^2 = 0, \tag{5.1}$$

where $\mathcal{M}_J$ denotes the tree level mass of a particle with spin J. In general, this theorem implies the existence of a superpartner lighter than its ordinary particle. In other words, this theorem rules out the scenario that supersymmetry is broken at the tree level and its breaking is communicated to the ordinary supermultiplets at the tree level.

Therefore, we have to consider the scenario in which supersymmetry is broken at the quantum level not at the classical level. In supersymmetric theories, if supersymmetry is unbroken at the classical level, it is unbroken perturbatively due to the non-renormalization theorem [6]. Supersymmetry can be broken by certain nonperturbative effects (instantons, gaugino condensation, confinement etc). This is a very nice feature for the hierarchy problem because nonperturbative effects relate the small scale (weak scale) to the large scale (Planck scale or GUT scale) exponentially [68].

How is supersymmetry breaking communicated to the real world once SUSY is broken? There are mainly two alternatives. One is supergravity (SUGRA) mediation, the other is gauge mediation. In this thesis, we focus on the latter case because it has many advantages phenomenologically compared to the former case. In the SUGRA mediation, there is no obvious reason to expect that sfermion masses are degenerate. For instance, the flavor non symmetric terms in the Kähler potential are not forbidden by any symmetry. These terms lead to flavor breaking effects in the soft terms. Thus, the degeneracy of the soft terms is not guaranteed in this scenario.

On the other hand, in the gauge mediation, since SUSY breaking is transmitted by the SM gauge interactions, the sfermion masses are degenerate automatically due to the flavor blindness of the gauge interaction. Although the flavor violation only arises through Yukawa coupling, these are suppressed enough because their effects can only appear at higher loop order due to super GIM mechanism. Furthermore, the number of parameters other than the SM parameters in the soft terms can be largely reduced. This leads to relations among soft masses, and the theory becomes highly predictive* .

---

* For more details, see [65].

Figure 5.1: Schematic structure of DNNS model

5.2 DNNS Model

First realistic model in this context was constructed by Dine, Nelson, Nir and Shirman [73]. This work was the beginning to attract much attention and interest for the gauge mediation of SUSY breaking, and to explore models along the various directions. It is instructive to review this model briefly.

The model consists of three sectors, namely, dynamical SUSY breaking sector (DSB sector), messenger sector and the observable sector as shown in Fig. 5.1. In DSB sector, SUSY is broken by non perturbative effects and its breaking effects are transmitted to the messenger sector through the radiative corrections by $U(1)_m$ gauge interactions, which is different from the hypercharge $U(1)$ in the SM gauge group. Then, its effects are transmitted to the observable sector through the radiative corrections by the SM gauge interactions. Note that the DSB sector is neutral under the SM gauge group and that the observable sector is neutral under $U(1)_m$. The DSB sector and the observable sector does not connect directly.

It is natural to ask why these two sectors are obliged to be separated, why SUSY breaking effects should not be transmitted directly to the observable sector. Let us digress for a while to explain the reason why they introduce the messenger sector. Of course, such an attempt had been made before by Affleck, Dine and Seiberg [60]. In order to embed the SM gauge group into the unbroken global symmetry group in the DSB sector, the gauge group in the DSB sector has to be very large[†] . This implies a

[†] Since most of SUSY breaking models are chiral, the size of the gauge group and that of the global symmetry group are interrelated through the gauge anomaly cancellation condition.

large number of extra fields (which play a role of the messenger fields[‡]) transforming nontrivially under the SM gauge group. As a result, the SM gauge couplings blow up slightly above the messenger scale, which is often below the GUT scale. For example, consider the DSB model based on $SU(N)$ (N:odd) with an antisymmetric tensor and $N-4$ antifundamentals. The smallest gauge group including usual $SU(5)$ GUT group in the unbroken global symmetry group is $SU(15)$, namely, there are 15 flavors messengers. Then the QCD coupling constant blows up at 10^{6-7} GeV. If we embedded $SU(2)_L \times U(1)_Y$ instead of $SU(3)_C \times SU(2)_L \times U(1)_Y$, the gauge coupling unification will be lost and the gluino will be too light because it is generated at a higher order.

Let us turn back to the model of Ref. [73]. Here we do not specify the DSB model concretely because what is now of interest is how to communicate SUSY breaking to the observable sector. Anyway, we assume that SUSY is broken in the DSB sector.

The superpotential in the messenger sector is

$$W_m = k_1 X \phi^+ \phi^- + \frac{1}{3} k_2 X^3 + k_3 X \Phi \bar{\Phi}, \tag{5.2}$$

where X is a singlet, $\phi^\pm$ have charge ± 1 under $U(1)_m$ and $\Phi, \bar{\Phi}$ are the messengers. The scalar potential in the messenger sector can be obtained

$$\begin{aligned} V_m \;=\; & k_1^2 |X|^2 (|\phi^+|^2 + |\phi^-|^2) + |k_1 \phi^+ \phi^- + k_2 X^2 + k_3 \Phi \bar{\Phi}|^2 \\ & + k_3^2 |X|^2 (|\Phi|^2 + |\bar{\Phi}|^2) + \frac{g_m^2}{2} (|\phi^+|^2 - |\phi^-|^2)^2 \\ & + M_{\phi^\pm}^2 (|\phi^+|^2 + |\phi^-|^2), \end{aligned} \tag{5.3}$$

where we denote the scalar components of the superfields by the same symbols as superfields. The first three terms come from F-term of W_m, the fourth term is $U(1)_m$ D-term and the last term gives the negative soft masses squared for $\phi^\pm$. Minimizing V_m in a certain range of parameters, non-zero VEVs for $\phi^\pm, X, F_X$ are obtained, while $\Phi, \bar{\Phi}$ VEVs are zero. It is crucial for the gauge mediation to have $\langle X \rangle \neq 0, \langle F_X \rangle \neq 0$ as follows.

[‡] Messenger field is defined as the field transforming nontrivially both under the DSB gauge group and the SM gauge group, and having non-zero SUSY mass and SUSY breaking mass.

Recall that SUSY breaking effects in the messenger sector are transmitted to the observable sector through loops by the SM gauge interactions. At one loop, we obtain gaugino masses

$$m_{\lambda_i} = c_i \frac{\alpha_i}{4\pi} \frac{\langle F_X \rangle}{\langle X \rangle}, \tag{5.4}$$

where $c_1 = 5/3, c_2 = c_3 = 1$. Sfermion masses are generated at two loops as

$$\tilde{m}^2 = 2 \left(\frac{\langle F_X \rangle}{\langle X \rangle} \right)^2 \left[C_3 \left(\frac{\alpha_3}{4\pi} \right)^2 + C_2 \left(\frac{\alpha_2}{4\pi} \right)^2 + \frac{5}{3} \left(\frac{Y}{2} \right)^2 \left(\frac{\alpha_1}{4\pi} \right)^2 \right]. \tag{5.5}$$

Here, $C_3 = 4/3$ for color triplets and zero for singlets, $C_2 = 3/4$ for weak doublets and zero for singlets, and Y is the hypercharge.

Although this model is a first phenomenologically viable model, there are some unsatisfactory points. First of all, many fields other than fields in the DSB sector and those in the observable sector are introduced in the messenger sector. This makes the model complicated. Second, SUSY breaking effects are not considered correctly. To be more precise, the superpotentials in the DSB sector, which has a dynamically generated superpotential, are not included in the superpotential of the messenger sector. It seems to be unnatural. To make matters worse, the vacuum discussed above turned out to be the only local minimum [74]. In other words, in the true vacuum, the SM gauge group is broken at the messenger scale, and SUSY breaking is not transmitted to the observable sector. Because of the above problems, the model of Ref. [73] became the driving force to simplifing, modifing the model or finding new models.

Summarizing the structure of the gauge mediated SUSY breaking, the following coupling is needed,

$$W = \lambda X \Phi \bar{\Phi}, \tag{5.6}$$

where X is a moduli field in DSB sector and Φ, $\bar{\Phi}$ are messengers. What is nontrivial when one constructs the model is that both $\langle X \rangle$ and $\langle F_X \rangle$ must be non-zero. If this is realized, SUSY breaking is communicated to the observable sector, and soft SUSY breaking masses are generated as

$$m_{\lambda_i} = c_i N \frac{\alpha_i}{4\pi} \frac{\langle F_X \rangle}{\langle X \rangle}, \tag{5.7}$$

$$\tilde{m}^2 = 2N\left(\frac{\langle F_X\rangle}{\langle X\rangle}\right)^2\left[C_3\left(\frac{\alpha_3}{4\pi}\right)^2 + C_2\left(\frac{\alpha_2}{4\pi}\right)^2 + \frac{5}{3}\left(\frac{Y}{2}\right)^2\left(\frac{\alpha_1}{4\pi}\right)^2\right], \tag{5.8}$$

where N is the flavor number of the messenger fields.

Chapter 6

Effective Messenger Sector Model

In the previous chapter, we pointed out some problems in the model of Dine, Nelson, Nir and Shirman [73]. In this chapter, we present the model whose messenger sector is the effective theory of DSB sector. It is also shown that SUSY breaking is communicated to the observable sector without breaking the SM gauge group.

This chapter is based on the work in collaboration with N. Haba and T. Matsuoka [69].

6.1 Messenger sector as the effective theory of DSB sector

Our model is based on the $SU(3) \times SU(2)$ model [60], which breaks supersymmetry dynamically. This model has the $SU(3) \times SU(2)$ gauge group, a global $U(1)_m$ and a non-anomalous global R-symmetry. In our model this $U(1)_m$ is gauged from the beginning. The representations and charges of the matter fields are summarized in Table 6.1. Note that the singlet superfield $\bar{E}$ is included to cancel $U(1)_m$ anomaly, and that vector-like superfields $f \equiv (q, l), \bar{f} \equiv (\bar{q}, \bar{l})$ are involved. Here we introduce the parameter r which specifies the $U(1)_R$-charges for each superfields and we assume that the sum of R-charge of f and $\bar{f}$ is 0. Under the SM gauge group $G_{\rm SM} \equiv SU(3)_C \times SU(2)_L \times U(1)_Y$, $Q, \bar{U}, \bar{D}, L$ and $\bar{E}$ are neutral, and $q, \bar{q}, l$ and $\bar{l}$

Particle	$SU(3)\times SU(2)$	$U(1)_m$	$U(1)_R$
Q	(3 , 2)	1/6	r
$\bar{U}$	($\bar{3}$, 1)	$-2/3$	$-4-4r$
$\bar{D}$	($\bar{3}$, 1)	1/3	$2+2r$
L	(1 , 2)	$-1/2$	$-3r$
$\bar{E}$	(1 , 1)	1	$8+6r$
f	(1 , 1)	0	r_f
$\bar{f}$	(1 , 1)	0	$r_{\bar{f}}$

Table 6.1: The representations and charges of matter fields in supersymmetry breaking sector

are transformed as

$$q:(\mathbf{3},\mathbf{1},-2/3),\quad \bar{q}:(\bar{\mathbf{3}},\mathbf{1},2/3),\quad l:(\mathbf{1},\mathbf{2},1),\quad \bar{l}:(\mathbf{1},\mathbf{2},-1), \tag{6.1}$$

respectively.

