String Theory on TsT-transformed Background

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Based on Work (arXiv:1207.5050[hep-th]) with
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AdS_3/CFT_2 and Deformations

AdS_3 / CFT_2 "The Most Powerful Holography"

ex) D1-D5/F1-NS5, MSW CFT ...

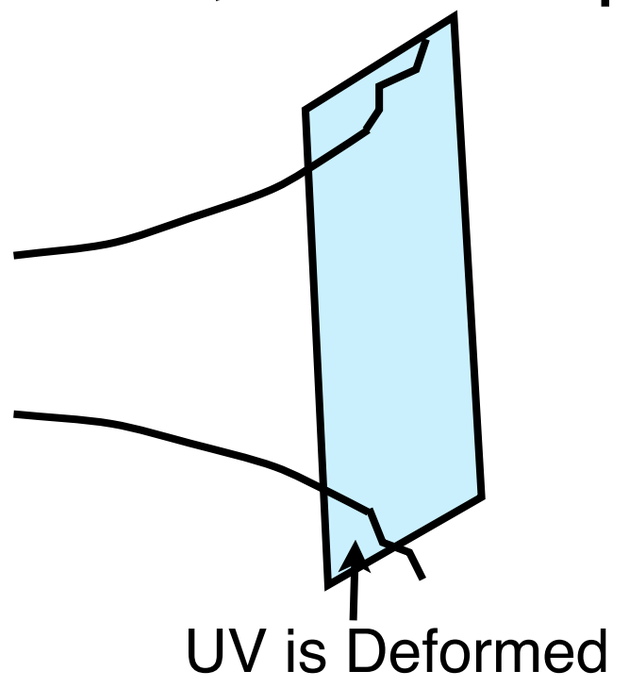
Power of Non-Chiral CFT_2

Two Virasoro Symmetries (+ Unitarity...)

ex) Modular Invariance, Cardy Formula, Bootstrap ...

Deformations

Symmetry Becomes Smaller
in General, but Holography
Might Still Work ...



Null-Warped AdS_3

Gravity Side

Son, Balasubramanian-McGreevy, Guica-Skenderis-Taylor-van Rees

Null-Warped AdS_3 = 3d Schrodinger Spacetime (with z=2)

$$ds^2 = -\frac{\lambda^2 du^2}{r^4} + \frac{2dudv + dr^2}{r^2} \qquad A = \lambda \frac{du}{r^2}$$

Isometry	Asymptotic Symmetry
SL(2,R)xU(1)	Virasoro x U(1) Kac-Moody

CFT Side = Chiral SL(2,R)xU(1) CFT

Add an Irrelevant Operator Sourced by a Massive Vector

$$S = S_{CFT} + \lambda \int O_v(z, \bar{z})$$

Chiral SL(2,R)xU(1) CFT_2

<u>Some Nice Properties</u>

1) Gravity Dual = Warped AdS_3

Anninos-Li-Song-Strominger

2) Infinite-Dim. Extension of Symmetry

→ At Least, One Virasoro + U(1) Kac-Moody

Hofman-Strominger

3)Stress-Energy Tensor, Correlators

ex) Fefferman-Graham Exp., Conformal Perturb. Holographic Renormalization, ...

Guica-Skenderis-Taylor-van Rees, Guica, van Rees ...

Looks Nice but Comprehensive Understanding is Still Poor...

Our Work

To Understand Chiral CFT_2 and its Holography via *String Theory*

Sketch

$AdS_3 x M_7$

"Nice" Deformation

Warped Geometry with SL(2,R)xU(1)

Put a Worldsheet

String on the Warped Geometry

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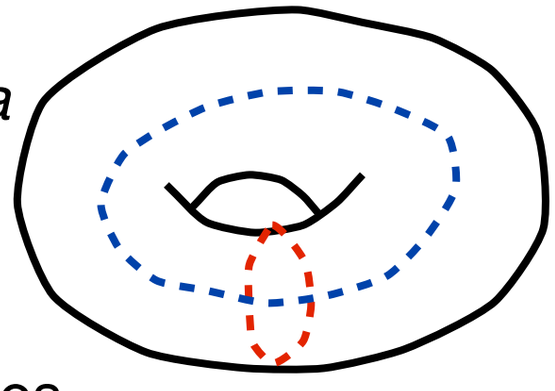
Engineering Warped Spacetime