The tree level superpotential consistent with the symmetries is

$$W_{\rm tree}=\lambda_1 Q\bar{D}L+\frac{\kappa'}{M^2}(Q\bar{D}L)f\bar{f}+\frac{\lambda_2'}{M^5}(Q\bar{U}L)\bar{E}(\det Q\bar{Q}), \tag{6.2}$$

where $\bar{Q}=(\bar{U},\bar{D})$. In Eq. (6.2), the first term is a renormalizable term which has been treated in Ref. [60]. The second and the third terms are non-renormalizable terms which will be at most cubic terms of the $SU(3)\times SU(2)$ gauge invariant operators* . The coupling constants λ_2' and κ' are taken to be of order $O(1)$. λ_1 is assumed to be very small so that the theory becomes weakly coupled.

To analyze the model we first consider the case in which the superpotential is absent. Under the condition that $g_3\gg g_2\gg g_1$, where g_3, g_2 and g_1 represent $SU(3), SU(2)$ and $U(1)_m$ gauge coupling constants, respectively, there exist the D-

* Other non-renormalizable terms are highly suppressed by the scale M, which will be fixed to the Planck scale later.

flat directions for $SU(3)$ and $SU(2)$ [60]

$$\langle Q\rangle = \begin{pmatrix} a & 0 \\ 0 & b \\ 0 & 0 \end{pmatrix}, \langle\bar{U}\rangle = \begin{pmatrix} a \\ 0 \\ 0 \end{pmatrix}, \langle\bar{D}\rangle = \begin{pmatrix} 0 \\ b \\ 0 \end{pmatrix}, \langle L\rangle = (0, \sqrt{a^2-b^2}). \tag{6.3}$$

Here a and b are taken as real and positive parameters with $a \geq b$. Gauge symmetries are completely broken along these flat directions and the one-instanton effect induces the non-perturbative superpotential [60]

$$W_{\rm dyn} = \frac{\Lambda_3^7}{\det Q\bar{Q}}, \tag{6.4}$$

where Λ_3 is the scale where $SU(3)$ gauge coupling blows up. Note that we consider the case $\Lambda_3 \gg \Lambda_2$, where Λ_2 is the scale where $SU(2)$ gauge coupling diverges † .

If we turn on the tree level superpotential, the flat directions are lifted. In our case VEVs of the fields are close to those of Ref. [60], $v \sim \Lambda_3/\lambda_1^{1/7} \gg \Lambda_3$. The vacuum energy is $V \sim \lambda_1^2 v^4 > 0$ and then supersymmetry is broken. The moduli space is described in terms of the $SU(3)\times SU(2)$ gauge invariant operators [61] listed in Table 6.2. From Eqs. (6.2) and (6.4) we have the effective superpotential below the scale Λ_3

$$\begin{aligned} W_{\rm eff} &= \lambda_1 X_1 + \frac{\Lambda_3^7}{X_3} + \frac{\kappa'}{M^2} X_1 f\bar{f} + \frac{\lambda_2'}{M^5} X_2 \bar{E} X_3, \\ &= \lambda_1 v^2 Y + \frac{\lambda_1 v^4}{X} + \kappa Y f\bar{f} + \lambda_2 PNX, \end{aligned} \tag{6.5}$$

where in the second equality we rescale the gauge invariant operators as $X_1 = v^2 Y, X_2 = v^2 N, X_3 = v^3 X$ and $\bar{E} = P$. Here we introduce the notations as $\kappa \equiv \kappa'(v/M)^2$ and $\lambda_2 \equiv \lambda_2'(v/M)^5$. Equation (6.5) represents the effective theory of the supersymmetry breaking sector. Since N and P have $U(1)_m$ charges -1 and $+1$, respectively, these superfields correspond to ϕ^- and ϕ^+ in the model of Ref. [73]. Contrary to Ref. [73] we add no superfields to the effective superpotential. In the present model we have $\langle F_Y\rangle \sim \lambda_1 v^2$. Then, the supersymmetry breaking is communicated to the messenger fields f and $\bar{f}$ through $\kappa Y f\bar{f}$ term. It is worth

† If we consider the case $\Lambda_2 \gg \Lambda_3$, supersymmetry is broken due to the quantum deformation of the moduli space [64].

	$U(1)_m$	$U(1)_R$
$X_1 = Q\bar{D}L$	0	2
$X_2 = Q\bar{U}L$	-1	$-4-6r$
$X_3 = \det Q\bar{Q}$	0	-2
$\bar{E}$	1	$8+6r$
$f\bar{f}$	0	0

Table 6.2: The gauge invariant operators which describe the moduli space

noting that the $U(1)_m$ gauge interaction and $\lambda_2 PNX$ term do not play an essential role in communicating the supersymmetry breaking to the messenger fields.

In order to analyze the scalar potential in our model, it is necessary to calculate the effective Kähler potential. Under the condition $\lambda_1 \ll 1$, we can calculate the effective Kähler potential using the procedure given by Poppitz and Randall [76] because the theory is weakly coupled and the gauge symmetries are completely broken. The effective Kähler potential is given by

$$K = 3\left(t + \frac{B}{t}\right), \tag{6.6}$$

where

$$\begin{aligned} t &\equiv (A+\sqrt{A^2-B^3})^{1/3} + (A-\sqrt{A^2-B^3})^{1/3}, \\ A &\equiv \frac{1}{2}(X_1^\dagger X_1 + X_2^\dagger X_2) = \frac{1}{2}v^4(Y^\dagger Y + N^\dagger N), \\ B &\equiv \frac{1}{3}(X_3^\dagger X_3)^{1/2} = \frac{1}{3}v^3(X^\dagger X)^{1/2}. \end{aligned} \tag{6.7}$$

The inverse of the effective Kähler metric is

$$K^{j^* i} = \begin{pmatrix} \left(\frac{t}{v^2}\right)^2 + \frac{2}{t}Y^\dagger Y & \frac{2}{t}Y^\dagger N & \frac{2}{t}Y^\dagger X \\ \frac{2}{t}N^\dagger Y & \left(\frac{t}{v^2}\right)^2 + \frac{2}{t}N^\dagger N & \frac{2}{t}N^\dagger X \\ \frac{2}{t}X^\dagger Y & \frac{2}{t}X^\dagger N & \frac{2t}{v^3}|X| + \frac{2}{t}X^\dagger X \end{pmatrix} \tag{6.8}$$

for $i, j = Y, N$ and X. Since $P(=\bar{E}), f$ and $\bar{f}$ have no $SU(3)$ charge, their components of the effective Kähler potential are assumed to be of canonical form. From

Eqs. (6.5) and (6.8) the scalar potential of the effective theory is given by

$$\begin{aligned}
V &= W_{j^*}K^{j^*i}W_i + \frac{g_1^2}{2}(|P|^2 - |N|^2)^2 + (M_P^2|P|^2 + M_N^2|N|^2), \\
&= \frac{2}{t}\left|\lambda_1 v^2 Y + \kappa Y f\bar{f} + 2\lambda_2 PNX - \frac{\lambda_1 v^4}{X}\right|^2 \\
&\quad + \left(\frac{t}{v^2}\right)^2 \left(\lambda_2^2|X|^2|P|^2 + |\lambda_1 v^2 + \kappa f\bar{f}|^2\right) + \frac{2t}{v^3}|X|\left|\lambda_2 PN - \frac{\lambda_1 v^4}{X^2}\right|^2 \\
&\quad + \lambda_2^2|N|^2|X|^2 + \kappa^2|Y|^2(|f|^2 + |\bar{f}|^2) \\
&\quad + \frac{g_1^2}{2}(|P|^2 - |N|^2)^2 + (M_P^2|P|^2 + M_N^2|N|^2),
\end{aligned} \tag{6.9}$$

where the second last term is $U(1)_m$ D-term, because $U(1)_m$ is gauged and $U(1)_m$ D-flatness condition is not imposed. The last term represents two-loop generated soft supersymmetry breaking mass term. Note that M_P^2 and M_N^2 are negative and of order $O\left(\left(\frac{g_1^2}{16\pi^2}\right)^2 \lambda_1^2 v^2\right)$ [73].

By minimizing the scalar potential (6.9) under the conditions‡

$$\kappa \gg \lambda_1, \quad \frac{g_1^2}{16\pi^2}\lambda_1 \gg \lambda_2, \tag{6.10}$$

we obtain VEVs which are in the vicinity of the $SU(3), SU(2)$ D-flat direction [60], namely

$$\begin{aligned}
&X = v_X + x, \quad |x| \ll v_X, \\
&Y = v_Y + y, \quad |y| \ll v_Y, \\
&|f|, |\bar{f}|, |P|, |N| \ll v,
\end{aligned} \tag{6.11}$$

where $v_X \equiv a^2b^2/v^3, v_Y \equiv a^2\sqrt{a^2-b^2}/v^2$ and x, y represent the fluctuation around v_X, v_Y, respectively. In the minimization it is important that the effective Kähler potential has of the non-canonical form. We can easily derive

$$\langle f\rangle = \langle \bar{f}\rangle = 0 \tag{6.12}$$

from the stationary conditions for f and $\bar{f}$. This shows that the SM gauge group is not broken at the minimum. By solving the minimization conditions with respect

‡ $\kappa \gg \lambda_1$ is a condition to ensure that the mass squared of the messengers is positive. $\frac{g_1^2}{16\pi^2}\lambda_1 \gg \lambda_2$ is a condition to ensure that $U(1)_m$ is broken.

to N and P we obtain

$$|\langle P\rangle| \simeq \frac{1}{g_1}\sqrt{-M_P^2} \sim \frac{g_1}{16\pi^2}\lambda_1 v, \tag{6.13}$$

$$|\langle N\rangle| \simeq \frac{\lambda_2}{\lambda_1}|\langle P\rangle| \sim \frac{g_1}{16\pi^2}\lambda_2 v \tag{6.14}$$

The order of VEVs of the fluctuation x and y is

$$|\langle x\rangle| \sim |\langle y\rangle| \sim O\left(\left(\frac{g_1^2}{16\pi^2}\right)^2 \lambda_2^2 v\right) \ll v_X, v_Y. \tag{6.15}$$

As a consequence, our analysis is found to be self-consistent.

6.2 The observable sector and estimation of the scales

Supersymmetry breaking in the messenger sector is transmitted to gauginos, squarks and sleptons in the observable sector radiatively through the interaction $\kappa Y f\bar{f}$ in Eq.(6.5). At one-loop we can obtain the masses for $SU(3)_C, SU(2)_L$ and $U(1)_Y$ gauginos

$$m_{\lambda_i} \sim \frac{g_i^2}{16\pi^2}\frac{\langle F_Y\rangle}{\langle Y\rangle} \sim \frac{g_i^2}{16\pi^2}\lambda_1 v, \tag{6.16}$$

where g_i stands for the corresponding gauge coupling of the standard model. Taking $m_{\lambda_i} = 10^{2.5\pm0.5}$GeV, we obtain

$$\lambda_1\left(\frac{v}{M_{\rm Planck}}\right) \sim 10^{-13.3\pm0.5}, \tag{6.17}$$

where $\frac{g_i^2}{16\pi^2} \simeq 10^{-2.5}$ is used. On the other hand, the soft supersymmetry breaking masses for squarks and sleptons are induced at two-loop. They are given by

$$m_{\phi_i}^2 \sim \left(\frac{g_i^2}{16\pi^2}\frac{\langle F_Y\rangle}{\langle Y\rangle}\right)^2 \sim \left(\frac{g_i^2}{16\pi^2}\lambda_1 v\right)^2. \tag{6.18}$$

Here we focus on the estimation of the various scales. In the supersymmetry breaking sector we have a scale M which suppresses the non-renormalizable interactions in Eq.(6.2). It is natural to take this scale M as $M_{\rm Planck}$. This implies that we have to take the effects of the gravitational interaction into account in calculating the masses for the gauginos, the squarks, and the sleptons. The scalar mass terms which are induced by gravity come from the D-term

$$\int d^4\theta \left(\frac{Q^\dagger Q}{M_{\rm Planck}^2} + \cdots \right) \Phi_i^\dagger \Phi_i, \tag{6.19}$$

where Φ_i are superfields of the standard model and i denotes flavor index. Therefore, the gravity-induced scalar masses are

$$\begin{aligned} m_{\phi_i}(\text{grav}) &\sim \frac{\langle F \rangle}{M_{\rm Planck}}, \\ &\sim \lambda_1 \left(\frac{v}{M_{\rm Planck}} \right)^2 M_{\rm Planck} \ll 10^{2.5\pm0.5}\text{GeV}, \end{aligned} \tag{6.20}$$

where $\langle F \rangle$ is VEV of the F-term in the supersymmetry breaking sector. The last inequality implies that we consider only the case in which the gravitational effects is negligible. From this inequality we obtain

$$\lambda_1 \left(\frac{v}{M_{\rm Planck}} \right)^2 \ll 10^{-15.8\pm0.5}, \tag{6.21}$$

where $M_{\rm Planck} \simeq 10^{18.3}$GeV is used.

On the other hand, gaugino mass terms which are induced by gravity arise via the term

$$\int d^2\theta \frac{X_1}{M_{\rm Planck}^3} W^\alpha W_\alpha = \int d^2\theta \left(\frac{v}{M_{\rm Planck}} \right)^2 \frac{Y}{M_{\rm Planck}} W^\alpha W_\alpha, \tag{6.22}$$

where W^α is a field strength superfield. Thus, the gravity-induced gaugino masses become

$$m_{\lambda_i}(\text{grav}) \sim \lambda_1 \left(\frac{v}{M_{\rm Planck}} \right)^4 M_{\rm Planck}. \tag{6.23}$$

From Eq. (6.17) and the inequality (6.21) we find that

$$m_{\lambda_i} \gg m_{\lambda_i}(\text{grav}). \tag{6.24}$$

Namely, the gauge-mediated contribution to the gaugino mass is dominant compared with the gravity-mediated contribution.

Taking the condition $\lambda_1 \ll \kappa \sim \left(\frac{v}{M_{\rm Planck}} \right)^2$ into account together with Eqs. (6.17) and (6.21), we obtain the allowed range of parameters

$$10^{-4.4\pm0.2} \ll \frac{v}{M_{\rm Planck}} \ll 10^{-2.5}, \tag{6.25}$$

$$10^{-10.8\pm0.5} \ll \lambda_1 \ll 10^{-8.9\pm0.3}. \tag{6.26}$$

If we take $\lambda_1 \sim 10^{-9.3}$ as an example, various scales in the model are determined as,

$$\begin{aligned} v &\sim 10^{14.3}\text{GeV}, \\ \Lambda &\sim 10^{13.0}\text{GeV}, \\ \sqrt{F} &\sim \sqrt{\lambda_1 v^2} \sim 10^{9.6}\text{GeV}, \\ m &\sim \kappa v \sim 10^{6.3}\text{GeV}, \\ m_{\lambda_i} &\sim 10^{2.5}\text{GeV}, \end{aligned} \tag{6.27}$$

where m means the mass of f and $\bar{f}$. Equation (6.20) also represents the order of the gravitino mass. In the present example the gravitino mass is $\sim 10^{1.5}$ GeV. We note that supersymmetry breaking scale $\sqrt{F}$ turns out to be the intermediate scale between the GUT scale and the soft supersymmetry breaking scale.

6.3 Discussion

We have shown that the messenger sector can be considered as the effective theory of supersymmetry breaking sector. No matter superfields and interactions are added in the messenger sector. All interactions in our messenger sector are derived from the supersymmetry breaking sector. Using this effective theory we have also shown that supersymmetry breaking can be communicated to the observable sector without breaking QCD color.

In the present framework, the essential role of communicating the supersymmetry breaking to the observable sector is played by the $\kappa Y f\bar{f}$ term in the effective superpotential. We do not need to rely on the $U(1)_m$ gauge interaction and also on the PNX term in the superpotential. This situation is in sharp contrast to that in Ref. [73]. In fact, when we do not introduce the field $P(=\bar{E})$ and the $U(1)_m$ gauge symmetry, we can make the model simpler. Even in that case the main point of our results remains unchanged.

There appear various scales in the present model. We have also estimated the gravitational effects in calculating masses for gauginos, squarks, and sleptons. It was found that gauge-mediated contributions are dominant compared with gravity-mediated contributions. As a typical example, we found that gauge symmetry breaking scale (v) in supersymmetry breaking sector is $10^{14.3}$ GeV, the scale of $SU(3)$ dynamics (Λ_3) in $SU(3) \times SU(2)$ model is $10^{13.0}$GeV, supersymmetry breaking scale ($\sqrt{F}$) is $10^{9.6}$ GeV and the mass of messenger fields (κv) is $10^{6.3}$ GeV.

In the observable sector of our model, as discussed in Ref. [73], the masses for gauginos are induced at one-loop through the standard model gauge interaction. On the other hand, the masses for squarks, sleptons are induced at two-loop through the standard model gauge interaction. Therefore, FCNC are naturally suppressed because these masses are proportional to the square of the flavor-blind standard model gauge coupling constants.

Chapter 7

A DGM Model with an Affine QMS

In the previous chapter, we discussed the effective messenger sector model, which simplify the model of Ref. [73] quite naturally. There has also been much progress to simplify the structure of the model along various lines [78]-[81].

The simplest idea of gauge mediated SUSY breaking is "Direct Gauge Mediation"(DGM). In this framework, the SM gauge group is coupled to the DSB sector directly, in other words, the SM gauge group is embedded into the flavor symmetry of the DSB sector, and the fields transforming both under the gauge group in the DSB sector and the SM gauge group communicate SUSY breaking to our world. As mentioned in the chapter 5, main problem in this scenario is that the SM gauge group, especially QCD color gauge group becomes asymptotically non-free and blows up below the GUT scale. Since the model which breaks SUSY dynamically has been increased, it is worth reconsidering this problem. Indeed, there are some DGM models which solve the problem [82]-[91].

However, there arises other problems in these DGM models.

- SUGRA contributions to soft masses dominate over or are comparable to the gauge mediation contributions because SUSY breaking scale is relatively high ($\sim 10^{10}$GeV) [82].
- If there are massive gauge multiplets charged under the SM gauge group (which

is referred to as "gauge messenger"), these give negative contributions to soft scalar masses squared [84, 85].

- If light scalars ($\sim 10^4$ GeV) charged under the SM gauge group exist, these also give negative contributions to soft scalar masses squared through 2-loop RGE [82, 83].

In this chapter, we present a new, simple model of direct gauge mediation of SUSY breaking with an Affine quantum moduli space which avoids the above problems. *

This chapter is based on the work [77].

7.1 A Model

The symmetries of our model are

$$SU(6)_1 \times SU(6)_2 \times [SU(6)], \tag{7.1}$$

where the first two $SU(6)_{1,2}$ are gauge groups and $SU(6)$ in the bracket is a global symmetry. We will later identify the subgroup of this $SU(6)$ with the SM gauge groups.

The field content of our model is as follows.

$$\begin{aligned} X &\sim (\begin{smallmatrix}\square\\\square\end{smallmatrix}, \mathbf{1}; \mathbf{1}), \\ \Sigma &\sim (\square, \bar{\square}; \mathbf{1}), \\ Q &\sim (\bar{\square}, \mathbf{1}; \square), \\ \bar{Q} &\sim (\mathbf{1}, \square; \bar{\square}). \end{aligned} \tag{7.2}$$

We note that this field content consists of two sectors. The first sector includes X only. This model is $SU(6) + \begin{smallmatrix}\square\\\square\end{smallmatrix}$, which has been originally discussed by Csáki, Schmaltz and Skiba [9] and also discussed recently in Ref. [92, 93]. According to Ref. [9], the low energy effective theory of this model has two branches, one is $W_{dyn} = 0$ ("an Affine quantum moduli space") and another is $W_{dyn} \neq 0$. The

* Our model has similar dynamics to the model in Ref. [90].

definition of Affine quantum moduli space is that the moduli space is given by gauge invariant polynomials with no relations among them and $W_{dyn} = 0$. Moreover, 't Hooft anomalies between fundamental fields and gauge invariant composites match at all points in the moduli space (including the origin). Therefore, if the appropriate gauge invariant operators to lift the flat directions are added to the superpotential, then we can expect the model to break SUSY due to confinement such as the model proposed by Intriligator, Seiberg and Shenker (ISS) [94]. We refer to this sector as SUSY breaking sector or DSB sector throughout this paper. We will utilize this branch to construct the model of direct gauge mediation. On the other hand, we will discuss later the case of $W_{dyn} \neq 0$.

The second sector contains $\Sigma, Q, \bar{Q}$. These superfields are necessary to communicate SUSY breaking effects to the observable sector. The field content for each gauge group is $SU(6)_{1,2} + 6(\square + \bar{\square})$, which is the special case of the model in Ref. [95].

Note also that this model is completely chiral, in other words, we cannot add mass terms for any field to the superpotential† .