TsT transformation

Lunin-Maldacena
Maldacena-Martelli-Tachikawa



Mix Up Two U(1)s of the Background

T) T-dual Along the Blue Circle
s) Take a Linear Combination of Blue and Red Circles
T) T-dual Along the Blue Circle Again

→ TsT of AdS_3xS^3 with RR Flux
= Warped AdS_3xS^3 (Direct Product)

Mauricio-Oz-Theisen, El-Showk-Guica, Song-Strominger...

Dual CFT : Dipole-Deformed CFT *Ganor ...*

→ TsT of AdS_3xS^3 with NSNS Flux
=Warped AdS_3xS^3 (Not Direct Product)
(Reduce to Null Warped AdS_3 in 3d)

Our Setup

Before Deformation = NSNS $AdS_3 \times S^3$

$$SL(2)_L \times SL(2,R)_R \times SU(2)_L \times SU(2)_R$$

$$j^- \sim \partial\mu \qquad\qquad \bar{k}^3 \sim \bar{\partial}\bar{\varphi}$$

Warped Background

λ :deformation parameter

$$ds^2 = Q\left(e^{2\rho}d\gamma d\bar{\gamma} + d\rho^2 + d\Omega_3^2 + \lambda e^{2\rho}d\bar{\gamma}(d\psi + \cos\theta d\phi)\right)$$

$$B = -\frac{Q}{4}\left(\cos\theta d\phi \wedge d\psi + 2e^{2\rho}d\gamma \wedge d\bar{\gamma} + 2\lambda e^{2\rho}(d\psi + \cos\theta d\psi) \wedge d\bar{\gamma}\right)$$

Isometry

$$U(1)_L \times SL(2,R)_R \times SU(2)_L \times U(1)_R$$

String Worldsheet

$$\mathcal{L} = \frac{Q}{2\pi}\left(e^{2\rho}\partial\bar{\gamma}(\bar{\partial}\gamma + \lambda(\bar{\partial}\psi + \cos\theta\bar{\partial}\phi)) + \partial\rho\bar{\partial}\rho + \frac{1}{4}(\bar{\partial}\psi + \cos\theta\bar{\partial}\phi)\partial\psi + \cdots\right)$$

$$\sim (AdS_3\ string) + \lambda j^- \bar{k}^3$$

Two Keys

1) String on TsT Background Has a Nice Property

Frolov, Alday-Arutyunov-Frolov
Russo, Tseytlin, Spradlin-Takayanagi-Volovich ...

2) String on AdS_3 with NS-NS flux is Well-Known (Free Field Rep. Near the Boundary)

Giveon-Kutasov-Seiberg, Kutasov-Seiberg,
de Boer-Ooguri-Robins-Tannenhauser, Maldacena-Ooguri, Teschner, Hosomichi-Okuyama-Satoh, Hikida-Hosomochi-Sugawara, Ishibashi-Okuyama-Satoh ...

TsT and Field Redefinition

For String on General TsT Backgruonds, *Alday-Arutyunov-Frolov*

String on TsT Background

|| Field Redefinition

String on Original Background
with Twisted B.C.

→ Twisted Boundary Condition

$$\hat{\gamma}(\sigma+2\pi)=\hat{\gamma}(\sigma)+\frac{2\pi\lambda}{Q}(\bar{q}-\lambda p)\quad \hat{\psi}(\sigma+2\pi)=\hat{\psi}(\sigma)+\frac{4\pi\lambda}{Q}p$$

→ Local Dynamics (OPE, WS Conserved Currents)
is Unchanged in Terms of New Variables

$$\mu(z)\hat{\gamma}(w)\sim-\log(z-w)\qquad \bar{\varphi}(\bar{z})\bar{\varphi}(\bar{w})\sim-(2/Q)\log(\bar{z}-\bar{w})$$