Here we take the following tree level superpotential

$$W = \lambda_1 \Sigma Q \bar{Q} + \frac{\lambda_2}{M_P} X^4 + \frac{\lambda_3}{M_P^3} \det\Sigma, \tag{7.3}$$

where $\lambda_{1,2,3}$ are the couplings of order unity and M_P is the reduced Planck scale. Although it is possible to add other nonrenormalizable terms to the superpotential, we forbid them by imposing additional symmetries. Note that explicit contraction of indices in X^4 are $X^4 \equiv X_{a_1 b_1 c_1} X_{d_1 e_1 f_1} X_{a_2 b_2 c_2} X_{d_2 e_2 f_2} \epsilon^{a_1 b_1 f_1 a_2 b_2 f_2} \epsilon^{d_1 e_1 c_1 d_2 e_2 c_2}$.

In the presence of the first two terms in the superpotential, there exist some classical flat directions: $v^6 \equiv \det\Sigma, B \equiv Q^6, B \equiv \bar{Q}^6, M \equiv XQ^3$.

Let us first discuss the direction $\langle \Sigma \rangle \neq 0$ (VEVs of other fields are zero) we are interested in, which corresponds to the direction $\det\Sigma \neq 0$;

$$\langle \Sigma \rangle = \mathrm{diag}(v, v, v, v, v, v). \tag{7.4}$$

Along this direction, the gauge group $SU(6)_1 \times SU(6)_2$ is broken to their diagonal

† X^2 vanish identically due to the Bose statistics of the superfield.

$SU(6)_D$ and $Q, \bar{Q}$ become massive since the superpotential includes the mass term $\lambda_1 \langle \Sigma \rangle Q\bar{Q}$. For large v, the low energy effective theory is $SU(6)_D + \begin{smallmatrix}\Box\\\Box\end{smallmatrix}$ + one singlet v. ‡ As mentioned earlier, the low energy dynamics of this model has already been discussed [28] and this model is one of the theories with an Affine quantum moduli space [93]. SUSY gauge theories based on simple groups which have an Affine quantum moduli space are completely classified by Dotti and Manohar [93] as in Table. 7.1. Here notations are as follows, G is the gauge group, ρ the matter representation

	G	ρ	d_M	μ	μ_{adj}	G_*
T1	$SU(2N)$	$\begin{smallmatrix}\Box\\\Box\end{smallmatrix} + \overline{\begin{smallmatrix}\Box\\\Box\end{smallmatrix}}$	$N+1$	$2N-2$	$2N$	$(SU(2))^N$
T2	$SU(6)$	$\begin{smallmatrix}\Box\\\Box\\\Box\end{smallmatrix}$	1	3	6	$SU(3)\times SU(3)$
T3	$Sp(2N), N\geq 2$	$\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$	$N-1$	$N-1$	$N+1$	$(SU(2))^N$
T4	$SO(N), N\geq 5$	$(N-4)\,\Box$	$\frac{1}{2}(N-4)(N-3)$	$N-4$	$N-2$	$SU(2)\times SU(2)$
T5	$SO(12)$	$2S$	7	8	10	$(SU(2))^3$
T6	$SO(14)$	S	1	8	12	$G_2 \times G_2$
T7	$SU(2)$	$\Box\Box\Box$	1	5	2	Z_3
T8	$SU(8)$	$\begin{smallmatrix}\Box\\\Box\\\Box\\\Box\end{smallmatrix}$	7	10	8	$(Z_2)^6$
T9	$Sp(8)$	$\begin{smallmatrix}\Box\\\Box\\\Box\\\Box\end{smallmatrix}$	6	7	5	$(Z_2)^6$
T10	$SO(N), N\geq 5$	$\Box\Box$	$N-1$	$N+2$	$N-2$	$(Z_2)^k$
T11	$SO(16)$	S	8	16	14	$(Z_2)^8$
S1	$SU(5)$	$\overline{\Box} + \begin{smallmatrix}\Box\\\Box\end{smallmatrix}$	0	2	5	$SU(5)$
S2	$SO(10)$	S	0	2	8	$SO(10)$

Table 7.1: All theories with an Affine quantum moduli space

(S denotes the spinor representation), d_M the number of gauge invariant composites, μ, μ_{adj} Dynkin index of matter representations and adjoint, for respectively, G_* the unbroken gauge group at D-flat directions where G is maximally broken. T1-T6 have $\mu < \mu_{adj}$, T7-T11 have $\mu > \mu_{adj}$. S1 and S2 have no gauge invariant composites.

‡ We use the same notation $\langle \Sigma \rangle$ for a singlet superfield.

In the present case, $U(1)_R$ anomaly of the fermionic component of X, gaugino and that of the fermionic component of X^4 are saturated. This implies that the theory is in the confining phase below the scale Λ_L (the strong coupling scale of $SU(6) + \begin{smallmatrix}\square\\\square\end{smallmatrix}$) and the low energy effective theory should be described in terms of the composite X^4. The form of the dynamically generated superpotential due to gaugino condensation from the subgroup $SU(3) \times SU(3)$ in $SU(6)$ is

$$W_{dyn} = (\omega^r - \omega^s)\Lambda_3^3, \tag{7.5}$$

where ω represents the cube root of unity $(r, s = 1, 2, 3)$ and Λ_3 means the strong coupling scale of pure $SU(3)$ super Yang-Mills theory. Naively, one expects that the superpotential is the sum $W = \Lambda_L^3(\omega^r + \omega^s)$, so the relative sign in Eq. (7.5) is non trivial and essential for $W_{dyn} = 0$.

We would like to digress here for a moment and outline the argument of Dotti and Manohar that how Eq. (7.5) is derived. The VEV $\langle X_{[123]}\rangle = v_1, \langle X_{[456]}\rangle = v_2$ $(|v_1| = |v_2|$ for D-flatness) breaks $SU(6) \to SU(3)_1 \times SU(3)_2$. One can easily see that the effective theory is pure $SU(3) \times SU(3)$ gauge theory (with singlet X^4) by counting degrees of freedom through Higgs mechanism $(20 - (35 - 8 - 8) = 1)$. The 1-loop matching of the gauge coupling between the high energy and the low energy is $\Lambda_{Li}^3 = \Lambda_{SU(6)}^5/v_i^2$. One can interchange the two $SU(3)$ groups by acting with the $SU(6)$ matrix

$$U = \begin{pmatrix} 0 & 0 & 0 & i & 0 & 0 \\ 0 & 0 & 0 & 0 & i & 0 \\ 0 & 0 & 0 & 0 & 0 & i \\ i & 0 & 0 & 0 & 0 & 0 \\ 0 & i & 0 & 0 & 0 & 0 \\ 0 & 0 & i & 0 & 0 & 0 \end{pmatrix}, \tag{7.6}$$

which maps $v_1 \to -iv_2, v_2 \to -iv_1$. Under this U, one finds that $\Lambda_{L1,2}^3 \to -\Lambda_{L2,1}^3$, therefore, the difference of each $SU(3)$ superpotential is $SU(6)$ invariant[§] .

[§] Also, for other theories with $\mu < \mu_{adj}$, these theories have multi branches, and $W_{dyn} = 0$ is obtained from the cancellation of gaugino condensations for each unbroken subgroup. On the other hand, in the theories with $\mu > \mu_{adj}$, W_{dyn} must be zero because the general

Let us turn back to the model of the direct gauge mediation. What is important here is that this model has $W_{dyn} = 0$ branch. In this situation, the effective superpotential becomes

$$W_{eff} = \frac{\lambda_2}{M_P} u, \quad u \equiv X^4. \tag{7.7}$$

Taking into account that the effective Kähler potential is

$$K_{eff} \sim u^\dagger u / |\Lambda_L|^6, \tag{7.8}$$

we obtain the following scalar potential.

$$V_{eff} = \left(\frac{\partial^2 K_{eff}}{\partial u^\dagger \partial u} \right)^{-1} \left| \frac{\partial W_{eff}}{\partial u} \right|^2 = \frac{\lambda_2^2}{M_P^2} \Lambda_L^6. \tag{7.9}$$

At the first glance, one may think that this model is the plateau model [84, 85, 91] since the scalar potential is flat. However, we have to recall that Λ_L depends on the scale v through 1-loop matching of the gauge couplings:

$$\left(\frac{\Lambda_1}{v} \right)^9 \left(\frac{\Lambda_2}{v} \right)^{12} = \left(\frac{\Lambda_L}{\lambda_1 v} \right)^{15}, \tag{7.10}$$

where $\Lambda_{1,2}$ denotes the dynamical scale of $SU(6)_1 + \begin{smallmatrix}\square\\\square\end{smallmatrix} + 6(\square + \bar{\square})$ and $SU(6)_2 + 6(\square + \bar{\square})$, respectively. Then, the scalar potential (7.9) becomes

$$V_{eff} = \frac{\lambda_2^2}{M_P^2} \left(\frac{\lambda_1^5 \Lambda_1^3 \Lambda_2^4}{v^2} \right)^{\frac{6}{5}} = \frac{\lambda_2^2}{M_P^2} \left(\frac{\lambda_1^5 \Lambda^7}{v^2} \right)^{\frac{6}{5}}, \tag{7.11}$$

where Λ is defined as $\Lambda^7 \equiv \Lambda_1^3 \Lambda_2^4$ for simplicity. This results in runaway behavior but the term $\det\Sigma$ in Eq. (7.3) stabilizes the scalar potential.

Indeed, by minimizing the scalar potential

$$V_{eff} = \frac{\lambda_2^2}{M_P^2} \left(\frac{\lambda_1^5 \Lambda^7}{v^2} \right)^{\frac{6}{5}} + \frac{36 \lambda_3^2}{M_P^6} v^{10}, \tag{7.12}$$

form of the superpotential consistent with the R symmetry is a sum of the term like $W = \Lambda^{(\mu - 3\mu_{adj})/(\mu - \mu_{adj})} \Pi_i \phi_i^{2\mu_i/(\mu - \mu_{adj})}$, where ϕ_i are elementary fields with index μ_i. The product of fields ϕ_i must be gauge and flavor invariant. For asymptotically free theories, $\mu < 3\mu_{adj}$, the power of Λ is negative if $\mu > \mu_{adj}$. In this case, W diverges in the weak coupling limit $\phi_i \to \infty, \Lambda \to 0$. Therefore, W must vanish.

we obtain

$$v \sim (\Lambda^{21} M_P^{10})^{\frac{1}{31}}, \quad F_v \sim \left(\frac{\Lambda^{105}}{M_P^{43}}\right)^{\frac{1}{31}}, \quad V_0 \sim \left(\frac{\Lambda^{210}}{M_P^{86}}\right)^{\frac{1}{31}}, \quad \frac{F_v}{v} \sim \left(\frac{\Lambda^{84}}{M_P^{53}}\right)^{\frac{1}{31}}. \tag{7.13}$$

Since the vacuum energy is non-zero, SUSY is certainly broken. This breaking effect is communicated to the observable sector as follows. Upon identifying the $SU(5)$ subgroup of the flavor group $SU(6)$ with usual GUT gauge group which includes the SM gauge groups, Q and $\bar{Q}$ behave as $\mathbf{5}+\bar{\mathbf{5}}$ messenger fields of which SUSY mass is $\lambda_1 v$ and SUSY breaking (mass)^2 is $\lambda_1 F_v$. They communicate SUSY breaking to the soft masses through loop diagrams of $Q, \bar{Q}$ in the usual way [72, 73]. Note also that since the original matter content is completely chiral, these vectorlike messengers are dynamically generated as a result of symmetry breaking. Furthermore, since Σ is a singlet for $SU(5)$, there is no gauge messengers and no light scalars charged under the SM gauge group, which gives a negative contribution to the soft (mass)^2.

Requiring $F_v/v \sim 10^4$ GeV to obtain the soft masses of order $10^{2\sim 3}$ GeV, we find

$$\begin{aligned} \Lambda &\sim 7\times 10^{12}\text{GeV}, \quad \Lambda_L \sim 1\times 10^{12}\text{GeV}, \\ v &\sim 3\times 10^{14}\text{GeV}, \quad \sqrt{F_v} \sim 1\times 10^{9}\text{GeV}, \end{aligned} \tag{7.14}$$

where we used $M_P = 2\times 10^{18}$ GeV.

We give some comments on the above scales in order. First of all, since $F_v < v^2$, the determinant of the mass squared matrix for messenger fields is positive, so the SM gauge groups are not broken at the minimum. The problem in Ref. [73] is avoided. Second, the messenger scale v is close to the GUT scale, so it is possible to preserve the perturbative unification in spite of six flavors of messengers. Third, one may worry about that the SUGRA contribution to the soft masses is comparable to or dominates over the gauge mediation contribution because SUSY breaking scale $\sqrt{F_v}$ is relatively large. If we require that the gravitino mass $m_{3/2}$, which is the typical scale of the SUGRA contribution, is less than 10 percent of the gluino mass (such that RGE induced squark mass squareds have degeneracy at 1 percent level),

$$m_{3/2} = \frac{F_v}{\sqrt{3}M_P} < 0.1\times 6\times \frac{\alpha_s}{4\pi}\frac{F_v}{v}, \tag{7.15}$$

then we find

$$v < 0.1 \times 6 \times \frac{\alpha_s}{4\pi}\sqrt{3}M_P \sim 2 \times 10^{16}\mathrm{GeV}. \tag{7.16}$$

In our model, the above requirement is clearly satisfied, so the SUGRA contribution is suppressed enough.

Recall here that there still remain classical flat directions $B, \bar{B}$ and M. We have to argue whether or not these directions are lifted quantum mechanically.

Along the B direction, $SU(6)_1$ and $SU(6)$ are completely broken. Then Σ and $\bar{Q}$ become massive, hence the low energy effective theory is $SU(6)_2 +$ singlets. The dynamical superpotential is

$$W_{dyn} = \Lambda_L^3 = \Lambda_2^2 B^{1/6}, \tag{7.17}$$

where we use the scale matching $\Lambda_L^3 = \Lambda_2^2 B^{1/6}$. This leads to non-zero constant vacuum energy. At one loop, the correction to Kähler potential makes the scalar potential stabilized near the origin [84, 85, 96].

Along the $\bar{B}$ direction, $SU(6)_2$ and $SU(6)$ are completely broken. Then Σ and Q become massive, hence the low energy effective theory is $SU(6)_1 + \begin{smallmatrix}\square\\\square\end{smallmatrix} +$ a singlet. The effective superpotential becomes ¶

$$W_{eff} = \frac{\lambda_2}{M_P} u \tag{7.18}$$

Using the canonical Kähler potential for u and the scale matching, we obtain the scalar potential

$$V_{eff} = \frac{\Lambda_L^6}{M_P^2} = \frac{\lambda_2^2}{M_P^2}\Lambda_1^{18/5}\bar{B}^{2/5}. \tag{7.19}$$

Clearly, this stabilizes $\bar{B}$ direction.

Along the M direction, $SU(6)_1$ and $SU(6)$ are completely broken. The low energy effective theory is $SU(6)_2 +$ singlets. This is the same effective theory along the B direction. Therefore, the M direction is also lifted.

We now briefly discuss the drawback of our model. In the case with $W_{dyn} \neq 0$, this W_{dyn} becomes a runaway potential for u, v. Using the matching condition

$$\left(\frac{\Lambda_1}{x}\right)^9 \left(\frac{\Lambda_2}{x}\right)^9 = \left(\frac{\Lambda_L}{x}\right)^{30}, \tag{7.20}$$

¶ We also assume here that the theory is in the $W_{dyn} = 0$ branch.

where x means a VEV of X, we obtain

$$W_{dyn} \sim \frac{\Lambda_L^5}{x^2} \sim \frac{\lambda_1^5 \Lambda_1^3 \Lambda_2^4}{u^{1/2} v^2}. \tag{7.21}$$

Since the effective Kähler potential is

$$K_{eff} \sim u^\dagger u / |\Lambda_L|^6 + v^\dagger v, \tag{7.22}$$

the effective scalar potential becomes

$$V_{eff} \sim \Lambda^6 \left(\frac{\lambda_2}{M_P} - \frac{\lambda_1^5 \Lambda^7}{2u^{3/2} v^2} \right)^2 + \left(\frac{6\lambda_3 v^5}{M_P^3} - \frac{2\lambda_1^5 \Lambda^7}{u^{1/2} v^3} \right)^2 . \tag{7.23}$$

Then one can easily see that SUSY vacuum exists ($u \sim (M_P \Lambda^{21})^{2/11}$, $v \sim (M_P^8 \Lambda^{14})^{1/22}$). Therefore, our model does not work in this case. The same situation occurs if we apply the model with $\mu < \mu_{adj}$ in Ref. [93] instead of $SU(6)+\begin{array}{|c|}\hline \\ \hline \\ \hline \end{array}$ model. On the other hand, if we apply the model with $\mu > \mu_{adj}$ in Ref. [93], the situation is different because $W_{dyn} \neq 0$ branch does not exist as mentioned before[‖] . For instance, it is an interesting challenge to construct a model with direct gauge mediation by using ISS model [94] as SUSY breaking sector.

7.2 Discussion

In this chapter, we have presented a new, simple model of direct gauge mediation of SUSY breaking with an Affine quantum moduli space. SUSY breaking is due to confinement, and is communicated to the observable sector by the SM gauge interactions. This model has no gauge messengers and no light scalars charged under the SM gauge group. These give a negative contribution to soft masses. Large VEV at the minimum is obtained by balancing the runaway potential and dimension six nonrenormalizable term. This makes it easy to preserve the perturbative unification. Enough suppression of the SUGRA contribution to soft masses can be shown. Moreover, we have seen that phenomenologically viable results are obtained. The

‖ Shirman [90] uses the model that has no flat directions as SUSY breaking sector in the first example. This model is also the case in which W_{dyn} necessarily vanishes.

drawback is that if the SUSY breaking model has $W_{dyn} \neq 0$ branch, this dynamically generated superpotential becomes runaway potential, then SUSY is unbroken. This problem is not so serious because this is model dependent.

Chapter 8

Conclusion

We have discussed the nonperturbative dynamics in $\mathcal{N}=1$ supersymmetric gauge theories. In the former part of this thesis, we focused on their theoretical aspects. Supersymmetric gauge theory is tractable compared to non SUSY theory thanks to holomorphy. The form of the superpotential can be determined analytically and exactly by symmetry and holomorphy. Since the superpotential is crucial to analyze the low energy behavior of the theory, we can extract informations about the strong coupling dynamics. Especially, the confining phase can be examined systematically, and exhibits rich phenomena for example, gaugino condensation, chiral symmetry breaking and confinement, which seems to be generic in $\mathcal{N}=1$ SUSY gauge theories. In this thesis, I have studied t he confining phase of SUSY $SO(12)$ gauge theory with one spinor and $N_f \leq 7$ vectors, which is motivated by the search for Spin group duality. Spin group duality has interesting features which are not included in Seiberg duality. From this point, I think that clarifying spin group duality may provide a clue to understand $\mathcal{N}=1$ duality. I have completely determined the low energy superpotentials, and it turned out that the confining phase structure is analogous to SUSY $SU(N_c)$ QCD, that is, no vacuum (ADS-type superpotentials), confinement with chiral symmetry breaking (quantum deformation of the moduli space), confinement without chiral symmetry breaking (s-confinement). Moreover, from this analysis I have conjectured that the dual gauge group must include $SU(N_f-6)$ as a subgroup. Although my analysis certainly gives a hint to find spin group duality, we need more insight and informations.

As another remarkable feature, a large number of $\mathcal{N}=1$ SUSY gauge theories with dual descriptions have been conjectured, which are equivalent to the original theory in the infrared limit, more precisely, which flow to the same infrared fixed point of the original theory. This also opens up an avenue to study the strong coupling physics because the strong coupling dynamics of one theory is described by the weak coupling dynamics of another theory. However, there are many uncertain points in the duality relations. For instance, we cannot in practice calculate the Green function of the strong coupling theory by calculating the Green function of the weak coupling theory because the explicit transformations of fields in both theories, such as two dimensional theories, are unknown, we know only the correspondence of their fields. Moreover, there is no systematic method to search for duality. In other words, it can be said that we know little about general properties of duality. Thus, there are many problems in duality, and we hope to clarify the origin of duality in future. In this thesis, for the second point mentioned above, I and S. Kitakado have found that a certain class of dualities ($Sp(2N_c)$ gauge theory with arbitrary matters and $SO(2N_c)$ gauge theory with vectors and its higher rank representations.) are interrelated through the negative dimensional group trick. This trick is useful and powerful in finding a new duality because if we know the $SO(Sp)$ group duality, we can easily obtain a new $Sp(SO)$ duality by a simple manipulation. This gives some informations on general structure of $\mathcal{N}=1$ SUSY duality.

In the latter part, phenomenological aspects of $\mathcal{N}=1$ SUSY gauge theories have been discussed, especially focusing on the gauge mediated SUSY breaking.

As our understanding of the low energy effective theory has advanced, a large number of models which breaks SUSY has been proposed, and its phenomenological applications have attracted much interest. As mentioned earlier, gauge mediated SUSY breaking has phenomenological advantages, which solve supersymmetric flavor problems naturally, compared to SUGRA mediated SUSY breaking. In this context, Dine *et al.* constructed the first interesting model, but this model does not seem to describe nature because of the complexity of the model and the color breaking minimum problem.