Vertex Operators

Physical Requirements $V_{p,\bar{q}}$

Momemtum/Charges

$$\hat{p}V_{p,\bar{q}} = pV_{p,\bar{q}} \qquad \hat{\bar{q}}V_{p,\bar{q}} = \bar{q}V_{p,\bar{q}}$$

Twisted Boundary Conditions

$$\hat{\gamma}(z)V_{p,\bar{q}}(w) \sim \frac{i\lambda}{Q}(\bar{q} - \lambda p)\log(z-w)V_{p,\bar{q}}(w)$$

$$\hat{\psi}(z)V_{p,\bar{q}}(w) \sim \frac{2i\lambda}{Q}p\log(\bar{z}-\bar{w})V_{p,\bar{q}}(w)$$

$\longrightarrow$ Vertex Operator

$$V_{p,\bar{q}} = V_0 e^{ip\hat{\gamma}} e^{i(\frac{\bar{q}}{2}-\lambda p)\bar{\varphi}} e^{-i\frac{\lambda}{Q}(\bar{q}-\lambda p)\mu}$$

Consistency is OK

String Spectrum

Looking Again the Vertex Operator

$$V_{p,\bar{q}} = V_0 e^{ip\hat{\gamma}} e^{i(\frac{\bar{q}}{2}-\lambda p)\bar{\varphi}} e^{-i\frac{\lambda}{Q}(\bar{q}-\lambda p)\mu} = V_0 e^{ip\hat{\gamma}+i\frac{\bar{q}}{2}\bar{\varphi}} e^{-i\lambda p\bar{\varphi}} e^{-i\frac{\lambda}{Q}(\bar{q}-\lambda p)\mu}$$

→ Deformation = (Momentum/Charge Dep.) Spectral Flow

cf) Spectral Flow for String on NS-NS AdS_3
= Flow from the Unwinding to Winding Sector *Maldacena-Ooguri*

On-Shell Condition

$$L_0 = \frac{-h(h-1)+J(J-1)}{Q-2} + \frac{(\lambda p)^2 - \lambda p\bar{q}}{Q-2} + (N-a) = 0$$

- Level Matching is Automatic
- Consistent with SUGRA Analysis

String on Warped Geometry : Defined by This Spectral Flow

Holography from String Worldsheet

"GKS formalism"

Giveon-Kutasov-Seiberg, Kutasov-Seiberg

(For AdS_3) Generators Acting on the Boundary
= WS Integrals of Vertex Ops. Dressed by Momentum

ex) $SU(2)_L$ Kac-Moody $\quad G^a(p) = \int \frac{d^2z}{\pi i} k^a \bar{\partial} e^{ip\hat{\gamma}}$

Based on Spectral Flowed Vertex Ops. + GKS

Right Virasoro	⟶	Virasoro (Untouched)
Left Virasoro	⟶	Global U(1)
Left SU(2) K-M	⟶	Global SU(2)
Right SU(2) K-M	⟶	U(1) Kac-Moody

Crossover Modes

Crossover Mode (For Both $SU(2)_L$ and $U(1)_L$)

Left Isometries Can Enhance to Infinite-Dim.
By Dressing with Right Momentum

ex) U(1) Crossover $\xi_C(\bar{p}) = e^{i\bar{p}\bar{\gamma}}\partial_\gamma$

$$[\bar{T}(\bar{p}), G_C(\bar{p}')] = i\bar{p}' G_C(\bar{p}+\bar{p}')$$

cf) Also appeared in varous context *Detournay-Compere, Strominger, Hartman-Strominger, Hofman-Strominger*

Key Ingredient to Understand Warped Holography!?

Note: Boundary Condition is Another Issue

→ Consistent Physical Spectrum is Chosen by B.C.

Summary

1) String on TsT-transformed F1-NS5
= “Spectral Flow” of $AdS_3 x S^3$ String

2) String Spectrum Consistent with SUGRA

3) Boundary Modes of String Worldsheet

- Virasoro+U(1) Kac-Moody (so far)
- Crossover Modes

Gauge Choice? More Complicated Operators?