We have presented two models in this thesis. One is the model which considers the messenger sector as the effective theory of the DSB sector, which has been constructed by N. Haba, T. Matsuoka and myself. This model simplified the complicated structure of the DNNS model naturally. We have shown that in the true vacuum, the color and electroweak symmetries are not broken at the scale higher than the weak scale. Moreover, we have estimated the various mass scales, which are phenomenologically desirable. This model has provided one direction to construct phenomenological viable models.

I have constructed a new and simple model of the DGM with an Affine quantum moduli space. This model has avoided some phenomenologically dangerous problems (the SUGRA mediation dominance over the gauge mediation, the negative sfermion mass squared problem) in earlier DGM models. Furthermore, the desirable mass scales have been obtained. Although it turned out that the minimum we are interested in is the only local minimum, it is not serious because this is a model dependent problem.

Although many models of gauge mediation have been proposed, it seems that we do not know well what type of models lead to what pattern of sparticle spectrum. In order to see which of the models describes the nature well, we have to investigate sparticle spectroscopy. Therefore, it will be interesting theoretically and phenomenologically if we can specify the model (the gauge group, matter content, messenger matter content etc.) from the sparticle analysis.

Finally, we comment on future directions. It will be most challenging to apply the methods and knowledge developed in $\mathcal{N} = 1$ supersymmetric gauge theories to non SUSY theories, especially to QCD. Further insights for color confinement may be gained through supersymmetry.

Instead of insisting on field theory, it will be interesting to study field theory dynamics using string theories or more generally using branes. Possibly, the information on the field theoretical dynamics may be interpreted in terms of the geometrical informations in their framework, "We may be able to see the dynamics".

Anyway, thanks to recent developments in supersymmetric gauge theories, we can now attack the strong coupling dynamics without hesitation. It seems (at least to me) that SUSY is indispensable to understand nature!

Acknowledgements

The author wishes to express his sincere gratitude to S. Kitakado and T. Matsuoka for stimulating discussions, various suggestions and careful reading of this thesis, and to N. Haba, K. Ito and J. Nishimura, who are collabolators, for many fruitful discussions. He is also grateful to H. Ikemori and K. Ito for valuable comments and careful reading of this thesis. Without their heartly encouragement, this work would not have been completed.

Finally, he expresses his gratitude for all members of the laboratory of the elementary particle physics (E-ken) in Nagoya University for their hospitality.

Appendix A

Superspace formalism

In this appendix, we would like to summarize the superspace formalism since it is used throughout this thesis, and for completeness of our discussion. For more details, see Refs. [97]-[101].

$\mathcal{N}=1$ SUSY algebra is generated by a momentum operator P_μ which are the generator of translation, and Weyl spinor operators $Q_\alpha, \bar{Q}_{\dot{\alpha}}$ which are the generator of SUSY* .

$$\begin{aligned} [Q_\alpha, P_\mu] &= \left[\bar{Q}_{\dot{\alpha}}, P_\mu\right] = [P_\mu, P_\nu] = 0, && \text{(A.1)} \\ \{Q_\alpha, Q_\beta\} &= \{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = 0, && \text{(A.2)} \\ \{Q_\alpha, \bar{Q}_{\dot{\beta}}\} &= 2\sigma^\mu_{\alpha\dot{\beta}} P_\mu, && \text{(A.3)} \end{aligned}$$

where $[\quad,\quad], \{\quad,\quad\}$ represent commutator, anticommutator, respectively. $\sigma^\mu_{\alpha\beta} = (\mathbf{1}, \sigma)$, here $\mathbf{1}$ is two by two unit matrix, and σ's are Pauli matrices.

The corresponding supertransformation group is

$$G(x, \theta, \bar{\theta}) = \exp\left(i(\theta Q + \bar{\theta}\bar{Q} - x^\mu P_\mu)\right), \tag{A.4}$$

$\theta_\alpha, \bar{\theta}_{\dot{\alpha}}$ are Grassmannian parameters.

Using Hausdorff formula $e^A e^B = \exp(A + B + \frac{1}{2}[A, B] + \cdots)$, and calculating the group product $G(x, \theta, \bar{\theta})G(a^\mu, \xi, \bar{\xi})$, we find

$$G(x, \theta, \bar{\theta})G(a^\mu, \xi, \bar{\xi}) = G(x^\mu + a^\mu - i\xi\sigma^\mu\bar{\theta} + i\theta\sigma^\mu\bar{\xi}, \theta + \xi, \bar{\theta} + \bar{\xi}). \tag{A.5}$$

* In this appendix, we use $g_{\mu\nu} = (1, -1, -1, -1)$.

We can read off the following transformations in parameter space.

$$(x, \theta, \bar{\theta}) \to (x^\mu + a^\mu - i\xi\sigma^\mu\bar{\theta} + i\theta\sigma^\mu\bar{\xi}, \theta + \xi, \bar{\theta} + \bar{\xi}). \tag{A.6}$$

Then, the transformations of the function $F(x, \theta, \bar{\theta})$ (referred to as "superfield") of $(x, \theta, \bar{\theta})$ (referred to as "superspace"),

$$F(x, \theta, \bar{\theta}) \to \exp\left(i(\xi Q + \bar{\xi}\bar{Q} - a^\mu P_\mu)\right) F(x, \theta, \bar{\theta}) \tag{A.7}$$

are generated by the following differential operators,

$$\begin{aligned} P_\mu &= i\partial_\mu, \\ iQ_\alpha &= \frac{\partial}{\partial\theta^\alpha} - i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu, \\ i\bar{Q}_{\dot{\alpha}} &= -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} + i\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu. \end{aligned} \tag{A.8}$$

Moreover, since the supercovariant derivatives on the superspace

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu, \tag{A.9}$$

$$\bar{D}_{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} - i\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu \tag{A.10}$$

generate the following SUSY algebra

$$\{D_\alpha, \bar{D}_{\dot{\alpha}}\} = 2i\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu, \tag{A.11}$$

$$\{D_\alpha, D_\beta\} = \{\bar{D}_{\dot{\alpha}}, \bar{D}_{\dot{\beta}}\} = 0, \tag{A.12}$$

$$\{D_\alpha, Q_{\dot{\beta}}\} = \{D_\alpha, \bar{Q}_{\dot{\beta}}\} = \{\bar{D}_{\dot{\alpha}}, Q_{\dot{\beta}}\} = \{\bar{D}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = 0, \tag{A.13}$$

and anticommute with $Q, \bar{Q}$. We obtain irreducible representations of SUSY algebra by imposing the constraint on the superfield using the supercovariant derivatives.

Chiral superfield, which is one of the irreducible representations, is defined by the condition

$$\bar{D}_{\dot{\alpha}}\Phi(x, \theta, \bar{\theta}) = 0. \tag{A.14}$$

Using the variable $y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta}$ we can expand the superfield Φ in $\theta, \bar{\theta}$ as follows.

$$\Phi = \phi(y) + \sqrt{2}\theta\psi(y) + \theta^2 F(y) \tag{A.15}$$

$$\begin{aligned} &= \phi(x) + \sqrt{2}\theta\psi(x) + \theta^2 F(x) + i\partial_\mu\phi(x)\theta\sigma^\mu\bar{\theta} - \frac{i}{\sqrt{2}}\theta^2\partial_\mu\psi(x)\sigma^\mu\bar{\theta} \\ &\quad -\frac{1}{4}\partial_\mu\partial^\mu\phi(x)\theta^2\bar{\theta}^2, \end{aligned} \tag{A.16}$$

where ϕ, F are complex scalar fields, ψ is a left-handed Weyl spinor field.

If we make an infinitesimal supertransformation

$$\delta\Phi = i(\xi Q + \bar{\xi}\bar{Q})\Phi, \tag{A.17}$$

from Eq. (A.8) we have

$$\begin{aligned} \delta\Phi &= \xi^\alpha \left(\frac{\partial}{\partial\theta^\alpha} - i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu \right) \Phi + \bar{\xi}^{\dot{\alpha}} \left(-\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} + i\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu \right) \Phi & \text{(A.18)} \\ &= \sqrt{2}\xi\psi + 2\xi\theta F + 2i\partial_\mu\phi\theta\sigma^\mu\bar{\xi} + \frac{i}{\sqrt{2}}\theta^2\partial_\mu\psi\sigma^\mu\bar{\xi} + \cdots, & \text{(A.19)} \end{aligned}$$

and from Eq. (A.15)

$$\delta\Phi = \delta\phi + \sqrt{2}\theta\delta\psi + \theta^2\delta F + \cdots. \tag{A.20}$$

Comparing the above two expressions, the transformation laws for the scalar, fermion, and the auxiliary field (referred to as F-term) are obtained, and these become irreducible representations.

$$\begin{aligned} \delta\phi &= \sqrt{2}\xi\psi, & \text{(A.21)} \\ \delta\psi &= \sqrt{2}\xi F + i\sqrt{2}\partial_\mu\phi\sigma^\mu\bar{\xi}, & \text{(A.22)} \\ \delta F &= \frac{i}{\sqrt{2}}\partial_\mu\psi\sigma^\mu\bar{\xi}. & \text{(A.23)} \end{aligned}$$

Since supertransformation of F-term leads to the total derivative, this is invariant under SUSY. It is also clear by definition that the product of the chiral superfields is the chiral superfield.

On the other hand, the superfield satisfying

$$D_\alpha\Phi^\dagger = 0 \tag{A.24}$$

is referred to as antichiral superfield, which is complex conjugate of the chiral superfield.

Next, the vector superfield is defined by

$$V = V^\dagger. \tag{A.25}$$

Explicit expansion in $\theta, \bar{\theta}$ is

$$\begin{aligned}
V(x,\theta,\bar{\theta}) &= C(x) + i\theta\chi(x) - i\bar{\theta}\bar{\chi}(x) + \frac{1}{2}i\theta^2[M(x) + iN(x)] \\
&\quad -\frac{1}{2}i\bar{\theta}^2[M(x) - iN(x)] + \theta\sigma^\mu\bar{\theta}V_\mu(x) \\
&\quad i\theta^2\bar{\theta}[\bar{\lambda}(x) + \frac{i}{2}\bar{\sigma}^\mu\partial_\mu\chi(x)] - i\bar{\theta}^2\theta[\lambda(x) + \frac{i}{2}\sigma^\mu\partial_\mu\bar{\chi}(x)] \\
&\quad \frac{1}{2}\theta^2\bar{\theta}^2[D - \frac{1}{2}\partial_\mu\partial^\mu C],
\end{aligned} \tag{A.26}$$

where C, M, N, D are real scalar fields, χ, λ are Weyl spinor field, and V_μ is a vector field. If we consider particularly as a vector superfield

$$\begin{aligned}
i(\Lambda - \Lambda^\dagger) &= i(\phi - \phi^\dagger) + i\sqrt{2}(\theta\psi - \bar{\theta}\bar{\psi}) + i\theta^2 F - i\bar{\theta}^2 F^\dagger \\
&\quad -\theta\sigma^\mu\bar{\theta}\partial_\mu(\phi + \phi^\dagger) - \frac{1}{\sqrt{2}}\theta^2\bar{\theta}\bar{\sigma}^\mu\partial_\mu\psi + \frac{1}{\sqrt{2}}\bar{\theta}^2\theta\sigma^\mu\partial_\mu\bar{\psi} \\
&\quad -\frac{i}{4}\theta^2\bar{\theta}^2\partial_\mu\partial^\mu(\phi - \phi^\dagger),
\end{aligned} \tag{A.27}$$

where Λ is a chiral superfield, and $\Lambda^\dagger$ is an antichiral superfield. Comparing with Eq. (A.26), we can derive

$$\begin{aligned}
C &= i(\phi - \phi^\dagger), \\
\chi &= \sqrt{2}\psi, \\
\frac{1}{2}(M + iN) &= F, \\
V_\mu &= -\partial_\mu(\phi + \phi^\dagger), \\
\lambda &= 0, \\
D &= 0.
\end{aligned} \tag{A.28}$$

It turns out that Eq. (A.27) corresponds to $U(1)$ gauge transformation from the fourth equality in Eq. (A.28). In fact, the vector superfield is transformed under $U(1)$ gauge transformation as follows,

$$V(x,\theta,\bar{\theta}) \to V'(x,\theta,\bar{\theta}) = V(x,\theta,\bar{\theta}) + i[\Lambda(x,\theta,\bar{\theta}) - \Lambda^\dagger(x,\theta,\bar{\theta})]. \tag{A.29}$$

C, M, N, χ are not physical degrees of freedom because these can be eliminated by the degrees of freedom of the gauge transformation. Wess-Zumino gauge, which eliminates them, is often used.

$$\begin{aligned} V_{WZ}(x,\theta,\bar\theta) &= \theta\sigma^\mu\bar\theta V_\mu(x) + i\theta^2\bar\theta\bar\lambda(x) - i\bar\theta^2\theta\lambda(x) + \frac{1}{2}\theta^2\bar\theta^2 D(x), \\ V_{WZ}^2(x,\theta,\bar\theta) &= -(\theta\sigma^\mu\bar\theta)(\bar\theta\bar\sigma^\nu\theta)V_\mu V_\nu = \frac{1}{2}\theta^2\bar\theta^2 V^\mu V_\mu, \\ V_{WZ}^3(x,\theta,\bar\theta) &= 0. \end{aligned} \tag{A.30}$$

Supertransforming the vector superfield,

$$\delta_\xi V = i(\xi Q + \bar\xi\bar Q)V. \tag{A.31}$$

The transformation laws of components are from Eqs. (A.8, A.30, A.31),

$$\delta_\xi \lambda_\alpha = -iD(x)\xi_\alpha - \frac{1}{2}(\sigma^\mu\bar\sigma^\nu)_\alpha^\beta \xi_\beta F_{\mu\nu}(x), \tag{A.32}$$

$$\delta_\xi V^\mu = i(\xi\sigma^\mu\bar\lambda(x) - \lambda(x)\sigma^\mu\bar\xi) - \partial^\mu(\xi\chi(x) + \bar\xi\bar\chi(x)), \tag{A.33}$$

$$\delta_\xi D = \partial_\mu(-\xi\sigma^\mu\bar\lambda(x) + \lambda(x)\sigma^\mu\bar\xi), \tag{A.34}$$

where $F_{\mu\nu} \equiv \partial_\mu V_\nu - \partial_\nu V_\mu$. Since

$$\delta_\xi F^{\mu\nu} = i\partial^\mu(\xi\sigma^\nu\bar\lambda - \lambda\sigma^\nu\xi) - i\partial^\nu(\xi\sigma^\mu\bar\lambda - \lambda\sigma^\mu\bar\xi), \tag{A.35}$$

$\lambda, \bar\lambda, V_{\mu\nu}, D$ make irreducible representation. In the vector superfield, as the coefficient ($D(x)$, referred to as D-term) of $\theta^2\bar\theta^2$ becomes the total derivative, D-term is supersymmetric.

The field strength superfield is defined as follows,

$$W_\alpha \equiv \frac{1}{4}\bar D^2 D_\alpha V \tag{A.36}$$

$$= i\lambda_\alpha(y) - [\delta_\alpha^\beta D(y) + \frac{i}{2}(\sigma^\mu\bar\sigma^\nu)_\alpha^\beta F_{\mu\nu}(y)]\theta_\beta + \theta^2\sigma^\mu_{\alpha\dot\alpha}\partial_\mu\bar\lambda^{\dot\alpha}(y). \tag{A.37}$$

One can easily derive $\bar D W_\alpha = 0$ because $\bar D^3 = 0$ due to the Grassmannian property of $\bar D$. In other words, W_α is a chiral superfield. Note that W_α is a spinor valued field. The gauge kinetic term can be written as

$$\int d^2\theta \frac{1}{4}W^\alpha W_\alpha + h.c. = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\lambda\sigma^\mu\partial_\mu\bar\lambda - \frac{1}{4}F^{\mu\nu}({}^*F_{\mu\nu}) + \frac{1}{2}D^2, \tag{A.38}$$

$${}^*F_{\mu\nu} \equiv \frac{i}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}, \tag{A.39}$$

where the first and the second terms are the kinetic term for the gauge field and gaugino, respectively. The third one is the anomaly term, and the last one is the term for the auxiliary fields. ${}^*F_{\mu\nu}$ is a dual of $F_{\mu\nu}$.

Charged matter fields under $U(1)$ gauge group are transformed as

$$\Phi \to e^{-i\Lambda}\Phi, \quad \Phi^\dagger \to \Phi^\dagger e^{i\Lambda^\dagger}, \tag{A.40}$$

so it is clear that the combination $\Phi^\dagger e^V \Phi$ is gauge invariant. Since this is a vector superfield, D-term is supersymmetric, that is,

$$\int d^2\theta d^2\bar{\theta}\Phi^\dagger e^V \Phi \tag{A.41}$$

is both supersymmetric and gauge invariant.

Extension to non Abelian gauge group is straightforward. Matter field is transformed under non Abelian gauge group transformation similar to Eq. (A.40), but $\Lambda \equiv \Lambda^a T^a, V \equiv V^a T^a$, Λ^a is a chiral superfield, V^a is a vector superfield, T^a is the generators of the representation that Φ belongs to. If the vector superfield is transformed as

$$e^V \to e^{-i\Lambda^\dagger} e^V e^{i\Lambda}, \tag{A.42}$$

Eq. (A.41) is both supersymmetric and gauge invariant under non Abelian gauge transformation as well.

In this case, the field strength superfield is defined as

$$W_\alpha \equiv \frac{i}{4}\bar{D}^2 e^{-V} D_\alpha e^V, \tag{A.43}$$

and is transformed as

$$W_\alpha \to e^{-i\Lambda} W_\alpha e^{i\Lambda}, \tag{A.44}$$

$$\int d^2\theta \frac{1}{4}\mathrm{Tr}(W^\alpha W_\alpha) + h.c. \tag{A.45}$$

is supersymmetric and gauge invariant.

Summing up the above content, the most general supersymmetric and gauge invariant Lagrangian is written as follows,

$$\mathcal{L} = \int d^4\theta \Phi^\dagger e^V \Phi + \left[\int d^2\theta \frac{1}{4}\mathrm{Tr}(W_\alpha W_\alpha) + h.c.\right] + \left[\int d^2\theta W(\Phi_i) + h.c.\right], \tag{A.46}$$

$$\begin{aligned} &= (\mathcal{D}_\mu\phi_i)^\dagger(\mathcal{D}^\mu\phi_i) + (\frac{i}{2}\bar{\psi}_i\bar{\sigma}^\mu\mathcal{D}_\mu\psi_i + h.c.) - \frac{1}{4}(F^a_{\mu\nu})^2 + (\frac{i}{2}\bar{\lambda}^a\bar{\sigma}^\mu\mathcal{D}_\mu\lambda^a + h.c.) \\ &\quad - \left(\frac{1}{2}\frac{\partial^2 W(\phi)}{\partial\phi_i\partial\phi_j}\psi_i\psi_j + i\sqrt{2}\frac{\partial D^a}{\partial\phi_i}\psi_i\lambda^a + h.c.\right) - V. \end{aligned} \tag{A.47}$$

Here we use Wess-Zumino gauge. W is referred to as superpotential, which is a holomorphic function of the chiral superfields. The covariant derivative, the field strength are given as

$$\mathcal{D}_\mu\phi_i = (\partial_\mu - igT^aV^a_\mu)\phi_i, \tag{A.48}$$

$$\mathcal{D}_\mu\psi_i = (\partial_\mu - igT^aV^a_\mu)\psi_i, \tag{A.49}$$

$$\mathcal{D}_\mu\lambda^a = \partial_\mu\lambda^a + gf^{abc}V^b_\mu\lambda^c, \tag{A.50}$$

$$F^a_{\mu\nu} = \partial_\mu V^a_\nu - \partial_\nu V^a_\mu + gf^{abc}V^b_\mu V^c_\nu. \tag{A.51}$$

V is the scalar potential, and is expressed as

$$V = \sum_i |F_i|^2 + \frac{1}{2}\sum_a (D^a)^2 \tag{A.52}$$

$$= \sum_i \left|\frac{\partial W}{\partial\phi_i}\right|^2 + \frac{1}{2}\sum_a (g^a\sum_i \phi_i^\dagger T^a\phi_i)^2, \tag{A.53}$$

where i denotes the flavor index, and the first term is F-term contribution, and the second term is D-term contribution.

Bibliography

[1] N. Seiberg and E. Witten, *Nucl. Phys.* **B426** (1994) 19; *Nucl. Phys.* **B431** (1994) 484

[2] N. Seiberg, *Nucl. Phys.* **B435** (1995) 129.

[3] K. Intriligator and N. Seiberg, *Nucl. Phys. Proc. Suppl.* **45BC** (1996) 1.

[4] M. Peskin, *Proc. Theoretical Advanced Study Institute in Elementary Particle Physics (TASI 96), Fields, Strings, and Duality, Boulder, CO, USA,* hep-th/9702094.

[5] M. Shifman, *Proc. Part. Nucl. Phys.* **39** (1997) 1; hep-th/9704114.

[6] M.T.Grisaru, W. Siegel and M. Roček, *Nucl. Phys.* **B159** (1979) 429.

[7] I. Affleck, M. Dine and N. Seiberg, *Nucl. Phys.* **B241** (1984) 493

[8] N. Seiberg, *Phys. Rev.* **D49** (1994) 6857.

[9] C. Csáki, M. Schmaltz and W. Skiba, *Phys. Rev. Lett.* **78** (1997) 799; C. Csáki, M. Schmaltz and W. Skiba, *Phys. Rev.* **D55** (1997) 7840.

[10] V. Novikov, M. Shifman, A. Vainstein and V. Zakhalov, *Nucl. Phys.* **B229** (1983) 381.

[11] M. Shifman and A. Vainstein, *Nucl. Phys.* **B277** (1986) 456; *Nucl. Phys.* **B359** (1991) 571.

[12] N. Arkani-Hamed and H. Murayama, hep-th/9707133.

[13] T. Banks and A. Zaks, *Nucl. Phys.* **B196** (1982) 189.

[14] C. Csáki and H. Murayama, hep-th/9804061.

[15] K. Intriligator and N. Seiberg, *Nucl. Phys.* **B444** (1995) 125.

[16] K. Intriligator and P. Pouliot, *Phys. Lett.* **B353** (1995) 471.

[17] K. Intriligator, R.G. Leigh and M.J. Strassler, *Nucl. Phys.* **B456** (1995) 567.

[18] D. Kutasov, *Phys. Lett.* **B351** (1995) 230.

[19] D. Kutasov and A. Schwimmer, *Phys. Lett.* **B354** (1995) 315.

[20] D. Kutasov, A. Schwimmer and N. Seiberg, *Nucl. Phys.* **B459** (1996) 455.

[21] J. Brodie, *Nucl. Phys.* **B478** (1996) 123.

[22] J. Brodie and M.J. Strassler, *Nucl. Phys.* **B524** (1998) 224.

[23] M. Berkooz, *Nucl. Phys.* **B452** (1995) 513.

[24] P. Pouliot, *Phys. Lett.* **B367** (1996) 151.

[25] T. Sakai, *Mod. Phys. Lett.* **A12** (1997) 1025.

[26] R.G. Leigh and M.J. Strassler, *Phys. Lett.* **B356** (1995) 492.

[27] K. Intriligator, *Nucl. Phys.* **B448** (1995) 187.

[28] C. Csáki, M. Schmaltz and W. Skiba, *Nucl. Phys.* **B487** (1997) 128.

[29] P. Cho and P. Kraus, *Phys. Rev.* **D54** (1996) 7640.

[30] M. Luty, M. Schmaltz and J. Terning, *Phys. Rev.* **D54** (1996) 7815.

[31] P. Pouliot, "Chirality versus Duality", Caltech theory seminar, September, 1996.

[32] P. Pouliot, *Phys. Lett.* **B359** (1995) 108.

[33] P. Pouliot and M.J. Strassler, *Phys. Lett.* **B370** (1996) 76.

[34] P. Pouliot and M.J. Strassler, *Phys. Lett.* **B375** (1996) 175.

[35] T. Kawano, *Prog. Theor. Phys.* **95** (1996) 963.

[36] P. Cho, *Phys. Rev.* **D56** (1997) 5260.

[37] M. Berkooz, P. Cho, P. Kraus and M.J. Strassler, *Phys. Rev.* **D56** (1997) 7166.

[38] S.B. Giddings and J.M. Pierre, *Phys. Rev.* **D52** (1995) 6065.

[39] I. Pesando, *Mod. Phys. Lett.* **A10** (1995) 1871.

[40] P. Ramond, *Phys. Lett.* **B390** (1997) 179.

[41] J. Distler and A. Karch, *Fortschr. Phys.* **45** (1997) 517.

[42] A. Karch, *Phys. Lett.* **B405** (1997) 280.

[43] R.G. Leigh and M.J. Strassler, *Nucl. Phys.* **B496** (1997) 132.

[44] C. Csáki, M. Schmaltz, W. Skiba and J. Terning, *Phys. Rev.* **D56** (1997) 1228.

[45] E. Poppitz, Y. Shadmi and S. Trivedi, *Nucl. Phys.* **B480** (1996) 125.

[46] N. Evans and M. Schmaltz, *Phys. Rev.* **D55** (1997) 3776.

[47] N. Maru and S. Kitakado, *Mod. Phys. Lett.* **A12** (1997) 691.

[48] G. 't Hooft, *in Recent Developments in Gauge Theories*, eds. G. 't Hooft *et al.* (Plenum Press, New York, 1980).

[49] R. Penrose, *Combinatorial Mathmatics and its Applications* (1971) p221. ed. D.Welsh (New York Academic Press).

[50] R.C. King, *Can.J.Math.* **23** (1971) 176; G.V. Dunne, *J.Phys. A: Math. Gen.* **22** (1989) 1719.

[51] G. Parisi and N. Sourlas, *Phys. Rev. Lett.* **43** (1979) 744.

[52] P. Cvitanović and A.D. Kennedy, *Phys. Scr.* **26** (1982) 5.

[53] P. Cvitanović, *Nucl. Phys.* **B188** (1981) 373.

[54] N. Maru, *Mod. Phys. Lett.* **A13** (1998) 1361.

[55] K. Intriligator, R.G. Leigh and N. Seiberg, *Phys. Rev.* **D50** (1994) 1092.

[56] P. Cho, *Phys. Lett.* **B400** (1997) 101.

[57] B. Grinstein and D.R. Nolte, *Phys. Rev.* **D57** (1998) 6471.

[58] P. Cho, *Phys. Rev.* **D57** (1998) 5214.

[59] M.J. Strassler, hep-th/9709081.

[60] I. Affleck, N. Seiberg and M. Dine, *Nucl. Phys.* **B256** (1985) 557.

[61] M. Luty and W. Taylor, *Phys. Rev.* **D53** (1996) 3399.

[62] R. Slansky, *Phys. Rep.* **79** (1981) 1.

[63] K-I. Izawa and T. Yanagida, *Prog. Theor. Phys.* **95** (1996) 829.

[64] K. Intriligator and S. Thomas, *Nucl. Phys.* **B473** (1996) 121.

[65] G.F. Giudice and R. Rattazzi, hep-ph/9801271.

[66] S. Ferrara, L. Girardello and F. Palumbo, *Phys. Rev.* **D20** (1979) 403.

[67] L. Alvarez-Gaumé and E. Witten, *Nucl. Phys.* **B234** (1984) 269.

[68] E. Witten, *Nucl. Phys.* **B188** (1981) 513.

[69] N. Haba, N. Maru and T. Matsuoka, *Nucl. Phys.* **B497** (1997) 31.

[70] E. Witten, *Nucl. Phys.* **B212** (1982) 253.

[71] H.P. Nilles, *Phys. Rep.* **110** (1984) 1.

[72] M. Dine and A.E. Nelson, *Phys. Rev.* **D48** (1993) 1277; M. Dine, A.E. Nelson, and Y. Shirman, *Phys. Rev.* **D51** (1995) 1362.

[73] M. Dine, A.E. Nelson, Y. Nir, and Y. Shirman, *Phys. Rev.* **D53** (1996) 2658.

[74] I. Dasgupta, B.A. Dobrescu, and L. Randall, *Nucl. Phys.* **B483** (1997) 95; N. Arkani-Hamed, C.D. Carone, L. Hall and H. Murayama, *Phys. Rev.* **D54** (1996) 7032.

[75] M. Dine, Y. Nir, and Y. Shirman, *Phys. Rev.* **D55** (1997) 1501; S.P. Martin, *Phys. Rev.* **D55** (1997) 3177; S. Dimopoulos and G.F. Giudice, *Phys. Lett.* **B393** (1997) 72; S. Dimopoulos, S. Thomas, and J.D. Wells, *Nucl. Phys.* **B488** (1997) 39.

[76] E. Poppitz and L. Randall, *Phys. Lett.* **B336** (1994) 402.

[77] N. Maru, *Phys. Lett.* **B436** (1998) 311.

[78] T. Hotta, K.-I. Izawa and T. Yanagida, *Phys. Rev.* **D55** (1997) 415.

[79] L. Randall, *Nucl. Phys.* **B495** (1997) 37.

[80] Y. Shadmi, *Phys. Lett.* **B405** (1997) 99.

[81] C. Csáki, L. Randall and W. Skiba, *Phys. Rev.* **D57** (1998) 383.

[82] E. Poppits and S. Trivedi, *Phys. Rev.* **D55** (1997) 5508.

[83] N. Arkani-Hamed, J. March-Russel and H. Murayama, *Nucl. Phys.* **B509** (1998) 3.

[84] H. Murayama, *Phys. Rev. Lett.* **79** (1997) 18.

[85] S. Dimopoulos, G. Dvali, R. Rattazzi and G. Giudice, *Nucl. Phys.* **B510** (1998) 12.

[86] M.A. Luty, *Phys. Lett.* **B414** (1997) 71.

[87] S. Dimopoulos, G. Dvali and R. Rattazzi, *Phys. Lett.* **B413** (1997) 336.

[88] K.-I. Izawa, Y. Nomura, K. Tobe and T. Yanagida, *Phys. Rev.* **D56** (1997) 2886; Y. Nomura, K. Tobe and T. Yanagida, *Phys. Lett.* **B425** (1998) 107.

[89] M.A. Luty and J. Terning, *Phys. Rev.* **D57** (1998) 6799; N. Arkani-Hamed, M.A. Luty and J. Terning, *Phys. Rev.* **D58** (1998) 15004.

[90] Y. Shirman, *Phys. Lett.* **B417** (1998) 281.

[91] K. Agashe, hep-ph/9804450.

[92] C. Csáki and H. Murayama, *Nucl. Phys.* **B515** (1998) 114.

[93] G. Dotti and A.V. Manohar, *Phys. Rev. Lett.* **80** (1998) 2758.

[94] K. Intriligator, N. Seiberg and S.H. Shenker, *Phys. Lett.* **B342** (1995) 152.

[95] E. Poppitz, Y. Shadmi and S. Trivedi, *Phys. Lett.* **B388** (1996) 561.

[96] Y. Shirman, *Phys. Lett.* **B389** (1996) 287.

[97] J. Bagger and J. Wess, "Supersymmetry and Supergravity", (Princeton University Press, 1983)

[98] P. West, "Introduction to Supersymmetry and Supergravity", (World Scientific, Singapore, 1986)

[99] S.J. Gates, M.T. Grisaru, M. Roček and W. Siegel, "Superspace" (The Benjamin/Cummings, 1983)

[100] M. Sohnius, "Introducing Supersymmetry", *Phys. Rep.* **128** (1985) 39.

[101] J. Lykken, *Proc. Theoretical Advanced Study Institute in Elementary Particle Physics (TASI 96), Fields, Strings, and Duality, Boulder, CO, USA,* hep-th/9612114.